From AdS$_3$ to Carroll fever and flat mania

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Highlights

First steps under Costas' supervision

November 1988 — CPHT — Ecole Polytechnique

A new *preprint* had arrived

CONSISTENCY OF STRING PROPAGATION ON CURVED SPACETIMES. AN SU(1, 1) BASED COUNTEREXAMPLE

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- $SU(1, 1) \sim SL(2, \mathbb{R}) \sim AdS_3$ exact WZW model
- issue: unitarity

How could that be?

COMMENTS ON SU(1, 1) STRING THEORY

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Unitarity restored with $k/2 \le j < 0$ (AdS radius $\sim \sqrt{-k\alpha'}$)

ON THE UNITARITY OF STRING PROPAGATION ON SU(1,1)

Noureddine Mohammedi

International Centre for Theoretical Physics, Trieste, Italy.

Same conclusion

Ambitious agenda

- complete & satisfactory proof of unitarity
- role of continuous series
- partition function
- string bound states
- string amplitudes

Prelude to the AdS advent?

Possibly … but off the main stream

1990 Holly Grail

quantizing 2-dim gravity — matrix models reloaded

Trigger: *Brézin & Kazakov, Douglas & Shenker, Gross & Migdal*

Framework: *non-critical strings → dynamical Liouville mode*

- string theory $\equiv$ 2-dim gravity plus matter fields
- discretization & description in terms of matrix models
- continuous double-scaling limit around multicritical points
  - contribution of *all* topologies
  - capturing the dynamics beyond perturbation: Painlevé I

Slogan: *we are on the route to understanding non-perturbative quantum gravity*

First common paper

An intriguing observation in the matrix-model maelstrom

Doubling of equations and universality in matrix models of random surfaces, PLB **247B** (1990) 363 — Bachas, Petropoulos

- arbitrary interaction potential in hermitian matrix models
  → *two* Painlevé I equations
- potential deep consequences for the non-perturbative dynamics of pure gravity

Subsequent collaboration after my PhD

In the discretized world

- *Topological models on the lattice and a remark on string theory cloning,* CMP **152** (1992) 191 — Bachas, Petropoulos
- *Quenched random graphs,* J. Phys. **A27** (1994) 6121 — Bachas, de Calan, Petropoulos

Back to AdS-related string/brane theory

Anti-de Sitter D-branes, JHEP **02** (2001) 025 — Bachas, Petropoulos

- rigorous analysis of D-branes within an exact string model
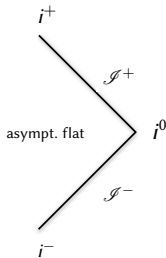- reservations on generic validity of Randall–Sundrum' phenomenological assumptions

Highlights

ASYMPTOTICALLY FLAT SPACETIMES IN 4 DIMENSIONS

- asymptotic symmetry group [Bondi, van der Burg, Metzner '62; Sachs '62]

  Lorentz $\ltimes$ Supertranslations $\longrightarrow 6 + \infty \equiv BMS_4$

- $BMS_4 \equiv$ Conformal group on the null boundary $\mathscr{I}^{\pm}$

i^+

$\mathscr{I}^+$

asympt. flat

i^0

$\mathscr{I}^-$

i^-

CLASSICAL NON-RELATIVISTIC LIMIT: $v/c \to 0$

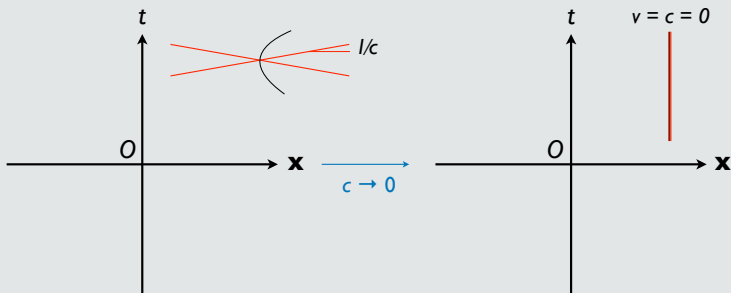- Poincaré group $\to$ Galilean group
- Minkowski spacetime $\to t \in \mathbb{R}$ & $\mathbf{x} \in \mathbb{E}_3$ Newton–Cartan

EXOTIC *ULTRA-LOCAL* LIMIT $v/c \to \infty$ [LÉVY-LEBLOND '65; SEN GUPTA '66]

- Poincaré group $\to$ Carroll group
- Minkowski spacetime $\to$ Carrollian spacetime

$c \to 0$

By law: *motion is forbidden*

…unless you allow for *tachyons* or kindred excitations…
…at least from a particle perspective — branes might be better…

CARROLLIAN SPACETIME

THROUGH THE LOOKING GLASS [Lewis Carroll 1871]

"Well, in our country," said Alice, still panting a little, "you'd generally get to somewhere else if you run very fast for a long time, as we've been doing."

"A slow sort of country!" said the Queen. "Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

- $ds^2 = 0 \times dt^2 + d\mathbf{x}^2$ $\quad \eta_{\mu\nu} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- isometries: Carroll group (Inönü–Wigner contr. of Poincaré)
- conformal isometries: BMS group

CARROLLIAN GEOMETRIES MATERIALIZE ON NULL HYPERSURFACES
- black-hole horizons
- null boundaries $\mathscr{I}^{\pm}$

Two main symptoms

Geometric structure *Ἀγεωμέτρητος μηδεὶς εἰσίτω* [Plato 427–348 BC]

- Carrollian manifolds
- Connections, torsion and curvature

Applications *πάντα ἔξεστιν ἀλλ᾽ οὐ πάντα συμφέρει* [1 Corinthians 10:23]

- Hard-core
  - dynamics and equations
  - 𝕮𝕮𝖆𝖗𝖗 field theories & representation theory
  - black-hole horizons
  - bulk from boundary reconstruction
  - flat-space holography investigation
- Entertaining
  - thermodynamics and statistical mechanics
  - exotic excitations and exotic fluids
  - cosmological applications, dark matter etc.
  - Stochastic resetting at zero resetting rate

Highlights

The revival of asymptotic symmetries

Where could BMS play a role?

In scattering processes

- soft theorems
- memory effects

general — beyond gravity

In gravity

Flat-space holography?

AᴅS ᴠᴇʀꜱᴜꜱ Fʟᴀᴛ: Hɪɢʜʟɪɢʜᴛꜱ

Eɪɴꜱᴛᴇɪɴ ꜱᴘᴀᴄᴇᴛɪᴍᴇꜱ

Finite solution space without generic gravitational radiation

AᴅS/CFT ᴇʟᴇᴍᴇɴᴛᴀʀʏ ꜰᴇᴀᴛᴜʀᴇꜱ

- incarnation of old 't Hooft & Susskind ideas on gravity dofs
- *fundamental theories:* type IIB string and $N = 4$ SYM
- *holographic:* dual field theory on a codim-1 time-like hypersurface — the conformal boundary $\mathscr{I}$ & vevs–sources
- boundary energy–momentum tensor plays a pivotal role

Fᴀᴄᴛꜱ ꜰᴏʀ ᴀꜱʏᴍᴘᴛᴏᴛɪᴄᴀʟʟʏ ꜰʟᴀᴛ ꜱᴘᴀᴄᴇᴛɪᴍᴇꜱ

- Ricci-flat spacetimes have infinite-dim solution space and generically gravitational radiation
- conformal boundary $\mathscr{I}^{\pm}$ is null hence Carrollian
- no clear concept of boundary energy–momentum tensor

Ideas for Flat-Space Holography

Following the AdS_4/CFT_3 paradigm

- dual CFT_3 on the Carrollian conformal boundary $\mathscr{I}^{\pm}$
- must be invariant under $\mathfrak{CCarr}_3 \equiv BMS_4$
- expected multi-sector (scattering, bound states & deep dofs)

possibly *non-local*, *non-unitary* or *non-holographic* dual

Utterly different path: $flat_4/CFT_2$ *celestial* approach

FRAMEWORK	• $\mathscr{S}_2 \equiv$ "spatial section" of the Carrollian bry.
	• 2-dim en.–mom. tensor $\sim \int_{\mathbb{C}} \int_{\mathbb{R}} \mathscr{N}_{ab}$ (news)
	• very exotic celestial CFT_2
FEATURES	• mostly designed for recasting *radiation* S-matrix on Minkowski & soft theorems
	• not fund. theor. — not bound to gravity — probably captures a sector only

could have been elaborated in the mid-80ies

Highlights

Working together

- common perspective: *think first write next* — a handful of projects never completed
- sometimes both strong characters — compromises necessary & reached

Impact

Well-perceived work

Personal satisfaction

Always a rewarding experience

CARROLL & FLAT-SPACE HOLOGRAPHY — PIPE DREAM?

A few steps might help decide

- analyse carefully AdS/CFT in the $\Lambda \to 0$ limit
- recasting celestial in the suitable framework — Carrollian

MORE GENERALLY

Big *and realistic* picture is missing

L'éclectique est un philosophe qui foulant au pied le préjugé, la tradition, l'ancienneté, le consentement universel, l'autorité, en un mot tout ce qui subjugue la foule des esprits, ose penser de lui-même, remonter aux principes généraux les plus clairs, les examiner, les discuter, n'admettre rien que sur le témoignage de son expérience et de sa raison ; et de toutes les philosophies, qu'il a analysées sans égard et sans partialité, s'en faire une particulière et domestique qui lui appartienne.

Denis Diderot

Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers, 1755.