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GW sourced by decay of massive particle from PBH evaporation

Based on arXiv: 2403.15269
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30TH VIETNAM SCHOOL OF PHYSICS:
PARTICLES AND DARK MATTER

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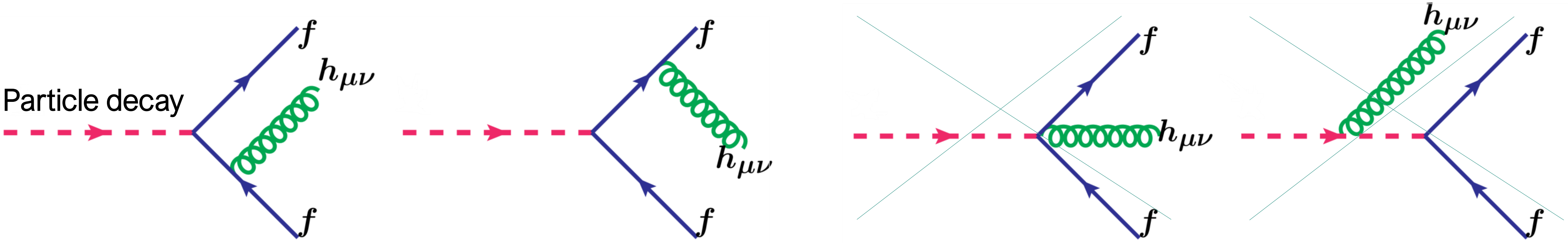
Motivation

Study for direct graviton emission :



Traceless condition for a massless graviton

Due to rest frame of decaying particle

[N. Bernal et al, 2301.11345; 2311.12694]

Graviton + Matter interaction

$$\mathcal{L}_{\text{eff}} \supset -\boxed{\frac{1}{M_p}} h_{\mu\nu} T^{\mu\nu}$$

Suppression!!

GW sourced through graviton bremsstrahlung would also be suppressed.

Motivation

Compensate suppression :

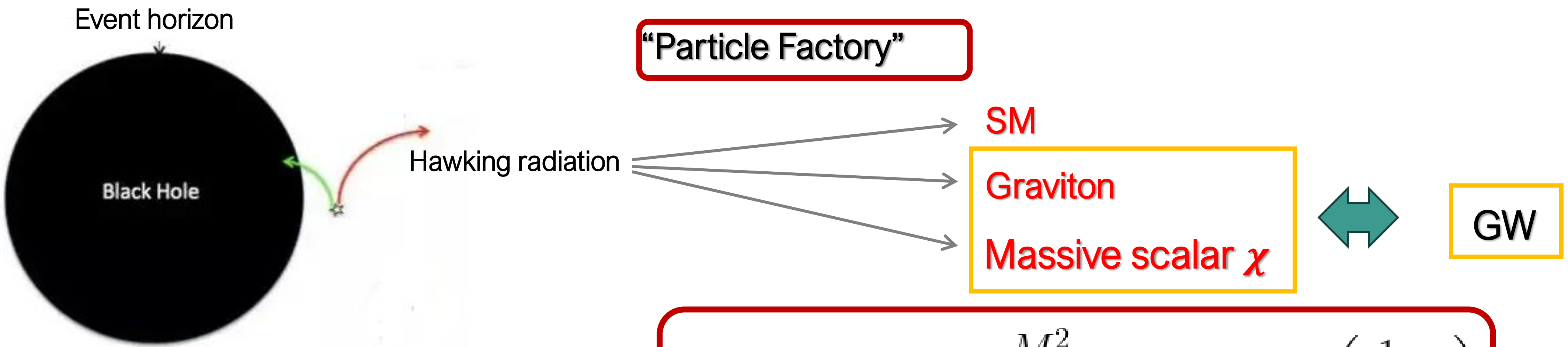
✓ $T^{\mu\nu}$ should be very large (considering inflation model) [N. Bernal et al, 2301.11345; 2311.12694]

✓ Sub-Planckian mass scale particle bremsstrahlung without suppression

How to produce such a massive particle?

Presence of PBH

- PBH could have produced in the early Universe due the collapse of large density perturbations during the radiation dominated era at the end of inflation. [Carr, et al. 2002.12778]



"Particle Factory"

SM

Graviton

Massive scalar χ

GW

$$\mathrm{R_s} = 2\mathrm{GM_{PBH}}$$

$$m_{\rm particle} \lesssim T_{\rm BH} = \frac{M_p^2}{M_{\rm BH}} \simeq 10^{13}\ {\rm GeV} \left(\frac{1\ {\rm g}}{\rm M_{BH}}\right)$$

Initial mass of the PBH is evaluated as :

$$M_{\rm in} = \gamma M_{\rm H} = \gamma M_p^2 t_{\rm in}$$

Numerical factor ~ 0.2

Particle horizon mass

Formation time

$$M_{\rm in} = \gamma \frac{4\pi}{3} \frac{\rho(T_{\rm in})}{H^3(T_{\rm in})}$$

[Gondolo, et al. 2009.02424
Morrison, et al. 1812.10606
Masina 2004.04730]

PBH evaporation

The energy spectrum of emitted particles with energy by a Schwarzschild BH:

$$\frac{d^2u_i(E,t)}{dEdt} = \frac{g_i}{2\pi^2}\frac{\sigma_{s_i}(M_{\rm BH},\mu_i,E_i)}{e^{E_i/T_{\rm BH}}-(-1)^{2s_i}}E_i^3$$

where

$$\sigma_{s_i} = \left(\frac{27}{64\pi}\frac{M_{\rm BH}^2}{M_p^4}\right)\psi_{s_i}(E)$$

[HAWKING1974, HAWKING1975]

The rate of PBH mass loss :

$$\frac{dM_{\rm BH}}{dt} = -\sum_i\int_0^\infty \frac{d^2u_i(E,t)}{dEdt}dE = -\varepsilon(M_{\rm BH})\frac{M_p^4}{M_{\rm BH}^2}$$

[Cheek et al, Phys. Rev. D 105, 015022
Phys. Rev. D 105, 015023]

Evaporation function depending on the grey-body factor

In Geometric-optic limit $\psi_{s_i}(E) = 1$:

$$M_{\rm BH}(t) = M_{\rm in}\left(1-\frac{t-t_i}{\tau}\right)^{1/3}$$

PBH lifetime: $\tau \simeq 2.66\times10^{-28}\ \mathrm{s}\left(\frac{100}{\mathrm{g}_*(\mathrm{T_{BH}})}\right)\left(\frac{\mathrm{M_{in}}}{1\ \mathrm{g}}\right)^3$

PBH evaporation temperature:

$$T_{\rm ev}|_{\rm MD} \simeq 3.55\times10^{10}\ \mathrm{GeV}\left(\frac{1\ \mathrm{g}}{M_{\rm in}}\right)^{3/2}$$

GW production from direct PBH evaporation

The rate of graviton emission for a collective population of evaporating PBHs :

$$\frac{d\rho_{\rm GW}}{dtdE} \simeq n_{\rm BH}(t)\frac{d^2u_{\rm GW}}{dtdE} \quad \text{where} \quad n_{\rm BH}(t) = n_{{\rm BH},i}\left(\frac{a_i}{a}\right)^3$$

- ❖ Comoving PBH number density remain conserved

Redshifting at the time of the complete evaporation of PBH :

$$\rho_{\rm GW} = \rho_{\rm GW,ev}\left(\frac{a_{\rm ev}}{a}\right)^4, \quad \omega = \omega_{\rm ev}\left(\frac{a_{\rm ev}}{a}\right)$$

$$\frac{d\rho_{\rm GW,ev}}{d\ln\omega_{\rm ev}} = \frac{27}{64\pi^3}\frac{M_{\rm in}^2}{M_p^4}n_{\rm BH}(t_i)\omega_{\rm ev}^4 \underbrace{\int_{t_i}^{t_{\rm ev}=t_i+\tau} dt\left(1-\frac{t-t_i}{\tau}\right)^{2/3}\frac{(a_i/a)^3}{e^{\omega_{\rm ev}a_{\rm ev}/aT_{\rm BH}}-1}}_{I(\omega_{\rm ev})}$$

GW production from direct PBH evaporation

The final relic abundance for GW at present:

$$h^2\Omega_{\rm GW} = \frac{1}{\rho_{\rm cr,0}h^{-2}}\frac{d\rho_{\rm GW,0}}{d\ln\omega_0} \quad \text{where} \quad \frac{d\rho_{\rm GW,0}}{d\ln\omega_0} = \frac{d\rho_{\rm GW,ev}}{d\ln\omega_{\rm ev}}\left(\frac{a_{\rm ev}}{a_0}\right)^4$$

$$\frac{a_{\rm ev}}{a_0} \simeq \frac{T_0}{T_{\rm ev}}\left(\frac{g_{*,s}(T_0)}{g_{*,s}(T_{\rm ev})}\right)^{1/3} \simeq 2.3\times 10^{-24}\left(\frac{M_{\rm in}}{1\ {\rm g}}\right)^{3/2}$$

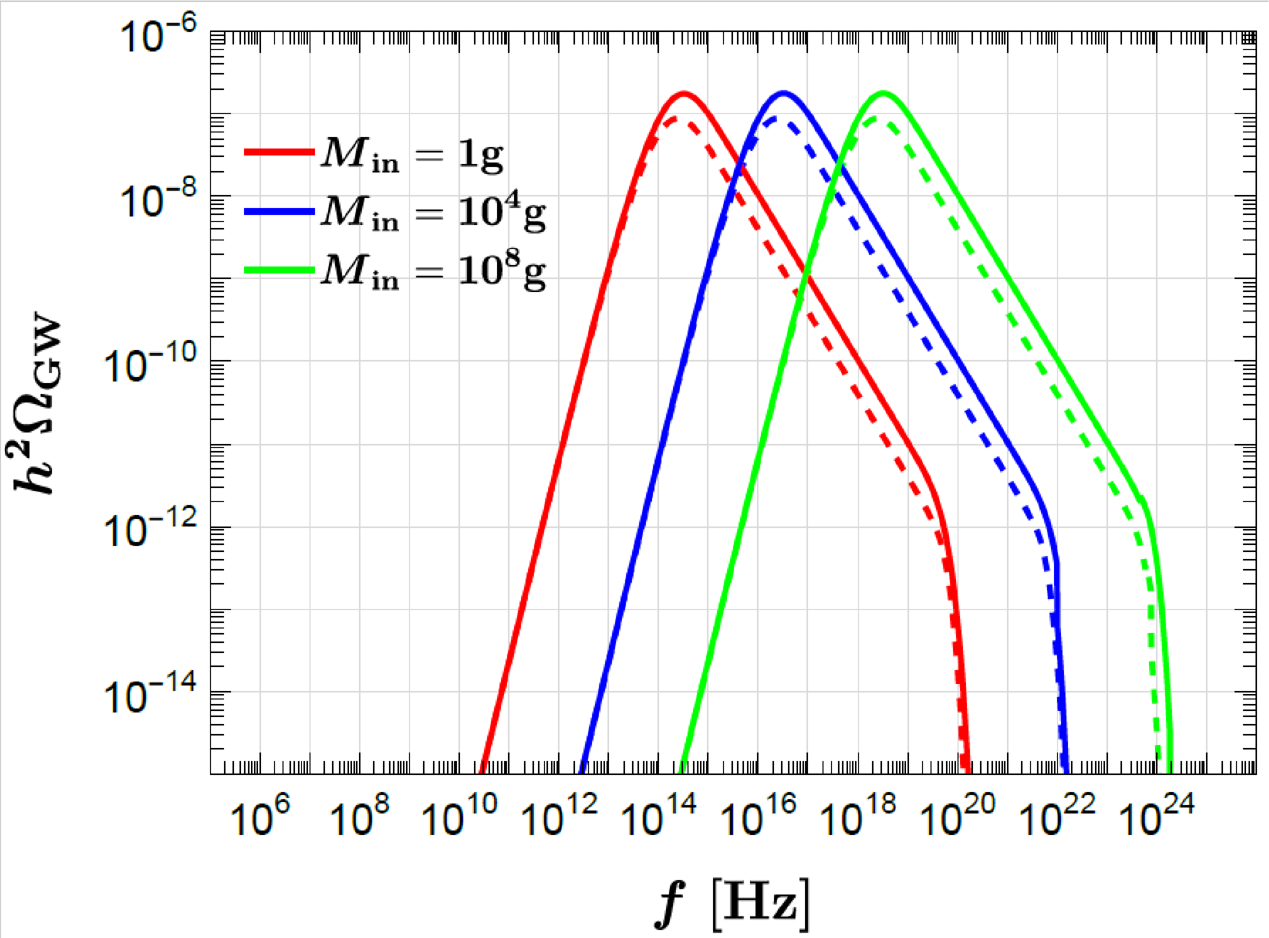
$$\frac{d\rho_{\rm GW,0}}{d\ln\omega_0} \simeq 6.2\times 10^{-77}\ {\rm GeV}\left(\frac{M_p}{M_{\rm in}}\right)^{1/2}\omega_0^4\ I(\omega_0)$$

$$I(\omega_0) = A^{-3}\int_{t_i}^{t_{\rm ev}} dt\left(1-\frac{t-t_i}{\tau}\right)^{2/3}\frac{t^{-2}}{\exp\left[\frac{\omega_0 M_{\rm in}}{AM_p^2}\ t^{-2/3}\left(1-\frac{t-t_i}{\tau}\right)^{1/3}\right]-1}$$

where $A = a_{\rm ev}/a_0\,(1/\tau)^{2/3}$

GW production from direct PBH evaporation

GW spectrum from direct evaporation of PBH



$$\boxed{f_{\rm peak} \simeq 1.6 \times 10^{13}\ {\rm Hz} \left(\frac{M_{\rm in}}{1\ {\rm g}}\right)^{1/2}}$$

- As the PBH mass increases, the peak frequency of the GW spectrum increases, consequently shifting the spectrum towards higher frequencies.

Massive scalar particle production

$$m_{\text{particle}} \lesssim T_{\text{BH}} = \frac{M_p^2}{M_{\text{BH}}} \simeq 10^{13}\ \text{GeV} \left(\frac{1\ \text{g}}{\text{M}_{\text{BH}}}\right)$$ ⟷ Sub-Planckian mass scale ?

- ✓ Produced from PBH evaporation at the late stage ($\boldsymbol{t_1 < \tau}$) of its lifetime as a consequence of the increase of Hawking temperature

$$t_1 = \tau \left[1 - \left(\frac{M_p^2}{M_{\text{in}} m_\chi}\right)^3\right] \quad \text{when } \boxed{\mathrm{T_{BH} = m_\chi}} \text{ or } \mathrm{M_{BH}(t_1) = M_p^2/m_\chi}$$

- ✓ To obtain large number density of the massive scalar particle we assume that the scalar **χ** can be long lived enough to decay after PBH evaporation

$$\tau_\chi \simeq 7 \times 10^{-26}\ \text{s} \left(\frac{10^{-7}}{y_f}\right)^2 \left(\frac{10^{-2} M_p}{m_\chi}\right) > \tau_{\text{BH}} \simeq 2.66 \times 10^{-28}\ \text{s} \left(\frac{100}{\mathrm{g_*(T_{BH})}}\right) \left(\frac{\mathrm{M_{in}}}{1\ \mathrm{g}}\right)^3$$

Massive scalar particle production

Long-lived χ

$$\frac{d\tilde{n}_\chi}{d\ln(a)} = \frac{\tilde{\rho}_{\rm BH}}{M_{\rm BH}}\boxed{\frac{\Gamma_{{\rm BH}\to\chi}}{H}} - \cancel{\frac{\Gamma_{\chi\to f\bar{f}}}{H}}\tilde{n}_\chi$$

The momentum-integrated emission rate

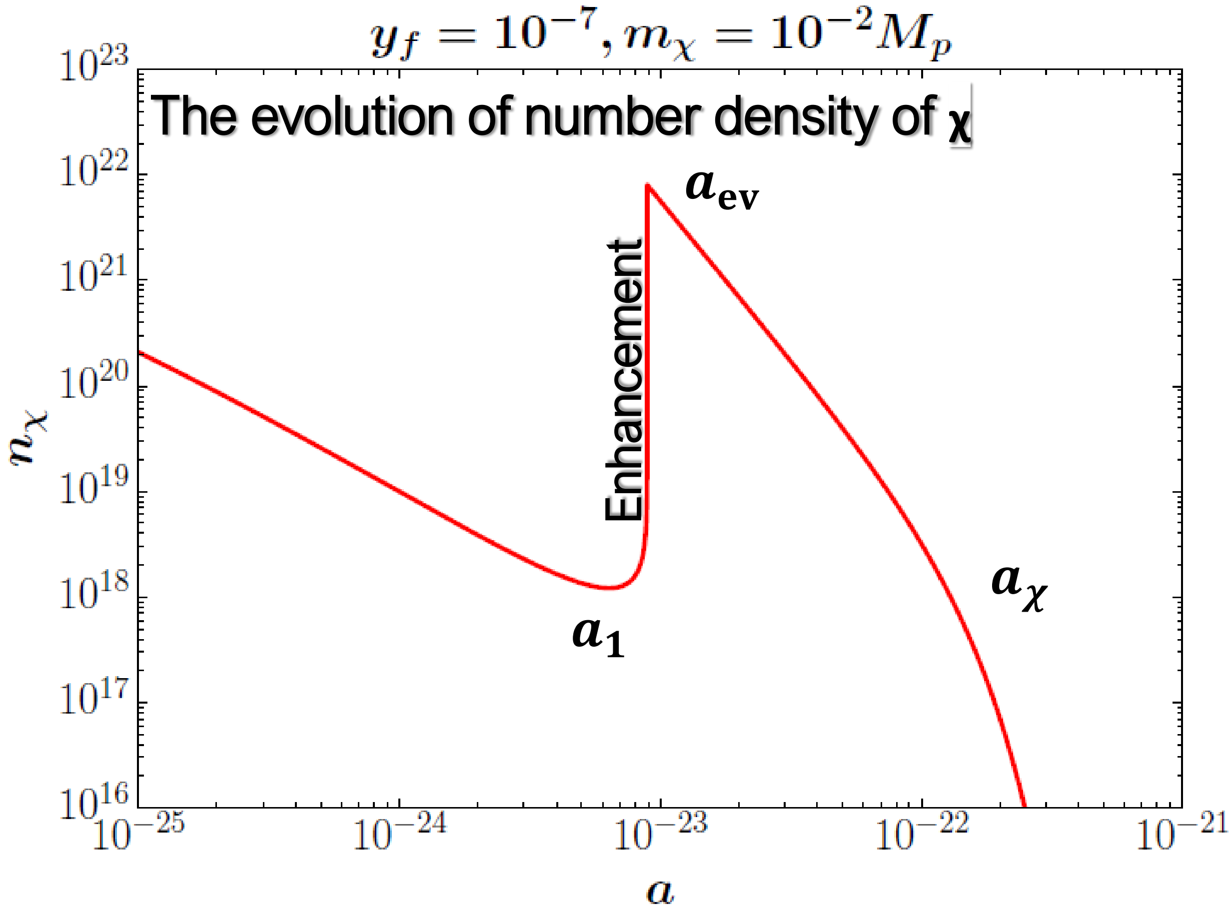
$$H^2 = \frac{8\pi}{3M_{\rm pl}^2}(\rho_r + \rho_{\rm BH})$$

$$\Gamma_{\chi\to f\bar{f}} = \frac{y_f^2}{8\pi}m_\chi$$

$$\Gamma_{{\rm BH}\to\chi}(t) = \frac{27g_\chi}{64\pi^3}\frac{M_p^2}{M_{\rm in}}\left(1-\frac{t-t_i}{\tau}\right)^{-1/3}\mathcal{G}(z)$$

$$\mathcal{G}(z) = [z{\rm Li}_2(e^{-z}) + {\rm Li}_3(e^{-z})\ \text{with z} = {\rm m}_\chi/{\rm T}_{\rm BH}$$

$$\boxed{n_\chi(a_{\rm ev}) \simeq 1.54\times 10^{21}\left(\frac{1g}{M_{\rm in}}\right)^7\left(\frac{10^{-2}M_p}{m_\chi}\right)^2}$$



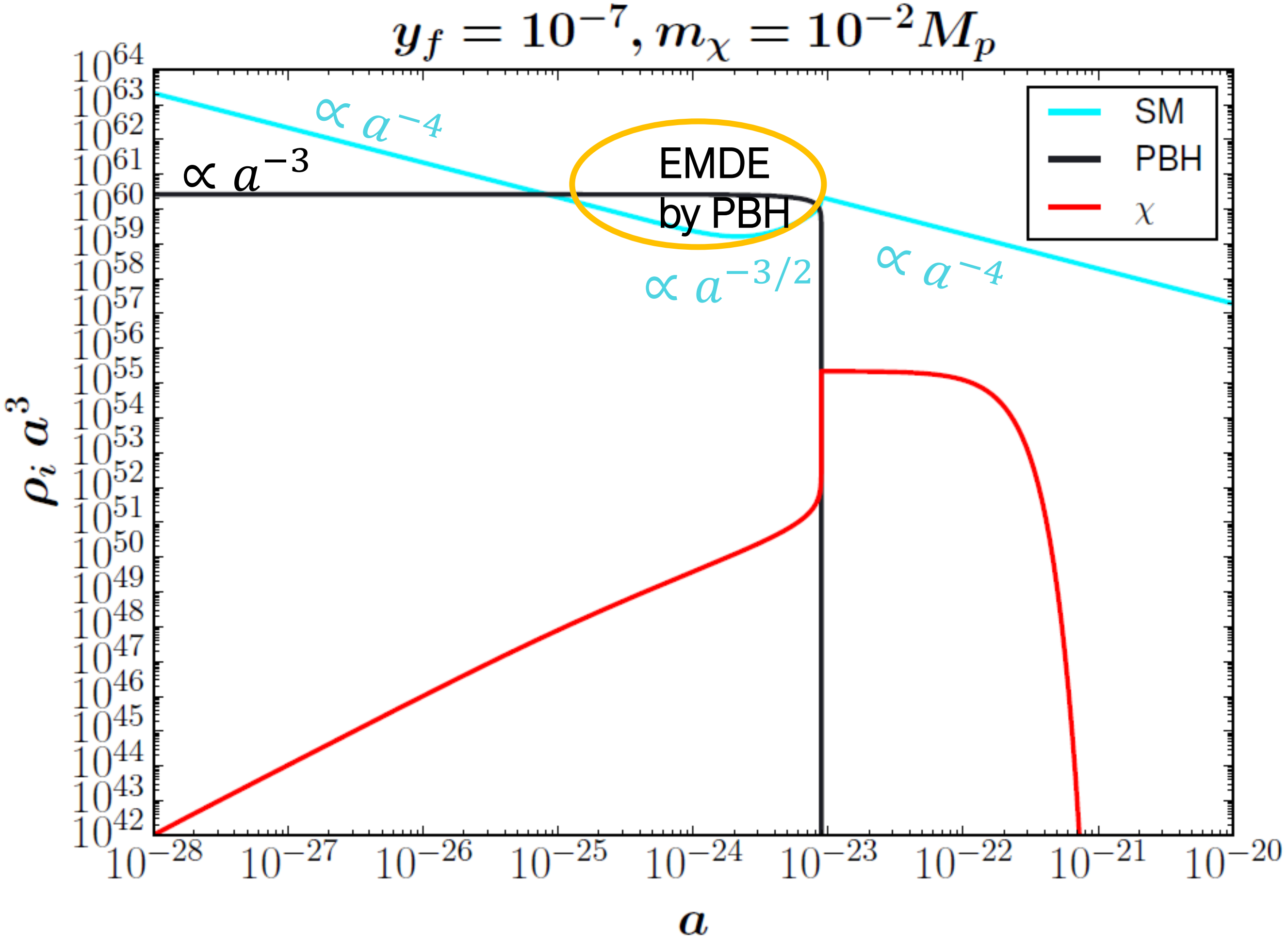
Energy density evolution

$$\frac{dM_{\rm BH}}{dt} = -\varepsilon(M_{\rm BH})\frac{\left(8\pi M_p^2\right)^2}{M_{\rm BH}^2}$$

$$\dot{\rho}_{\rm BH} + 3H\rho_{\rm BH} = \frac{\rho_{\rm BH}}{M_{\rm BH}}\frac{dM_{\rm BH}}{dt},$$

$$\dot{\rho}_{\rm r} + 4H\rho_{\rm r} = -\frac{\varepsilon_{\rm SM}(M_{\rm BH})}{\varepsilon(M_{\rm BH})}\frac{\rho_{\rm BH}}{M_{\rm BH}}\frac{dM_{\rm BH}}{dt}$$

The fraction of SM particles from evaporation



GW production from massive scalar particle

Boltzmann equation for the evolution of the energy density of gravitons produced in the bremsstrahlung of the scalar χ :

$$\frac{d}{da}\left(a^4\frac{d\rho_{\rm GW}}{d\ln E_{\rm GW}}\right)=\frac{n_\chi(a_{\rm ev})a_{\rm ev}^3}{H}\boxed{\frac{d\Gamma_{\chi\to{\rm GW}}}{dE_{\rm GW}}}E_{\rm GW}^2$$

The differential decay rate of scalar particle

[S.Kanemura, et al. 2310.12023]

The generation of GW can occur from $\boldsymbol{a_{\rm ev}}$ to $\boldsymbol{a_\chi}$:

$$\frac{d\rho_{\rm GW}(a_\chi)}{d\ln E_{\rm GW}}\simeq 2.3\times10^{58}\,\frac{y_f^2 m_\chi M_p^9}{M_{\rm in}^7}E_{\rm GW}F(E_{\rm GW}/m_\chi)\frac{a_{\rm ev}^3}{a_0^2 a_\chi}\left[1-\left(\frac{a_{\rm ev}}{a_\chi}\right)^3\right]$$

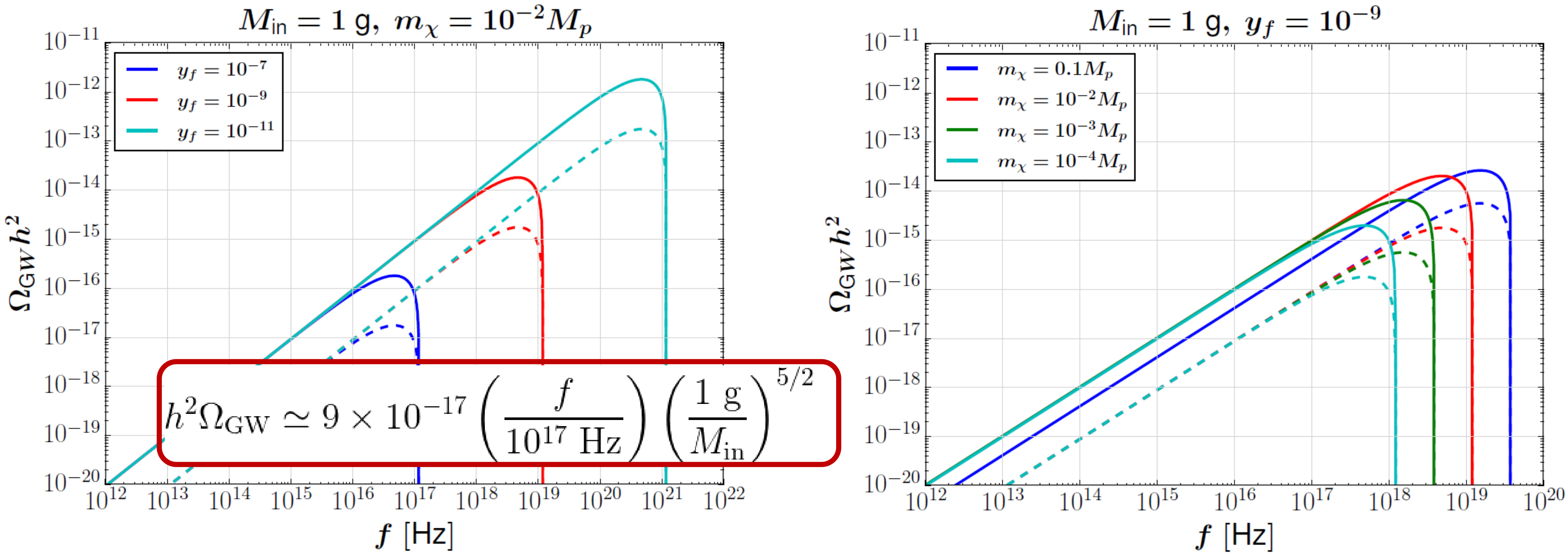
The relic abundance of gravitational wave at the present :

$$h^2\Omega_{\rm GW}=\frac{1}{\rho_{{\rm cr},0}h^{-2}}\frac{d\rho_{\rm GW}(a_\chi)}{d\ln E_{\rm GW}}\left(\frac{a_\chi}{a_0}\right)^4\ \text{where}\ {\rm E_{GW}}=2\pi{\rm f}({\rm a_0}/{\rm a_\chi})\ \Leftrightarrow\ \frac{a_\chi}{a_0}\simeq\frac{T_0}{T_\chi}\left(\frac{g_{*,s}(T_0)}{g_{*,s}(T_\chi)}\right)^{1/3}$$

$$\text{RD}:\quad {\rm H}=1/2\tau_\chi\ \Leftrightarrow\ T_\chi\simeq\frac{3y_f}{4\pi}\left(\frac{M_p m_\chi}{g_*(T_\chi)^{1/2}}\right)^{1/2}$$

GW production from massive scalar particle

Gravitational wave spectrums for graviton bremsstrahlung from decay of massive scalar



$$h^2 \Omega_{\rm GW} \simeq 9 \times 10^{-17} \left(\frac{f}{10^{17}\ {\rm Hz}} \right) \left(\frac{1\ {\rm g}}{M_{\rm in}} \right)^{5/2}$$

$$f_{\rm peak} = \frac{m_\chi}{4\pi} \left(\frac{a_\chi}{a_0} \right) \simeq 1.2 \times 10^{17}\ {\rm Hz} \left(\frac{10^{-7}}{y_f} \right) \left(\frac{m_\chi}{10^{-2} M_p} \right)^{1/2}$$

GW contribution to dark radiation

$$\rho_{\rm rad} = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} (N_{\rm eff}^{\rm SM} + \boxed{\Delta N_{\rm eff}})\right] ; \qquad \text{since } \rho_{\rm GW} \propto a^{-4}$$

Gravitational Wave contribution to $\Delta \boldsymbol{N_{eff}}$:

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_{\rm GW}}{\rho_\gamma} = \int df\, f^{-1} \Omega_{\rm GW}(f) = \frac{120}{7\pi^2}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_{\rm cr,0}}{T_0^4} \Omega_{\rm GW}^{\rm max}$$

$$\boxed{h^2 \Omega_{\rm GW}^{\rm max} \lesssim 5.6269 \times 10^{-6} \Delta N_{\rm eff}}$$

Indirect probe of GW spectrum:

Planck: $\Delta N_{\rm eff} < 0.30$ [1807.06209]

CMB-S4: $\Delta N_{\rm eff} \lesssim 0.06$ [1610.02743]

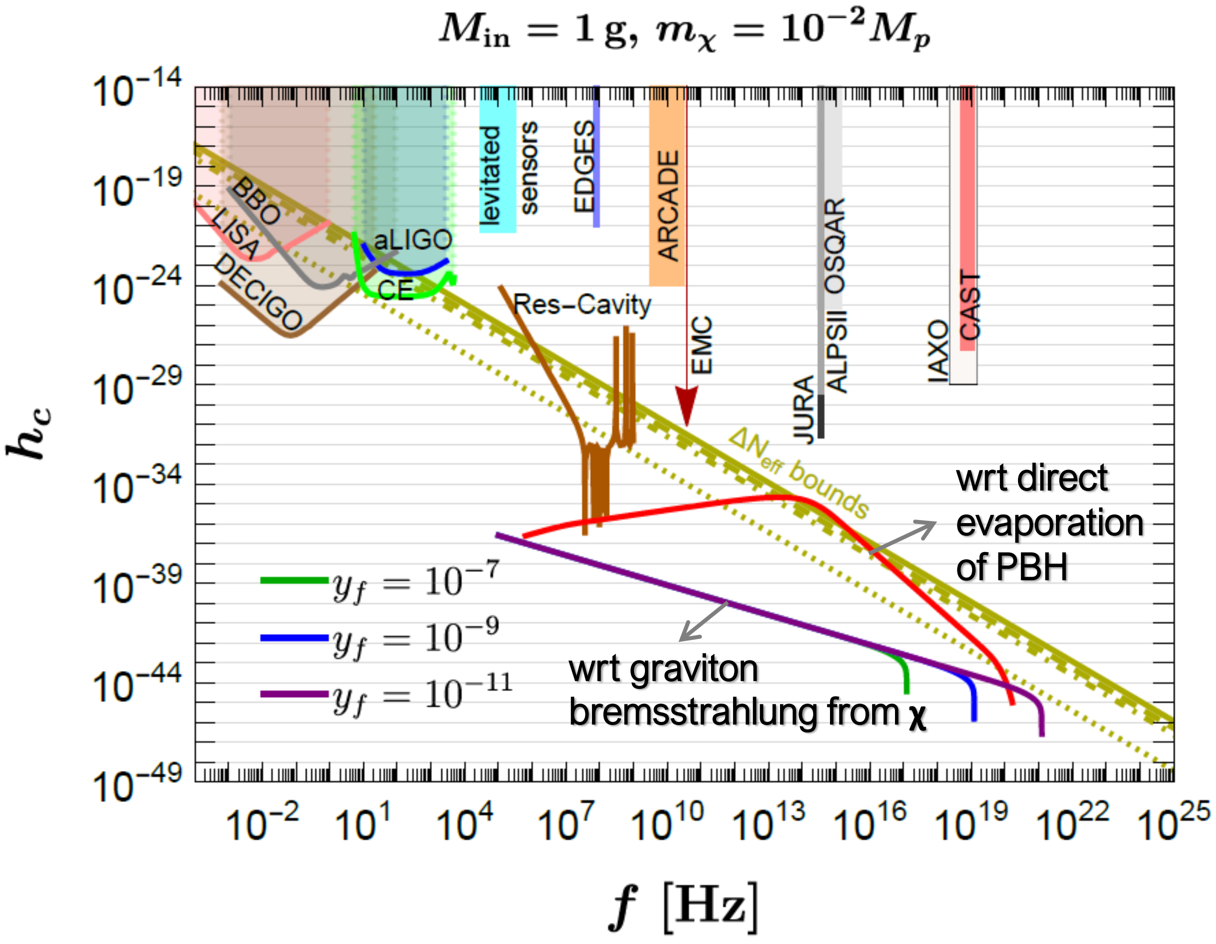
EUCLID: $\Delta N_{\rm eff} \lesssim 0.013$ [1110.3193]

CMB-CVL: $\Delta N_{\rm eff} \lesssim 3.1 \times 10^{-6}$ [1903.11843]

Fisher matrix analysis of cosmic variance limited (CVL) CMB polarization measurement

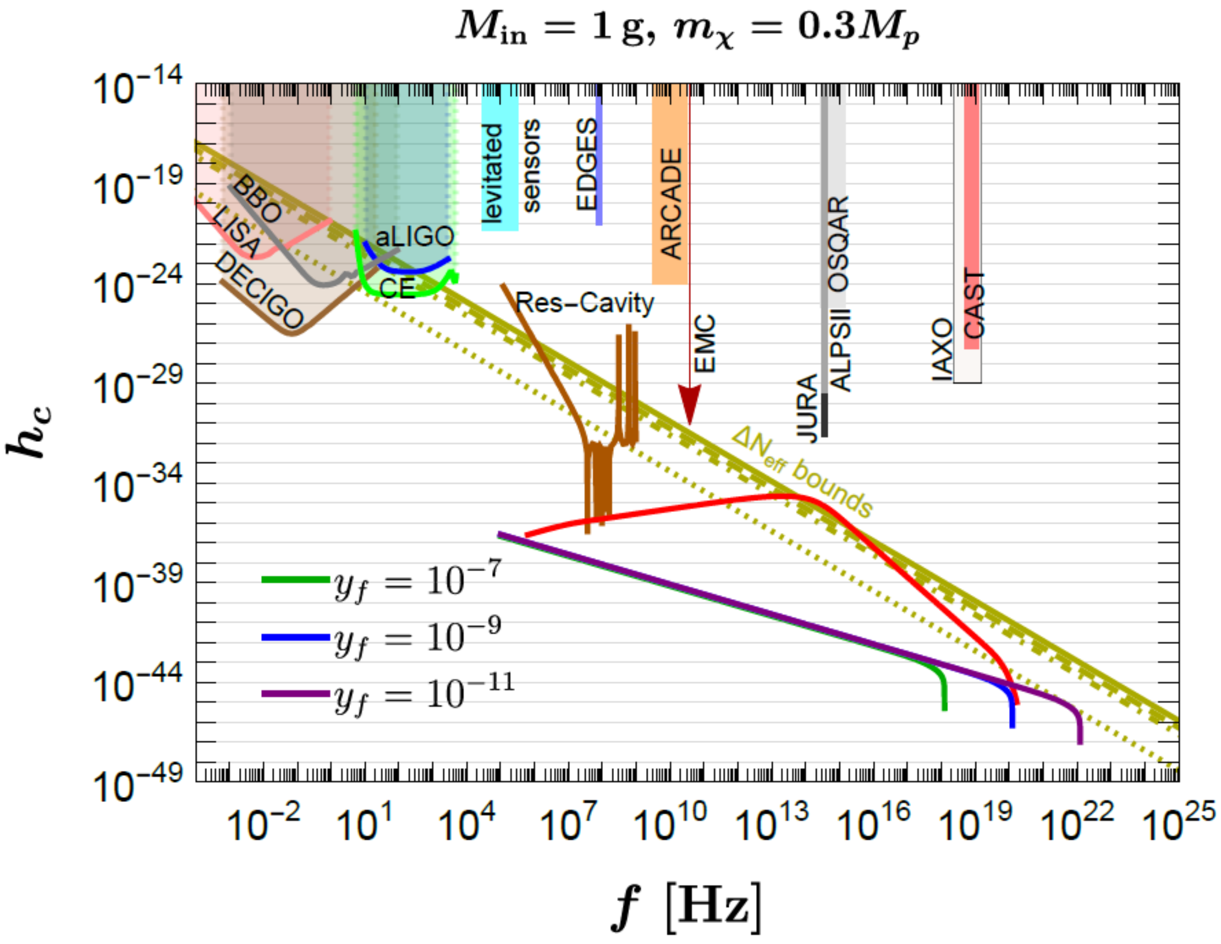
Gravitational wave characteristic strain with both contributions

$$h_c = f^{-1}\sqrt{\frac{3H_0^2}{4\pi^2}\Omega_{\rm GW}} \simeq 8.93\times 10^{-19}\sqrt{\Omega_{\rm GW}h^2}\left(\frac{\rm Hz}{f}\right)$$


$M_{\rm in} = 1\,{\rm g},\ m_\chi = 10^{-2}M_p$

wrt direct evaporation of PBH

wrt graviton bremsstrahlung from χ


$M_{\rm in} = 1\,{\rm g},\ m_\chi = 0.3M_p$

Conclusion

- We have explored the production of stochastic gravitational waves not only through the direct evaporation of PBHs but also via the bremsstrahlung process during the decay of a massive scalar particle **χ**.
- Both contributions give two distinct spectral signatures which are possible to detect by future resonant cavity detectors.
- In our model, a novel source of GW production in bremsstrahlung process can especially account for the high-frequency GW scenarios, but it remains below the sensitivity of the present detectors.
- Moreover, GWs generating from the decay of massive scalar particles, characterized by sub-Planckian mass scales and small Yukawa coupling, could significantly contribute to dark radiation. These contributions are expected to fall within the detection thresholds of CMB-CVL.

Cảm ơn!