Axions from the Quantum Gravity Viewpoint

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Outline

1. **Warm-up: magnetic symmetries and monopoles**
2. **Instanton number symmetry**
3. **Examples in quantum gravity**
4. **Axion models and cosmology: overview**
5. **Eliminating axion isocurvature: a new approach**

(When) does QG require axions to exist?

Which kinds of axion models are plausible?

Warm-Up: Magnetic Symmetry and Monopoles

Magnetic Symmetry and Monopoles

A first example: U(1) gauge theory has a **topological** U(1) symmetry, with current $\frac{1}{2\pi}F$. This is a $(d-3)$-form symmetry, acting on 't Hooft operators.

Conserved due to the **Bianchi identity**: $\mathrm{d}F = 0$.

How can we eliminate it?

1. **Gauge it**: add a $\frac{1}{2\pi}B^{(d-2)} \wedge F$ coupling to a dynamical $(d-2)$-form gauge field. This **gives the photon a mass** and changes the IR physics.

2. **Break it (explicitly)**: add dynamical magnetic monopoles, $\frac{1}{2\pi}\mathrm{d}F = J_{\text{mag}}$.

Gauging vs Breaking

The magnetic symmetry is implemented by a family of topological surface operators $U_{g=\mathrm{e}^{\mathrm{i}\alpha}}(\Sigma) = \exp\left(\mathrm{i}\alpha \int_{\Sigma} \frac{1}{2\pi} F\right)$.

Gauging:

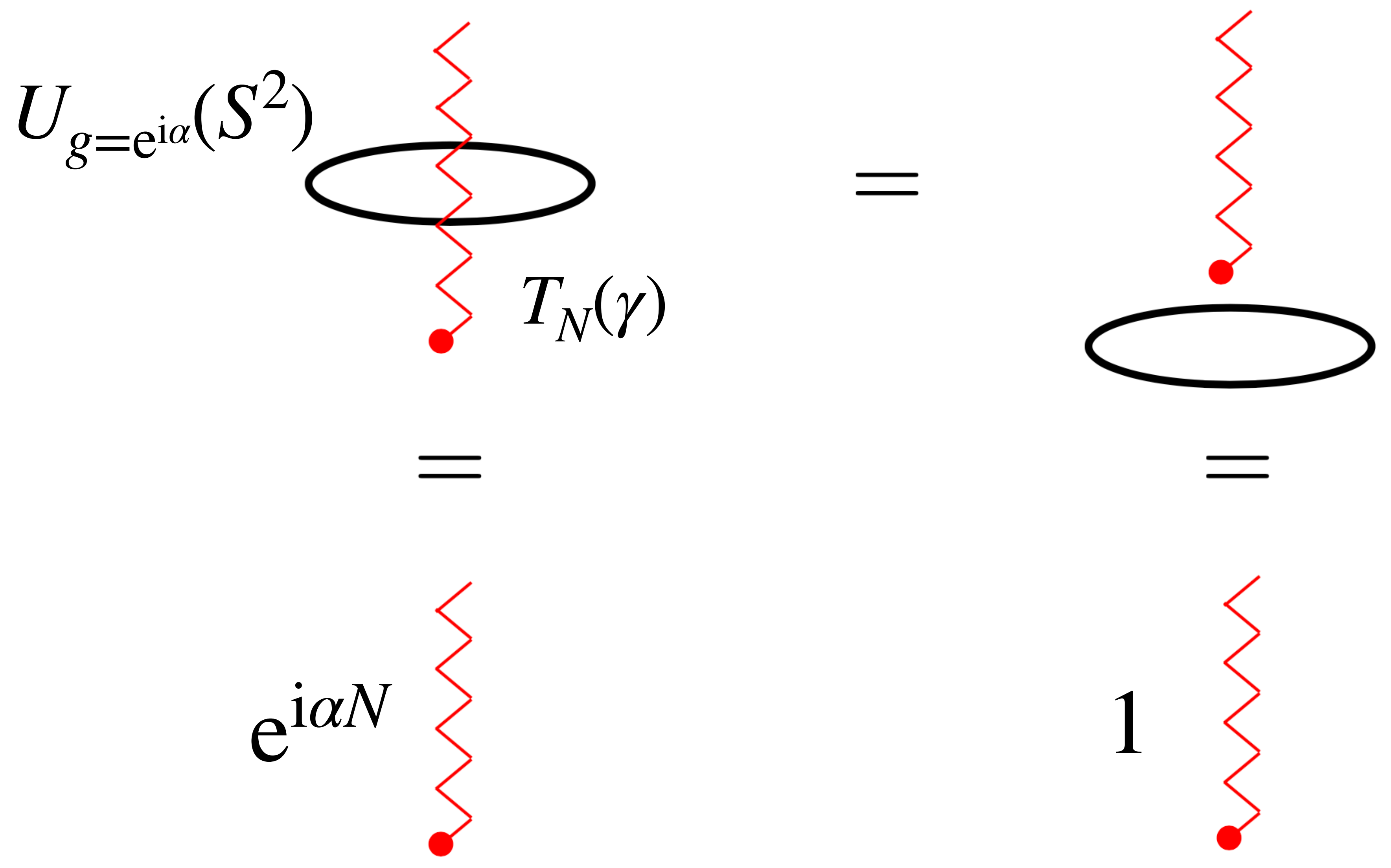
$$\frac{1}{2\pi} F = \frac{1}{g^2} \mathrm{d} \star H^{(d-1)}$$

Current *exact*, $U_g(\Sigma)$ becomes *trivial* on-shell.

Breaking:

’t Hooft line can *end*.

$U_g(\Sigma)$ nontrivial but not topological.



Gauging and Breaking from Low-Energy EFT

When we **gauge** a continuous symmetry, often has a big impact on spectrum of low-energy EFT.

- gauge 0-form symmetry ⇒ propagating gauge field
- gauge 1-form magnetic symmetry ⇒ gauge field mass, decouples

When we **break** a continuous symmetry, effect may not be visible in EFT of light modes. However there is often some **heavy object** describable in EFT that makes the breaking visible.

- break 1-form symmetry ⇒ monopole exists

Prediction: Magnetic Monopoles Exist

This example is instructive because it shows that **the principle of no global symmetries** in QG has a **direct real-world implication.**

Photon is massless ⇒ magnetic 1-form symmetry was not gauged.

Symmetry must be broken ⇒ magnetic monopoles exist.

The magnetic monopoles could be very heavy, so this is not immediately useful as a guide to experiment. But it is an important proof of principle.

Strategy:

1. Identify a symmetry.

2. Categorize ways to **gauge** or **break** it.

3. Understand which are possible in the real world.

Instanton Number Symmetry

Topological Symmetries in Gauge Theory, 1.

Gauge theories have many topological symmetries.

From the mathematical viewpoint, these correspond to *characteristic classes* of the gauge bundle.

A familiar example arises in non-abelian gauge theory:

$$\begin{aligned}
\mathrm{d}\,\mathrm{tr}(F\wedge F) &= \mathrm{tr}(\mathrm{d}F\wedge F + F\wedge \mathrm{d}F)\\
&= \mathrm{tr}\Big((\mathrm{d}F + [A,F])\wedge F + F\wedge(\mathrm{d}F+[A,F])\Big)\\
&= \mathrm{tr}(\mathrm{d}_A F\wedge F + F\wedge \mathrm{d}_A F) = 0
\end{aligned}$$

This shows that $\mathrm{tr}(F\wedge F)$ is a conserved current due to the non-abelian Bianchi identity $\mathrm{d}_A F = 0$. It generates a $(d-5)$-form **instanton number symmetry**.

Topological Symmetries in Gauge Theory, 2.

More generally, we have a family of conservation laws, $\mathrm{d}\,\mathrm{tr}\left(\bigwedge^k F\right) = 0$

Here $\bigwedge^k F$ denotes $F \wedge F \wedge \cdots \wedge F$, with *k* copies of F.

We call such a U(1) global symmetry a ***Chern-Weil global symmetry***. The currents (appropriately normalized) integrate to integers. E.g., for U(1) or SU(N),

$$\int_M \frac{1}{8\pi^2}\mathrm{tr}(F \wedge F) \in \mathbb{Z},$$

for any closed spin 4-manifold M.

w/ Ben Heidenreich, Jake McNamara, Miguel Montero, Tom Rudelius, Irene Valenzue

arXiv:2012.00009 [hep-th]

Topological Symmetries in Gauge Theory, 3.

More general examples also arise, e.g., in $\mathrm{PSU}(N) \cong \mathrm{SU}(N)/\mathbb{Z}_N$ gauge theory, there is a $\mathbb{Z}_N$-valued "Stiefel-Whitney class" $w_2(A) \in H_2(X, \mathbb{Z}_N)$. It is closed, $\mathrm{d}(w_2) = 0$, so it generates a $\mathbb{Z}_N$ $(d-3)$-form global symmetry.

This is the "magnetic symmetry" of $\mathrm{PSU}(N)$ gauge theory. It is broken by monopoles carrying a $\mathbb{Z}_N$ charge:

$$\mathrm{d}w_2 = J_{\mathrm{mag}}\,.$$

We can also form lower-form symmetries along the lines of $w_2 \cup w_2$, and so on.

In general, G gauge theory (for compact, connected G) has a $(d-3)$-form magnetic symmetry group $\pi_1(G)^\vee$, the Pontryagin dual of the fundamental group.

Monopole Breaking of $F \wedge F$ Symmetry, 1.

Given $\frac{1}{2\pi}\mathrm{d}F = J_{\mathrm{mag}}$, monopoles also break $J_{\mathrm{inst}} = \frac{1}{8\pi^2}F \wedge F$:

$$\mathrm{d}J_{\mathrm{inst}} = \mathrm{d}\left(\frac{1}{8\pi^2}F \wedge F\right) = \frac{1}{4\pi^2}F \wedge \mathrm{d}F = \frac{1}{2\pi}F \wedge J_{\mathrm{mag}}\,.$$

This generalizes to other gauge groups G that are not simply connected!

$$\mathrm{PSU}(N): \quad N_{\mathrm{inst}} = \int_M \frac{1}{8\pi^2}\mathrm{tr}(F \wedge F) \equiv \frac{N-1}{N}\int_M \frac{1}{2} w_2 \cup w_2 \mod 1$$

Thus, $\mathrm{PSU}(N)$ monopoles **break** instanton number symmetry:

$$\Delta N_{\mathrm{inst}} \sim \frac{1}{N}\int w_2 \cup J_{\mathrm{mag}} \mod 1.$$

w/ Daniel Aloni, Eduardo García-Valdecasas, Motoo Suzuki

work in progress

Monopole Breaking of $F \wedge F$ Symmetry, 2.

In $d = 4$, do monopoles break $J_{\text{inst}} = \frac{1}{8\pi^2} F \wedge F$? $\mathrm{d}J_{\text{inst}} = 0$, trivially.

There is still a sense in which "(-1)-form" instanton number symmetry is broken: **monopoles obstruct a coupling to a background axion field**.

Start from action with gauge field and axion θ:

$$S = \int \left(-\frac{1}{2e^2} F \wedge \star F - \frac{f^2}{2} \mathrm{d}\theta \wedge \star \mathrm{d}\theta + \frac{1}{8\pi^2} \theta F \wedge F \right).$$

Monopole Breaking of $F \wedge F$ Symmetry, 3.

Ordinarily we dualize a U(1) gauge field A to the magnetic dual A_{M} via

$$\frac{1}{2\pi}F_{\mathrm{M}} \equiv \frac{1}{2\pi}\mathrm{d}A_{\mathrm{M}} = -\frac{1}{e^2}\star F,$$

which makes sense because, away from sources, $\mathrm{d}\star F = 0$. In the presence of the axion coupling, however, we have

$$\frac{1}{e^2}\mathrm{d}\star F = \frac{1}{4\pi^2}\mathrm{d}\theta \wedge F\,.$$

We say that A_{M} has a “modified Bianchi identity,” and we must define it differently:

$$\frac{1}{2\pi}\mathrm{d}A_{\mathrm{M}} = -\frac{1}{e^2}\star F + \frac{1}{4\pi^2}\theta F\,.$$

Monopole Breaking of $F \wedge F$ Symmetry, 3.

We find the magnetic gauge field A_{M} by solving:

$$\frac{1}{2\pi}\mathrm{d}A_{\mathrm{M}} = -\frac{1}{e^2}\star F + \frac{1}{4\pi^2}\theta F\,.$$

But θ is a gauge field, and F_{M} is not gauge invariant! If $\theta \mapsto \theta + 2\pi n,$ then our solution changes, e.g.,

$$A_{\mathrm{M}} \mapsto A_{\mathrm{M}} + nA\,.$$ **This is the Witten effect!**

This means that a monopole worldvolume action of the form

$$S_{\mathrm{M}} = \int_{\Gamma} \left(T \star_{\Gamma} 1 + A_M\right)$$

is **not invariant** under θ background gauge transformations; hence, the symmetry generated by J was **broken**.

Dyon Restoration of $F \wedge F$ Symmetry, 1.

Now suppose on the monopole we have a **dyon degree of freedom**

$$\sigma \cong \sigma + 2\pi, \quad A \mapsto A + \mathrm{d}\alpha, \sigma \mapsto \sigma - \alpha, \quad \mathrm{d}_A\sigma \equiv \mathrm{d}\sigma + A$$

We can have a monopole worldvolume action coupling to the background field θ:

$$S_{\mathrm{M}} = \int_\Gamma \left(T \star_\Gamma 1 + A_M - \frac{1}{2\ell^2}\mathrm{d}_A\sigma \wedge \star \mathrm{d}_A\sigma + \frac{1}{2\pi}\theta\, \mathrm{d}_A\sigma \right)$$

This restores the invariance:

$$\theta \mapsto \theta + 2\pi n: \quad A_{\mathrm{M}} + \frac{1}{2\pi}\theta\, \left(\mathrm{d}\sigma - A\right) \mapsto \left(A_{\mathrm{M}} + nA\right) + \frac{1}{2\pi}(\theta + 2\pi n)\left(\mathrm{d}\sigma - A\right)$$

$$\delta S_{\mathrm{M}} = \int_\Gamma n\, \mathrm{d}\sigma \quad \Rightarrow \quad \exp(\mathrm{i}\,\delta S_{\mathrm{M}}) = 1.$$

(related: Witten, 1979; Callan & Harvey, 1985 ["anomaly inflow"]; Fukuda & Yonekura, 2020)

Dyon Restoration of $F \wedge F$ Symmetry, 3.

Even though the monopole worldvolume action is not invariant under $\theta \mapsto \theta + 2\pi$, $\exp(\mathrm{i}S_{\mathrm{M}})$ is invariant.

We can thus couple the **monopole with a dyon mode** to a background axion field θ. Another way to say this is that there is an **improved symmetry current:**

$$J_{\mathrm{imp}} = \frac{1}{8\pi^2} F \wedge F + \frac{1}{2\pi} \mathrm{d}_A \sigma \wedge J_{\mathrm{mag}} \,.$$

In the $d > 4$ case we can check this is conserved:

$$\mathrm{d}J_{\mathrm{imp}} = \frac{1}{4\pi^2} F \wedge \mathrm{d}F + \frac{1}{2\pi} \mathrm{d}(\mathrm{d}_A \sigma) \wedge J_{\mathrm{mag}} = \frac{1}{2\pi} F \wedge J_{\mathrm{mag}} - \frac{1}{2\pi} F \wedge J_{\mathrm{mag}} = 0.$$

Monopoles and $F \wedge F$ Symmetry

Summary for U(1) case

- A magnetic monopole always **breaks** the symmetry with current $F \wedge F$.
- **If** the monopole has a dyon mode, there is an **improved symmetry** that is a linear combination of $F \wedge F$ and the localized $\mathrm{d}_A\sigma$ term. This symmetry can be **gauged**. In $d = 4$, this means coupling to an axion.
- Whether or not the dyon mode exists depends on the UV completion, but
- the dyon mode can be described in the monopole worldvolume EFT.

This generalizes to other gauge groups G that are not simply connected!

Examples in Quantum Gravity

Examples

I'll now survey some examples in string theory with gauge fields, where we can see whether or not the instanton number symmetry is gauged, i.e., is there a $\int C^{(d-4)}\mathrm{tr}(F \wedge F)$ Chern-Simons term or not?

We'll see **examples of breaking** and **examples of gauging**

Keep an eye on:

- Existence of "magnetic" defects playing a role in symmetry breaking or restoration
- Mass scale of electrically charged states

Example: Type IIB on Rigid Calabi-Yau

(Cecotti & Vafa, 1808.03483)

A CY with rigid complex structure is one with $h^{2,1} = 0$: the only 3-cycles are the holomorphic Ω and anti-holomorphic $\overline{\Omega}$. We obtain a 4d U(1) gauge field from $A = \int_\Omega C_4$.

The gauge coupling and θ angle are both frozen, because there are no vector multiplets to provide moduli. Cecotti & Vafa argued that $\theta = 0$ or $\theta = \pi$.

Magnetic monopole: a D3 brane wrapping $\overline{\Omega} \Rightarrow$ breaks $F \wedge F$ symmetry.

Electrically charged particle: a D3 brane wrapping Ω.

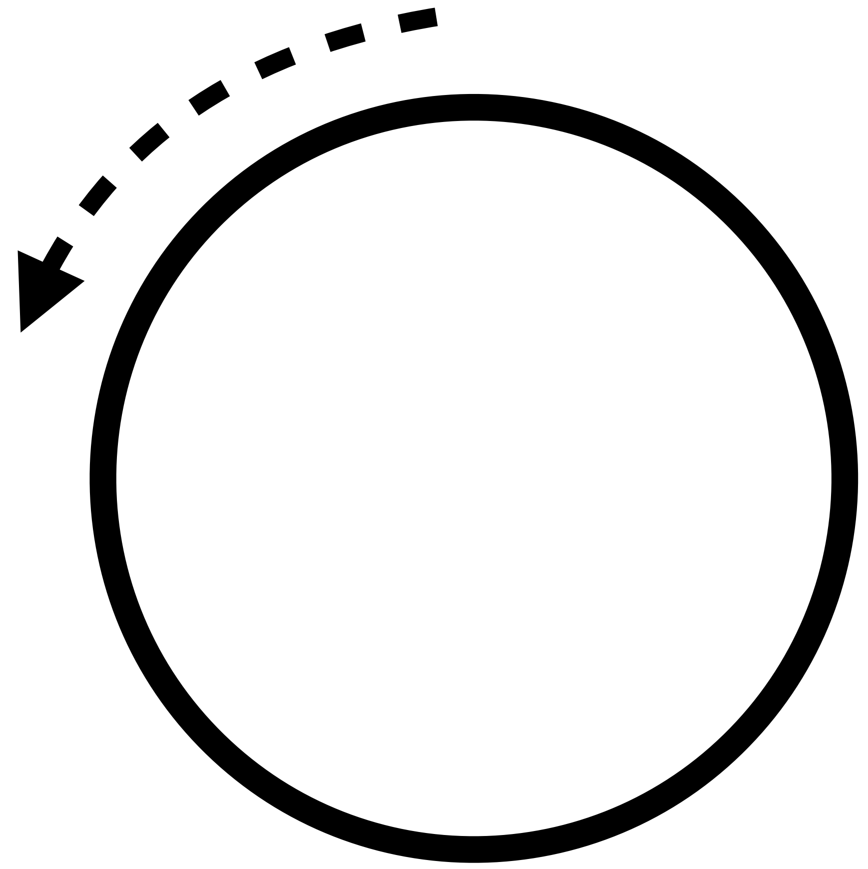
Both are very heavy: $\sim M_{\text{string}}\sqrt{\mathscr{V}} \sim M_{\text{Pl}}$.

Example: U(1) Kaluza-Klein theory

Compactify pure d-dimensional gravity on a circle of radius R. One obtains a U(1) gauge theory in $(d-1)$ dimensions. There is no $C^{(d-5)} \wedge F \wedge F$ term in the effective action.

Magnetic monopole: the Kaluza-Klein monopole, which has tension $\pi R M_{\mathrm{Pl}}^{(d-2)/2}$. Has no dyonic excitation with electric KK charge, and explicitly breaks the $F \wedge F$ symmetry.

Electrically charged particle: Kaluza-Klein graviton, with mass $1/R$.



The d-dimensional EFT breaks down at the mass scale of the electrically charged particle, which is *also* the core radius of the monopole.

Example: Heterotic string theory

In heterotic string theory, the gauge group is $\mathrm{Spin}(32)/\mathbb{Z}_2$ or $(E_8 \times E_8) \rtimes \mathbb{Z}_2$. In both cases, the **instanton number symmetry is gauged by $B^{(6)}$** (the magnetic dual of $B^{(2)}$); this follows from the modified Bianchi identity in the Green-Schwarz mechanism.

Focus on $(E_8 \times E_8) \rtimes \mathbb{Z}_2$. Two groups with permutation symmetry; the combination $\mathrm{tr}_1(F \wedge F) + \mathrm{tr}_2(F \wedge F)$ is gauged but the combination $\mathrm{tr}_1(F \wedge F) - \mathrm{tr}_2(F \wedge F)$ is (explicitly) broken. There are no monopoles. This current is not $\mathbb{Z}_2$ gauge invariant, but still could couple to a $\mathbb{Z}_2$-odd $B'^{(6)}$ gauge field. Why not?*

Is there a defect that makes its breaking visible? There is a $\mathbb{Z}_2$ "twist vortex" (7-brane). Need a clearer general theory of such examples of explicit breaking.

(thanks to Jake McNamara for emphasizing this question to me)*

Example: Gauge fields on D-branes

In string theory, gauge fields can live on a stack of D*p*-branes, which have a (*p*+1)-dimensional worldvolume. In these cases, we always find that the Chern-Weil current $\mathrm{tr}(F \wedge F)$ is gauged by a closed string $(p-3)$-form Ramond-Ramond field:

$$C^{(p-3)} \wedge \mathrm{tr}(F \wedge F)$$

So far, so good. But this field actually propagates into the bulk, where it couples to lower-dimensional membranes, so a more complete story is:

$$C^{(p-3)} \wedge \left[\mathrm{tr}(F \wedge F) \wedge J_{Dp} + J_{D(p-4)}\right]$$

Where J_{Dq} is a $(9-q)$-form (the number of delta functions needed to localize on the brane).

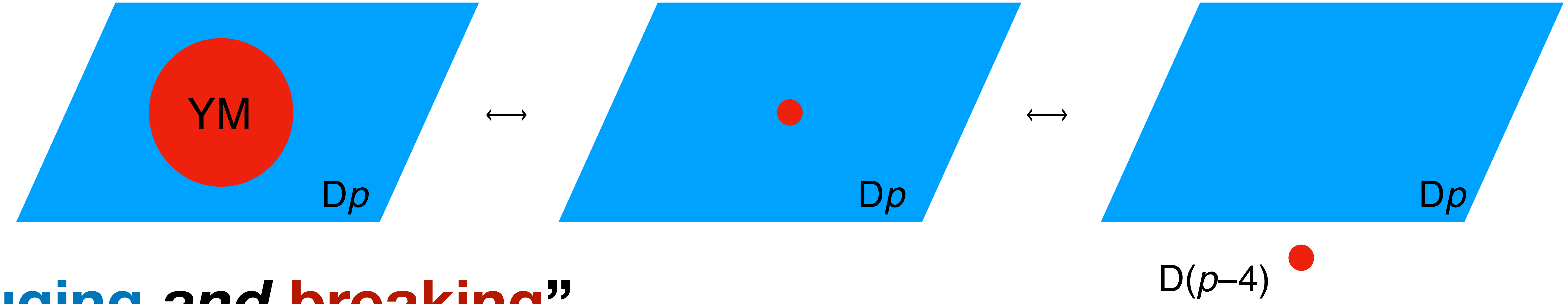
Example: Gauge fields on D-branes

If the closed string gauge field $C^{(p-3)}$ is gauging the current in brackets,

$$C^{(p-3)} \wedge \left[\mathrm{tr}(F \wedge F) \wedge J_{Dp} + J_{D(p-4)}\right]$$

then **what happens to the other linear combination of these two conserved currents?**

The answer is a well-known effect in string theory: **zero-size Yang-Mills instantons on the D*p*-brane are *the same thing* as D(*p* – 4)-branes.**
(Witten '95; Douglas '95; Green, Harvey, Moore '96).



"**Gauging *and* breaking**"

Example: Gauge fields on D6-branes

Consider the specific case where we get 4d gauge fields from D6-branes wrapped on a 3-cycle α, and they coupled to an axion $\theta = \int_\alpha C^{(3)}$. We could try to **decouple** this axion using the 10d term $\frac{1}{8\pi^2}\int C^{(3)} \wedge \mathrm{d}C^{(3)} \wedge H^{(3)}$, turning on a flux $\int_\beta H^{(3)}$ through an intersecting cycle β. In 4d this turns into a large $\frac{1}{2\pi}\theta F_4$ tree-level axion mass. The instanton number symmetry is effectively **broken**: instantons can dissolve into F_4 flux.

However, there is a catch: this **obstructs the existence of chiral fermions** charged under the gauge fields on the cycle α!

Example: Gauge fields on D6-branes

Chiral fermions live at the intersection of D6 branes on α and D6 branes on an intersecting cycle β. If $\int_\beta H^{(3)} \neq 0$, then we *cannot* wrap D6 branes on β! The gauge field A on such D6 branes has a Stueckelberg coupling involving $\mathrm{d}A - B^{(2)}$, which dualizes to a term $\frac{1}{2\pi}\int_{M^{3,1}\times\beta} A^{(4)}_{\mathrm{M}} \wedge H^{(3)}$.

The $H^{(3)}$ flux gives a tadpole in 4d for the magnetic gauge field, which is inconsistent.

Thus, the D6-brane example has a light axion coupled to gauge fields whenever there are chiral fermions charged under the gauge field.

Patterns in Examples

In cases where we understand the breaking of instanton number symmetry to be due to monopoles, electrically charged particles are heavy — they have mass at the cutoff scale.

In Standard Model-like examples with light chiral matter, we find axions.

This pattern is at least partly explained by the Callan-Rubakov effect: to define consistent boundary conditions for light charged fermions at the core of a magnetic monopole, we couple them to a localized dyon mode.

The $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ example suggests that the full story is more complex: e.g., what if there is a “twin” Standard Model and only one axion for both? (considered in phenomenological models: Rubakov ’97; Berezhiani, Gianfagna, Giannotti ’00; Dimopoulos, Hook, Huang, Marques-Tavares ’16)

Classifying the Options

The current $\mathrm{tr}(F \wedge F)$ is either **gauged** or **explicitly broken**.

Gauged: axion *or* fermion with anomalous but otherwise unbroken chiral symmetry $\Rightarrow$ classic Strong CP solutions.

Explicitly broken: visible with defect in EFT. This defect could be…

Monopole: only if $\pi_1(G) \neq 1$ *and* no chiral fermions

Twist vortex: only if $\pi_0(G) \neq 1$. Characterize better?

Something else? How to be exhaustive?

For explicit breaking, θ is frozen to discrete possible values $\Rightarrow$ look for new Strong CP solutions? (or known ones, e.g., twin axion)

Axion Models and Cosmology: Overview

Axion Models at a Glance

	Pseudo-Nambu-Goldstone for 4d $U(1)_{PQ}$	Zero mode of gauge field in higher dimensions
"Pre-inflation" scenario	Quality problem Isocurvature problem	(Quality problem) Isocurvature problem
Post-inflation PQ transition	Quality problem Domain wall problem Stable relic problem	**Not possible** (no linearly realized PQ symmetry to break) (maybe similar late-time physics from other initial conditions?)

Post-Inflation Axion Cosmology

4d U(1) PQ symmetry spontaneously broken after inflation.

- Axion randomized, strings form (Kibble-Zurek)
- QCD phase transition: axion domain walls form
- String-wall network destroys itself ($N_{\rm DW} = 1$)

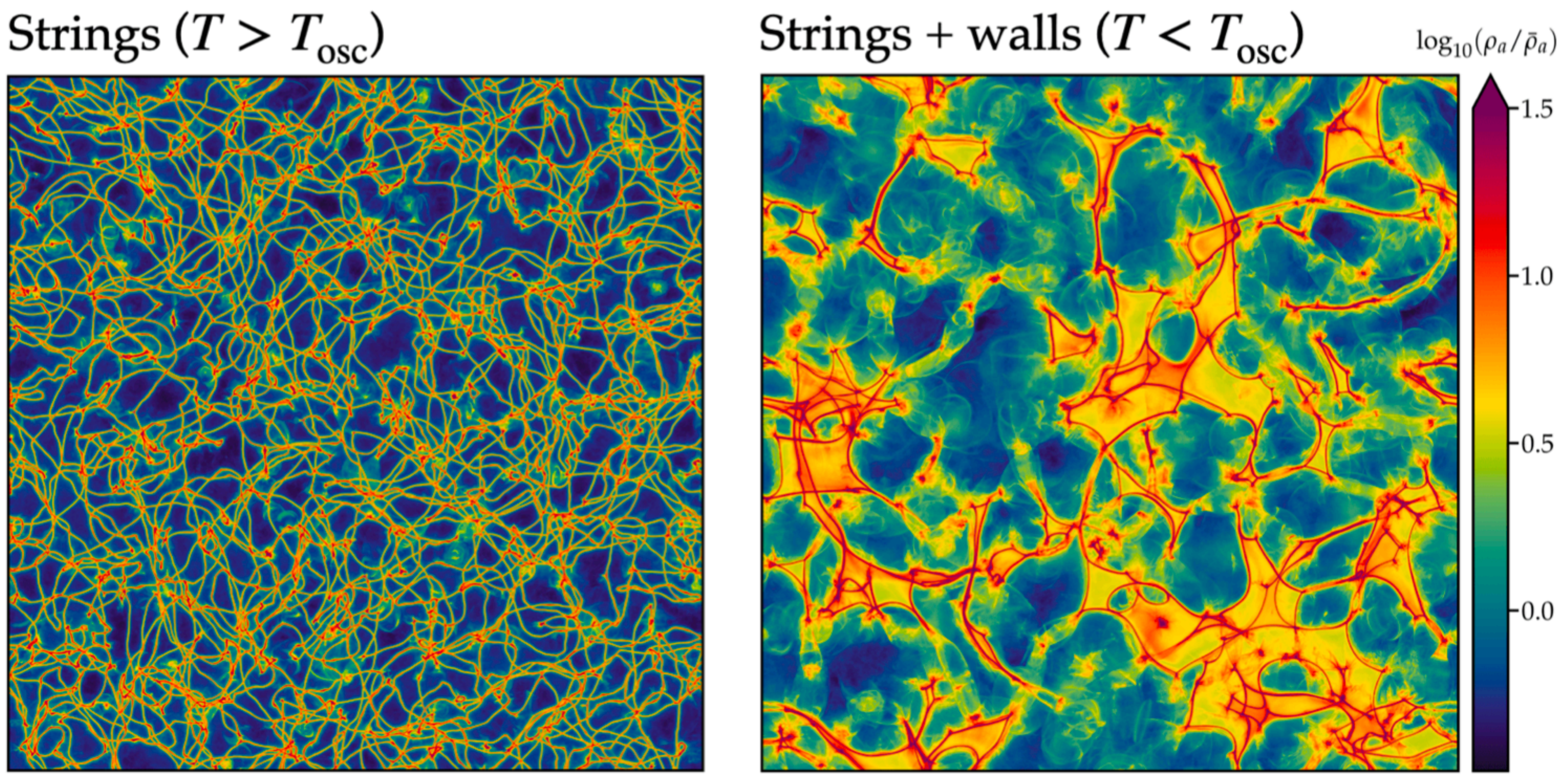


Figure from **Ciaran O'Hare**'s lectures on axion cosmology, arXiv:2403.17697 [hep-ph]
Using code from **Alejandro Vaquero, Javier Redondo, Julia Stadler**, arXiv:1809.09241

Axion dark matter relic abundance dominantly from axion emission from string network, as well as misalignment.

Detailed simulations, e.g., Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi arXiv:2108.05368

Post-Inflation: Axion Domain Wall Problem

Domain walls can end on strings if

$$\int \frac{k_G}{8\pi^2}\theta\,\mathrm{tr}(F \wedge F)$$

has minimal coupling $N_{\mathrm{DW}} = |k_G| = 1$.

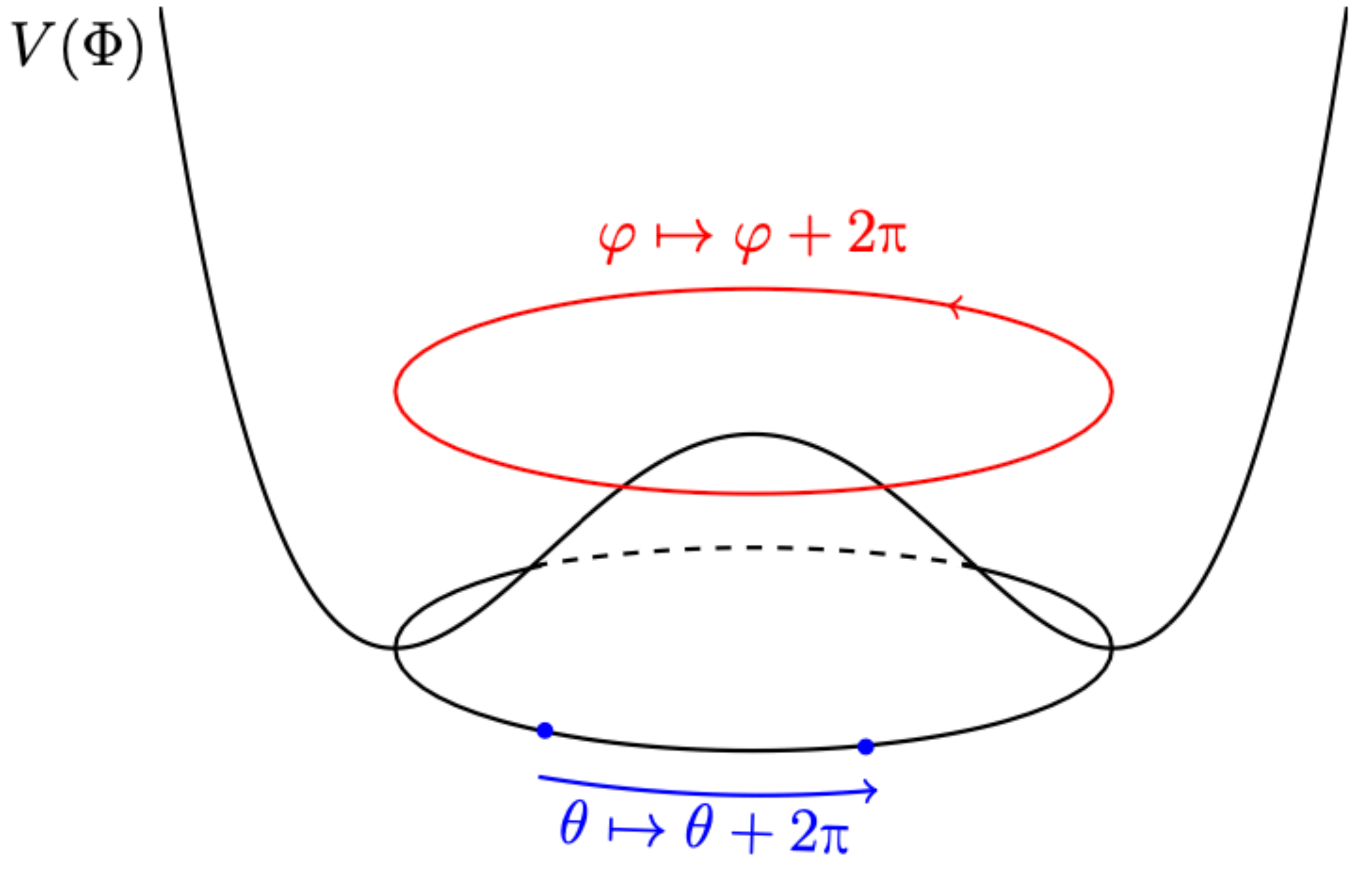
But such strings may not form, or may not be elementary! Tension w/ models for quality problem

Example:

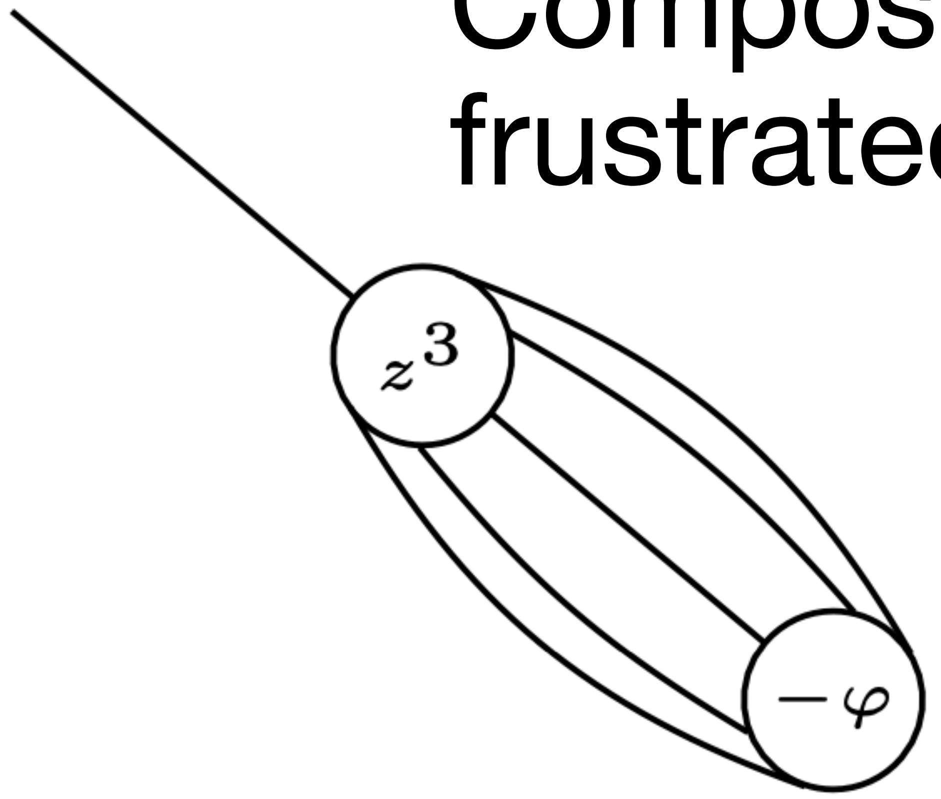
$\mathbb{Z}_p$ symmetry,

$\Phi(x) = fe^{i\varphi(x)} = fe^{i\theta(x)/p}$

Kibble-Zurek: string of winding p



Composite string — frustrated network



Hard to find convincing models!


w/ Qianshu Lu, Zhiquan Sun

arXiv:2312.07650 [hep-ph]

“Pre-Inflation”: Axion Isocurvature Problem

A light scalar during inflation fluctuates by $\delta\varphi \sim H_I/(2\pi)$. Fluctuations independent of inflaton fluctuations $\Rightarrow$ ***isocurvature***, strongly constrained.

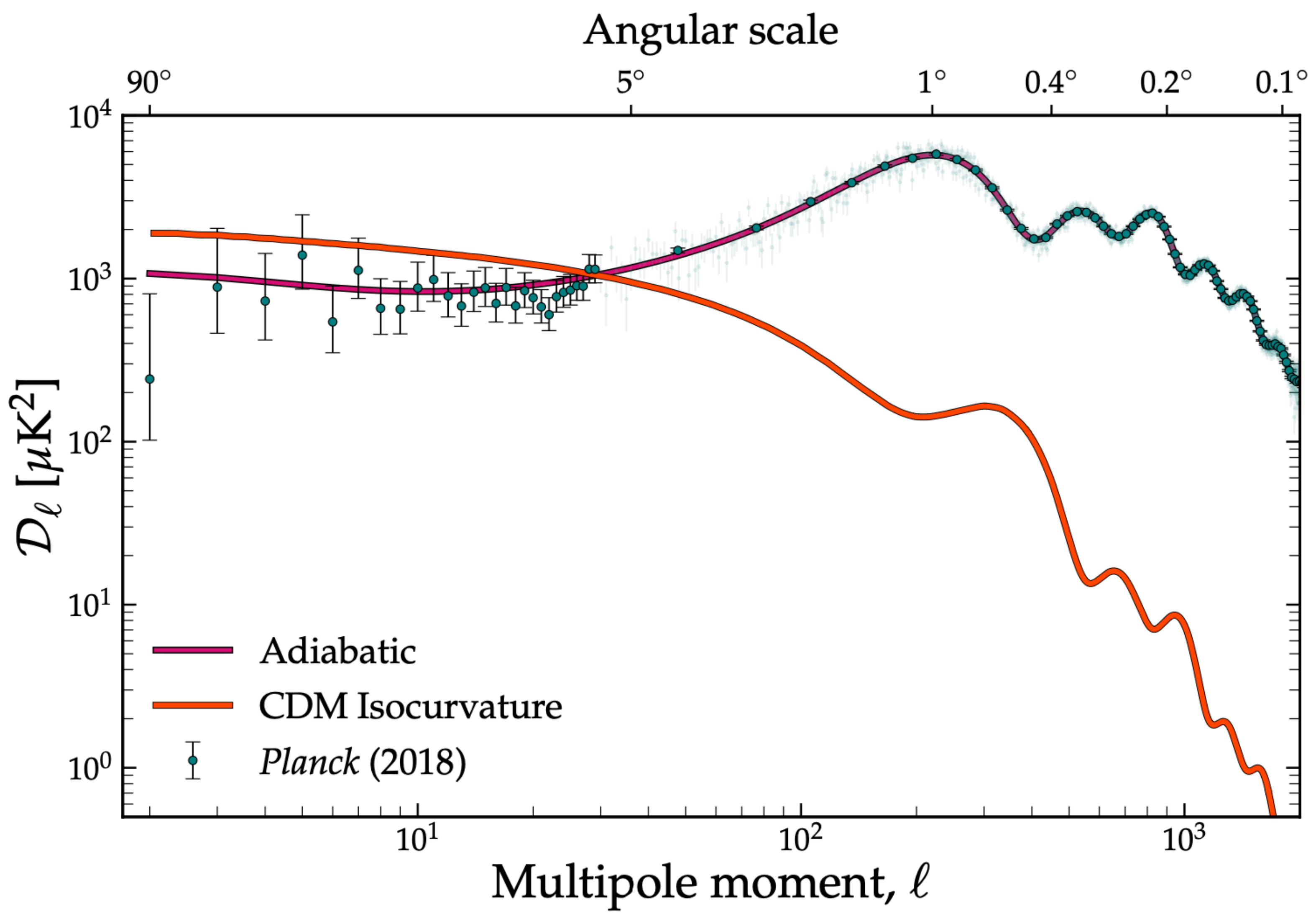


Figure from **Ciaran O’Hare**’s lectures on axion cosmology, arXiv:2403.17697 [hep-ph]

Leads to a bound

$$H_I \lesssim 3 \times 10^7\,\mathrm{GeV}\,\frac{f_I}{10^{12}\,\mathrm{GeV}}$$

which is much stronger than the observational bound from (lack of) tensor modes,

$$H_I \lesssim 10^{13}\,\mathrm{GeV}\,.$$

Is a bound a **problem**? Not a sharp one, but the simplest and most natural inflation models are large-field (hence high-scale).

Solutions to the Axion Isocurvature Problem

Several ideas have been discussed in the literature for opening up a wider range of H_I for a given axion decay constant. Broadly,

- Turn on larger $|\Phi|^2$ term during inflation — back to post-inflation.
- **Dynamical axion mass**, heavier than H_I during inflation, e.g., make QCD very strongly coupled so Λ_{QCD}/f is not small. (Dvali '95, …)

  [Awkward to continuously change *exponentially tiny* number to O(1)!]

- **Dynamical axion decay constant,** $f_I \gg f_a$ to relax bound (Linde/Lyth '90, …)

String pheno: time-varying modulus can lead to both of the last two.

Rest of this talk: a new variation on dynamical axion mass.

Eliminating Axion Isocurvature: A New Approach

w/ Prish Chakraborty, Junyi Cheng, Zekai Wang
expected to appear on arxiv this summer

Monodromy Mass vs. Isocurvature

An axion $\theta \cong \theta + 2\pi$ can get a large ("monodromy") mass from a Chern-Simons coupling to a 4-form field strength $F^{(4)} = \mathrm{d}C^{(3)}$:

$$S = \int -\frac{1}{2} f^2 |\mathrm{d}\theta|^2 - \frac{1}{2g^2} |F^{(4)}|^2 + \frac{n}{2\pi} \theta F^{(4)}, \quad n \in \mathbb{Z}\,.$$

[Kallosh, Linde, Linde, Susskind '95; Gabadadze '99; Silverstein, Westphal '08; Kaloper, Sorbo '08; ....]

Axion mass: $m_{\hat{\theta}} = \frac{n}{2\pi} \frac{g}{f}\,.$

Main idea:

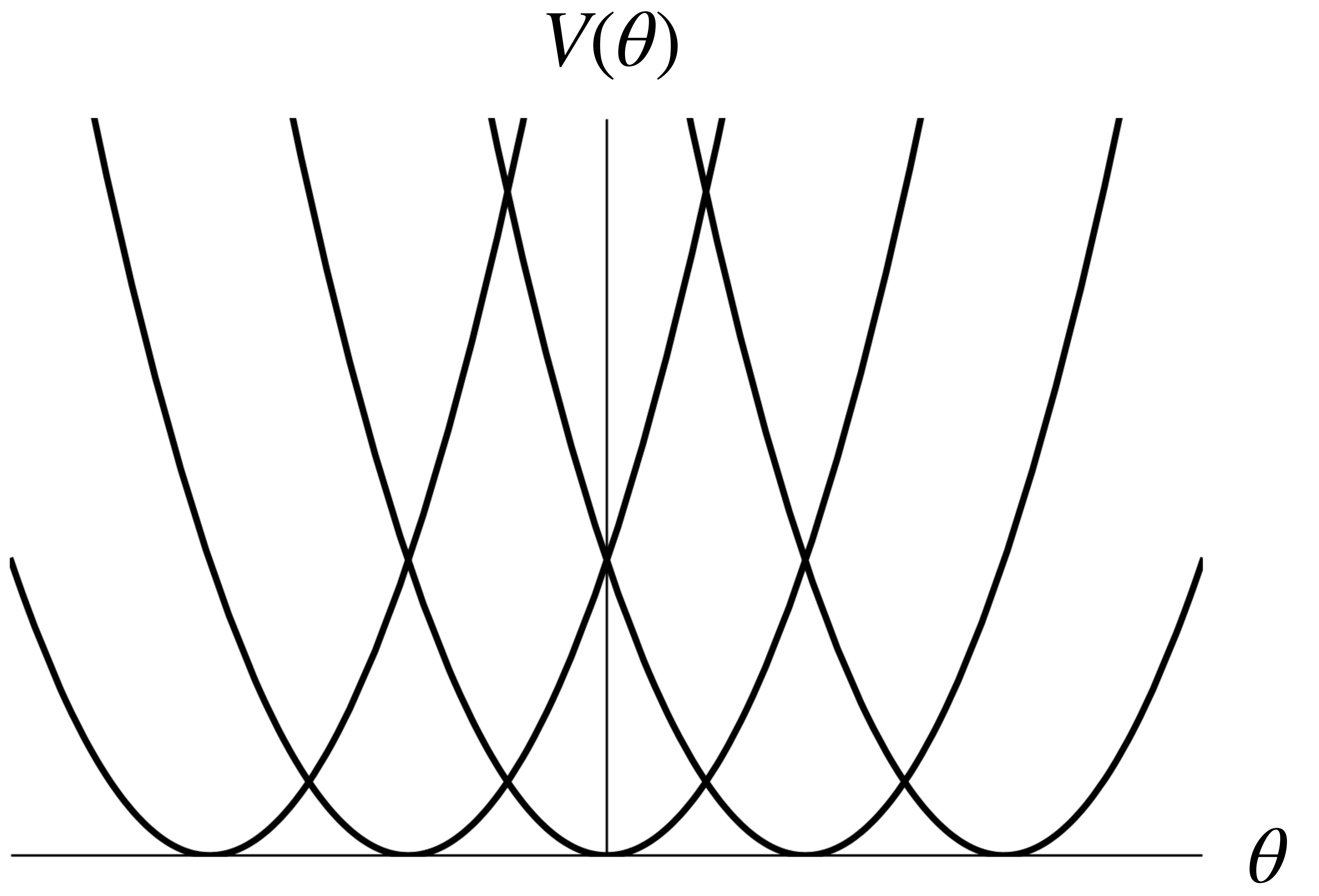
If $n \in \mathbb{Z}$ is a ***dynamical integer***, it could be nonzero during inflation (heavy axion, no isocurvature) and zero today (standard axion).

Change between them with a ***first-order phase transition***.

Avoiding confusion

$$S = \int -\frac{1}{2} f^2 |\mathrm{d}\theta|^2 - \frac{1}{2g^2} |F^{(4)}|^2 + \frac{n}{2\pi} \theta F^{(4)}, \quad n \in \mathbb{Z}\,.$$

The monodromy potential $V(\theta)$ has infinitely many branches labeled by an integer $j = \frac{1}{e_4^2} \star F^{(4)} - \frac{n}{2\pi}\theta$, and a gauge invariance $\theta \mapsto \theta + 2\pi,\, j \mapsto j - n\,.$



j, the $C^{(3)}$ electric field, is **always dynamical**. It is **not** the dynamical integer n that we wish to change in cosmology.

Making n Dynamical

Idea: the integer n is flux of higher-dimensional gauge field, $n = \frac{1}{2\pi}\int_{\Sigma^{(q+1)}} \mathrm{d}A^{(q)}$

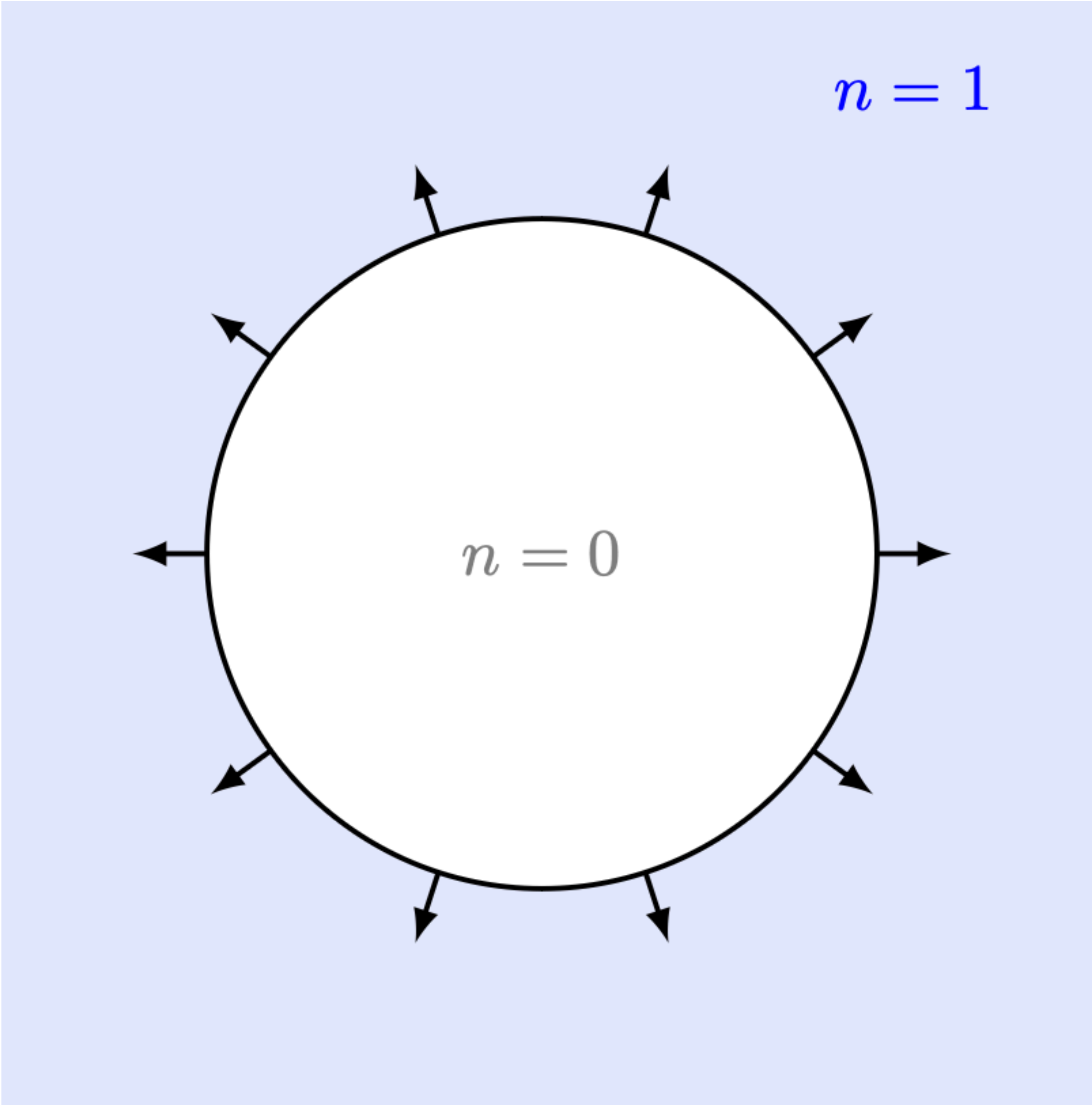
Extra-dimensional axion $\theta = \int_{\Lambda^{(p)}} C^{(p)}$

Chern-Simons in $n = p + q + s + 1$ extra dims:

$$\frac{1}{4\pi^2}\int_{M^{(4)}\times Y^{(n}} C^{(p)} \wedge \mathrm{d}A^{(q)} \wedge \mathrm{d}C^{(3+s)}$$

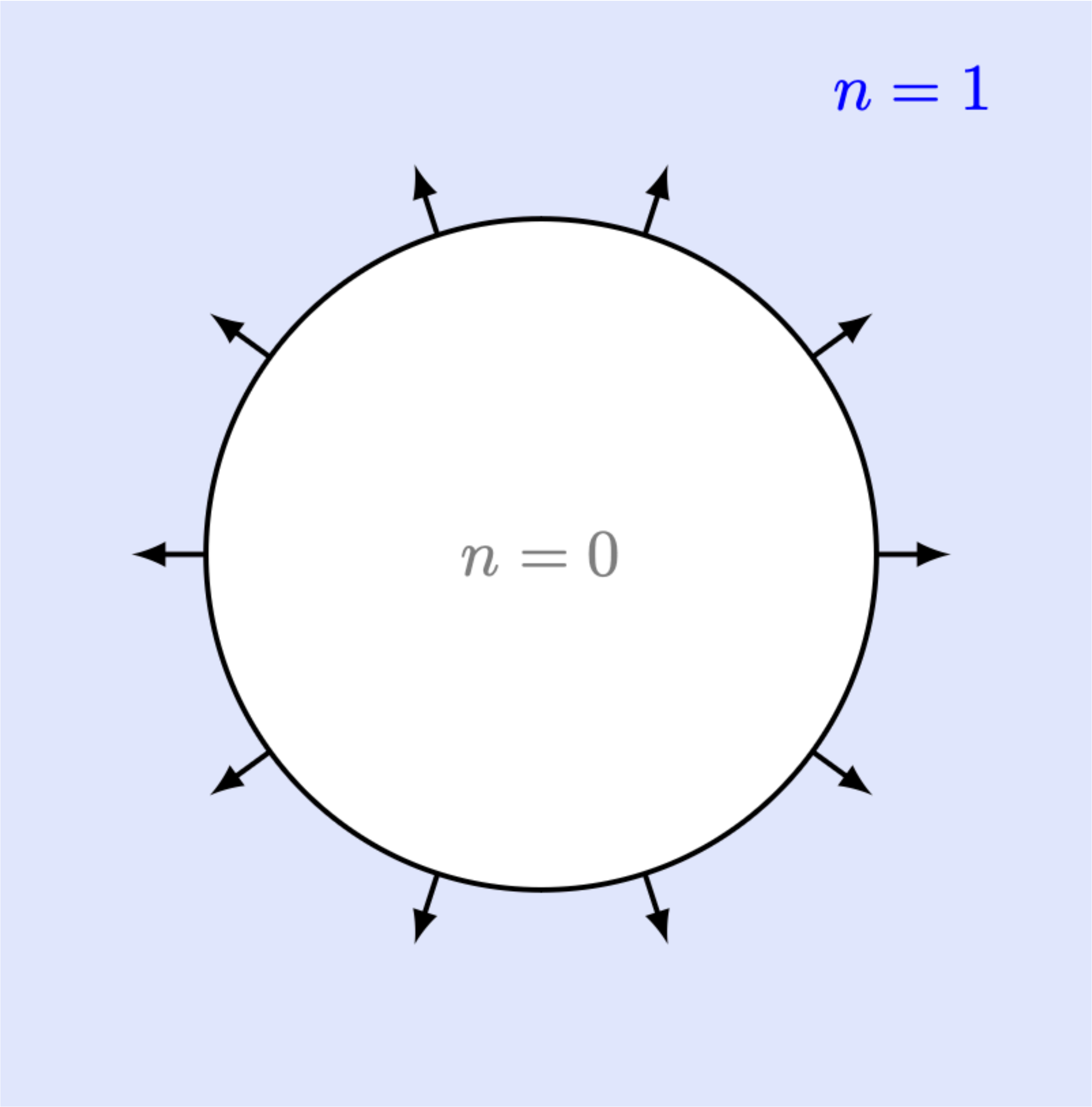
$$\Downarrow$$

$$\int_{M_4} \frac{n}{2\pi}\theta F^{(4)}$$



Flux Tunneling

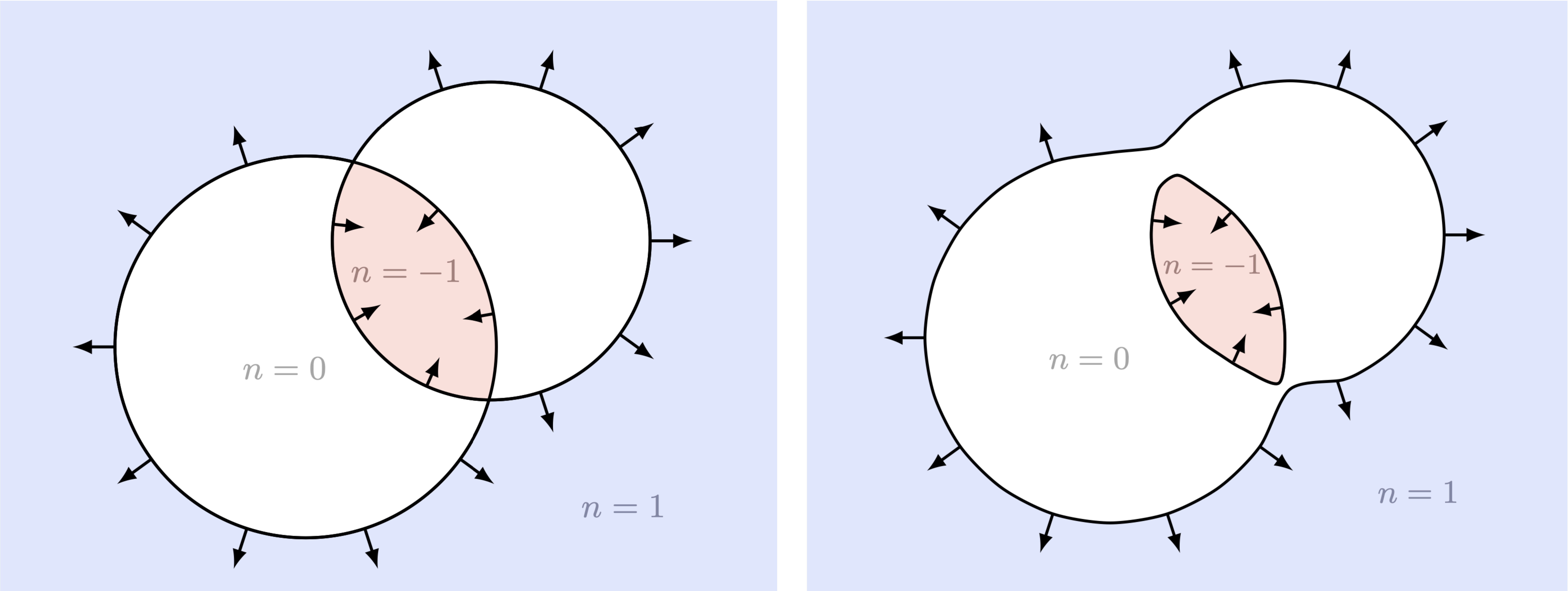
Our tunneling process must change the flux $n = \frac{1}{2\pi} \int_{\Sigma^{(q+1)}} \mathrm{d}A^{(q)}$.



This can only happen by nucleating a dynamical **magnetically charged brane** for $A^{(q)}$. This has $4 + n - q = 3 + r + s$ dimensions. Wrapping the $r + s$ internal dimensions transverse to $\Sigma^{(q+1)}$, we have a **domain wall** in (3+1)d.

(see, e.g., Blanco-Pillado, Schwartz-Perlov, Vilenkin ’09, but details differ — we do *not* want a Freund-Rubin compactification, our flux is through a cycle in a larger geometry)

Bubble Mergers



Provided the $n = 0$ state has lowest vacuum energy, we expect colliding branes to reconnect and the $n \neq 0$ regions to collapse.

Flux Tunneling at the End of Inflation?

Need to suppress isocurvature: $|n| > 0$ during inflation. Drops to 0 after.

Vacuum energy contribution $V(n)$ from flux energy density: could that provide the energy driving inflation?

"Graceful exit" problem of old inflation: need to make bubble nucleation rate time-dependent. $\Gamma(t) < H(t)^4$ until some critical time t_*.
Scenarios:

- Inflaton ϕ affects $\Gamma(t)$, e.g., brane tension $\mathscr{T}(\phi)$ dynamical.
- Tunneling as inflation is ending, $H(t)$ starts to drop rapidly.

$$S = \int -\frac{Z_n}{2}|\mathrm{d}\phi|^2 - \frac{1}{2}f_n(\phi)^2|\mathrm{d}\theta|^2 - \frac{1}{2g_n(\phi)^2}|F^{(4)}|^2 + \frac{n}{2\pi}\theta F^{(4)} + V(\phi, n) + h(\phi)\mathscr{T}\delta^{(2)}(M)$$

String Theory Embedding?

All the *ingredients* exist in string theory, e.g.:

Type IIA model with D6 branes, $\theta = \int_\alpha C^{(3)}$, dynamical integer $n = \int_\beta H^{(3)}$, axion mass from $C^{(3)} \wedge \mathrm{d}C^{(3)} \wedge H^{(3)}$.

The bubble wall is an NS5 brane wrapped on β.

Inside the wall: D6's wrapped on β for realizing Standard Model.
Outside the wall: obstructed by $H^{(3)}$ flux on β.

Dynamical emergence of chirality after inflation? (Potential implications for baryogenesis, Festina Lente bound, ….)

Summary

- Axions play an important role in quantum gravity, by gauging instanton number symmetry. The alternative, explicit breaking, can happen but known examples are not SM-like.
- Conventional 4d QCD axion models face serious cosmological challenges.
- Extra-dimensional axions primarily face the axion isocurvature problem: difficult to combine with high-scale inflation.
- Possible scenario: time-dependent moduli fields after inflation change the value of the decay constant.
- **Novel scenario:** first-order phase transition from large tree-level axion mass during inflation to zero mass afterward. Implications for reheating, gravitational waves, and more. Can we find a realistic version of this scenario?