Energy Reflection and Transmission at 2D Holographic Interfaces

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Phys.Rev.Lett. 125 (2020) 23, 231602

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Phys.Rev.Lett. 131 (2023) 2, 021601

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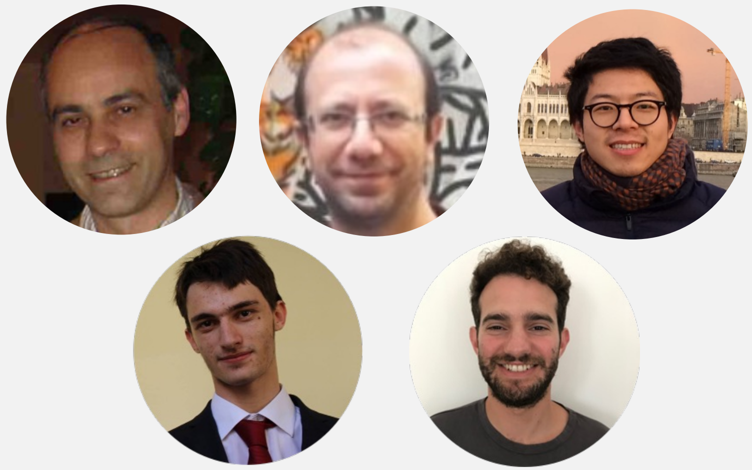


ENS Summer Institute

24 June 2024

Outline

(1) Conformal interfaces

(2) Universality of energy transmission (in 2d)

(3) Transmission in holography

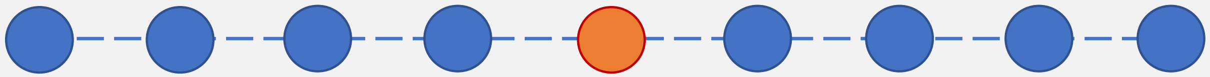
- (3a) The thin brane model
- (3b) The thick brane model (Janus)

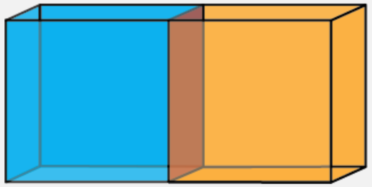
(4) Summary and outlook

(1) Conformal Interfaces

Conformal Interfaces

- **Interfaces** – codimension one extended objects which split the system into two





- **Conformal Interfaces** – separate two critical systems and preserve a large subgroup of the conformal symmetry $SO(d,1) \subset SO(d+1,1)$

- In 2d these are **impurities** which preserve one copy of the Virasoro algebra

What are conformal interfaces good for?

- Condensed matter physics: Junction of quantum wires [Wong, Affleck, 1993], line or surface defects in the critical 2D or 3D Ising models [Oshikawa, Affleck, 1997]…
- Holography: dynamical branes in AdS [Karch, Randall, 2000] [DeWolfe, Freedman, Ooguri, 2001], supergravity solutions (Janus) [Bak, Gutperle, Hirano, 2007]
- Playgrounds for computations in quantum information: Islands in black hole evaporation [Almheiri, Engelhardt, Marolf, Maxfield, 2019] [Penington, 2019] [Almheiri, Mahajan, Maldacena, Zhao, 2019] [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini, 2019]
- Monotonicity theorems along the RG flow: g-theorem, b-theorem [Affleck, Ludwig, 1991] [Jensen, O'Bannon 2015]
- And many more….

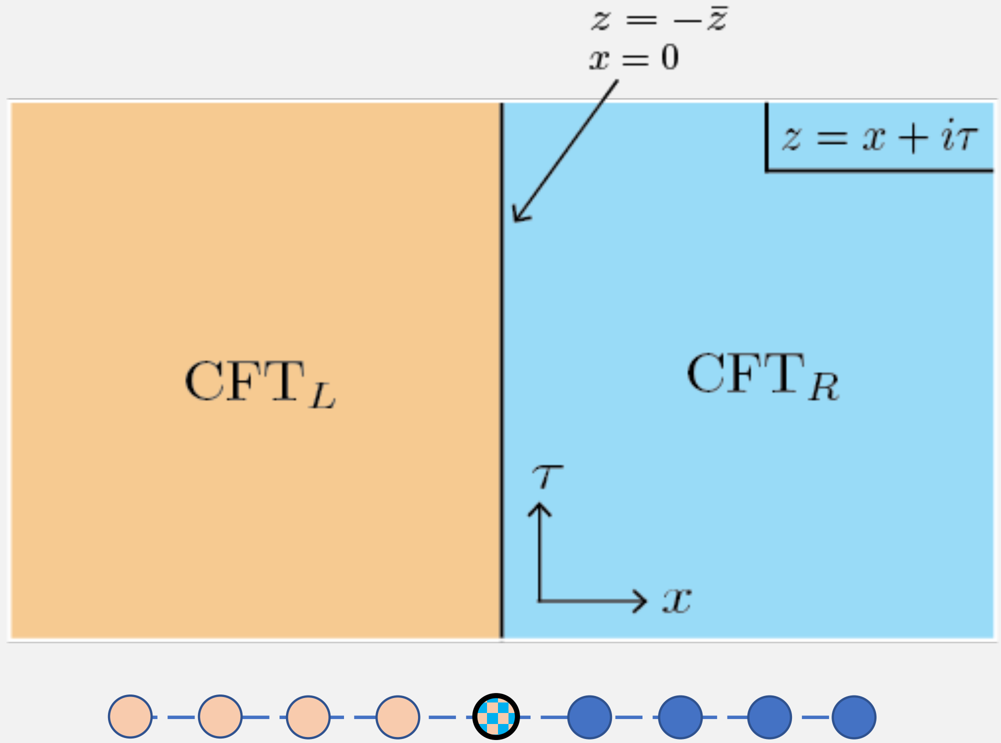
Conformal Interfaces in 2d

- Preserves Virasoro generators that do not displace the interface

$$L_n + (-1)^n \bar{L}_n$$

- Energy conservation implies a gluing condition ($T_{x\tau}$ is continuous)

$$T_L - \bar{T}_L \Big|_{x=0^-} = T_R - \bar{T}_R \Big|_{x=0^+}$$



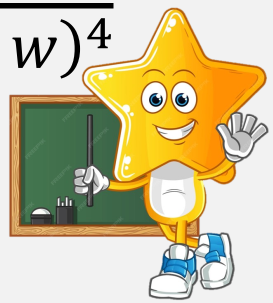
Conformal Interfaces in 2d

- 2pt functions of stress tensor completely fixed by conformal symmetry

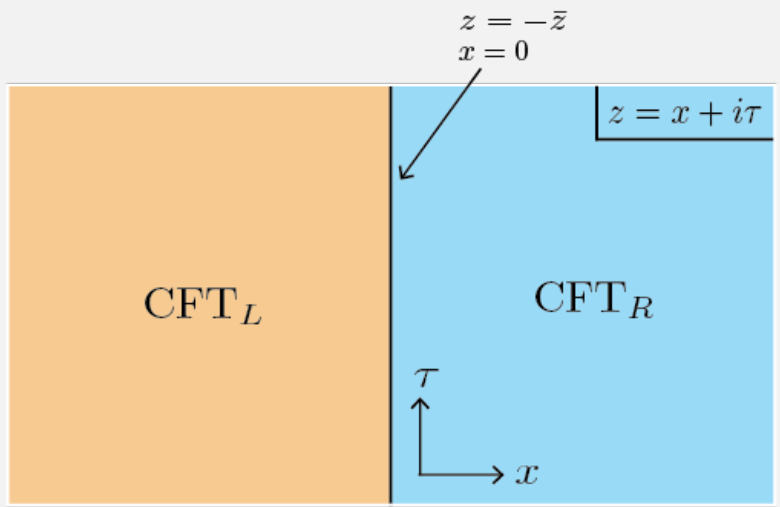
$$\langle T_L(z) T_L(w) \rangle_I = \frac{c_L/2}{(z-w)^4} \qquad \langle T_R(z) T_R(w) \rangle_I = \frac{c_R/2}{(z-w)^4}$$

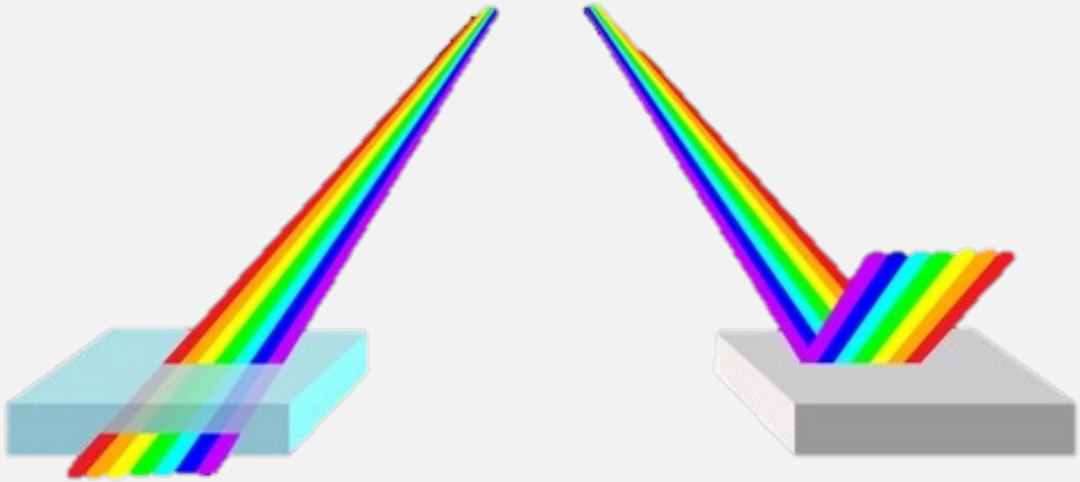
- New coefficient in left/right correlations

$$\langle T_L(z) T_R(w) \rangle_I = \frac{c_{LR}/2}{(z-w)^4}$$



- 3pt functions are also fixed
- But 4pt point functions are not!





(2) Universality of energy transmission and reflection (in 2d)

Energy Reflection and Transmission

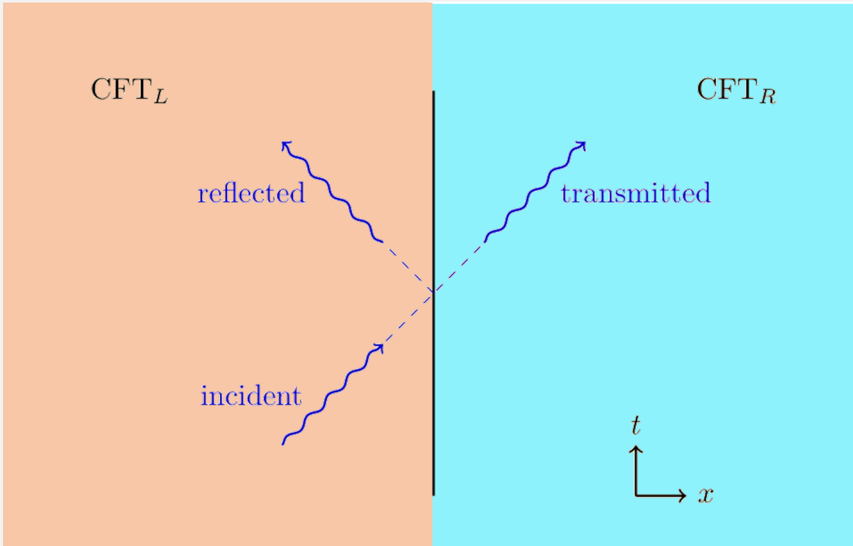
- Scattering experiment

$$\mathcal{T} = \frac{transmitted\ energy}{incident\ energy} \qquad \mathcal{R} = \frac{reflected\ energy}{incident\ energy}$$

- Different transmission from left and right

$$\mathcal{T}_{L(R)} \qquad \mathcal{R}_{L(R)}$$

- Universality – scattered and reflected energy is completely independent* of the details of the incoming excitation



Quella, Runkel, Watts (2007)
Meineri, Penedones, Rousset (2019)

* as long as no more than one spin 2 conserved quasi-primary is present

Energy Reflection and Transmission

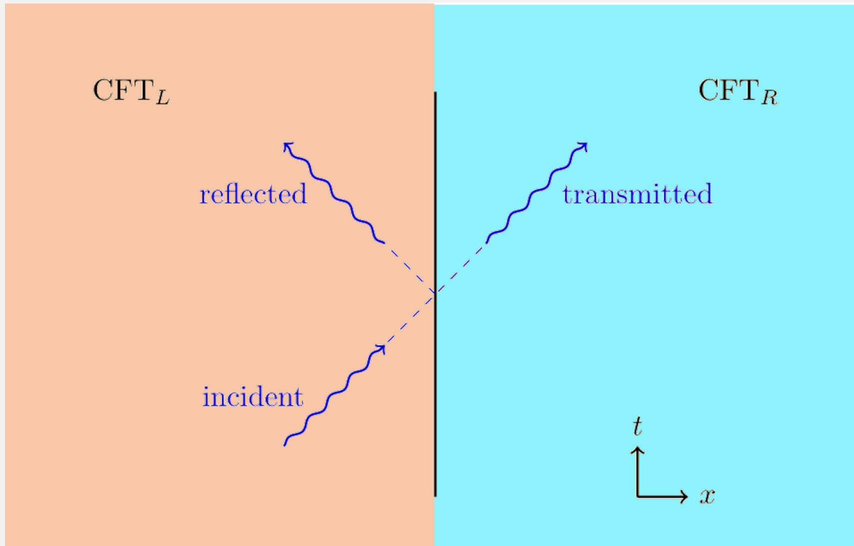
$$\mathcal{T}_L = \frac{c_{LR}}{c_L} \qquad \mathcal{T}_R = \frac{c_{LR}}{c_R} \qquad \mathcal{R}_{L(R)} = 1 - \mathcal{T}_{L(R)}$$

ANEC implies $0 \leq \mathcal{T}, \mathcal{R} \leq 1 \quad \Rightarrow \quad 0 \leq c_{LR} \leq \min(c_L, c_R)$

* Recent claims Karch [2404.01515] at al. $c_{LR} \leq c_{eff} \leq \min(c_L, c_R)$

$$0 \leq \mathcal{T}_L \leq min\left(1, \frac{c_R}{c_L}\right)$$

$$0 \leq \mathcal{T}_R \leq min\left(1, \frac{c_L}{c_R}\right)$$



Can't fully transmit from higher to lower central charge.

Quella, Runkel, Watts (2007)
Meineri, Penedones, Rousset (2019)

(3) Energy reflection and transmission in holography

Goal of this Talk: Holographically compute the transmission coefficient

Conduct a holographic scattering experiment to find

$$\mathcal{T}_{L(R)} \quad \leftrightarrow \quad c_{LR}$$

Two models:

- Thin brane model: AdS_2 brane in AdS_3
  [Bachas, Chapman, Ge, Policastro, 2020] [Baig, Karch, 2022]
- Thick brane model: continuous geometry with dilaton (Janus AdS_3)
  [Bachas, SB, Chapman, Policastro, Schwartzman, 2023]

Why?

- Understand better the models.
- Properties of transmission and reflection at strong coupling/large central charge?

(3a) The thin brane model

The thin brane model - bottom-up approach

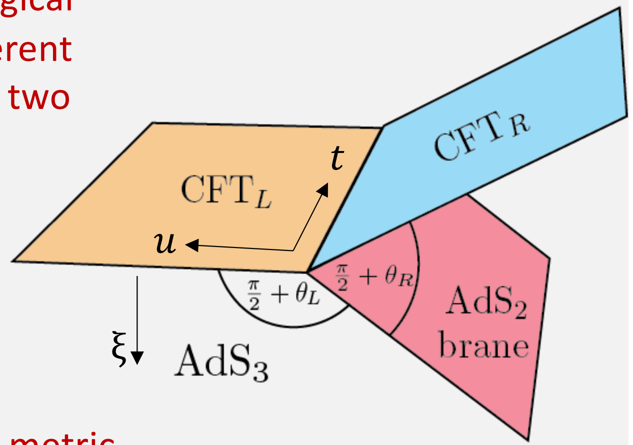
- Thin AdS_2 brane in AdS_3

$$S = \frac{1}{16\pi G_N}\int d^3x_L\sqrt{-g}\left(R + \frac{2}{\ell_L^2}\right) + \frac{1}{16\pi G_N}\int d^3x_R\sqrt{-g}\left(R + \frac{2}{\ell_R^2}\right) - \sigma\int d^2x\sqrt{-\gamma}$$

Two different cosmological constants encode different central charges on the two sides via $c_{L,R} = \frac{3\ell_{L,R}}{2G_N}$

Brane tension

Induced metric



- Solve Einstein equations in the left/right

$$ds^2_{L(R)} = \frac{\ell^2_{L(R)}}{\xi^2_{L(R)}}\left[-dt^2_{L(R)} + d\xi^2_{L(R)} + du^2_{L(R)}\right]$$

- Israel matching conditions determine the location of the brane [Israel, 1966]

$$\gamma_{L,\alpha\beta} = \gamma_{R,\alpha\beta} \qquad K^R_{\alpha\beta} - K^L_{\alpha\beta} = -8\pi G\sigma\gamma_{\alpha\beta}$$

The thin brane model - bottom-up approach

- Stable solutions with a thin AdS_2 brane exist as long as

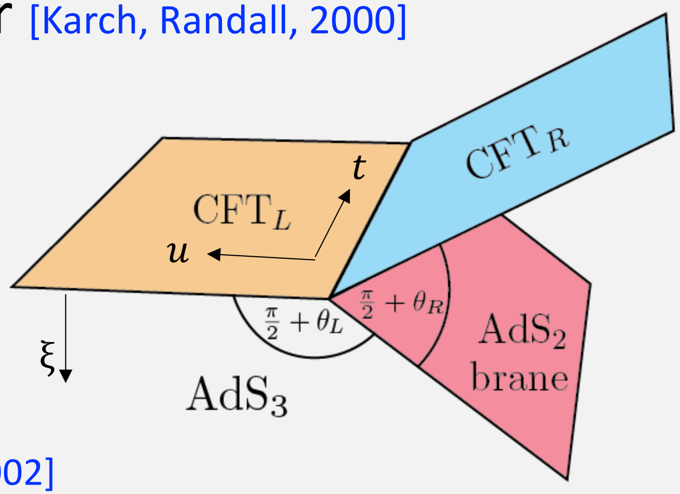
$$\left|\frac{1}{\ell_R}-\frac{1}{\ell_L}\right| \leq 8\pi G_N \sigma \leq \frac{1}{\ell_R}+\frac{1}{\ell_L}$$

$$\tan\theta \equiv \frac{u}{\xi}$$

  - Lower bound: no bubble nucleation [Coleman, De Luccia, 1980]
  - Upper bound: brane geometry becomes de Sitter [Karch, Randall, 2000]

- The solution consists of two patches of AdS_3 connected along an AdS_2 brane with

$$\frac{\ell_L}{\cos\theta_L} = \frac{\ell_R}{\cos\theta_R} = \frac{\tan\theta_L + \tan\theta_R}{8\pi G\sigma}$$



Interface

- ~~Boundary~~ entropy is fixed by the tension.

[Bachas, 2002]
[Azeyanagi, Karch, Takayanagi, and Thompson, 2007]
[Anous, Meineri, Pelliconi, Sonner 2022]

[Costas Bachas, **SC**, Dongsheng Ge, Giuseppe Policastro 2020]

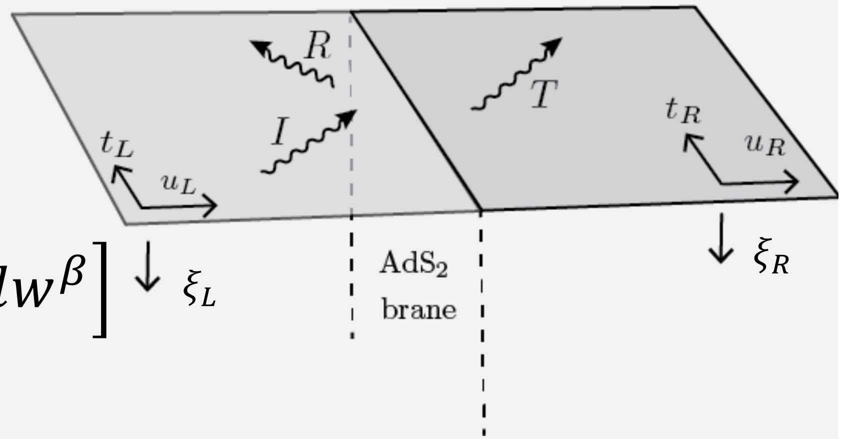
Holographic scattering experiment

- Bulk solution corresponding to a scattering experiment?
  Stress tensor with left and right moving waves

$$\langle T^L_{\alpha\beta}\rangle dx^\alpha_L dx^\beta_L = \epsilon\left[1\, e^{i\omega(t_L-u_L)} d(t_L-u_L)^2 + \mathcal{R}_L e^{i\omega(t_L+u_L)} d(t_L+u_L)^2\right] + c.c.$$

$$\langle T^R_{\alpha\beta}\rangle d\mathrm{x}^\alpha_\mathrm{R} dx^\beta_R = \epsilon\, \mathcal{T}_L e^{i\omega(t_R-u_R)} d(t_R-u_R)^2 + c.c$$

- 3d Bulk solution is completely fixed in FG gauge



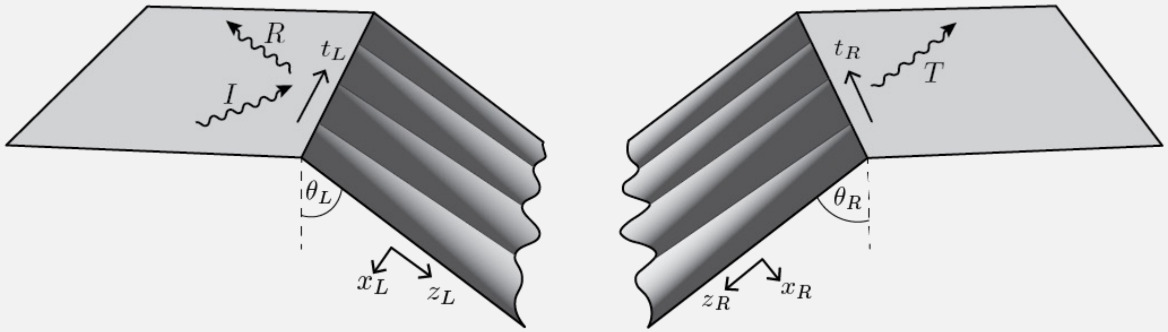
$$ds^2 = \frac{\ell^2}{\xi^2}\left[d\xi^2 + \left(g^{(0)}_{\alpha\beta} + \frac{\xi^2}{\ell^2} g^{(2)}_{\alpha\beta} + \frac{\xi^4}{4\ell^4} g^{(4)}_{\alpha\beta}\right) dw^\alpha dw^\beta\right] \downarrow \xi_L$$

$$g^{(2)}_{\alpha\beta} = 4G_\mathrm{N}\ell\langle T_{\alpha\beta}\rangle \qquad g^{(4)} = g^{(2)}\left(g^{(0)}\right)^{-1} g^{(2)}$$

Characteristic frequency ω

The brane fluctuates

- Without the metric perturbation the two sides were matched along a brane with angles θ_L and θ_R
- The perturbation changes the shape of the brane



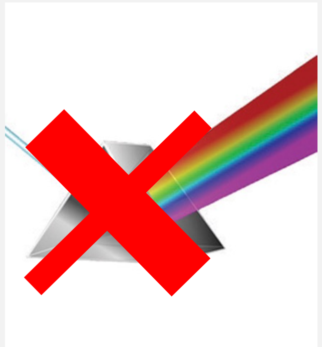
- Method:
  - Impose Israel matching conditions+boundary conditions

$$\Rightarrow \; \mathcal{R}_L + \mathcal{T}_L = 1$$

  - Impose no-outgoing wave condition at the horizon (in the IR).

Transmission in the single brane model

$$\mathcal{T}_{L(R)} = \frac{2}{\ell_{L,R}}\left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G_N \sigma\right]^{-1}$$



- Monotonically decreases with the tension
- Transmission in empty AdS_3
$$\sigma = 0, \quad \ell_L = \ell_R \quad \Rightarrow \quad \mathcal{T}_{L(R)} = 1$$
- **Universality:** the result does not depend on the frequency
- Holographic model has only one parameter
$\Rightarrow$ Transmission and boundary entropy $\log g$ both fixed in terms of the tension! Is this generic for strongly coupled theories?

Transmission in the single brane model

$$\mathcal{T}_{L(R)} = \frac{2}{\ell_{L,R}}\left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G_N \sigma\right]^{-1}$$

- Allowed range of tensions

$$\left|\frac{1}{\ell_R} - \frac{1}{\ell_L}\right| \leq 8\pi G\sigma \leq \frac{1}{\ell_R} + \frac{1}{\ell_L}$$

Provides bounds on the transport

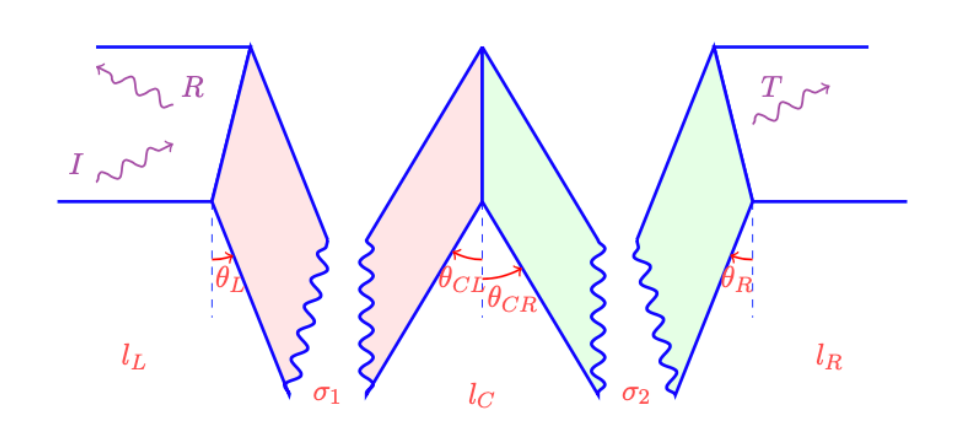
$$\frac{c_R}{c_R + c_L} \leq \mathcal{T}_L \leq min\left(1, \frac{c_R}{c_L}\right),$$

ANEC: $0 \leq \mathcal{T}_L \leq min\left(1, \frac{c_R}{c_L}\right)$

- Consistent with ANEC, but lower bound is stronger.
- Total reflection ($\mathcal{T}_L$=0) when $c_R/c_L \to 0$ (BCFT limit)
- Is this generic for strongly coupled theories? (we will see that no)

Transmission in double brane model

Fuse two branes and perform the same computation [Baig, Karch, 2022]



$$\mathcal{T}_{L(R)} = \frac{2}{\ell_{L,R}}\left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G_N(\sigma_1 + \sigma_2)\right]^{-1}$$

- Additive in the tensions ⇒ for N branes $\sum^N \sigma_i$
- Does not depend on ℓ_c
- $\log g$ depends on ℓ_c ⇒ Transmission and $\log g$ can vary independently!
- Consistent with ANEC – lower bound can be realized by sending $\ell_c \to 0$

Shortcomings of the thin brane

- It is discontinuous – delta function localized energy
- Bottum-up approach - we do not know the dual field theory

Can we find these results for smooth gravity solutions/top-down models?

(3b) The thick brane model (Janus)

Smooth ICFT holographic models

- Einstein gravity coupled to a dilaton

$$S = \frac{1}{16\pi G_N}\int d^{n+1}x\,\sqrt{-g}\,\left(R - 2\partial_\mu\phi\partial^\mu\phi - 2V(\phi)\right)$$

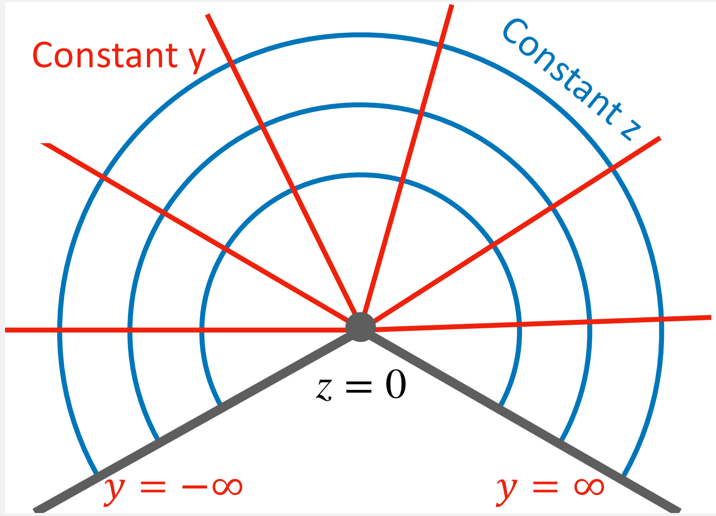
- Continuous geometries dual to vacuum states of ICFT

$$ds^2 = dy^2 + a^2(y)\,\bar{\gamma}_{\alpha\beta}dx^\alpha dx^\beta, \qquad \phi = \phi(y),$$

  with $\bar{\gamma}_{\alpha\beta}dx^\alpha dx^\beta$ the metric on AdS$_n$ slices

- Empty AdS: $V(\phi) = -1/\ell^2$

$$\phi(y) = 0, \qquad a(y) = \ell\cosh\left(\frac{y}{\ell}\right)$$



- Double Wick rotation $y \to it$ and AdS$_n$ $\to$ EAdS$_n$ leads to FRW geometry for open universe coupled to an inflaton.

Janus – a specific example

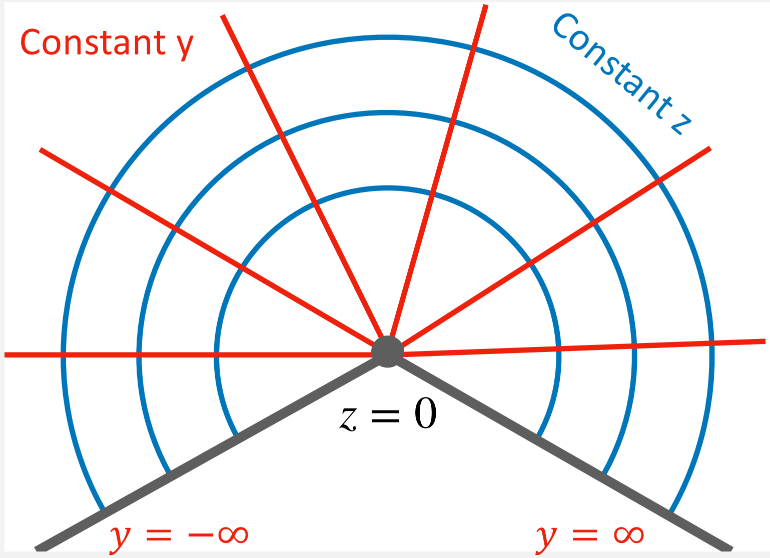
- Special case $V(\phi) = -1/L^2$: non-supersymmetric 3d Janus AdS solution [Freedman, Nunez, Schnabl, Skenderis, 2003][Bak, Gutperle, Hirano, 2007]

$$ds^2 = dy^2 + a^2(y)\left(\frac{-dt^2+dz^2}{z^2}\right)$$

$$a(y) = \frac{L}{\sqrt{2}}\left[1 + (1-b)\cosh\left(\frac{2y}{L}\right)\right]^{1/2}$$

$$\phi(y) = \phi_0 + \frac{1}{\sqrt{2}}\log\left[\frac{\sqrt{2-b}+\sqrt{b}\tanh\left(\frac{y}{L}\right)}{\sqrt{2-b}-\sqrt{b}\tanh\left(\frac{y}{L}\right)}\right]$$

- $b = 0$ recovers empty AdS.
- Can be embedded in type IIB SUGRA on $AdS_3 \times S^3 \times M_4$.



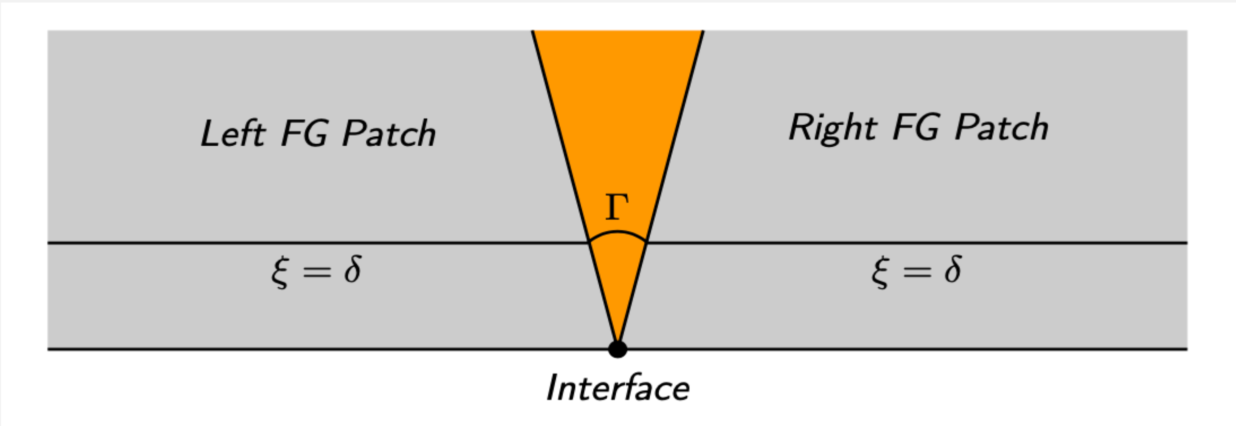
Perturbation with plane waves is difficult

Method:

- Add a perturbation for the stress-tensor at the boundary
- Solve the Einstein's equations with perturbation

Problems:

- Fefferman-Graham coordinates are not defined everywhere
  [Papadimitriou, Skenderis, 2004]
- Hard to study Einstein's equations

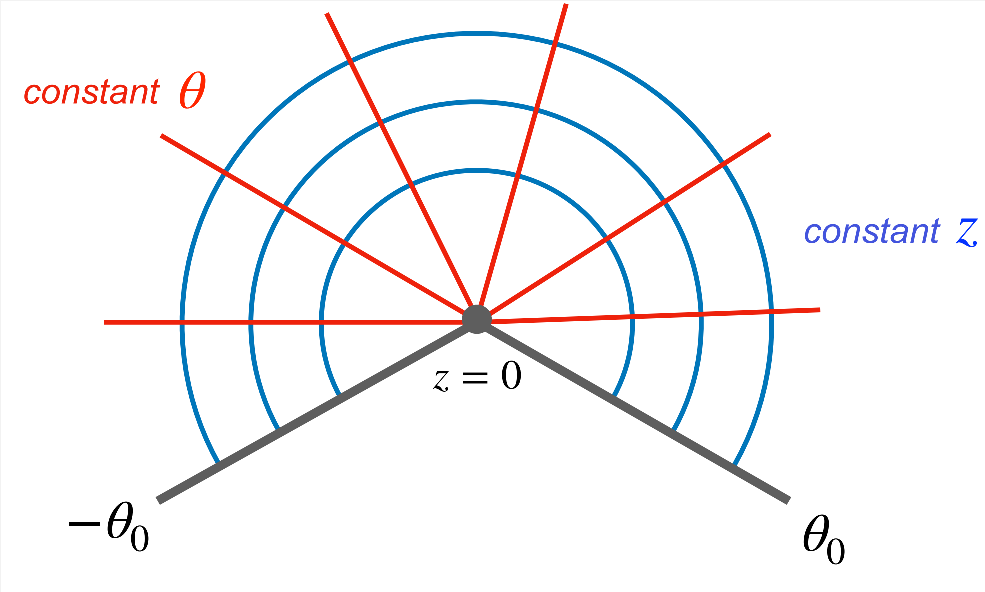


Discrete geometries are simpler!

- Our geometries are in fact very similar to empty AdS $d\theta \equiv \frac{dy}{a(y)}$

$$ds^2 = a(\theta)\left(d\theta^2 + \frac{-dt^2 + dz^2}{z^2}\right),$$
$$\phi = \phi(\theta),$$

I will keep changing between θ and y, they are really the same thing

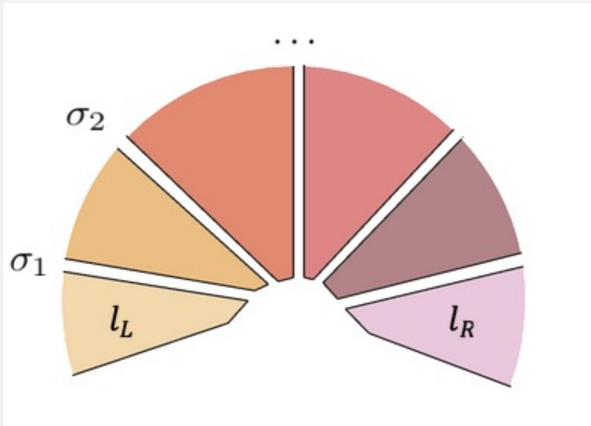


- $a(\theta) = \ell/\cos(\theta)$ – empty AdS
- This means that we can treat them as many small slices of AdS_{n+1} with different radii!

[Bachas, Baiguera, **SC**, Policastro, Schwartzman, 2023]

Discrete geometries are simpler!

- Consider a pizza geometry with multiple branes and use additivity



$$\mathcal{T}_{L(R)} = \frac{2}{\ell_{L,R}} \left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G_N \sum_i \sigma_i \right]^{-1}$$

- Take the continuum limit: $\sum_i \sigma_i \to \int_{-\infty}^{\infty} \frac{d\sigma}{dy} dy$

Discretization method

- Take a collection of empty AdS_3 regions

$$ds_j^2 = \tilde{a}_j(\theta)\left(d\theta^2 + \frac{-dt^2 + dz^2}{z^2}\right)$$

$$\tilde{a}_j(\theta) = \frac{\ell_j}{\cos(\theta - \delta_j)} \quad \text{for} \quad (\mathrm{j}-1)\epsilon < \theta < j\epsilon$$

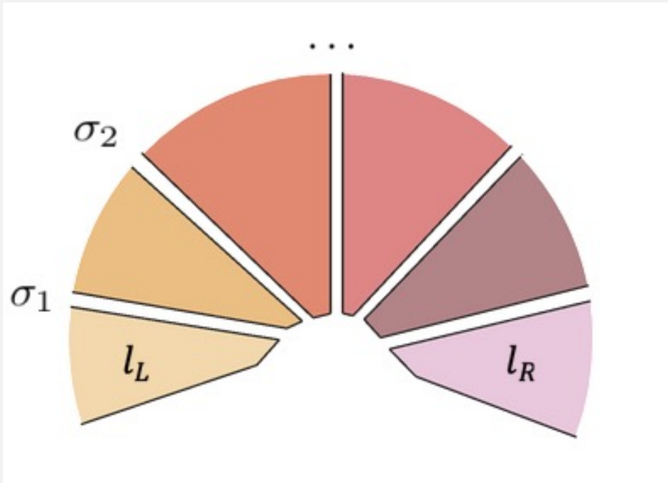
- Impose:
  - Israel matching conditions.
  - recover the original $a(\theta = j\epsilon) = \tilde{a}_j(j\epsilon)$

- continuum limit $a(\theta), \ell(\theta), \delta(\theta)$

- Result is simple

$$\frac{d\sigma}{dy} = \left(\frac{d\phi}{dy}\right)^2, \frac{1}{\ell(y)^2} = \frac{1}{2}\left(\frac{d\phi}{dy}\right)^2 - V(\phi)$$

- Integrate to obtain the transmission!



Transmission of Janus interface

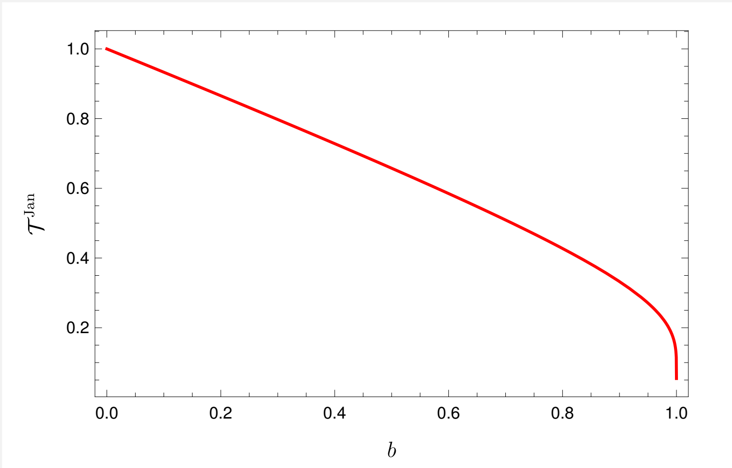
$$\mathcal{T}^{\mathrm{Jan}} = \frac{1}{2}\sqrt{b(2-b)}\left[\operatorname{arctanh}\left(\sqrt{\frac{b}{2-b}}\right)\right]^{-1}$$

- Monotonically decreasing function of the deformation parameter b
- Transmission in empty AdS_3

$$b = 0 \;\Rightarrow\; \mathcal{T}^{\mathrm{Jan}} = 1$$

- Infinitely strongly coupled case (linear dilaton)

$$b \to 1 \;\Rightarrow\; \mathcal{T}^{\mathrm{Jan}} \to 0$$



ANEC Bounds for smooth geometries

- Stability window for tension in continuum limit

$$\left|\frac{1}{\ell_R}-\frac{1}{\ell_L}\right| \leq 8\pi G_N \sigma \leq \frac{1}{\ell_R}+\frac{1}{\ell_L} \quad \Rightarrow \left|\frac{d}{dy}\frac{1}{\ell(y)}\right| \leq 8\pi G_N \frac{d\sigma}{dy} \leq \infty$$

Satisfied on the equations of motion✅

- Bounds of tension imply

$$0 \leq \mathcal{T}_L \leq min\left(1, \frac{c_R}{c_L}\right)$$

Same bounds given by ANEC! ✅

- Janus case: $c_L = c_R$ gives

$$0 \leq \mathcal{T}_L \leq 1$$

Equivalence between discrete branes and dilaton

- Discretized geometry solves Einstein's equations with source

$$T_{\mu\nu}^{mat} = -\Lambda(y) g_{\mu\nu} - \frac{d\sigma}{dy} \Pi_{\mu\nu}$$

$$\Lambda(y) = -\frac{1}{\ell(y)^2}, \qquad \Pi_{\mu\nu} = g_{\mu\nu} - \hat{n}_\mu \hat{n}_\nu$$

- In the continuum limit needs to converge to smooth solution

$$T_{\mu\nu}^{mat} = -\left(\partial^\rho \phi \partial_\rho \phi\right) \Pi_{\mu\nu} + g_{\mu\nu} \left(\frac{1}{2} \partial^\rho \phi \partial_\rho \phi - V(\phi)\right)$$

- Map to each other for the background solution:

$$\frac{d\sigma}{dy} = \left(\frac{d\phi}{dy}\right)^2 \qquad , \frac{1}{\ell(y)^2} = \frac{1}{2}\left(\frac{d\phi}{dy}\right)^2 - V(\phi)$$

Equivalence still holds after perturbation

- Universality: scattering experiment can be prepared with 2d transverse traceless modes

$$ds^2 = dy^2 + a^2(y)\left(\bar{\gamma}_{\alpha\beta} + h_{\alpha\beta}\right)dx^\alpha dx^\beta$$

$$\bar{\gamma}^{\alpha\beta}h_{\alpha\beta} = 0, \qquad \bar{\nabla}^\alpha h_{\alpha\beta} = 0$$

- Can solve for a given frequency

$$h_{\pm\pm}(\boldsymbol{x}|y) = e^{i\omega\left(x^0 \pm x^1\right)}\left[A_\pm^\omega + B_\pm^\omega \int^y \frac{d\tilde{y}}{a(\tilde{y})^2}\right]$$

- To complete the calculation, should impose boundary conditions for the scattering and the no-outgoing wave condition.
- equation of motion only depend on the scale factor $a(y)$

⇒ Discretization should not change the result in the continuum limit!

(4) Summary and outlook

Summary and Outlook

- Energy reflection and transmission are universal in 2d conformal interfaces
- In the thin brane model the transmission is fixed by the tension, just like the boundary entropy
- Bounds by ANEC satisfied, but can’t achieve complete reflection
- In general boundary entropy and energy transmission will differ; Full reflection can be achieved (e.g., 2 brane models)
- General technique to compute transmission coefficients for smooth holographic ICFTs with Einstein-dilaton action, including Janus
- Discretization provides a simpler method than a direct computation

Future developments

- Energy transfer in general dimensions? (new subtleties!) Is there anything universal? Other codimensions?
- Holographic check of universality?
- Holographic transport of electric charge? In thermal states?
- Application to cosmology: propagation of gravity waves, particle production
- Relation between energy and information transfer (Karch bound?)
- Monotonicity theorems along RG flows?

Lots to explore!

Thank you for listening!