Axion strings:

Gravitational waves (and axion stars)

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Axions (=ALP)

- Axion a, shift symmetry $a \to a + c$, candidate axions generic in high energy theories

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This talk: the dynamics of simple field theory axions as a first step

Caution: Likely to be many important differences (production of strings, core structure, cosmological history, KK modes, etc.) in more realistic theories

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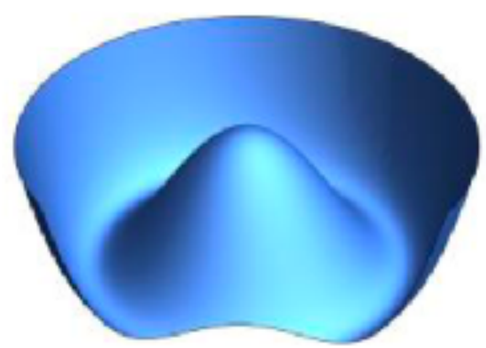
$$\mathcal{L} = |\partial_\mu \phi|^2 - \frac{m_r^2}{2v^2}\left(|\phi|^2 - \frac{v^2}{2}\right)^2$$

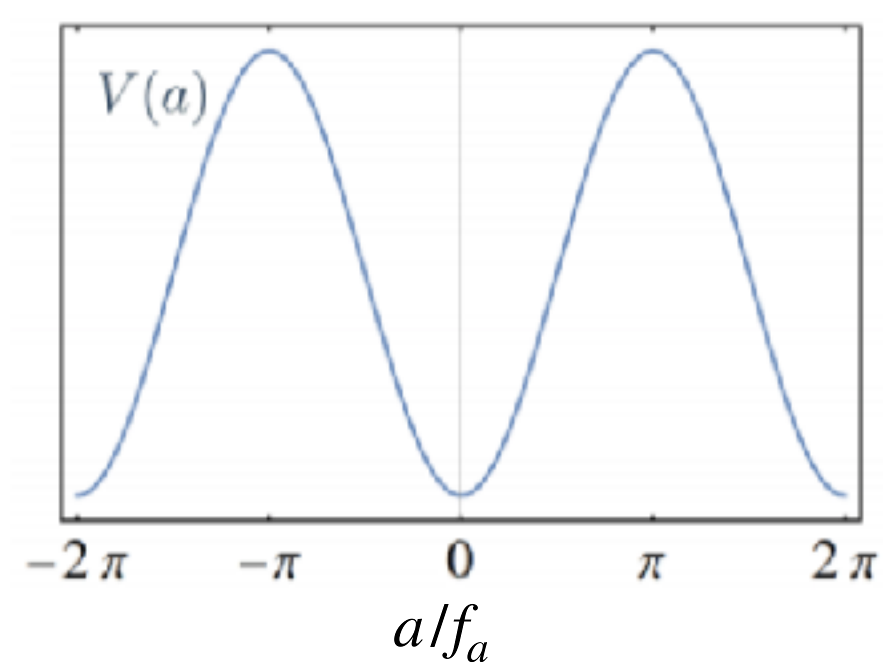
$$\phi = \frac{(v+r)}{\sqrt{2}} e^{ia/v}$$



Axion decay constant f_a such that

$a \cong a + 2\pi f_a$

Assume $m_r \sim f_a$



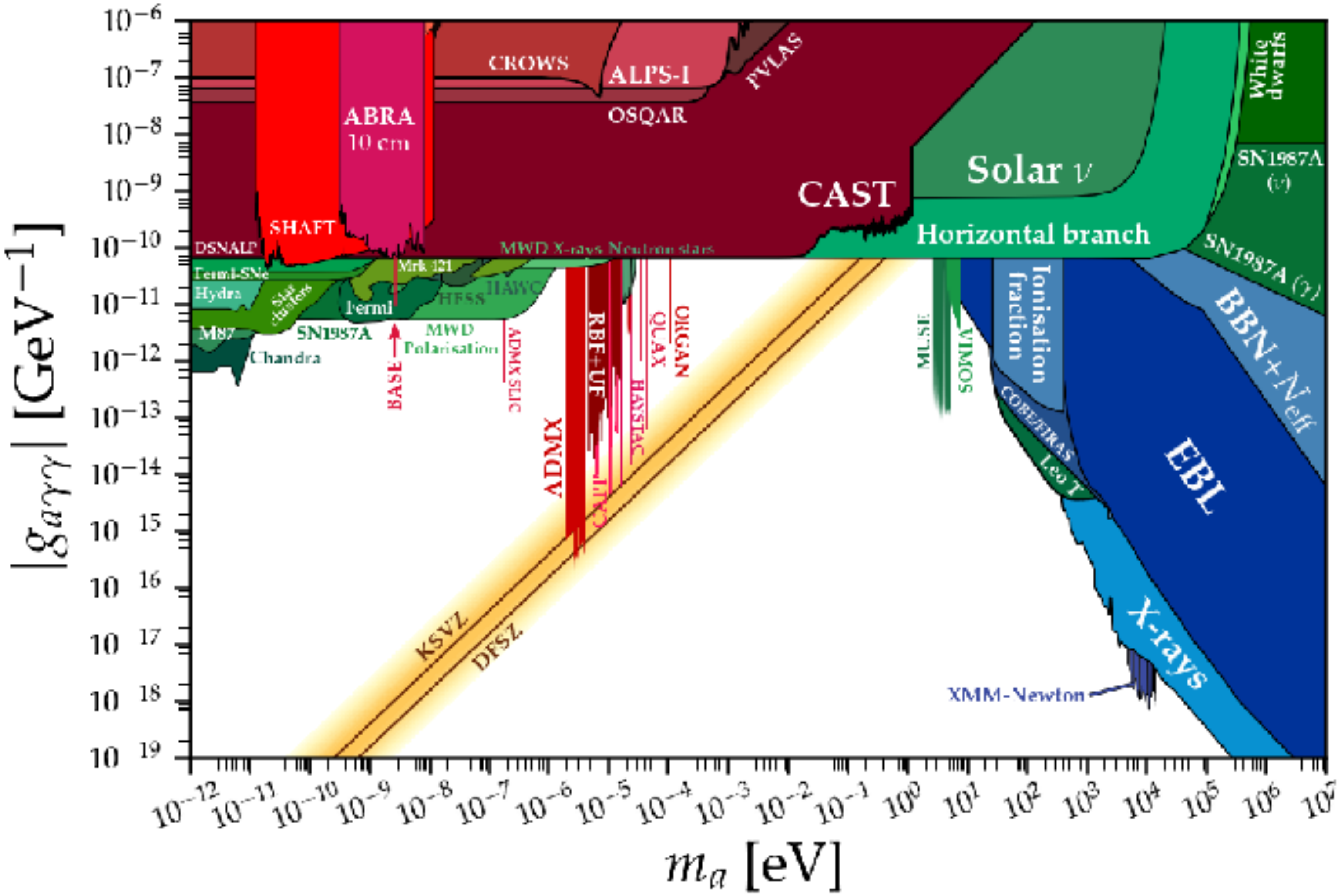
$\theta = a/f_a$

Axion mass m_a

$(N_W = 1)$

Searches

$$\mathscr{L} \supset -\frac{1}{4} g_{a\gamma\gamma} a \; F_{\mu\nu}\tilde{F}^{\mu\nu}$$



https://cajohare.github.io/AxionLimits

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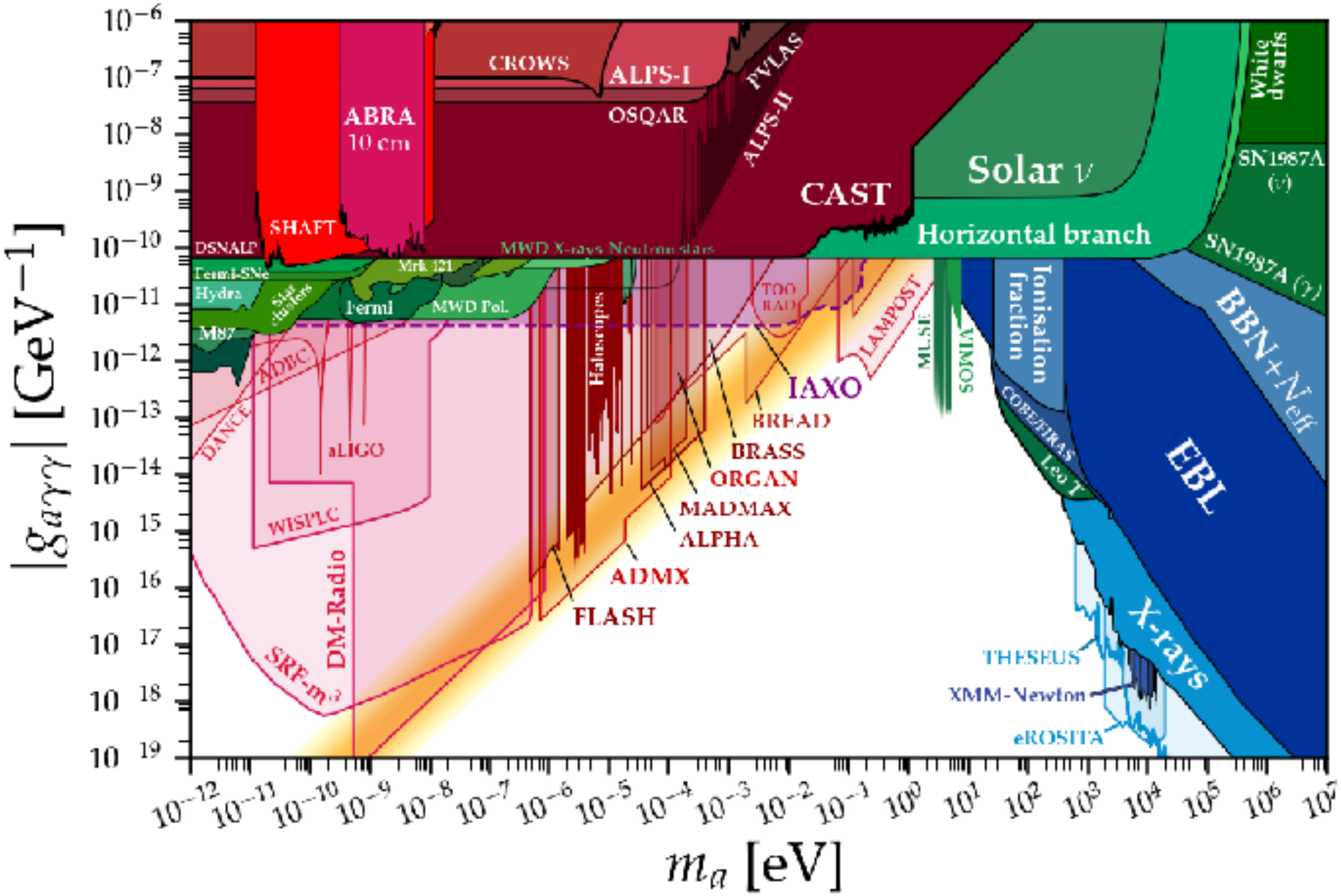

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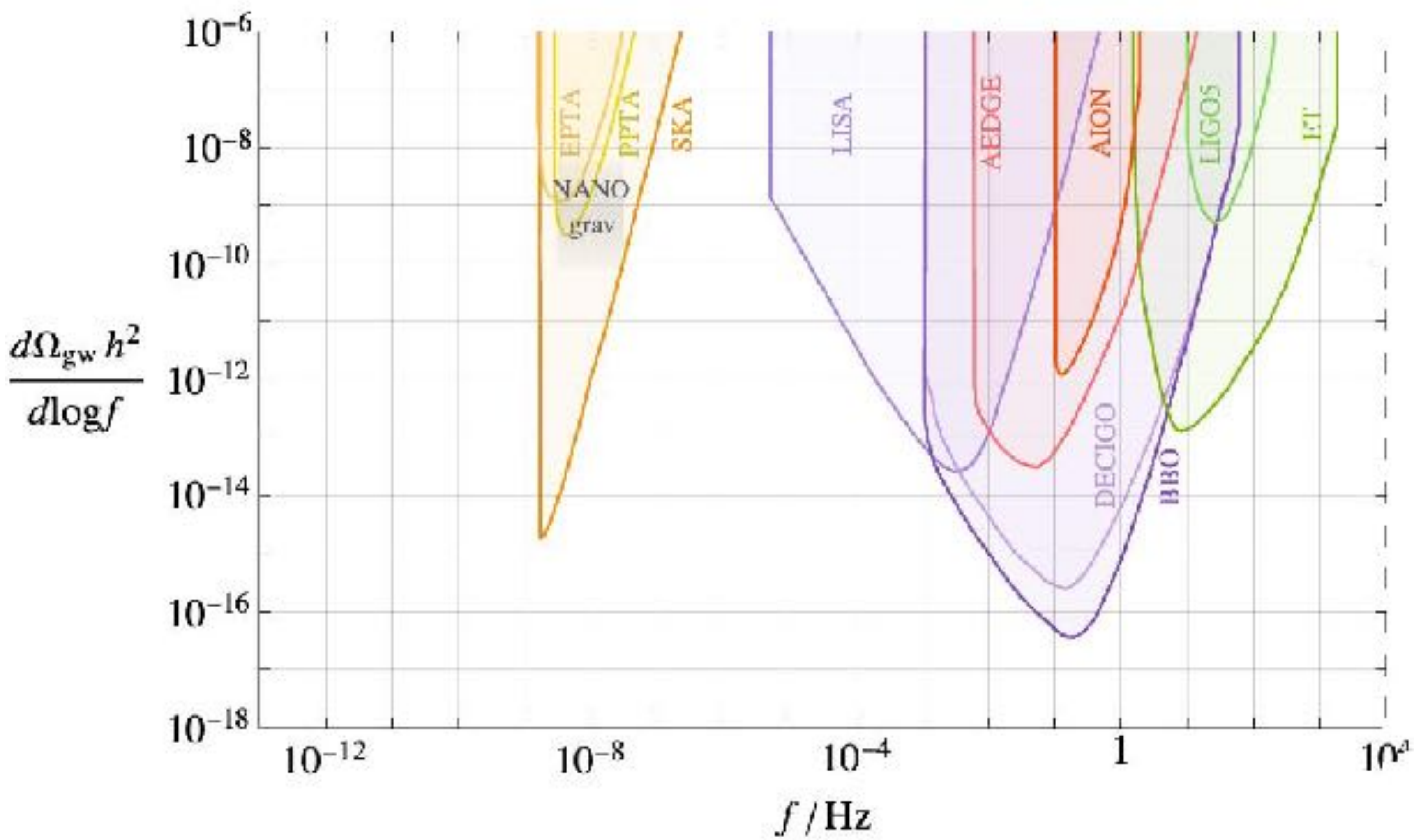
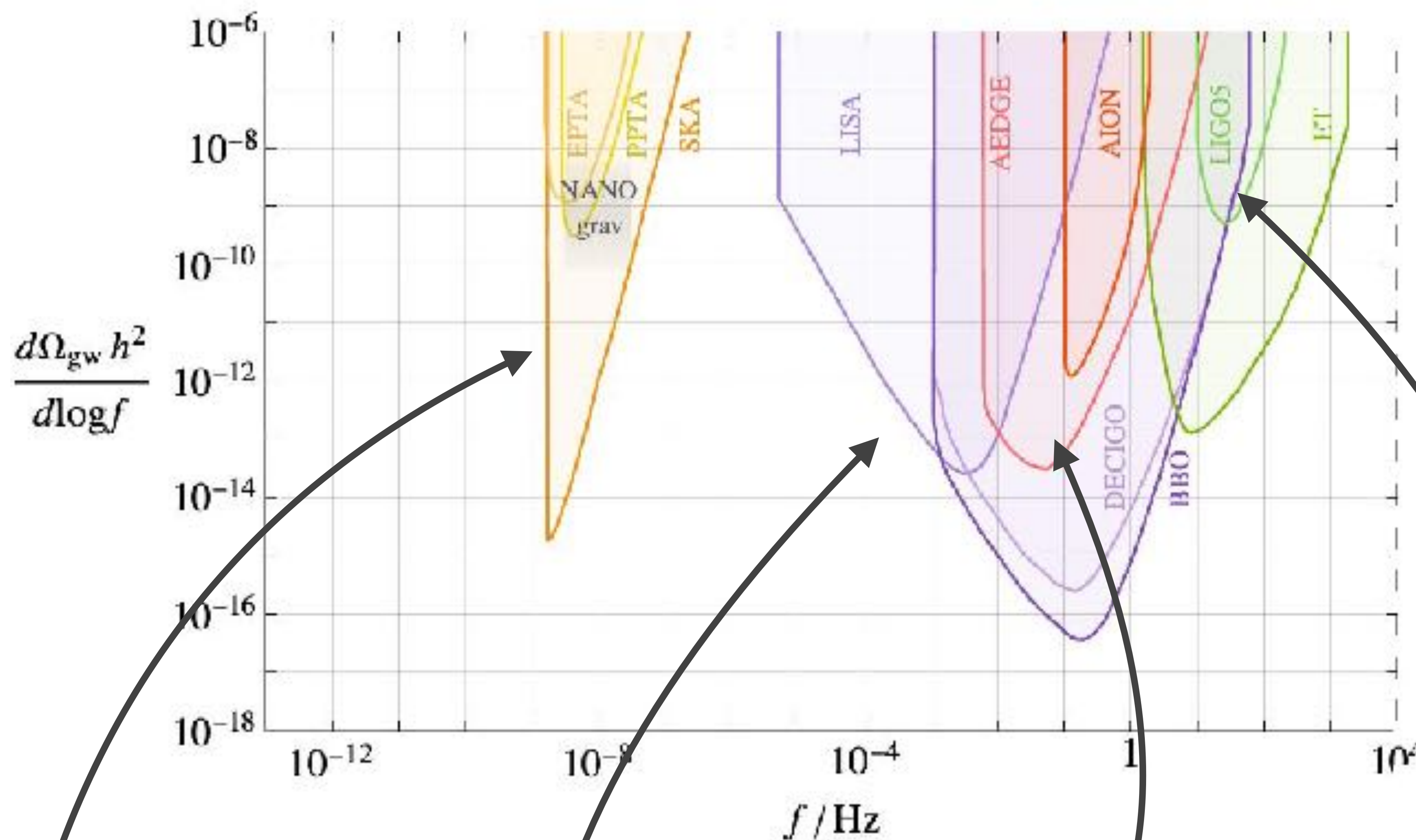
Gravitational wave searches



Caution:
Many experimental challenges to overcome

Gravitational wave searches



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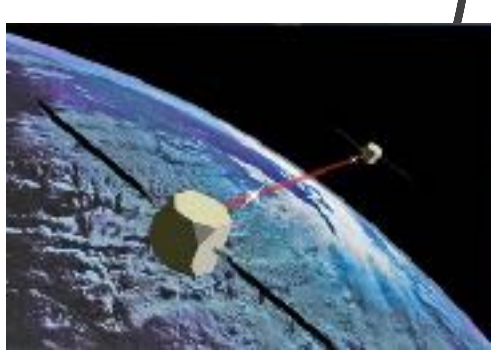

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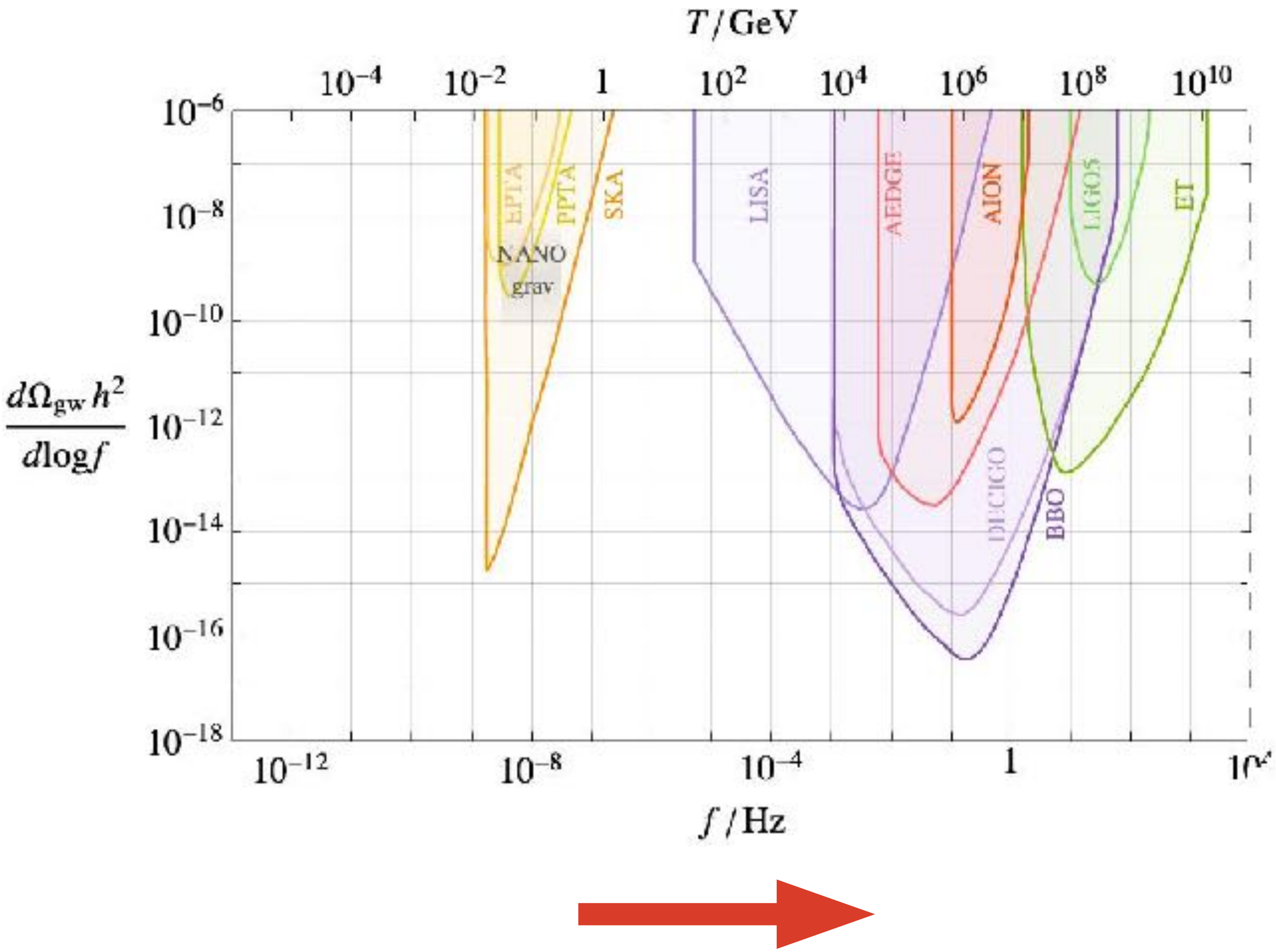


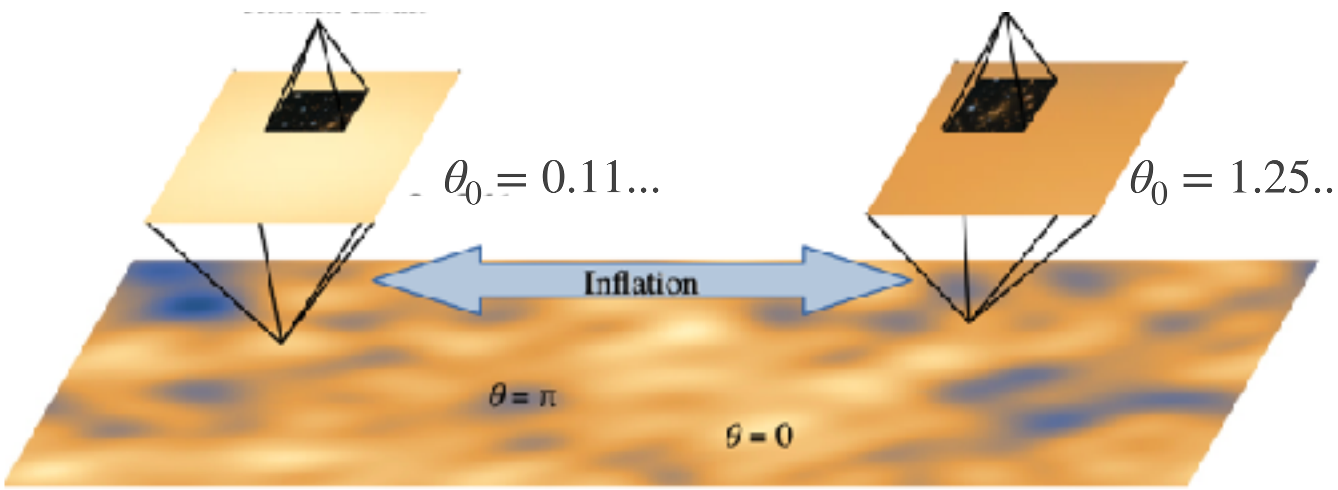
The early universe

Caution:
Many experimental challenges to overcome

Initial conditions

Pre-inflationary

Observable universe Observable universe



Initial conditions

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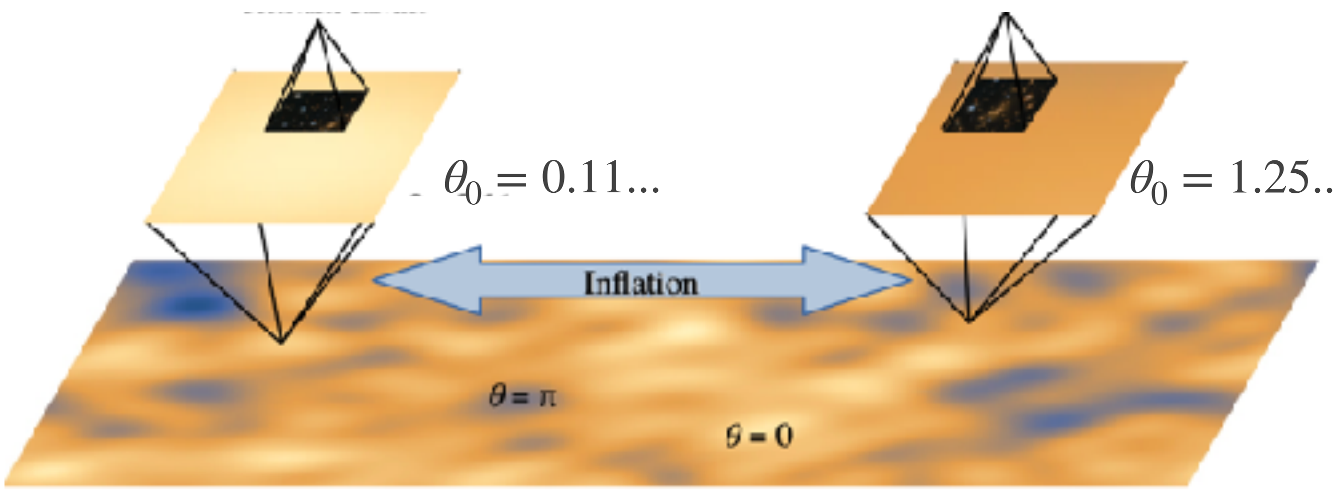
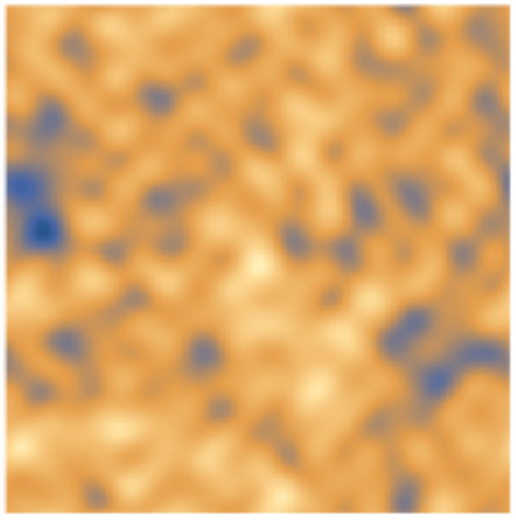
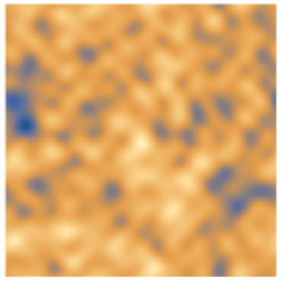


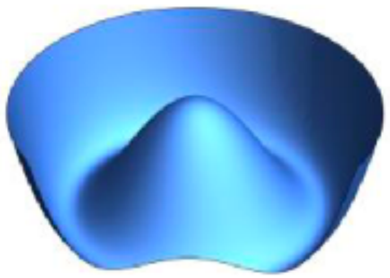
Post-inflationary

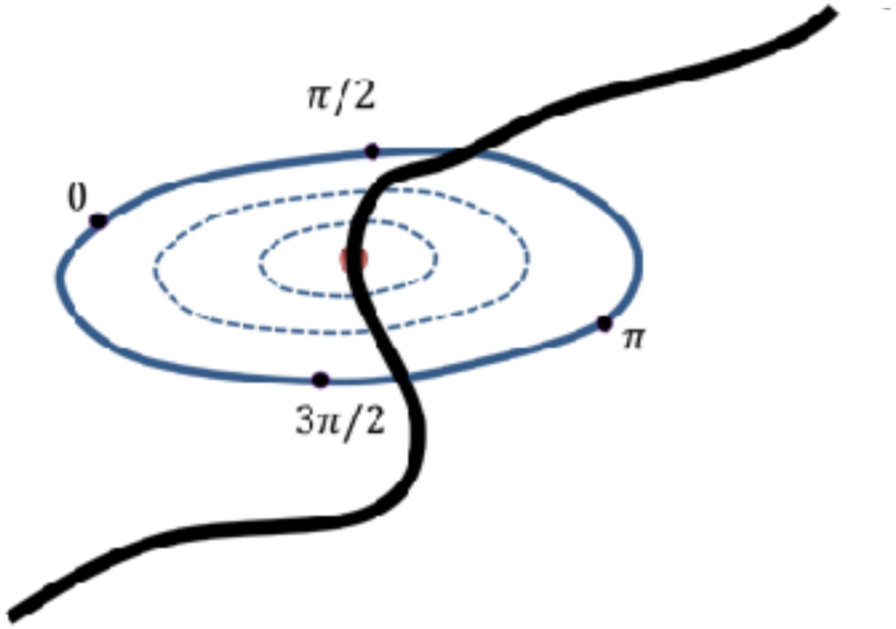
Observable universe



Topological strings







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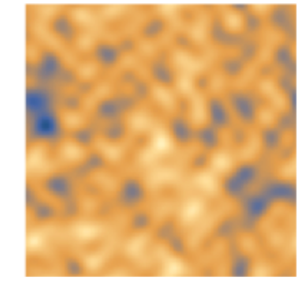
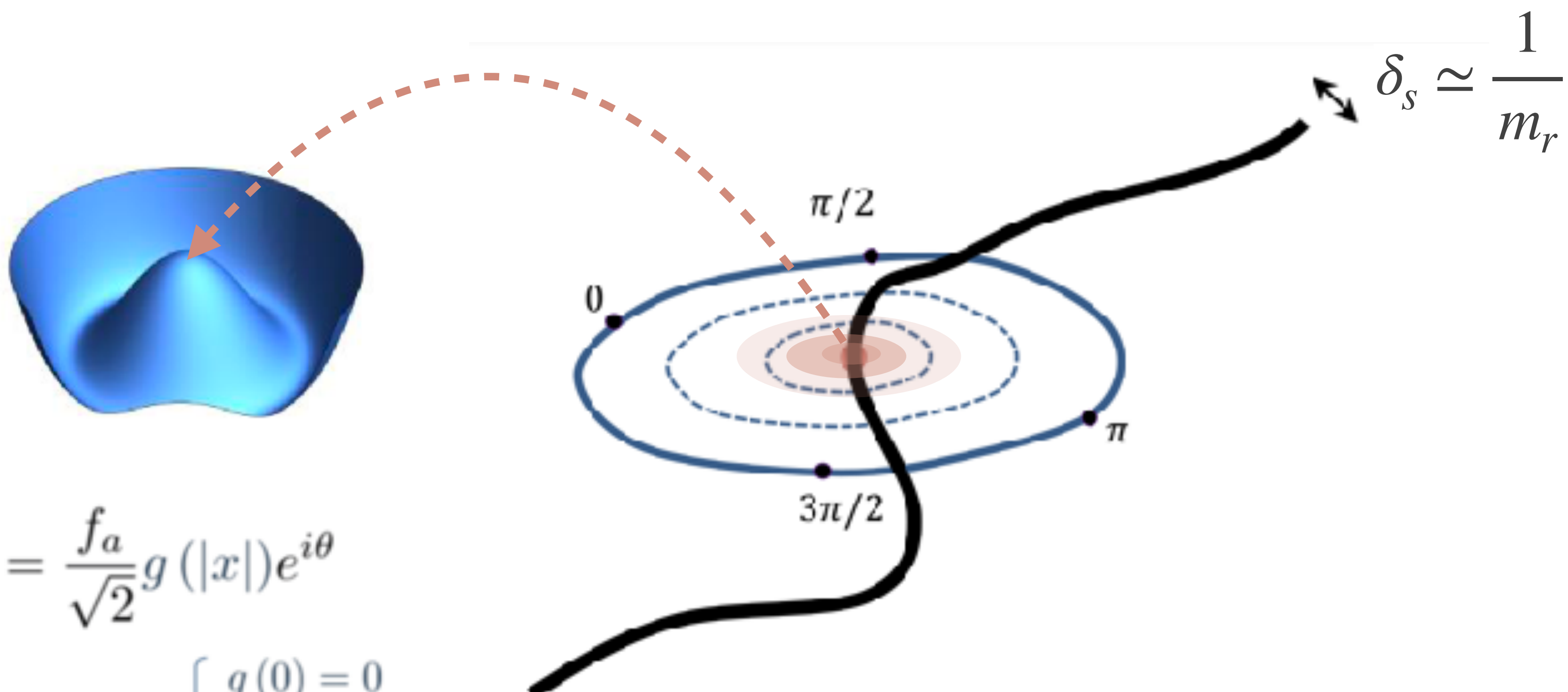
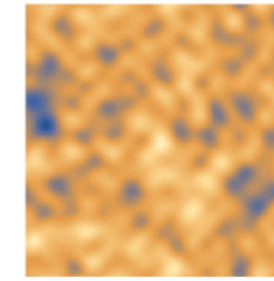


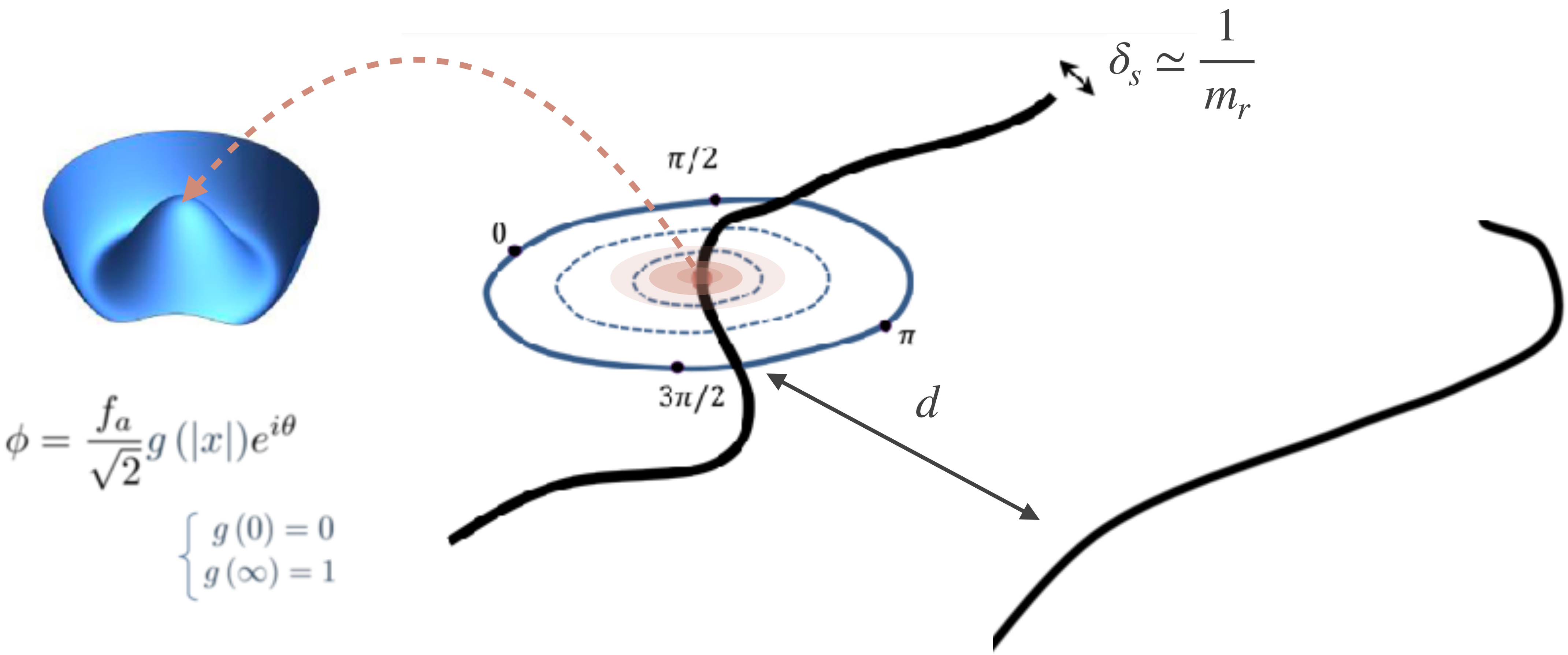


$$\phi = \frac{f_a}{\sqrt{2}} g\left(|x|\right) e^{i\theta}$$

$$\begin{cases} g\left(0\right) = 0 \\ g\left(\infty\right) = 1 \end{cases}$$

Topological strings



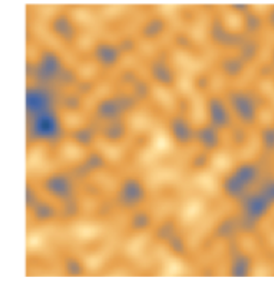


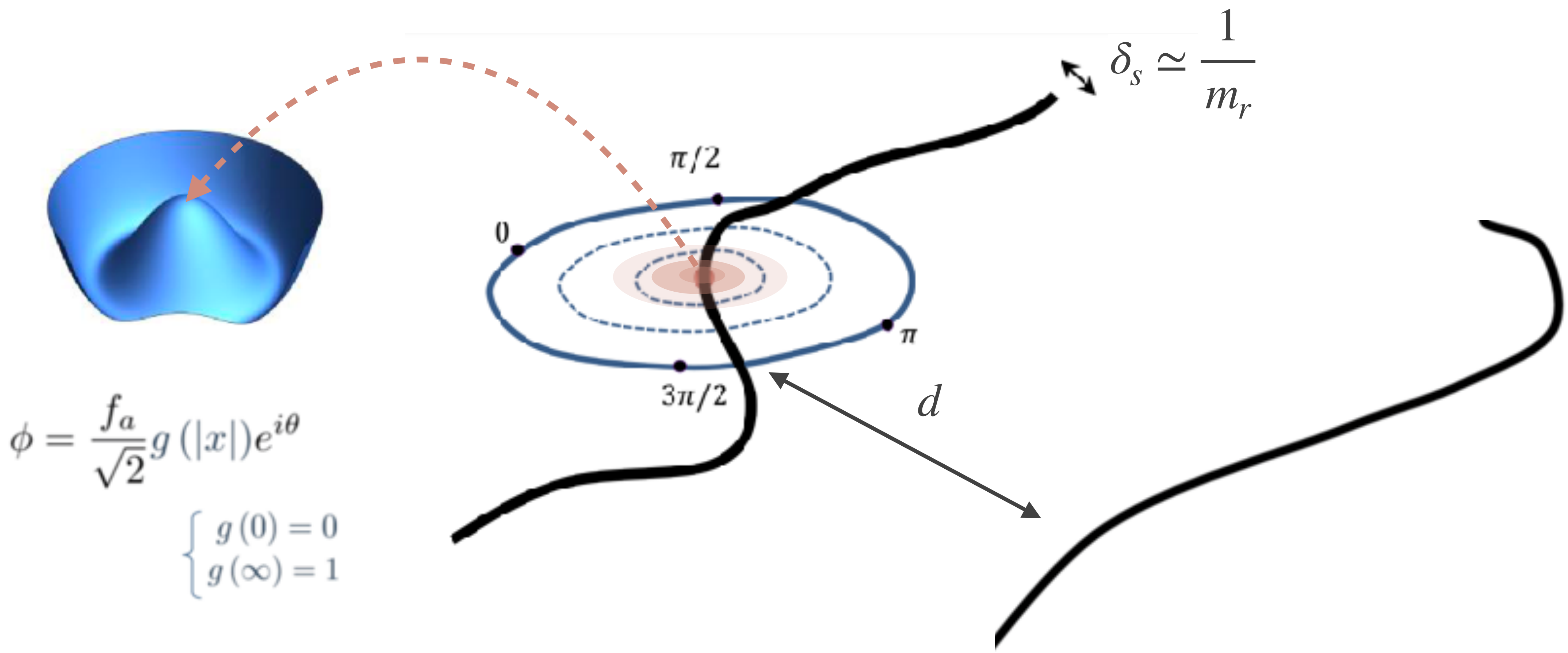
$$\mu = \frac{E}{L} \sim \pi f_a^2 \log \frac{d}{m_r^{-1}}$$

Core

gradient

Topological strings





$$\mu = \frac{E}{L} \sim \pi f_a^2 \log \frac{d}{m_r^{-1}} \sim \pi f_a^2 \log \frac{m_r}{H}$$

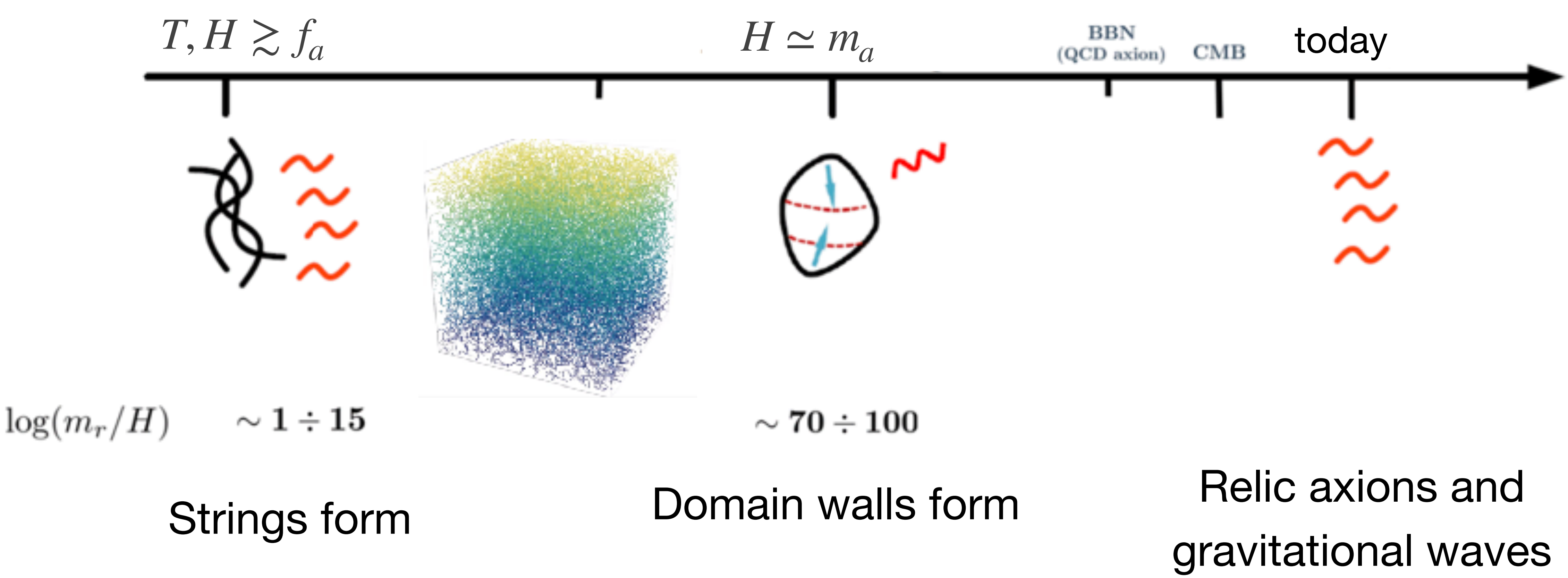
Core

gradient

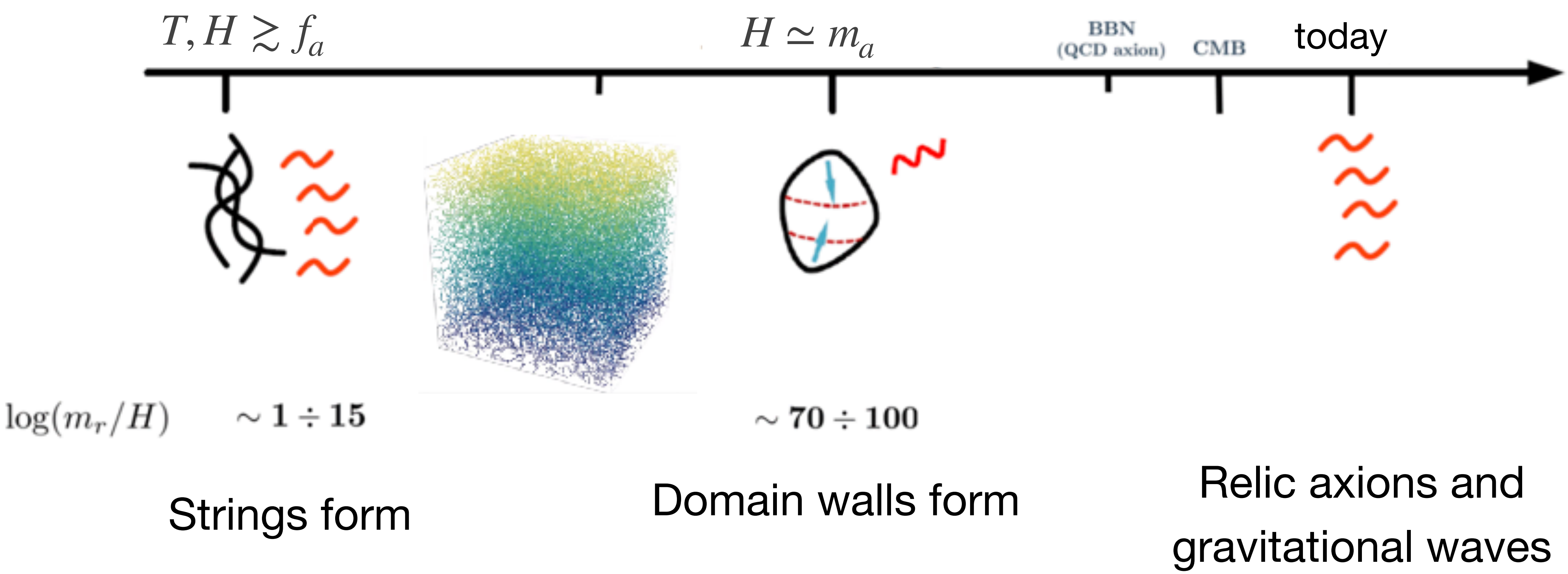
$H \sim T^2/M_{\rm Pl}$

Grows logarithmically with time

Full evolution



Full evolution

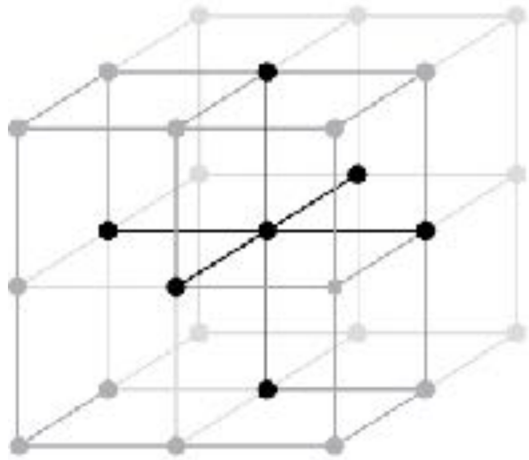


Dynamics:

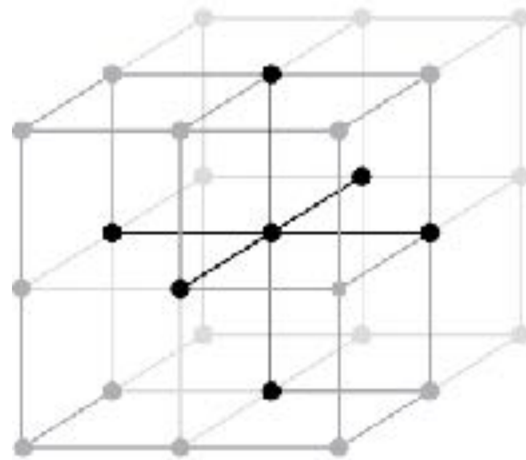
- nonlinear

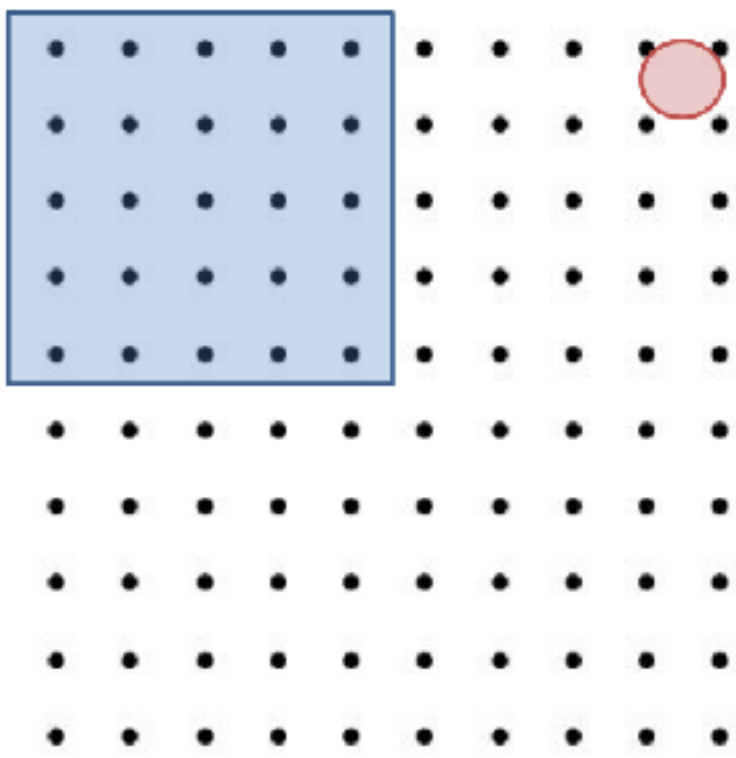
- large scale separation

Simulations



Simulations

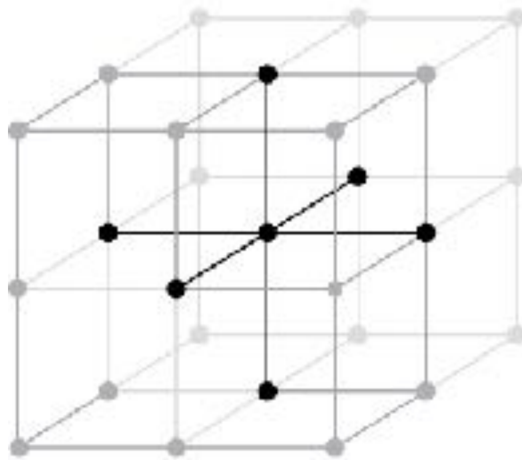


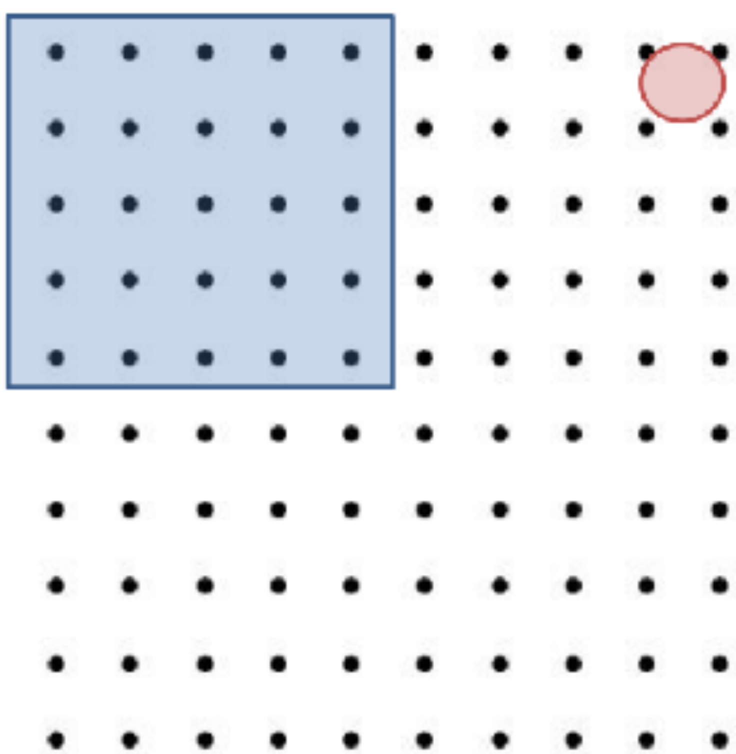


- a few lattice points per string core
- a few Hubble patches

Memory constraints → max 5000^3 grid points

Simulations





- a few lattice points per string core
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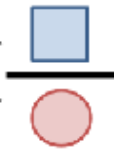
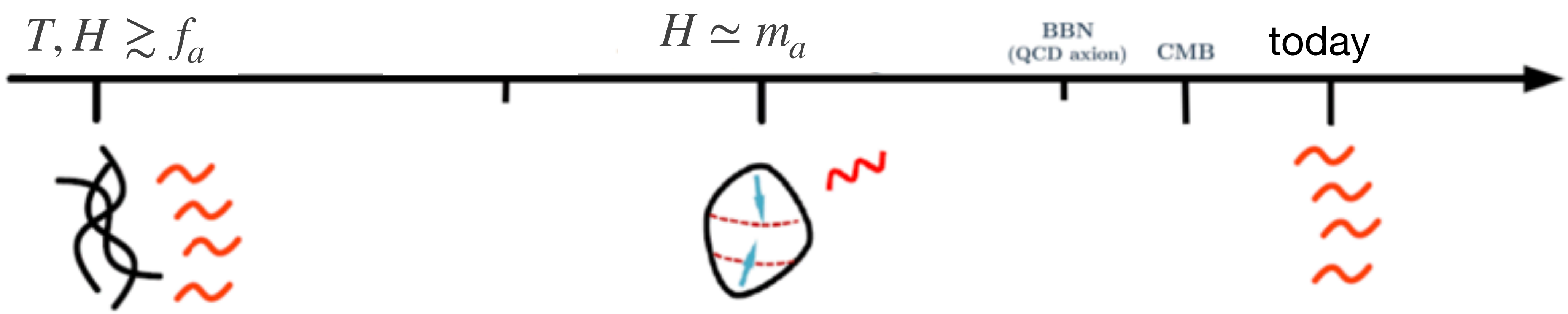
Memory constraints → max 5000^3 grid points

Simulations $\log(m_r/H) \leq \log($$) \lesssim 8$

Physical $\log(m_r/H) \sim 70$

Scaling regime

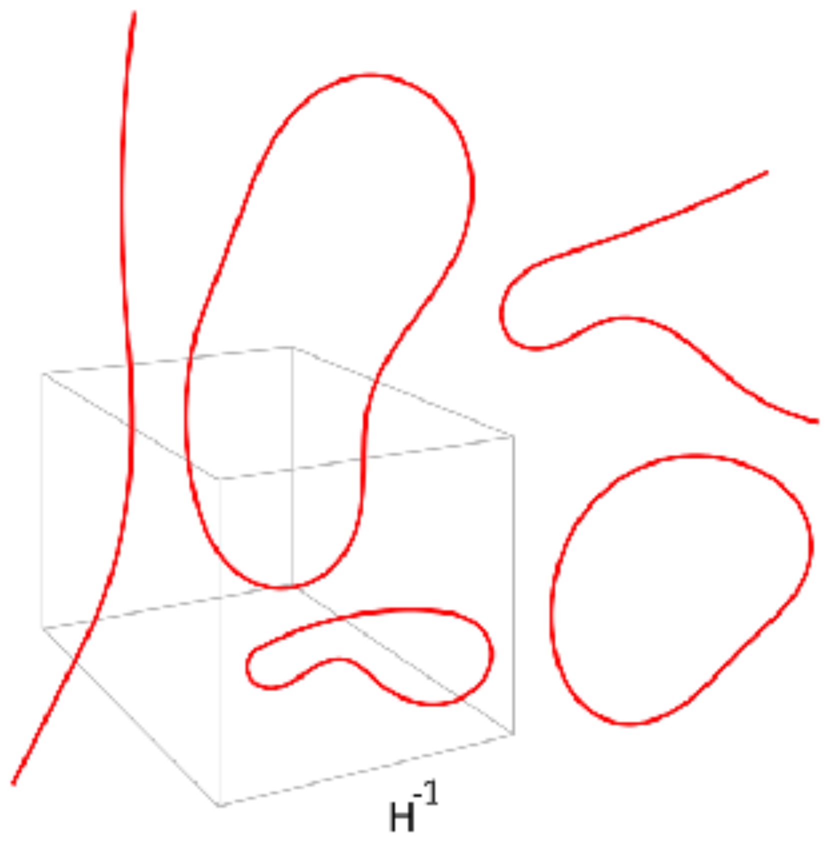


$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70 \div 100}$

Simulate →

Extrapolate

Scaling



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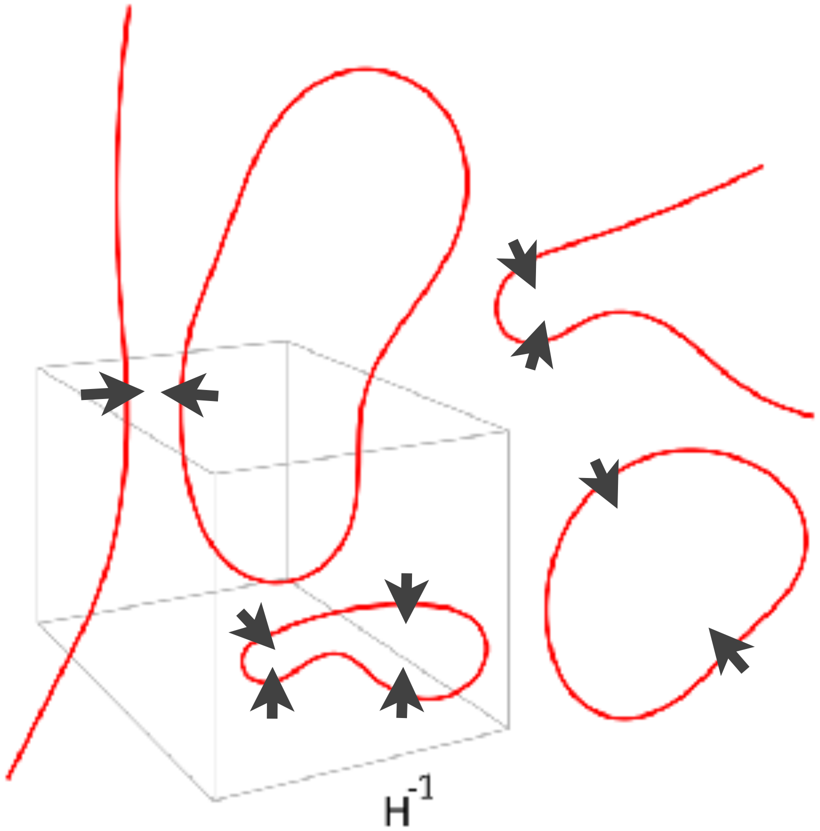

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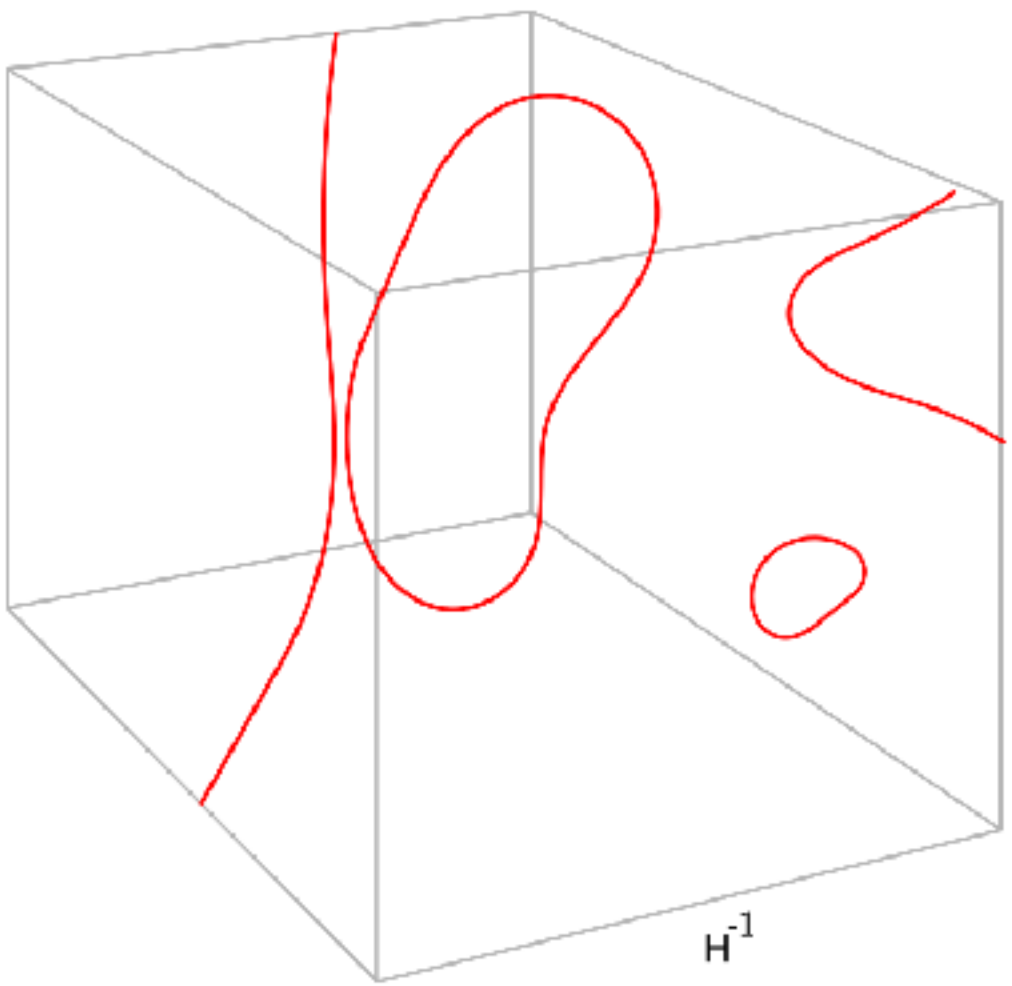
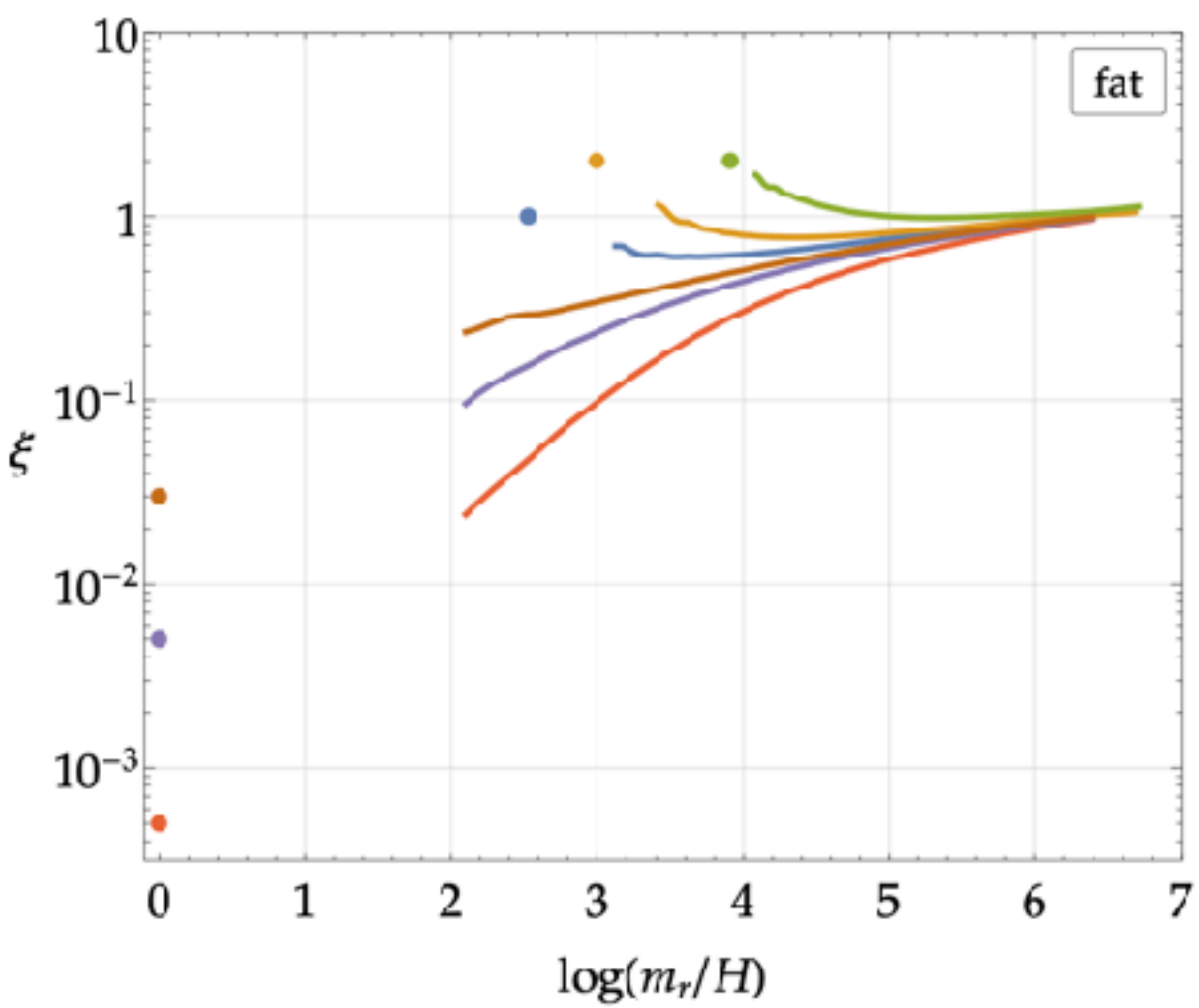


$\xi(t)$ = Length of string in Hubble lengths per Hubble volume

$\mu(t)$ = string tension $\simeq \pi f_a^2 \log(m_r/H)$

Scaling



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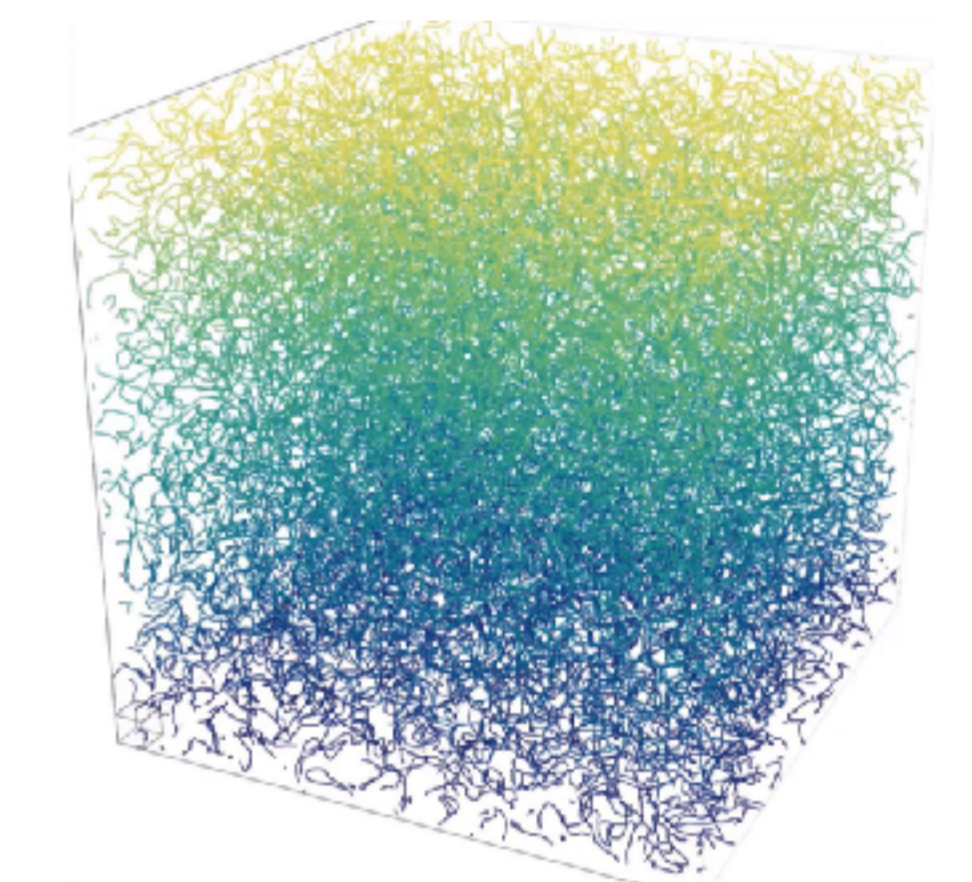

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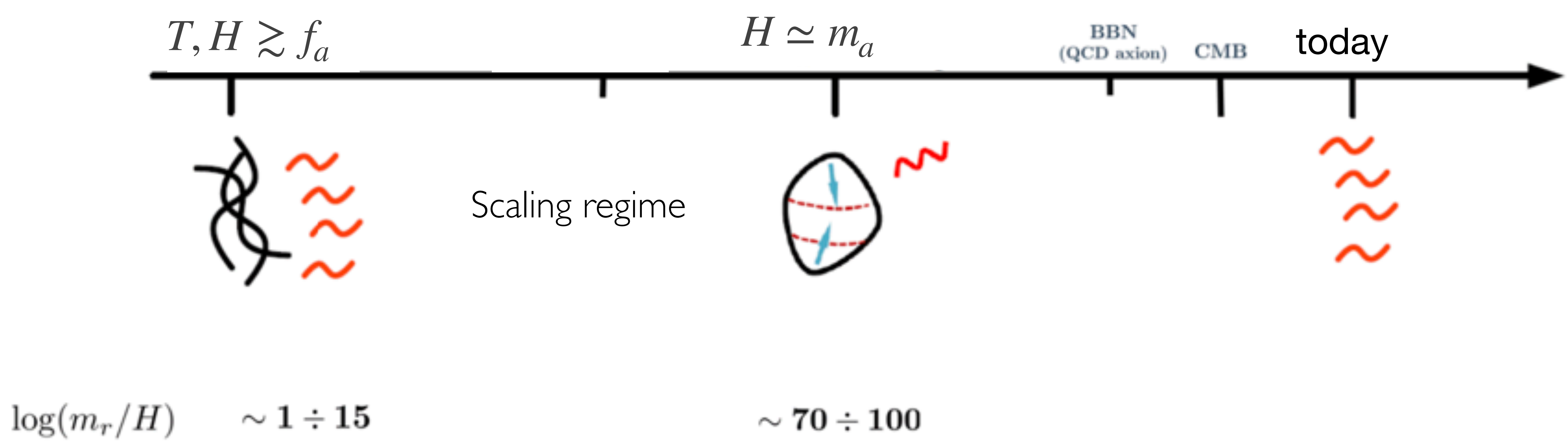

Simulate →

Extrapolate

Need:

1. Energy emitted into GWs
2. Momentum distribution

Energy emitted

$$\Gamma \simeq \frac{\xi(t)\mu(t)}{t^3}$$

Energy emitted

1. **Energy emitted into GWs**
2. Momentum distribution

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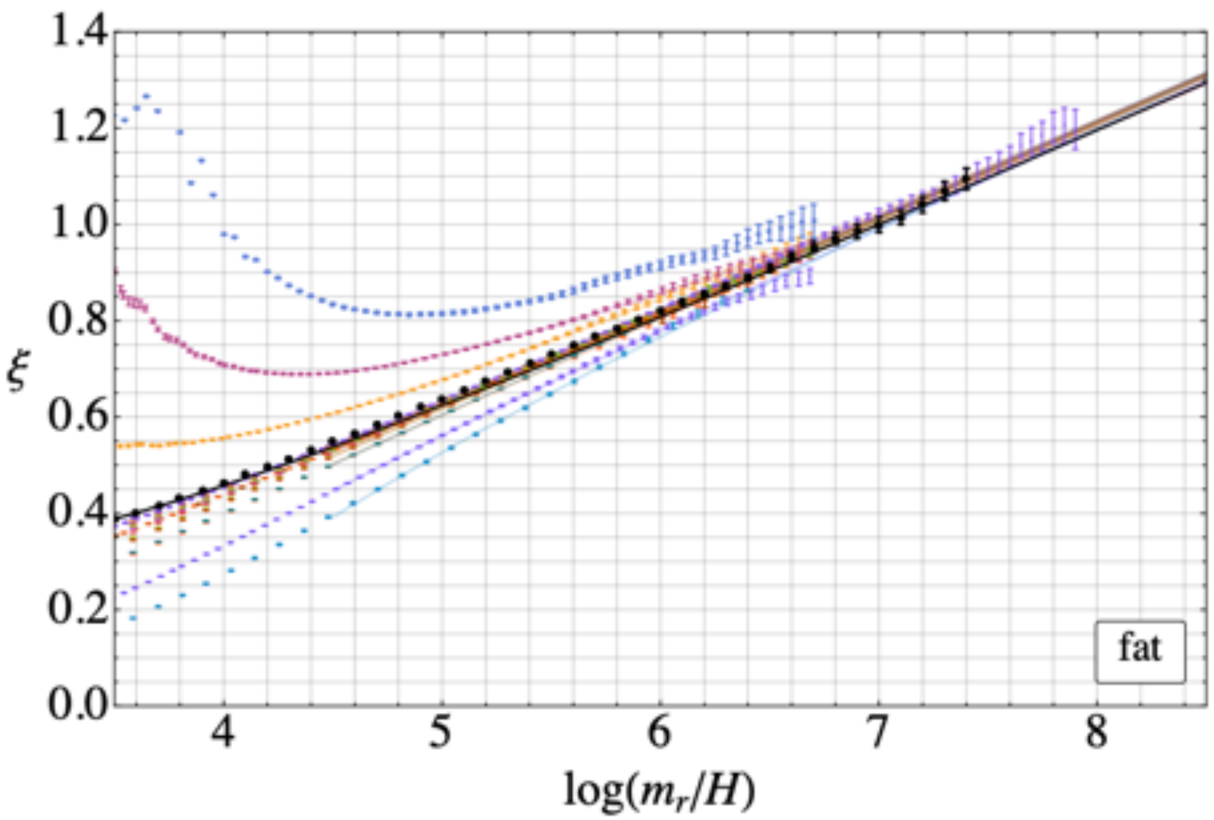
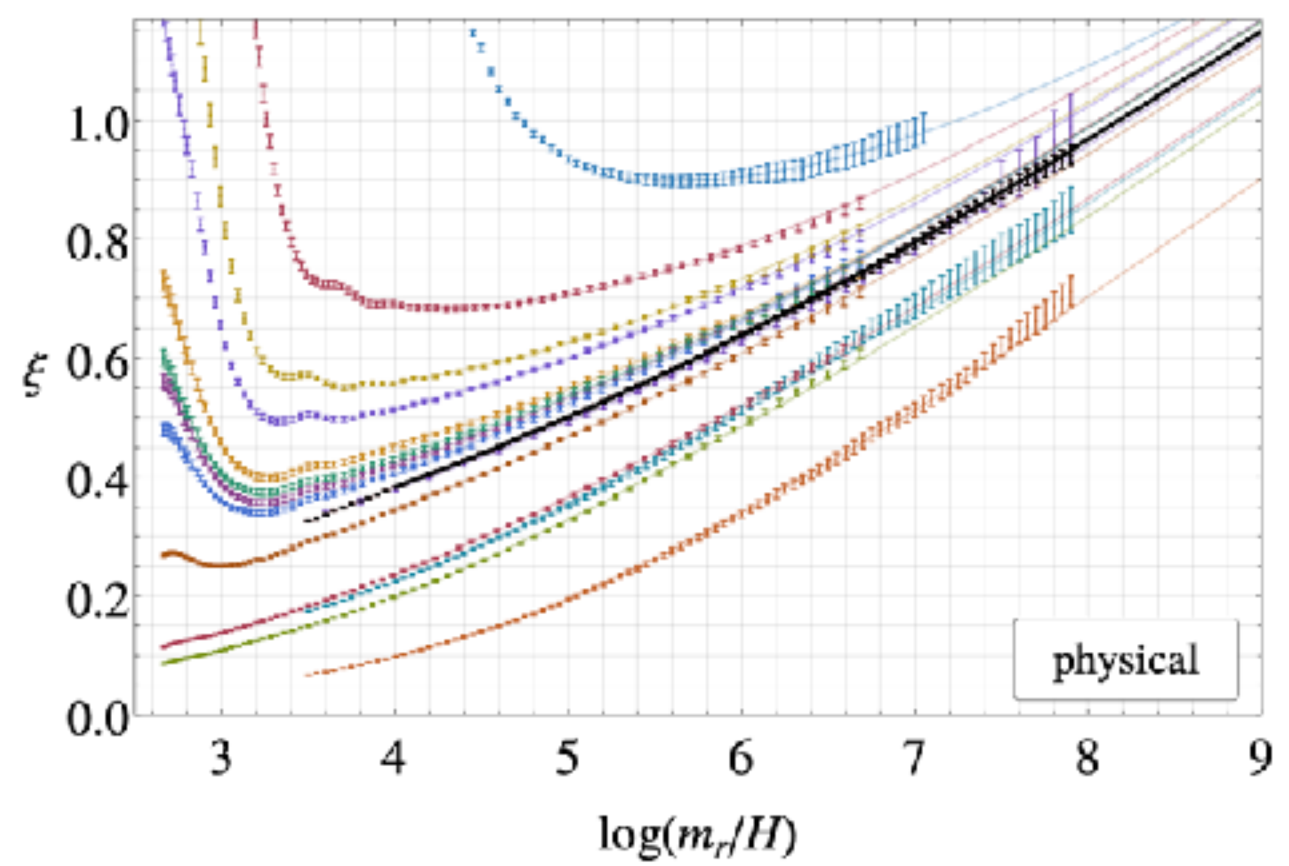


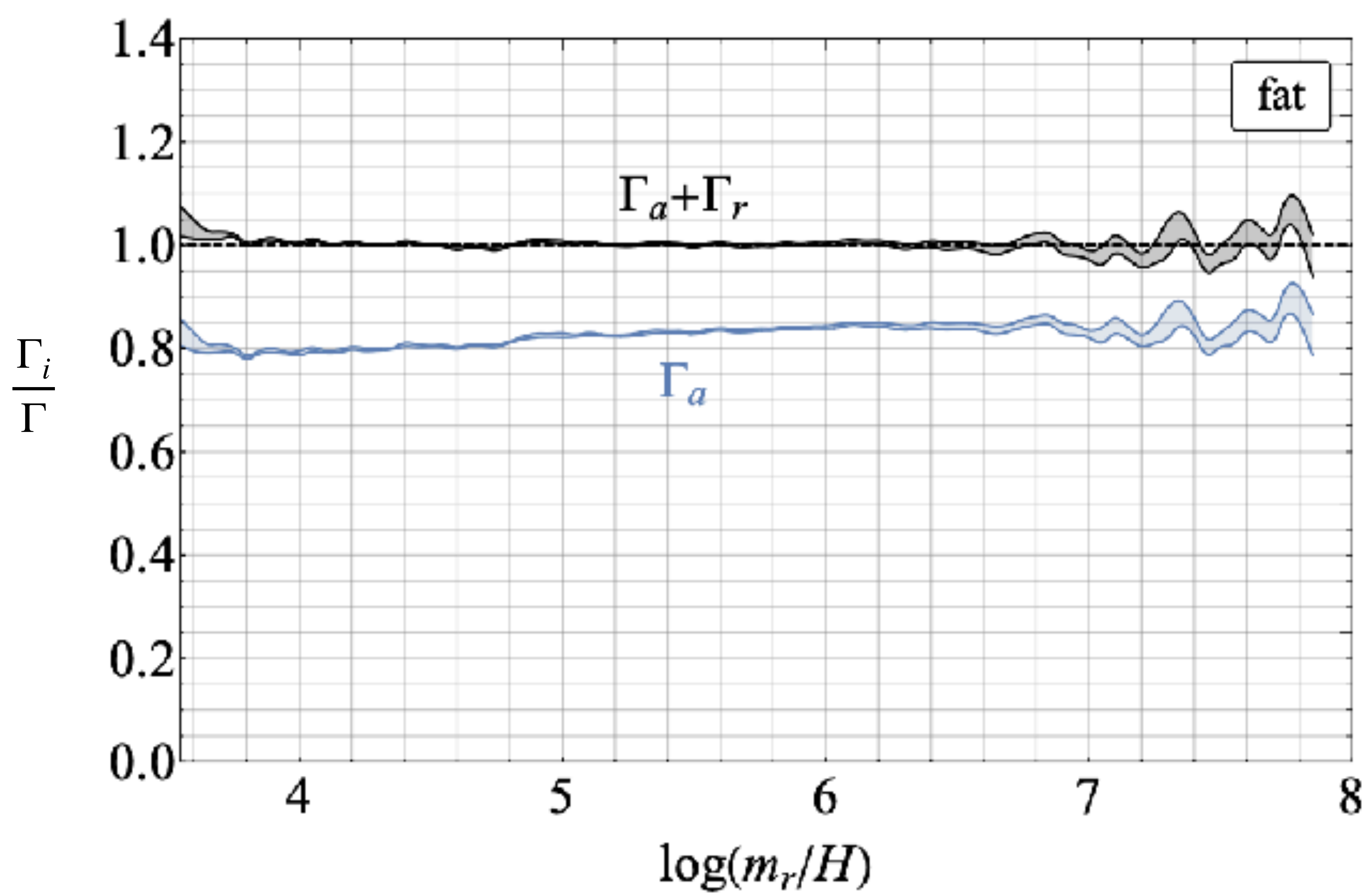


$$\xi = c_1 \log + c_0 + \frac{c_{-1}}{\log} + \frac{c_{-2}}{\log^2}$$

Energy emitted

1. **Energy emitted into GWs**
2. Momentum distribution

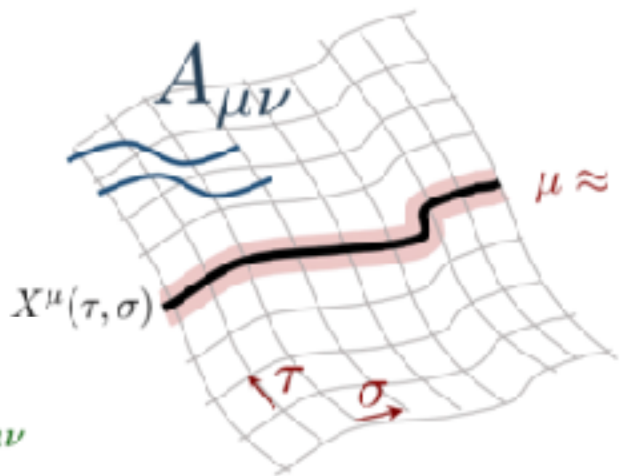
$$\Gamma \simeq \frac{\xi(t)\mu(t)}{t^3}$$



String EFT

1. **Energy emitted into GWs**
2. Momentum distribution

$a \longleftrightarrow A_{\mu\nu}$ $\partial A \sim F^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma}\partial_\sigma a$



Axion-string coupling

$$S_{\rm EFT} = -\mu \int d\tau d\sigma \sqrt{-\gamma} - \frac{1}{6}\int d^4x \, (\partial A)^2 + 2\pi f_a \int d\tau d\sigma \, \partial_\tau X^\mu \partial_\sigma X^\nu A_{\mu\nu}$$

Nambu-Goto action

$\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu$

Axion kinetic term

Axion-string interaction (Kalb-Ramond action)

[Lund & Regge, 1976]
[Davis & Shellard, 1988]

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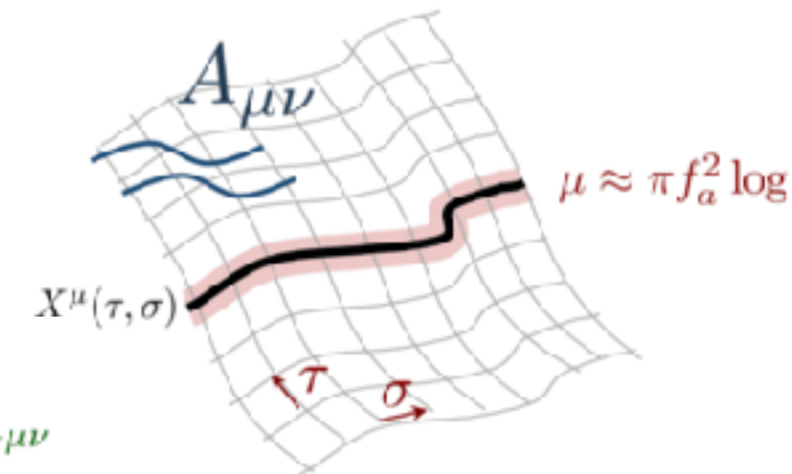

Axion-string coupling

$$S_{\rm EFT} = \underbrace{-\mu \int d\tau d\sigma \, \sqrt{-\gamma}}_{\text{Nambu-Goto action}} \; \underbrace{-\tfrac{1}{6}\int d^4x \, (\partial A)^2}_{\text{Axion kinetic term}} \; \underbrace{+ 2\pi f_a \int d\tau d\sigma \, \partial_\tau X^\mu \partial_\sigma X^\nu A_{\mu\nu}}_{\text{Axion-string interaction (Kalb-Ramond action)}}$$

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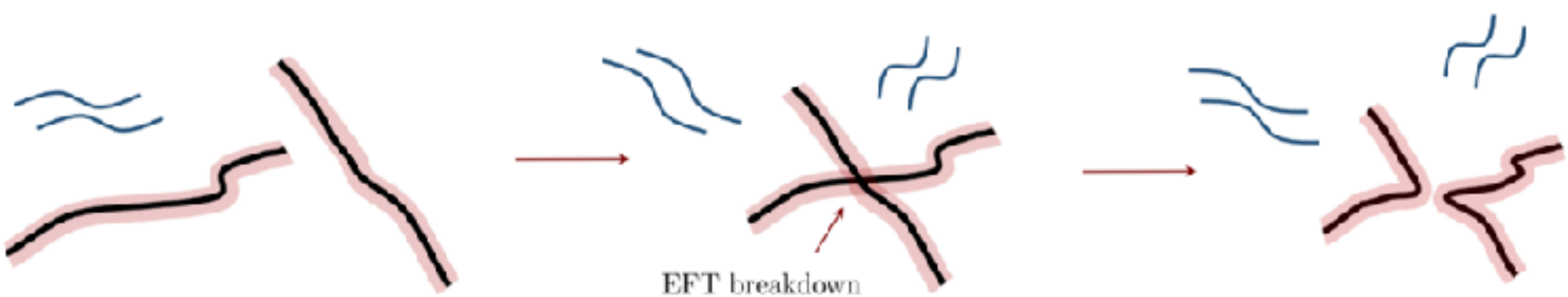
Axion-string interaction (Kalb-Ramond action)

[Lund & Regge, 1976]
[Davis & Shellard, 1988]

EoM:

$$\begin{cases} \mu(\ddot{X}^\mu - X''^\mu) = 2\pi f_a F^{\mu\nu\rho} \dot{X}_\nu X'_\rho \\ \Box_x A^{\mu\nu} = 2\pi f_a \int d\sigma \, \dot{X}^{[\mu} X'^{\nu]} \delta^3(\vec{x} - \vec{X}) \end{cases}$$



String EFT

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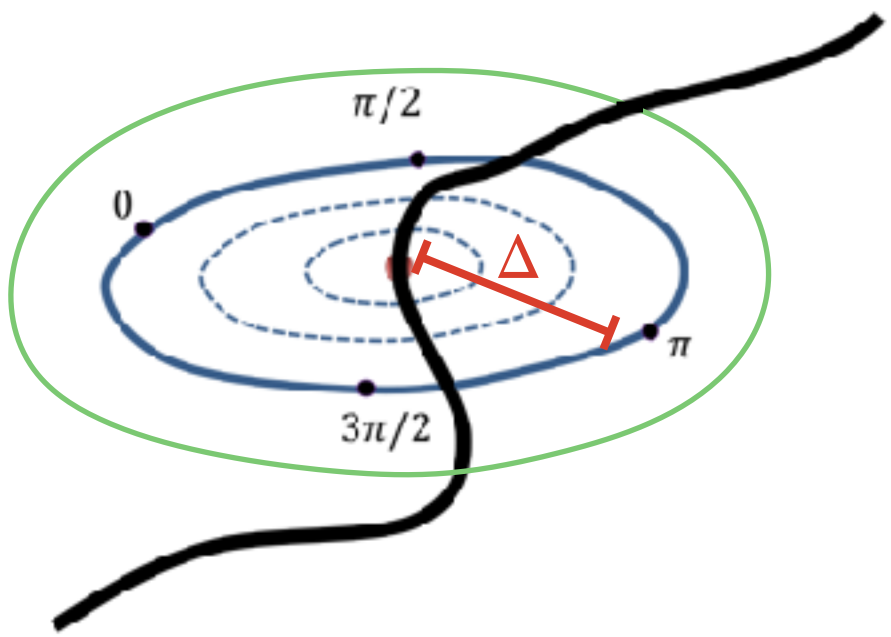
2. Momentum distribution



$$\mu(\Delta) = \pi f_a^2 \log\left(\Delta / m_r^{-1}\right)$$

[Lund & Regge, 1976]
also [Horn, Nicolis, Penco] in the context of superfluids

String EFT

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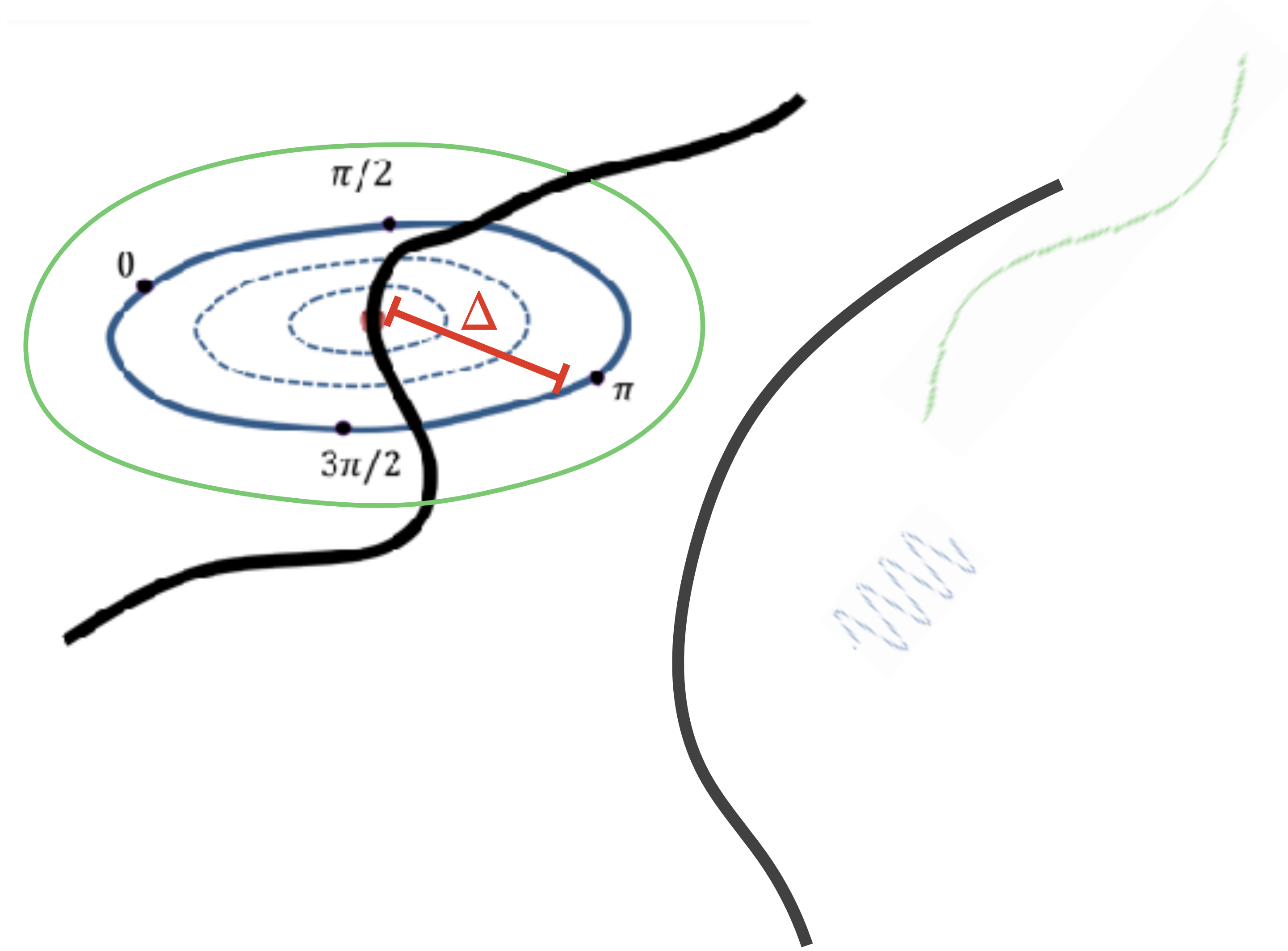

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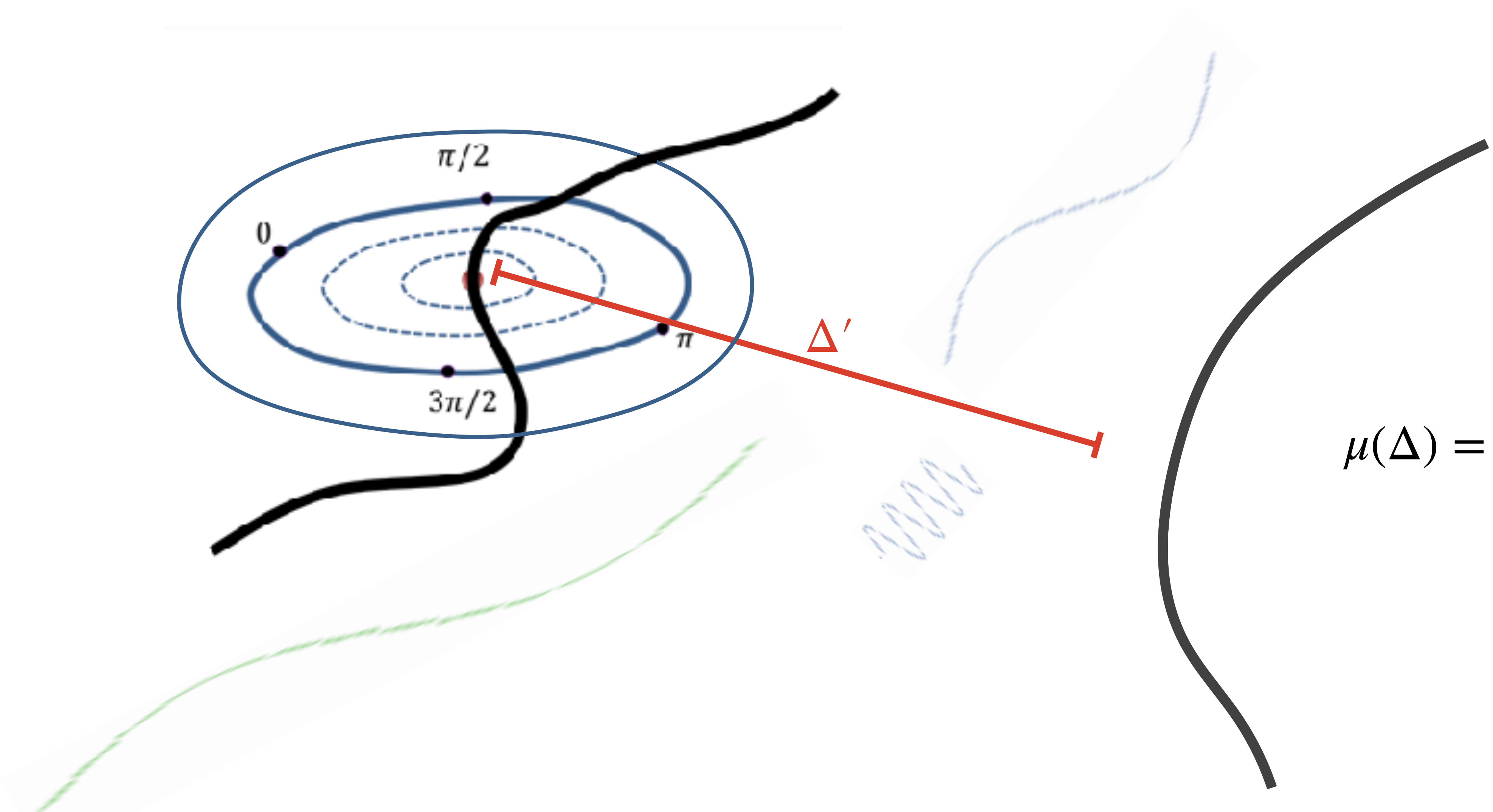

$$\mu(\Delta) = \pi f_a^2 \log\left(\Delta/m_r^{-1}\right)$$

$$\mu(\Delta') = \mu(\Delta) + (g^2/2\pi)\log(\Delta'/\Delta) = \mu(\Delta) + \pi f_a^2 \log(\Delta'/\Delta)$$

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String EFT

1. **Energy emitted into GWs**
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$$\Box_x A^{\mu\nu} = 2\pi f_a \int d\sigma\, \dot{X}^{[\mu} X'^{\nu]}\, \delta^3(\vec{x} - \vec{X})$$

Einstein Eq. $\quad \Box_x h^{\mu\nu} = 16\pi G \left(T_s^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} T^{\lambda}_{s\lambda} \right)$ $\quad T_s^{\mu\nu} = \mu \int d\sigma\, \left(\dot{X}^{\mu}\dot{X}^{\nu} - X'^{\mu}X'^{\nu} \right)\, \delta^3(\vec{x} - \vec{X})$

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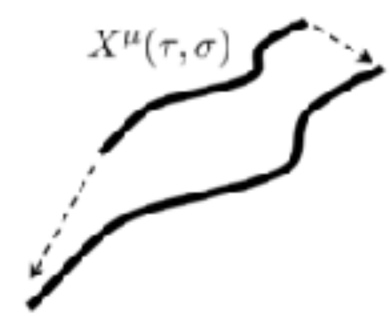
$$\frac{dE_a}{dt} = \underbrace{r_a[X]}\, f_a^2 \qquad \frac{dE_g}{dt} = \underbrace{r_g[X]}\, G\mu^2$$

Dimensionless functionals of shape of string trajectory



$$r_g[X] = \int \frac{d\Omega}{2\pi} \left\{ \left[\int d\sigma \partial_t (\dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu) \right]^2 - \left[\frac{1}{2} \int d\sigma \partial_t (\dot{X}^2 - X'^2) \right]^2 \right\}$$

String EFT

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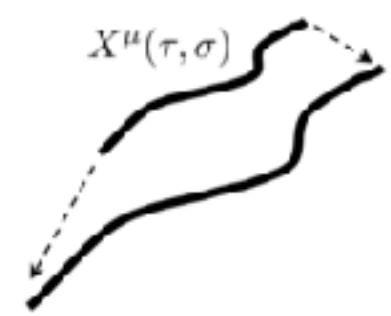

$$\frac{\Gamma_g}{\Gamma_a} = \underbrace{\frac{r_g[X]}{r_a[X]}}_{\equiv r} \frac{G\mu^2}{f_a^2}$$

$$\equiv r \quad = \text{const}$$

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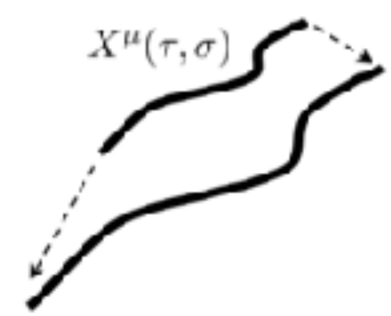
$$\frac{dE_a}{dt} = r_a[X]\, f_a^2 \qquad \frac{dE_g}{dt} = r_g[X]\, G\mu^2$$

Dimensionless functionals of shape of string trajectory



$$\frac{\Gamma_g}{\Gamma_a} = \underbrace{\frac{r_g[X]}{r_a[X]}}_{\equiv r} \frac{G\mu^2}{f_a^2} \qquad = \text{const}$$

$$\longrightarrow \quad \boxed{\Gamma_g = r \frac{G\mu^2}{f_a^2} \Gamma} \propto \frac{\log^4}{t^3}$$

$\xi\mu/t^3$

Simulations

1. **Energy emitted into GWs**
2. Momentum distribution

Instantaneous emission

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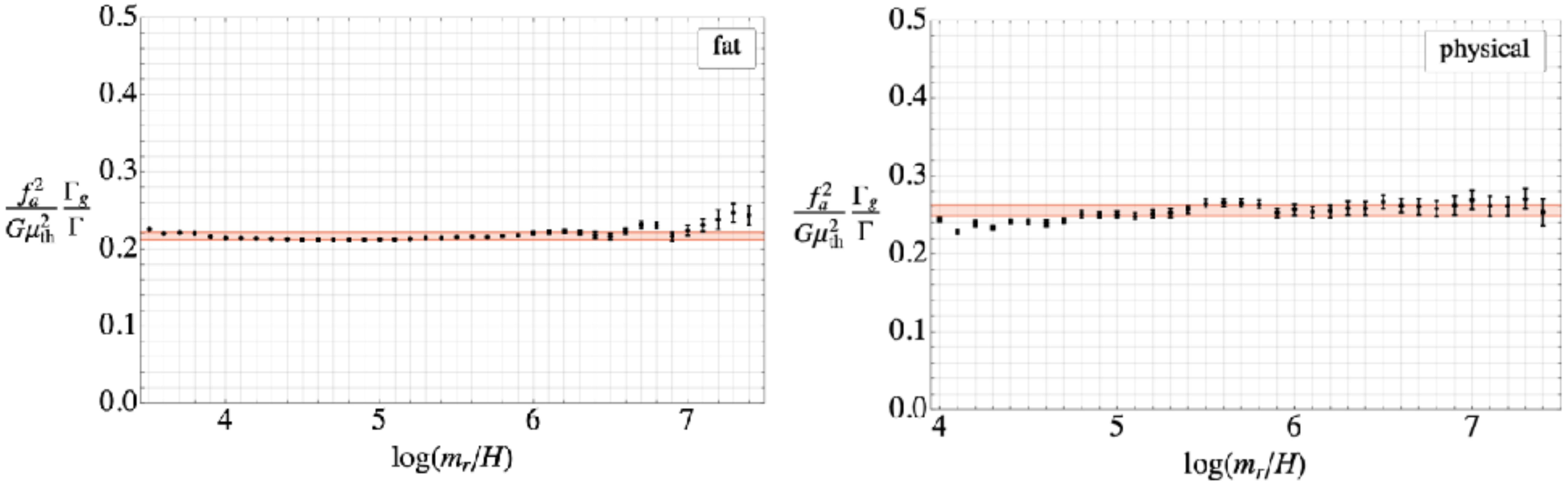
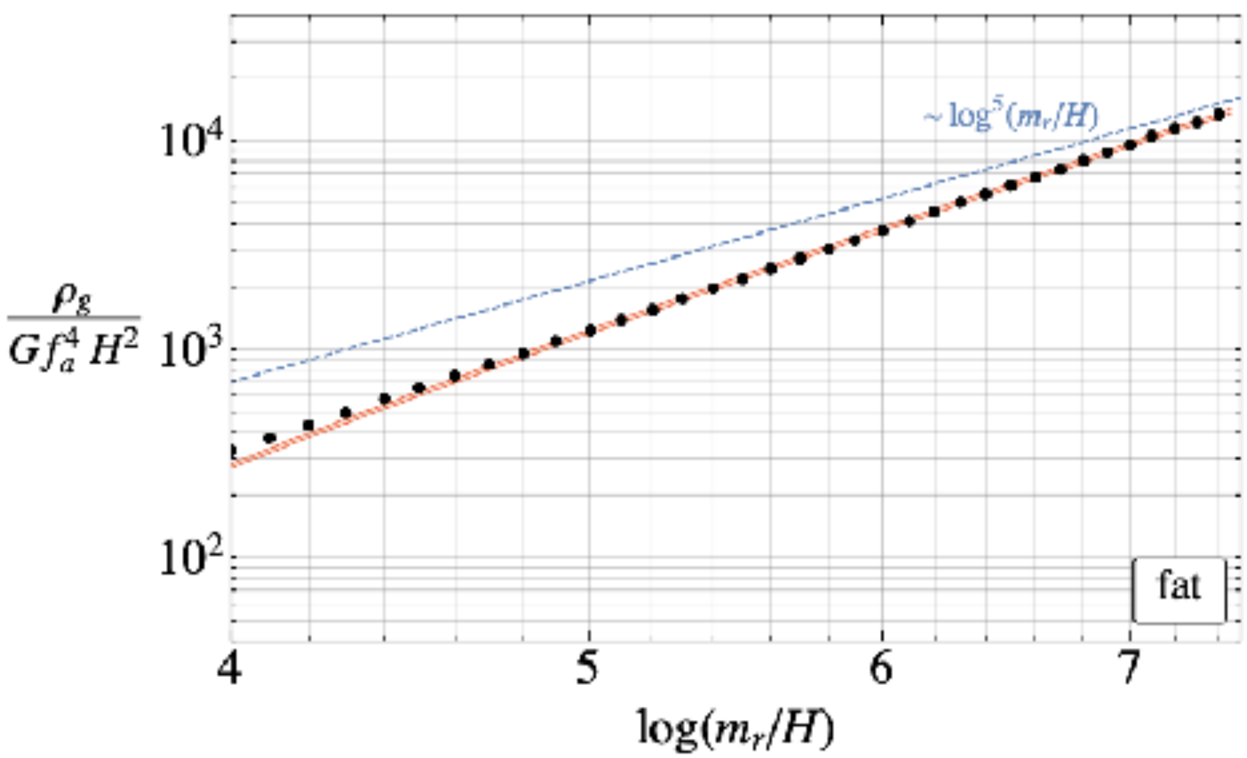


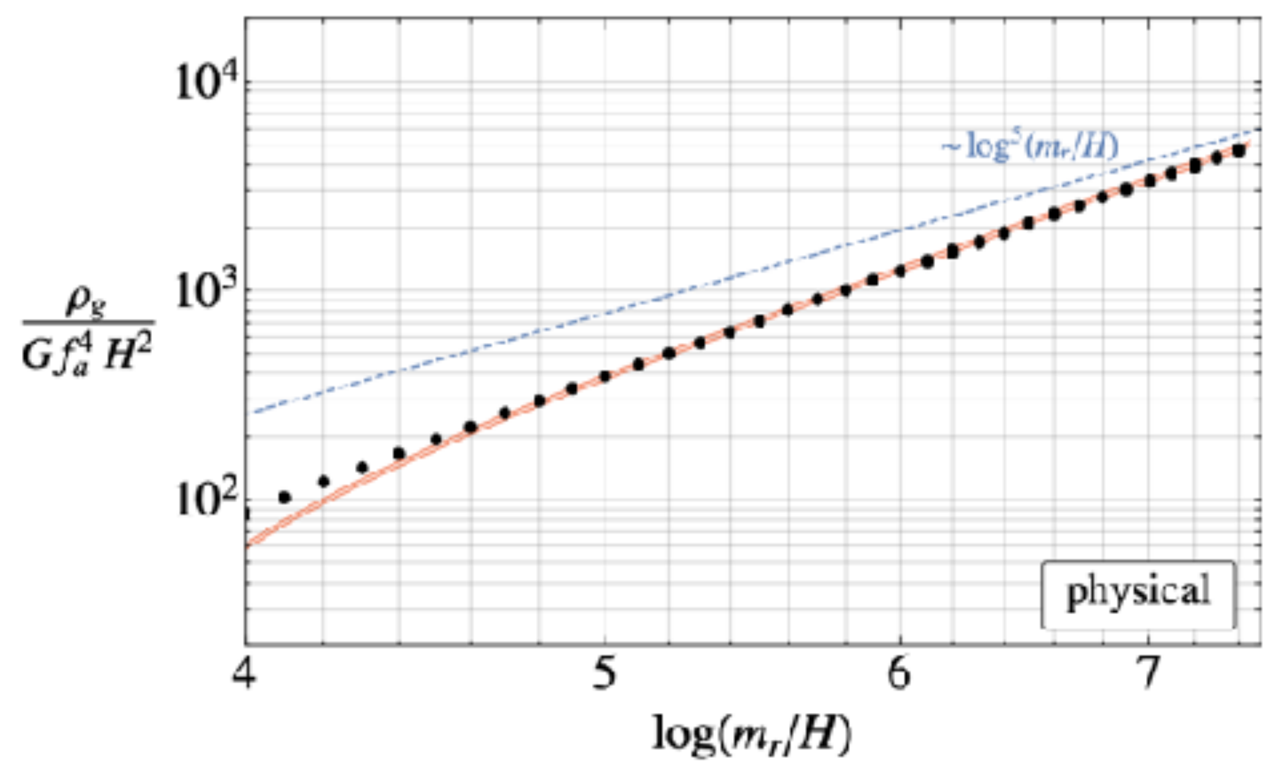
Simulations

1. Energy emitted into GWs
2. Momentum distribution

Total energy

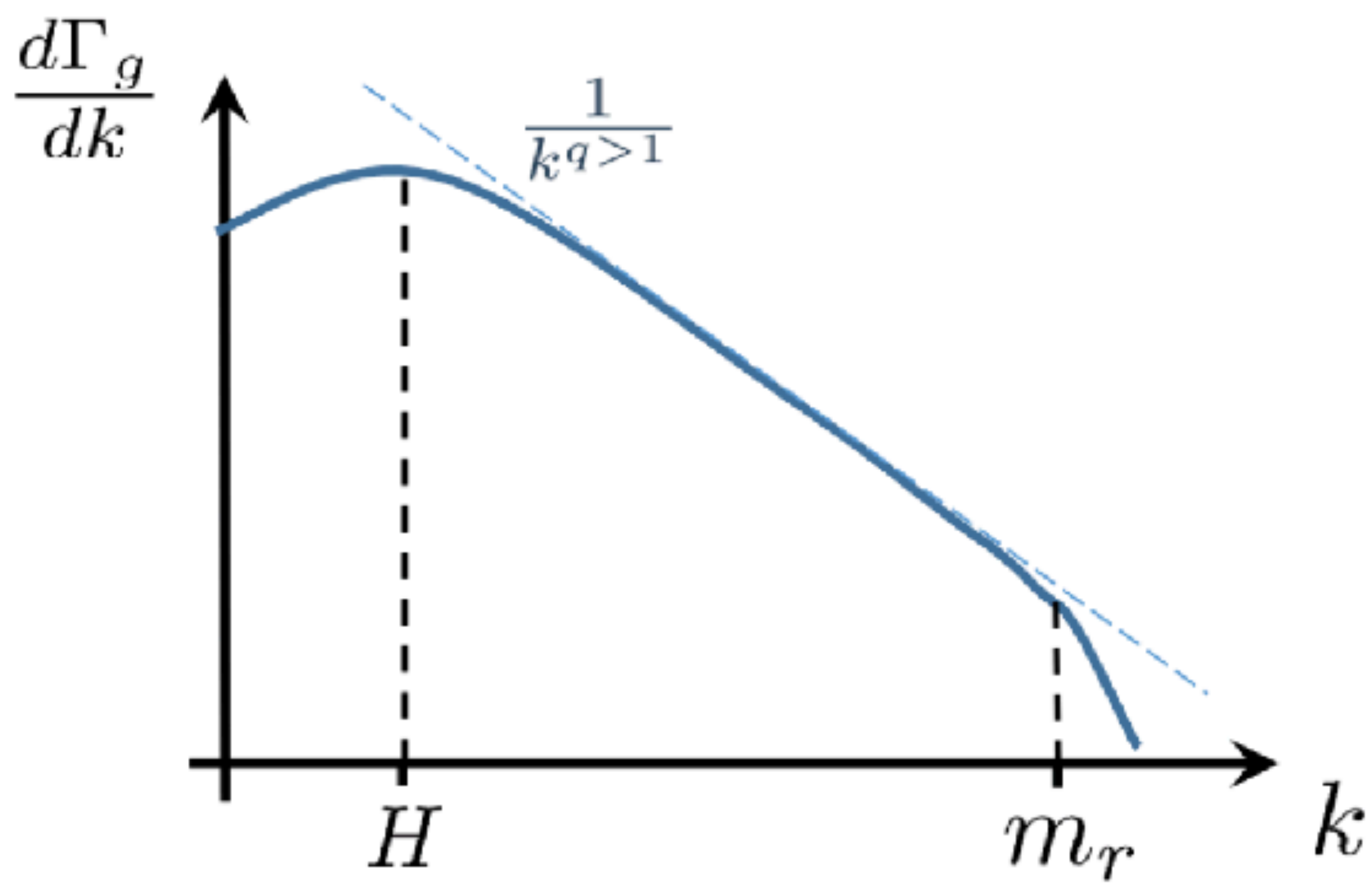
$$\rho_g(t) \propto \int dt' \frac{\log'^4}{t'^3} \left(\frac{R(t')}{R(t)} \right)^4 \rightarrow \log^5$$





GW spectrum

1. Energy emitted into GWs
2. **Momentum distribution**



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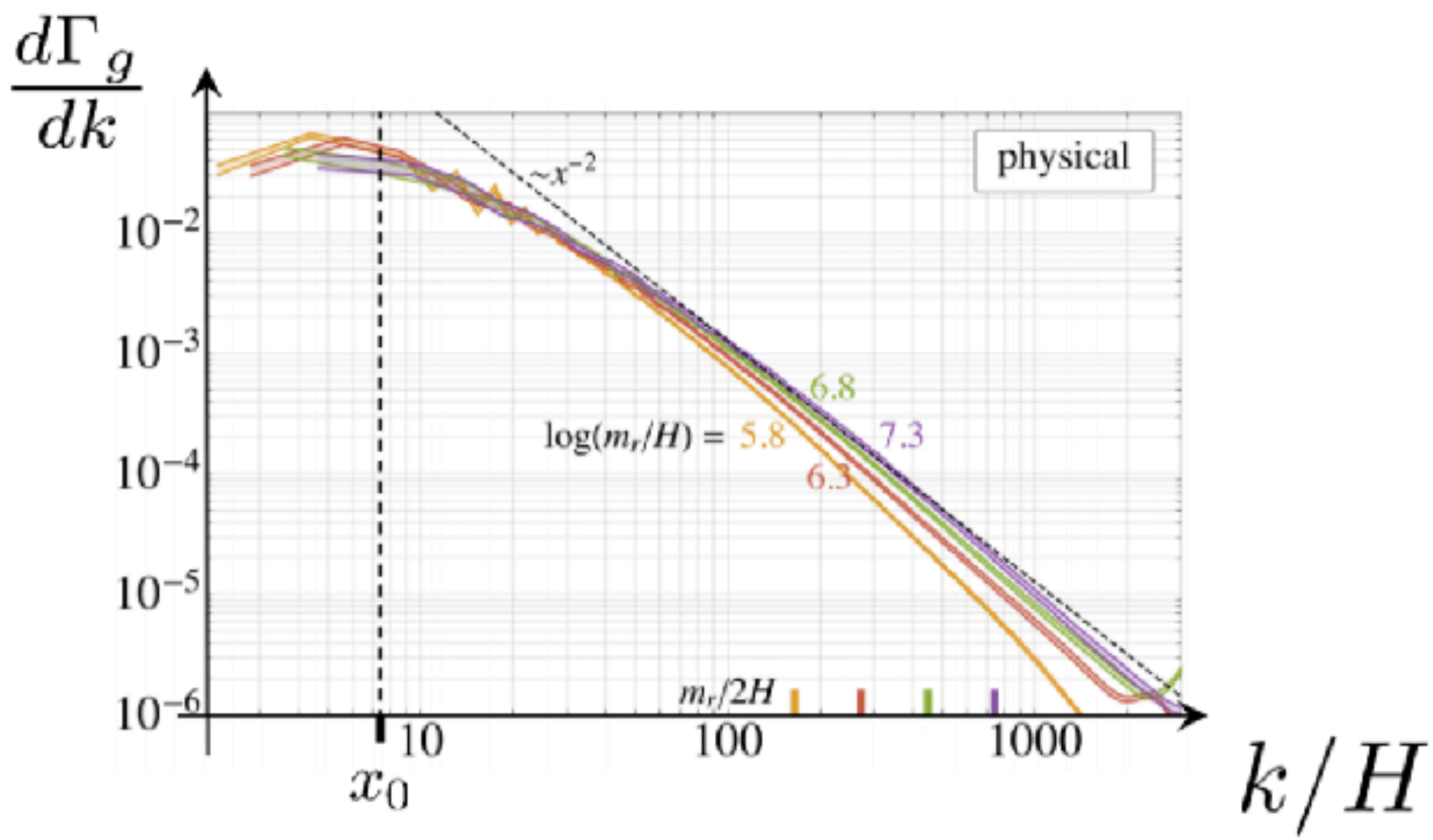
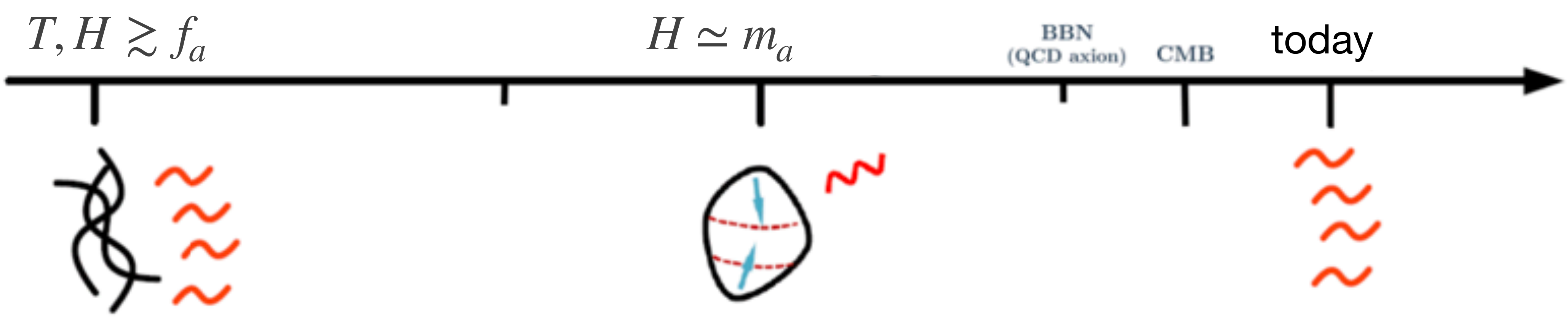
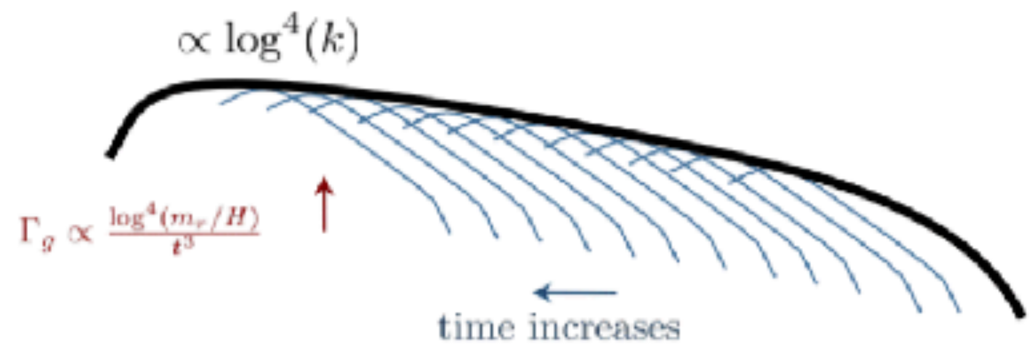


Spectrum today

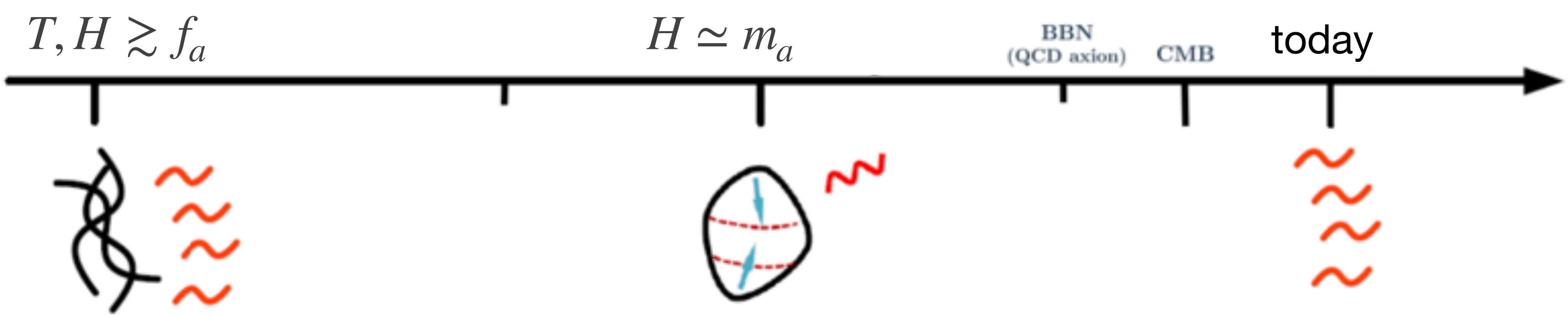


$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70 \div 100}$

$$\frac{\partial \rho_g}{\partial \log k} = \int dt' \, \frac{d\Gamma'}{d \log k} \left(\frac{R'}{R} \right)^4$$



Spectrum today



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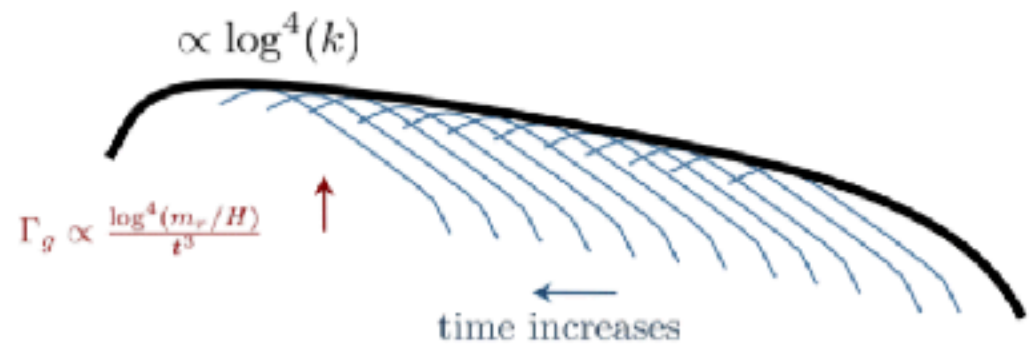
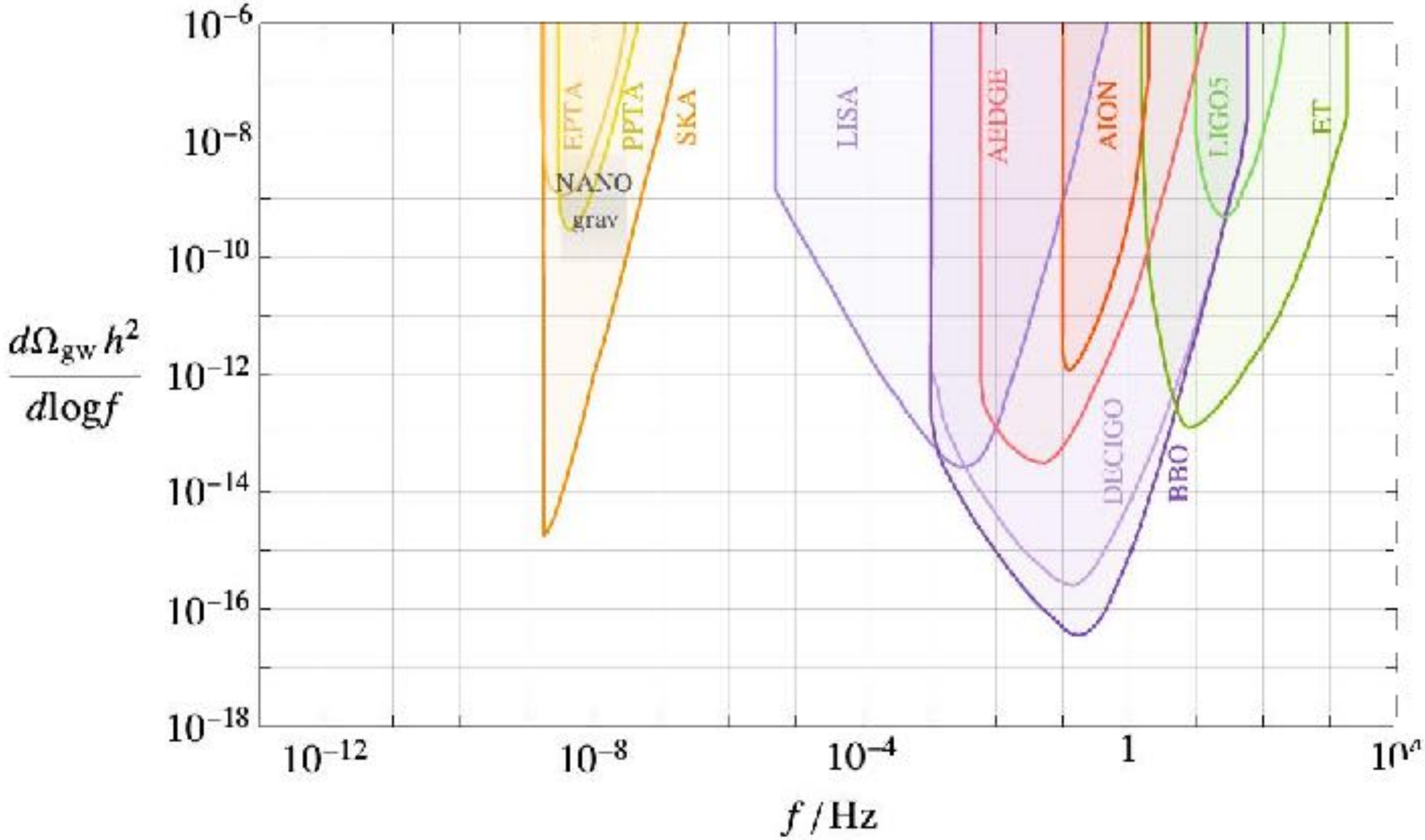


$$\frac{d\Omega_{\rm gw} h^2}{d\log f} \simeq 10^{-15} \left(\frac{r}{0.26}\right) \left(\frac{f_a}{10^{14}\mathrm{GeV}}\right)^4 \left(\frac{10}{g_f}\right)^{\frac{1}{3}} \left\{1 + 0.12 \log\left[\left(\frac{m_r}{10^{14}\mathrm{GeV}}\right)\left(\frac{10^{-8}\mathrm{Hz}}{f}\right)^2\right]\right\}^4$$

Spectrum today



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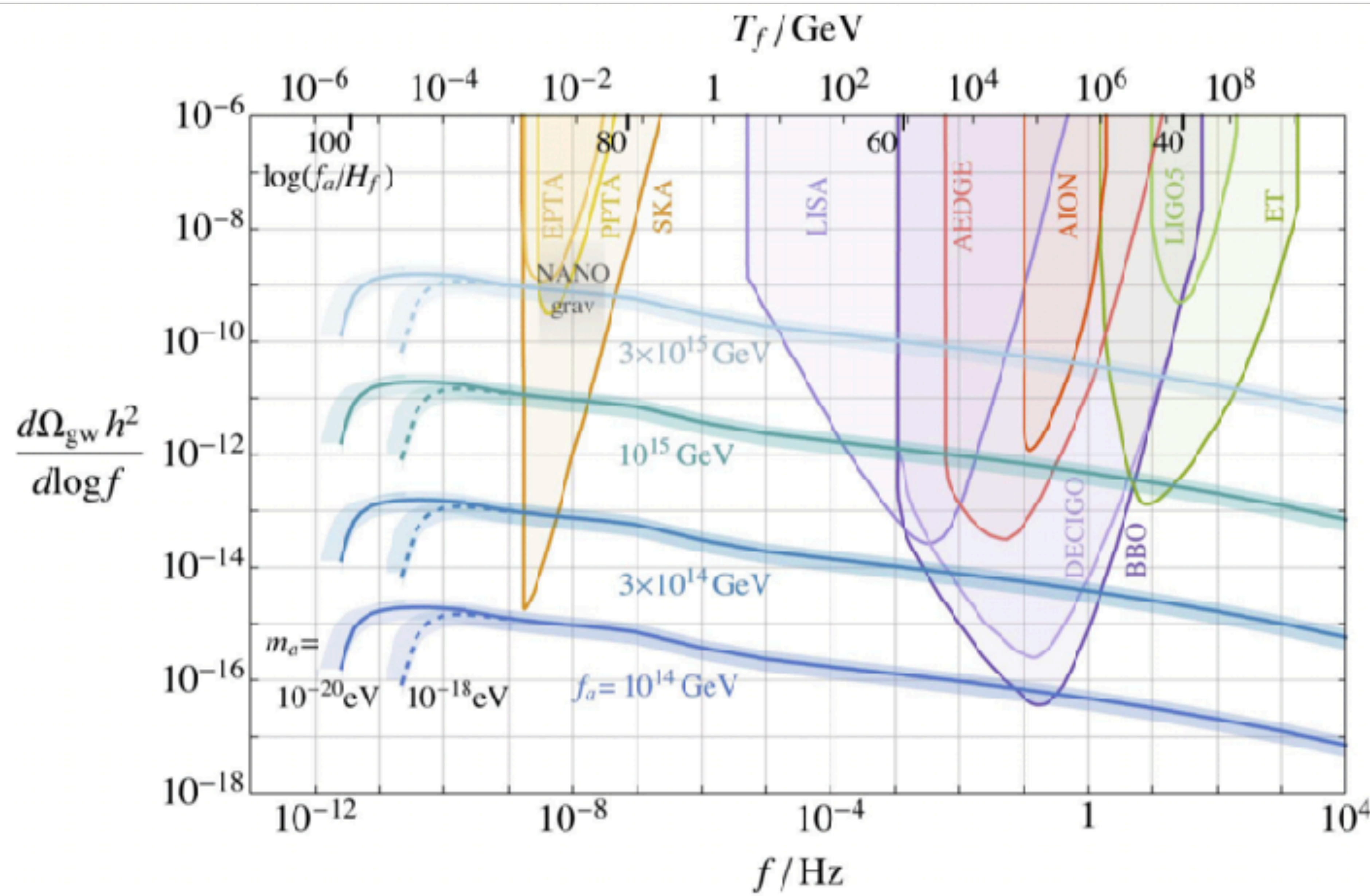

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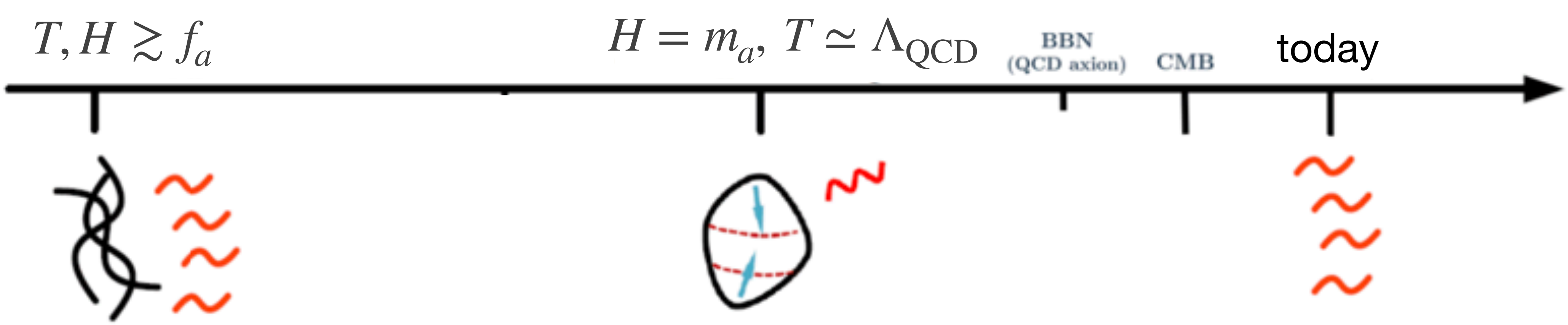
Axion stars

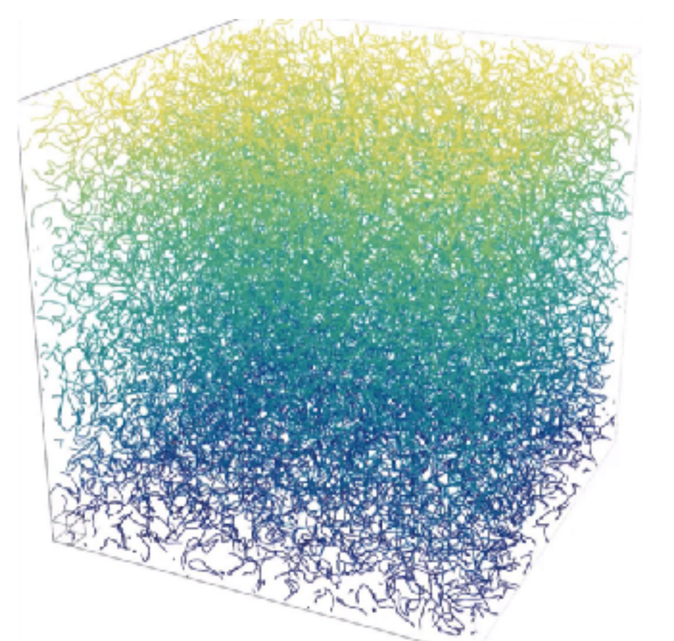
Full evolution

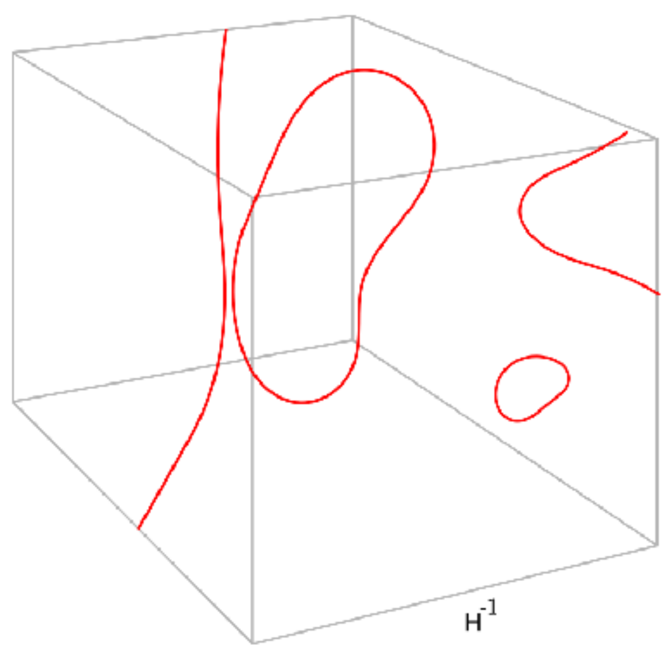


Strings form

Strings destroyed

Relic axions





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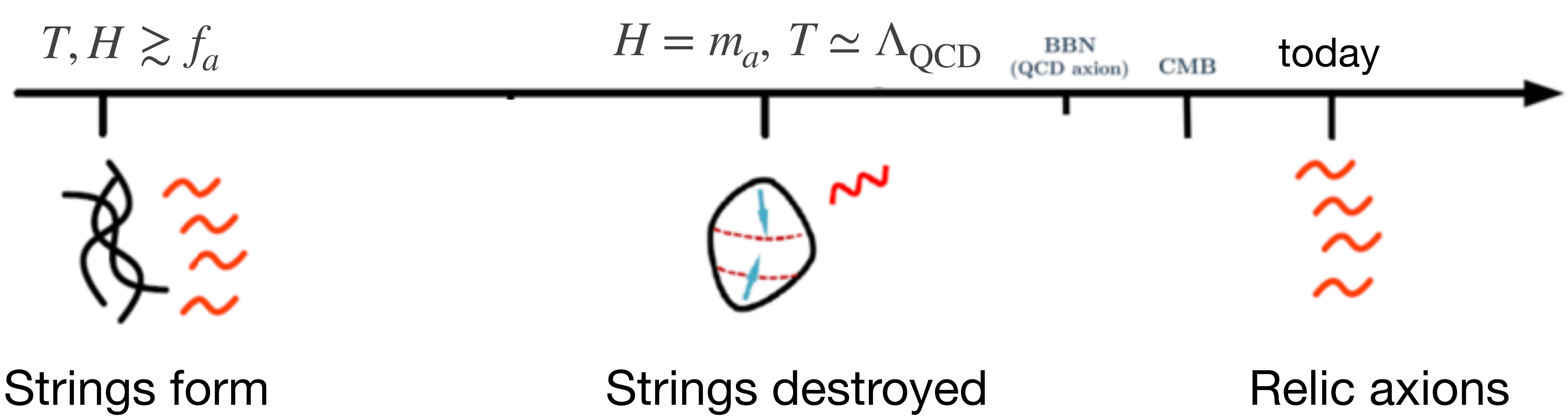
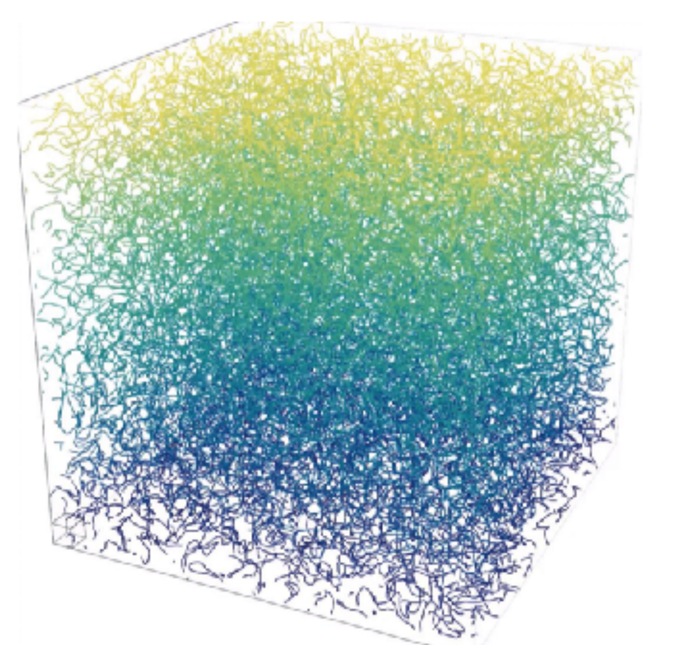
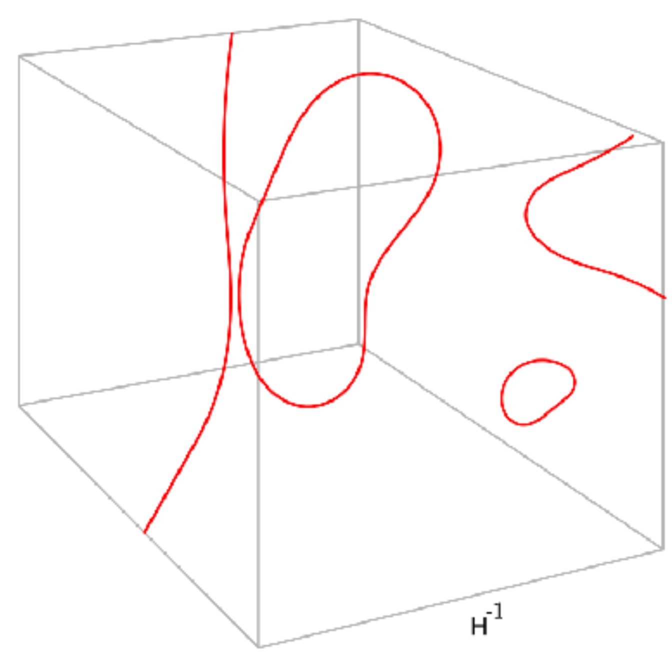




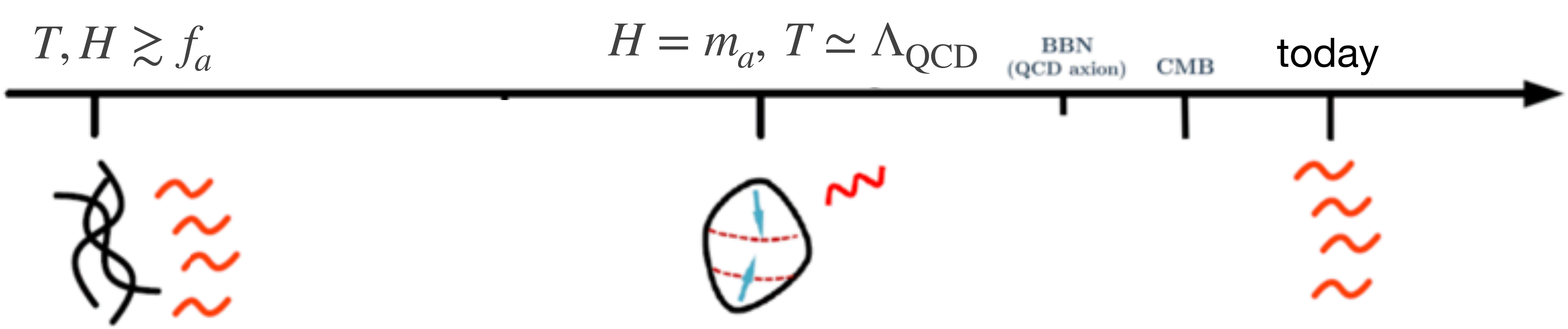


$\Omega_{DM} \simeq 0.23 \implies$

$f_a \lesssim 10^{10}$ GeV

$m_a \gtrsim 0.5$ meV

Full evolution



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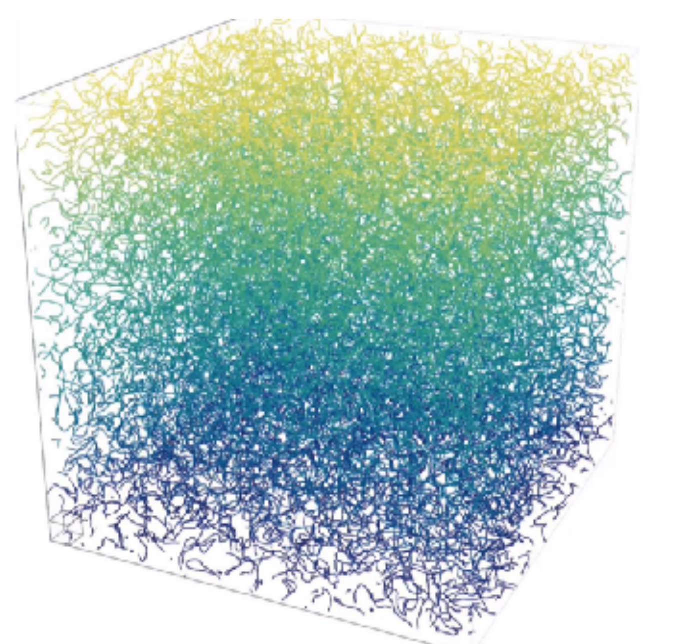
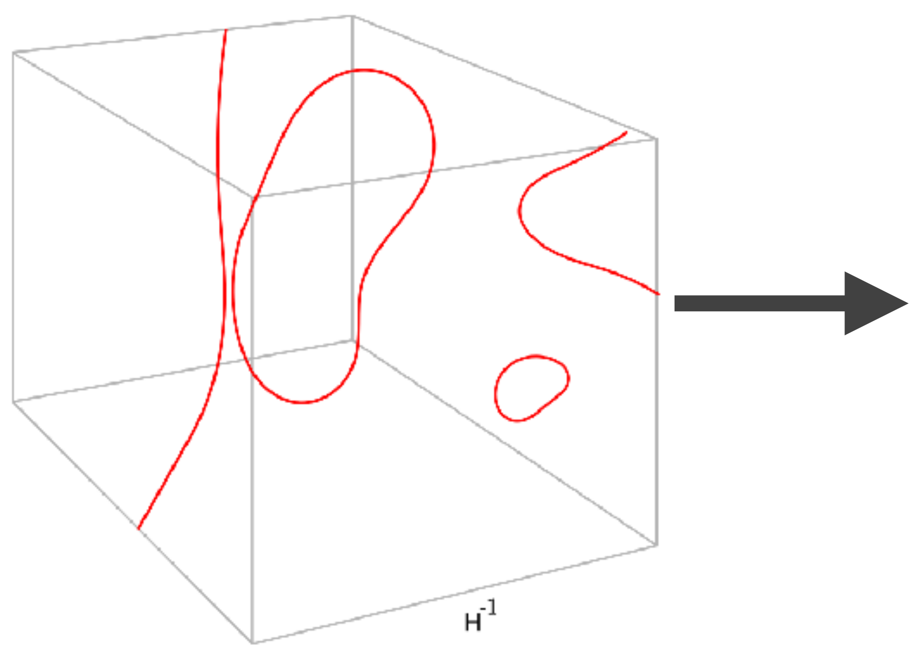
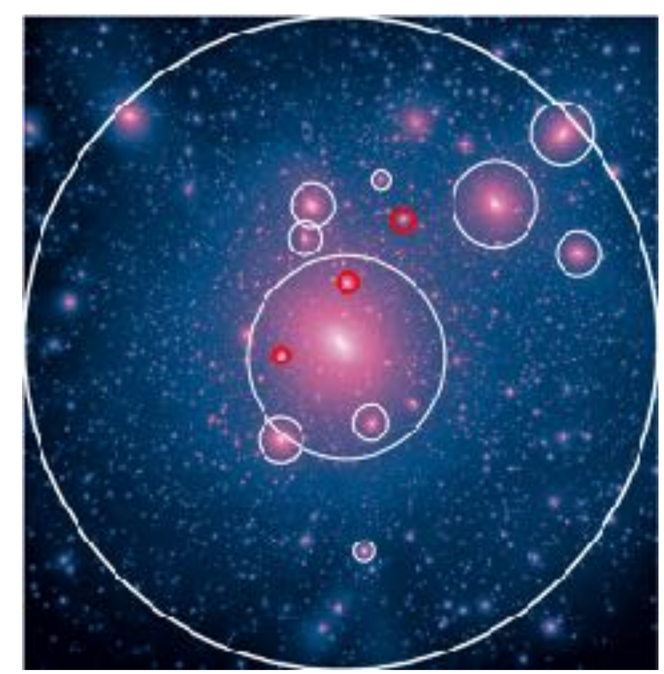






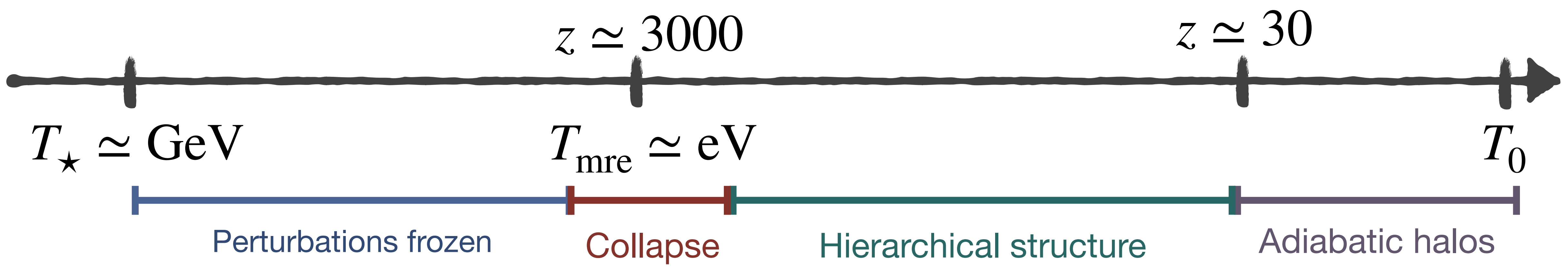
[Eggemeier et al]

Axions with spatial fluctuations

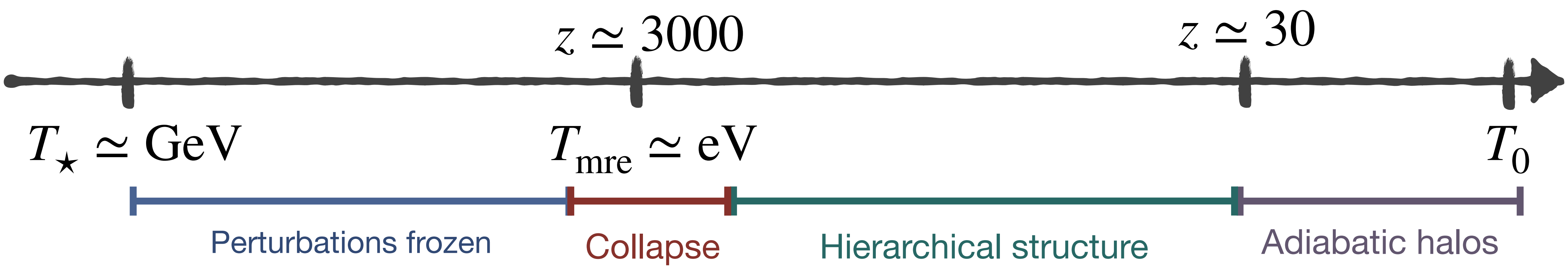
Dark matter substructure

“Axion miniclusters”

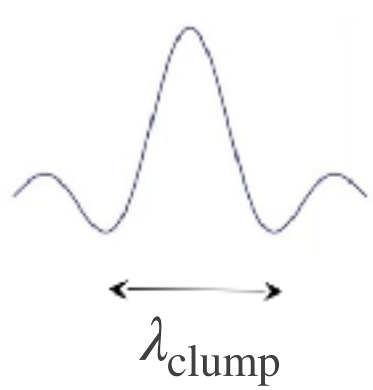
Standard picture



Standard picture



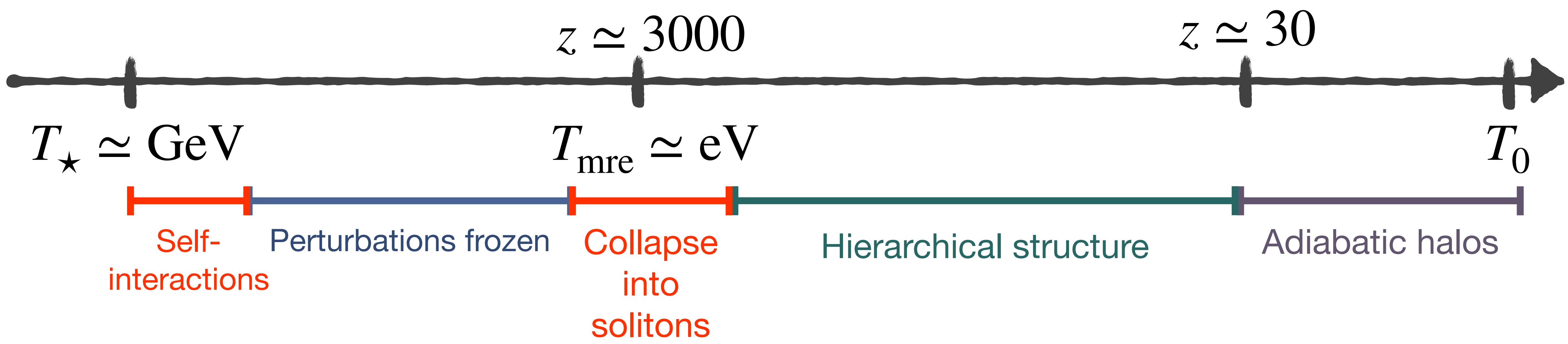
Wave effects at matter radiation equality



$$\lambda_{\rm dB} = \frac{1}{m_a v} = \frac{1}{m_a (GM/\lambda_{\rm clump})^{1/2}} = \frac{1}{\lambda_{\rm clump}(4\pi G\rho m_a^2)^{1/2}}$$

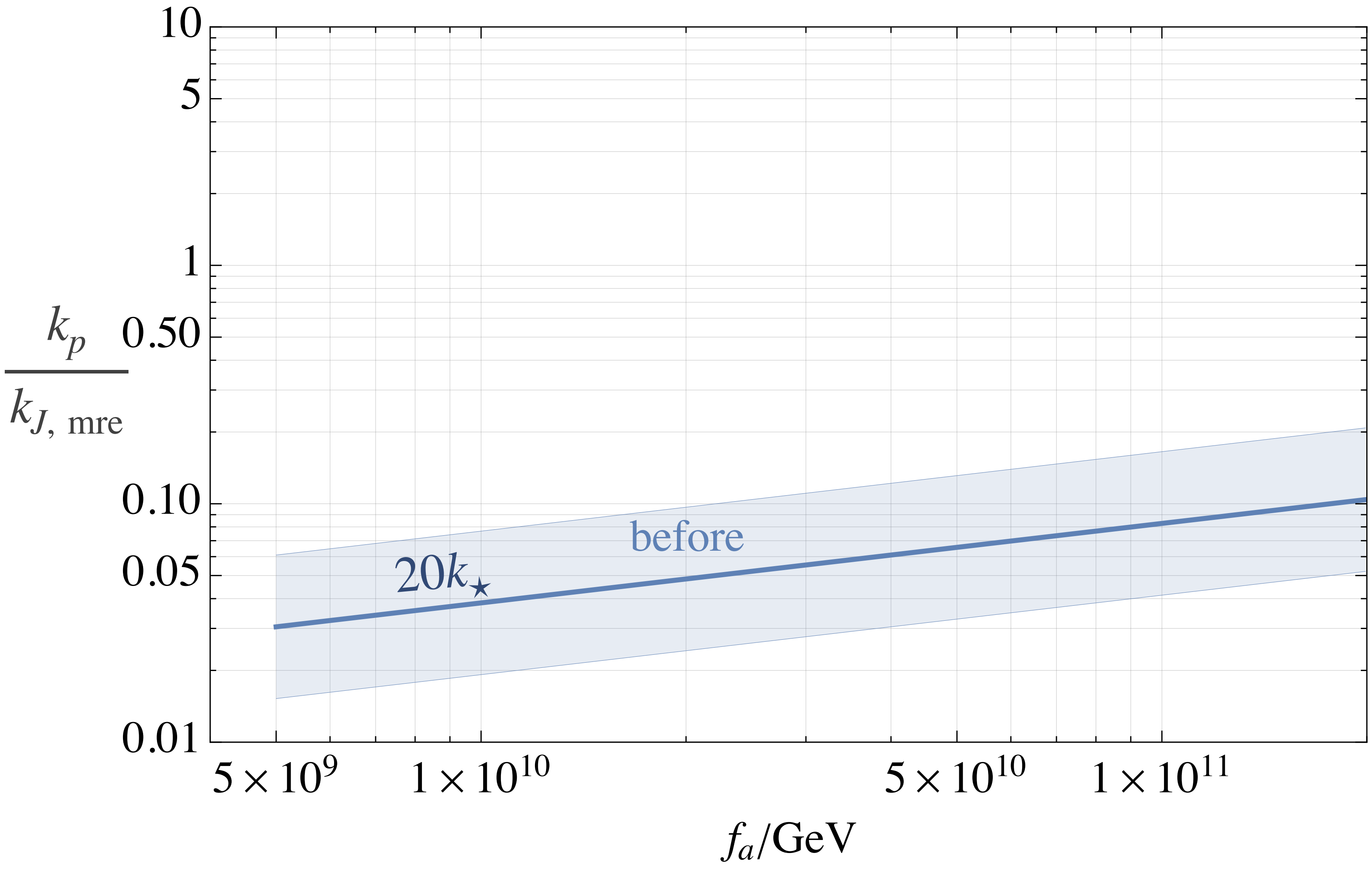
"Quantum" Jeans scale: $\lambda_J \simeq (G\rho m_a^2)^{1/4}$ $\quad k_J/R = (16\pi G\rho m_a^2)^{1/4}$

New aspects


$z \simeq 3000$
$z \simeq 30$
$T_\star \simeq \text{GeV}$
$T_{\rm mre} \simeq \text{eV}$
T_0
Self-interactions
Perturbations frozen
Collapse into solitons
Hierarchical structure
Adiabatic halos

Self-interactions

$$\frac{k_\star}{k_{J,\mathrm{eq}}} \simeq 0.002 \left(\frac{f_a}{10^{10}\ \mathrm{GeV}} \right)^{1/3}$$



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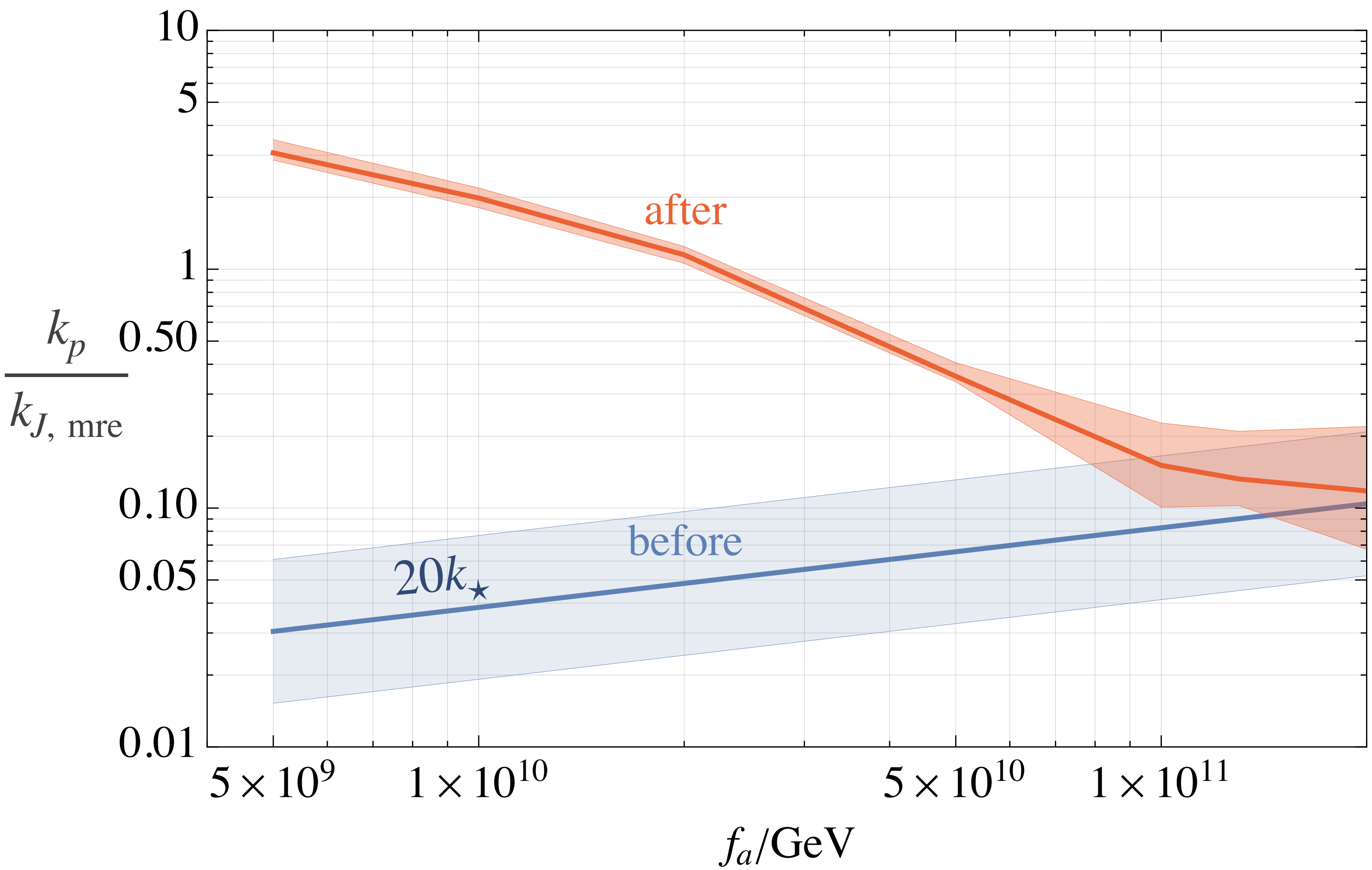
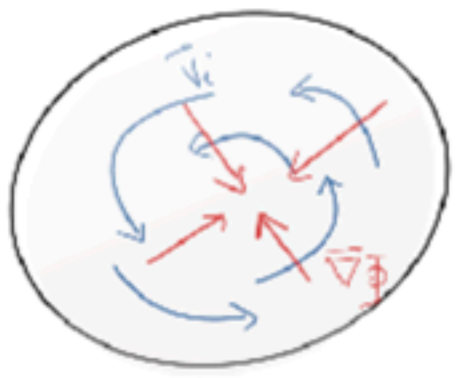


Halos vs solitons

Halos

$\Phi_Q \simeq 0$

→ gravitational potential balanced by velocity term



Angular momentum "supports" the gravitational potential

Halos vs solitons

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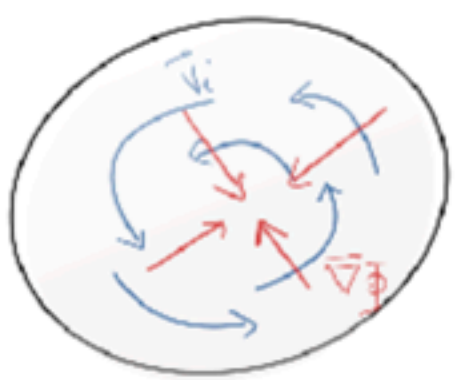
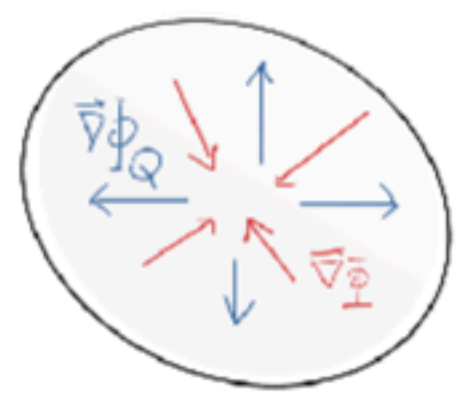


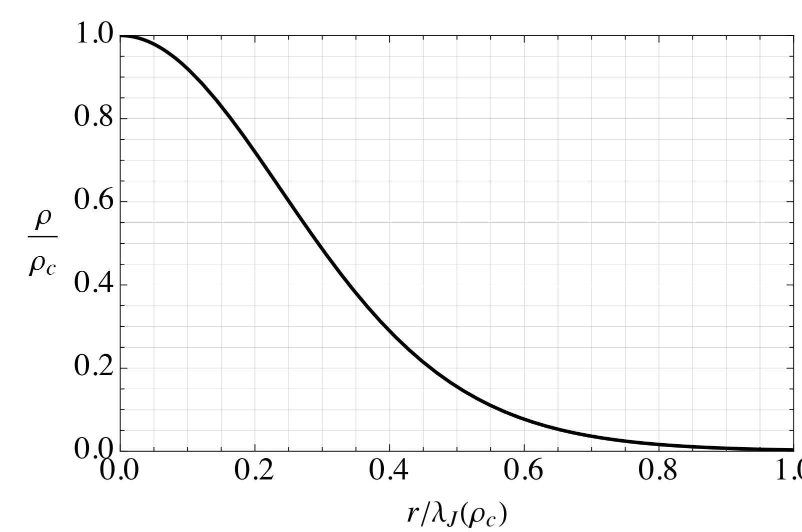
Angular momentum “supports” the gravitational potential

Soliton

$\Phi_Q = -\Phi \qquad \vec{v} = 0$

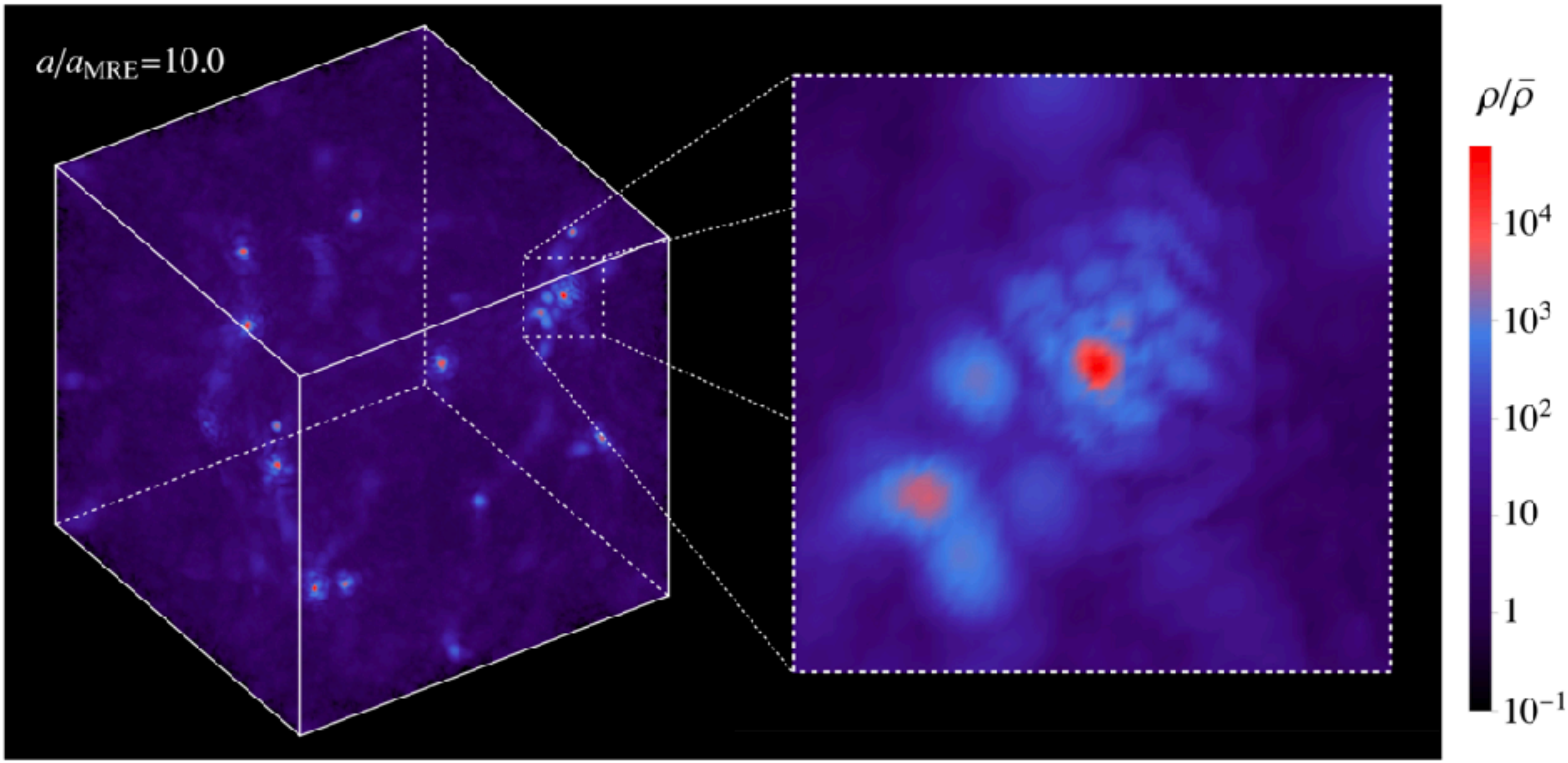
→ gravitational potential balanced by quantum pressure *“Axion star”*



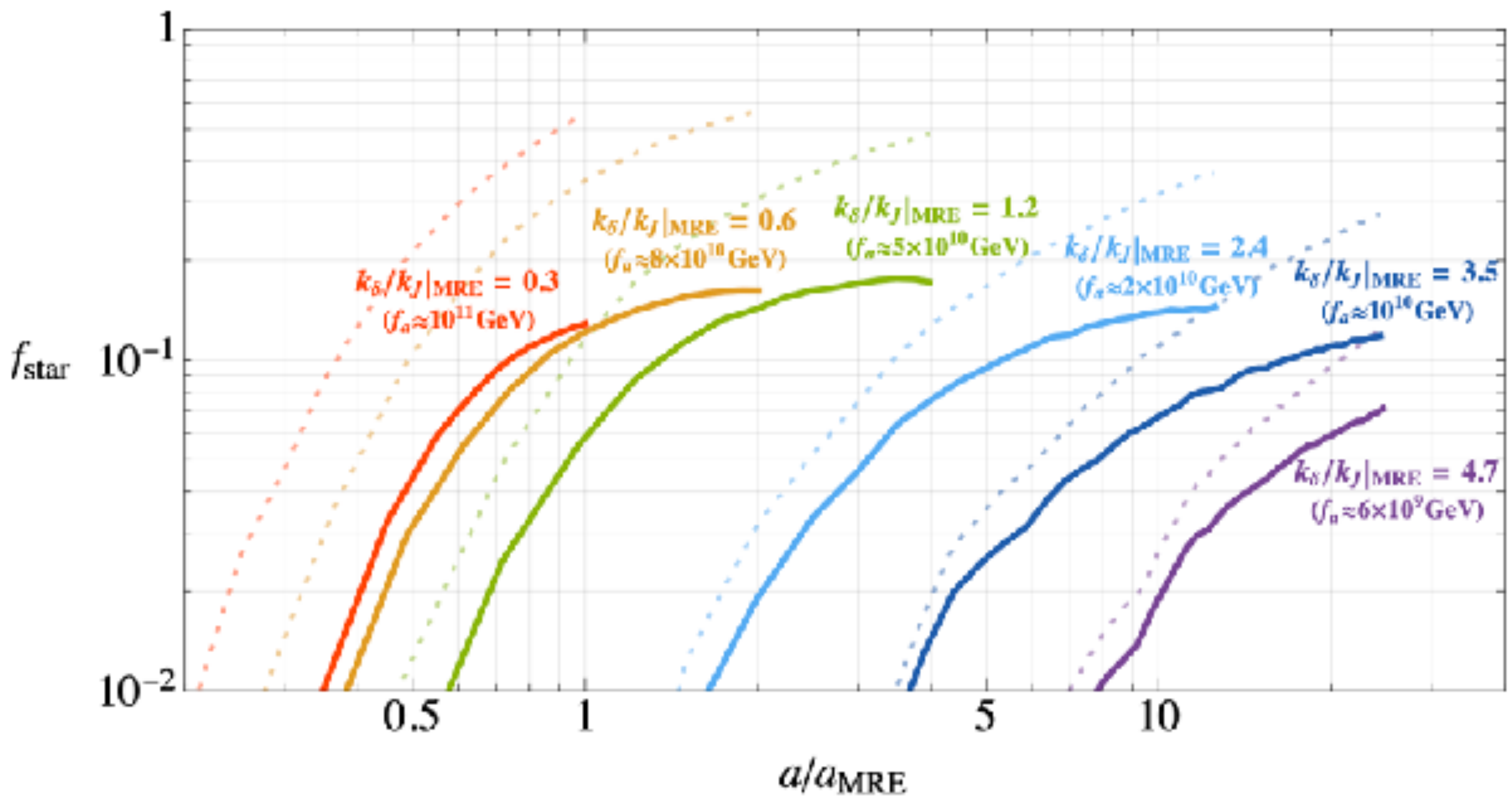


Quantum pressure “supports” the gravitational potential

Simulations

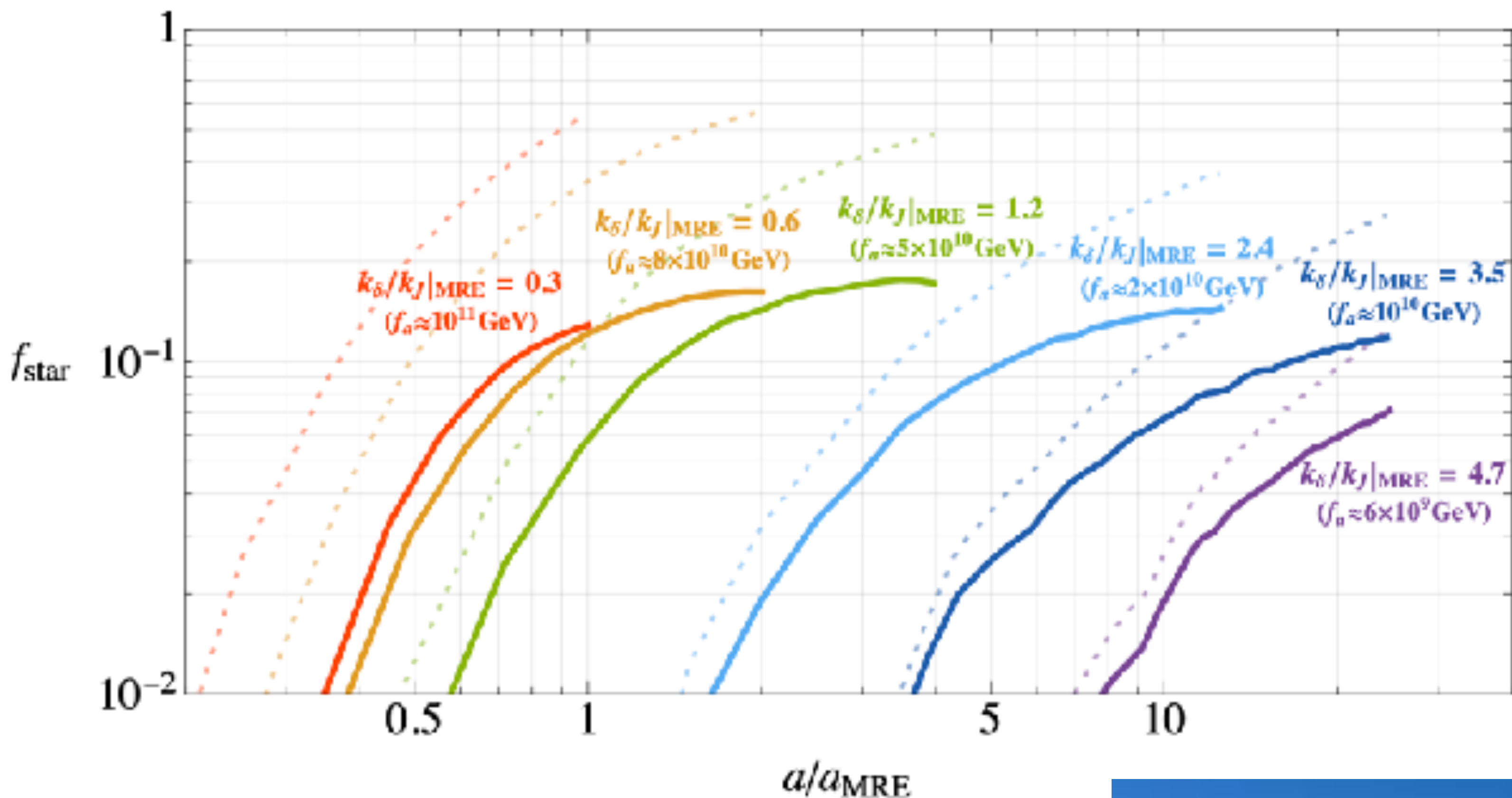


Properties of the substructure



$$R_{0.1} \simeq 4.2 \times 10^6\ {\rm km} \left(\frac{f_a}{10^{10}\ {\rm GeV}}\right)^2 \left(\frac{10^{-19} M_\odot}{M_s}\right)$$

Properties of the substructure



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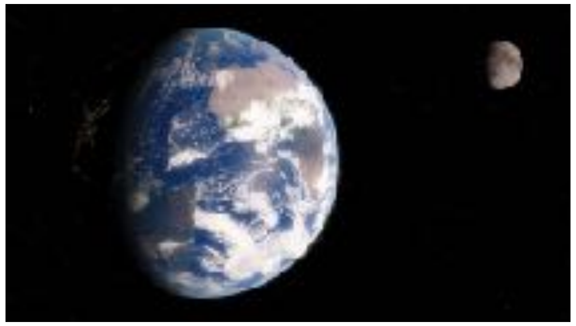

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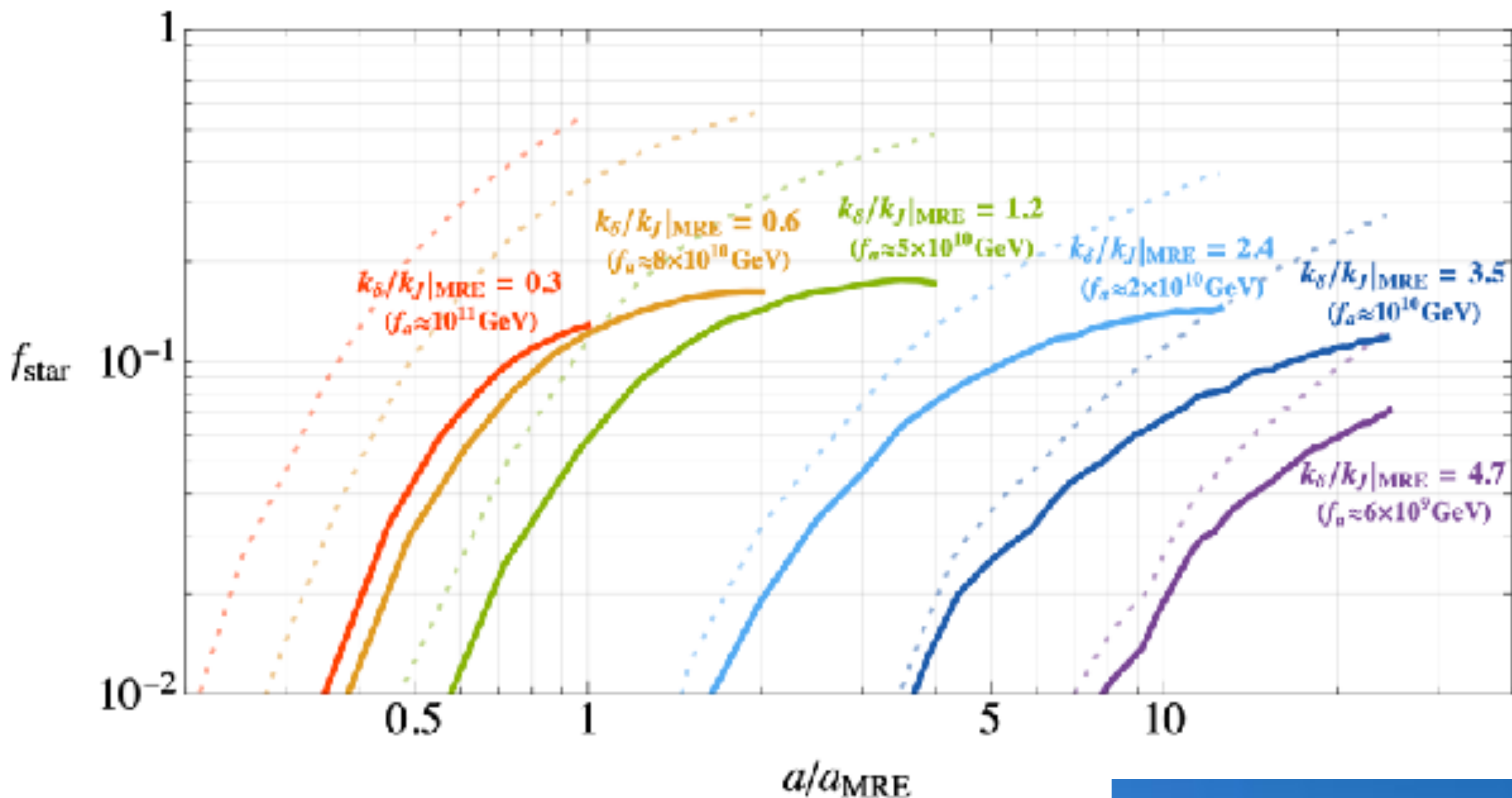


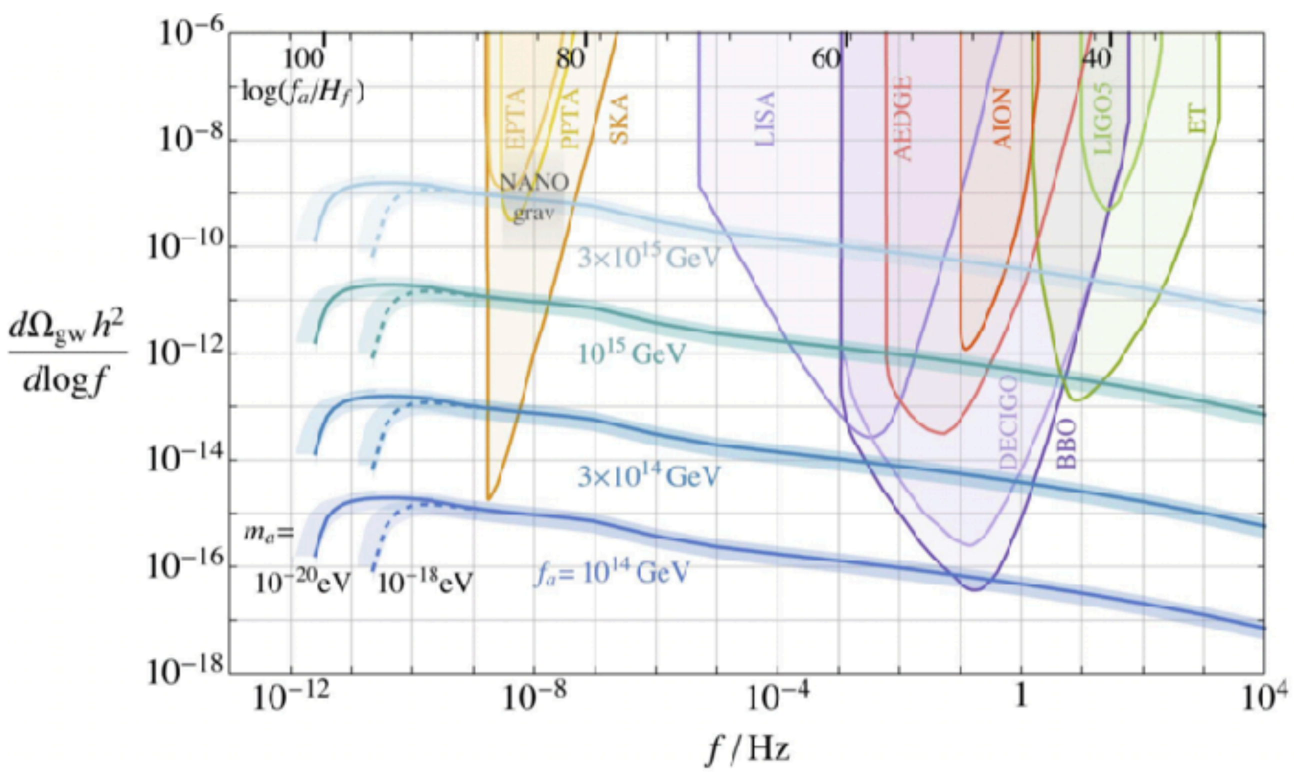
$$R_{0.1} \simeq 4.2\times10^6\ \text{km}\left(\frac{f_a}{10^{10}\ \text{GeV}}\right)^2\left(\frac{10^{-19}M_\odot}{M_s}\right)$$

$$\tau_\oplus = 5\ \text{yrs}\left(\frac{R_{0.1}}{R}\right)^2\left(\frac{0.1}{f_{\text{star}}}\right)\left(\frac{\bar{M}_s}{10^{-19}M_\odot}\right)^3\left(\frac{10^{10}\ \text{GeV}}{f_a}\right)^4$$

$$\Delta t \simeq \frac{2R_{0.1}}{v_r}\sqrt{1-\frac{R^2}{R_{0.1}^2}} = 8\ \text{hrs}\left(\frac{10^{-19}M_\odot}{\bar{M}_s}\right)\left(\frac{f_a}{10^{10}\ \text{GeV}}\right)^2\sqrt{1-\frac{R^2}{R_{0.1}^2}}$$

Summary

- GW spectrum from axion strings has a $\Gamma_g \propto \log^4$ scaling violation
- Enhances the spectrum at low frequencies, observable for $f_a > 10^{14}$ GeV



- Simple post-inflationary QCD axion:

  $\simeq 20\,\%$ of dark matter in axion stars soon after MRE