A Holographic View of the QCD axion

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Strong CP problem

$$\mathcal{L}_{QCD} \supset \theta G\tilde{G}$$

CP-odd term

Basis independent: $\bar{\theta} = \theta + \arg(\det \mathcal{M}_q)$

Observable effect

Neutron electric dipole moment: $d_N \simeq (5 \times 10^{-16} e \cdot \text{cm})\bar{\theta}$

$|d_N| \lesssim 3 \times 10^{-26} e \cdot \text{cm}$ ⇨ $\bar{\theta} \lesssim 10^{-10}$

Why is $\bar{\theta}$ so small?

$\bar{\theta}$ does not appear to be "anthropic"

Our Universe possible for $0 \lesssim \bar{\theta} \lesssim 0.1$ [Lee,Meissner,Olive,Shifman,Vonk: 2006.12321]

Dynamical Solution: PQ mechanism and axion

[Peccei, Quinn 1977; Weinberg 1978; Wilczek 1978]

Axion: $\Phi = \frac{1}{\sqrt{2}}(f_a + \rho)e^{i\frac{a}{f_a}}$ $U(1)_{\rm PQ}$ spontaneously broken $\langle\Phi\rangle = \frac{f_a}{\sqrt{2}}$

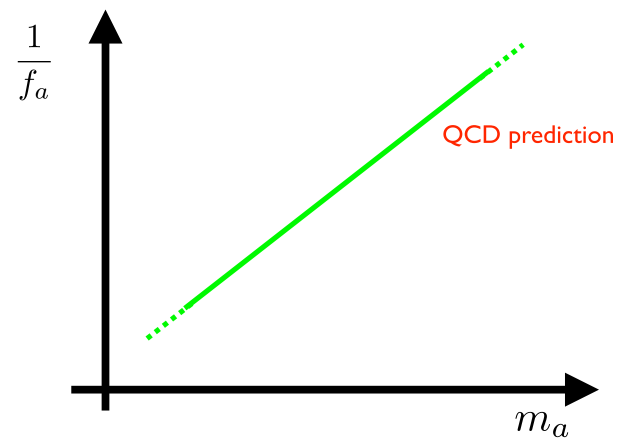
$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \frac{1}{4}a g_{a\gamma\gamma}F\tilde{F} + \frac{1}{f_a}J^\mu\partial_\mu a$$

Axion potential: $V_{\rm QCD}(a) \simeq -m_a^2 f_a^2 \cos\left(\frac{a}{f_a} + \bar{\theta}\right) + \ldots$ ⇨ *minimum:* $\theta_{\rm eff} \equiv \frac{a}{f_a} + \bar{\theta} = 0$! solves strong CP problem!

$$m_a^2 = \frac{\mathcal{T}}{f_a^2} \qquad \mathcal{T} \equiv -i\int d^4x\, \langle 0|\mathrm{T}\left[\frac{1}{32\pi^2}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}(x),\ \frac{1}{32\pi^2}G^b_{\rho\sigma}\widetilde{G}^{b\rho\sigma}(0)\right]|0\rangle$$

topological susceptibility

$$m_a^2 f_a^2 \sim m_q \Lambda^3_{\rm QCD} = \text{constant}$$

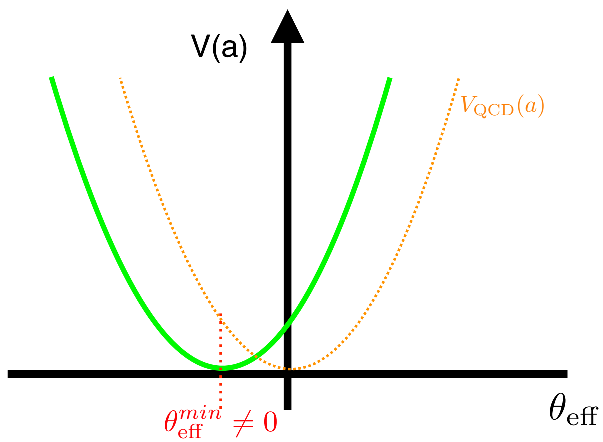


Axion quality:

[Georgi, Hall, Wise 1981; Giddings, Strominger 1988; S.J.Rey 1989; Holman et al 1992; March-Russell, Kamionkowski 1992]

Gravitational violation of $U(1)_{\rm PQ}$

$$\frac{|c_n|e^{i\delta_n}}{M_P^{n-4}}\Phi^n + h.c. \quad \Rightarrow \quad V(a) = V_{\rm QCD}(a) - \frac{|c_n| f_a^n}{M_P^{n-4}}\cos\left(\frac{na}{f_a}+\delta_n\right)$$



⇒ *minimum:* $\theta_{\rm eff} \neq 0$!

axion quality problem

Terms must be suppressed to very high order! $(c_n \sim 1, n \geq 10)$

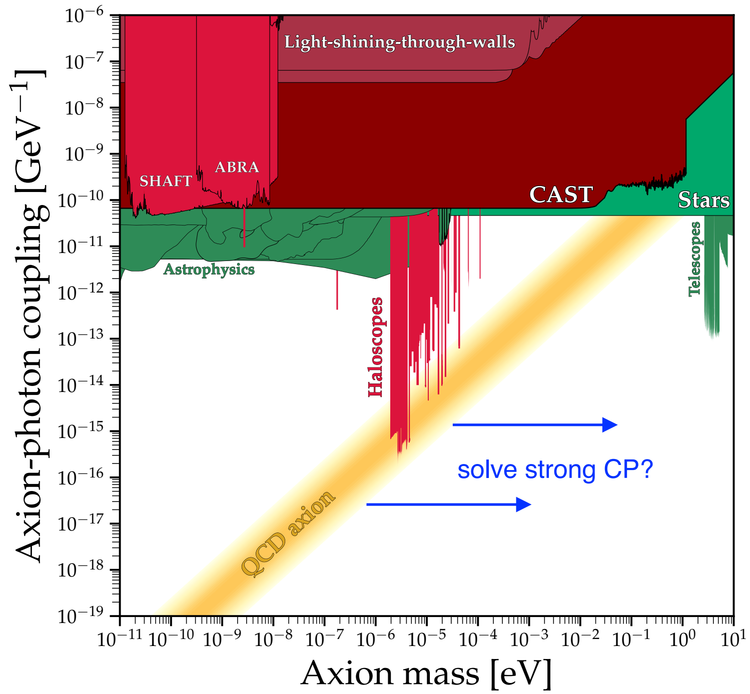
[Note: gravitational instantons $c_n \sim e^{-S} \longrightarrow \; S \geq 200$ but $S \sim \log\frac{M_P}{f_a\sqrt{g_s}} \sim \frac{3}{2}\log\frac{1}{g_s}$ for dynamical radial mode]

[Alvey, Escudero: 2009.03917]

Questions

- How to solve the axion quality problem?
- Can the QCD axion mass be different?

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi}\frac{1}{f_a}$$

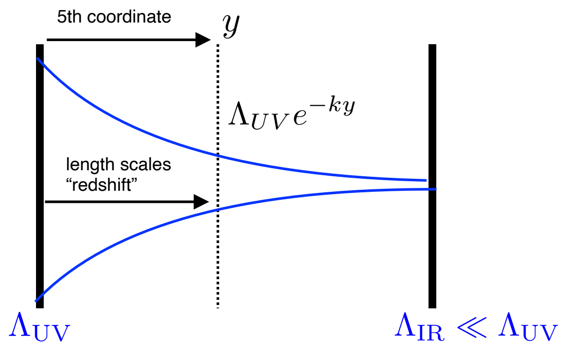


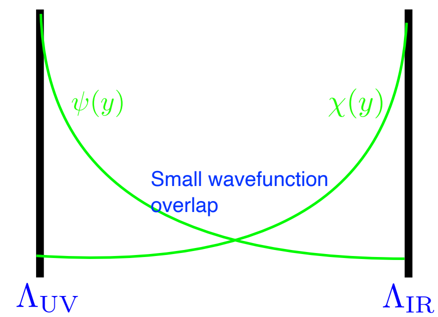
➡ Use 5th dimension to address these questions!

Why use the 5th dimension?

◆ Can generate a hierarchy of scales and small couplings!

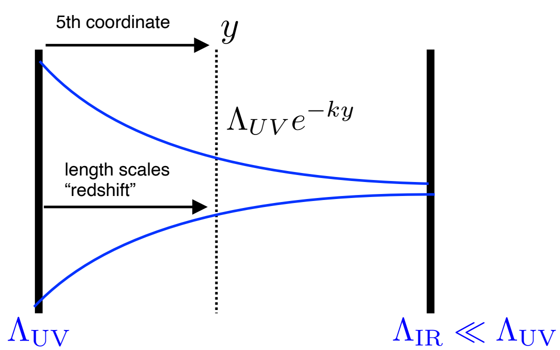
5D AdS



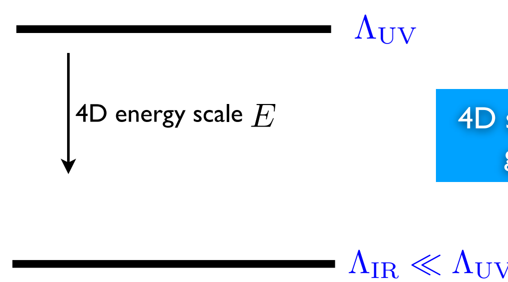


◆ "Warped" dimension is dual to 4D strong dynamics!

5D AdS



⟺



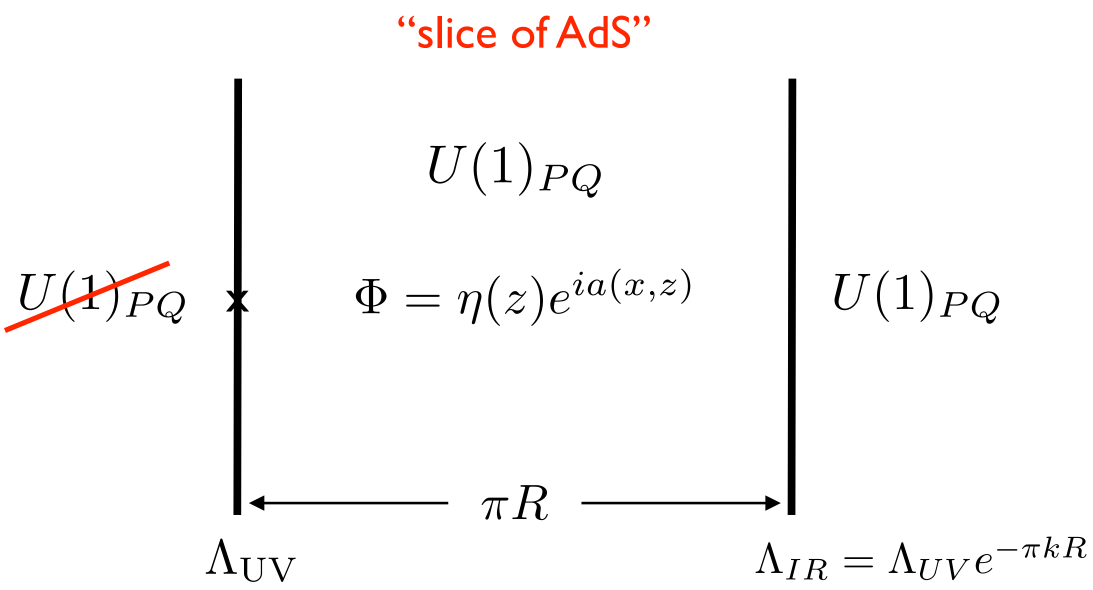
4D strongly-coupled gauge theory

"Holographic view"

I. Axion Quality Problem [Cox, TG, Nguyen 1911.09385]

5D metric: $ds^2 = A^2(z)(dx^2 + dz^2) \equiv g_{MN}dx^M dx^N$ $\quad A(z) = \frac{1}{kz}$

AdS curvature scale



[Randall, Sundrum '99]
[Dienes, Dudas, TG '99]
[Choi '03; Flacke, Gripaios, March-Russell, Maybury '06]

$$m_\Phi^2 = \Delta(\Delta - 4)k^2$$

"Bulk mass"

Explains PQ scale

PQ symmetry breaking ⇒

$$\eta(z) = k^{3/2}(\lambda(kz)^{4-\Delta} + \sigma(kz)^{\Delta})$$

"Bulk VEV"

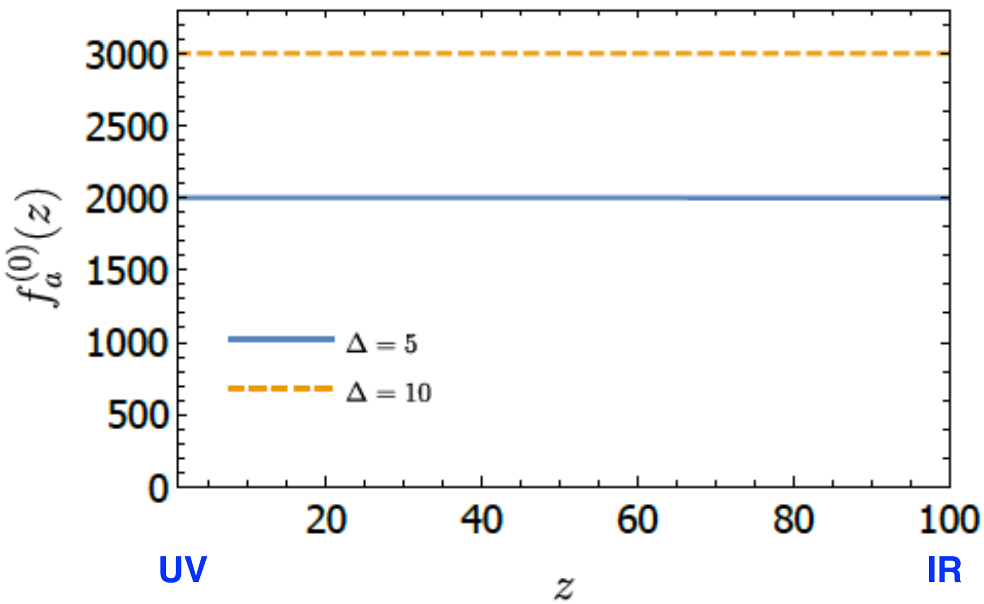
$$\lambda = \frac{\ell_{UV}}{\Delta - 4 + b_{UV}}(kz_{UV})^{\Delta-4},$$ "explicit" breaking

$$\sigma = \sqrt{v_{IR}^2 - \frac{\Delta}{2\lambda_{IR}}}\,(kz_{IR})^{-\Delta} \equiv \sigma_0\,(kz_{IR})^{-\Delta}$$ "spontaneous" breaking

No UV PQ-breaking $(\lambda = 0)$

$(z_{IR} \gg z_{UV})$

$$f_a^{(0)}(z) \simeq \frac{z_{IR}}{\sigma_0}\sqrt{\Delta - 1}\left(1 + \frac{g_5^2 k\sigma_0^2}{4\Delta(\Delta-1)}\left(\frac{(\Delta-1)^2}{2\Delta-1} + \frac{z^2}{z_{IR}^2}\left(\left(\frac{z}{z_{IR}}\right)^{2(\Delta-1)} - \Delta\right)\right) + \mathcal{O}(\sigma_0^4)\right)$$



Global $U(1)_{PQ}$ symmetry:

$$a^{(0)}(x^\mu) \to a^{(0)}(x^\mu) + \alpha_0$$

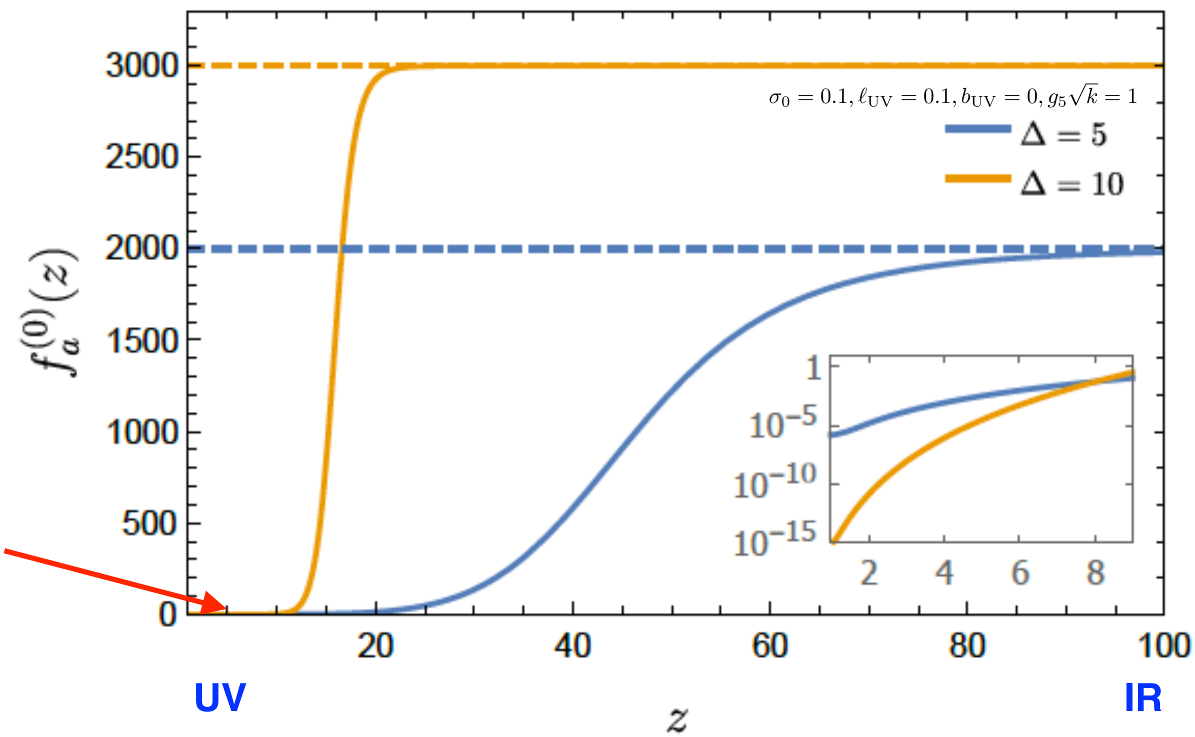
Massless axion

UV PQ-breaking $(\lambda \neq 0)$

$$U_{UV}(\Phi) \supset -2\ell_{UV} k^{5/2}\eta\cos(a) = -2\ell_{UV} k^{5/2}\eta\left(1-\frac{1}{2}a^2+\dots\right)$$

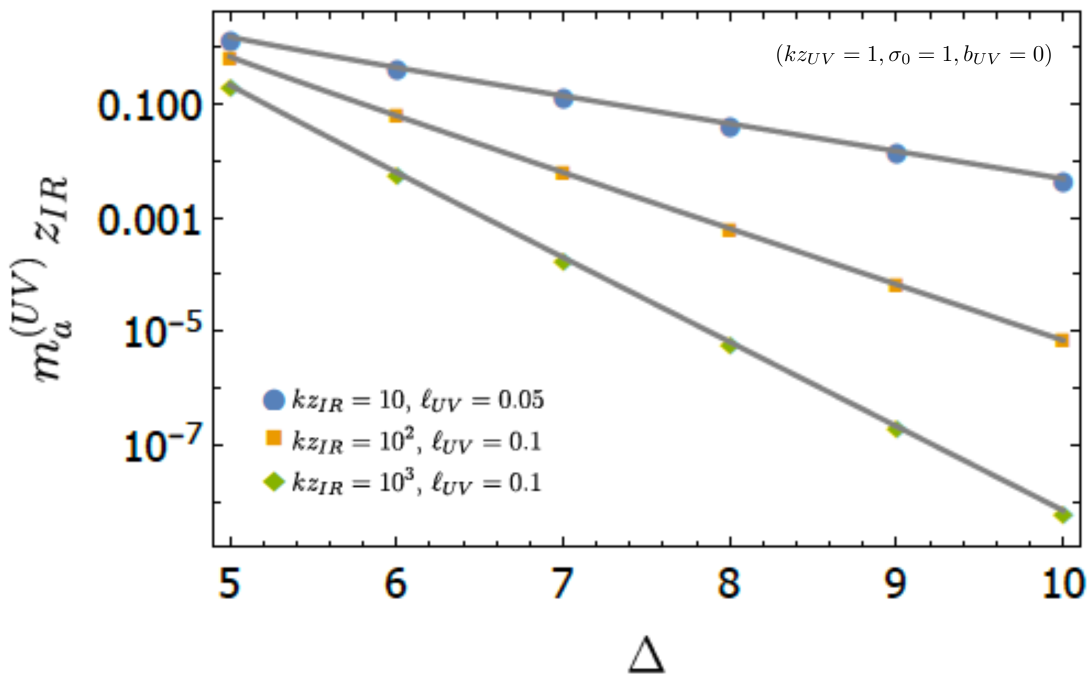
$(z_{IR} \gg z_{UV})$

$$f_a^{(0)}(z) \simeq z_{IR}\frac{k^{3/2}}{\eta(z)}\sqrt{\Delta-1}\left(\frac{z}{z_{IR}}\right)^{\Delta}\left[1+\frac{2\lambda(\Delta-2)(kz_{UV})^{\Delta}(kz)^{2(2-\Delta)}}{\ell_{UV}+2\sigma_0(\Delta-2)(z_{UV}/z_{IR})^{\Delta}}\right]$$



Axion profile suppressed!

UV axion mass $(\lambda \neq 0)$



UV axion mass
suppressed for large Δ

Bulk Chern-Simons term: $-\frac{\kappa}{32\pi^2}\int_{z_{UV}}^{z_{IR}} d^5x\, \epsilon^{MNPQR} V_M G^a_{NP} G^a_{QR}$ ← generates axion-gluon coupling

$$(m_a^{(UV)})^2 = \frac{4\ell_{UV}\sigma_0(\Delta-2)}{\kappa^2(\Delta-4+b_{UV})}\left(\frac{\kappa\sqrt{\Delta-1}}{\sigma_0}\right)^{\Delta}\left(\frac{F_a}{\Lambda_{UV}}\right)^{\Delta-4} F_a^2$$

where

$$F_a \simeq \frac{1}{\kappa}\frac{\sigma_0}{\sqrt{\Delta-1}} z_{IR}^{-1}$$

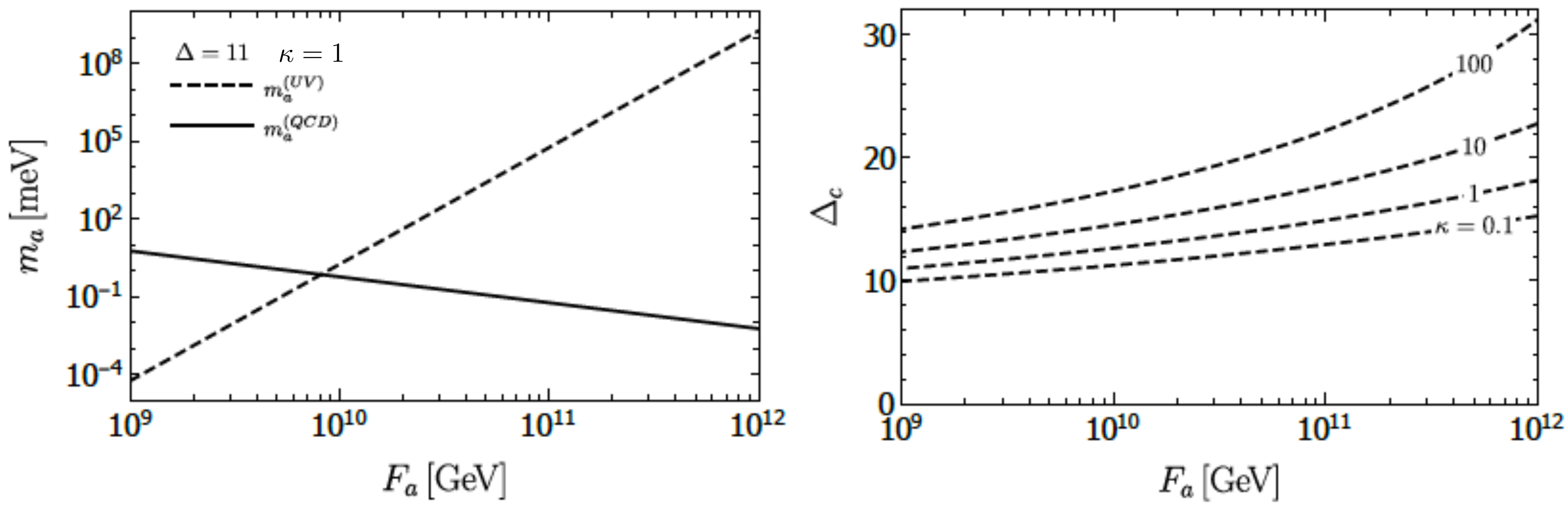
Axion potential:

UV explicit PQ violation

$$V(a^{(0)}) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \bar{\theta}\right) - (m_a^{(UV)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \delta\right)$$

relative phase

Require: $(m_a^{(UV)})^2 \lesssim 10^{-10}\,(m_a^{(QCD)})^2$



10^9 GeV $\lesssim F_a \lesssim 10^{12}$ GeV $\Rightarrow$ $\Delta_c \gtrsim 10$ ← **Solves axion-quality problem!**

AdS/CFT dictionary

5D		4D
local $U(1)_{PQ}$ symmetry	$\leftrightarrow$	global $U(1)_{PQ}$ symmetry
$\Phi,\ m_\Phi^2 = \Delta(\Delta-4)k^2$	$\leftrightarrow$	$\mathcal{O}$, dimension Δ
$\Lambda_{\rm UV} e^{-\pi k R} = \Lambda_{\rm IR}$	$\leftrightarrow$	$\Lambda_{\rm IR} = \Lambda_{\rm UV} e^{-\frac{8\pi^2}{b_{\rm CFT} g^2}}$
σ	$\leftrightarrow$	$\langle \mathcal{O} \rangle$
λ	$\leftrightarrow$	$\lambda \phi_0 \mathcal{O}$

Holographic interpretation:

5D axion, local U(1) PQ symmetry ⟺ 4D composite axion, accidental global U(1) PQ symmetry

Axion quality: $\frac{|c_n| e^{i\delta_n}}{M_P^{n-4}} \Phi^n \quad (n \gtrsim 10) \quad \leftrightarrow \quad \frac{c_\Delta}{M_P^{\Delta-4}} \mathcal{O} \sim \frac{c_\Delta}{M_P^{\Delta-4}} (\psi_i \psi_j \ldots \ldots \psi_k) \quad (\Delta \gtrsim 10)$

due to gauge + Lorentz symmetry!

Examples: [Kim 1985; Choi, Kim 1985];
[Randall1992];[Cheng, Kaplan 2001]; [Redi, Sato 2016]; [Lillard, Tait 2018]; [Contino,Podo, Revello: 2112.09635]

4D Model

[Gavela, Ibe, Quilez, Yanagida: 1812.08174] [Cox, TG, Paul: 2310.08557]

New strong gauge group

	$SU(5)$	$SU(3)_c$
$\psi_{\bar{5}}$	$\bar{\mathbf{5}}$	$\mathbf{R}$
ψ_{10}	$\mathbf{10}$	$\mathbf{R}$

Global symmetries

	$SU(n)_{\bar{5}}$	$SU(n)_{10}$	$U(1)_{B-L} \equiv U(1)_{PQ}$
$\psi_{\bar{5}}$	$\square$	$\mathbf{1}$	-3
ψ_{10}	$\mathbf{1}$	$\square$	1

Chiral condensate: $\langle \mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}} \rangle \sim \Lambda_5^6 \quad \Longrightarrow \quad SU(n)_{\bar{5}} \times SU(n)_{10} \longrightarrow G \supset SU(3)_c$

PQ condensate: $\langle \bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10} \rangle \sim \Lambda_5^9$ ← dimension 9! (due to gauge and Lorentz invariance)

composite axion!

$$\Rightarrow \quad \mathcal{L}_{\cancel{PQ}} = \frac{c_{\cancel{PQ}}}{4\pi} \frac{(\Lambda_5 f_{\rm PQ}^2)^3}{M_{\rm Pl}^5} e^{-2i\frac{a}{f_{\rm PQ}}} + {\rm h.c.} \qquad f_{\rm PQ} = \Lambda_5/g_*, \quad 1 \lesssim g_* \lesssim 4\pi.$$

Composite sterile neutrinos: $\Psi_1 \equiv \psi_{\bar{5}}^\dagger \psi_{\bar{5}}^\dagger \psi_{10}^\dagger\,, \;\; \Psi_2 \equiv \psi_{\bar{5}}^\dagger \psi_{10} \psi_{10}$

$$\mathcal{L}_{\rm EFT} = \frac{\tilde{\xi}_{ij}}{\Lambda_L^3} L_i H \left(\psi_{\bar{5}} \psi_{\bar{5}} \psi_{10}\right)_j + \frac{\tilde{\xi}'_{ij}}{\Lambda_L^3} L_i H \left(\psi_{\bar{5}} \psi_{10}^\dagger \psi_{10}^\dagger\right)_j + {\rm h.c.} \quad \Rightarrow \quad m_{\nu,i}^{\rm active} = \lambda_{\nu,i} \left(\frac{\Lambda_5 f_{\rm PQ}^2}{\Lambda_L^3}\right)^2 \frac{v^2}{2\Lambda_5}$$

Common origin for the see-saw mass and PQ scales!

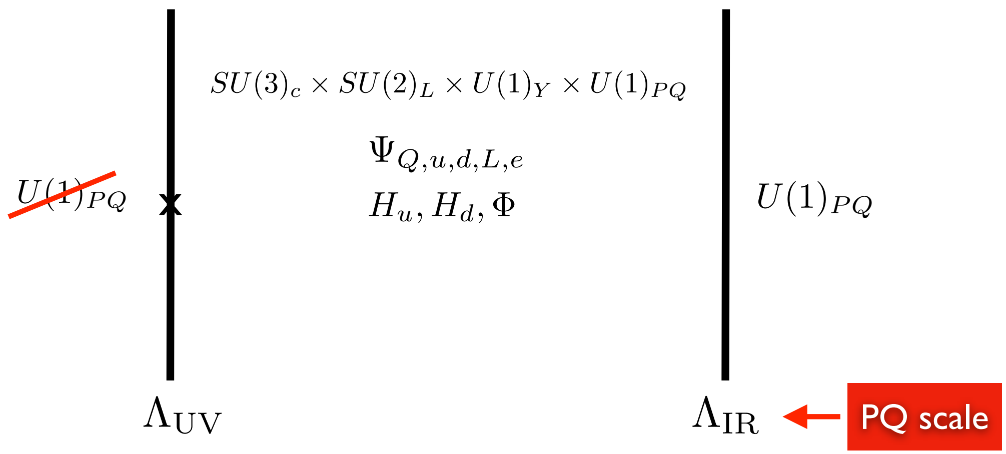
$$\frac{m_{\nu,i}^{\rm active}}{m_a} \sim \frac{\lambda_{\nu,i}}{\mathcal{N} g_*^5} \left(\frac{13.2\,\Lambda_5}{\Lambda_L}\right)^6$$

i.e. $m_{\nu,i}^{\rm active} \sim m_a$

Flavored Warped Axion [Bonnefoy, Cox, Dudas, TG, Nguyen: 2012.09728]

DFSZ-like axion model with bulk Standard Model fermions:

[Also Wilczek 1982; Ema,Hamaguchi,Moroi,Nakayama 2016
Calibbi,Goertz,Redigolo,Ziegler,Zupan 2016; Bonnefoy,Dudas,Pokorski 2019]



Axion-fermion couplings:

$$i\int d^4x\,\frac{\partial_\mu a^0}{2F_a}\left(\bar{u}_i\gamma^\mu\left((c_u^V)_{ij}-(c_u^A)_{ij}\gamma^5\right)u_j\right)$$

$$A_L^u m_u^{ij} A_R^{u\dagger} = m_{u_i}$$

(similarly for down-type quarks and leptons)

Overlap between axion and fermion profiles

$$(c_u^{V,A})_{ij} = X_{H_u}\int_{z_{UV}}^{z_{IR}}\frac{dz}{(kz)^4}\,g_a^0(z)\left((A_R^u)_{ik}(f^0_{U_{kR}})^2(A_R^{u\dagger})_{kj} \mp (A_L^u)_{ik}(f^0_{Q_{kL}})^2(A_L^{u\dagger})_{kj}\right)$$

Numerical results

Flavor-violating couplings:

$$\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}$$

$$F^A_{u,d,\ell} \approx \mathcal{O}(F^V_{u,d,\ell})$$

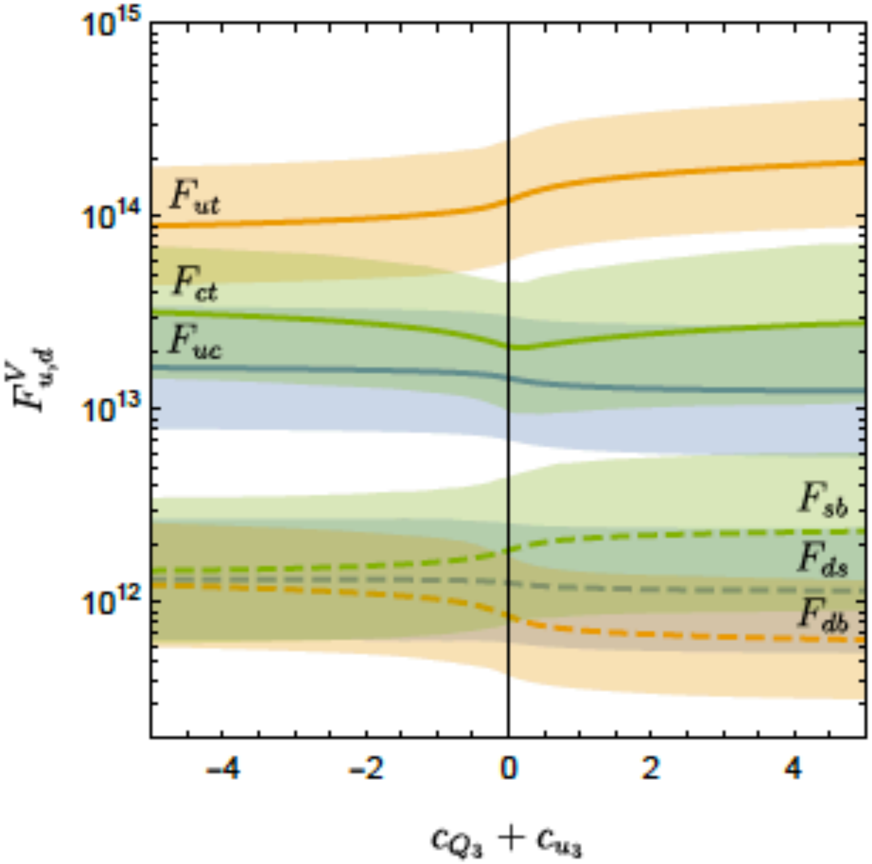
$kz_{\rm IR} = 10^{10}, g_5^2 k = 1$

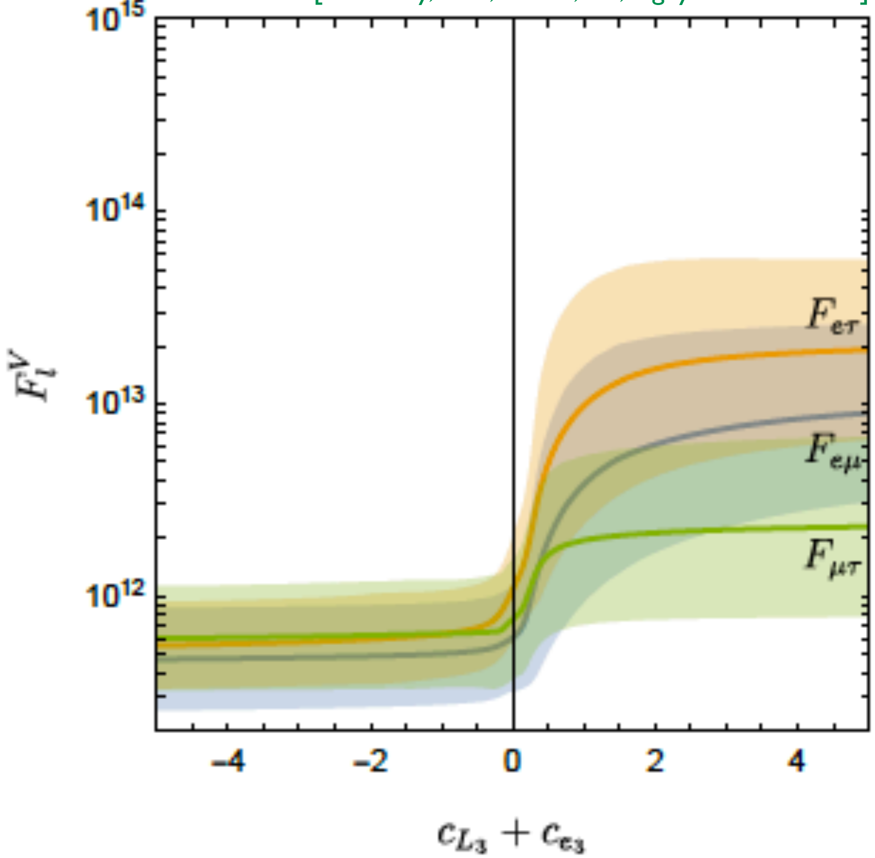
$\Delta = 10 \quad \sigma_0 = 3$

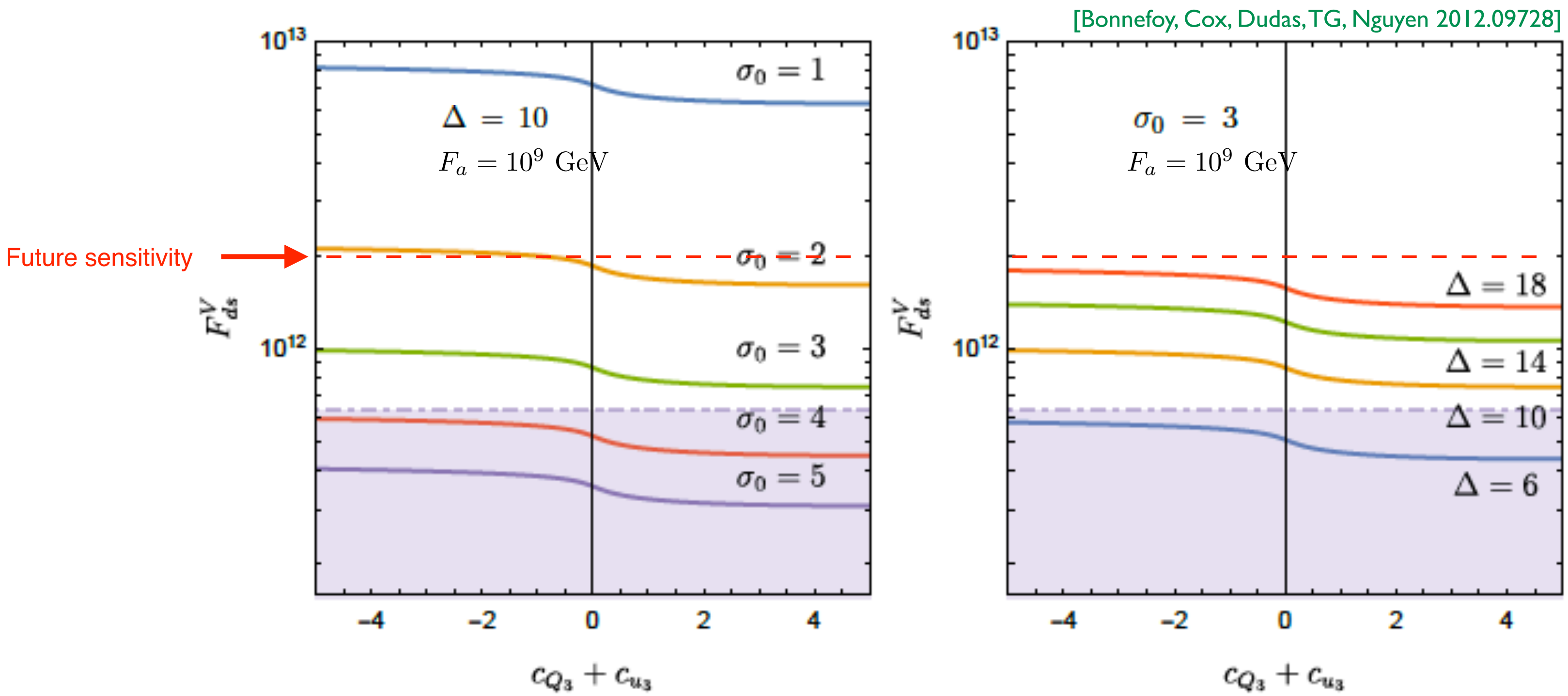
$F_a \simeq 10^9\,\text{GeV}.$

Scan over $y^{(5)}_{u,d,e} \sim 1$

[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]







Experimental limits : $(F_d^V)_{12} \gtrsim 6.8 \times 10^{11}\,\mathrm{GeV}$ $(K^+ \to \pi^+ a \text{ decays})$

[Martin Camalich, Pospelov, Vuong, Ziegler, Zupan 2002.04623]

[$(8.3 \times 10^{11}\ \mathrm{GeV})$ Bauer, Neubert, Renner, Schnubel, Thamm 2102.13112; 2110.10698]

$$\sigma_0 \gtrsim 4, \quad \Delta \gtrsim 6$$

2. Axion mass from 5D small instantons

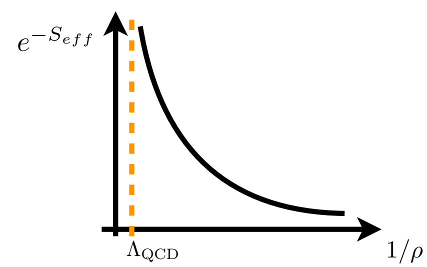
QCD axion mass:

$$m_a^2 = \frac{\mathcal{T}}{f_a^2} \qquad \mathcal{T} \equiv -i \int d^4x \, \langle 0 | \mathrm{T} \left[\frac{1}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}(x), \; \frac{1}{32\pi^2} G^b_{\rho\sigma} \widetilde{G}^{b\rho\sigma}(0) \right] | 0 \rangle$$

"topological susceptibility"

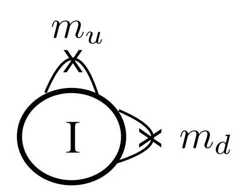
Dilute instanton gas approximation:

$$\mathcal{T} \propto \int \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha_s(1/\rho)}}$$



QCD asymptotically free $\Rightarrow$ $\mathcal{T} \propto \Lambda_{QCD}^4$

Fermion zero modes: $(\rho m_f)^{N_f} \longrightarrow$ suppression $\frac{\prod_f m_f}{\Lambda_{QCD}^{N_f}}$

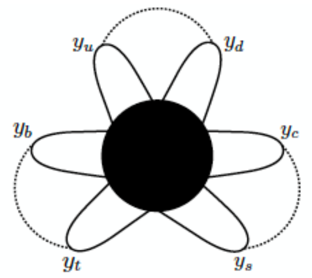


$$\Rightarrow \quad \boxed{m_{a,QCD}^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}}$$

How to enhance QCD axion mass?

- Change QCD coupling in UV $\alpha_s(1/\rho) \sim 1$ "Small instantons" $\rho \sim 1/\Lambda_{UV}$

- Close fermion loops with Higgs boson



$$\kappa_f = \frac{y_u}{4\pi}\frac{y_d}{4\pi}\frac{y_c}{4\pi}\frac{y_s}{4\pi}\frac{y_t}{4\pi}\frac{y_b}{4\pi} \approx 10^{-23}$$

(otherwise $\frac{m_u m_d m_c m_s m_b m_t}{\Lambda_{UV}^6}$)

$$\Rightarrow \quad m_a^2 f_a^2 \sim m_q \Lambda_{\rm QCD}^3 + \Lambda_I^4$$

new contribution

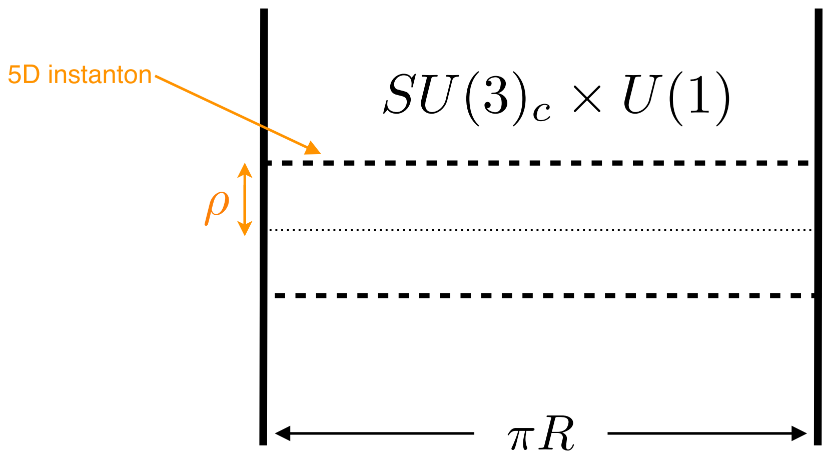
where $\Lambda_I \gg \Lambda_{\rm QCD}$

Use 5th dimension to make QCD axion heavy!

Warm-up: QCD in 5D

Flat space 5D metric: $ds^2 = dx^2 + dy^2$

$$S_5 = -\int d^4x \int_0^L dy \left(\frac{1}{4g_5^2} \mathrm{Tr}[G_{MN}^2] + \frac{b_{CS}}{32\pi^2} \varepsilon^{MNRST} B_M \mathrm{Tr}[G_{NR} G_{ST}] + \frac{1}{4g_5^2} F_{MN}^2 + \dots \right),$$



$A_\mu^{(0)}(x)$ QCD gluon

$B_5^{(0)}(x) \equiv a(x)$ QCD axion

5D instanton: $A_\mu^a(x,y) = A_\mu^{(I)a}(x) = \frac{2\eta_{a\mu\nu}(x-x_0)_\nu}{(x-x_0)^2+\rho^2}, \quad A_5^a(x,y) = 0$

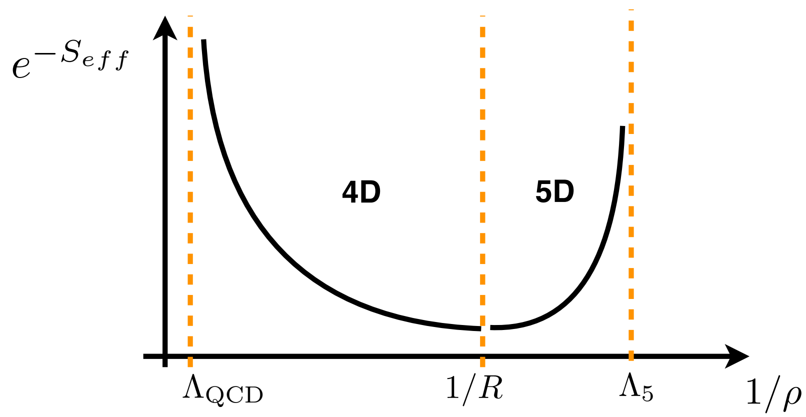
$$\Rightarrow \quad S_5^{(I)} = \frac{8\pi^3 R}{g_5^2} = \frac{2\pi}{\alpha_s}$$

Finite action

5D small instantons

[Poppitz, Shirman '02] [TG, Khoze, Pomarol, Shirman: 2001.05610]

Fluctuations + Kaluza-Klein contributions



small instantons!

$$\mathcal{T} \sim \int_{1/\Lambda_5}^{R} \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/R)}\right)^6 e^{-S_{\rm eff}} \equiv \frac{K}{R^4} \propto m_a^2 f_a^2$$

$$\Rightarrow \quad S_{\rm eff} = \frac{2\pi}{\alpha_s(1/R)} - 3\xi(R/\rho)\frac{R}{\rho} + b_0 \ln\frac{R}{\rho}$$

power law term!

$\xi(R/\rho) \sim 1/3$

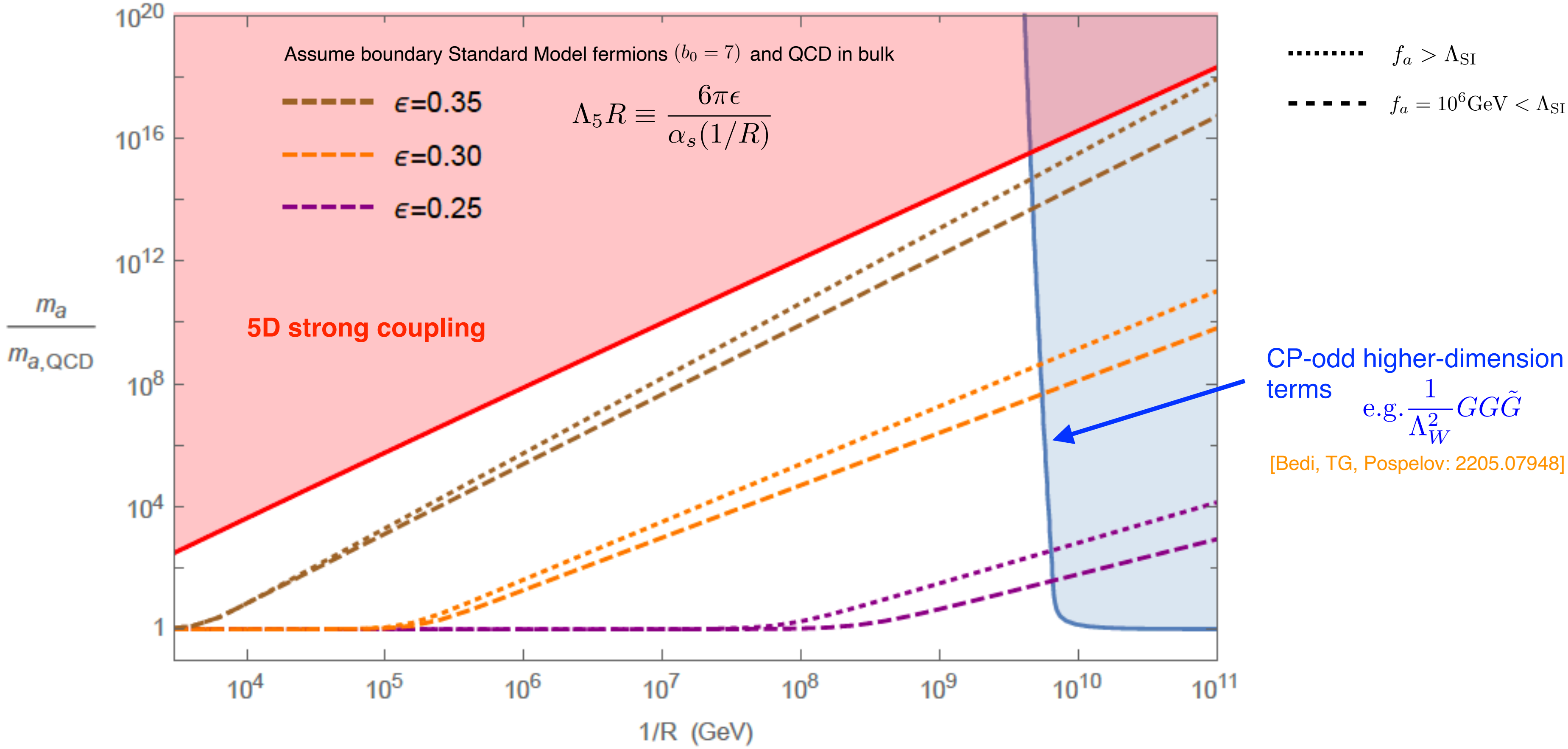
$R/\rho \gtrsim 20$

$$\Rightarrow \quad K \simeq C[3] \left(\frac{2\pi}{\alpha_s(1/R)}\right)^6 (\Lambda_5 R)^{3-b_0} e^{-\frac{2\pi}{\alpha_s(1/R)} + \Lambda_5 R}$$

power law contribution can overcome suppression

$$\text{Valid up to} \quad \frac{g_5^2 \Lambda_5}{24\pi^3} \sim 1 \quad \text{or} \quad \Lambda_5 R \lesssim \frac{6\pi}{\alpha_s}$$

[TG, Khoze, Pomarol, Shirman: 2001.05610]



➪ Small 5D instantons can dominate for $\frac{1}{R} \gtrsim 100\ \text{TeV}$

Enhanced EDMs [Bedi, TG, Pospelov: 2205.07948]

Higher-dimension CP-odd sources are enhanced by small instantons

Weinberg operator: $\mathcal{L} \supset \frac{1}{\Lambda_W^2} G G \widetilde{G}$

[Also fermion operator: $\mathcal{L} \supset \sum_{ijkl} \frac{\lambda_{ijkl}}{\Lambda_F^2} \bar{\psi}_i \psi_j \bar{\psi}_k \psi_l$ [see also Bedi,TG,Grojean,Guedes, Kley,Vuong: 2402.09361]]

$\Rightarrow$ $$V(a) = \chi_W(0)\left(\frac{a}{f_a}\right) + \frac{1}{2}\chi(0)\left(\frac{a}{f_a}\right)^2$$

where $$\chi_W(0) = -i \lim_{k\to 0} \int d^4x\, e^{ikx} \left\langle 0 \left| T\left\{ \frac{1}{32\pi^2} G\widetilde{G}(x), \frac{1}{\Lambda_W^2} GG\widetilde{G}(0) \right\} \right| 0 \right\rangle$$

$\Rightarrow$ $$\left\langle \frac{a}{f_a} \right\rangle \equiv \theta_{\rm ind} = -\frac{\chi_W(0)}{\chi(0)} \propto \frac{\Lambda_{SI}^2}{\Lambda_W^2}$$ $\leftarrow$ Enhanced by $\frac{\Lambda_{SI}}{\Lambda_{QCD}}$

Neutron EDM: $|\theta_{\rm ind}| \lesssim 10^{-10}$ $\Rightarrow$ $\frac{\Lambda_{SI}}{\Lambda_W} \lesssim 10^{-6}$

5D AdS small instantons

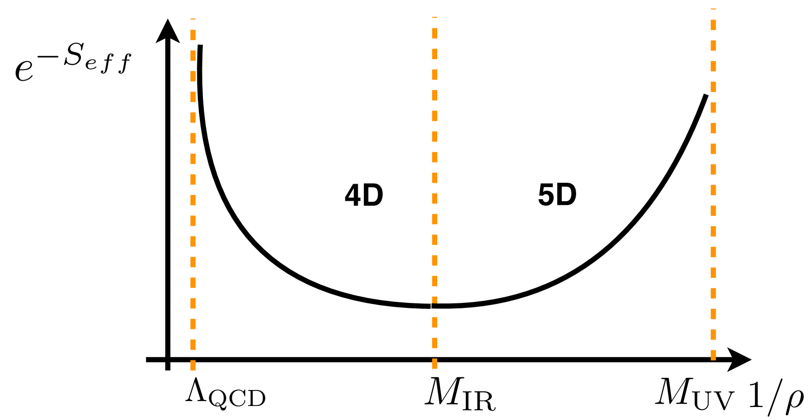
small instantons

Axion mass: $m_a^2 f_a^2 = \mathcal{T} \sim \int_{1/M_{\rm UV}}^{1/M_{\rm IR}} \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(M_{\rm IR})}\right)^6 e^{-\frac{2\pi}{\alpha_s(1/\rho)}} \equiv \kappa M_{\rm UV}^4$ where $\alpha_s(M_{\rm UV}) = 1$

QCD coupling in AdS5: $$\frac{2\pi}{\alpha_s(1/\rho)} = \frac{2\pi}{\alpha_s(M_{\rm IR})} - \left(b_0 + \frac{8\pi^2}{g_5^2 k}\right) \log\left(\frac{\rho^{-1}}{M_{\rm IR}}\right)$$

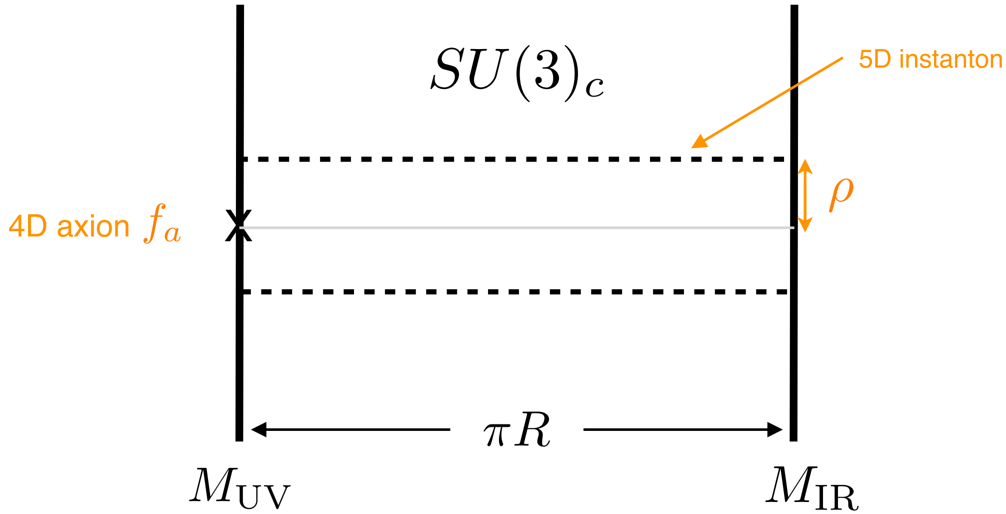
$b_0 = -\frac{11}{3}N_c + \frac{2}{3}N_f = -7$

AdS5 contribution > 0!



Thus, for $\rho^{-1} \gg M_{\rm IR}$ obtain $\frac{2\pi}{\alpha_s(1/\rho)} \ll \frac{2\pi}{\alpha_s(M_{\rm IR})}$ ⇨ axion mass enhancement?

QCD instanton in 5D AdS [TG, Pomarol: 2110.01762]



5D instanton: $A_\mu^{(\rm cl)}(x,z) = -i(\partial_\mu \Omega)\Omega^{-1} \times \frac{x^2}{x^2+\rho^2}$, $\quad A_5(x,z)=0, \quad (\Omega \in SU(2))$

$$\Rightarrow \quad S_5^{(I)}[A_\mu^{(\rm cl)}(x,z)] = -\frac{1}{2g_5^2}\int d^4x \int_{1/M_{\rm UV}}^{1/M_{\rm IR}} dz\sqrt{g}\,{\rm Tr}\left[G_{MN}^{(\rm cl)}G^{MN\,(\rm cl)}\right]$$

$$= \frac{8\pi^2}{g_5^2 k}\log\frac{M_{\rm UV}}{M_{\rm IR}} = \frac{2\pi}{\alpha_s(M_{\rm IR})}$$

← Finite action $\neq \frac{2\pi}{\alpha_s(1/\rho)}$!!

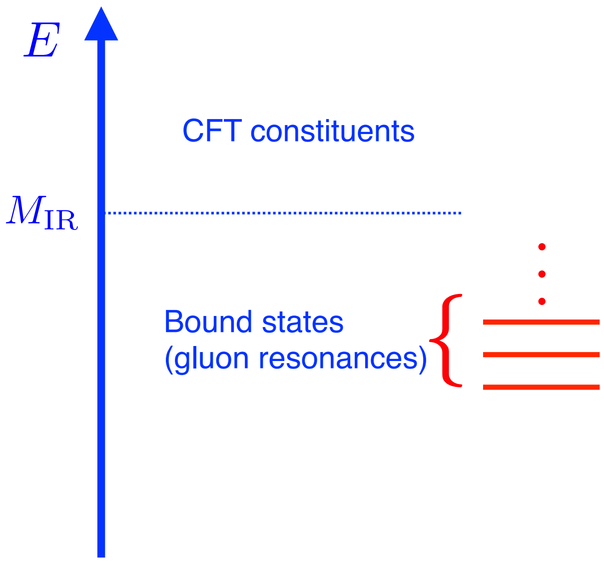
Why did this happen?

Holographic interpretation:

QCD gauge coupling:

$$\frac{2\pi}{\alpha_s(1/\rho)} = \frac{2\pi}{\alpha_s(M_{\rm IR})} - \Big(\overbrace{b_0}^{= -\frac{11}{3}N_c + \frac{2}{3}N_f = -7} + \underbrace{\frac{8\pi^2}{g_5^2 k}}_{\text{5D contribution } (\equiv b_{\rm CFT})}\Big) \log\left(\frac{\rho^{-1}}{M_{\rm IR}}\right)$$

[Pomarol '00; Arkani-Hamed, Porrati, Randall '00; Goldberger, Rothstein '02]



➡ CFT constituents must be bifundamental $SU(3)_c \times SU(N_{\rm CFT})$

➡ CFT constituents = bifundamental (colored) **matter** (no spin 1!)

➡ $b_{\rm CFT} \sim N_{\rm CFT} \gg 1$ dominates asymptotically-free QCD contribution!

Two possibilities:

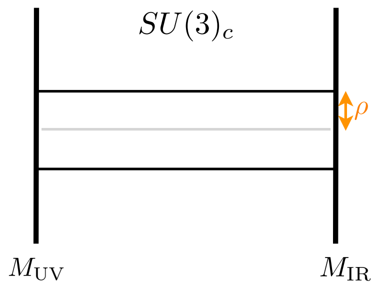
(i) Colored fermion constituents:

chiral suppression from CFT fermions!

$$m_a^2 f_a^2 = \mathcal{T} \sim \int_{1/M_{\rm UV}}^{1/M_{\rm IR}} \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(M_{\rm IR})}\right)^6 e^{-\frac{2\pi}{\alpha_s(1/\rho)}} (\rho M_{\rm IR})^{b_{\rm CFT}}$$

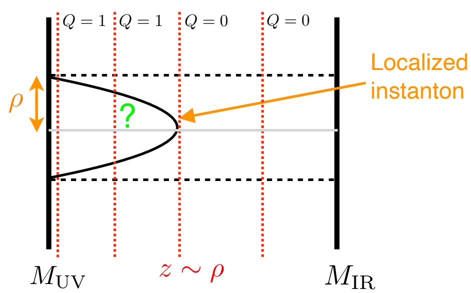
$$= e^{-\frac{2\pi}{\alpha_s(M_{\rm IR})}}$$

No axion mass enhancement!



$\leftrightarrow$ CFT contains colored fermions

(ii) Colored scalar constituents: No chiral suppression ⇨ Axion mass enhanced!



However, topological charge conservation suggests **no** localized instanton!

➡ CFT dual *necessarily* contains colored fermions !?!

Can be avoided with singular configurations or constrained instantons? [Work in progress]

Other possibilities:

- ***Strong QCD*** [Holdom, Peskin 1982] [Flynn, Randall 1987]

- ***Enlarge QCD color***

$$SU(3+N') \to SU(3)_c \times SU(N')$$

[Dimopoulos, Susskind '79; Dimopoulos '79]

$$SU(3+N) \times SU(N)' \to SU(3)_c \times SU(N)_D$$

[TG, Nagata, Shifman: 1604.01127]
[Gaillard, Gavela, Houtz, Quilez,del Rey: 1805.06465]
[Valenti, Vecchi, Xu: 2206.04077]

$$SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_k \to SU(3)_c$$

[Agrawal, Howe 1710.04213]
[Fuentes-Martin,Reig,Vicente 1907.02550]
[Csaki, Ruhdorfer, Shirman 1912.02197]

- ***Mirror QCD*** [Rubakov '97] [Berezhiani, Gianfagna, Gianotti '00]
[Dimopoulos, Hook, Huang, Marques-Tavares: 1606.03097]
[Hook, Kumar, Liu, Sundrum: 1911.12364]
[Dunsky, Hall, Harigaya: 2302.04274]

⇨ ***Axion mass is sensitive to UV completion!***

Questions/Future Work

- Generalize to z-dependent bulk Higgs VEVs
  — could enhance specific axion-fermion couplings
- Dark matter ALPs with axion-fermion couplings?
  — incorporate SM fermion mass hierarchy
- Construct 4D dual models with $\Delta \geq 10$
  — warped compactifications in string theory
- Small instantons in weakly-gauged holographic models

  [TG, Pomarol: 2110.01762] $$A_\mu^a(x,z) = 2\eta_{\mu\nu}^a \frac{x_\nu}{x^2} \frac{(x^2+z^2)^2}{x^2\rho^2+(x^2+z^2)^2}$$ **New "localized" instanton anti-instanton solution!**

  — other solutions that give axion mass enhancement?
- ……..

Summary

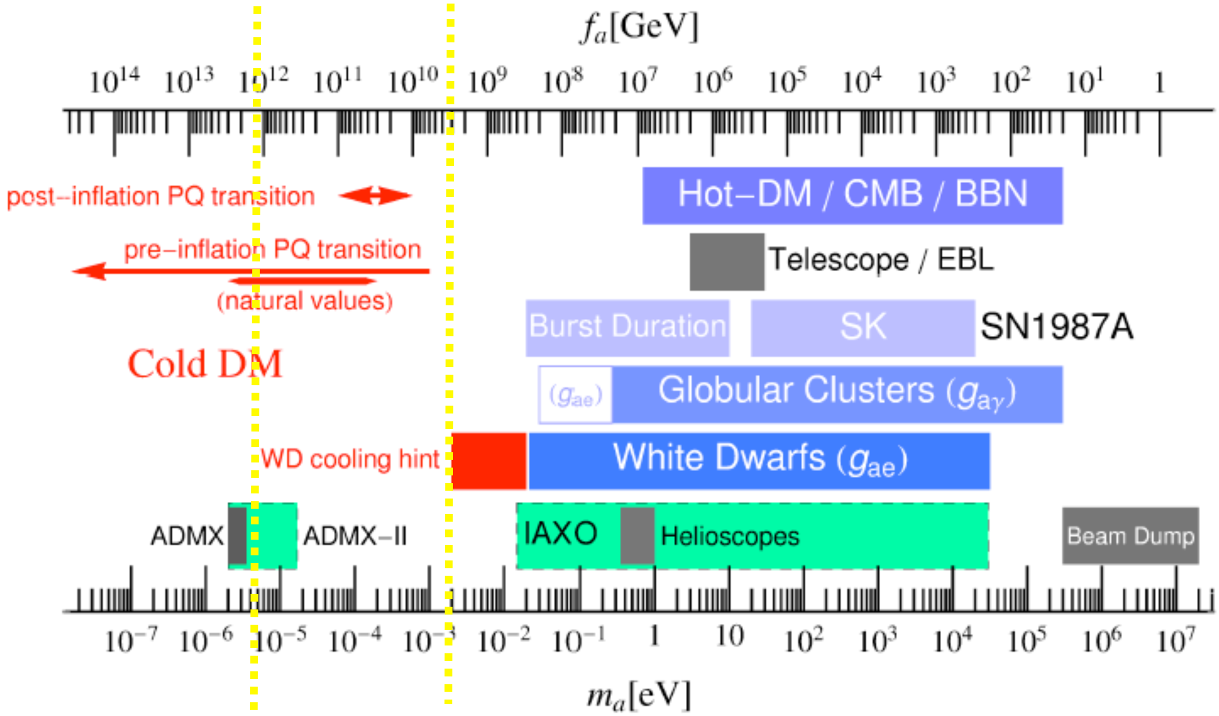
- Axion quality problem can be solved in 5D warped dimension
  - — *dual to 4D dynamical axion with accidental PQ symmetry*

- "Flavored" warped axion
  - — *solves axion quality and explains fermion mass hierarchy*
  - — *predicts flavor-violating axion-fermion couplings*
  - — *light sterile neutrinos*
- 5D small instantons
  - — *can enhance axion mass and not spoil strong CP solution*
  - — *axion mass could be a sensitive probe of UV physics!*

- Holographic view provides *complete* framework to study QCD axion!

Extra Slides

QCD axion limits

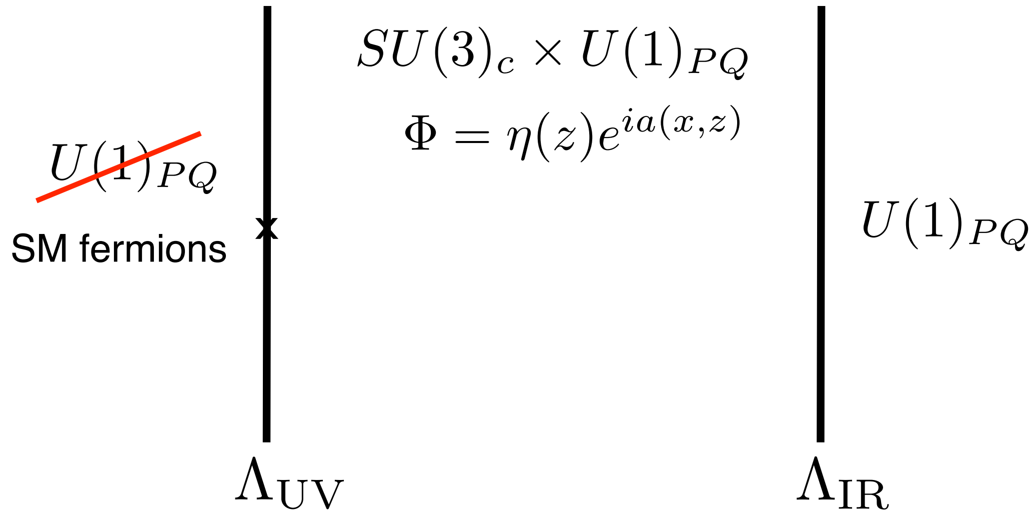


$$10^9\ \text{GeV} \lesssim f_a \lesssim 10^{12}\ \text{GeV}$$

$$\frac{1}{f_a} J^\mu \partial_\mu a$$ ⇨ axion weakly-coupled - “invisible”

$$m_a^2 f_a^2 \sim \tfrac{1}{8}\Lambda_{\rm QCD}^4$$ ⇨ $$10^{-5}\ \text{eV} \lesssim m_a \lesssim 10^{-3}\ \text{eV}$$

Axion-Gluon Coupling



Bulk Chern-Simons term: $$-\frac{\kappa}{32\pi^2}\int_{z_{UV}}^{z_{IR}} d^5x\, \epsilon^{MNPQR} V_M G^a_{NP} G^a_{QR}$$ ← **generates axion-gluon coupling**

Under 5D gauge transformation: $V_M \to V_M + \partial_M \alpha$

$$\Rightarrow \quad \delta S = -\frac{\kappa}{32\pi^2}\left[\int d^4x\, \alpha(x^\mu, z)\, \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}\right]_{z_{UV}}^{z_{IR}}$$

Add IR boundary term:

$$\frac{\kappa}{32\pi^2}\int d^4x\, a\, G\tilde{G}\Big|_{z_{IR}}$$

Obtain:

$$\mathcal{S}_{eff} = \int d^4x \left(\frac{1}{2}a^{(0)}\left(\Box - m_a^2\right)a^{(0)} + \frac{g_s^2}{32\pi^2 F_a}a^{(0)}G\tilde{G}\right)$$

where $F_a \simeq \dfrac{1}{\kappa}\dfrac{\sigma_0}{\sqrt{\Delta-1}}z_{IR}^{-1}$

$$\Rightarrow \quad (m_a^{(UV)})^2 = \frac{4\ell_{UV}\sigma_0(\Delta-2)}{\kappa^2(\Delta-4+b_{UV})}\left(\frac{\kappa\sqrt{\Delta-1}}{\sigma_0}\right)^{\Delta}\left(\frac{F_a}{\Lambda_{UV}}\right)^{\Delta-4}F_a^2$$

(suppression for $F_a \ll \Lambda_{UV}$ and $\Delta > 4$)

Numerical results

$$\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}$$

$kz_{\rm IR} = 10^{10}, g_5^2 k = 1$

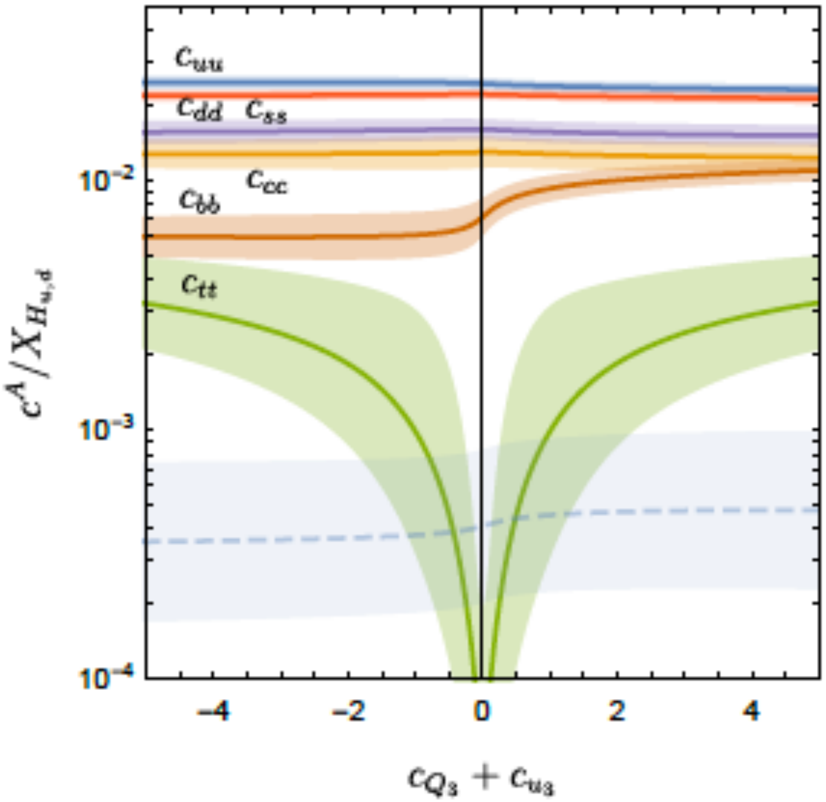
$\Delta = 10 \quad \sigma_0 = 3$

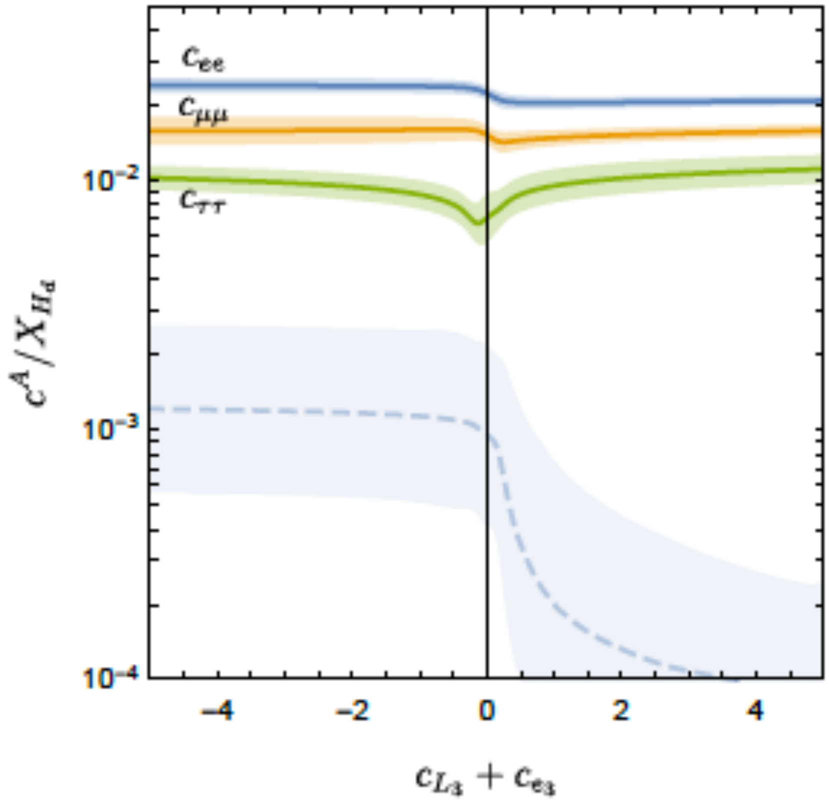
$F_a \simeq 10^9\,{\rm GeV}.$

Scan over $y^{(5)}_{u,d,e} \sim 1$

Flavor-preserving couplings:

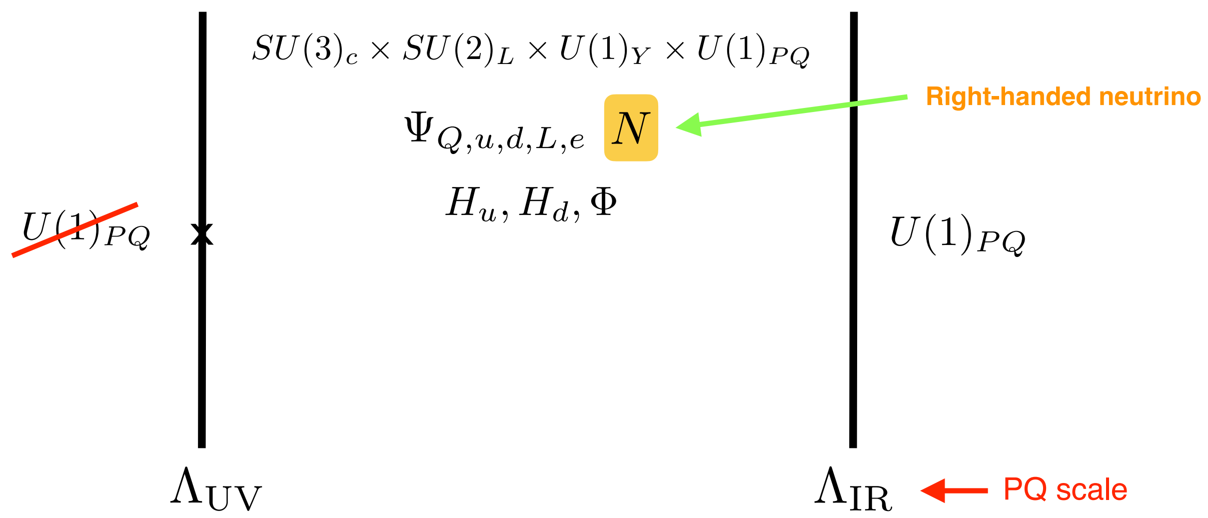
[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]





Neutrino-Axion Model [Cox, TG, Nguyen: 2107.14018]

[Also Mohapatra, Senjanovic 1983; Holman, Lazarides, Shafi 1983; Langacker, Peccei, Yanagida 1986; Bertolini et al 2015; Clarke, Volkas 2016]



Bulk Yukawa coupling: $$\frac{1}{\sqrt{k}}\left(y^{(5)}_{\nu,ij}\overline{L_i}N_jH_u + y^{(5)}_{e,ij}\overline{L_i}E_jH_d + \text{h.c.}\right)$$ ← PQ charge of N forbids bulk Majorana terms

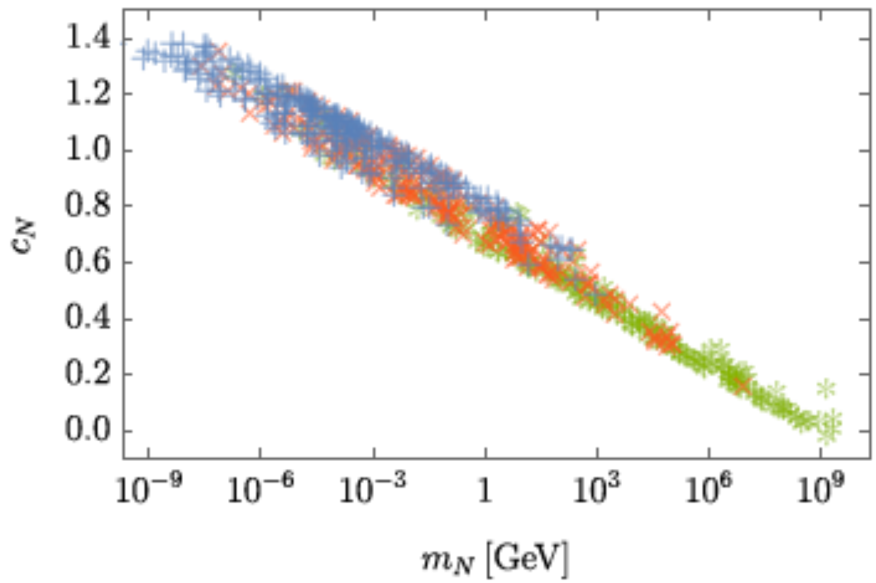
UV boundary: $$\frac{1}{2}\left(b_{N,ij}\overline{N^c_i}N_j + \frac{y^{(5)}_{N,ij}}{k^{3/2}}\Phi\overline{N^c_i}N_j + \text{h.c.}\right)$$

Predictions:

Parameters chosen to explain axion quality, neutrino mass differences and PMNS angles

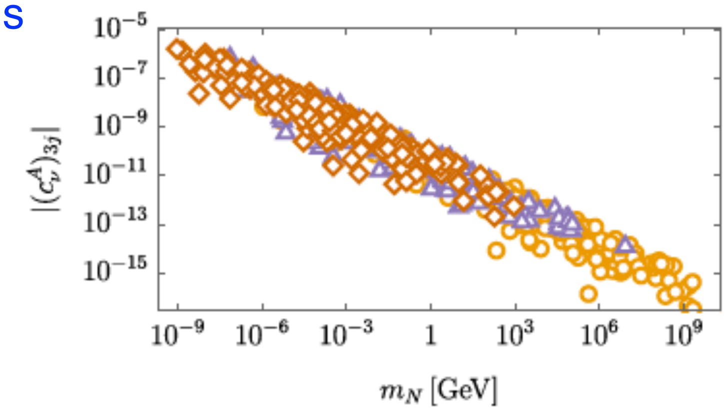
$\sigma_0 = 0.1, \lambda = 0.1, \Delta = 10, \tan\beta = 3, kz_{\rm IR} = 10^7$ ⇒ $F_a \simeq 8.12 \times 10^9$ GeV

✦ Light sterile neutrinos!



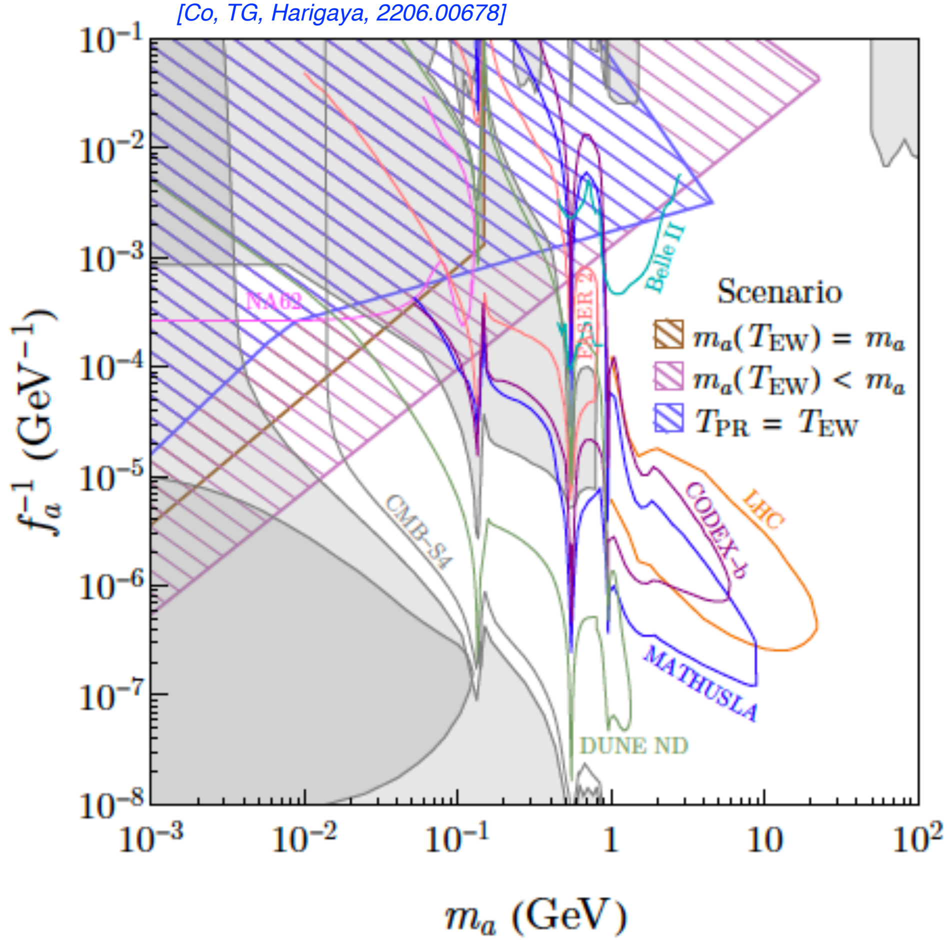
⇒ $1\ {\rm eV} \lesssim m_N \lesssim 10^9\ {\rm GeV}$

✦ Axion-neutrino couplings



⇒ Probe local DM halo with neutrino oscillations NEW
[TG, Shkerin: 2305.06441]

[Co, TG, Harigaya, 2206.00678]



Heavy QCD axion solves strong CP + baryon asymmetry!