Defects in fully extended TQFT – a tour in pictures

Ilka Brunner, LMU München

Paris, Costas Bachas day, 26. June 2024

Fusion of conformal interfaces

C. Bachas♯ and I. Brunner♭

♯ Laboratoire de Physique Théorique de l'Ecole Normale Supérieure [1]
24 rue Lhomond, 75231 Paris cedex, France

♭ Institut für Theoretische Physik, ETH-Hönggerberg
8093 Zürich, Switzerland

Abstract

We study the fusion of conformal interfaces in the $c = 1$ conformal field theory. We uncover an elegant structure reminiscent of that of black holes in supersymmetric theories. The role of the BPS black holes is played by topological interfaces, which (a) minimize the entropy function, (b) fix through an attractor mechanism one or both of the bulk radii, and (c) are (marginally) stable under splitting. One significant difference is that the conserved charges are logarithms of natural numbers, rather than vectors in a charge lattice, as for BPS states. Besides potential applications to condensed-matter physics and number theory, these results point to the existence of large solution-generating algebras in string theory.

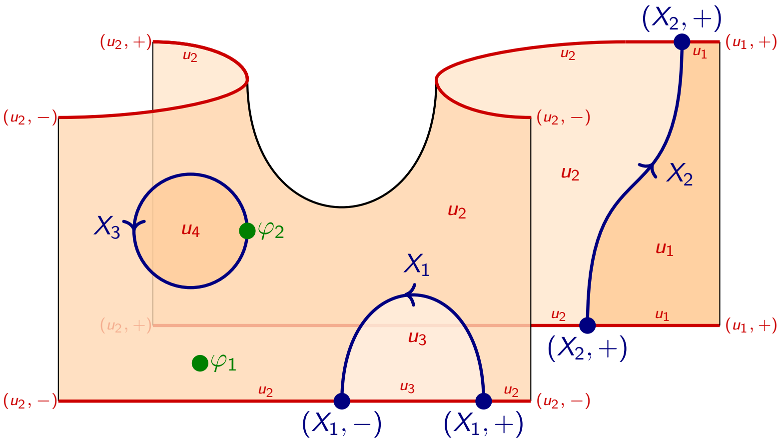
Plan

- **Plan**: TQFT are very simple models of QFT
  Axiomatic formulation Atiyah, Segal
  Defects, Extended TQFT with defects
- **Example**: Rozansky Witten with affine target $T^*\mathbb{C}^n$
- Use cobordism hypothesis (Lurie, Freed, Baez, Dolan) in a constructive manner, in an example.

Work with N. Carqueville, P. Fragkos, D. Roggenkamp

Aim

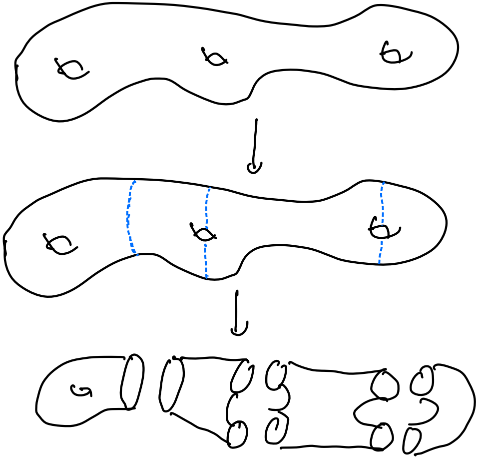
$$\mathcal{Z}\colon \mathrm{Bord}^{\mathrm{def}}_{2,1,0}(\mathbb{D}) \to \mathcal{C} \tag{1}$$



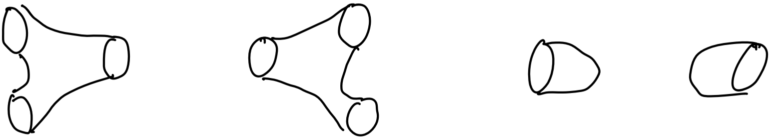
.

Axiomatic formulation of TQFT

Computation of path integral on M



Basic building blocks for 2d TQFT



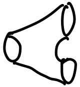
Build up any surface from these ingredients.

"bordisms"

TQFT are functors

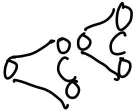
Z :  $\longrightarrow V$

 $\longrightarrow V \otimes V$

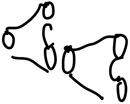
 $\longrightarrow (V \otimes V \to V)$

 $\longrightarrow V \otimes V \to \mathbb{C}$

 $\longrightarrow \mathrm{id} : V \to V$



$\parallel$ associativity



1 dimensional TQFT

0-dimensions: points $+\bullet$, $-\bullet$

emptyset $\emptyset$

1-dimensions: 1-manifolds connecting 0-manifolds,

co-evaluation: $\emptyset \to pt_+ \sqcup pt_-$

evaluation: $pt \sqcup pt \to \emptyset$

identity on a pt

identity on $pt \sqcup pt$

snake relation

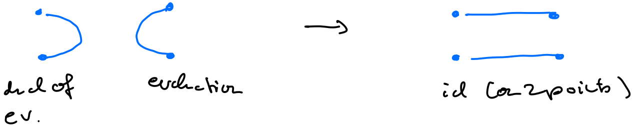
TQFT fixed by the image of one of the points, $\mathcal{Z}(+)$.

Extended TQFT

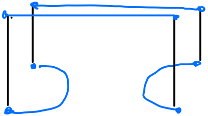
- So far: Manifolds of dimension $d-1$ and d
- In (fully) extended TQFT we want to consider manifolds of any codimension!
- Start with points
- can be connected by lines
- Collection of lines can in turn be connected by 2-dimensional surfaces
- (...)
- There are again basic building blocks as well as gluing relations between them
- Higher category, in 2 dimensions a 2 category

Lurie, Freed, Baez, Dolan...

Building blocks, 2d

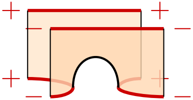


2-d interpolating surface

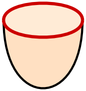


This is
"the evaluation on the evaluation"
ev_{ev}
(a 2-morphism)

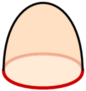
Building blocks, 2d: The adjunction 2-morphisms



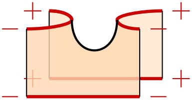
$$= \mathrm{ev}_{\widetilde{\mathrm{ev}}_+} \tag{2}$$



$$= \widetilde{\mathrm{coev}}_{\widetilde{\mathrm{coev}}_+} \,, \tag{3}$$

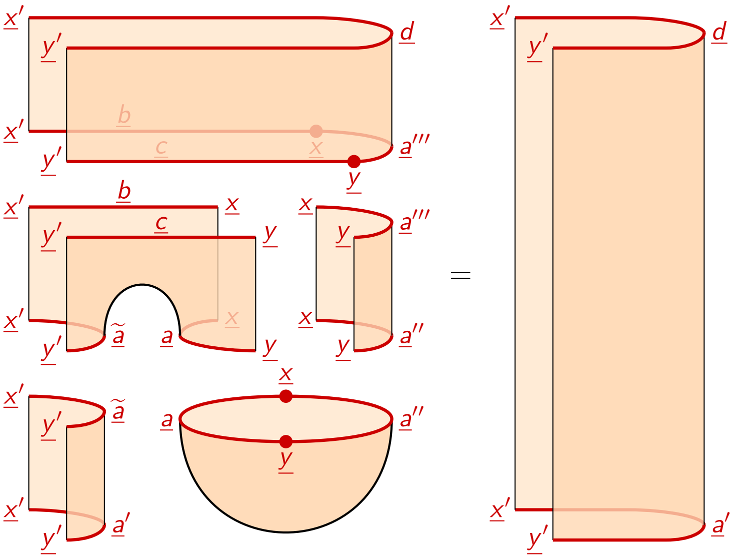


$$= \mathrm{ev}_{\widetilde{\mathrm{coev}}_+} \,, \tag{4}$$



$$= \widetilde{\mathrm{coev}}_{\widetilde{\mathrm{ev}}_+} \tag{5}$$

...and a snake-condition



.

(6)

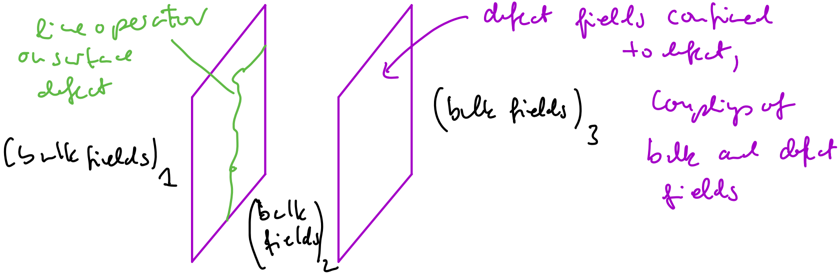
Extended TQFT

- We have introduced a higher category of bordisms.
- (i.e. start from points, look at lines between them, consider surfaces between the lines ...)
- An extended TQFT is a functor between this category and a target category that needs to have the same structure

$$\mathcal{Z} : \mathrm{Bord}_{(2,1,0)} \to \mathcal{C}$$

- Cobordism hypothesis:
  Fully dualizable objects $\in \mathcal{C} \leftrightarrow$ extended TQFTs
- Suitable target category come for example from defects

Defects



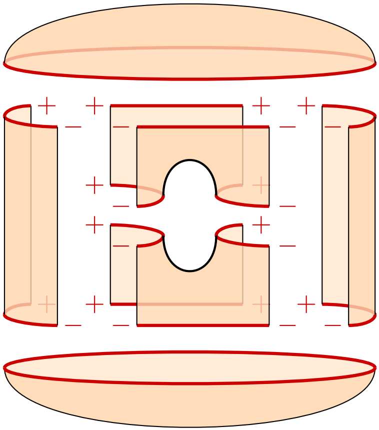
- objects: Theories (images of the points $\in \mathrm{Bord}_{(2,1,0)}$)
- 1-morphisms: Domain walls (images of the lines $\in \mathrm{Bord}_{(2,1,0)}$)
- 2- morphisms: Line operators (images of the surfaces $\in \mathrm{Bord}_{(2,1,0)}$)

Now we can compute anything!

IB, N. Carqueville, P. Fragkos, D. Roggenkamp

at least in the topological subsector, up to dimension 2, for affine RW or models under equally good control

$$\mathcal{Z}(T^2) = \mathcal{Z}\left(\vphantom{\int} \right.$$



$$\left. \vphantom{\int} \right)$$

$$= \widetilde{\mathrm{ev}}_{\widetilde{\mathrm{ev}}_+} \cdot \left[1_{\widetilde{\mathrm{ev}}_u} \circ \left(\mathrm{ev}_{\widetilde{\mathrm{ev}}_u} \cdot \widetilde{\mathrm{coev}}_{\widetilde{\mathrm{ev}}_u} \right) \circ 1_{\mathrm{coev}_u} \right] \cdot \mathrm{coev}_{\widetilde{\mathrm{ev}}_u} \, . \qquad (7)$$

Genus g state spaces

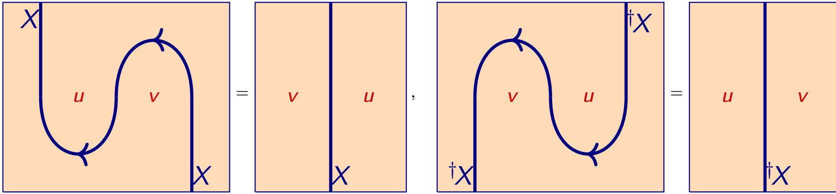
- Evaluate the functor on any genus g surface without boundary
- Example: 2-category coming from RW models with affine target
- $$\mathcal{H}_{\Sigma_g} \cong H^\bullet\big(\mathcal{Z}_n(\Sigma_g)\big) \cong \big(\mathbb{C} \oplus \mathbb{C}[1]\big)^{\otimes 2g} \otimes_{\mathbb{C}} \mathbb{C}[\underline{a}, \underline{x}]$$

Interpretation?

- 3d $\mathcal{N} = 4$ hypermultiplets, Rozansky-Witten twist
- Scalars $\to \mathbb{C}[\underline{a}, \underline{x}]$
- Fermions $\to$ vectors after twist $\to \big(\mathbb{C} \oplus \mathbb{C}[1]\big)^{\otimes 2g}$

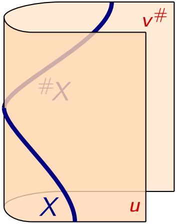
Including defects on the bordisms

- Defect data: bulk, codim 1 defects, codim 2 defects
- Look at stratified bordisms and label the different strata by the defect data.
- Defects need to satisfy conditions, for example

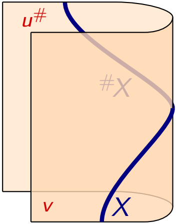


(8)

or...(existence of Ω_X...)

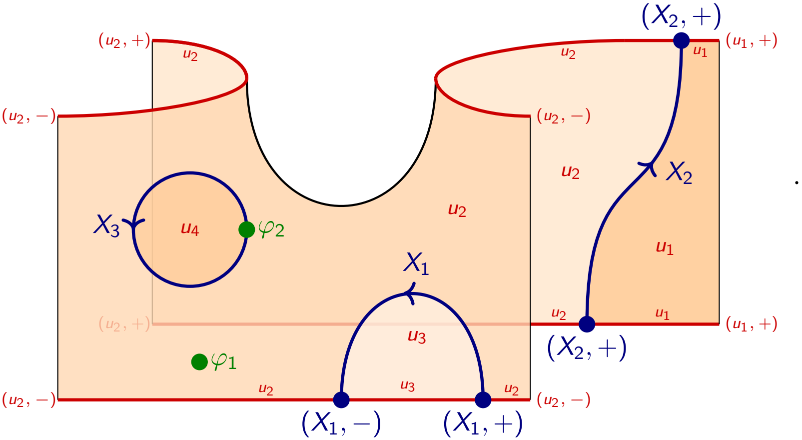


$$= \Omega_X\colon\ \mathrm{ev}_v \circ \left(1_{v^\#} \,\Box\, X\right) \xrightarrow{\cong} \mathrm{ev}_u \circ \left({}^\# X \,\Box\, 1_u\right)\,, \tag{9}$$



$$= \Omega'_X\colon\ \left(1_{u^\#} \,\Box\, X\right) \circ \widetilde{\mathrm{coev}}_u \xrightarrow{\cong} \left(X^\# \,\Box\, 1_v\right) \circ \widetilde{\mathrm{coev}}_v \tag{10}$$

Example...



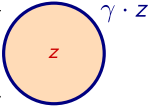
(11)

Use graphical calculus

Barrett-Meusburger-Schaumann

Disk state spaces

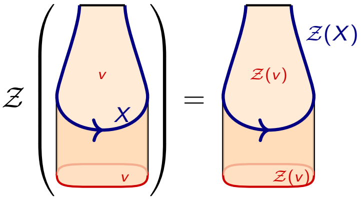
- We can include boundaries: Defect between a theory and a trivial theory
- Here for example $(\gamma; W(z,\gamma)) : \emptyset \to z$,

$$\mathcal{Z}\Big($$


$$\Big) \cong \mathbb{C}[\gamma]\,.$$

- Dirichlet condition on the z variable

and more



(12)

This is the bulk-boundary map ...
and indeed one can formulate an analog of open-closed TQFT including Cardy's condition (Hirzebruch-Riemann-Roch)

Some results on affine Rozansky Witten models

- Applying the cobordism hypothesis systematically, we construct a unique extended TQFT for each number of variables.
- Compute for example state spaces on any orientable surface, with or without boundary
- Include defects: Evaluate any stratified bordism.
- Include gradings: R-symmetry, flavor symmetry
- Example/application: Symmetries
- Use symmetry defects to model flat background gauge fields
- Example/application boundaries
- Construct an open-closed TQFT, where e.g intersection numbers get replaced by spaces