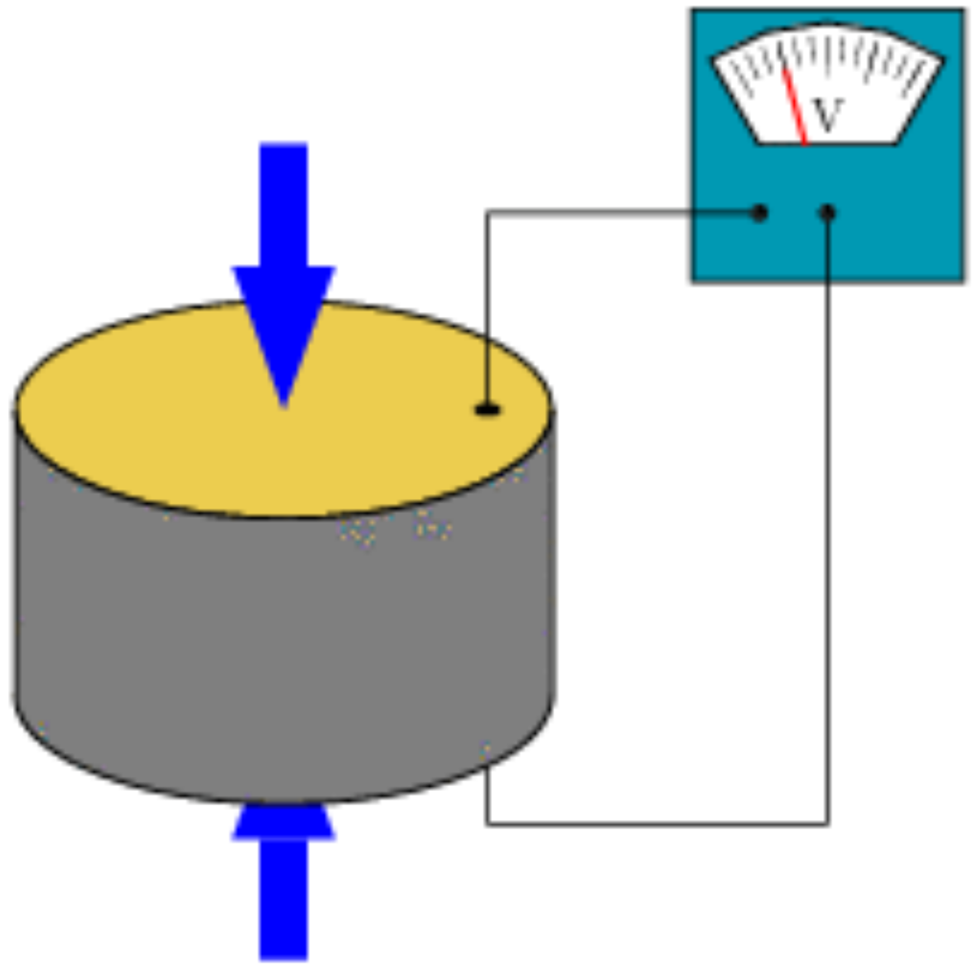
The Piezoaxionic Effect: Dark Matter Detection & New Forces



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Outline

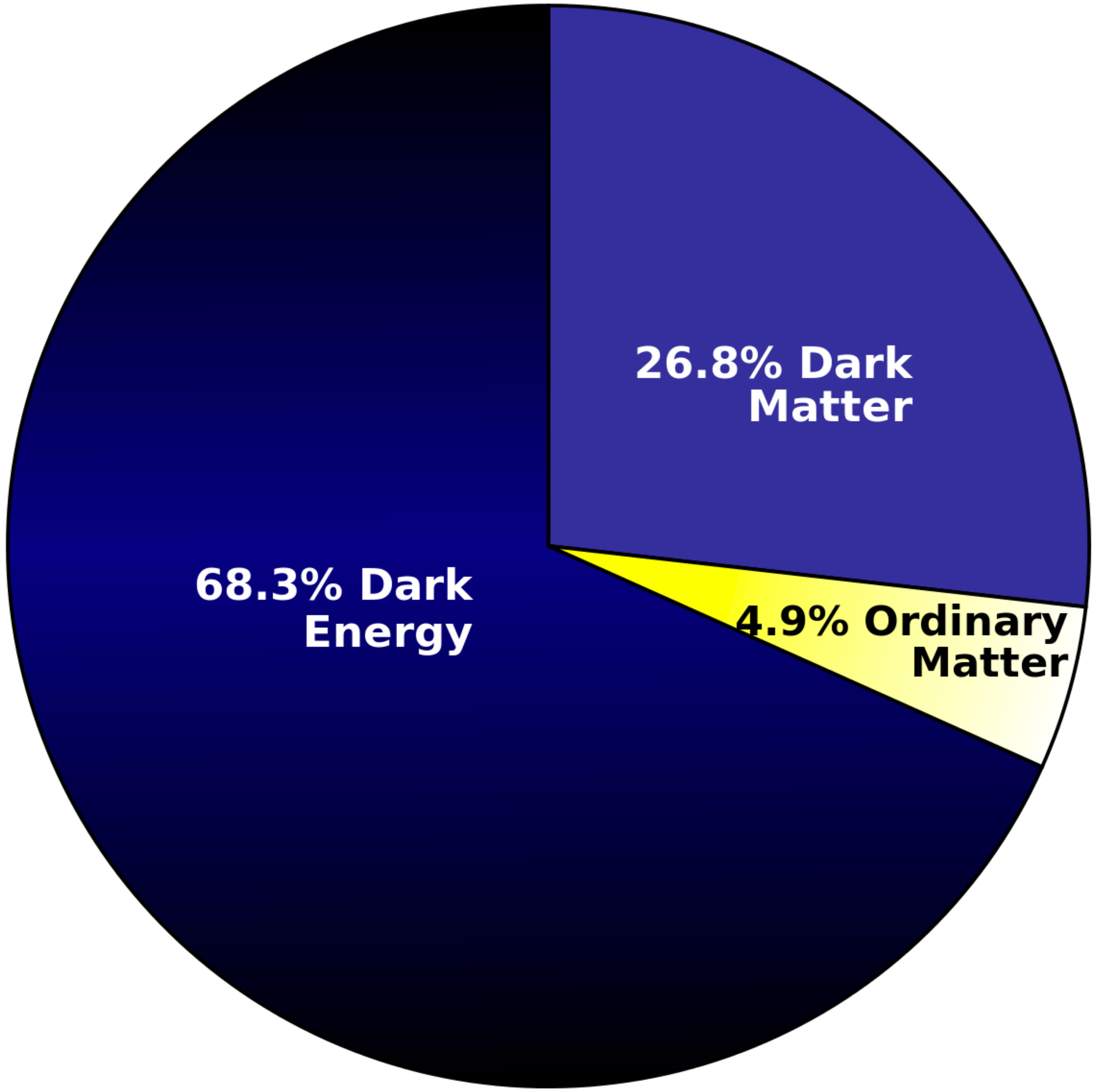
1. **Axions and the strong CP problem**

2. **The Piezoaxionic dark matter effect**
(w/ A. Arvanitaki and K. Van Tilburg)

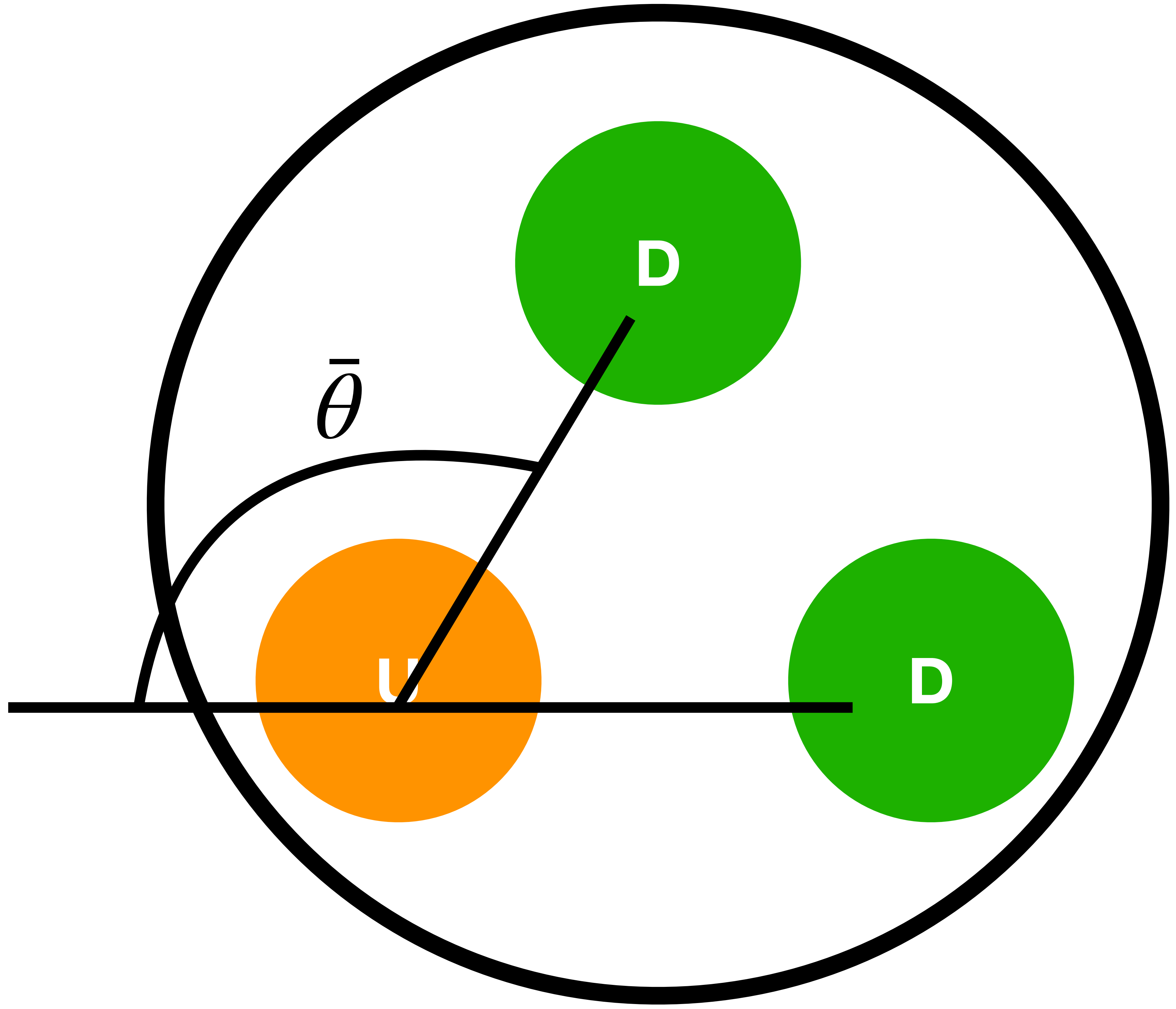
3**. The Piezoaxionic force**
(w/ A. Arvanitaki, J. Engel, A. Geraci, D. Stilwell and K. Van Tilburg)

Dark Matter

- What is the DM made of?
- What problems does it solve in the Standard Model?
- How is it produced?
- How do we detect it?



The Strong CP Problem



$$r_{neutron} \simeq 10^{-15}m$$

$$|d_{n,SM}| \simeq 10^{-18}\,\bar{\theta} \cdot\; e \cdot m$$

Experimentally, $|d_n| \lesssim 10^{-28} \cdot e \cdot \mathrm{m}$

$$|\bar{\theta}| < 10^{-10}$$

The Strong CP Problem

$d = 0$ means that both P and T symmetries are preserved

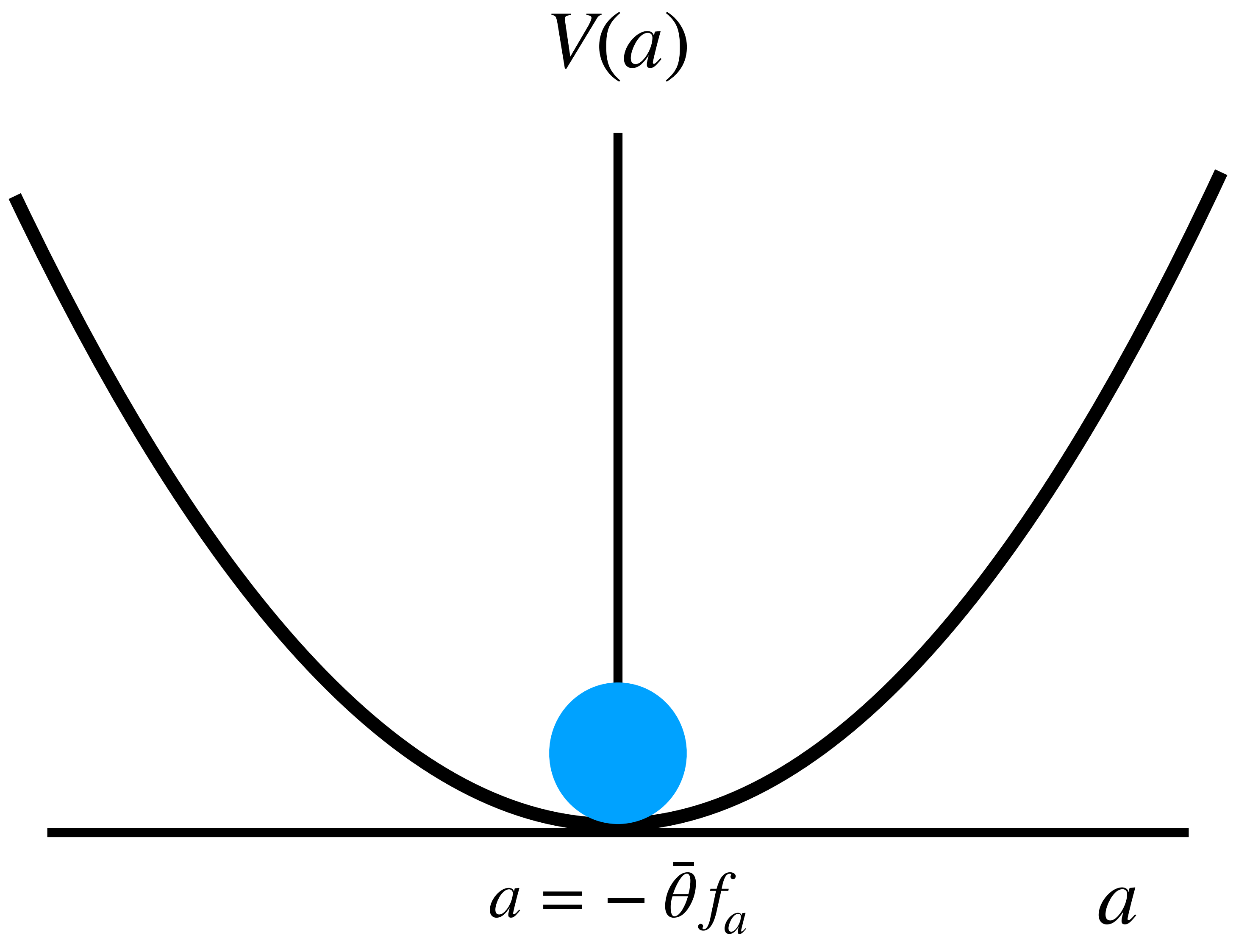
The electroweak sector violates P and T, so why not QCD?

QCD Axion

- $\mathscr{L} \supset \frac{\theta_0}{32\pi^2} G\tilde{G}$
- $\bar{\theta} = \theta_0 +$ arg det$[M_q]$
- Introduce the axion field $a(x)$ coupled to QCD:

  $$\mathscr{L} \supset \frac{a(x)}{32\pi^2 f_a} \tilde{G}G$$

  (f_a = axion decay constant)



- Minimum of axion potential *dynamically* solves strong CP problem:

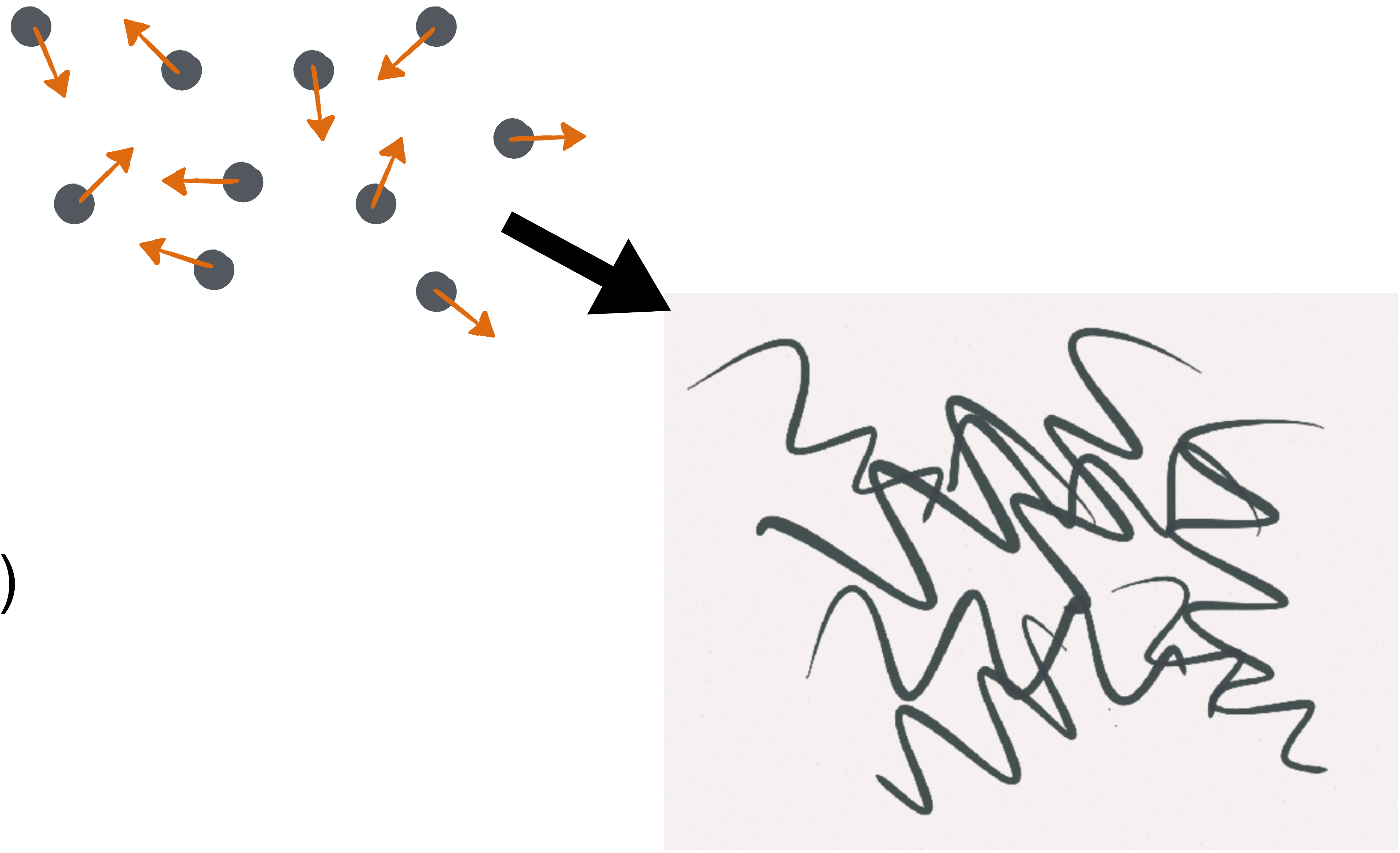
  $$\theta_{eff} = \langle \frac{a(x)}{f_a} \rangle + \bar{\theta} = 0$$

Wavy Dark Matter

Bosonic DM has wave-like properties when $n_{DM} > \frac{1}{\lambda_{DM}^3}$. In our galaxy: $m_{DM} < 1eV$.

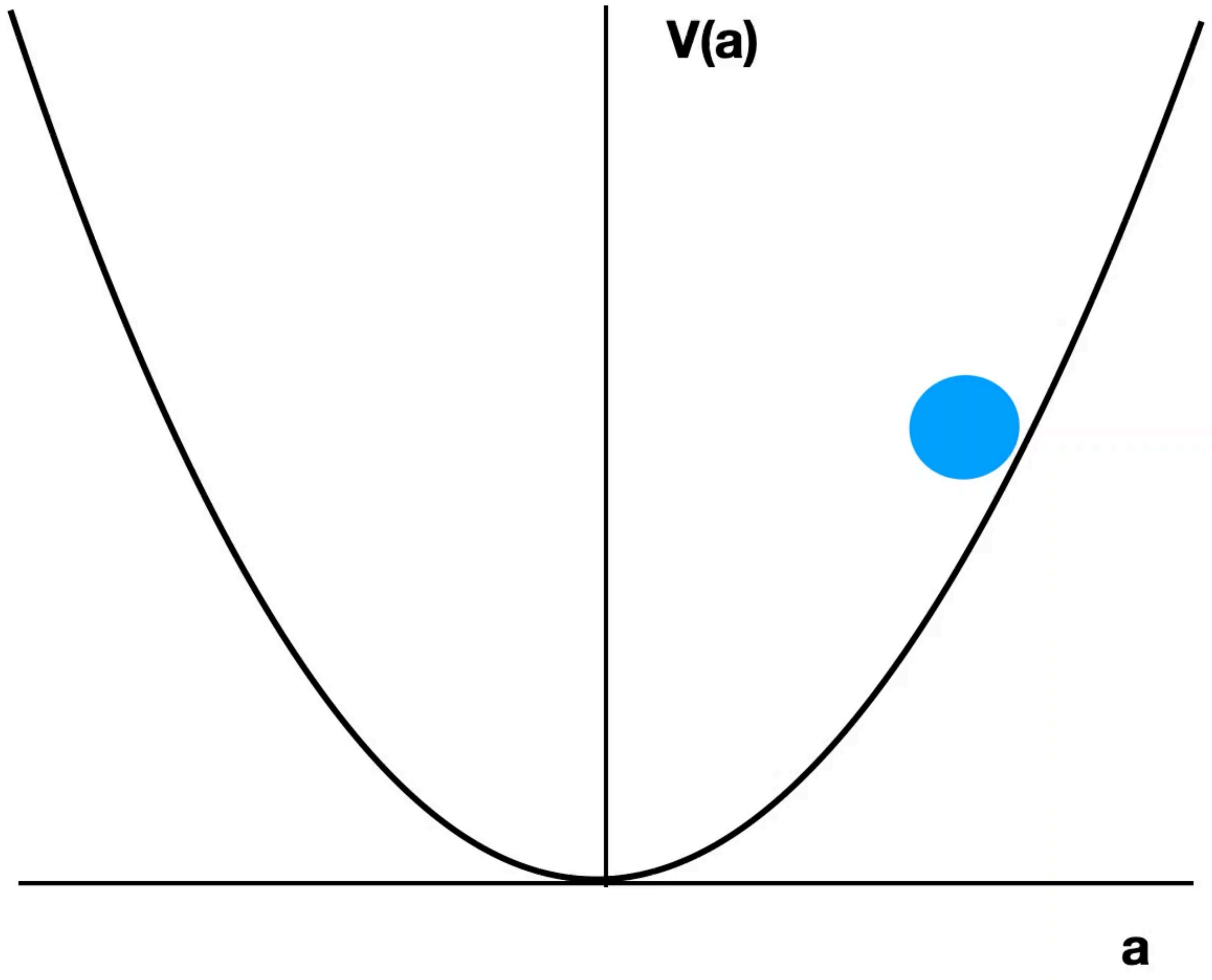
- Locally, $a(t) \approx a_0 \cos \frac{m_a c^2}{\hbar} t$
- Amplitude $a_0 \propto \frac{\sqrt{\rho_{DM}}}{m_a}$
- Small frequency spread (coherence)

$$\delta\omega_a \approx \frac{v^2}{\hbar} \omega_a \approx 10^{-6} \omega_a$$



Misalignment Mechanism

(Other production mechanisms possible)



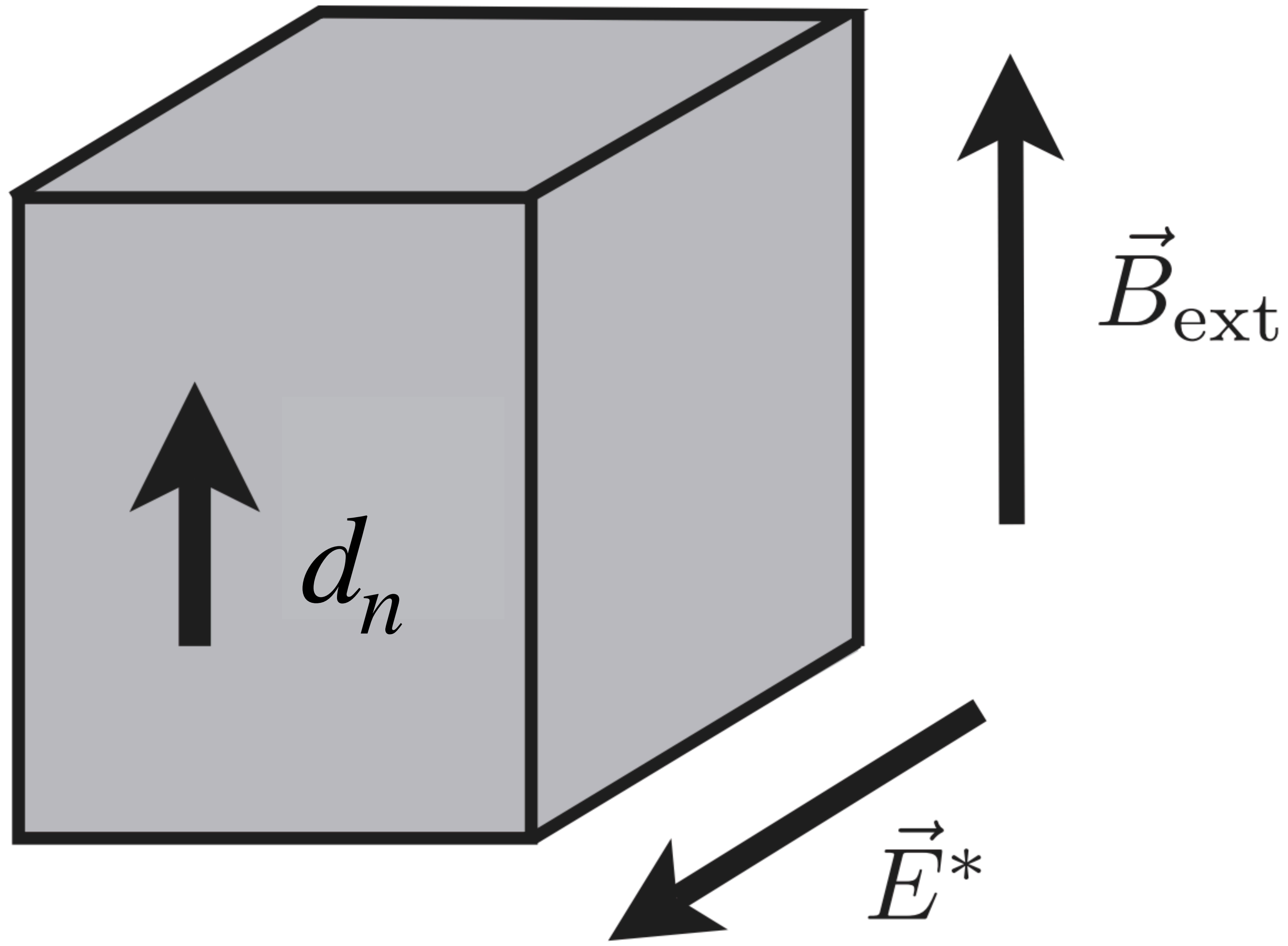
- $\ddot{a} + 3H(T)\dot{a} + m(T)^2 a = 0$
  (H = Hubble parameter)

- $m < 3H$: frozen

- $m > 3H$: oscillates around minimum

- $\frac{\rho_a}{\rho_{total}} = 0.25 < \theta^2_{initial} > \left(\frac{f_a}{5 \times 10^{12} \text{GeV}} \right)^{7/6}$
  and scales as a^{-3}

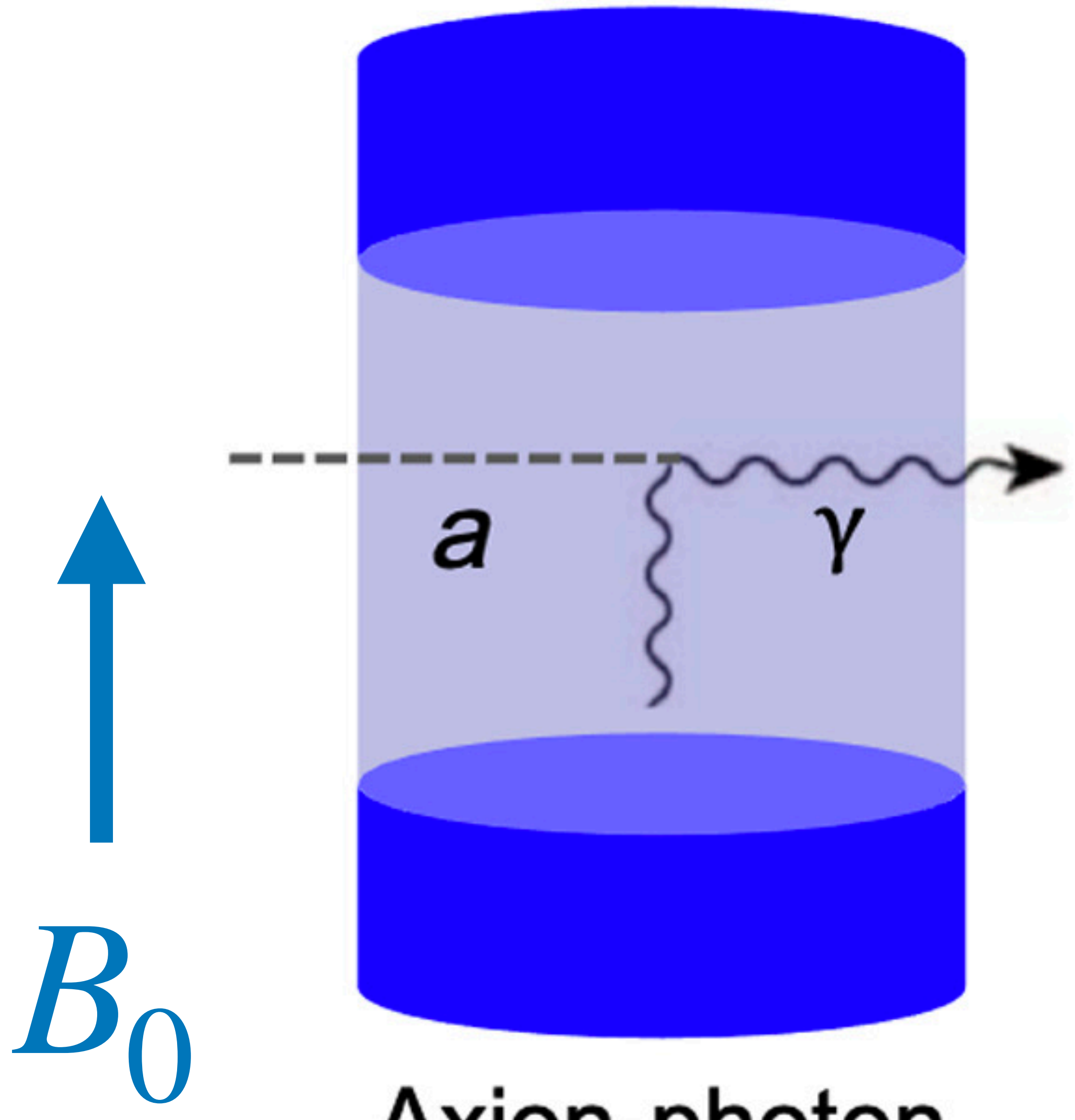
Gluon Couplings: $\mathscr{L} \supset \frac{\alpha_s^2}{32\pi^2}\frac{a}{f_a}\mathrm{G}\tilde{\mathrm{G}}$



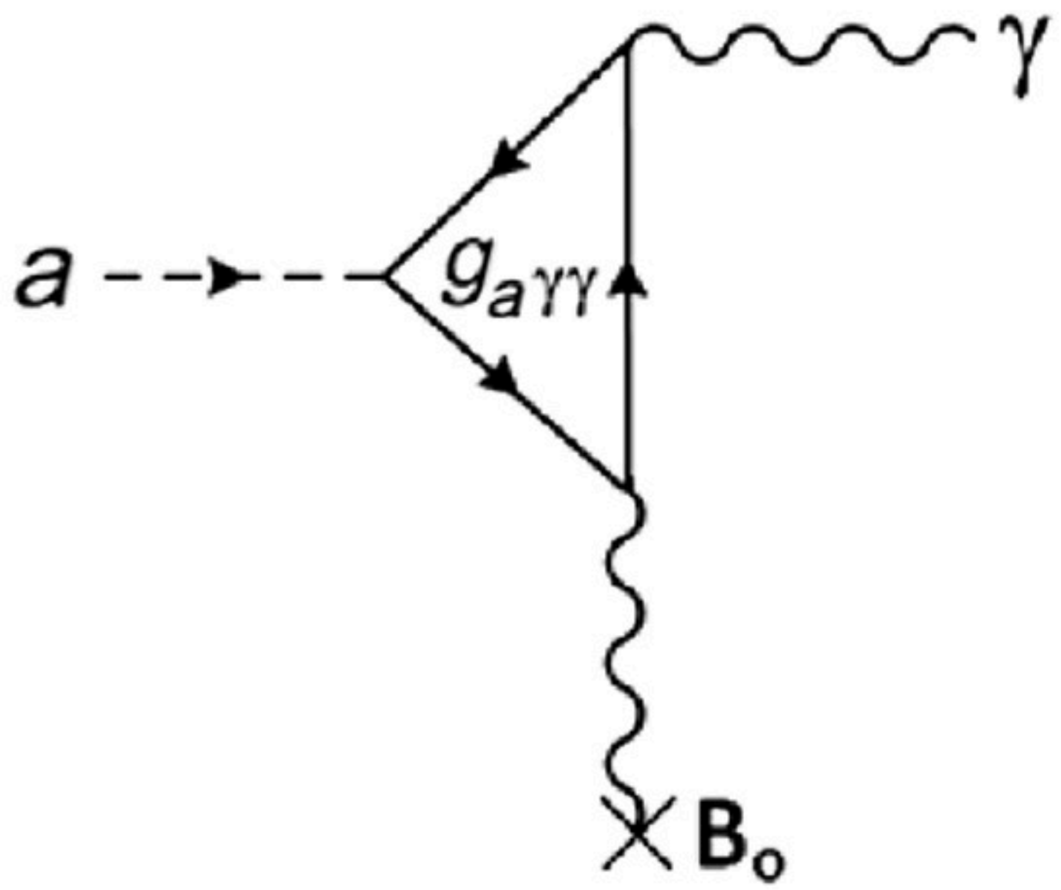
$$\tau = d_n(t) \times E^*$$

CASPER-electric

Photon Couplings: $\mathscr{L} \supset -\frac{g_{a\gamma\gamma}}{4} a F\tilde{F}$



Axion-photon interaction



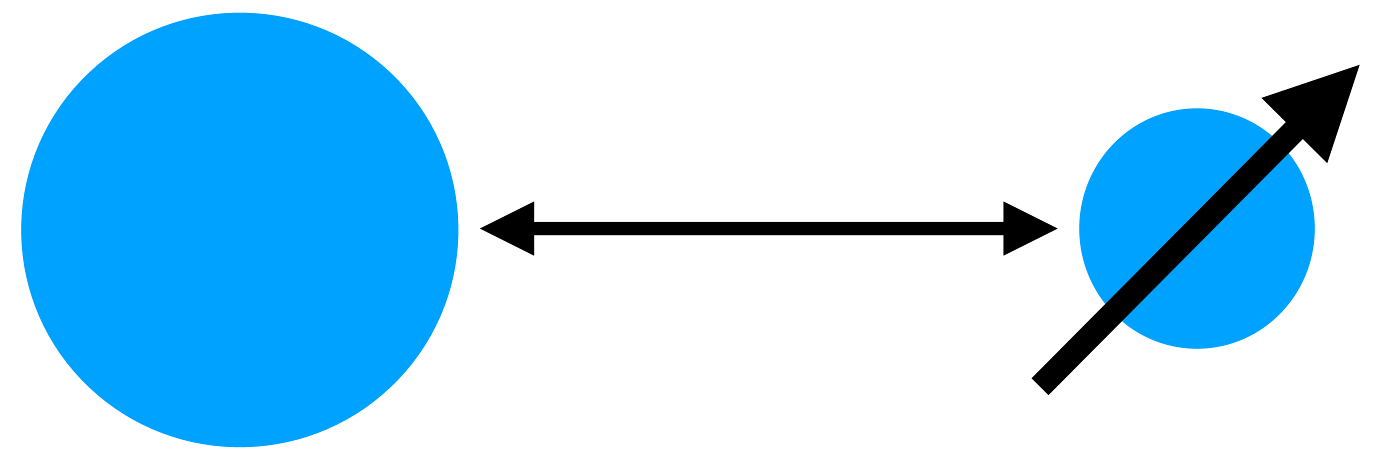
<u>ADMX</u>, DM Radio, CAST, IAXO (solar), ALPS…

Image: Semertzidis et al. 2022

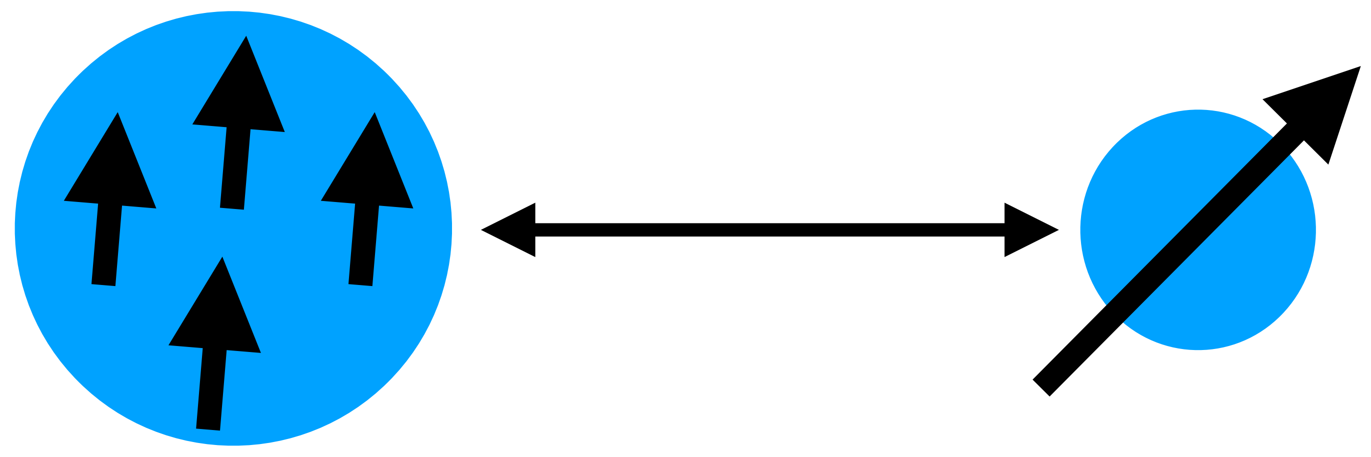
Fermion Couplings:

$$-g_s a\bar{\psi}\psi + \frac{g_p}{2m_\psi}\partial_\mu a\bar{\psi}\gamma^\mu\gamma_5\psi \xrightarrow{\text{NR limit}} \frac{g_p}{2m_\psi}\sigma\cdot\left[\nabla a + \dot{a}\frac{\mathbf{p}_\psi}{m_\psi}\right]$$

monopole-dipole forces

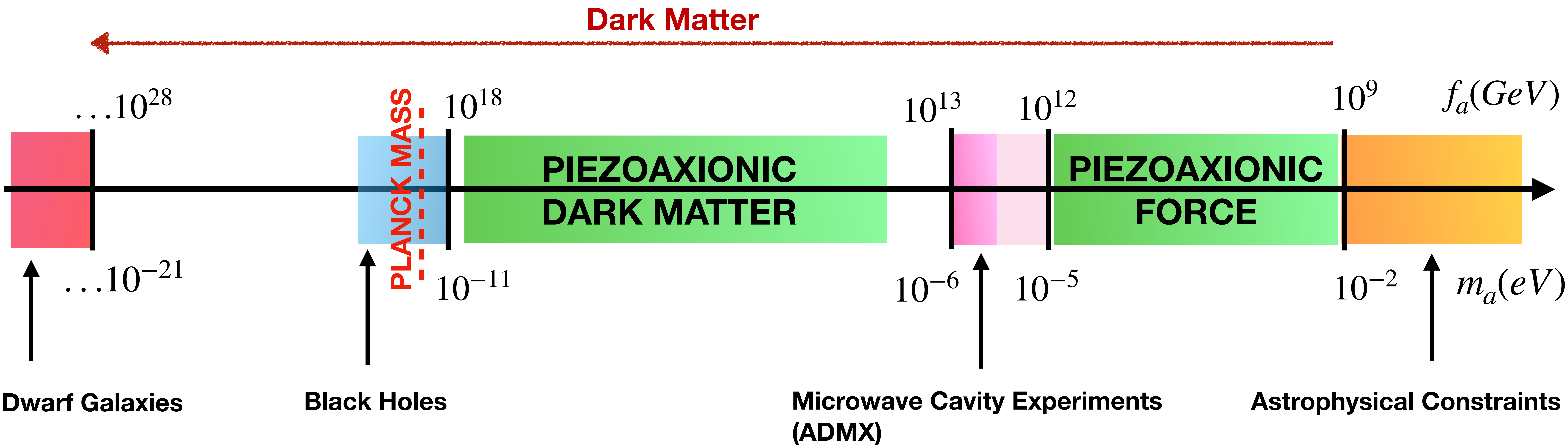


dipole-dipole forces



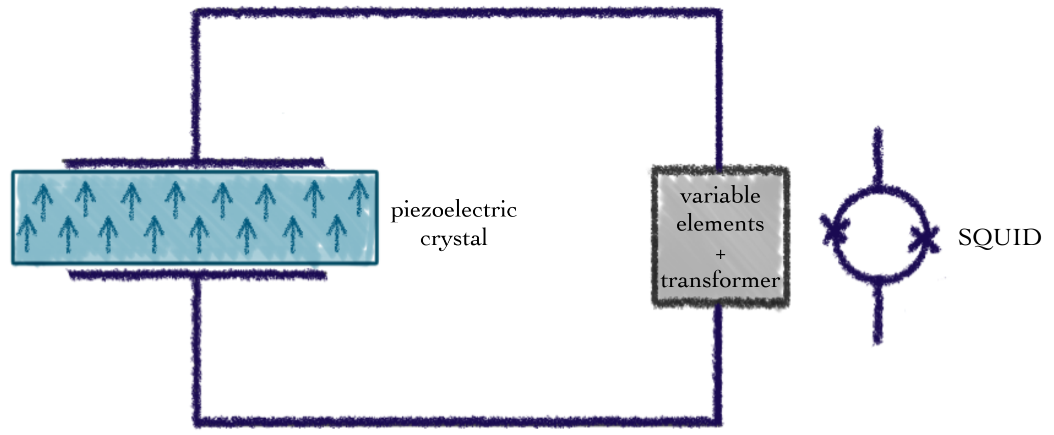
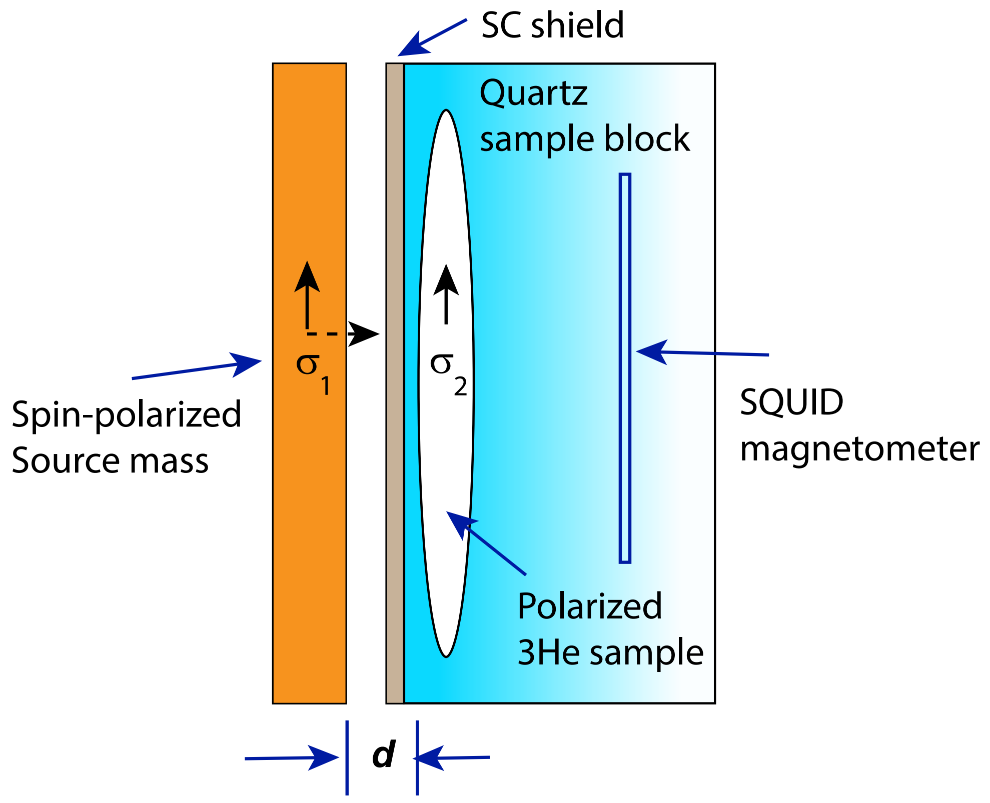
ARIADNE, CASPEr-wind, QUAX

Where is the QCD axion?

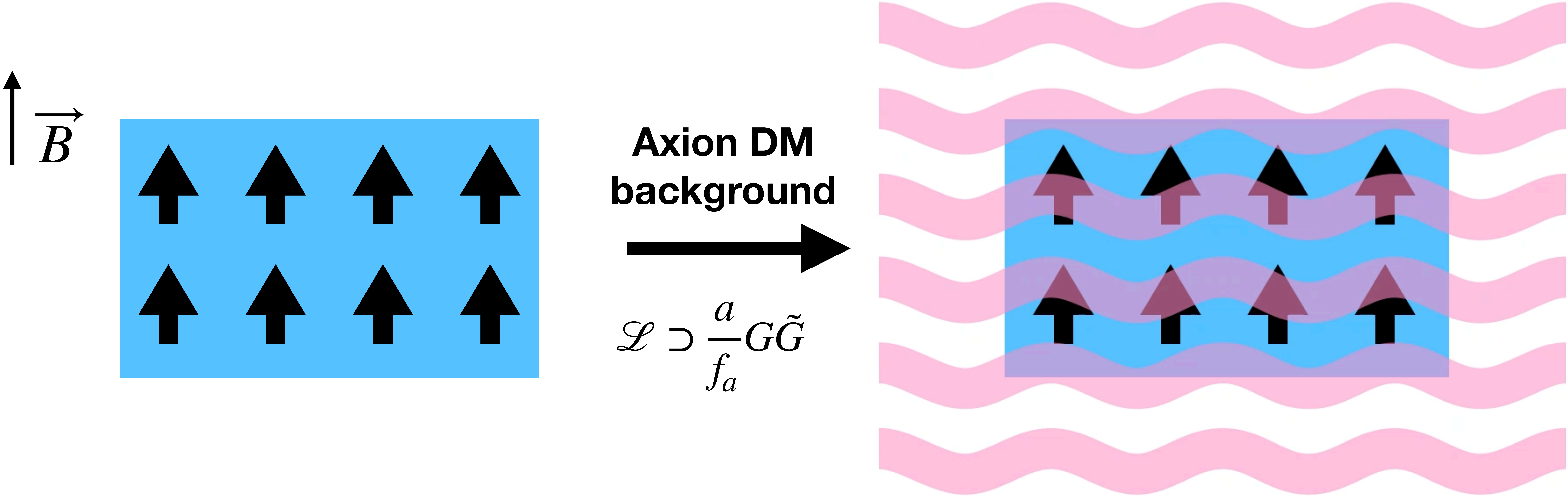


$$m_a \sim 6 \times 10^{-11} eV \left(\frac{10^{17} GeV}{f_a} \right)$$

The Piezoaxionic Effect

	**Gluon Coupling**	$10^{-11}eV$ to $10^{-7}eV$	**Must be DM**
	**Gluon Coupling** **Fermion Coupling**	$10^{-5}eV$ to $10^{-2}\,eV$	**Doesn't need to be DM**

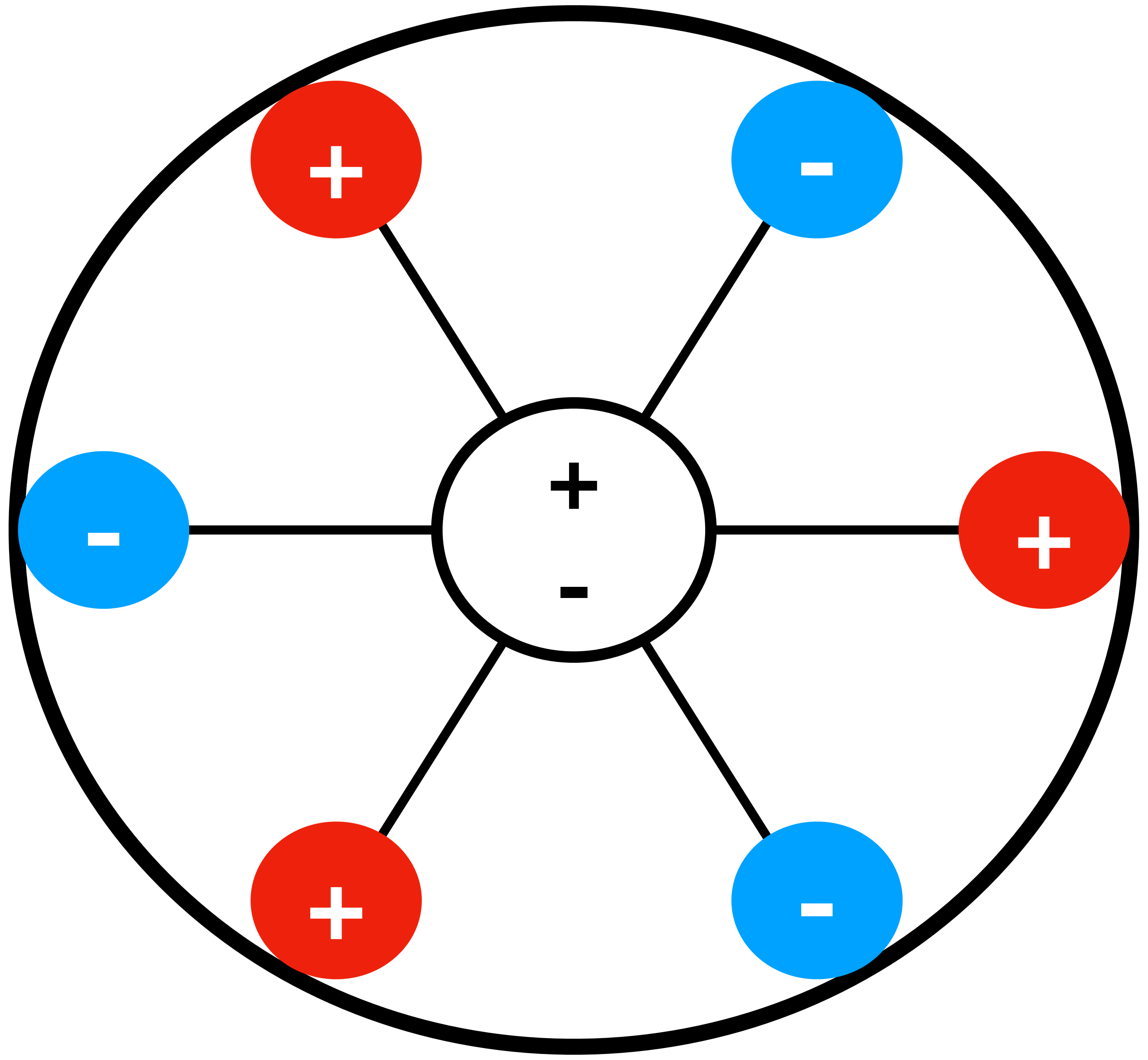
The Piezoaxionic Effect:
Dark matter detection



Axion DM background

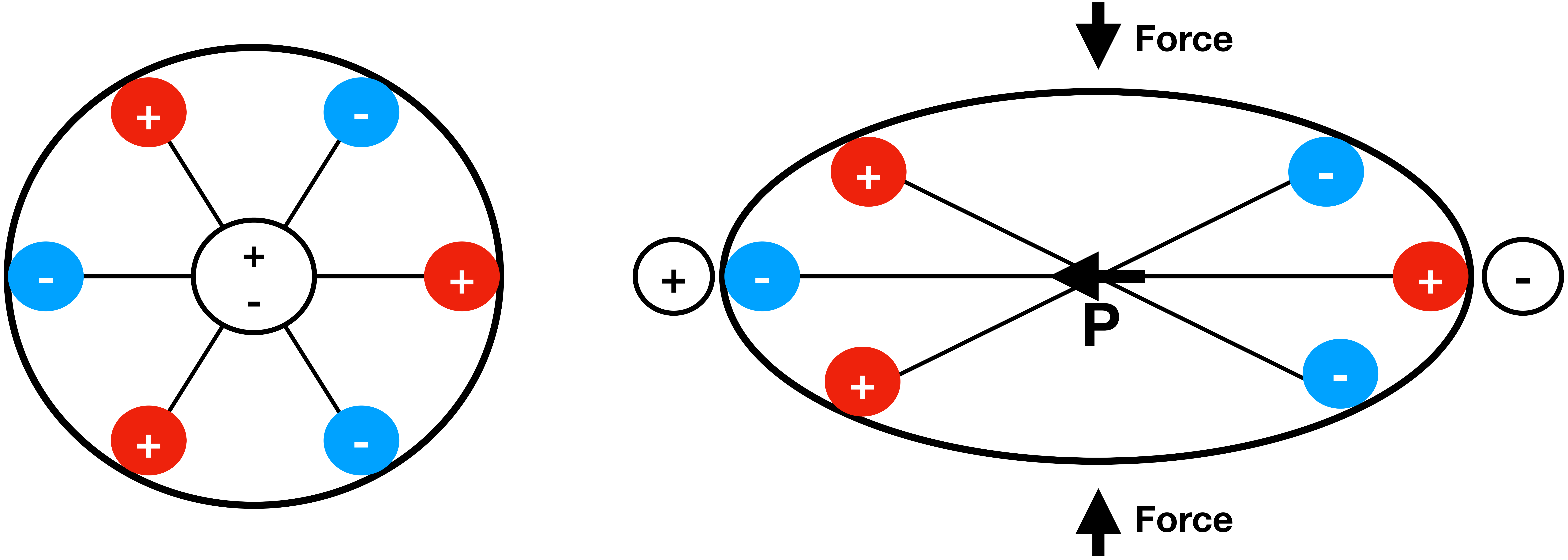
$$\mathscr{L} \supset \frac{a}{f_a} G\tilde{G}$$

Piezoelectric Crystals



- Crystal structure breaks parity symmetry $(x, y, z) \neq (-x, -y, -z)$

Piezoelectric Crystals



- Crystal structure breaks parity symmetry $(x, y, z) \neq (-x, -y, -z)$

- Deformation causes electric dipole moment across unit cell (and vice versa).

Constitutive Equations for Piezoelectricity

$$\theta_a(t) \equiv \frac{a(t)}{f_a}$$

$$\underline{\text{Stress}} = +\, \underline{\overset{\substack{\text{Stiffness}\\ \downarrow}}{c}} \cdot \underline{\text{Strain}} - \underline{\overset{\substack{\text{Piezoelectric}\\ \downarrow}}{h}} \cdot \underline{\text{Electric Displacement}} - \underline{\overset{\substack{\text{Piezoaxionic}\\ \downarrow}}{\xi}}\, \underline{\underline{\theta_a(t)}} \cdot \underline{\text{Nuclear Spin Direction}}$$

$$\underline{\text{Electric Field}} = -\, \underline{h} \cdot \underline{\text{Strain}} + \frac{1}{\underset{\substack{\uparrow\\ \text{Permittivity}}}{\underline{\epsilon}}} \cdot \underline{\text{Electric Displacement}} - \underset{\substack{\uparrow\\ \text{Electroaxionic}}}{\underline{\zeta}}\, \underline{\underline{\theta_a(t)}} \cdot \underline{\text{Nuclear Spin Direction}}$$

parity even
parity odd
time-reversal odd

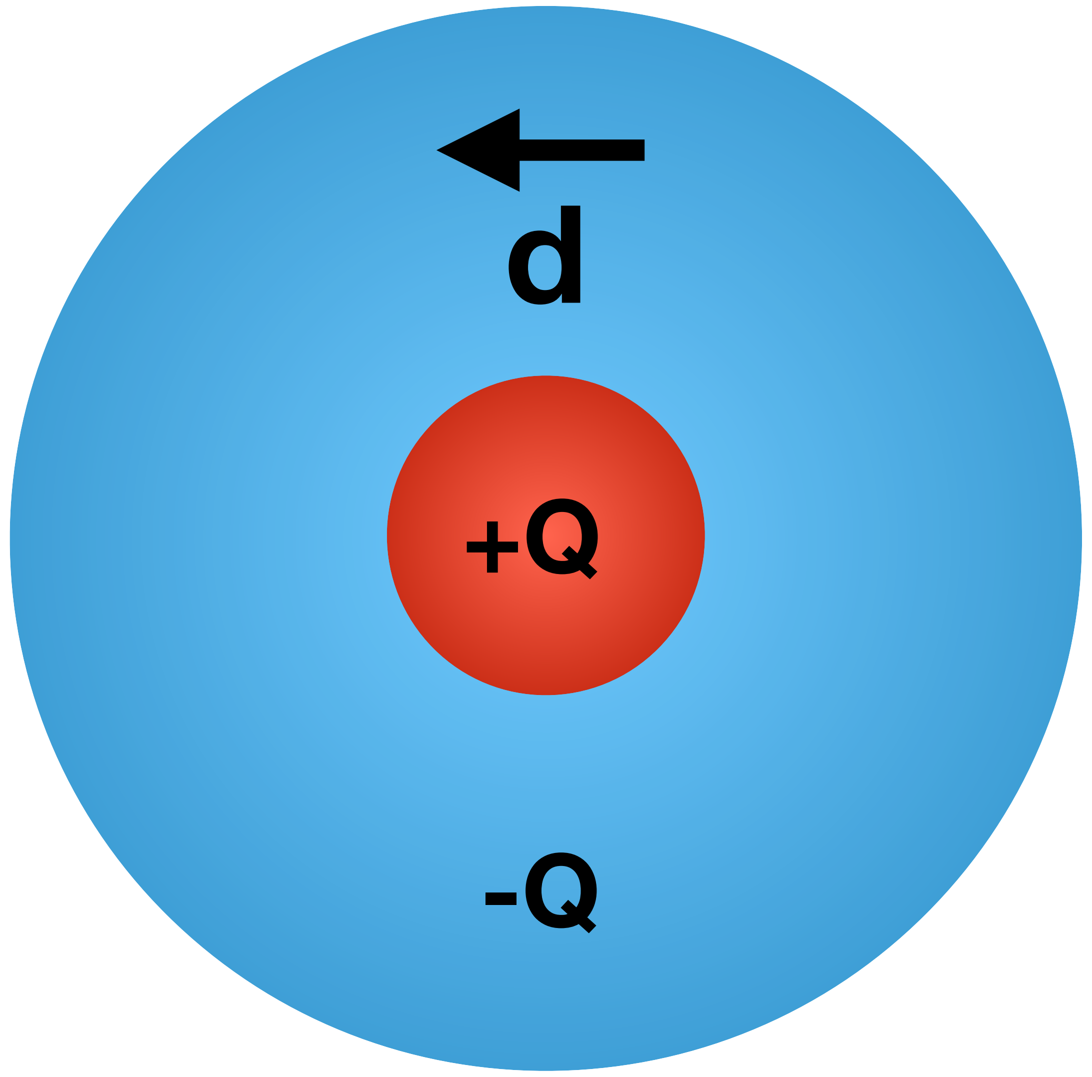
The piezoaxionic tensor ξ is **ODD** under parity, and can only be present in piezoelectric materials.

The electroaxionic tensor ζ is **EVEN** under parity, and can be present in all dielectrics.

We will focus on ξ in this talk!

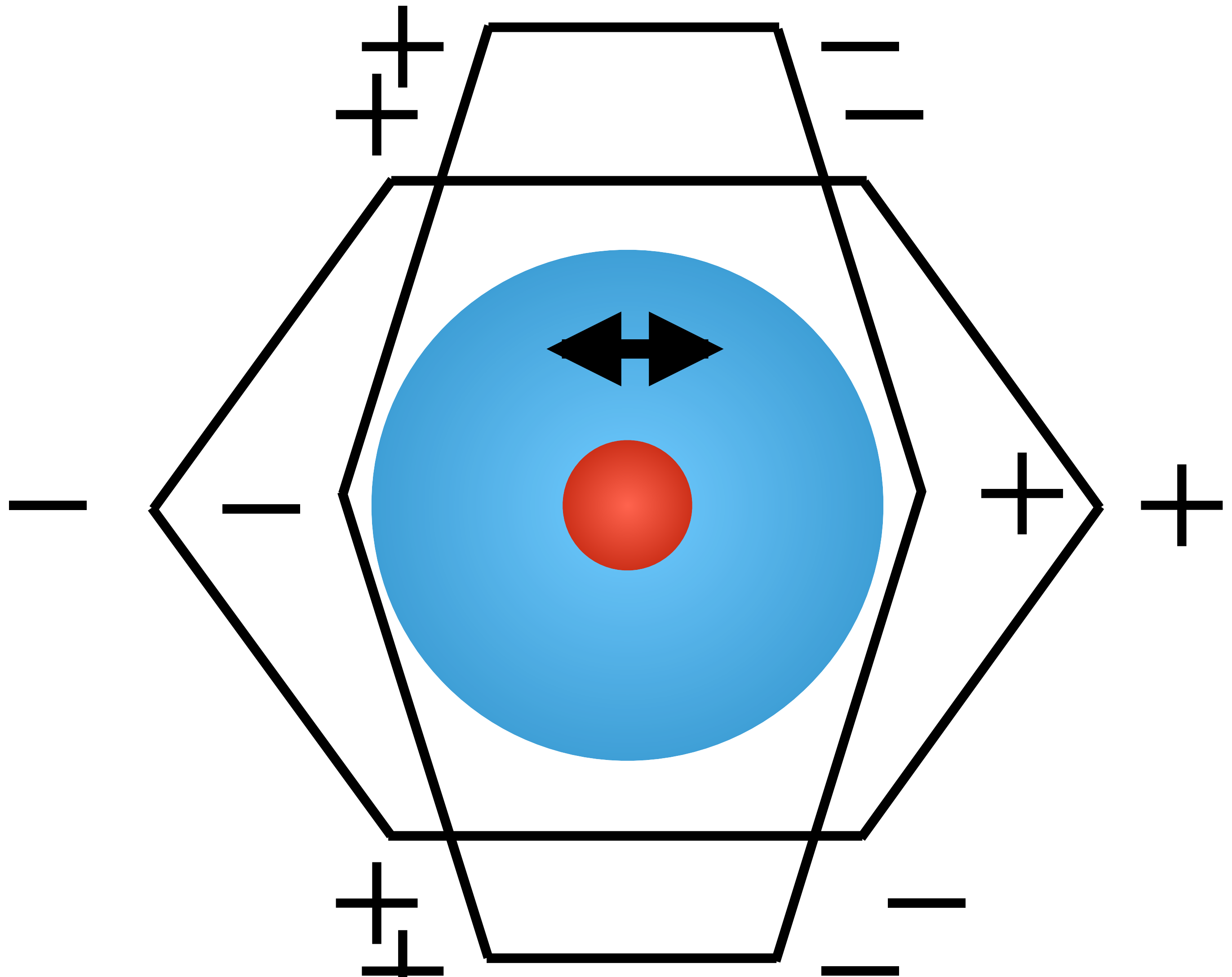
How big is the piezoaxionic tensor?

Step 1. QCD axion dark matter induces an ***oscillating*** nuclear electric dipole moment (EDM):



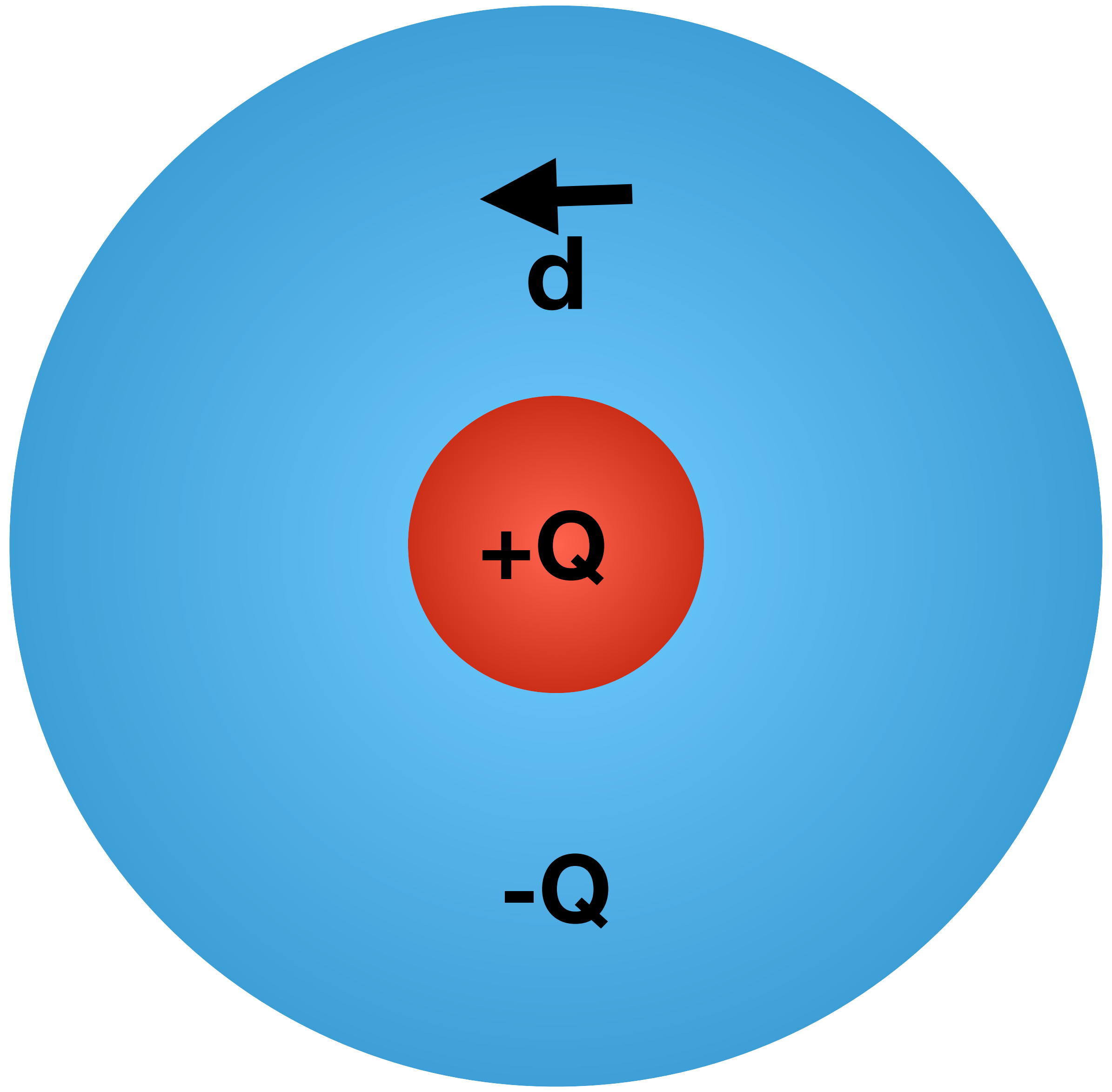
$$d_n \sim 10^{-16} \frac{\sqrt{\rho_{DM}}}{m_a f_a} \cos m_a t \cdot \text{e} \cdot \text{cm}$$

Step 2. EDM generates an oscillating stress on unit cell:



The caveat: Schiff's theorem

If we treat an atom as a system of **static, point-like** particles, nuclear EDM is perfectly shielded by electron cloud [*Schiff* 1963].



Resolution: finite size effects

Schiff Moment

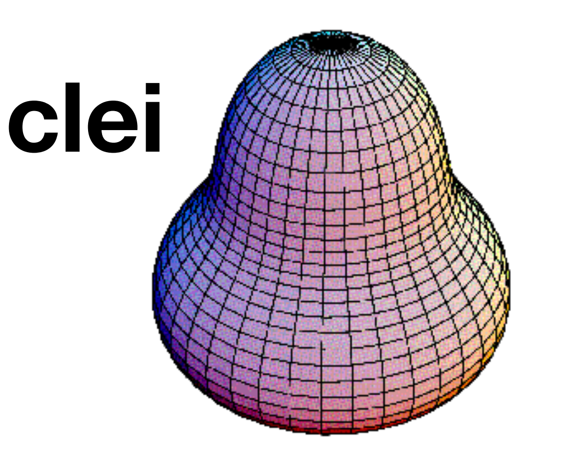
$$V_e = 4\pi e\, \mathsf{S} \cdot \nabla(\delta_e(\mathbf{r}))$$

$$\mathsf{S} \sim e\frac{\bar{\theta}_a}{m_N}R_0^2 \propto A^{2/3}$$

non-deformed nuclei

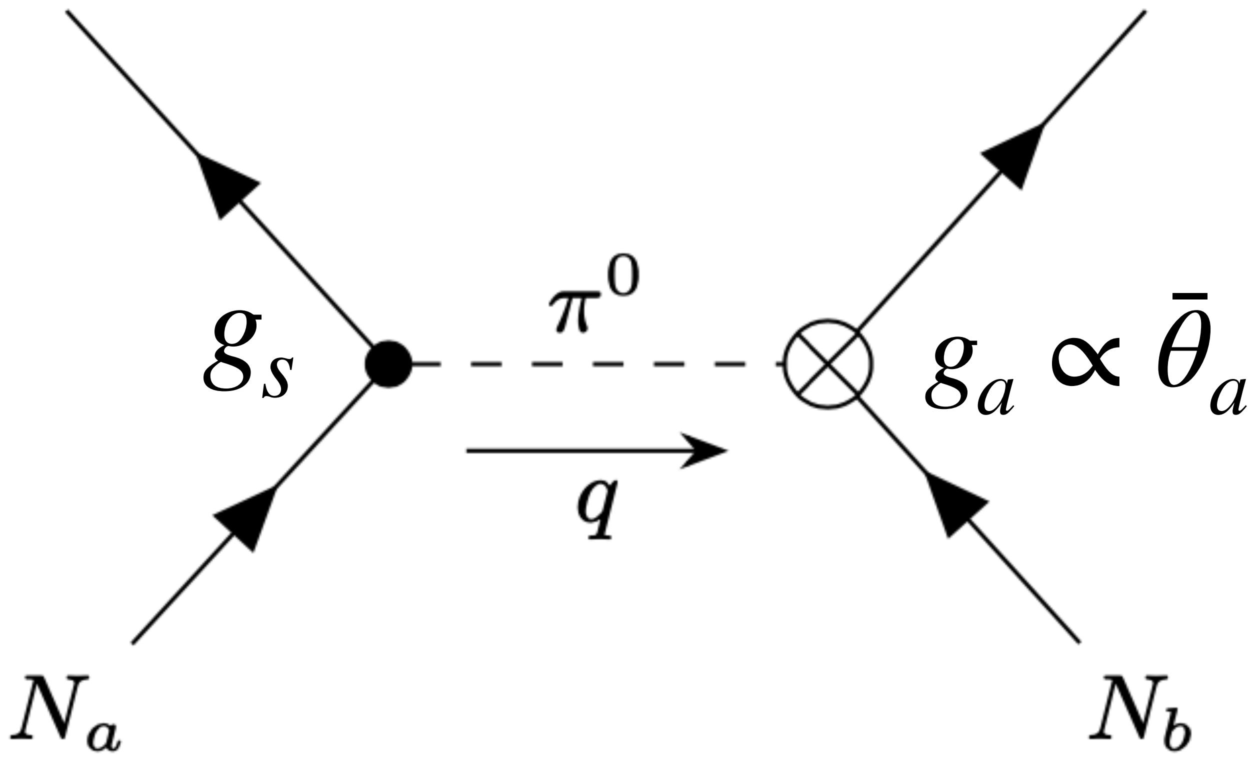
$$\mathsf{S} \sim eZ\frac{\bar{\theta}_a}{m_N}R_0^2 \propto Z\, A^{2/3}$$

pear shaped nuclei



$$\sim (0.01 - 1) \times \bar{\theta}_a\, e\, fm^3$$

Pion-nucleon forces:



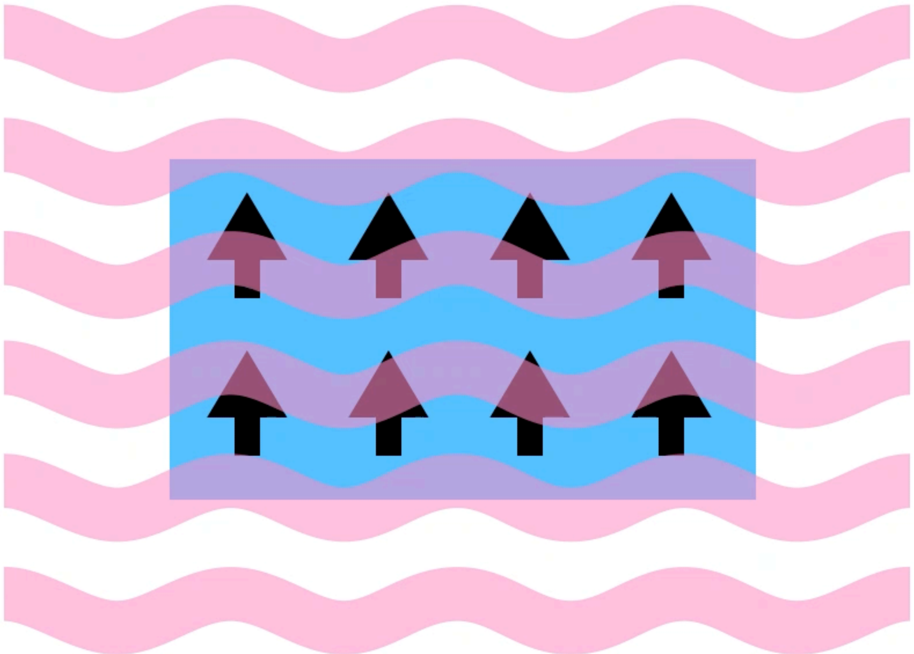
- In a piezoelectric crystal, the ground state electron wave function is a mixture of opposite parity orbitals ϵ_s and ϵ_p:

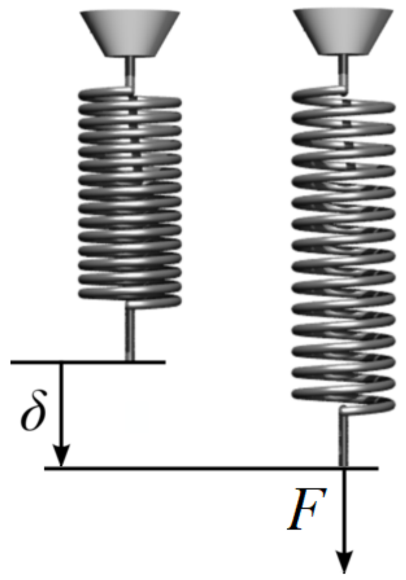
$$|\psi\rangle_e = \epsilon_s\,|\,s\rangle + \epsilon_p|p\rangle$$

- The piezoaxionic tensor can be estimated as:

$$\xi \sim \partial_{Strain}\frac{\langle H_{Schiff}\rangle}{V_{cell}} \simeq \frac{Z^2}{a_0^4}\frac{d\mathsf{S}}{d\theta_a} \times \frac{N_{\mathsf{S}}}{V_{cell}}\boxed{\frac{\partial(\epsilon_s\epsilon_p^*)}{\partial_{Strain}}}$$

$\sim \mathcal{O}(1)$ factor
Bigger in strongly piezoelectric materials





elastic stiffness tensor

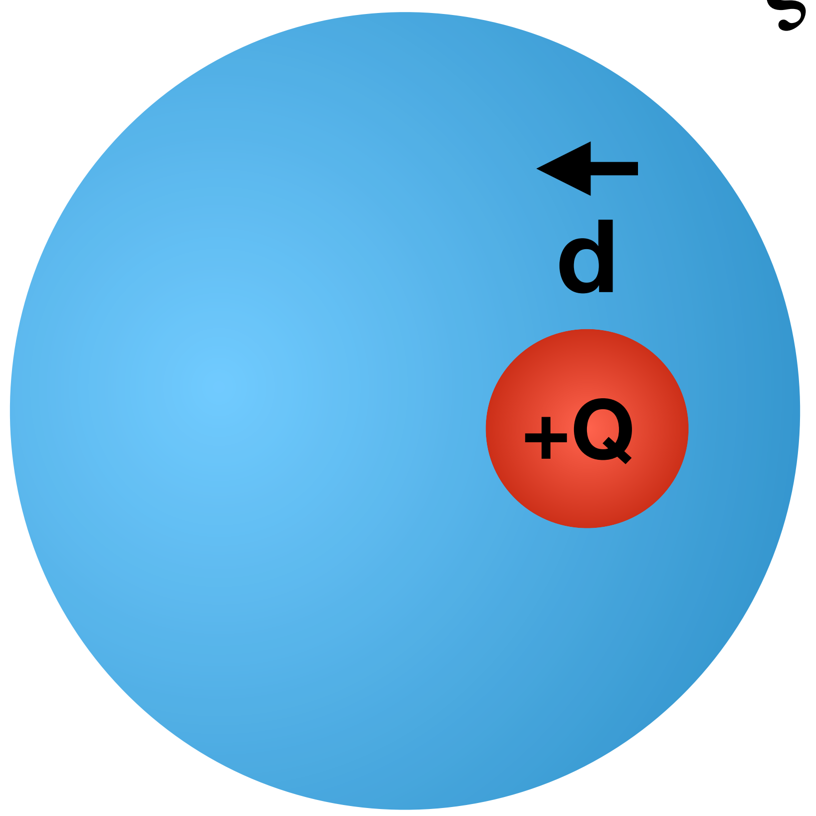
strain = $\frac{\Delta L}{L}$

Axion theta angle $\propto \frac{\sqrt{\rho_a}}{m_a f_a}$

$$S = |\xi c^{-1} \hat{I} \bar{\theta}_a| \sim 10^{-26}$$

ξ = **Piezoaxionic tensor**

$\hat{I}$ = **nuclear spin direction**



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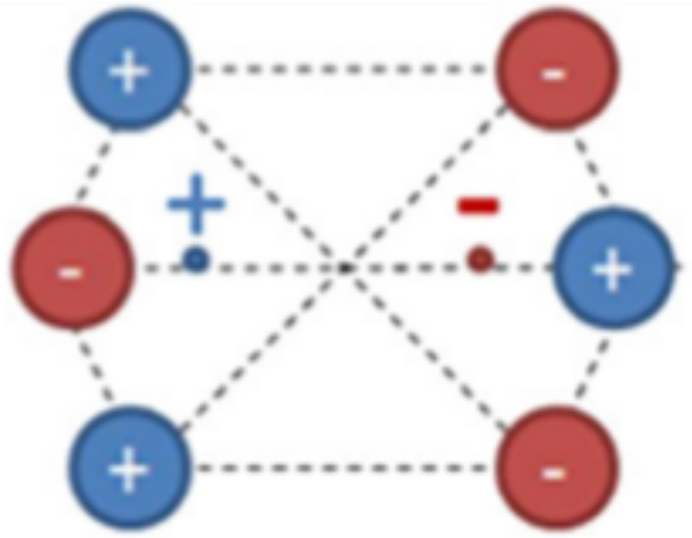
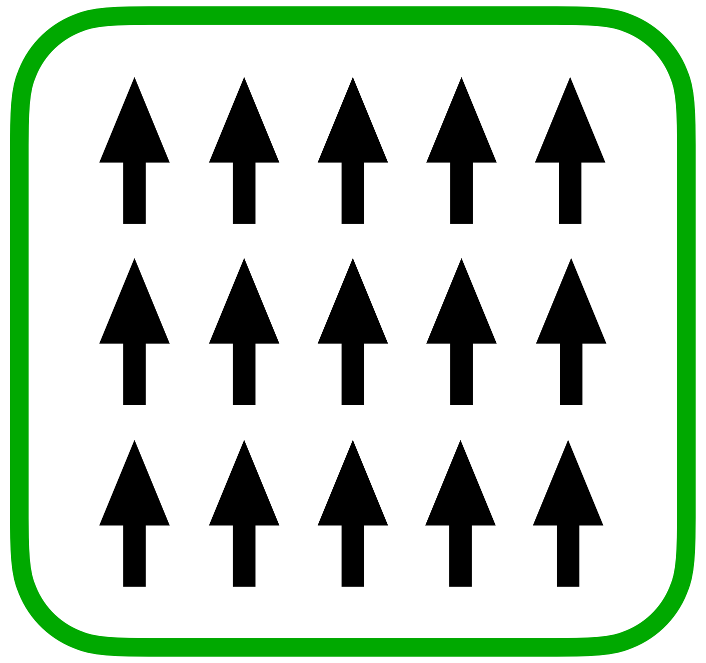
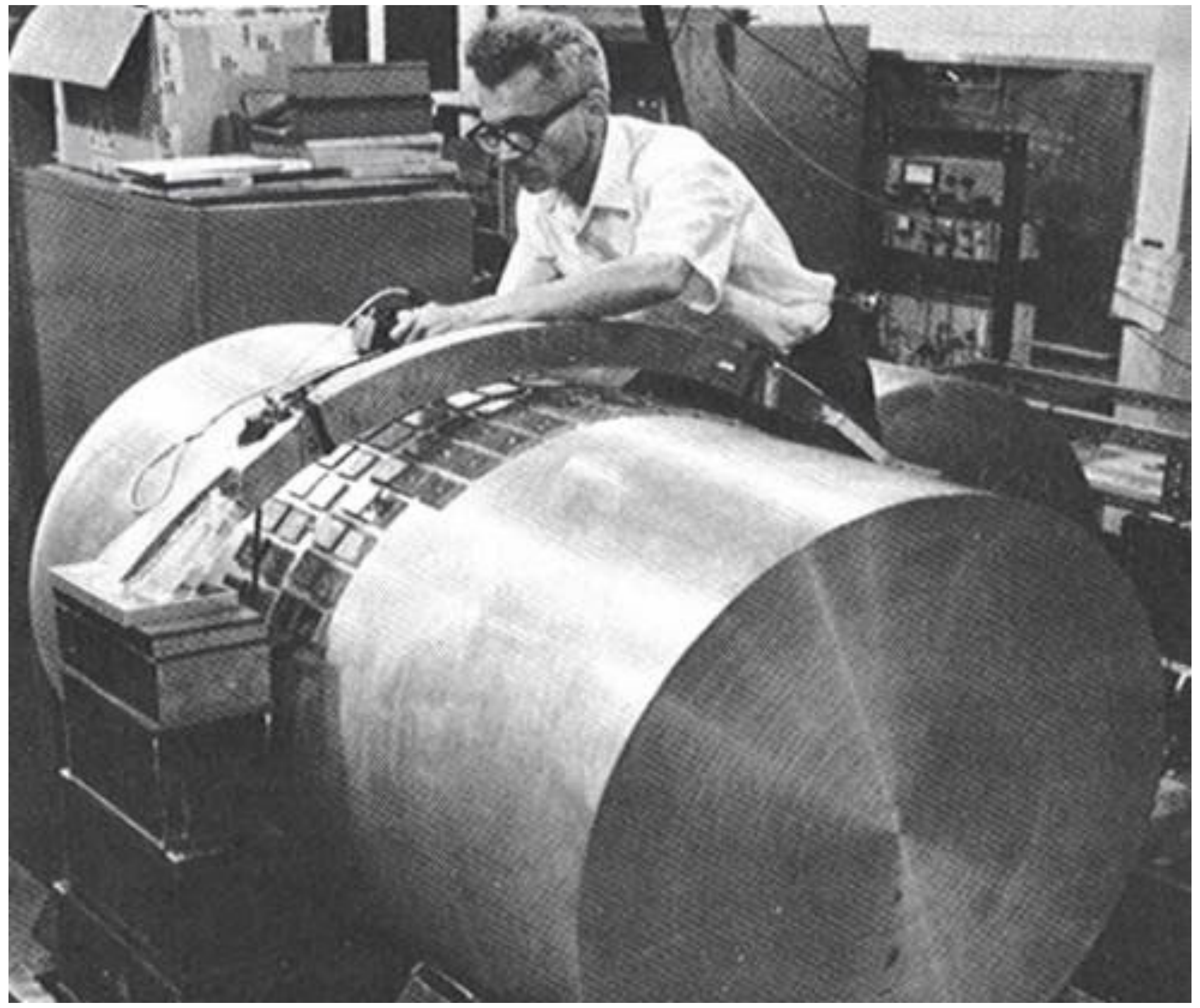




Schiff potential

Piezoelectric factor

Resonant Mass Detectors



In the 1960's:
Weber Bar, $S \sim 10^{-17}$

AURIGA, NAUTILUS,
MiniGrail, $S \sim 10^{-25}$

Goryachev et al. 2014
$S \sim 10^{-22}$

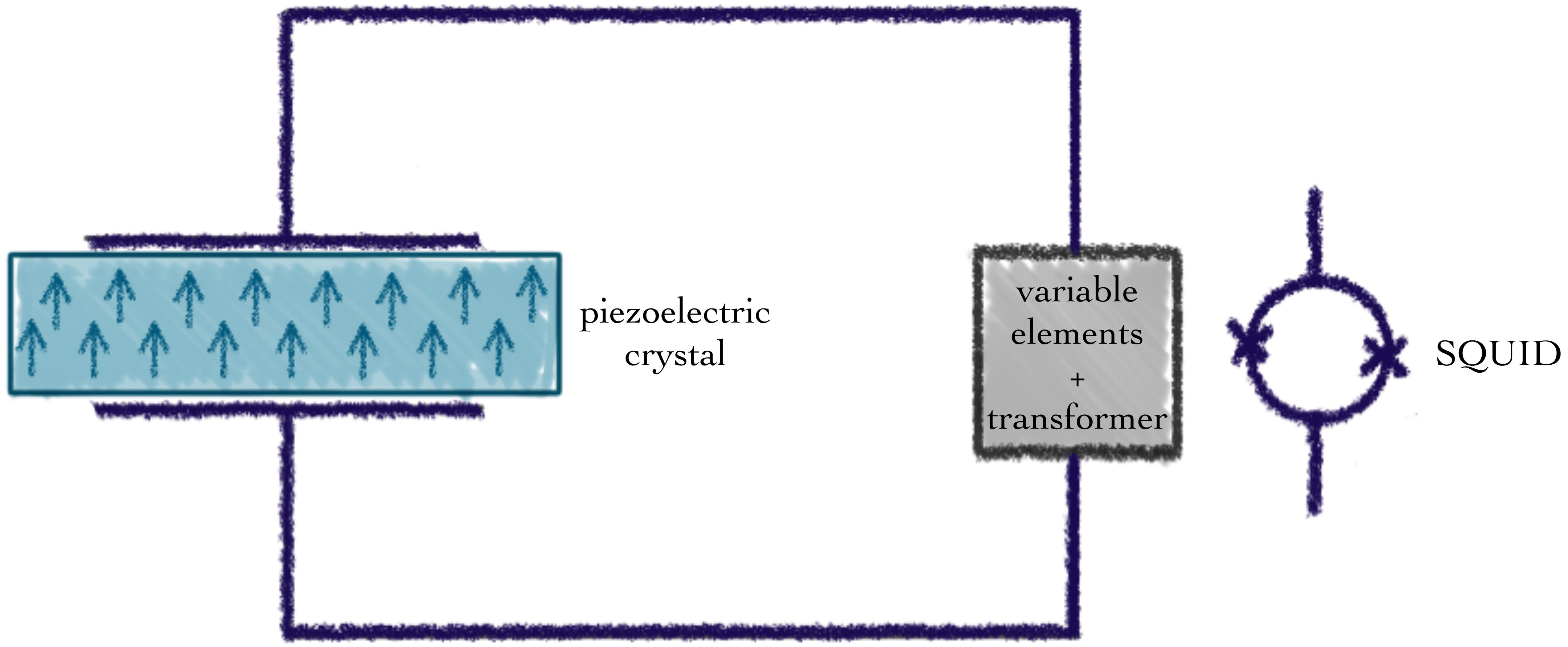
$0.1 - 1kHz$

$MHz - GHz$

PEARL

(**P**iezoelectric **E**xperiment for **A**xions with **R**esonant crysta**L**s)

1. Find a piezoelectric material with low mechanical noise and big Schiff moments
2. Cool to $\sim\ mK$
3. Align nuclear spins using a magnetic field
4. Measure tiny oscillating voltage using a SQUID



Backgrounds:

Fluctuating nuclear spins

Small effect

Fluctuating magnetic impurities in material

$\lesssim$ppm

Magnetization noise

→ fictitious EMF

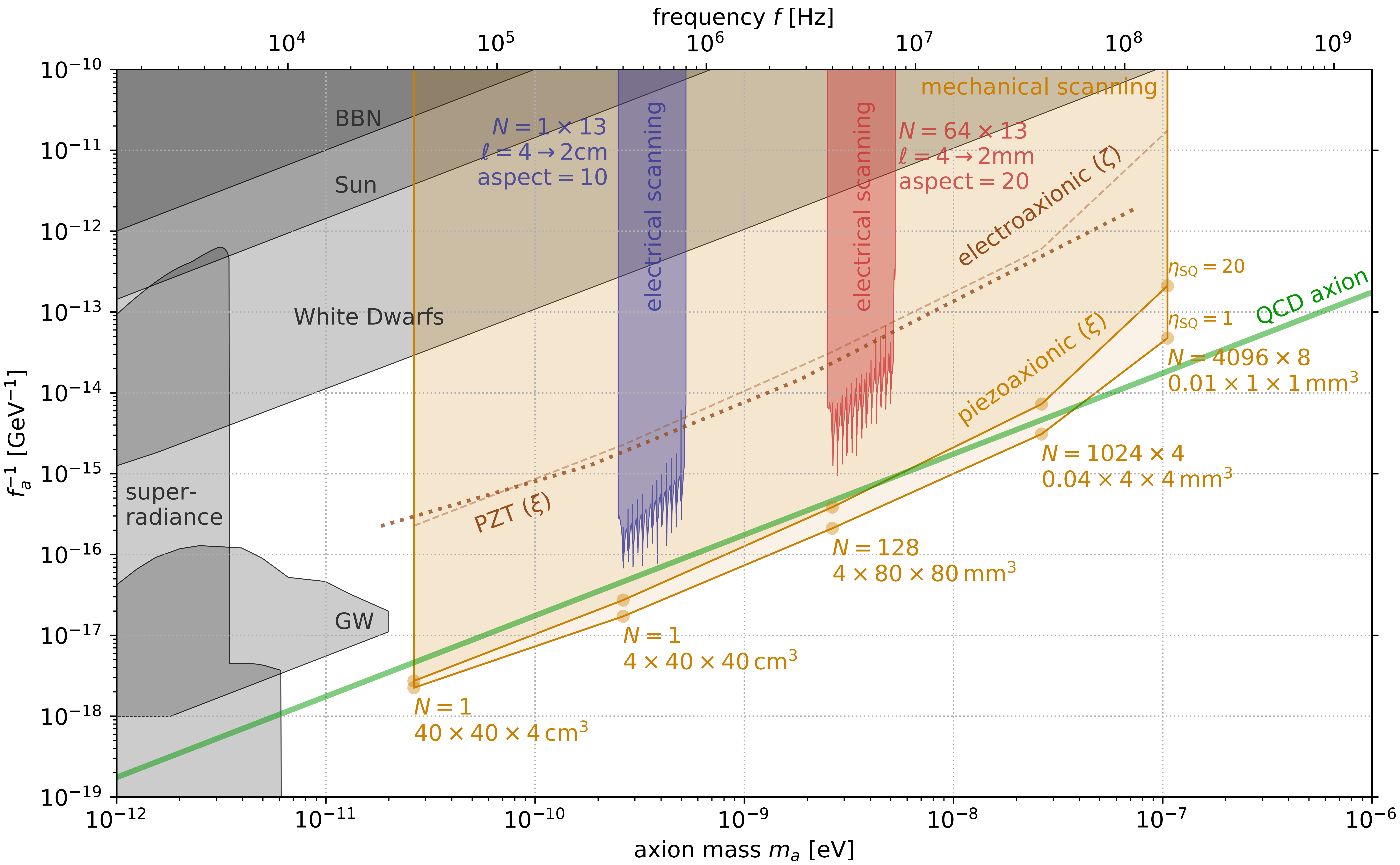
Vibrational noise

Systematic, demonstrated at AURIGA

Noise:

Thermal noise limited, main sources: crystal mechanical noise and SQUID noise

Idealized Forecast



***parameter space above QCD axion line tuned in mass and vacuum alignment**

BBN: K. Blum, R. T. D'Agnolo, M. Lisanti, B. R. Safdi (2014)

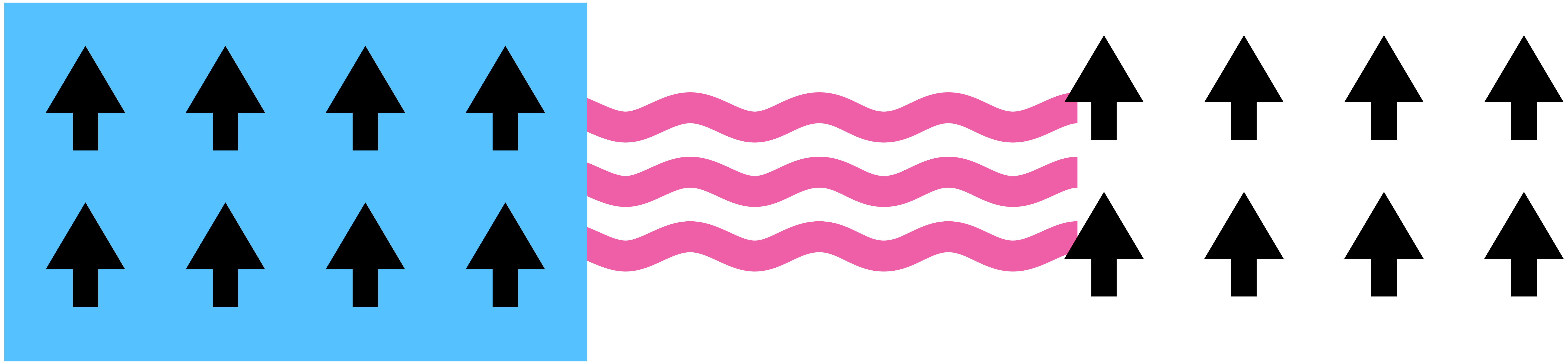
Sun: A. Hook, J. Huang (2018)

WDs: R Balkin, J Serra, K Springmann, S Stelzl, A Weiler (2022)

Superradiance: A. Arvanitaki, S. Dubovsky (2011)

GWs: J. Zhang, Z. Lyu, J. Huang, M. C. Johnson, L. Sagunski, M. Sakellariadou, H. Yang (2021).

The Piezoaxionic Force

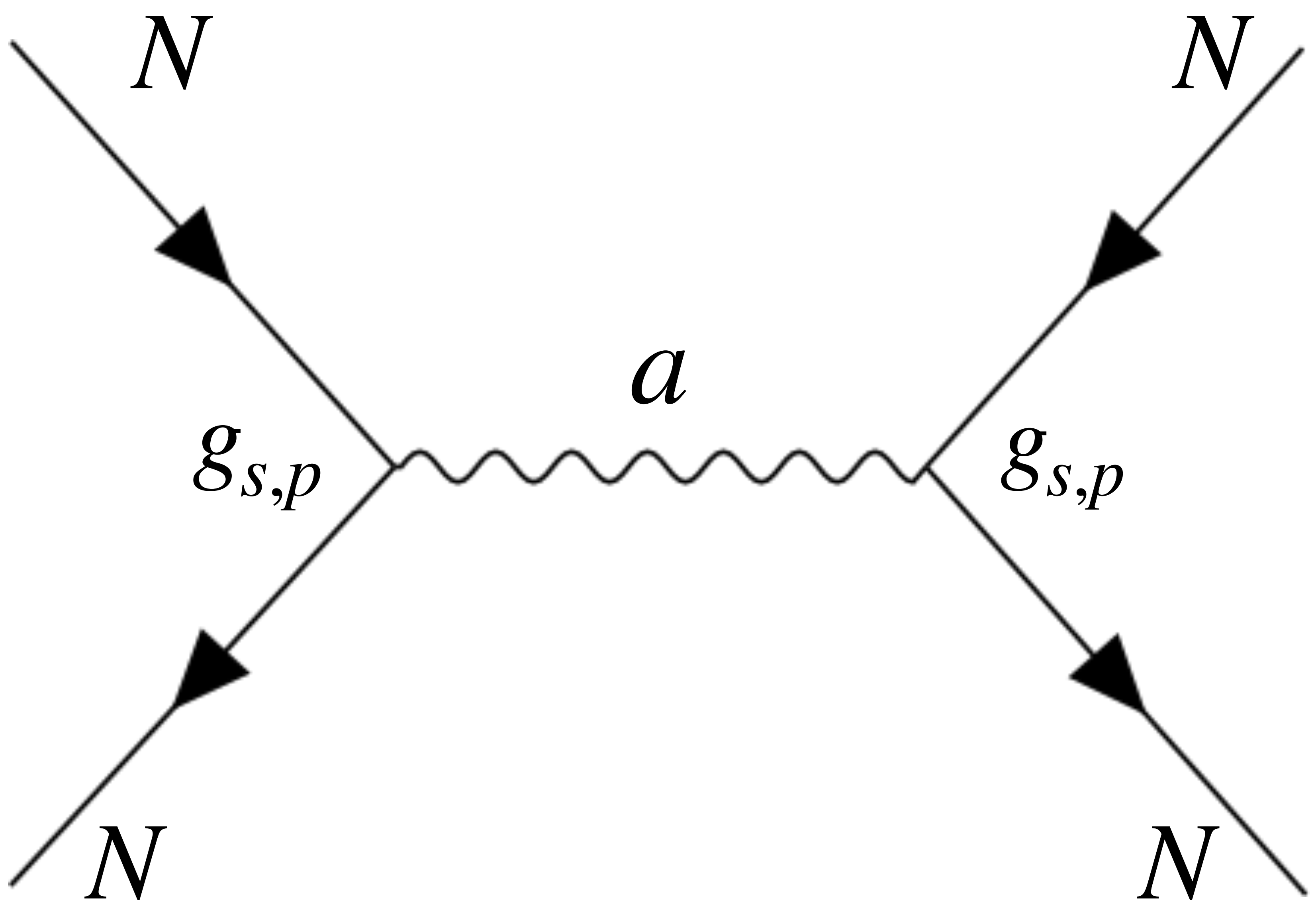


$$\mathscr{L} \supset \frac{a}{f_a} G\tilde{G}$$

$$\mathscr{L} \supset \frac{g_p}{2m_\psi} \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi$$

Axion-mediated forces

$$\mathscr{L} \supset \frac{(\partial a)^2}{2} - \frac{m_a^2 a^2}{2} - g_s a \bar{N} N + \frac{g_p}{2 m_N} \partial_\mu a \bar{N} \gamma^\mu \gamma_5 N$$



$$U_{sp} = \frac{g_s g_p}{8 \pi m_N} \left(\frac{m_a}{r} + \frac{1}{r^2} \right) e^{-r\, m_a} (\hat{\sigma} \cdot \hat{r})$$

Axion-mediated forces

$$\mathcal{L} \supset \frac{(\partial a)^2}{2} - \frac{m_a^2 a^2}{2} - \boxed{g_s} a\bar{N}N + \frac{\boxed{g_p}}{2m_N}\partial_\mu a\bar{N}\gamma^\mu\gamma_5 N$$

P & T odd

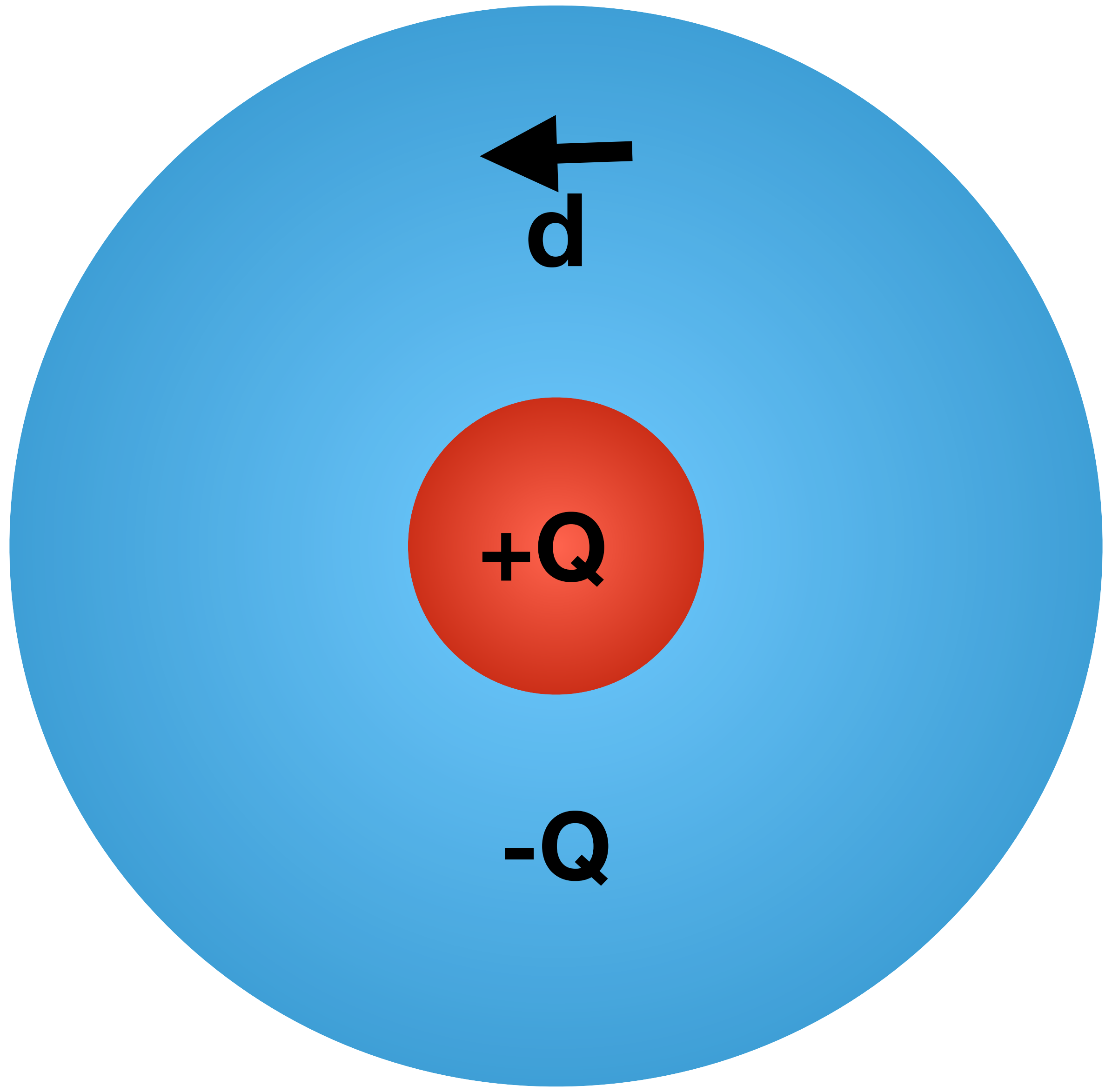
P & T even

$$g_s \sim 10^{-30}\,\frac{10^9\,GeV}{f_a}$$

(from CKM)

Idea: what if P and T violation comes from piezoelectric crystal?

Schiff's theorem, again



Resolution: finite size effects

Electrostatic (scalar) potential:
Nuclear Schiff Moment

Magnetic (vector) potential:
Nuclear Magnetic Quadrupole Moment

Schiff Moment

- In a piezoelectric crystal, the ground state electron wave function is a mixture of opposite parity orbitals ϵ_s and ϵ_p:

$$|\psi\rangle_e = \epsilon_s|s\rangle + \epsilon_p|p\rangle$$

Number density of Schiff moments

Electronic matrix element

Nuclear spin polarization

$$\rho_S = n_S \frac{4\pi e}{f_a}\frac{\partial S}{\partial \theta_a}\epsilon_s\epsilon_p^* \boxed{\mathcal{M}_S} \cdot \hat{\mathbf{I}} \quad + c.c$$

$\neq 0$ **in a ferroelectric crystal**

Effective in-medium energy density

Magnetic Quadrupole Moment

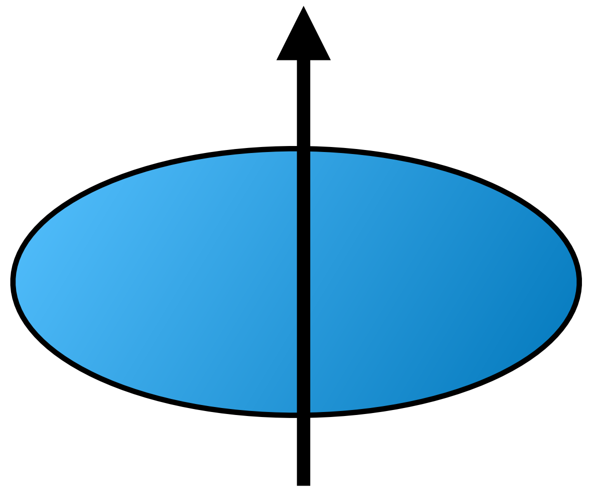
$$V_{\mathrm{M}} = \frac{e\,\mathrm{M}}{4I(2I-1)}\left[I_m I_n + I_n I_m - \frac{2}{3}\delta_{mn} I(I+1)\right] \times t_{mn}(\sigma_e, \hat{r}_e)$$

$$\mathrm{M} \sim 10\frac{\bar{\theta}_a}{m_N}\mu_N$$

non-deformed nuclei

$$\mathrm{M} \sim Z^{2/3}\,10\frac{\bar{\theta}_a}{m_N}\mu_N$$

rugby-ball shaped nuclei



$$\sim (0.1-1) \times \bar{\theta}_a\, e\, fm^2$$

$\mu_n = \dfrac{e}{2m_p}$ **= nuclear magnetic moment**

MQM: Atomic Mixing

$$|\psi\rangle_e = \epsilon_s \,|\, s\rangle + \epsilon_p |p\rangle$$

Density of MQM nuclei

Electronic matrix element

$$\rho_{\mathrm{M}} = e\, n_{\mathrm{M}} \frac{\partial \mathrm{M}}{\partial \theta_a} t_{mn}(I_N) \times \epsilon_s \epsilon_p^* A_{mn}(\sigma_e, \hat{r}_e) \quad + c\,.\,c\,.$$

$\neq 0$ in a magnetic piezoelectric crystal

Effective in-medium energy density

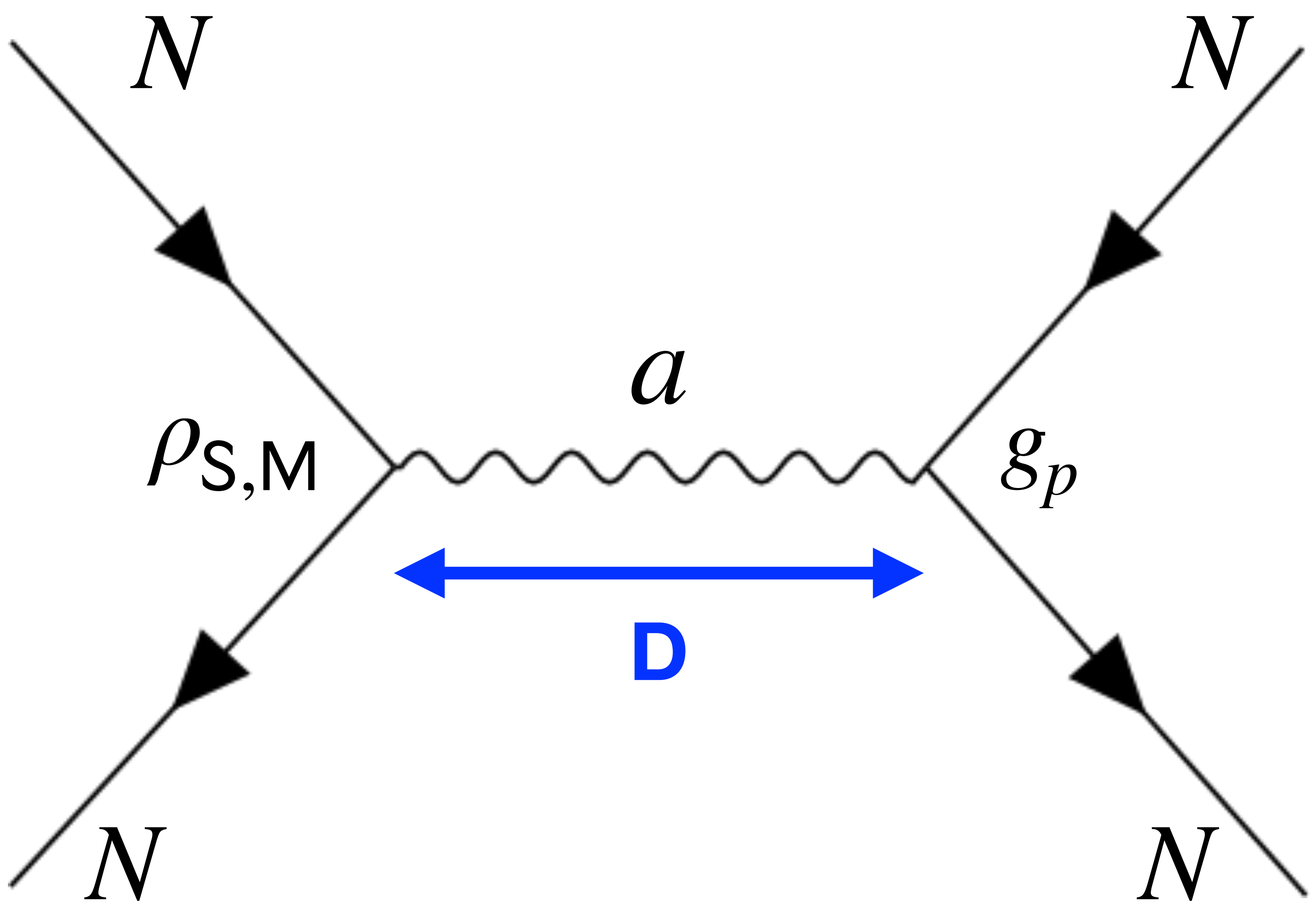
Nuclear quadrupole tensor

Take a uniform slab of piezoelectric crystal with thickness h:

$$\left(\Box + m_a^2\right) a\,(t, \mathbf{x}) = -\frac{\rho_{\mathrm{S}} + \rho_{\mathrm{M}}}{f_a} \equiv j(t, \mathbf{x})$$

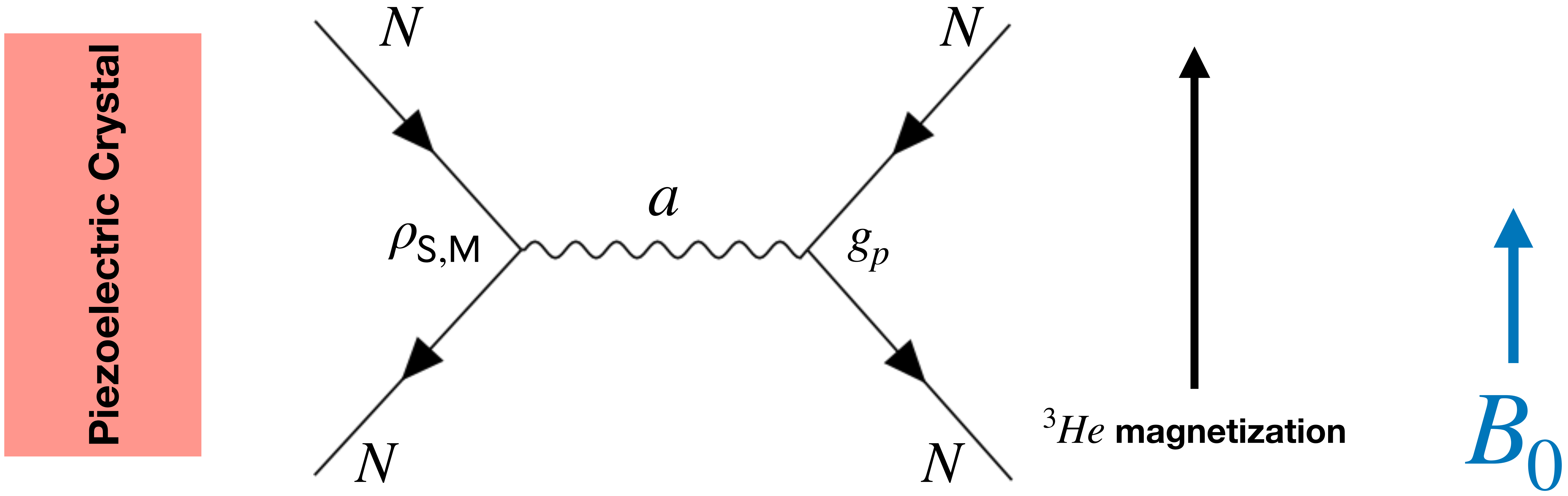
$$\nabla \bar{\theta}_a \simeq -\hat{\mathbf{D}} \frac{j}{2m_a f_a} e^{-m_a D} \left(1 - e^{-m_a h}\right)$$

$$H \supset -\frac{g_p}{m_N} \sigma_N \cdot \nabla \bar{\theta}_a, \qquad g_p \equiv \frac{c_N\, m_N}{f_a}$$



Like a B-field, but unaffected by magnetic shielding!

Nuclear Magnetic Resonance

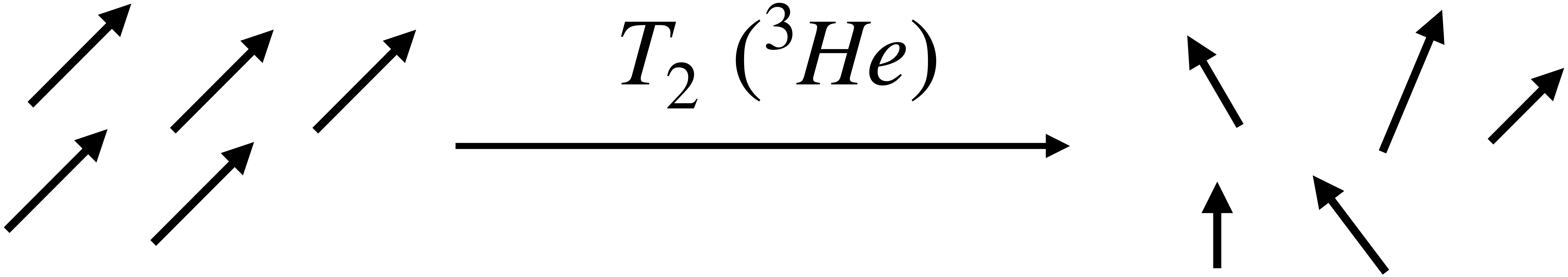


By moving the crystal at the Larmor frequency $\omega = -\gamma_N B_0$, we pick up a resonantly enhanced, off-axis magnetization

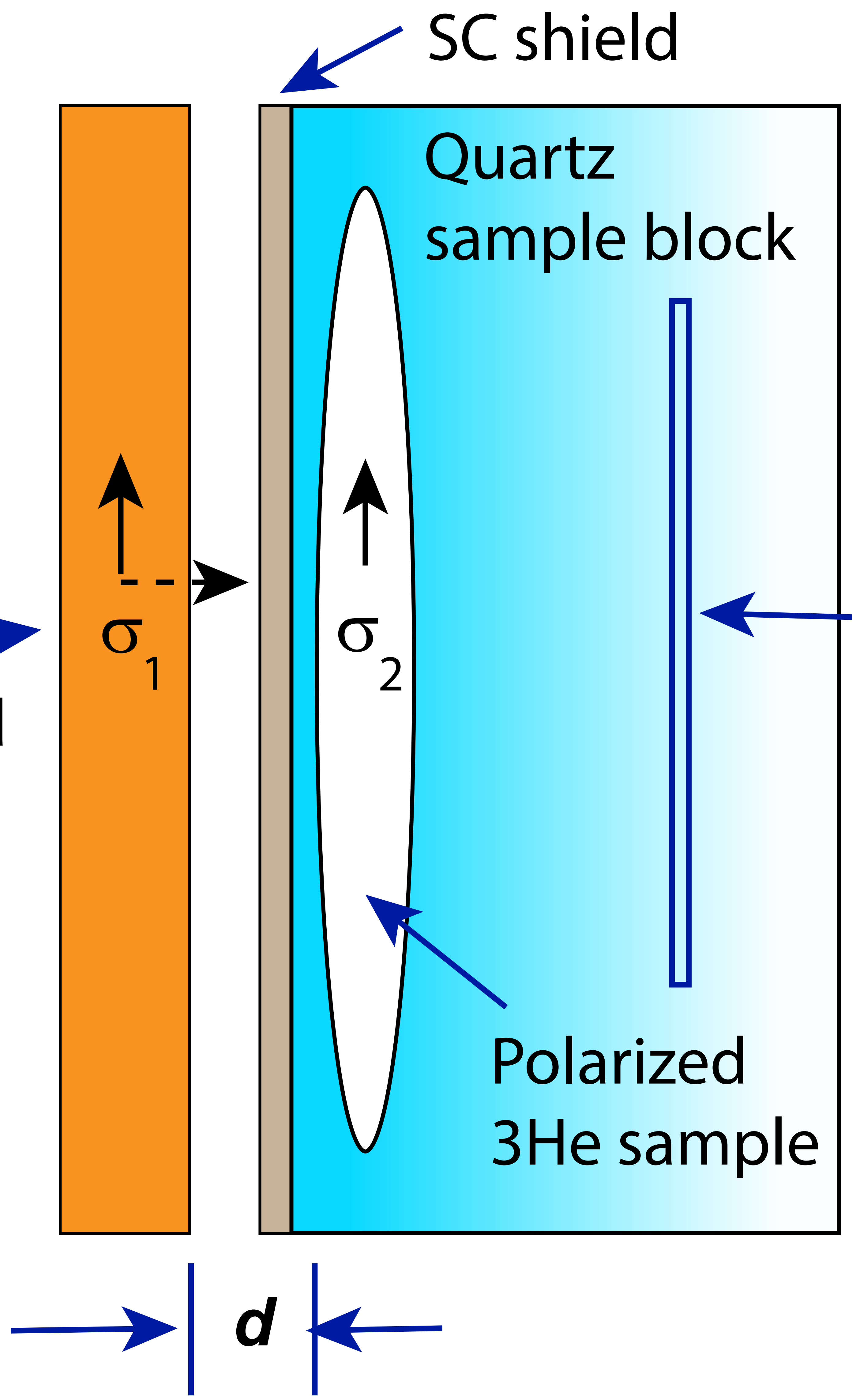
The separation between the crystal and NMR sample sets the range of axion masses that we are sensitive to.

Noise

The main noise source is transverse spin projection noise from the sample itself



$$B_{\text{min}} \approx 10^{-20}\ \text{T} \times \sqrt{\left(\frac{b}{1\ \text{Hz}}\right)\left(\frac{1\ mm^3}{V_{^3He}}\right)\left(\frac{10^{22}\ cm^{-3}}{n_{^3He}}\right)\left(\frac{1000\ s}{T_{2_{^3He}}}\right)}$$



SC shield

Quartz
sample block

σ_1

σ_2

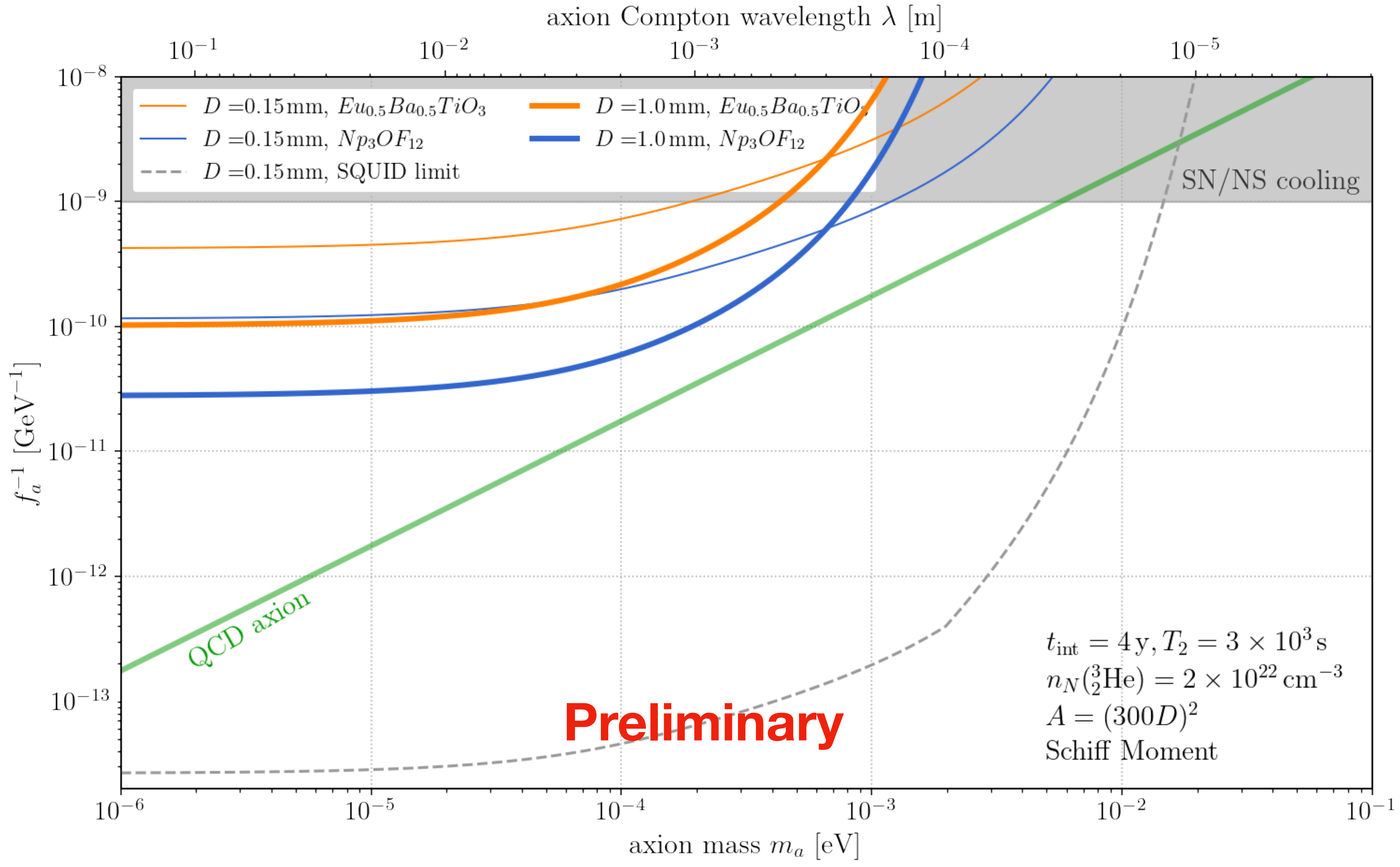
Spin-polarized
Source mass

SQUID
magnetometer

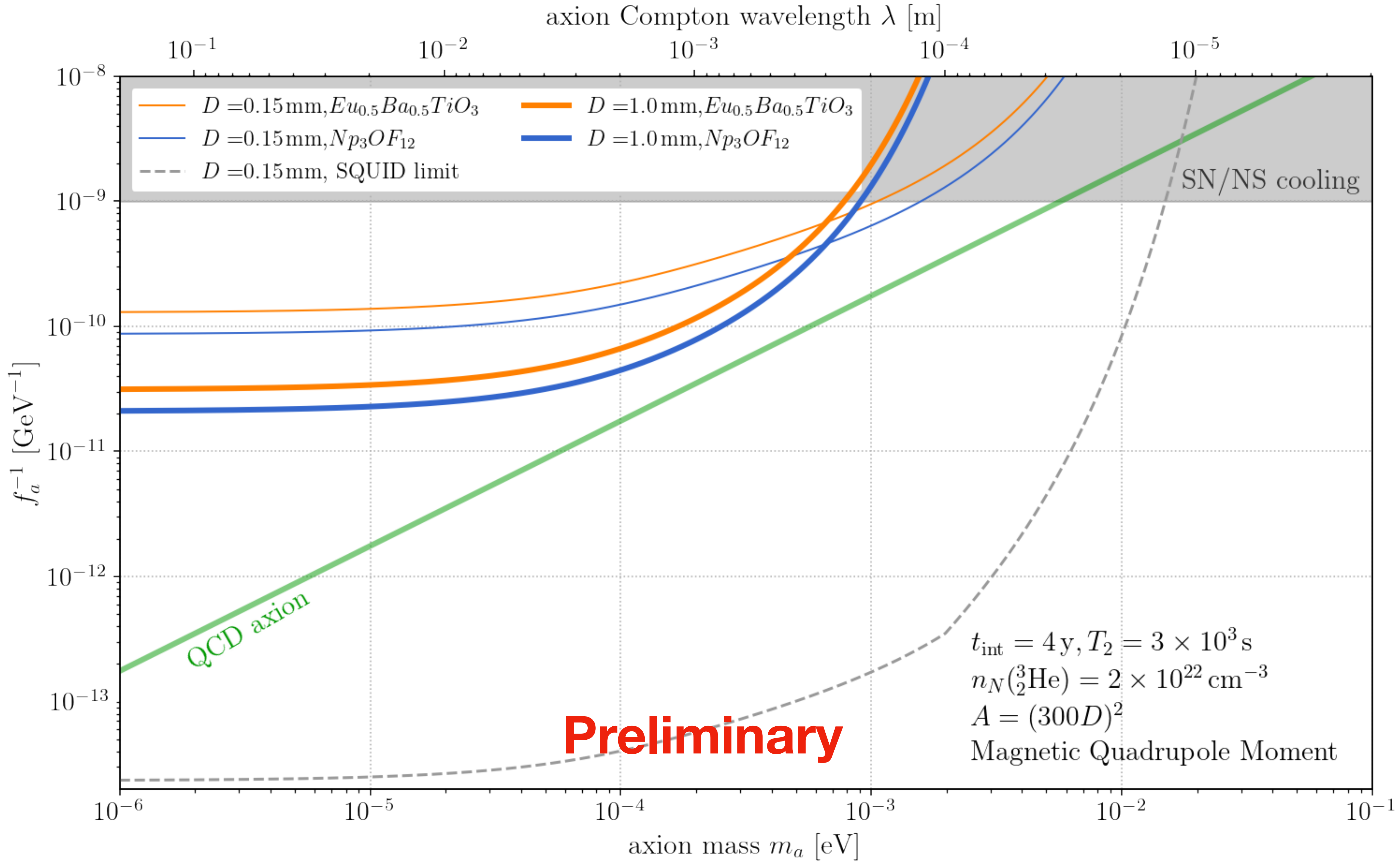
Polarized
3He sample

d

Schiff moment:

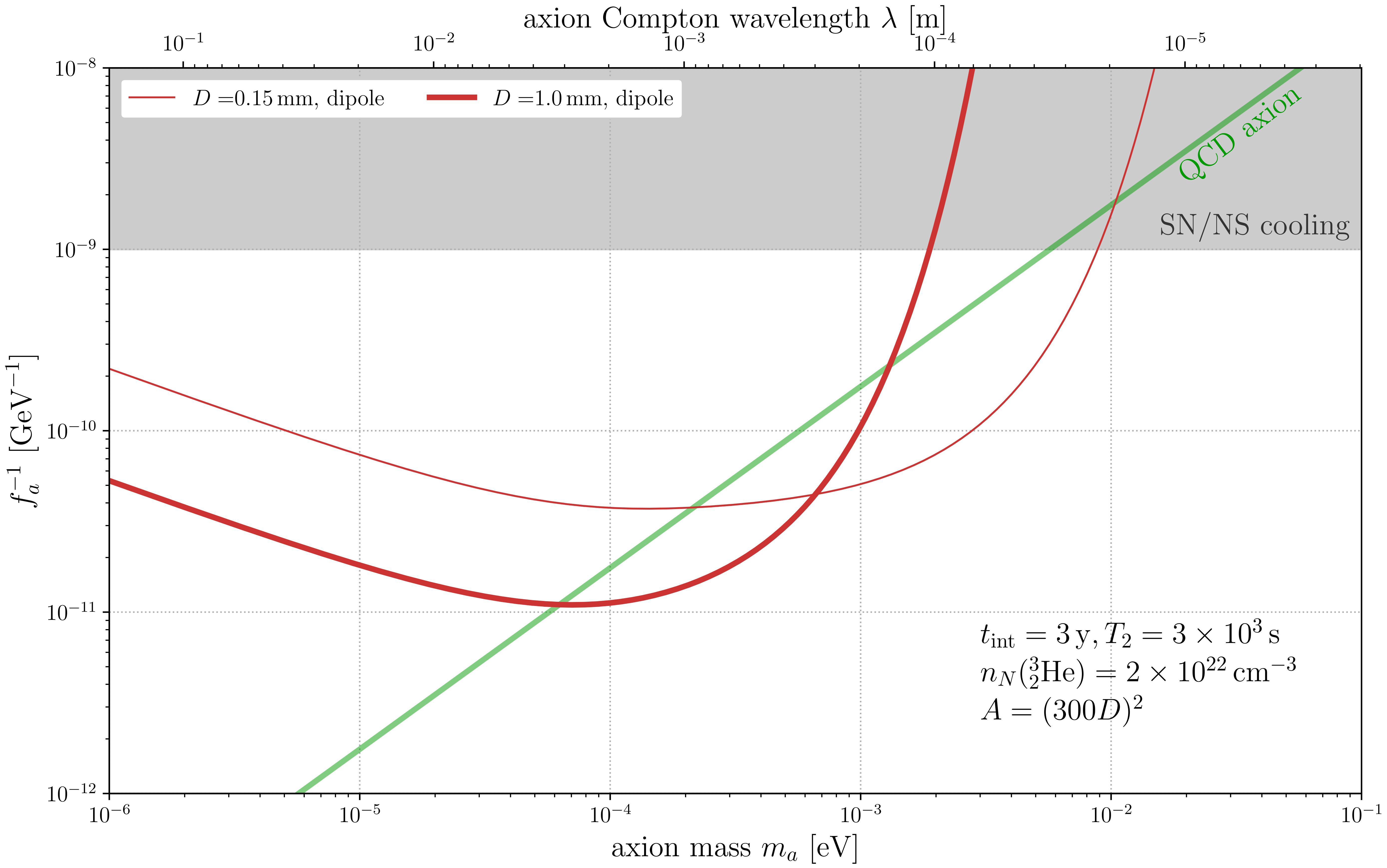


MQM:



***parameter space above QCD axion line tuned in mass and vacuum alignment**

Dipole-Dipole



What’s next for the Piezoaxionic Effect?

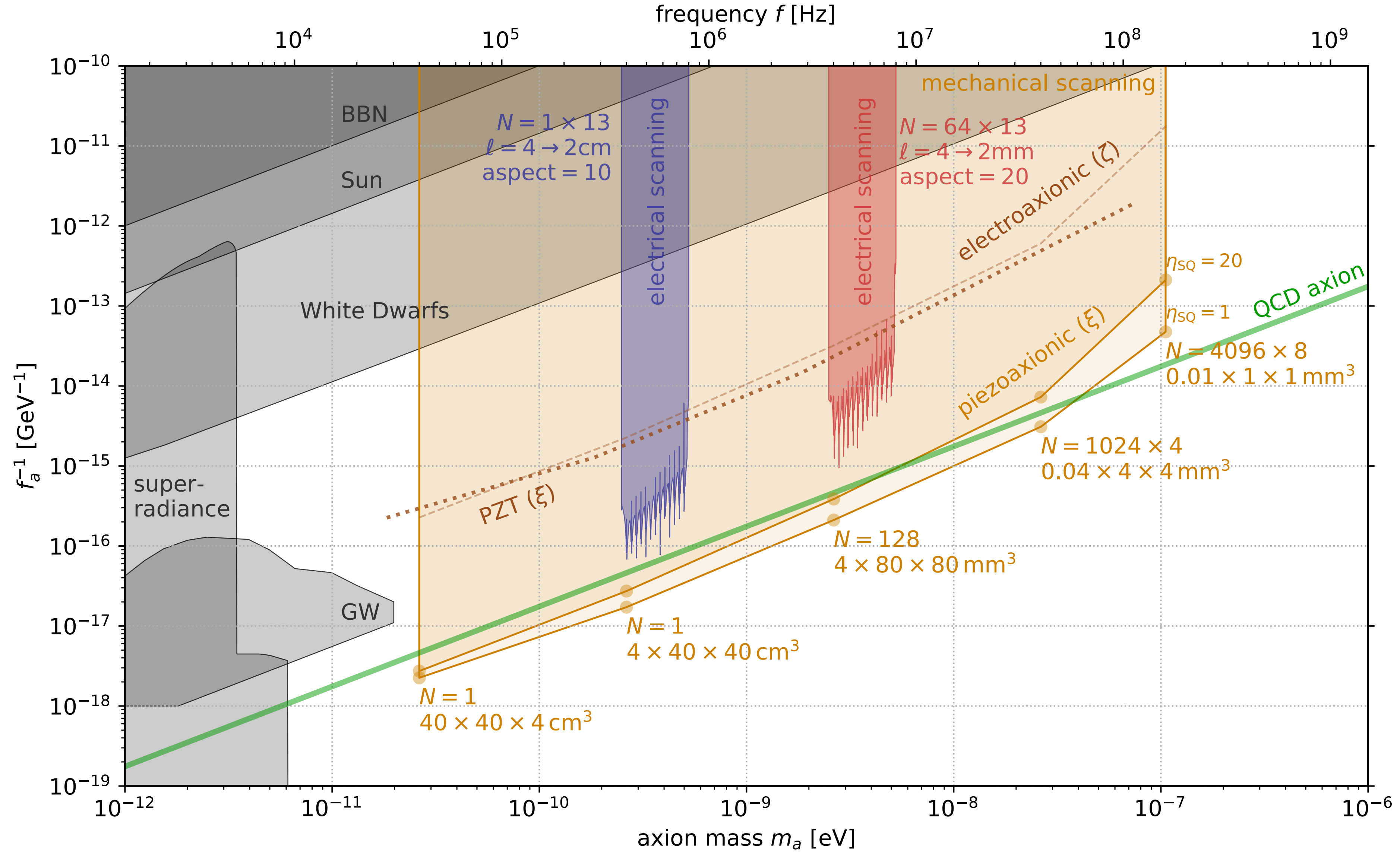
- In progress: precise Schiff moment calculations for stable, deformed nuclei
- Density functional theory (DFT) calculations of electron wave functions near nuclei
- Further experimental investigation of candidate materials (e.g. mechanical quality factors)
- Search for model dependent axion-electron coupling
- Higher masses for dark matter ($\sim meV$), single-phonon regime (w/ I. Bloch, D. Carney, S. Knapen, G. Marocco)
- New ideas for mechanical scanning

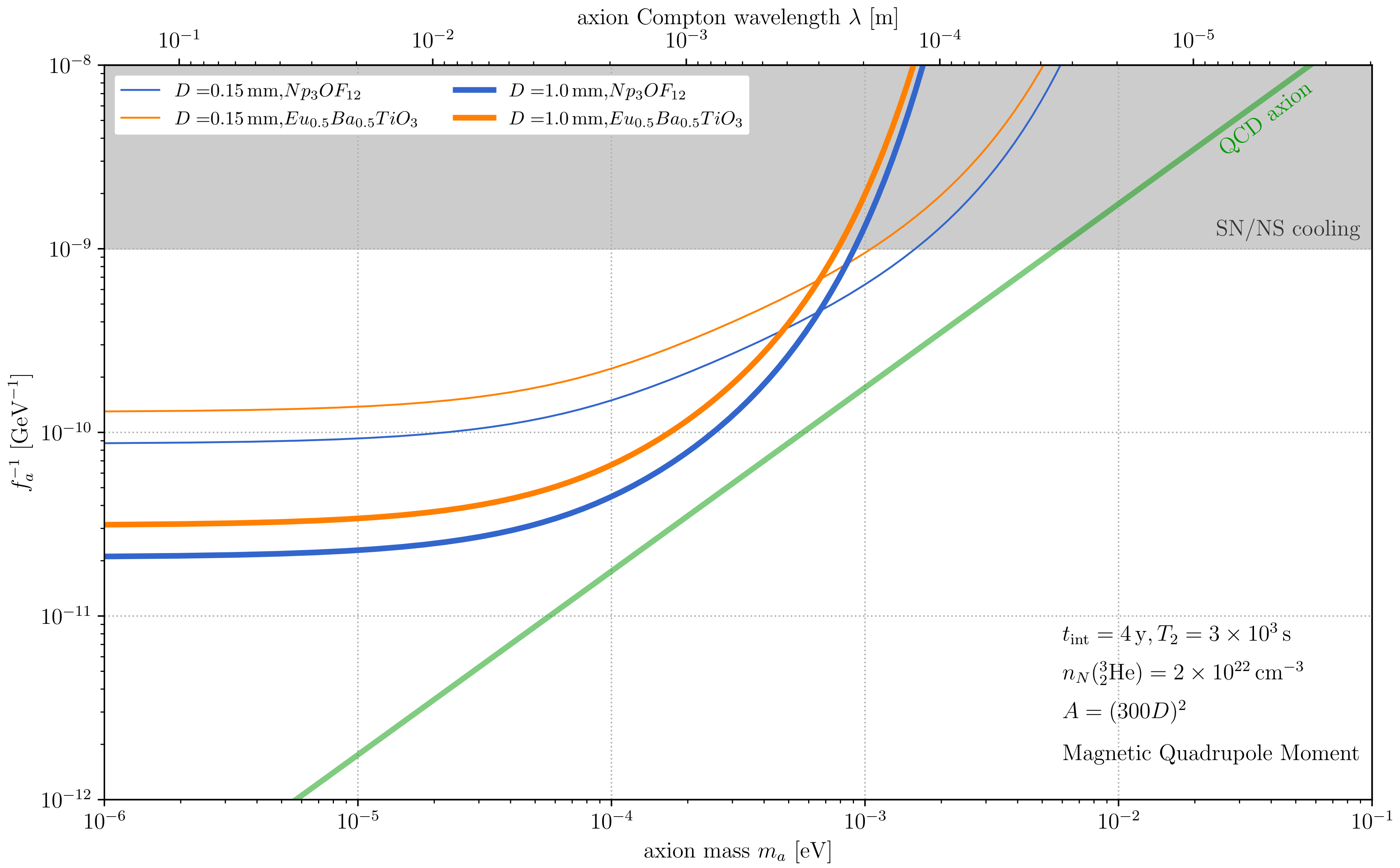
Summary

- QCD axion DM can excite vibrational modes in piezoelectric crystals via its model-independent coupling to gluons.

- Piezoelectric crystals can source QCD axion mediated forces also via their model independent coupling to gluons, that could be detected using an NMR sample.

- Complimentary to cavity experiments





Materials

Piezoelectric make up a large class of materials - 20 out of 32 symmetry groups!

- High density of nuclei with large Schiff moments and low radioactivity
- Good acoustic properties (high Q-factor)
- Strong piezoelectric properties
- Structural similarity to well-known resonator crystals.

Class	Candidates	Similar Crystals
32	$\mathrm{Na}\mathbf{Dy}\mathrm{H_2S_2O_9}$	$\mathrm{SiO_2}$ (quartz)
		$\mathrm{Ga_5La_3SiO_{14}}$ (langasite)
	$\mathbf{Bi}\mathrm{PO_4}$	$\mathrm{GaPO_4}$ (gallium orthophosphate)
3m	$\mathbf{U}\mathrm{OF_4}$	tourmaline
	$\mathbf{U}\mathrm{Cd}$	$\mathrm{LiNbO_3}$ (lithium niobate)
4mm	$\mathbf{Dy}\mathrm{Si_3Ir}$	$\mathrm{Li_2B_4O_7}$ (lithium tetraborate)
	$\mathbf{Dy}\mathrm{AgSe_2}$	
$\bar{4}2$m	$\mathbf{Dy}\mathrm{AgTe_2}$	$\mathrm{NH_6PO_4}$ (ADP)
	$\mathbf{Dy}_2\mathrm{Be_2GeO_7}$	$\mathrm{KH_2PO_4}$ (KDP)
mm2	$\mathbf{U}\mathrm{CO_5}$	$\mathrm{Ba_2NaNb_5O_{15}}$ (barium sodium niobate)

Candidate materials collected from the database at https://materialsproject.org/

Scanning

- Grow a series of crystals of different thicknesses
- Vary *electrical* resonance frequency using capacitor and inductor