Celebration of Costas Bachas

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Null Brane Intersections

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Abstract: We study pairs of planar D-branes intersecting on null hypersurfaces, and other related configurations. These are supersymmetric and have finite energy density. They provide open-string analogues of the parabolic orbifold and of the null-fluxbrane backgrounds for closed superstrings. We derive the spectrum of open strings, showing in particular that if the D-branes are shifted in a spectator dimension so that they do not intersect, the open strings joining them have no asymptotic states. As a result, a single non-BPS excitation can in this case catalyze a condensation of massless modes, changing significantly the underlying supersymmetric vacuum state. We argue that a similar phenomenon can modify the null cosmological singularity of the time-dependent orbifolds. This is a stringy mechanism, distinct from black-hole formation and other strong gravitational instabilities, and one that should dominate at weak string coupling. A by-product of our analysis is a new understanding of the appearance of 1/4 BPS threshold bound states, at special points in the moduli space of toroidally-compactified type-II string theory.

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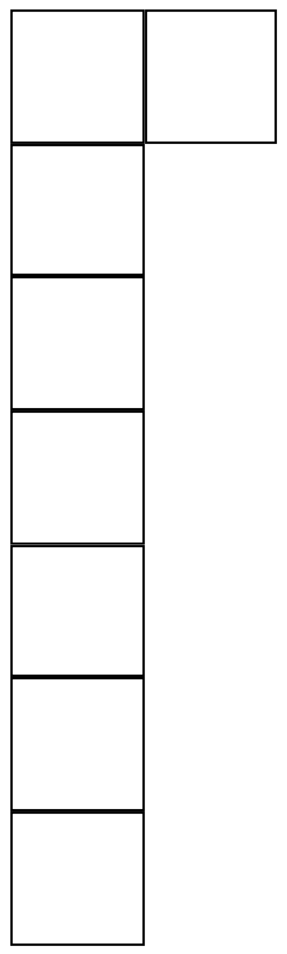
Magnetic Charges in Gravity

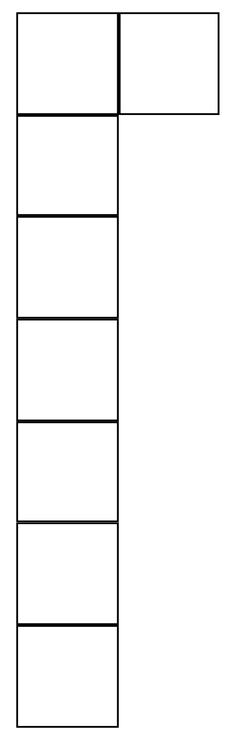
2310.18441

Gravity and Magnetic Charges

- NUT charges in D=4
- KK Monopole charge: BPS charge carried by KK monopoles
- D-5 form charge arising in D>4 Superalgebra. Explicit form calculated from the super-algebra [CH '97]
- Dual "gravity" charge with [D-4,1] hook Young tableau?
- Arises: Exceptional generalised geometry, E_{10}, E_{11} etc



Gravity and Magnetic Charges



KK monopoles vital yet mysterious part of BPS spectrum

Aim: better understanding of their charges

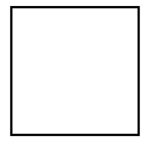
Maxwell theory:

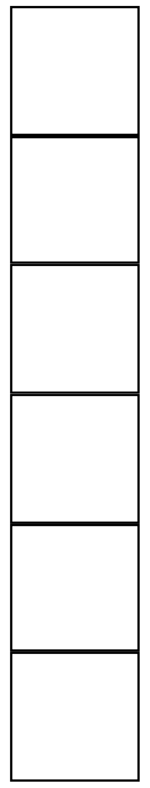
Magnetic charge : electric charge for dual formulation in terms of dual photon $\tilde{A}$

Construct "electric" charges for dual graviton and interpret as magnetic charges for graviton

Magnetic charges: typically integral of closed form. E.g. $\int dA$

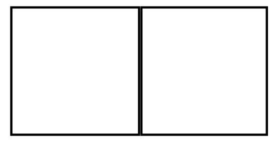
Duality of Free Fields in D>4 dimensions

Photon A_μ 

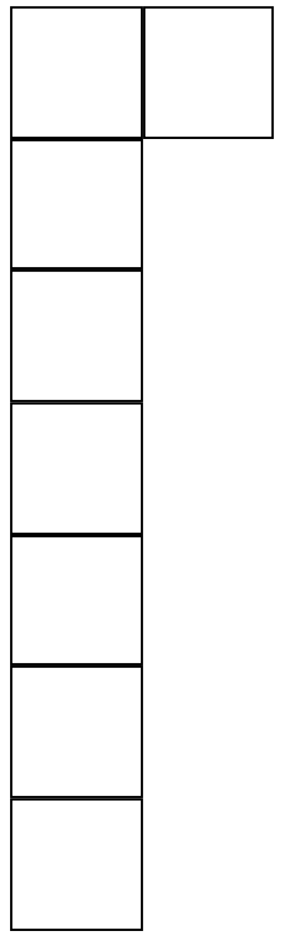
Dual Photon n-form $\tilde{A}_{\mu_1 \dots \mu_n}$ 

$n = D - 3$

Graviton

$h_{\mu\nu}$ [1,1] 

Dual Graviton

$D_{\mu_1 \mu_2 \dots \mu_n \mid \nu}$ [n,1] 

CH 2000

Duality interchanges field equations and Bianchi identities

Gravitational Duality

CH 2000

Field strength $S_{\mu\nu\ldots\rho\,|\,\sigma\tau} = \partial_{[\mu}D_{\nu\ldots\rho]\,|\,|\,[\sigma,\tau]}$ [D-2,2] $S \sim d_L d_R D$

Dual to graviton: $S_{\mu_1\mu_2\ldots\mu_{n+1}\,|\,\nu\rho} = \frac{1}{2}\epsilon_{\mu_1\mu_2\ldots\mu_{n+1}\alpha\beta}R^{\alpha\beta}{}_{\nu\rho}$ $S = *R$

$$R_{\mu\,\nu} = 0 \quad \leftrightarrow \quad S_{[\mu_1\mu_2\ldots\mu_{n+1}\,\nu]\rho} = 0$$

$$R_{[\mu\nu\,\sigma]\tau} = 0 \quad \leftrightarrow \quad S'_{\mu_1\mu_2\ldots\mu_n\,\nu} = 0$$

Trace $S'_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = S_{\mu_1\mu_2\ldots\mu_n\rho\,|\,\nu}{}^{\rho}$

Gravity Magnetic Charges: Noether Charges of Graviton

Linearised gravity Abbott and Deser

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad G^L_{\mu\nu}(h) = T_{\mu\nu}$$

Gauge symmetry $\quad \delta h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$

Killing vector $\quad \partial_{(\mu}k_{\nu)} = 0$

Invariance $\quad \delta h_{\mu\nu} = 0 \qquad \xi_\mu = \alpha k_\mu$

Constant 0-form parameter α: global symm

Noether current $\quad j_\mu[k] = T_{\mu\nu}k^\nu$

Gravity Magnetic Charges: Noether Charges of Graviton

Linearised gravity

Secondary 2-form current $J[k]$

$$j_\mu[k] = \partial^\nu J_{\mu\nu}[k]$$

$J[k]$ conserved in regions where $j = 0$

ADM 2-form $J[k]$ $$J[k] \sim k\partial h + h\partial k$$

Noether charge $$Q[k] = \int_\Sigma * j[k] = \int_{\partial\Sigma} * J[k]$$

e.g. Σ is region of spatial hyper-surface bounded by $\partial\Sigma$

Topological charge: $Q[k]$ unchanged under deformations of $\partial\Sigma$ that do not pass through regions with matter where $j \neq 0$

Charges in Gauge Theories

Similar structure to Gravity

Isometries/invariances

For a given configuration, the gauge transformations preserving that configuration give an "isometry group", typically finite dimensional.

Killing gauge parameters, Noether charges

Global 0-form symmetry, 1-form Noether current j, secondary 2-form current J

Noether charge

$$Q = \int_{\Sigma} * j = \int_{\partial\Sigma} * J$$

Antisymmetric Tensor Gauge Theories

D dimensions, general spacetime

p -form A $\quad F = dA$

Gauge symmetry $\quad \delta A = d\lambda \quad$ Reducible: exact $\sigma = d\rho$ don't act

Isometries/invariances

Killing gauge parameters: closed $p - 1$ forms $\sigma \quad d\sigma = 0$

Modulo exact, so isometries correspond to cohomology classes

Global symmetry: $\lambda = \alpha\ \sigma$

$$dF = *\tilde{I}, \qquad d*F = *I$$

p-form electric current I, (D - p - 2)-form magnetic current $\tilde{I}$

Conserved 1-form current

$$*j = (\sigma \wedge *I)$$

$$j_\mu = \frac{1}{(n-1)!} I_{\mu\nu_1\ldots\nu_{n-1}} \sigma^{\nu_1\ldots\nu_{n-1}}$$

$$j_\mu[\sigma] = \partial^\nu J_{\mu\nu}[\sigma] \qquad J[\sigma] = \sigma \wedge *F$$

$$Q[\sigma] = \int_\Sigma *j[\sigma] = \int_{\partial\Sigma} \sigma \wedge *F$$

Conserved charge for each cohomology class

$$Q[\sigma + d\alpha] = Q[\sigma]$$

Dual (D-p-3)—form potential $\tilde{A}$ $\quad * F = d\tilde{A}$

Gauge symmetry $\quad \delta\tilde{A} = d\tilde{\lambda}$

Reducible: exact $\tilde{\lambda} = d\tilde{\alpha}$ don't act

Invariance: closed $\tilde{\sigma}$, $d\tilde{\sigma} = 0$

$$Q[\tilde{\sigma}] = \int_{\Sigma} \tilde{\sigma} \wedge *\tilde{J} = \int_{\partial\Sigma} \tilde{\sigma} \wedge F$$

Conserved charges corresponding to Killing gauge parameter cohomology classes

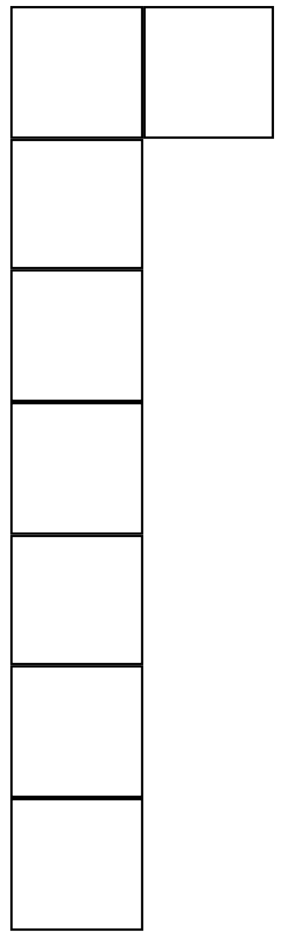
Dual Graviton in D Dimensions

Dual graviton

$D_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$

[n,1] bi-form

$n \equiv D - 3$



Two gauge symmetries.

Curtright

Parameters:

$\alpha_{\mu_1\ldots\mu_{n-1}\,|\,\rho}$ $[n-1,1]$ bi-form

$\beta_{\mu_1\ldots\mu_n}$ $[n,0]$ n-form

$$\delta D_{\mu\nu\ldots\sigma\,|\,\rho} = \partial_{[\mu}\alpha_{\nu\ldots\sigma]\,|\,\rho} + \partial_\rho\beta_{\mu\nu\ldots\sigma} - \partial_{[\rho}\beta_{\mu\nu\ldots\sigma]}$$

Field strength $S_{\mu\nu\ldots\rho\,|\,\sigma\tau} = \partial_{[\mu}D_{\nu\ldots\rho]\,|\,[\sigma,\tau]}$ [n+1,2] $S \sim d_L d_R D$

Two types of Noether charge for two types of symmetery

Killing Tensors

$$\delta D_{\mu\nu\ldots\sigma\,|\,\rho} = \partial_{[\mu}\alpha_{\nu\ldots\sigma]\,|\,\rho} + \partial_\rho \beta_{\mu\nu\ldots\sigma} - \partial_{[\rho}\beta_{\mu\nu\ldots\sigma]}$$

“Dual isometries” if

1) parameter α is [n-1,1] generalised Killing tensor $\kappa_{\mu_1\ldots\mu_{n-1}\,|\,\rho}$ satisfying

$$\partial_{[\mu}\kappa_{\nu\ldots\sigma]\rho} = 0$$

2) parameter β given by a Killing-Yano tensor, i.e, an n-form $\lambda_{\mu_1\ldots\mu_n}$ satisfying

$$\partial_\rho \lambda_{\mu\nu\ldots\sigma} - \partial_{[\rho}\lambda_{\mu\nu\ldots\sigma]} = 0$$

Noether charges $\quad Q[\kappa], \quad Q[\lambda]$

Construction of Magnetic Charges

- Source for dual graviton: conserved $[n,1]$ current $U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$
- Conserved 1-form currents: $j[\kappa] \sim \kappa\,.\,U,\;\; j[\lambda] \sim \lambda\,.\,U$
- Secondary 2-form currents $J[\kappa], J[\lambda]$ $\qquad d^\dagger J = j$
- Charges $Q[\kappa], Q[\lambda]$ $\qquad Q = \int_\Sigma * j = \int_{\partial\Sigma} * J$
- In regions without sources, rewrite 2-forms J in terms of $h_{\mu\nu}$ instead of $D_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$
- Find $J = \star\, dZ$

$$Q[\kappa] = \int dZ[\kappa] \qquad\qquad Q[\lambda] = \int dZ[\lambda]$$

$$Q[\kappa] = \int dZ[\kappa] \qquad\qquad Q[\lambda] = \int dZ[\lambda]$$

$$Z[\kappa]_{\mu_1 \ldots \mu_{d-3}} = h_{[\mu_1}{}^{\tau} \kappa_{\mu_2 \mu_2 \ldots \mu_{d-3}] \,|\, \tau} \qquad\qquad Z_{\mu_1 \ldots \mu_n}[\lambda] = h^{\rho}{}_{[\mu_1} \lambda_{\mu_2 \ldots \mu_{d-3}]\rho}$$

Charge depends on closed κ

$$\partial_{[\mu} \kappa_{\nu \ldots \sigma]\rho} = 0 \qquad\qquad d_L \kappa = 0$$

modulo exact κ

$$\kappa_{\mu_1 \ldots \mu_{n-1} \,|\, \rho} \sim \kappa_{\mu_1 \ldots \mu_{n-1} \,|\, \rho} + \partial_{[\mu_1} \sigma_{\mu_2 \ldots \mu_{n-1}] \,|\, \rho} \qquad\qquad \kappa \sim \kappa + d_L \sigma = 0$$

Topology

Charges vanish if $h_{\mu\nu}$ globally defined. Non-trivial if graviton has Dirac strings/defined locally patches with non-trivial transition functions

Sources

$T_{\mu\nu}$ source for $R_{\mu\nu}$ or $S_{[\mu_1\mu_2\ldots\mu_{n+1}\nu]\rho}$

$U_{\mu_1\mu_2\ldots\mu_n|\nu}$ source for $S'_{\mu_1\mu_2\ldots\mu_n\nu}$ or $R_{[\mu\nu\sigma]\tau}$

$T_{\mu\nu}$ gives regular $h_{\mu\nu}$, but Dirac strings in $D_{\mu_1\mu_2\ldots\mu_n|\nu}$

$U_{\mu_1\mu_2\ldots\mu_n|\nu}$ gives regular $D_{\mu_1\mu_2\ldots\mu_n|\nu}$, but Dirac strings in $h_{\mu\nu}$

In regions with $T_{\mu\nu} = 0$, can write theory in terms of $h_{\mu\nu}$

In regions with $U_{\mu_1\mu_2\ldots\mu_n|\nu} = 0$, can write theory in terms of $D_{\mu_1\mu_2\ldots\mu_n|\nu}$

Charges on $\mathbb{R}^{d,1} \times T^m$

p -form A

$$Q[\lambda] = \int_{\partial\Sigma} \lambda \wedge *F$$

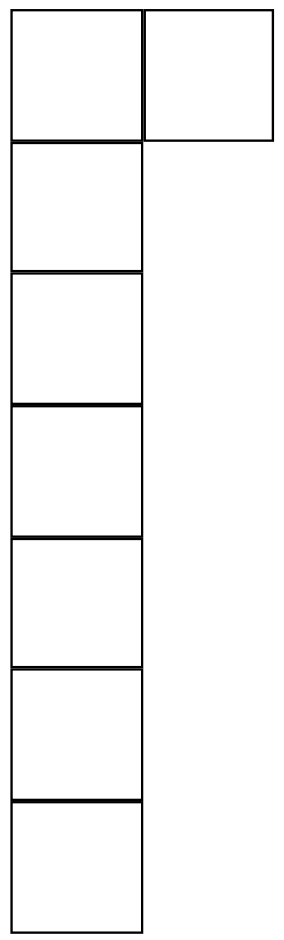
Take λ constant $p-1$ form on T^m $(m \geq p-1)$

$$Q[\lambda] = \frac{1}{(p-1)!}\lambda_{\mu_1\ldots\mu_{p-1}}\mathscr{Z}^{\mu_1\ldots\mu_{p-1}}$$

$$k_\mu = V_\mu + \Lambda_{\mu\nu}x^\nu \qquad Q[k] = V_\mu P^\mu + \frac{1}{2}\Lambda_{\mu\nu}J^{\mu\nu}$$

For constant κ

$$Q[\kappa] = \frac{1}{(n-1)!}\tilde{P}^{\mu_1\dots\mu_{n-1}|\rho}\kappa_{\mu_1\dots\mu_{n-1}|\rho}$$



gives $[D-4,1]$ Hook Charge $\tilde{P}^{\mu_1\dots\mu_{n-1}|\rho}$

For $\kappa_{\mu_1\dots\mu_{n-1}\,|\,\rho} = \alpha_{[\mu_1\dots\mu_{n-2}}V_{\mu_{n-1}]}V_\rho$

with V, α constant forms on T^m, $V \wedge \alpha = 0$

$$Q[\kappa] = \frac{1}{(D-5)!}K^{\mu_1\dots\mu_{D-5}}\alpha_{\mu_1\dots\mu_{D-5}}$$

$D-5$ form Charge $K_{\mu_1\dots\mu_{D-5}}$

BPS Charge for (linearised) KK monopole

Coordinates on T^m: (y, u^a)

$$\kappa \sim dy \wedge du^{a_1} \wedge du^{a_2} \dots \wedge du^{a_{D-5}} \otimes dy$$

One circle has enhanced role: becomes twisted circle in non-linear solution

Covariant Constructions

CH, Hutt and Lindstrom 2401.17361, 2405.08876

- ADM type expressions not covariant, problematic if $h_{\mu\nu}$ not globally defined
- Covariant current

$$J_{\mu\nu} = R_{\mu\nu\rho\sigma} K^{\rho\sigma}$$

conserved if $K_{\mu\nu}$ is Conformal Killing Yano tensor Penrose, 1982

- Integrate to get conserved charges
- Conformal Killing Yano tensors related to Killing vectors, Killing tensors, e.g. $k^{\mu} = \partial_{\nu} K^{\mu\nu}$
- Penrose type charges related to electric and magnetic gravitational charges.
- Covariant expressions that reduce to ADM-type expressions when $h_{\mu\nu}$ globally defined

Outlook

- Penrose currents used to study generalised symmetries of graviton
- More general covariant constructions of charges **CH, Hutt and Lindstrom**
- Generalisation to non-linear theory: only special spacetimes have Killing tensors
- General case: integral at spatial or null infinity, asymptotic Killing tensors
- Topology of graviton field constrains topology of spacetime

Bacchanal

Celebration of Bachas

Bacchanalia

Best wishes, Costas!