René Magritte (1967)

FLAT SPACE HOLOGRAPHY

Tracking celestial & Carrollian theories

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Topics in Quantum Gravity, Collège de France
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Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes ($\Lambda = 0$)?

→ These spacetimes are relevant from collider physics … to astrophysics (< cosmological scales)

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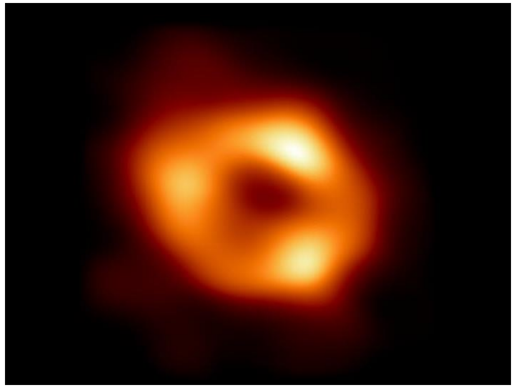
$$\boxed{S_{BH} = \frac{\mathcal{A}c^3}{4G\hbar}}$$

[Bekenstein][Hawking]

$\mathcal{A}$: event horizon area



Event Horizon Telescope

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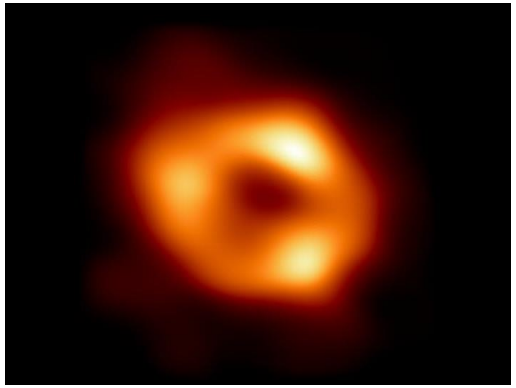

Event Horizon Telescope

↓

Holography beyond Anti-de Sitter/CFT?

$$\Lambda < 0$$

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Early attempts:

[Susskind ‘99][Polchinski ‘99][Giddings ’99]
[de Boer, Solodukhin ‘03][Arcioni, Dappiaggi ’03 ’04]
[Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06]…

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…and even earlier

[Penrose ‘76][Newman ’76]

→ aimed at a reconstruction of the bulk spacetime from quantities defined only at null infinity $\mathscr{I}$

General Relativity and Gravitation, Vol. 7, No. 1 (1976), pp. 107–111

Heaven and Its Properties

EZRA T. NEWMAN

Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213

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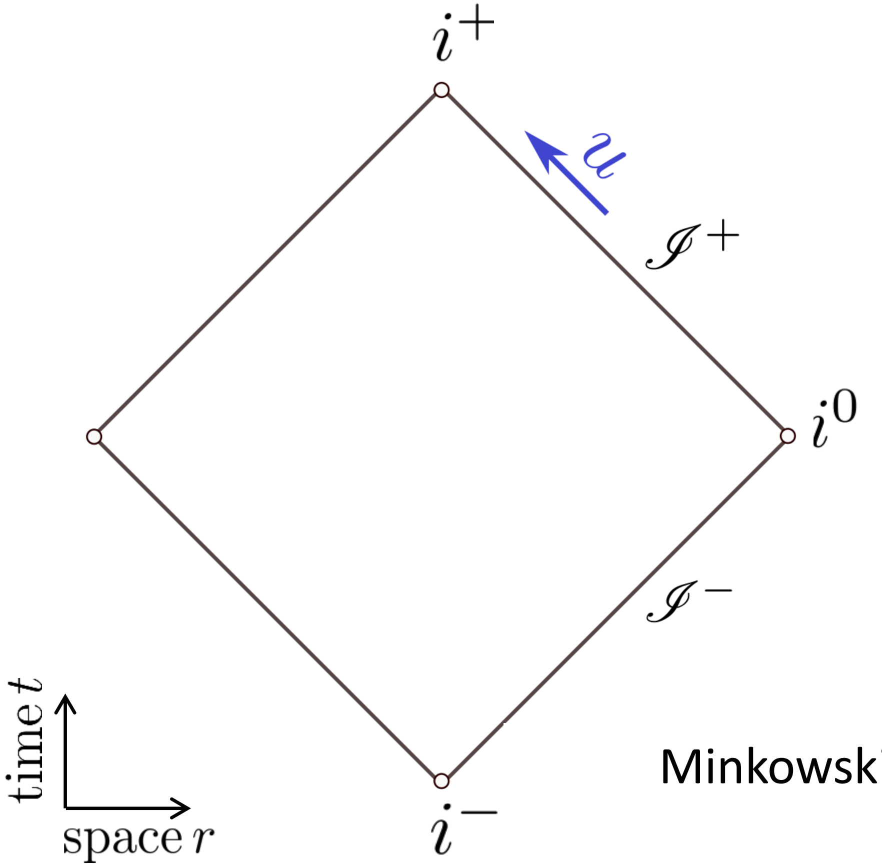
Main obstructions/difficulties:



Minkowski

Flat space holography

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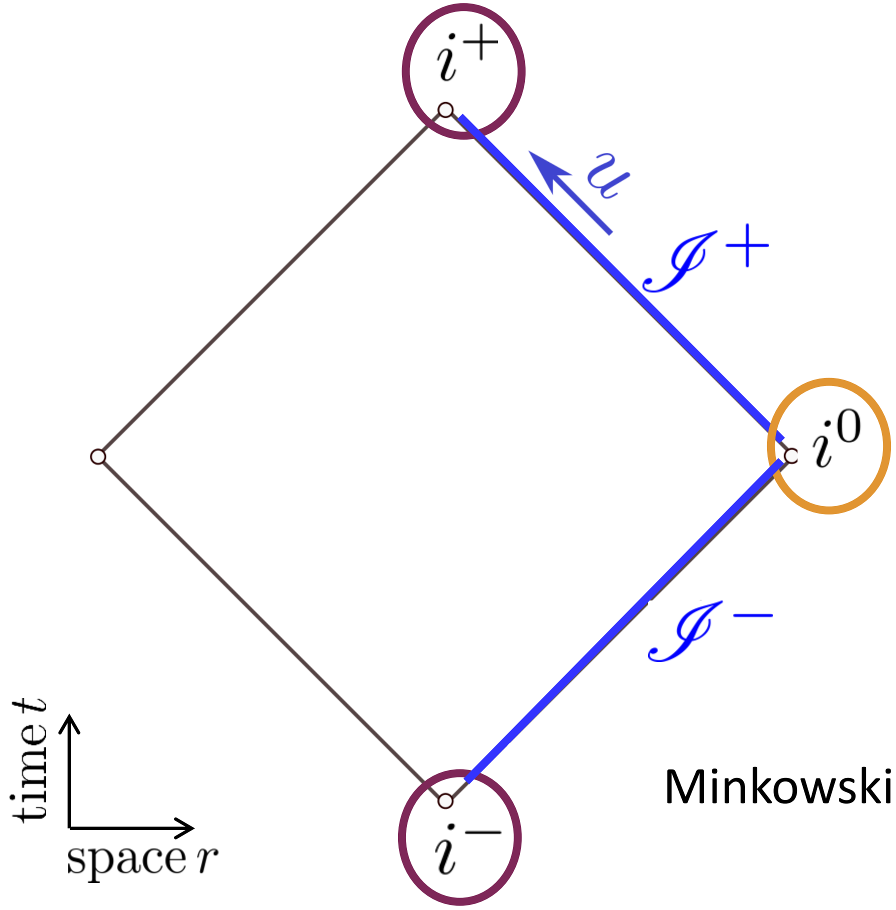
1 The conformal **boundary** includes

future/past timelike infinity
future/past null infinity
spatial infinity



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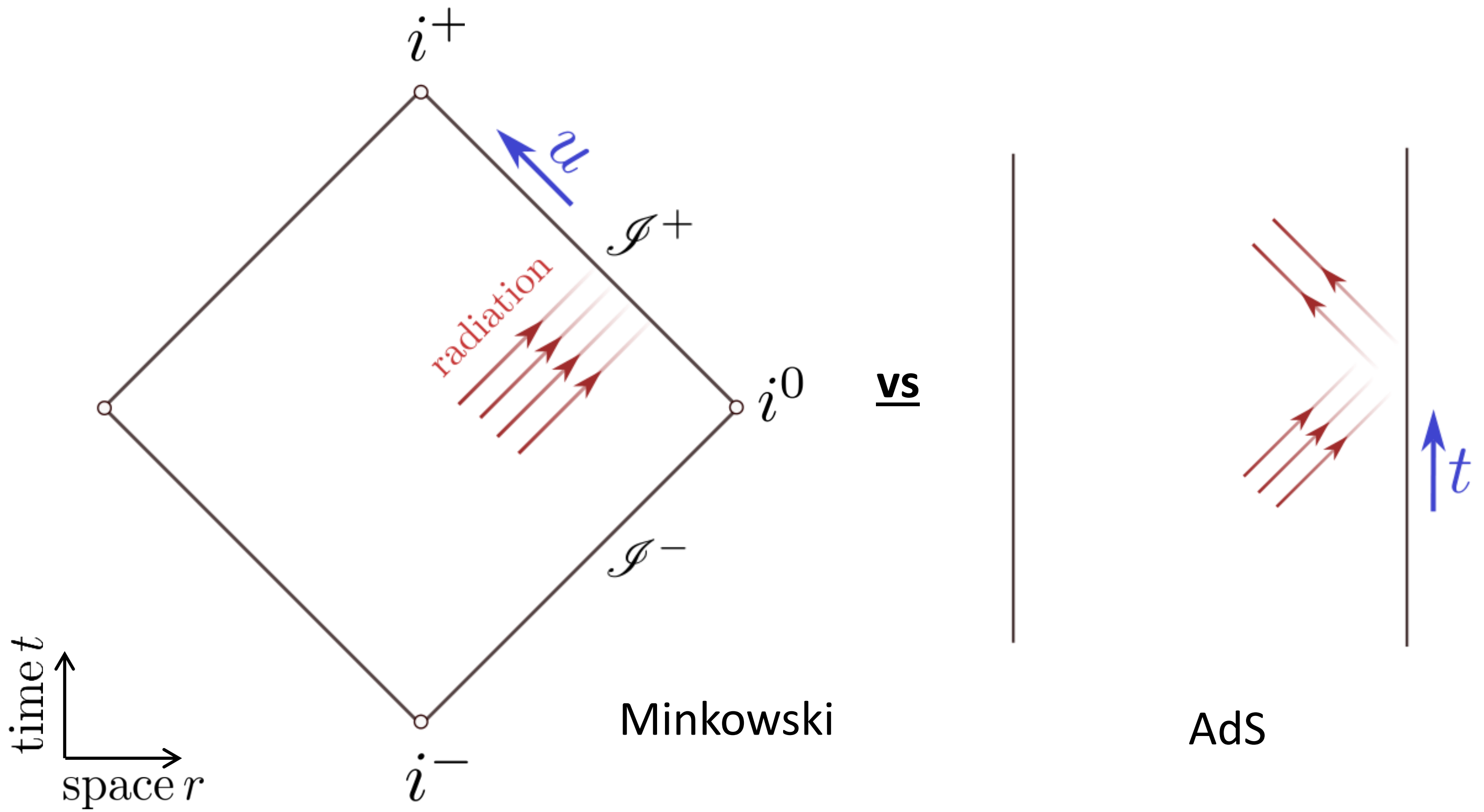

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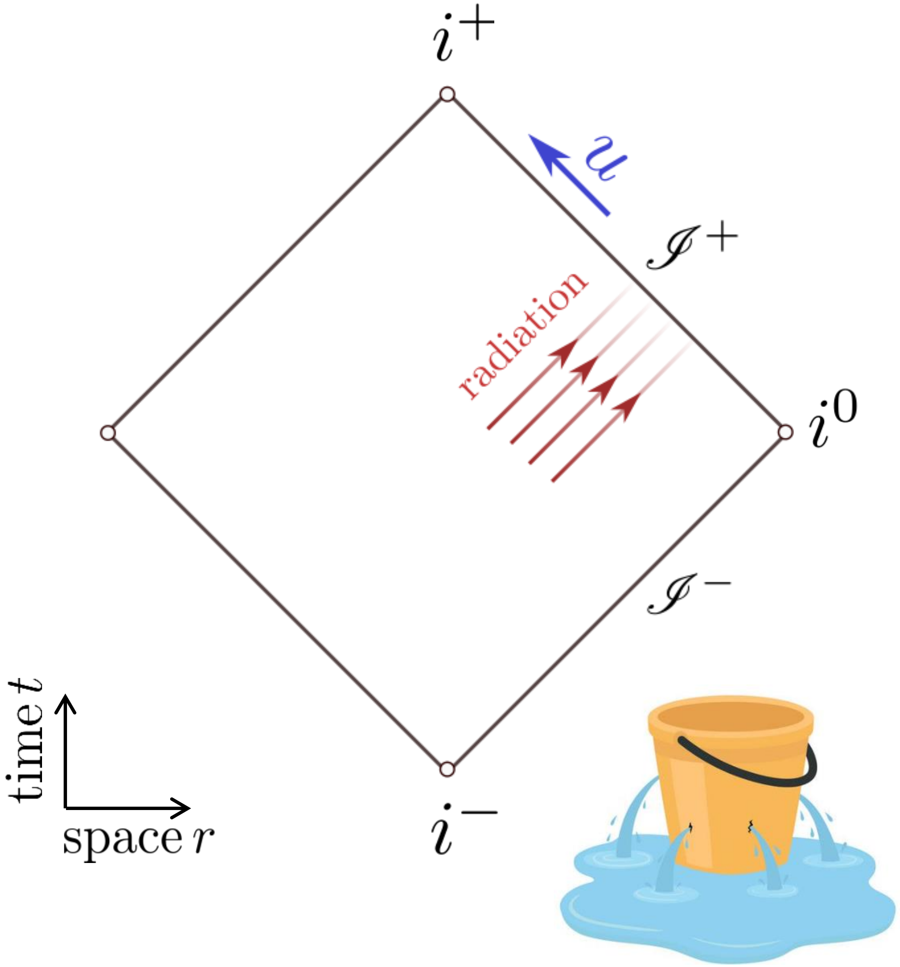
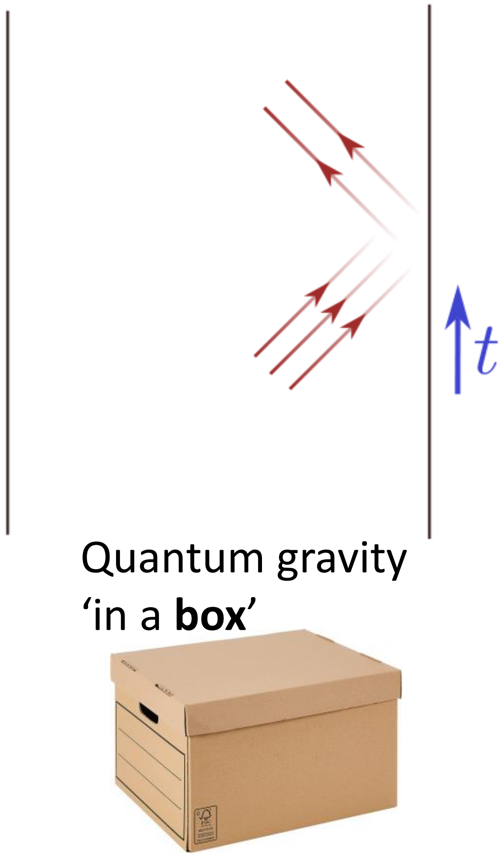


vs



Quantum gravity
'in a **box**'

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

--> Road map: symmetries

What are the symmetries of asymptotically flat spacetimes?

Flat space holography

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What are the symmetries of asymptotically flat spacetimes?

what was expected

Poincaré

what was found

Bondi-Metzner-Sachs ('62)

Flat space holography

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Road map: symmetries

What are the symmetries of asymptotically flat spacetimes?

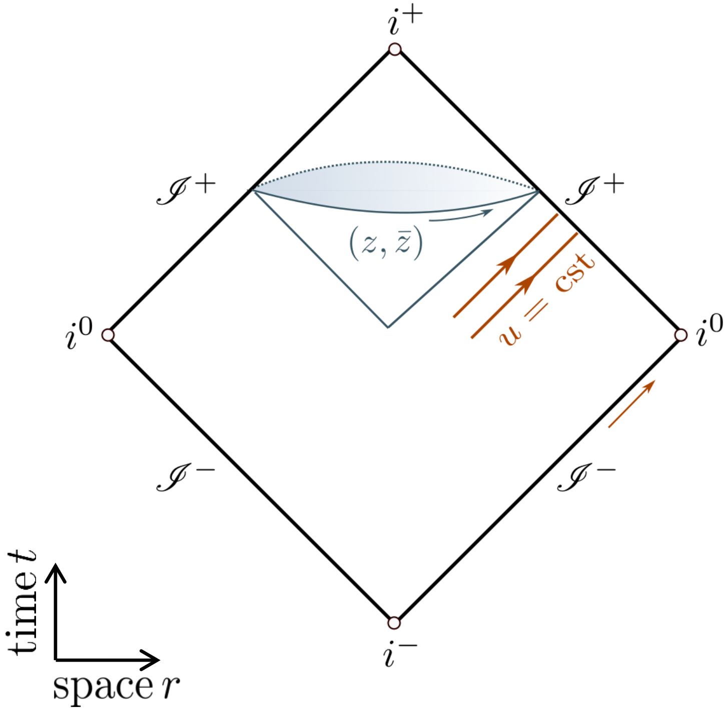
→ infinite-dimensional extension of Poincaré!

4 Poincaré translations

↓ *Symmetry enhancement*

∞ BMS supertranslations

[Bondi, van der Burg, Metzner ’62] [Sachs ’62]



$$\xi = \boxed{\mathcal{T}(z, \bar{z})}\partial_u + \cdots$$

arbitrary function
on the celestial sphere

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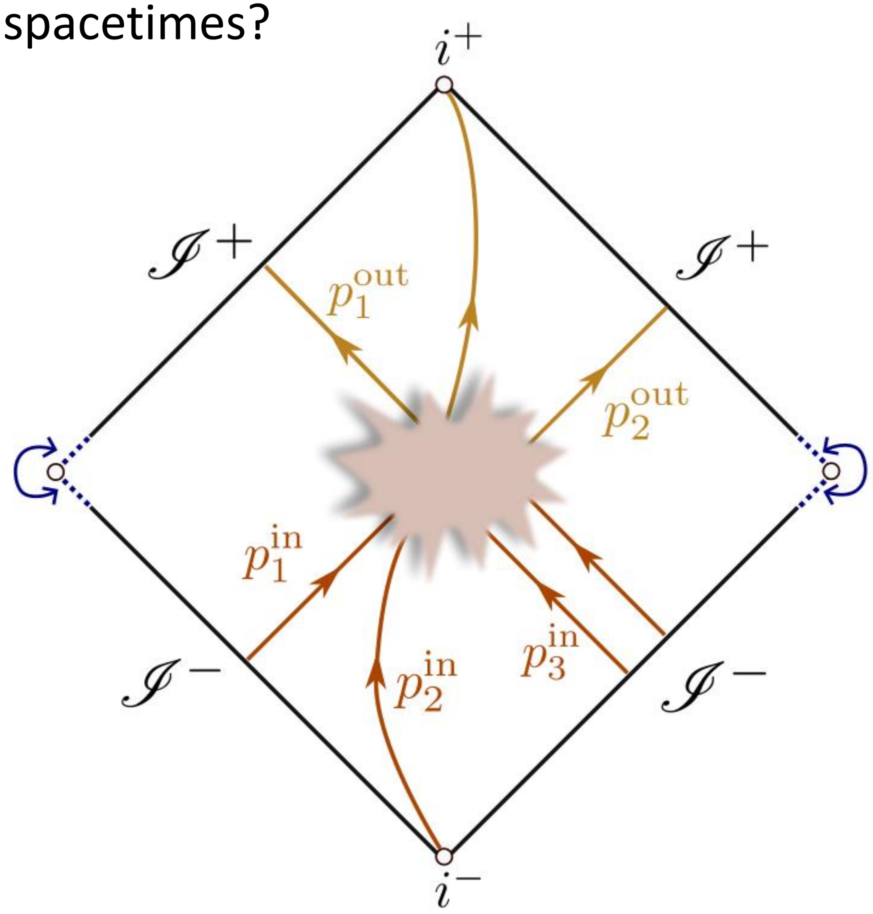
While BMS symmetries were originally disregarded, it was realized (50 years later) that they

- constrain the gravitational **S-matrix**



$$\langle \text{out}|S|\text{in}\rangle$$

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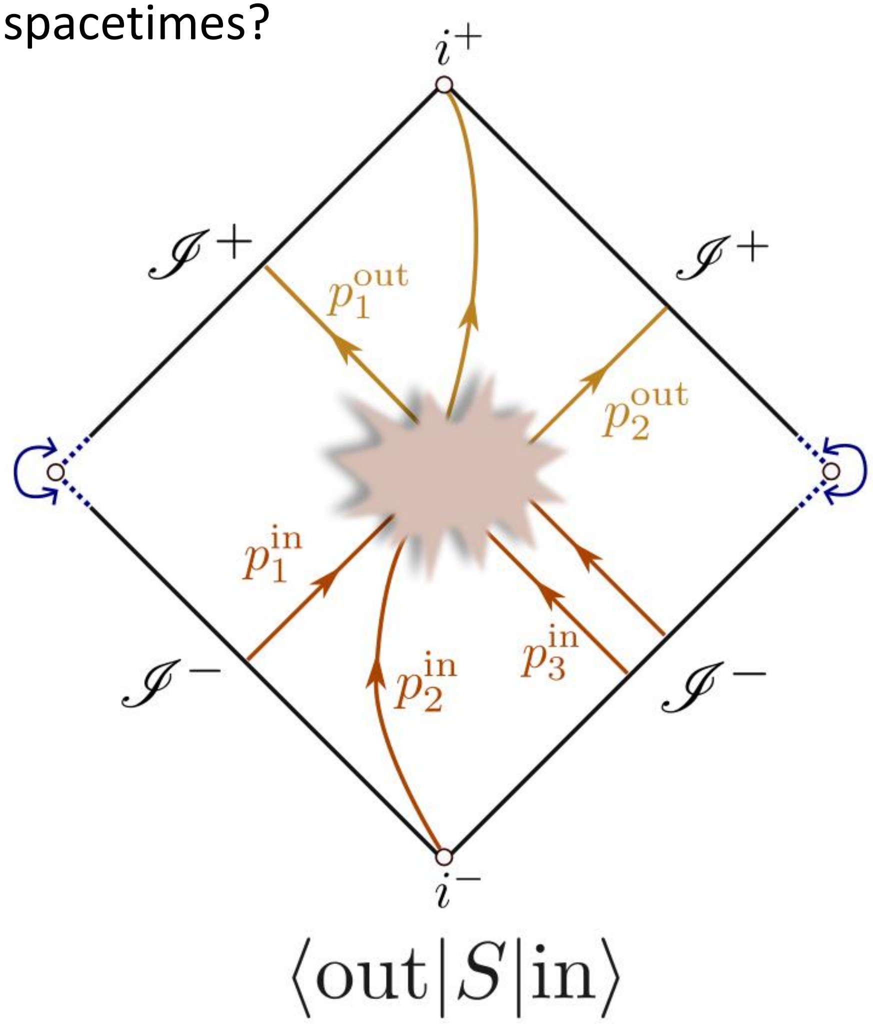

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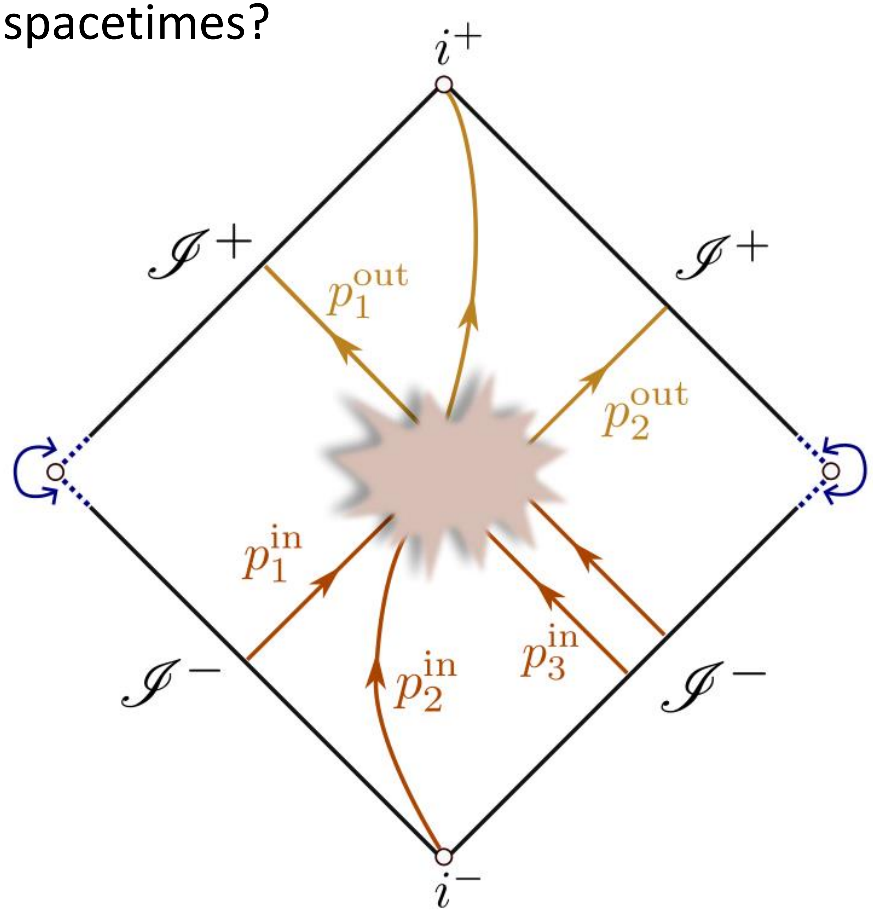
- allow further extensions, including the local **conformal** group



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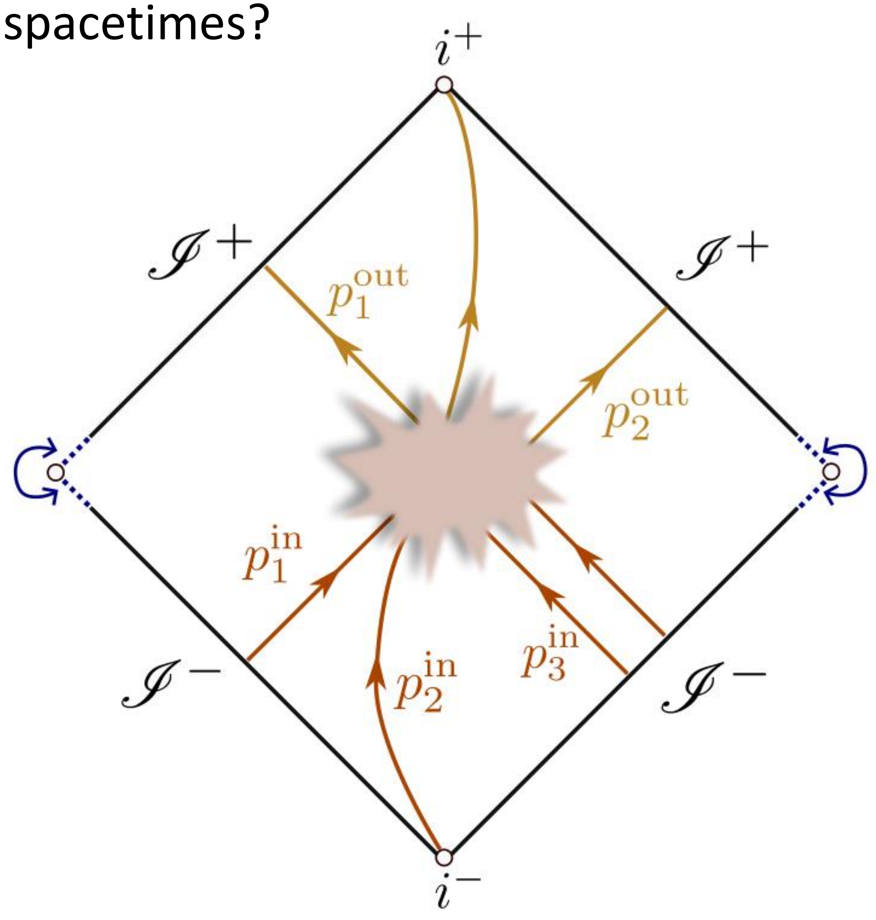
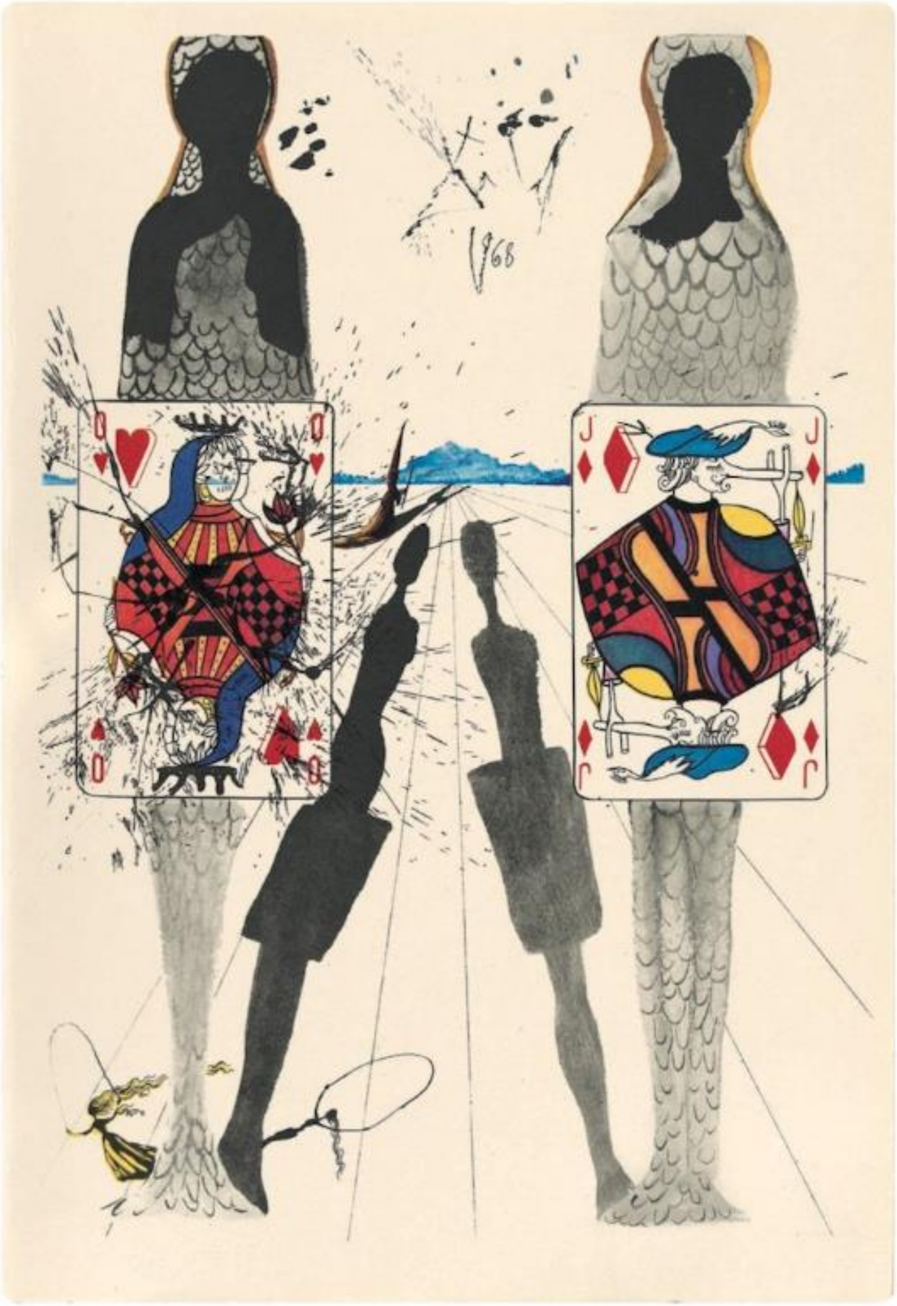
revival / proposals for **flat holography**



$$\langle \text{out}|S|\text{in}\rangle$$

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

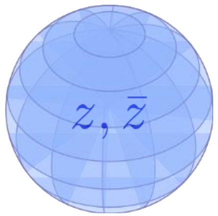
1. Celestial holography
2. Carrollian holography
3. $\mathscr{L}w_{1+\infty}$ symmetries
4. Final remarks

Flat space holography: on which boundary?

two natural boundaries/proposals

celestial sphere

Euclidean 2-sphere



4d bulk/2d holography: ‘celestial holography’

Dual: 2d ‘celestial CFT’

[de Boer, Solodukhin ’03][Pasterski, Shao, Strominger ’17] [Pasterski, Shao ’17]
[Cheung, de la Fuente, Sundrum’17][…]

Flat space holography: on which boundary?

two natural boundaries/proposals

null infinity

lighlike 3d hypersurface

$$\mathscr{I} = \mathbb{R} \times S^2$$

4d bulk/3d holography: ‘Carroll holography’

Dual: 3d ‘BMS field theory’

[Arcioni, Dappiaggi ’03 ’04] [Adamo, Casali, Skinner ‘14] [Bagchi, Basu, Kakkar, Melhra ’16] [Ciambelli, Marteau, Petkou, Petropoulos, Siampos] [LD, Fiorucci, Herfray, Ruzziconi ’22][Bagchi, Banerjee, Basu, Dutta ‘22][…]

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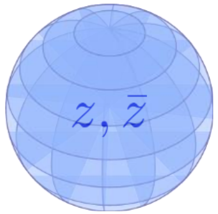

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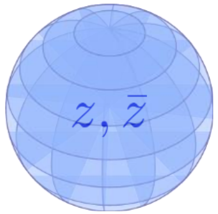
Features: closer to AdS/CFT ☺

treatment of fluxes ☹

celestial sphere

Euclidean 2-sphere



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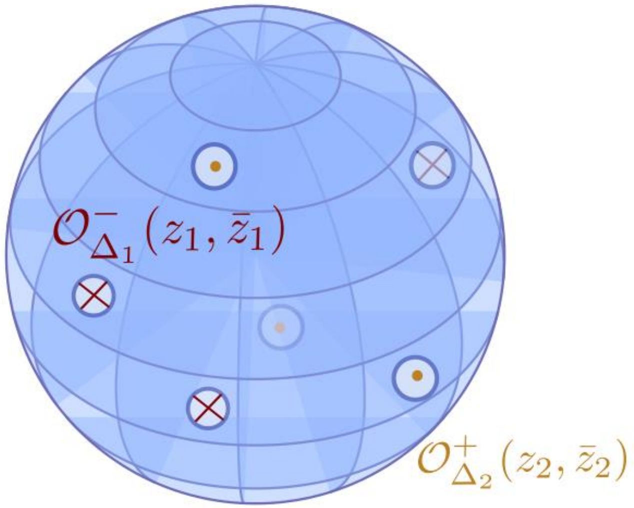
Features: powerful CFT techniques at hand ☺

role of translations obscured ☹



$$\langle \mathcal{O}^{\pm}_{\Delta_1}(z_1,\bar{z}_1)\dots\mathcal{O}^{\pm}_{\Delta_N}(z_N,\bar{z}_N)\rangle$$

Celestial Holography

'If you look up at the sky on a clear cloudless night, you appear to see a hemispherical dome above you, punctuated by myriads of stars.'

R. Penrose, *The road to reality*, 2004

Celestial Holography

Celestial Holography

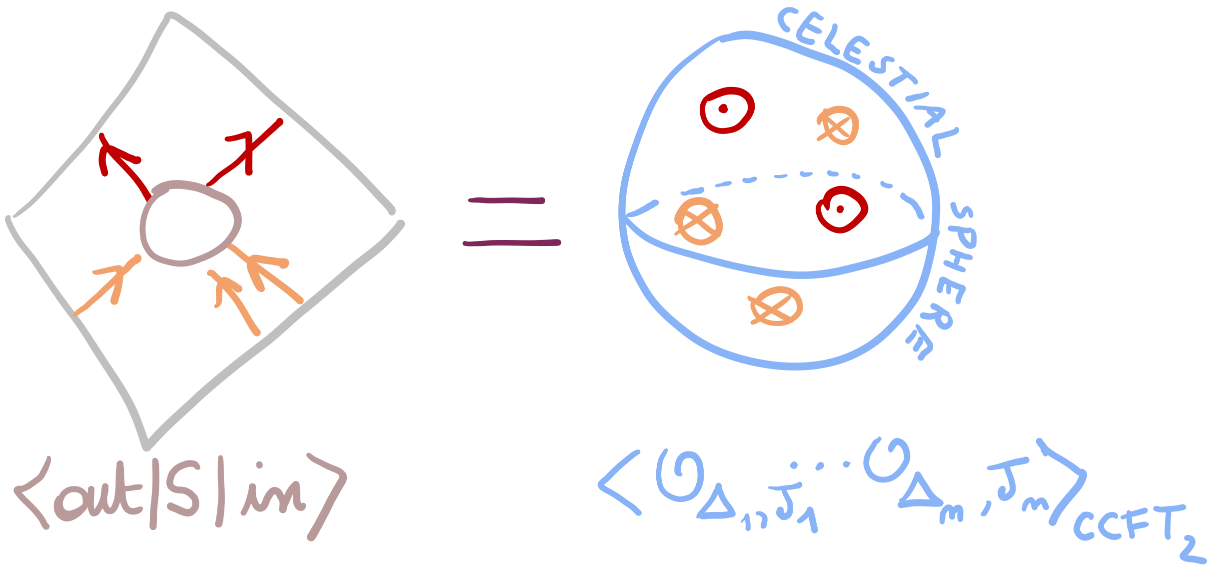
The 4d spacetime S-matrix is encoded in a 2d ‘Celestial Conformal Field Theory’

momentum of a massless particle

$$p^\mu = \omega q^\mu(z, \bar{z})$$

ω: energy

$(z, \bar{z})$: a point on $\mathcal{CS}^2$



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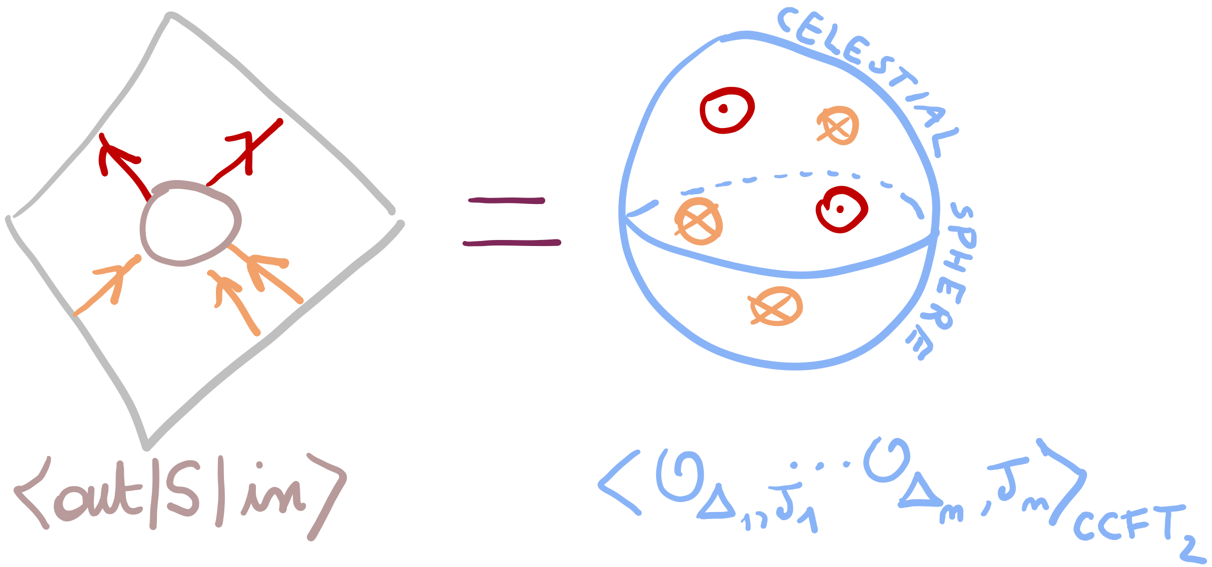

Simple idea: make conformal properties manifest

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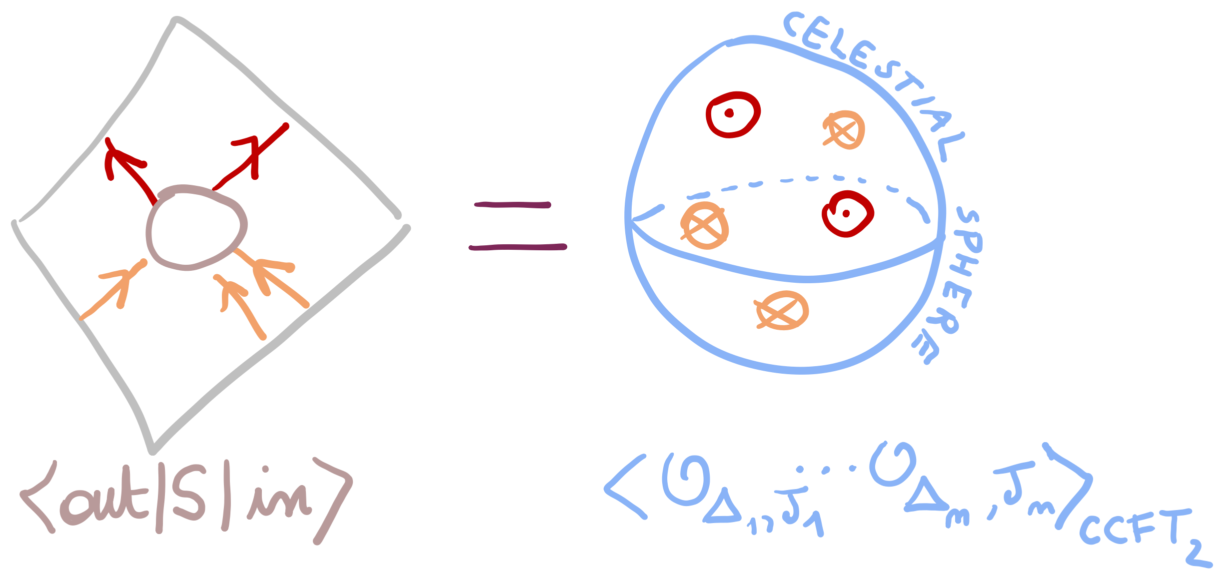


Simple idea: make conformal properties manifest

→ Plane waves are mapped to

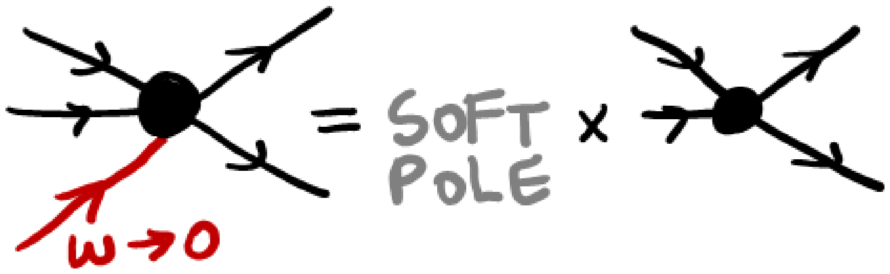
$$\Psi^{\pm}_{\Delta}(X; z, \bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta - 1} e^{\pm i p \cdot X}$$

$$\boxed{\Psi_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \Psi_{h,\bar{h}}(z,\bar{z})}$$

Primary field of weight $\Delta = h + \bar{h}$

Celestial currents

The soft sector of celestial CFT is captured by 2d celestial currents.



$$\mathcal{A}_{n+1} \overset{\omega\to 0}{\sim} \frac{1}{\omega}\mathcal{A}_n$$

[Weinberg ‘65][…]

Celestial currents

$$\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^{h}\left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}}\mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

The soft sector of celestial CFT is captured by 2d celestial currents.

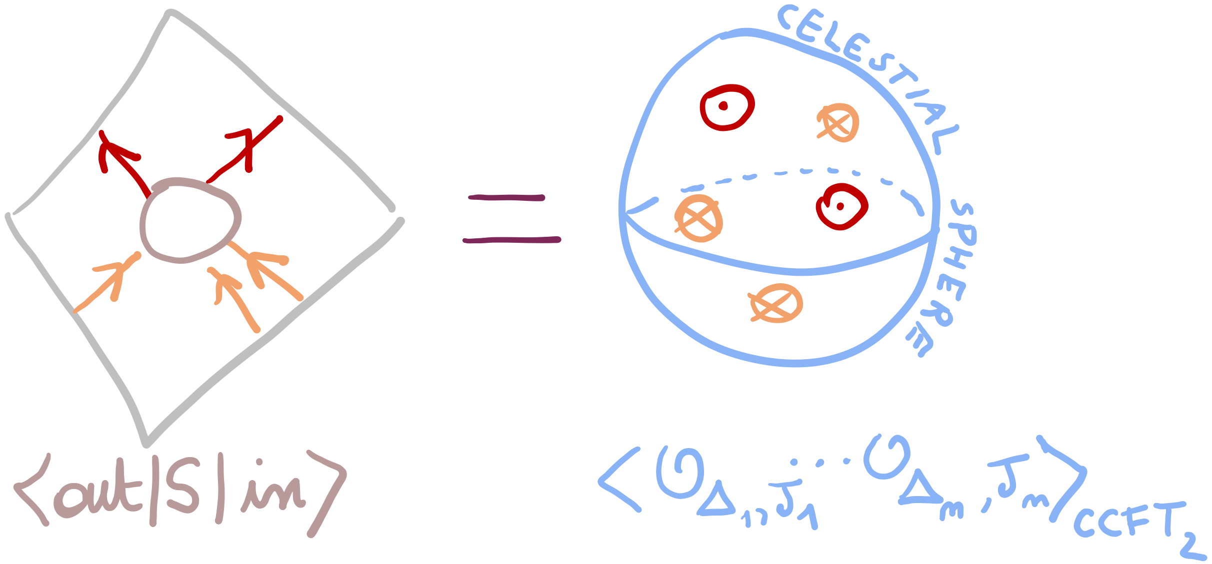
$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

[Kapec, Mitra, Raclariu, Strominger '16] [Cheung, de la Fuente, Sundrum '17]
[LD, Puhm, Strominger '18] [Fotopoulos, Stieberger,Taylor '20] …

Asymptotic symmetry	Ward identity	Weight	2d Celestial current
'large gauge' $\delta A_z = D_z\epsilon$	**Soft photon** theorem	$\Delta \to 1$ $(h,\bar{h}) = (1,0)$	$J(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h,\bar{h}}(w,\bar{w})$
supertranslations $\delta C_{zz} = D_z^2 f$	**Soft graviton** theorem	$\Delta \to 1$ $(\frac{3}{2},\frac{1}{2})$	$P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$
superrotations $\delta C_{zz} = uD_z^3 Y^z$	**Sub-leading** soft graviton theorem	$\Delta \to 2$ $(2,0)$	$T(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{h}{(z-w)^2}\mathcal{O}_{h,\bar{h}}(w,\bar{w}) + \frac{\partial\mathcal{O}_{h,\bar{h}}(w,\bar{w})}{z-w}$ 2d stress tensor!

$g_{zz} = rC_{zz} + \dots$

Celestial Holography

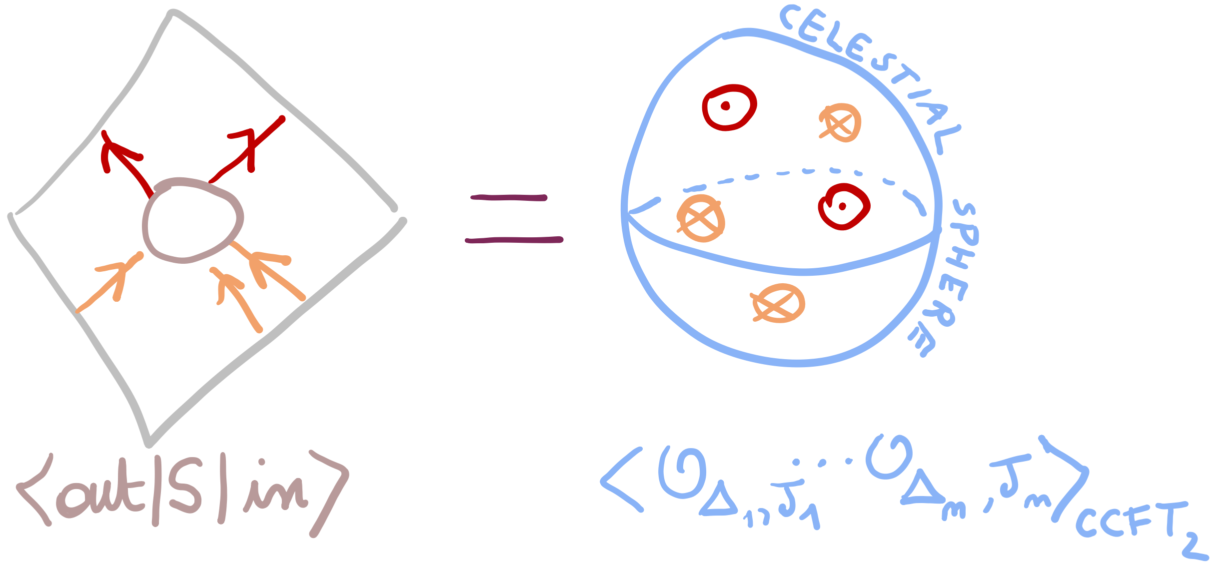


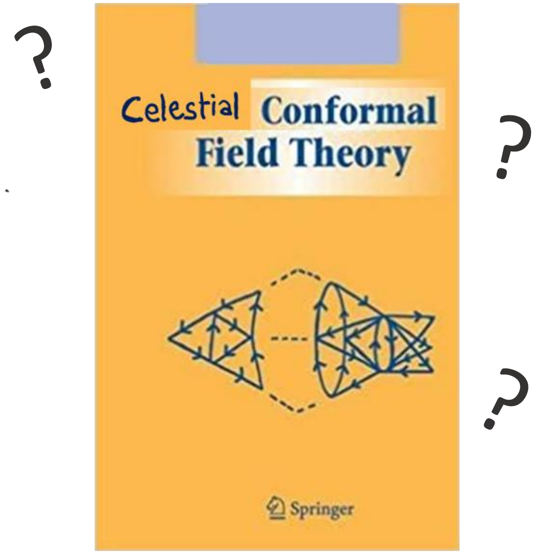
collinear limits $p_1^\mu \parallel p_2^\mu$ — celestial **OPEs**

low point amplitudes — kinematic singularities

asymptotic symmetries — *2d* currents

Celestial Holography





collinear limits $p_1^\mu \parallel p_2^\mu$

low point amplitudes

asymptotic symmetries

celestial **OPEs**

kinematic singularities

2*d* currents

spectrum? non-unitary?

...

Carrollian Holography

'We're all mad here'

Lewis Carroll, *Alice's Adventures in Wonderland*

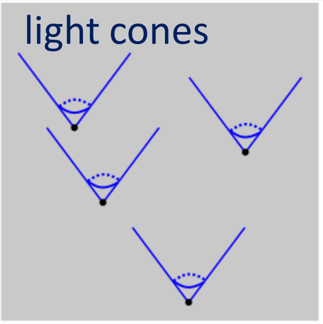
Carrollian physics

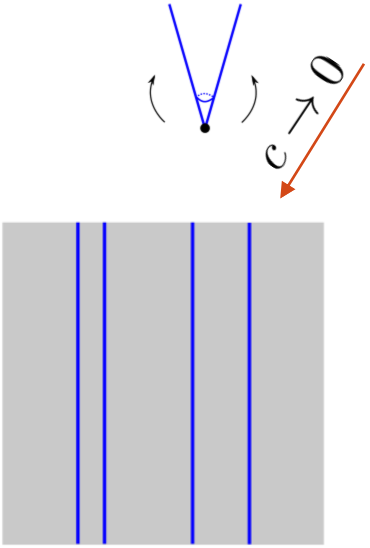
- 1965: A curiosity of Lévy-Leblond (also independently by Sen Gupta 1966)

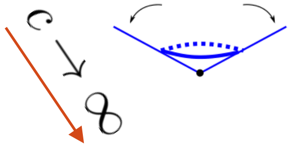
  The $c \to \infty$ limit of the Poincaré group leads to the Galilean group.

  But what if we take the $c \to 0$ *limit instead?*

  → '**Carroll** group'

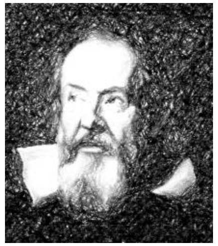


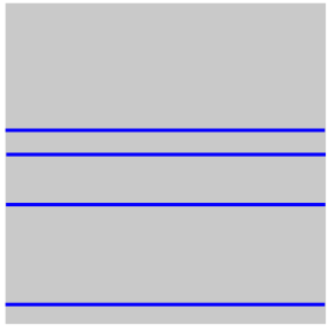




"Alice's Adventures in Wonderland"
Lewis Carroll (1865)

Carrollian spacetime
(space is absolute)





Galilean spacetime
(time is absolute)

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- Weird features… but (lately) found to be relevant for

  - Hamiltonian analysis of GR [Henneaux ‘79]
  - fluid/gravity correspondence
    [Ciambelli, Marteau, Petkou, Petropoulos, Siampos ‘18]
    [de Boer, Hartong, Obers, Sybesma, Vandoren ‘22]
  - black hole near-horizon physics [Penna‘18][LD, Marteau ‘18]
  - cosmology [de Boer, Hartong, Obers, Sybesma, Vandoren ‘22]
  - …flat space holography

BMS = conformal Carrollian symmetries

- BMS symmetries = **conformal** symmetries of a **Carrollian** structure at null infinity

  [Geroch][Penrose][Henneaux][Duval, Gibbons, Horvathy][Hartong][Ciambelli, Leigh, Marteau, Petropoulos][Bekaert, Morand][Herfray]…

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Carrollian geometry

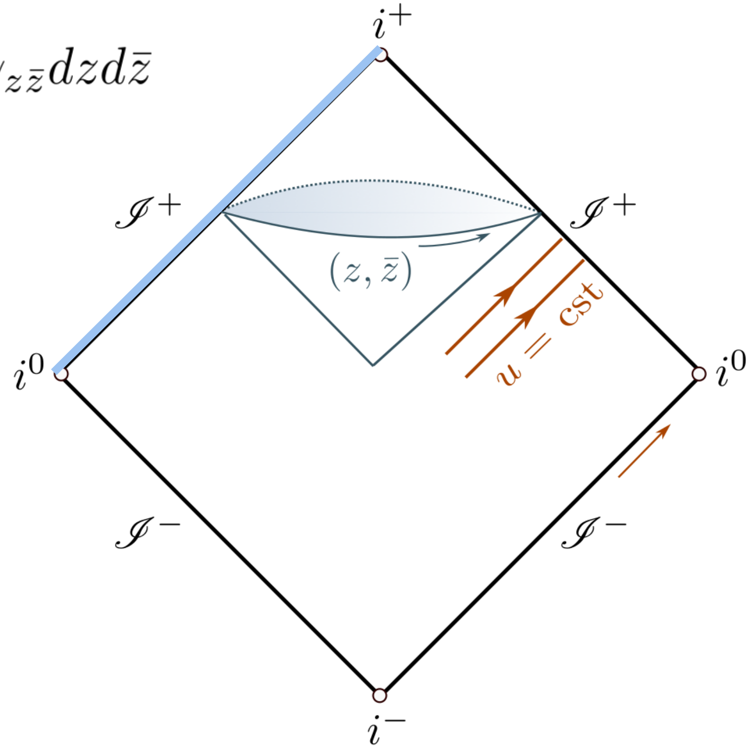
q_{ab} : a *degenerate* metric

a vector field satisfying $q_{ab}n^b = 0$

$\mathscr{I}^+$ $x^a = (u, z, \bar{z})$

$$q_{ab}dx^a dx^b = 0 \times du^2 + 2\gamma_{z\bar{z}}dzd\bar{z}$$

$$n = \partial_u$$



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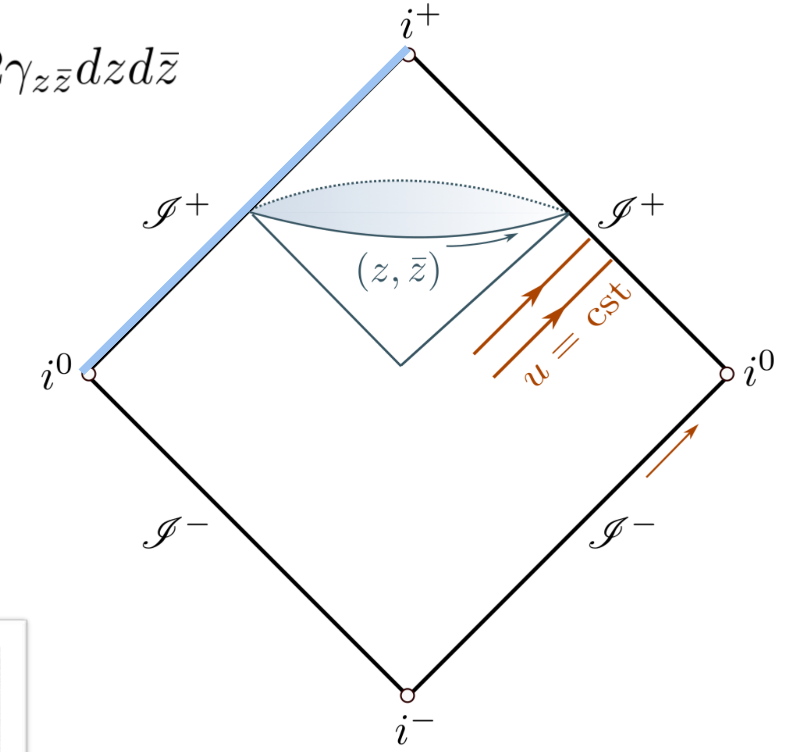


Conformal Carrollian symmetries:

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab} \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$$

$$\alpha := \frac{1}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})$$

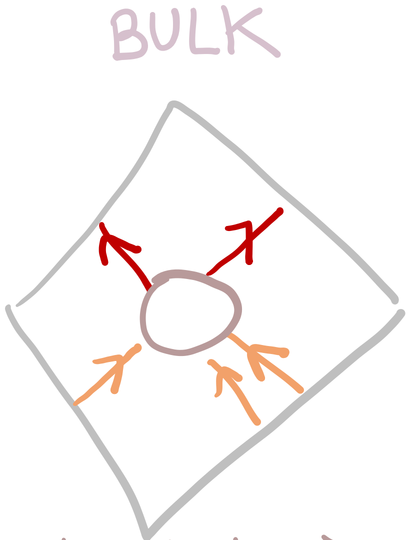
$$\bar{\xi} = \left[\mathcal{T} + \frac{u}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

$$\boxed{\mathfrak{CCarr}_d = \mathfrak{bms}_{d+1}}$$

Carrollian 'dictionary'

Observables: S-matrix elements as correlators of a '**Carrollian**' field theory

[LD, Fiorucci, Herfray, Ruzziconi '22]



$$\langle \text{out}|S|\text{in}\rangle$$

$=$

$$\left\langle \sigma_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}_1)\ldots\sigma_{k_m,\bar{k}_m}(u_m,z_m,\bar{z}_m)\right\rangle_{\text{Carr}}$$

Field-operator map
(for outgoing massless spin s field)

$$\Phi^{(s)}(X) \overset{\mathscr{I}^+}{\sim} r^{s-1}\,\sigma^{\text{out}}_{k,\bar{k}}(u,z,\bar{z})$$

Carrollian 'dictionary'

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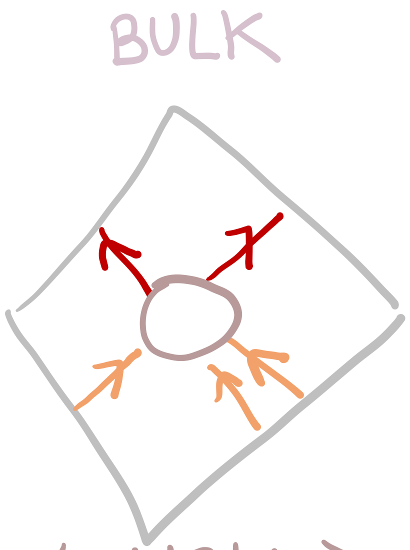

$$\langle out|S|in\rangle = \left\langle \sigma_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}_1)\dots\sigma_{k_m,\bar{k}_m}(u_m,z_m,\bar{z}_m)\right\rangle_{Carr}$$

Field-operator map
(for outgoing massless spin s field)

$$\Phi^{(s)}(X) \overset{\mathscr{I}^+}{\sim} r^{s-1}\boxed{\sigma^{\text{out}}_{k,\bar{k}}(u,z,\bar{z})}$$

transform as a 'conformal Carrollian primary' of weights $(k,\bar{k})$

$$\delta_{\bar{\xi}}\sigma_{k,\bar{k}} = \left[\left(\mathcal{T}+\tfrac{u}{2}(\partial\mathcal{Y}+\bar{\partial}\bar{\mathcal{Y}})\right)\partial_u+\mathcal{Y}\partial+\bar{\mathcal{Y}}\bar{\partial}+k\,\partial\mathcal{Y}+\bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\sigma_{k,\bar{k}}$$

Carrollian ‘dictionary’

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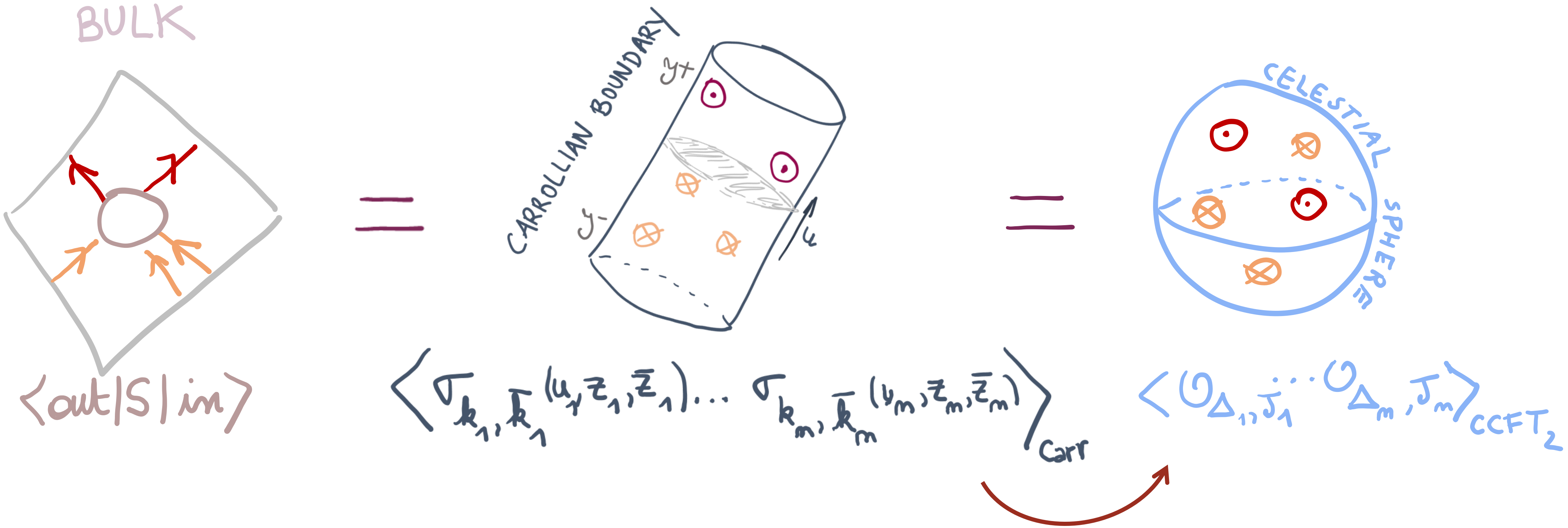


$$\mathcal{O}^{\text{out}}_{(\Delta,J)}(z,\bar{z}) = \lim_{\epsilon\to 0^+} \int_{-\infty}^{+\infty} \frac{du}{(u+i\epsilon)^{\Delta}}\, \sigma^{\text{out}}_{(k,\bar{k})}(u,z,\bar{z})$$

Carrollian – **celestial** operator map

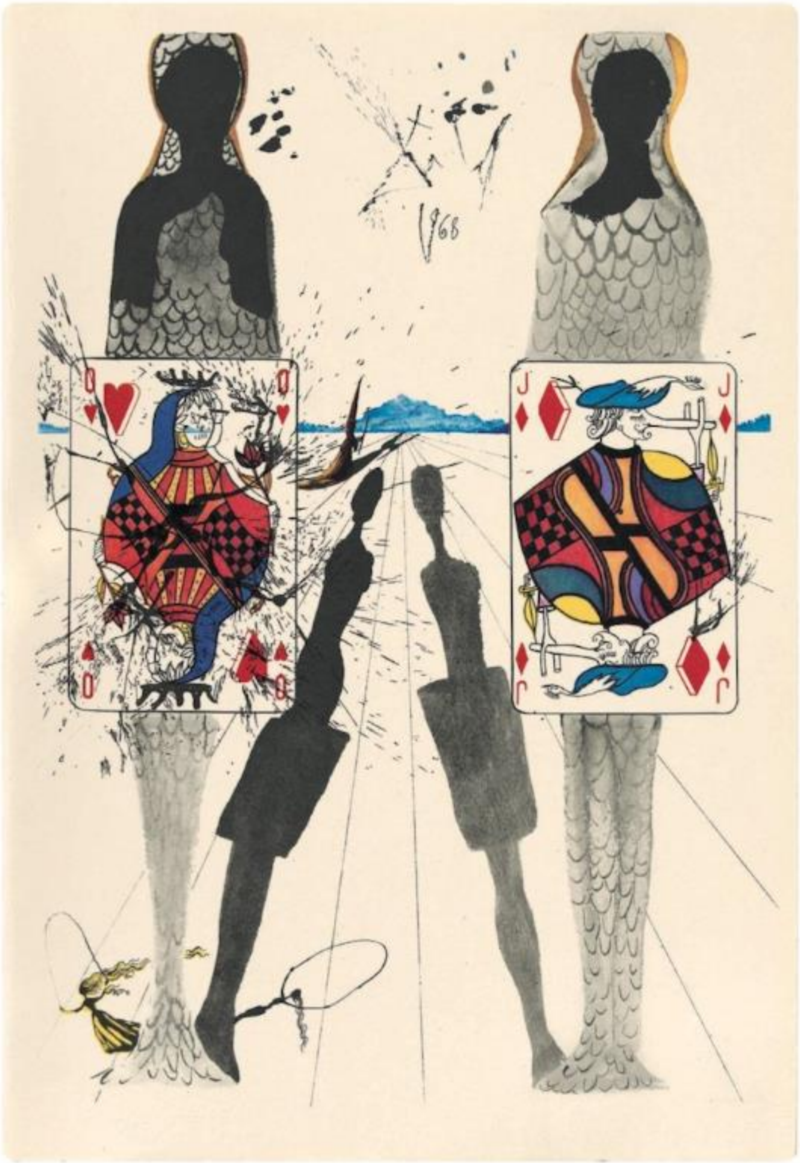
Just a change of basis?

Is this really holography?

Is this useful?

Can we learn something we did not know already?

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



$\mathscr{L}w_{1+\infty}$ symmetries

$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

- Celestial operators of integer conformal dimension give rise to 2d currents

$$H^k(z,\bar{z}) := \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\varepsilon,+2} \qquad k = 2, 1, 0, -1, \dots$$

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Celestial graviton OPE

- $$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\kappa}{2}\frac{1}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{(\bar{z}_{12})^{n+1}}{n!}\bar{\partial}^n\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)$$

[Guevara, Himwich, Pate, Strominger ‘21]

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[Guevara, Himwich, Pate, Strominger ‘21]

$$H^k(z,\bar z) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar z^{n+\frac{k-2}{2}}}$$

the holomorphic modes close the algebra

$$\left[H^k_m, H^l_n\right] = -\frac{\kappa}{2}\left[n(2-k)-m(2-l)\right]\frac{(\frac{2-k}{2}-m+\frac{2-l}{2}-n-1)!}{(\frac{2-k}{2}-m)!(\frac{2-l}{2}-n)!}\frac{(\frac{2-k}{2}+m+\frac{2-l}{2}+n-1)!}{(\frac{2-k}{2}+m)!(\frac{2-l}{2}+n)!}H^{k+l}_{m+n},$$

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$$H^k(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H_n^k(z)}{\bar{z}^{n+\frac{k-2}{2}}}$$ redefining $$w_n^p = \frac{1}{\kappa}(p-n-1)!(p+n-1)!\, H_n^{-2p+4}$$

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redefining $$w_n^p = \frac{1}{\kappa}(p-n-1)!(p+n-1)!\, H_n^{-2p+4}$$

$$\boxed{[w_m^p, w_n^q] = [m(q-1) - n(p-1)]\, w_{m+n}^{p+q-2}}$$

$$p = 1, \frac{3}{2}, 2, \frac{5}{2}, \dots \qquad 1-p \le m \le p-1$$

`wedge'

Virasoro

(super)translations

$\mathscr{L}w_{1+\infty}$ symmetries in celestial CFT

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$$H^k(z,\bar z) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar z^{n+\frac{k-2}{2}}}$$

redefining

$$w^p_n = \frac{1}{\kappa}(p-n-1)!(p+n-1)!\, H^{-2p+4}_n$$

$$\boxed{[w^p_m, w^q_n] = [m(q-1)-n(p-1)]\, w^{p+q-2}_{m+n}}$$

$$p = 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots \qquad 1-p \le m \le p-1$$

`wedge'

Virasoro

(super)translations

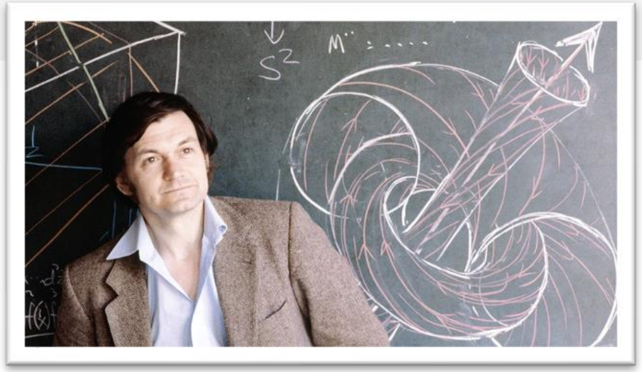
The infinite tower of celestial currents organizes into a single $\mathscr{L}w_{1+\infty}$ algebra ! [Strominger '21]

$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity

- … but how do these symmetries act on the boundary fields ?

[LD, Herfray, Freidel '24]

$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity



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→ Go to twistor space !

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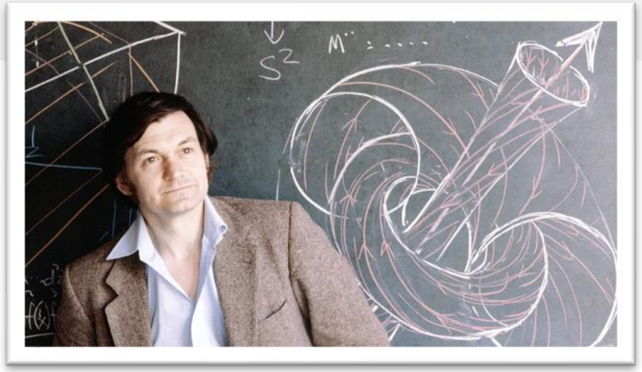


- … but how do these symmetries act on the boundary fields ?

→ Go to twistor space ! [LD, Herfray, Freidel '24]

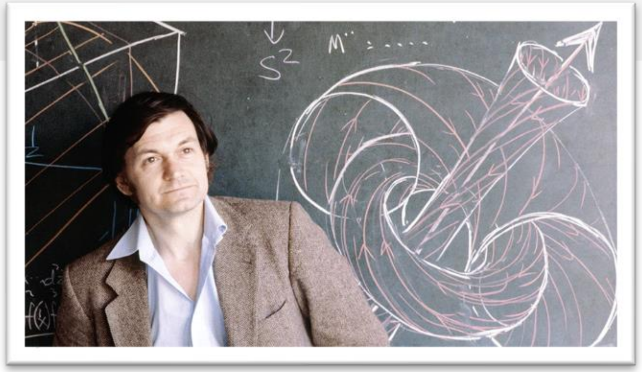
- The $\mathscr{L}w_{1+\infty}$ algebra has a very natural implementation in twistor space [Penrose '76] [Boyer, Plebanski '85][Adamo, Mason, Sharma '22]

$$[Z^A] = (\mu^{\dot\alpha}, \lambda_\alpha(z)) \quad \in \mathbb{CP}^3$$

$$g = g_0(z) + g_{\dot\alpha}(z)\mu^{\dot\alpha} + g_{\dot\alpha\dot\beta}(z)\mu^{\dot\alpha}\mu^{\dot\beta} + \dots$$

$$\{g_1, g_2\} = \epsilon^{\dot\alpha\dot\beta}\frac{\partial g_1}{\partial\mu^{\dot\alpha}}\frac{\partial g_2}{\partial\mu^{\dot\beta}}$$

$\mathscr{L}w_{1+\infty}$ symmetries seen from null infinity



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$\mathscr{I}$

BOUNDARY

$$\sigma(u, z, \bar{z})$$

Carrollian field of weights

$$(k, \bar{k}) = \left(\tfrac{1-s}{2}, \tfrac{1+s}{2}\right)$$

Large r expansion / Kirchoff-d'Adhémar formula

BULK

$$\Phi^{(s)}(X)$$

zero-rest mass field of helicity s

$\mathbb{M}$

[Eastwood, Tod '82]

[LD, Herfray, Freidel '24]

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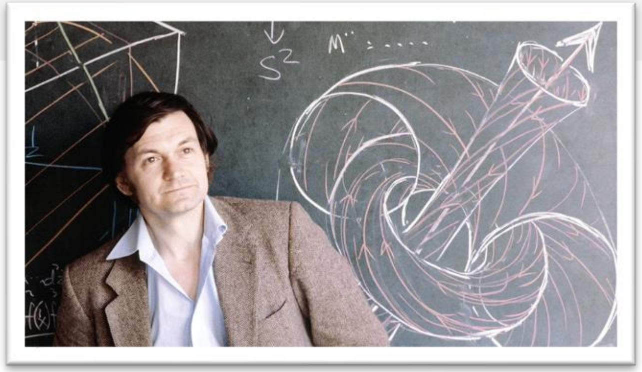

$\mathscr{I}$

BOUNDARY

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$(k, \bar{k}) = (\frac{1-s}{2}, \frac{1+s}{2})$

Twistor lift

TWISTOR SPACE

twistor representative

$H^{0,1}(\mathbb{PT}, \mathcal{O}(2s-2))$

$\mathbb{PT}$

BULK

$\Phi^{(s)}(X)$

zero-rest mass field of helicity s

$\mathbb{M}$

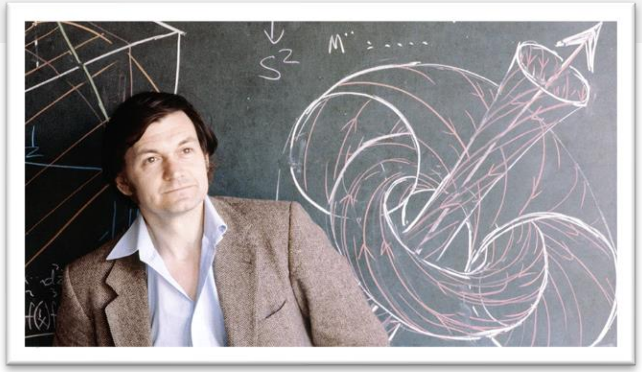
Large r expansion / Kirchoff-d'Adhémar formula

Penrose transform [Penrose '69]

[Eastwood, Tod '82]

[LD, Herfray, Freidel '24]

$\mathscr{L}w_{1+\infty}$ symmetries



- … but how do these symmetries act on the boundary fields ?

$\mathscr{I}$

$$\bar{\sigma}(u,\lambda)$$

$$\Big\downarrow {\scriptstyle Lw_{1+\infty}}$$

GOAL: $\delta\bar{\sigma}(u,\lambda)$?

$\mathscr{L}w_{1+\infty}$ symmetries

- … but how do these symmetries act on the boundary fields ?

$\mathscr{I}$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $\mathbb{PT}$

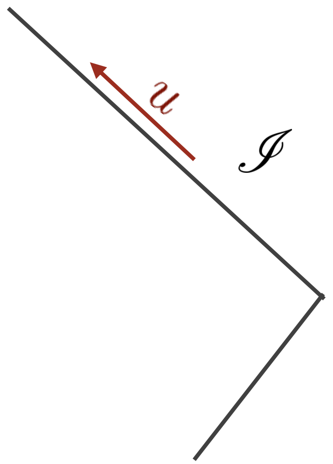
$$\begin{tikzcd}
\bar{\sigma}(u,\lambda) \arrow[r] \arrow[d, "Lw_{1+\infty}"'] & h(u,\lambda) = \partial_u^{-1}\,\bar{\sigma} \arrow[r, "(i)", "\text{twistor lift}"'] & \mathbf{h} = h(u=\mu\lambda,\lambda)\, D\bar{\lambda} \arrow[d, "Lw_{1+\infty}"'] \\
\delta\bar{\sigma}(u,\lambda) & \delta\Phi(x) = \displaystyle\int_{\mathbb{CP}^1} \frac{\partial^2 \delta\mathbf{h}}{\partial\mu\partial\mu} \arrow[l, "\text{large } r"', "(iii)"] & \delta\mathbf{h} = \{g,\mathbf{h}\} \arrow[l, "\text{Penrose transform}"', "(ii)"]
\end{tikzcd}$$

$Z^A = (\mu^{\dot{\alpha}}, \lambda_\alpha)$
$\in \mathbb{C}^4$

[LD, Herfray, Freidel '24]

$\mathscr{L}w_{1+\infty}$ symmetries

- …how do these symmetries act on the boundary fields ?



Main result:

$$\delta_n \bar{\sigma} = \sum_{\ell=0}^{n} \bar{\partial}^{n-\ell}\left(g_{\dot{\alpha}(n)} \bar{\lambda}^{\dot{\alpha}(n)}\right) \frac{\ell}{(n-\ell)!} \partial_u^3 \left(u^{n-\ell}\, \partial_u^{-1-\ell}\, \bar{\partial}^{\ell-1} \bar{\sigma}\right) \qquad s=+2$$

$$\delta_n \sigma = \sum_{\ell=0}^{n} \bar{\partial}^{n-\ell}\left(g_{\dot{\alpha}(n)} \bar{\lambda}^{\dot{\alpha}(n)}\right) \frac{\ell}{(n-\ell)!} \partial_u^{-1} \left(u^{n-\ell}\, \partial_u^{3-\ell}\, \bar{\partial}^{\ell-1} \sigma\right) \qquad s=-2$$

$\mathscr{L}w_{1+\infty}$ generators

$n = 0, 1, 2, \ldots$

non-local action
(vs local in twistor space)

[LD, Herfray, Freidel '24]

✓ explicit match with [Freidel, Raclariu, Pranzetti '21]

In summary

- $\mathscr{L}w_{1+\infty}$ symmetries organize an infinite tower of celestial currents at tree level

- We derived the explicit realization of these symmetries for Carrollian fields at null infinity.

  The action of these symmetries is local in twistor space but non-local in spacetime

→ What is the faith of these symmetries beyond tree level ?

Final remarks:

IR divergences and loop corrections

The problem of IR divergences

- The S-matrix is plagued with infrared (IR) divergences

  ⟶ set all conventional Fock basis S-matrix elements to zero

  In practice, dealt with by working with inclusive cross-sections

  [Yennie, Frautschi, Suura '61][Weinberg '65]

The problem of IR divergences

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  $\longrightarrow$ set all conventional Fock basis S-matrix elements to zero

  In practice, dealt with by working with inclusive cross-sections

  [Yennie, Frautschi, Suura ‘61][Weinberg ‘65]

- We now understand that
  IR divergences = penalties to pay for *violating BMS conservation laws* !

  [Kapec, Perry, Raclariu, Strominger ‘17][Choi, Akhoury]

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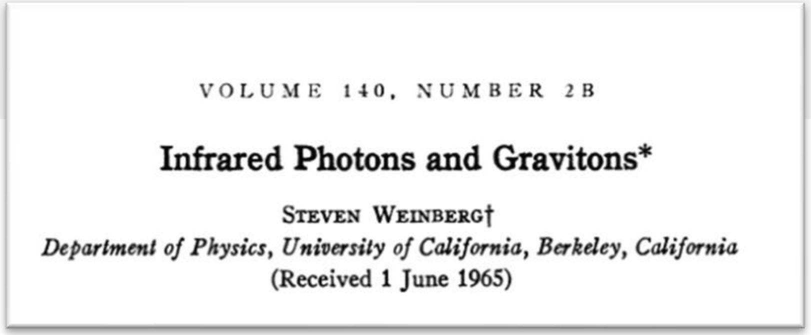
  [Kapec, Perry, Raclariu, Strominger '17][Choi, Akhoury]

  Indeed, BMS symmetries act on the vacuum as $$\delta C_{zz}^{vac} = \partial_z^2 C^{(0)}(z,\bar{z}) \neq 0$$

  supertranslation Goldstone boson

  → need to account for transitions between in and out vacuua

Soft factorization


VOLUME 140, NUMBER 2B

Infrared Photons and Gravitons*

STEVEN WEINBERG†
Department of Physics, University of California, Berkeley, California
(Received 1 June 1965)

- Weinberg showed that IR divergences factorize

$$\boxed{\mathcal{A}_n = \mathcal{A}_{\text{soft}}\, \mathcal{A}_{\text{finite}}}$$

(IR divergences from soft virtual gravitons exchanged between external legs)

$$\mathcal{A}_{\text{soft}} = \exp\left[\frac{1}{\epsilon}\frac{\kappa^2}{2(8\pi)^2}\sum_{ij}\eta_i\eta_j\, m_i m_j \frac{1+\beta_{ij}^2}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}\left(i\pi\delta_{\eta_i,\eta_j} - \frac{1}{2}\ln\frac{1+\beta_{ij}}{1-\beta_{ij}}\right)\right]$$

$\eta_i = \pm 1$ (in or out)

$$\beta_{ij} = \sqrt{1-(\hat{p}_i\cdot\hat{p}_j)^{-2}}$$

Soft factorization

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$$\boxed{\mathcal{A}_n = \mathcal{A}_{\text{soft}}\,\mathcal{A}_{\text{finite}}}$$

- In terms of celestial operators, this IR factorization is

$$\boxed{\mathcal{A}_n = \langle \mathcal{W}_1 \ldots \mathcal{W}_n \rangle \, \langle \tilde{\mathcal{O}}_1 \ldots \tilde{\mathcal{O}}_n \rangle}$$

[Himwich, Narayanan, Pate, Paul, Strominger '20]

soft (IR divergent)

hard (IR finite) operators

massless particle $p = \omega \hat{q}$: $\mathcal{W}(p) = e^{i\omega C^{(0)}(\hat{q})}$

$C^{(0)}(\hat{q})$: supertranslation Goldstone

massive particle $p = m\hat{p}$: $\mathcal{W}(p) = \exp\left[\frac{im}{2}\int d^2\hat{q}\,\mathcal{G}(\hat{p};\hat{q})\,C^{(0)}(\hat{q})\right]$

bulk-to-boundary operator

Loop corrections to soft theorems?

Loop corrections to soft theorems

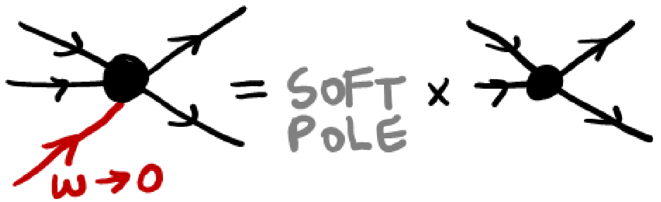
- Tree-level soft graviton theorem
  (power series expansion in the soft momentum $q = \omega \hat{q}$)

[Weinberg '65]
[Cachazo, Strominger '14]

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1} S_n^{(0)} + \omega^0 S_n^{(1)}\right] \mathcal{A}_n + \mathcal{O}(\omega)$$

leading   subleading

$$S_n^{(0)} = \frac{\kappa}{2}\sum_{i=1}^{n} \frac{p_i^\mu p_i^\nu \varepsilon_{\mu\nu}(\hat{q})}{p_i \cdot \hat{q}}$$



$$S_n^{(1)} = -\frac{i\kappa}{2}\sum_{i=1}^{n} \frac{p_i^\mu \,\varepsilon_{\mu\nu}(\hat{q})\, q_\lambda}{p_i \cdot q} \left(J_i^{\lambda\nu} + S_i^{\lambda\nu}\right)$$

$$\kappa = \sqrt{32\pi G}$$

Logarithmic soft theorems

- One-loop corrections generate logarithmic corrections!

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4} \ln\omega\, S_n^{(\ln)}\right] \mathcal{A}_n + \mathcal{O}(\omega^0)$$

dominate over the subleading term

[Laddha, Sen ’18 ‘19] [Sahoo, Sen ‘18] [Saha, Sahoo, Sen ‘19][Krishna, Sahoo ‘23]
[Ciafaloni, Colferai, Veneziano ‘18] [Addazi, Bianchi, Veneziano ’19]
[di Vecchia, Heissenberg, Russo, Veneziano ‘23][Alessio, di Vecchia ‘24]

Logarithmic soft theorems

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$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4} \ln\omega\, S_n^{(\ln)}\right] \mathcal{A}_n + \mathcal{O}(\omega^0)$$

$$\begin{aligned} S_n^{(\ln)} &= \frac{i\kappa}{8\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{p_i\cdot q} \sum_j \delta_{\eta,\eta_j}\, q\cdot p_j \\ &+ \frac{i\kappa}{16\pi} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{p_i\cdot q} \sum_j \delta_{\eta_i,\eta_j} (p_i\cdot p_j)\,(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho) \frac{2(p_i\cdot p_j)^2 - 3p_i^2 p_j^2}{\left[(p_i\cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}} \\ &- \frac{\kappa}{8\pi^2} \sum_i \frac{\varepsilon_{\mu\nu} p_i^\nu p_i^\nu}{p_i\cdot q} \sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j| \\ &- \frac{\kappa}{32\pi^2} \sum_i \frac{p_i^\mu \varepsilon_{\mu\nu} q_\lambda}{p_i\cdot q} \left(p_i^\lambda \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\lambda}}\right) \sum_j \frac{2(p_i\cdot p_j)^2 - p_i^2 p_j^2}{[(p_i\cdot p_j)^2 - p_i^2 p_j^2]^{1/2}} \ln\left(\frac{p_i\cdot p_j + \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}{p_i\cdot p_j - \sqrt{(p_i\cdot p_j)^2 - p_i^2 p_j^2}}\right) \end{aligned}$$

Logarithmic soft theorems

- One-loop corrections generate logarithmic corrections! [Sahoo, Sen '18][…]

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}\, S_n^{(0)} - \frac{\kappa^2}{4} \ln\omega\, S_n^{(\ln)}\right] \mathcal{A}_n + \mathcal{O}(\omega^0)$$

$$S_n^{(\ln)} = \frac{i\kappa}{8\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{p_i\cdot q}\sum_j \delta_{\eta,\eta_j}\, q\cdot p_j$$
$$+\frac{i\kappa}{16\pi}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu q_\rho}{p_i\cdot q}\sum_j \delta_{\eta_i,\eta_j}(p_i\cdot p_j)\,(p_i^\mu p_j^\rho - p_j^\mu p_i^\rho)\frac{2(p_i\cdot p_j)^2-3p_i^2p_j^2}{\left[(p_i\cdot p_j)^2-p_i^2p_j^2\right]^{3/2}}$$
$$-\frac{\kappa}{8\pi^2}\sum_i \frac{\varepsilon_{\mu\nu}p_i^\nu p_i^\nu}{p_i\cdot q}\sum_j q\cdot p_j \ln|\hat{q}\cdot\hat{p}_j|$$
$$-\frac{\kappa}{32\pi^2}\sum_i \frac{p_i^\mu\varepsilon_{\mu\nu}q_\lambda}{p_i\cdot q}\left(p_i^\lambda\frac{\partial}{\partial p_{i\nu}} - p_i^\nu\frac{\partial}{\partial p_{i\lambda}}\right)\sum_j \frac{2(p_i\cdot p_j)^2-p_i^2p_j^2}{[(p_i\cdot p_j)^2-p_i^2p_j^2]^{1/2}}\ln\left(\frac{p_i\cdot p_j+\sqrt{(p_i\cdot p_j)^2-p_i^2p_j^2}}{p_i\cdot p_j-\sqrt{(p_i\cdot p_j)^2-p_i^2p_j^2}}\right)$$

[Agrawal, LD, Nguyen, Ruzziconi '23]

Can be exactly **derived** from superrotation **symmetry** conservation!

Key ingredient: **Goldstone modes** and **dressing** at timelike infinity

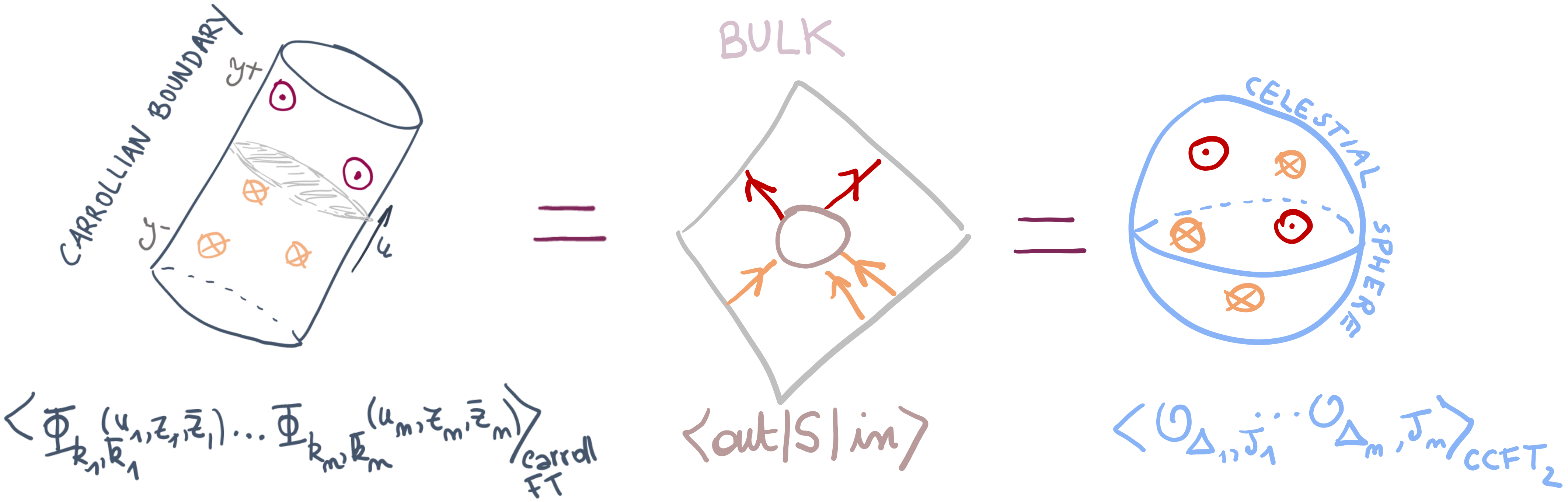
Summary and outlook

Summary and outlook

Celestial CFT living on the celestial sphere

$\updownarrow$ $\longleftrightarrow$ quantum gravity in flat spacetime

Conformal Carrollian field theory living at null infinity



Summary and outlook

Celestial **CFT** living on the celestial sphere

↕ ⟷ quantum gravity in flat spacetime

Conformal Carrollian field theory living at null infinity

What is a **CCFT**?

→ **B**eyond kinematics? **T**op-down constructions?

full tower of currents
link with AdS/CFT, dS/CFT
building representations
log corrections
bootstrapping CCFT
higher dimensions
massive particles
relationship to string theory
adding black holes
…

BMS symmetries in the sky

LD, Boris Goncharov & Jan Harms [*Phys. Rev. Lett.* 2024]

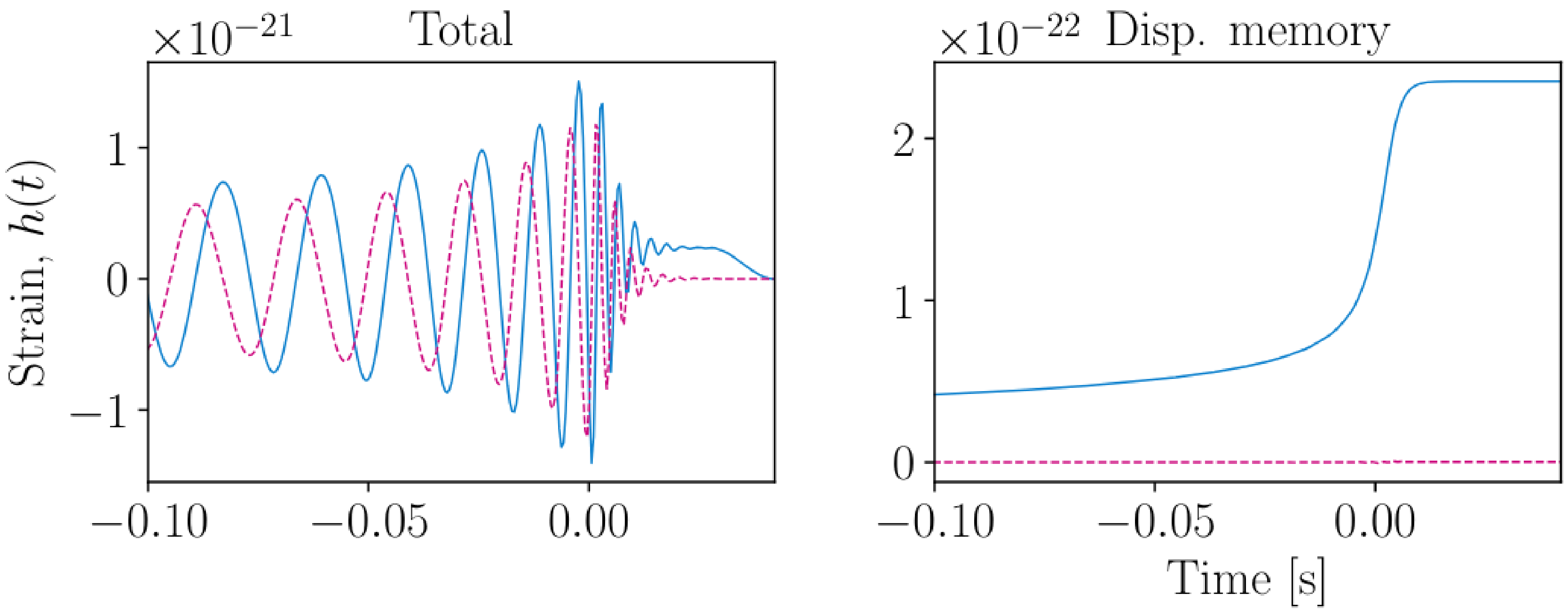
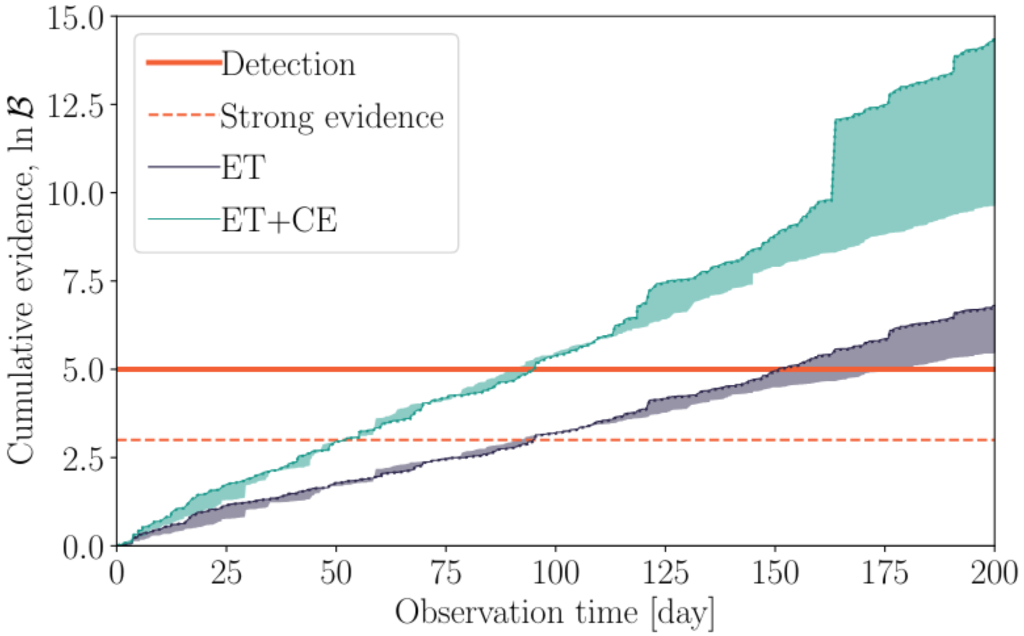


FIG. 4: Demonstration of the GW memory contribution to strain from a merger of two non-spinning BBHs in the extended BMS scenario, $(m_1, m_2, \theta_{jn}, z) = (30\ M_\odot, 30\ M_\odot, \pi/3, 0.06)$. Solid lines show h_+, dashed lines show $h_\times$.

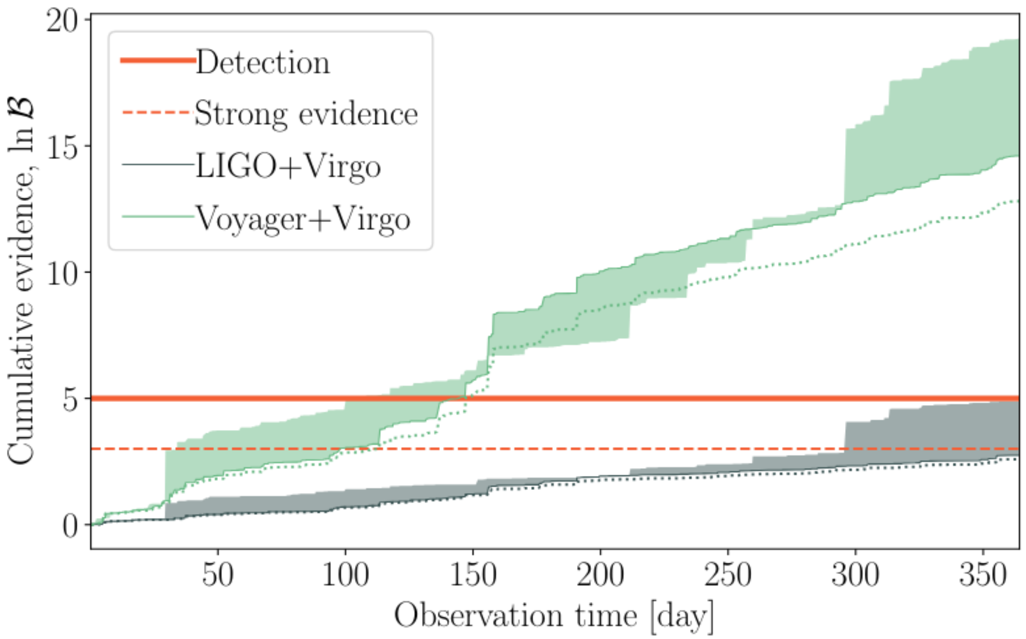
BMS symmetries in the sky

LD, Boris Goncharov & Jan Harms [*Phys. Rev. Lett.* 2024]

Model selection between standard and extended BMS symmetries.



Einstein Telescope (ET) and Cosmic Explorer (CE)



LIGO and VIRGO

Summary and outlook

amplitudes

gravitational waves observation

conformal field theory

twistor theory

asymptotic symmetries

quantum field theory

hydrodynamics

mathematical GR

Summary and outlook

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Thank you!