ICISE

The 30th Vietnam School of Physics: Particle and Dark Matter

Department of Theoretical Physics, University of Science
Vietnam National University, HCM city

Student Seminar

Theoretical studies the $e^+e^- \to \nu\bar{\nu}H$ process in the ILC

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VSOP - 2024

Overview

1. Introduction

The field of Physics has witnessed remarkable advancements in recent times, with the emergence of numerous novel theories and models.

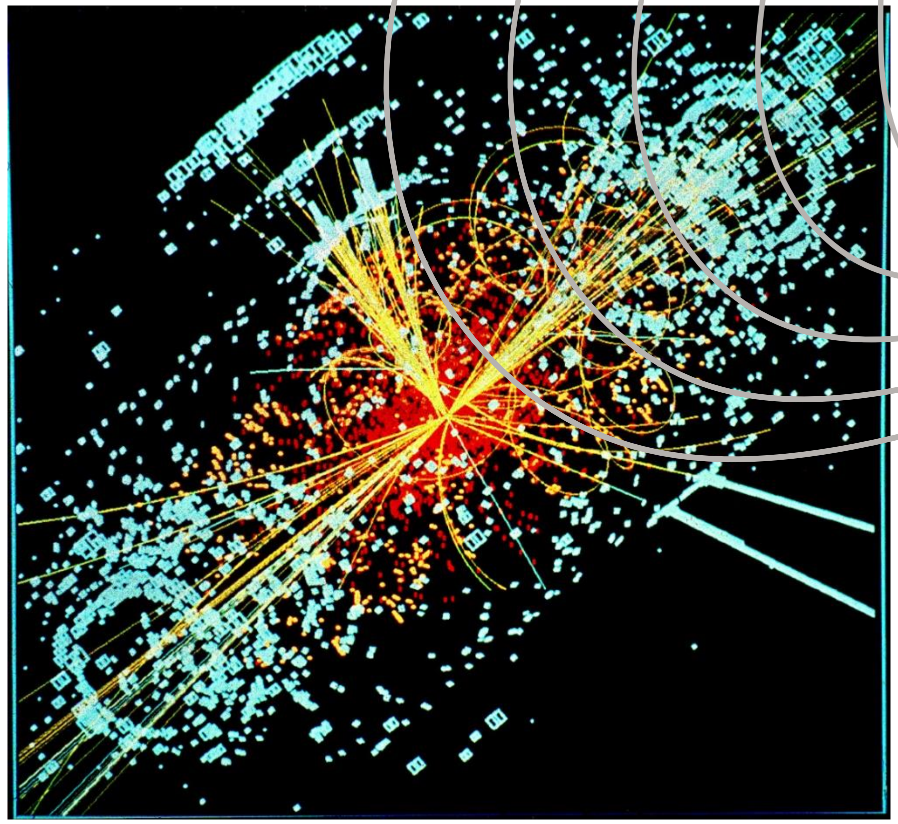


Figure 1. Simulating the Higgs boson decay event after the collision of protons occurring at the CERN.

The Standard Model, a mathematical framework that describes the fundamental interactions of nature, has garnered significant attention and interest.

The Standard Model has been extensively validated through numerous experiments, particularly at high-energy particle accelerators such as ILC, LHC, RHIC, ATK,...

2. Objective

Starting from the Standard Model, this seminar will focus on studying the process $e^+e^- \to \nu\bar{\nu}H$.

The project will be carried out:

Calculate the scattering amplitude $|\mathcal{M}|^2$.
Construct the phase space for the scattering process.

Perform numerical calculations of scattering cross sections, obtain results and discuss physical distributions.

→ Contributes to providing more useful information to verify the Standard Model

3. Theoreticle Framework

3.1. Overview of the Standard Model

The Standard Model is a Physics model built based on Quantum Field Theory through the group symmetry property $SU_c(3) \times SU_L(2) \times U_Y(1)$, this model is to describe the physical particles and interactions between them.

Table 1: Elementary particles in the Standard Model

	I	II	III
Lepton	e	μ	τ
	ν_e	ν_μ	ν_τ
Quark	u	c	t
	d	s	b

Gauge boson	Higgs boson
$W^\pm$ Z γ g	H

The Standard Model of Particle Physics

3.2. Lagrangian for the Standard Model

The Lagrangian of the Standard Model is given by

$$\mathcal{L}_M = \mathcal{L}_{Kin} + \mathcal{L}_{YM} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

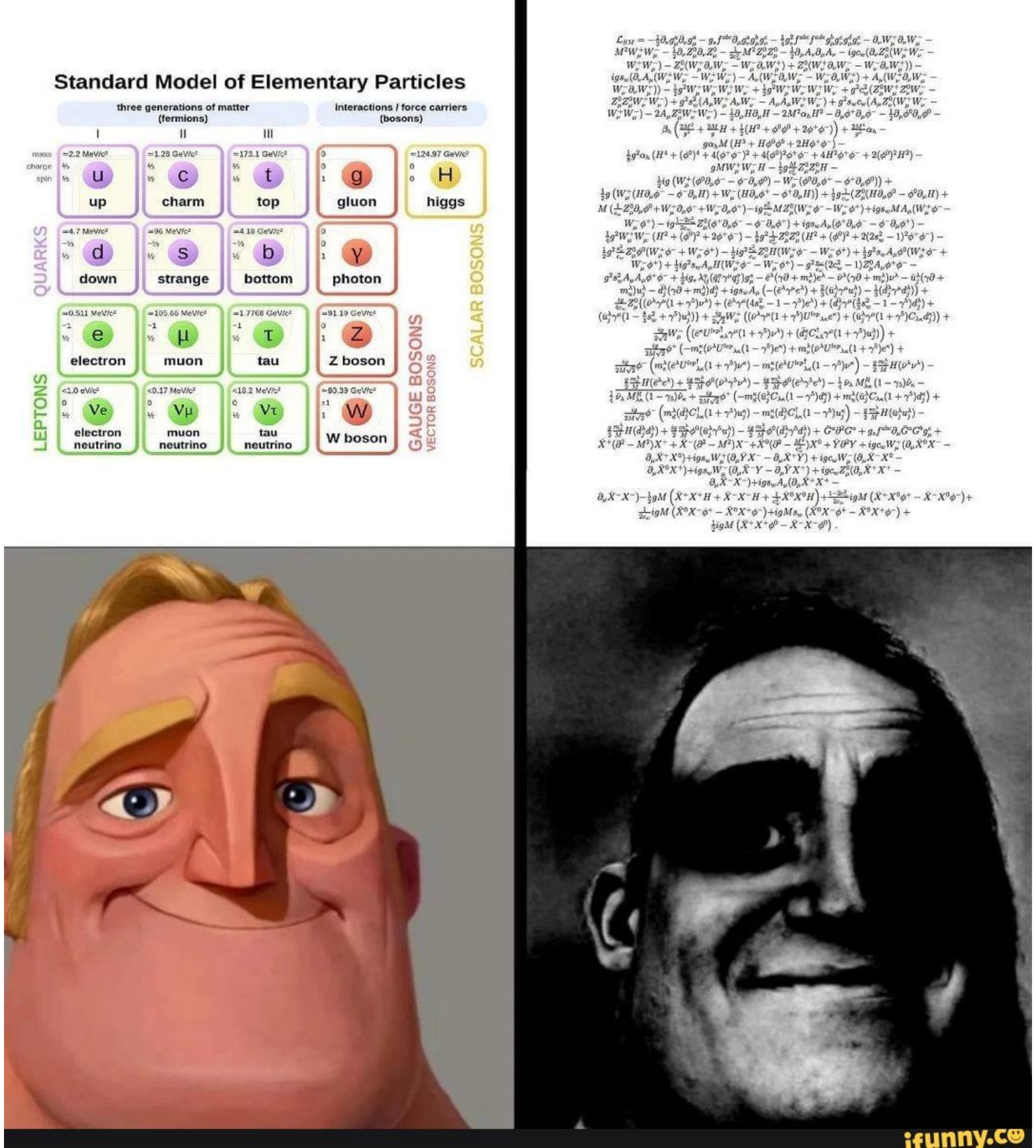
The Lagrangian of the Standard Model consist of components

$$\mathcal{L}_{Kin} = i\bar{Q}^i_L \gamma^\mu D_\mu Q^i_L + i\bar{u}^i_R \gamma^\mu D_\mu u^i_\mu + i\bar{d}^i_R \gamma^\mu D_\mu d^i_R + i\bar{L}^i \gamma^\mu D_\mu L^i + i\bar{e}^i_R \gamma^\mu D_\mu e^i_R$$

$$\mathcal{L}_{YM} = -\frac{1}{4} W^{\mu\nu} W_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

$$\mathcal{L}_{Yukawa} = -h_e(\bar{L}^i \phi e^i_R + h.c) - h_d(\bar{Q}^i \phi d^i_R + h.c) - h_u(\bar{Q}^i \phi_c u^i_R + h.c)$$


Standard Model of Elementary Particles
three generations of matter (fermions)
interactions / force carriers (bosons)
I
II
III
mass
charge
spin
QUARKS
LEPTONS
GAUGE BOSONS
VECTOR BOSONS
SCALAR BOSONS
≈2.2 MeV/c²
u
up
≈1.28 GeV/c²
c
charm
≈173.1 GeV/c²
t
top
g
gluon
≈124.97 GeV/c²
H
higgs
≈4.7 MeV/c²
d
down
≈96 MeV/c²
s
strange
≈4.18 GeV/c²
b
bottom
γ
photon
≈0.511 MeV/c²
e
electron
≈105.66 MeV/c²
μ
muon
≈1.7768 GeV/c²
τ
tau
≈91.19 GeV/c²
Z
Z boson
<1.0 eV/c²
νe
electron neutrino
<0.17 MeV/c²
νμ
muon neutrino
<18.2 MeV/c²
ντ
tau neutrino
≈80.39 GeV/c²
W
W boson
ifunny.co

4. Methodology

In this project, we will use the Mathematica with FeynArts and FormCalc package to calculation program and numerical solution using Monte Carlo to perform calculations for the scattering process $e^+e^- \rightarrow \nu\bar{\nu}H$.

Mathematica

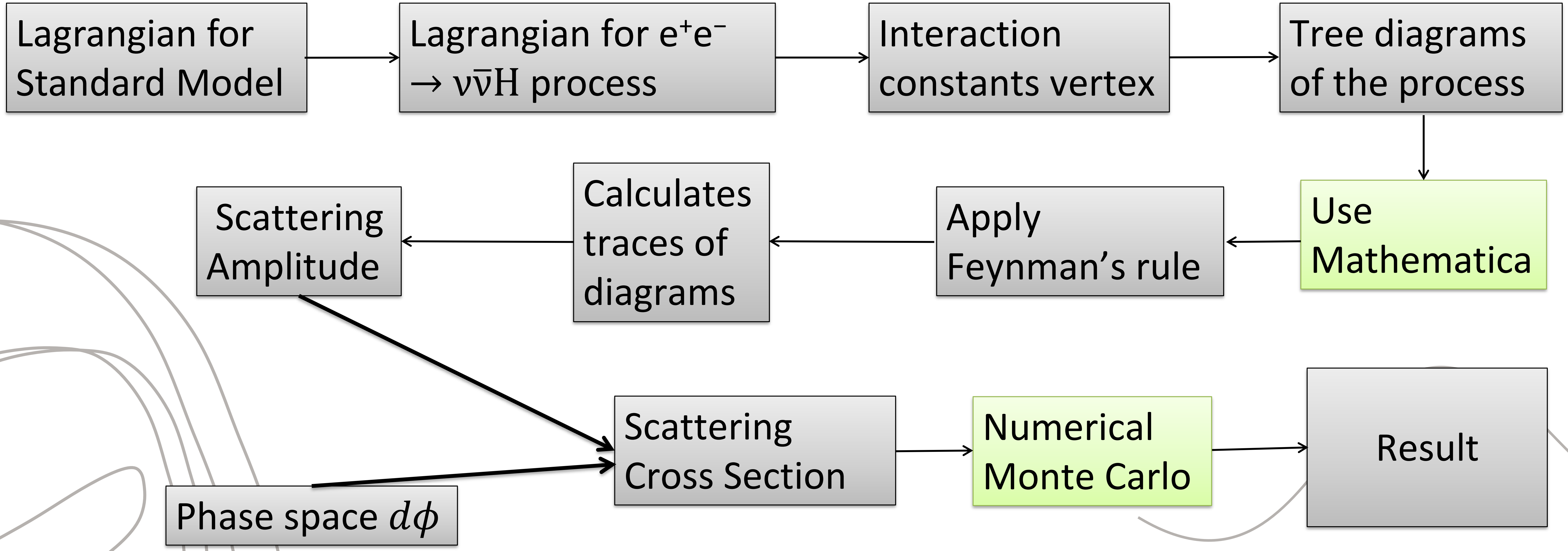
Tool to calculate complex expressions, used to calculate diagrams of the scattering process

Monte Carlo

Random number seeding method plays an important role in numerical solution for multi-dimensional integral

5. Implementation

Going from the Lagrangian for the Standard Model, use Mathematica to calculate the process diagrams to find the scattering amplitude $|\mathcal{M}|^2$ of the $e^+e^- \to \nu\bar{\nu}H$ process. Then build the phase space to give the scattering cross section. Finally, proceed to numerically solve the process using Monte Carlo.



6. Calculate

The scattering cross sector of process $e^+e^- \to \nu\bar{\nu}H$ has form

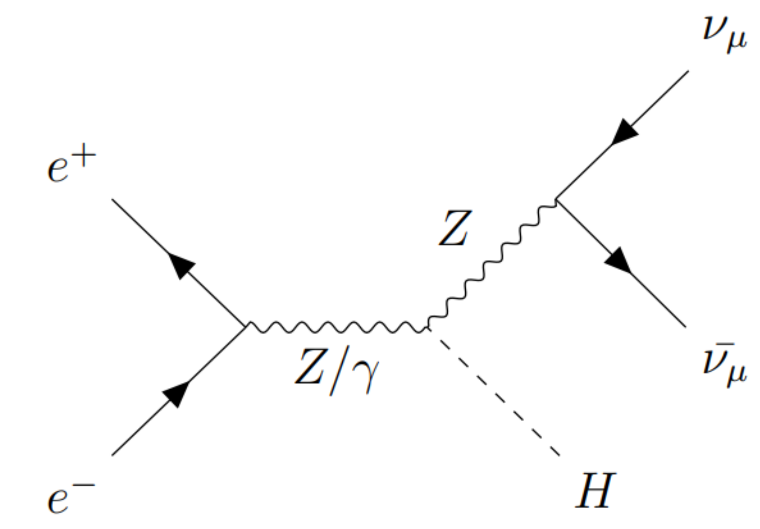
$$\sigma = \int d\phi_3 |\mathcal{M}|^2$$

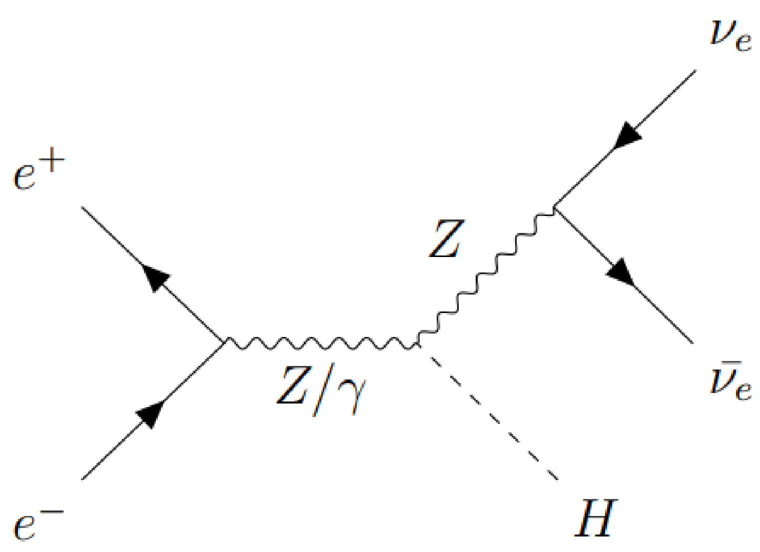
with

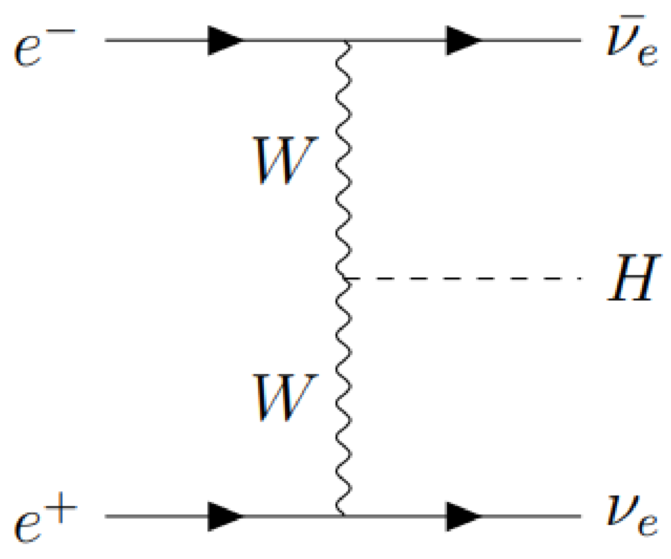
$$d\Phi_3 = J dx_1 dx_2 dx_3 dx_4 dx_5$$

Perform calculations for the process $e^+e^- \to \nu\bar{\nu}H$ the input parameters: m_W = 80.3767; m_Z = 91.1876; $m_e = m_\nu$ = 0; Γ_Z = 2.4952; $\sqrt{s} = 500$. With the integration area $\Omega = [0,1]^5$ for the independent variables x_1, x_2, x_3, x_4, x_5.

6.1. Calculate the scattering amplitude $|\mathcal{M}|^2$







We have the relation for the diagram $e^+e^- \rightarrow \nu_\mu\overline{\nu_\mu}H$

$$|\mathcal{M}_1|^2 = \frac{16\alpha^3 M_W^2 \pi^3 \left(4p_1p_3 \times p_2p_4 \times sin^4\theta_W + p_1p_4 \times p_2p_3\left[1 - 2sin^2\theta_W\right]^2\right)}{cos^8\theta_W\left((M_Z^2 - 2p_3p_4)^2 + \Gamma_Z^2 M_Z^2\right)(M_Z^2 - s)^2 sin^6\theta_W}$$

We have the relation for the diagram $e^+e^- \rightarrow \nu_e\overline{\nu_e}H$

$$|\mathcal{M}_2|^2 = 16\alpha^3 M_W^2 \pi^3 \Bigg(p_1p_4 \times p2p3(M_W^4 + 4p_1p_3 \times p_2p_4 + 2M_W^2(p_1p_3 + p_2p_4) - 2cos^4\theta_W$$
$$\times(M_Z^2 - 2p_3p_4)(M_Z^2 - s))^2 - 4(M_W^2 + 2p_1p_3)p_1p_4 \times p_2p_3(M_W^2 + 2p_2p_4)(M_W^4 + 4p_1p_3$$
$$\times p_2p_4 + 2M_W^2(p_1p_3 + p_2p_4) - 2cos^4\theta_W(M_Z^2 - 2p_3p_4)(M_Z^2 - s))sin^2\theta_W + 4(M_W^2 + 2p_1p_3)^2$$
$$\times(M_W^2 + 2p_2p_4)^2(p_1p_4 \times p_2p_3 + p_1p_3 \times p_2p_4)sin^4\theta_W \Bigg) \Big/ \Bigg(cos^8\theta_W(M_W^2 + 2p_1p_3)^2$$
$$\times(M_W^2 + 2p_2p_4)^2\left[(M_Z^2 - 2p_3p_4)^2 + \Gamma_Z^2 M_Z^2\right](M_Z^2 - s)^2 sin^6\theta_W \Bigg)$$

So we have amplitude

$$|\mathcal{M}|^2 = 2|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2$$

6.2. Phase space for the scattering process

Phase space for $e^+e^- \to \nu\bar{\nu}H$ process

$$d\Phi_3 = \frac{1}{8^2(2\pi)^5} dm_{34}^2 dcos\theta_5 d\phi_5 dcos\theta_{34} d\phi_{34} \frac{\sqrt{\lambda(s, m_{34}^2, m_5^2)}}{s} \frac{\sqrt{\lambda(m_{34}^2, m_3^2, m_4^2)}}{m_{34}^2}$$

Transform expressions by selecting distributions

$$p(m_{34}^2) = \begin{cases} \frac{1}{q_{max} - q_{min}}, & q_{min} \leq m_{34}^2 \leq q_{max} \\ 0, & \neq \end{cases}$$

$$p(cos\theta_5) = \begin{cases} \frac{1}{2}, & cos\theta_5 \in [-1, 1] \\ 0, & \neq \end{cases}$$

$$p(cos\theta_{34}) = \begin{cases} \frac{1}{2}, & cos\theta_{34} \in [-1, 1] \\ 0, & \neq \end{cases}$$

with $$(m_3 + m_4)^2 \leq m_{34}^2 \leq (\sqrt{s} - m_5)^2$$

$$p(\phi_5) = \begin{cases} \frac{1}{2\pi}, & \phi_5 \in [0, 2\pi] \\ 0, & \neq \end{cases}$$

$$p(\phi_{34}) = \begin{cases} \frac{1}{2\pi}, & \phi_{34} \in [0, 2\pi] \\ 0, & \neq \end{cases}$$

Let J be the Jacobian quantity, now we have the Jacobian corresponding to the 2-particles phase space of the process $p_1 + p_2 \to p_{34} + p_5$

$$J_1 = \frac{1}{4\pi} \frac{\sqrt{\lambda(s, m_{34}^2, m_5^2)}}{2s}$$

We have the Jacobian corresponding to the phase space of two particles in the process $p_{34} = p_3 + p_4$

$$J_2 = \frac{1}{4\pi} \frac{\sqrt{\lambda(m_{34}^2, m_3^2, m_4^2)}}{2m_{34}^2}$$

We get the 3-particle phase space of the scattering process rewritten in terms of the 5 independent variables with a new integration region $\Omega = [0,1]^5$

$$d\Phi_3 = J dx_1 dx_2 dx_3 dx_4 dx_5 \qquad \text{with} \qquad J = \frac{q_{max} - q_{min}}{2\pi} J_1 J_2$$

7. Result

7.1. Scattering Cross Section

m_H [GeV]	σ_{tree}[fb]	σ [fb]	δ [%]
150	61.12	61.08	-0.06
200	37.33	37.44	0.29
250	21.17	21.13	-0.18
300	10.76	10.83	0.65
350	4.603	4.605	0.05

Table 2: Scattering cross section of the $e^+e^- \to \nu\bar{\nu}H$ process under the dependence of the Higgs mass at $\sqrt{s} = 500$ GeV

https://arxiv.org/abs/hep-ph/0211268

Note: Result at 2002

7.2. Distribution

Compare the distribution for the $e^+e^- \rightarrow \nu\bar{\nu}H$ process at m_H = 350 [GeV], $\sqrt{s} = 500$ GeV with previous research.

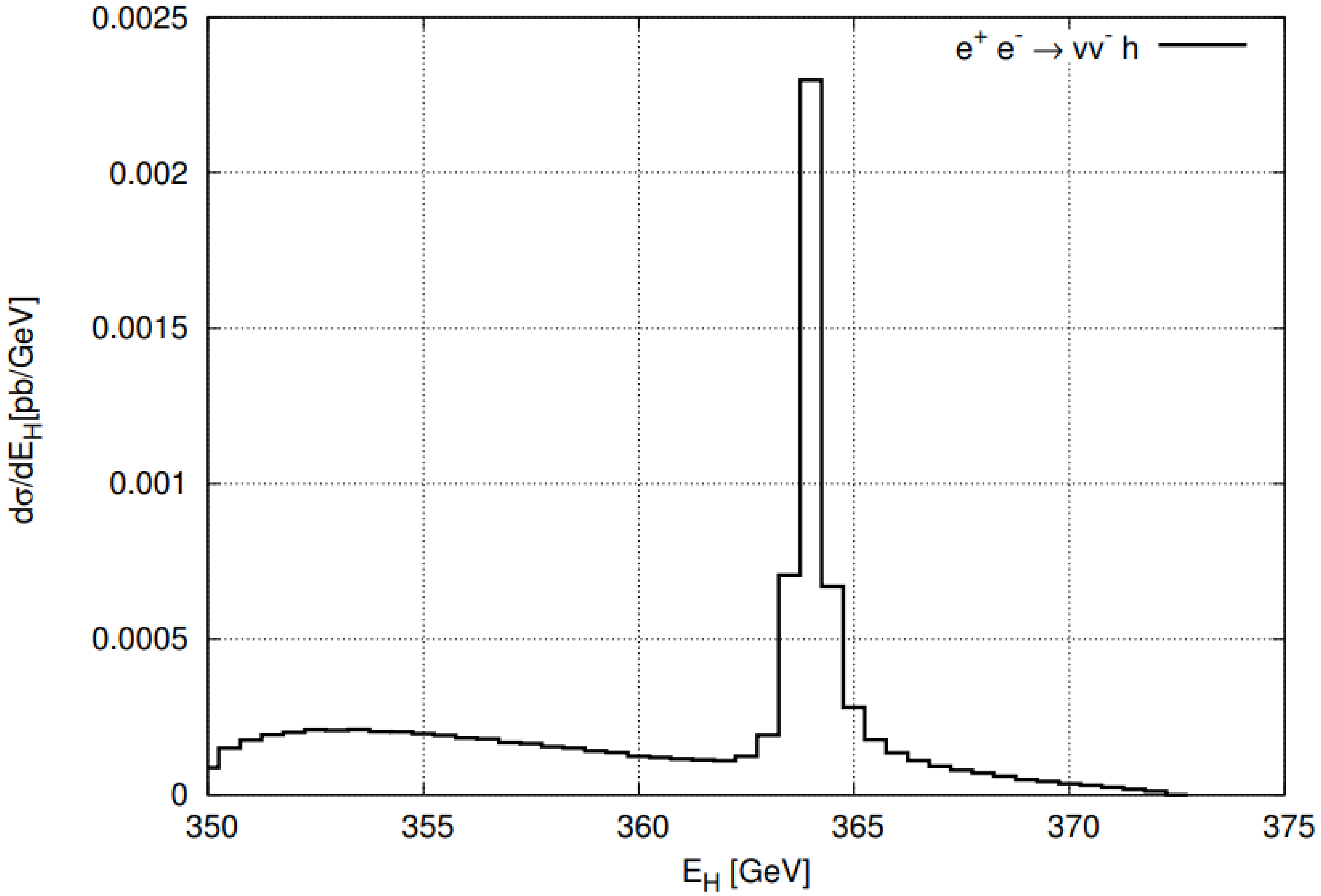
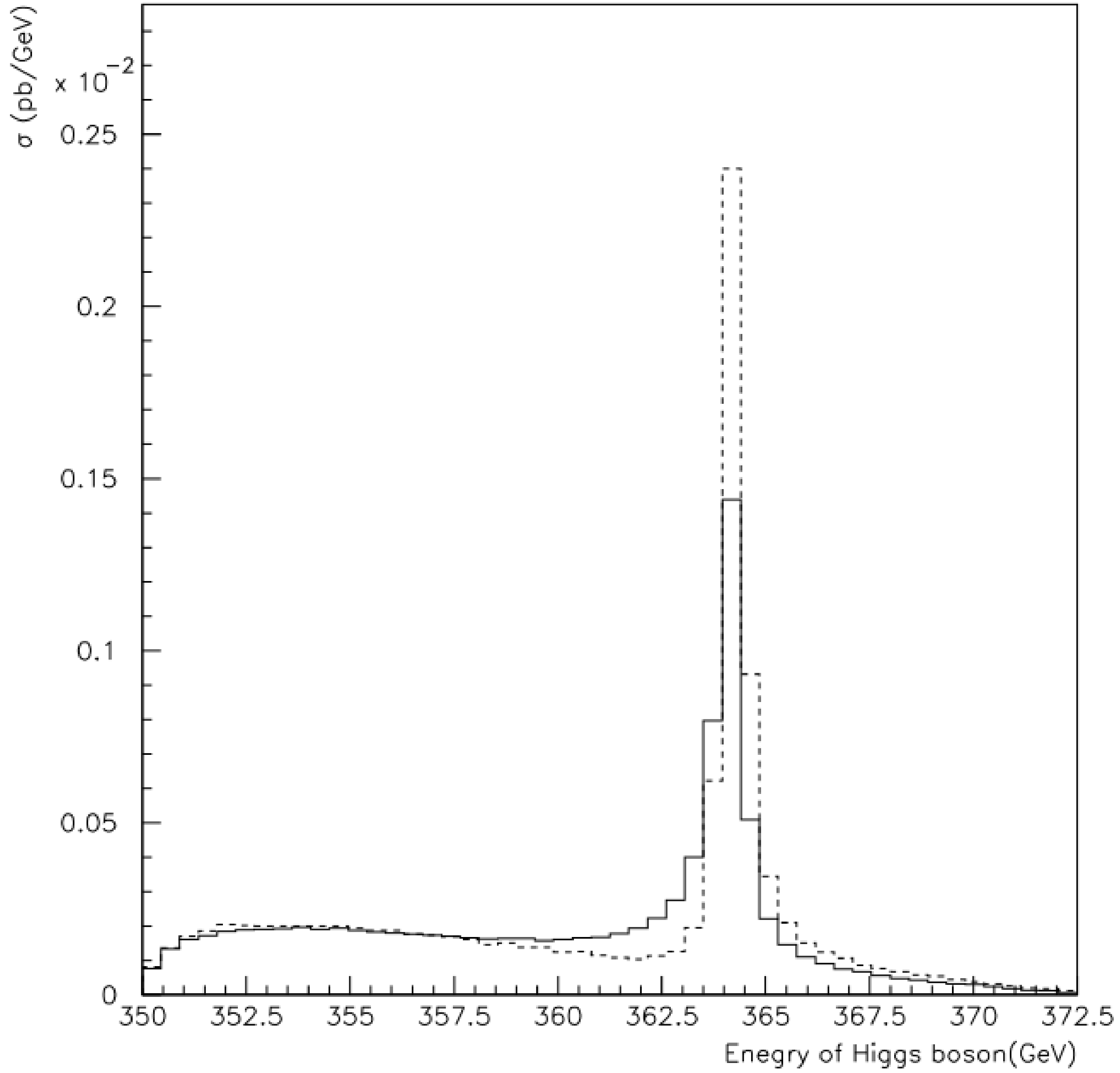


Figure 2. Higgs energy distribution



https://arxiv.org/abs/hep-ph/0211268

Result at 2002

7.2. Distribution

Result of Higgs boson energy and $m_{\nu\bar{\nu}}$ for the $e^+e^- \rightarrow \nu\bar{\nu}H$ process at m_H = 350 [GeV] with $\sqrt{s} = 500$ GeV

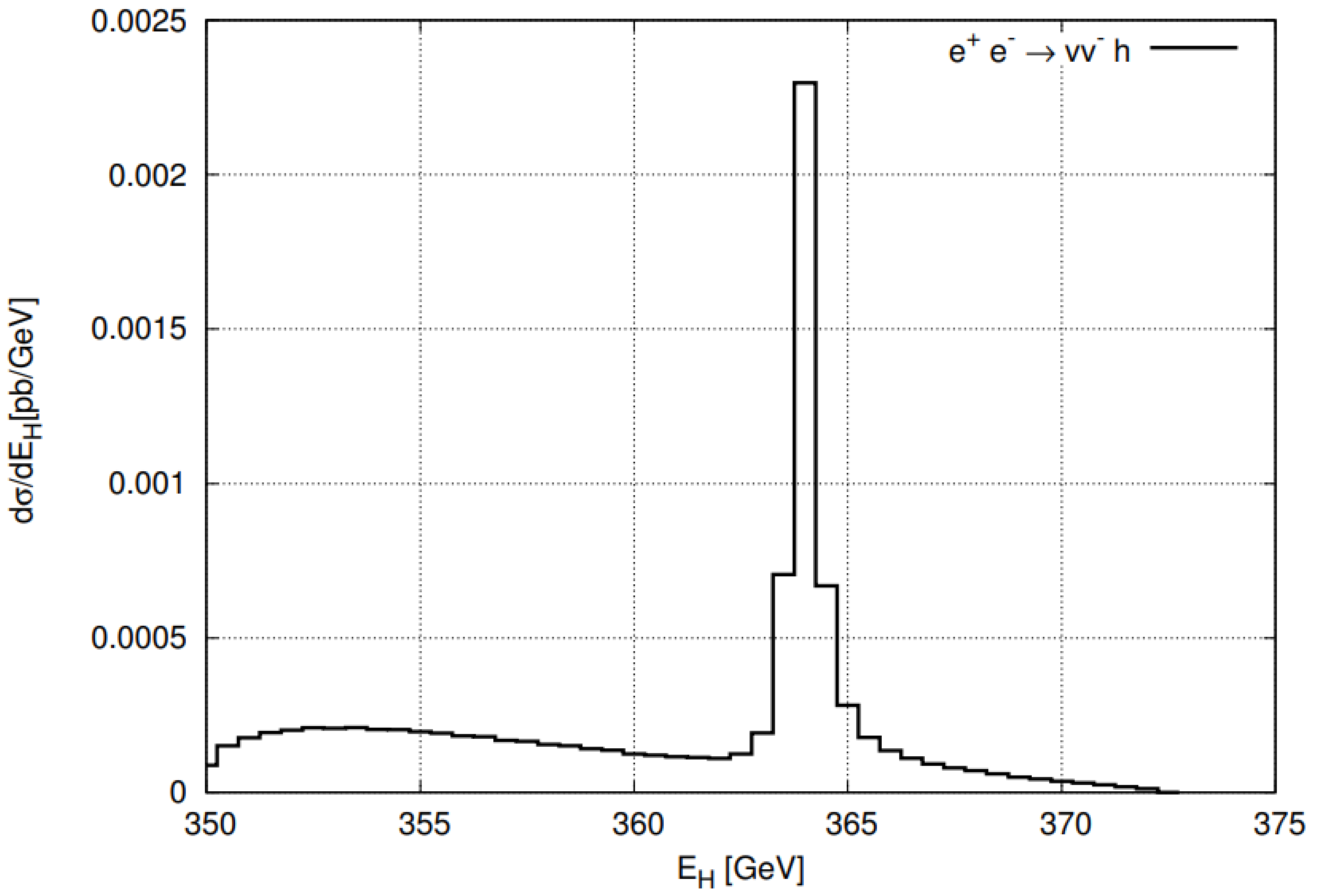


Figure 2. Higgs energy distribution

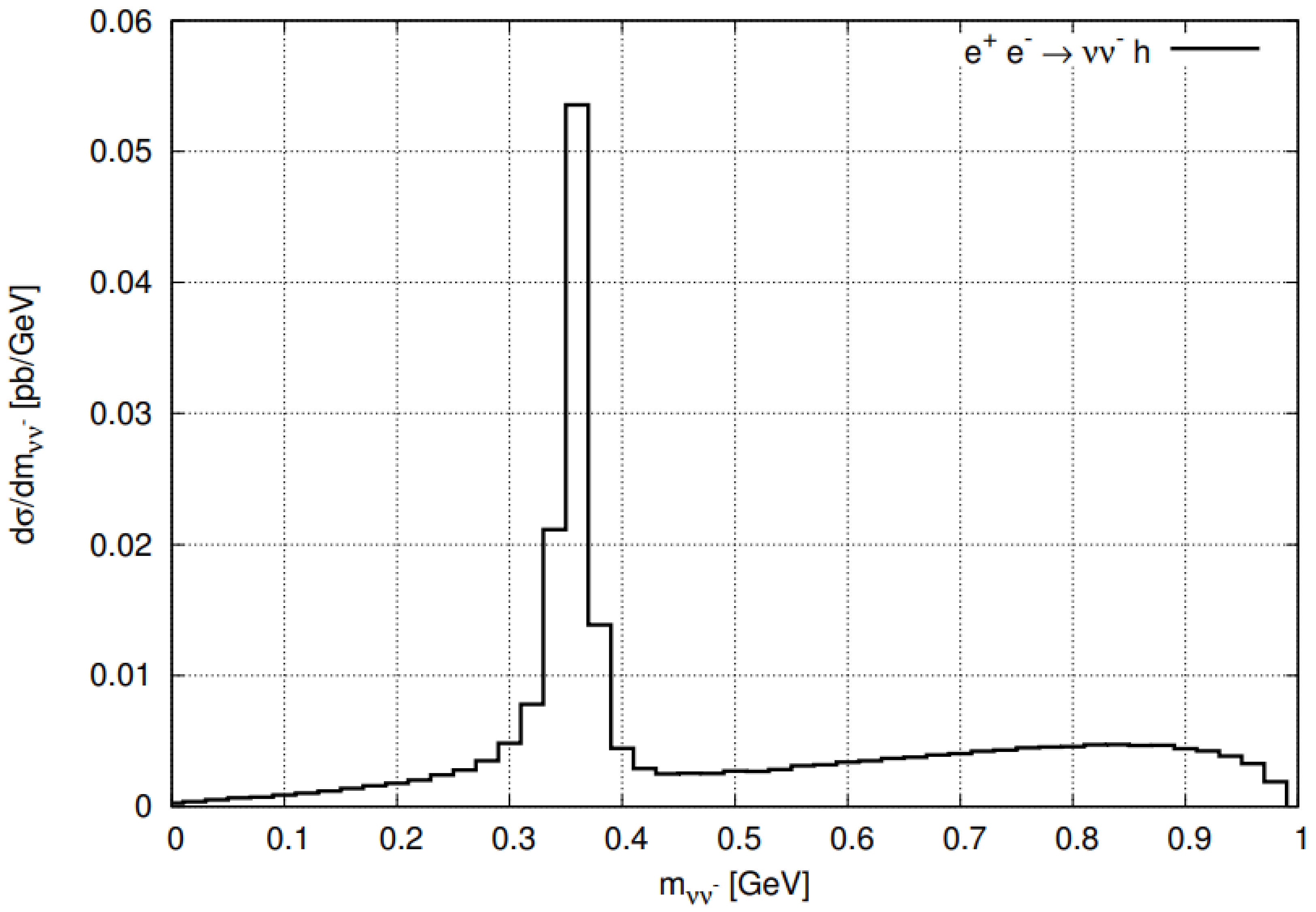


Figure 3. Energy distribution of $m_{\nu\bar{\nu}}$

7.2. Distribution

Perform calculations with Higgs boson mass m_H = 125 [GeV] for the $e^+e^- \to \nu\bar{\nu}H$ process with $\sqrt{s} = 500$ GeV.

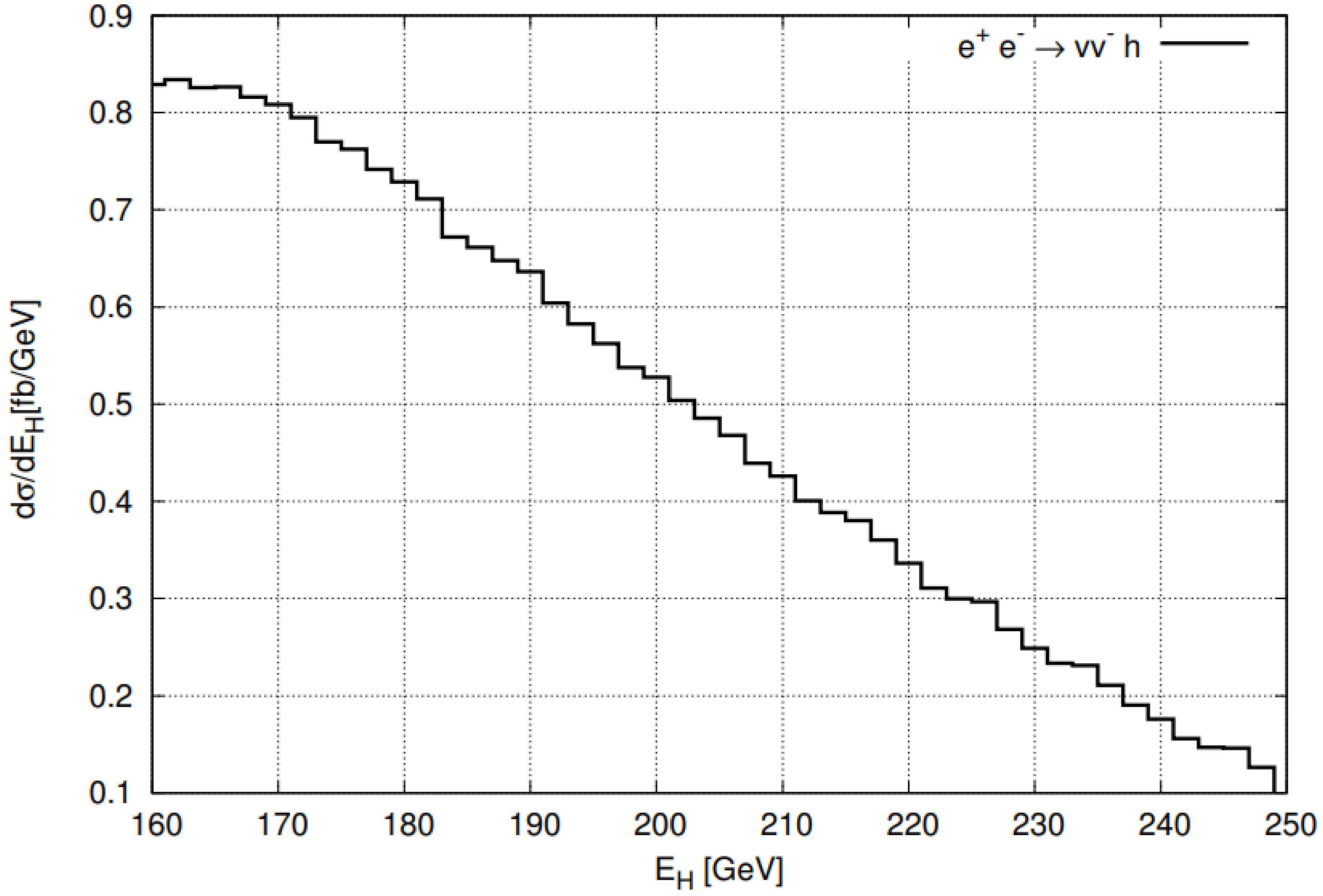
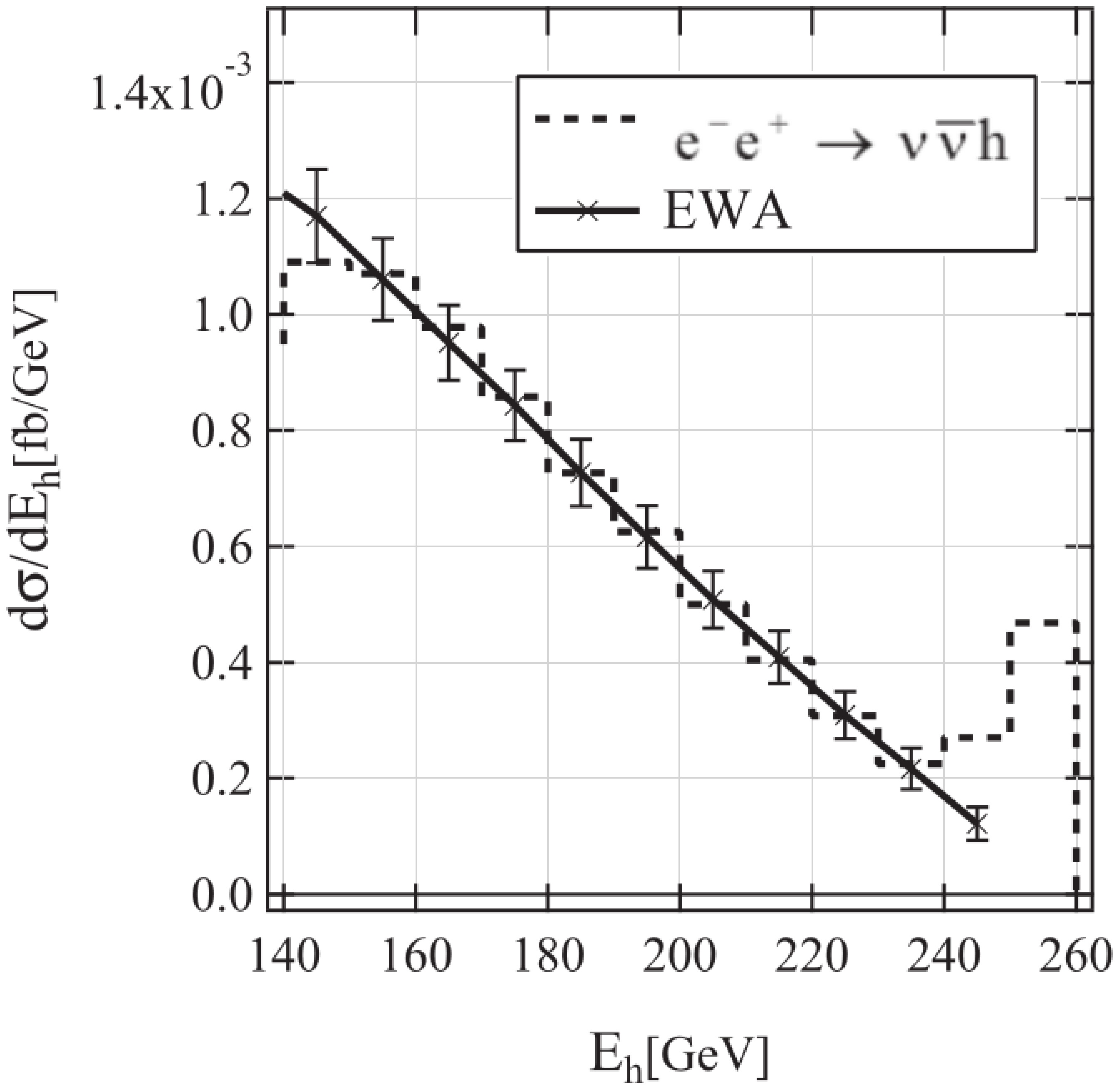


Figure 4. Higgs energy distribution at m_H = 125 [GeV]



https://arxiv.org/abs/1807.01911

Result at 2018

8. Summary

In summary, we have performed calculations to verify the Standard Model, studying the Physical properties through $e^+e^- \rightarrow \nu\bar{\nu}H$ process. Form here, we get the comparison results and Physical distributions necessary to discuss the topic achieved.

1. Calculate the scattering amplitude $|\mathcal{M}|^2$ for $e^+e^- \rightarrow \nu\bar{\nu}H$ process
2. Apply phase space to build an expression to calculate the scattering cross section for the process
3. Calculate the scattering cross sections using the Monte Carlo numerical solution method
4. Comparison with existing results and present the distribution

→ Through these steps, we can verify the theoretical results from the Standard Model and compare them with results obtained form other studies

Thank you for watching