# Gravitational waves from domain wall collapses and dark matter in the SM with a complex scalar

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# VSOP-30

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# Outline

- "SM very successful but still problematic"
- Spotaneous Symmetry Breaking (SSB) can lead to some topological defect: **Domain wall** (DW), monopole, cosmic string,... They give some cosmologycal effects.
- Experimental data from the detector: NanoGrav, IPTA,... (GW), LZ,... (DM)

Consider a scalar field in $\phi^4$ theory, with $\phi$ is a real field:

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4. \qquad (1)$$

We have the EOM on the $z$ direction:

$$\frac{\partial^2\phi}{\partial z^2} = \frac{\partial V}{\partial \phi}. \qquad (2)$$

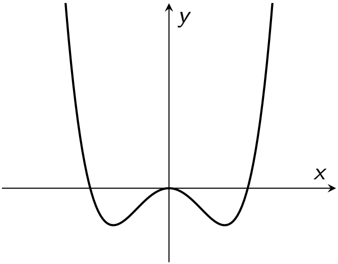


Figure 1: Potential for SSB

The potential is divided into two domains,vacuum expectation value (VEV) $v$ and $-v$. The transition from domain has VEV $-v$ to $v$, the boundary conditions:

$$\lim_{z\to\infty} \phi(z) = v, \qquad (3)$$

$$\lim_{z\to-\infty} \phi(z) = -v. \qquad (4)$$

The wall tension is:

$$\sigma_{\text{DW}} = \int_{-\infty}^{\infty} dz \mathcal{E} = \int_{-\infty}^{\infty} dz \left( \frac{1}{2} \left( \frac{\partial \phi}{\partial z} \right)^2 + V(\phi) \right). \tag{5}$$

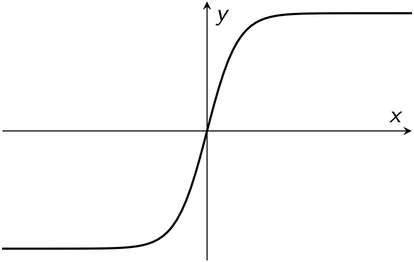


Figure 2: Domain wall solution

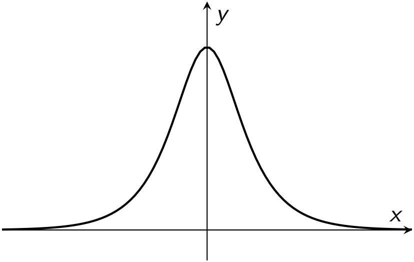


Figure 3: Energy density

In the energy spectrum Fig.(3), the energy is concentrated at the origin→ the DW must be unstable.

The Model:

$$V(\Phi,\mathbb{S}) = \mu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 + \frac{\delta_2}{2}|\Phi|^2|\mathbb{S}|^2 + \frac{b_2}{2}|\mathbb{S}|^2 + \frac{d_2}{4}|\mathbb{S}|^4 + \left(a_1\mathbb{S} + \frac{b_1}{4}\mathbb{S}^2 + \text{h.c.}\right). \tag{6}$$

Our model potential with $\Phi$ is SM doublet Higgs and $\mathbb{S}$ is complex scalar:

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}, \quad \mathbb{S} = \frac{1}{\sqrt{2}}(v_S + S + i\chi), \tag{7}$$

- The first line is $U(1)$ symmetry.
- $b_1$ breaking the $U(1)$
- $a_1$ term is added to break the $\mathcal{Z}_2$ symmetry in the $S$ direction. This term be considered later in collapsion of the DW.
- $a_1, b_1$ are the real paprameters.

Minimization condition:

$$\left\langle \frac{\partial V}{\partial h} \right\rangle = \mu^2 v \frac{\lambda}{4} v^3 + \frac{\delta_2}{4} v v_S^2 = 0, \quad \left\langle \frac{\partial V}{\partial S} \right\rangle = \frac{\delta_2}{4} v^2 v_S + \frac{b_2}{2} v_S + \frac{d_2}{4} v_S^3 + \sqrt{2} a_1 + \frac{b_1}{2} v_S = 0. \tag{8}$$

The mass matrix:

$$M^2 = \begin{pmatrix} \frac{\lambda}{2} v^2 & \frac{\delta_2}{2} v v_S & 0 \\ \frac{\delta_2}{2} v v_S & \frac{d_2}{2} v_S^2 - \sqrt{2} \frac{a_1}{v_S} & 0 \\ 0 & 0 & -b_1 - \sqrt{2} \frac{a_1}{v_S} \end{pmatrix}. \tag{9}$$

CP-odd scalar mass:

$$m_\chi^2 = -b_1 - \sqrt{2} \frac{a_1}{v_S}, \tag{10}$$

Diagonalize the mass matrix

$$O^T M^2 O = \mathrm{diag}(m_{h_1}^2, m_{h_2}^2), \quad O = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}. \tag{11}$$

Theoretical constraints: Perturbative unitarity, Global minimum, Stability of the tree-level potential.

The classical field:

$$\langle H(z)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi(z) \end{pmatrix}, \qquad \langle S(z)\rangle = \frac{\phi_S(z)}{\sqrt{2}}. \tag{12}$$

EOMs:

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{\partial V}{\partial \phi}, \quad \frac{\partial^2 \phi_S}{\partial z^2} = \frac{\partial V}{\partial \phi_S}, \tag{13}$$

with the boundary condition:

$$\lim_{z\to\pm\infty} \phi(z) = v, \quad \lim_{z\to\pm\infty} \phi_S(z) = \pm v_S, \tag{14}$$

and the tension:

$$\sigma_{DW} = \int dz \left[\sigma_{\rm DW}^{\rm kin} + \sigma_{\rm DW}^{\rm pot}\right]. \tag{15}$$

In the $|\alpha| \ll 1$ limit, DW reduced to that in the $\phi^4$:

$$\phi_S(z) = v_S \tanh\left(\sqrt{\frac{d_2}{8}} v_S z\right), \tag{16}$$

and the tension can approx as:

$$\sigma_{\rm DW} \approx \frac{d_2}{8} v_S^3 \int d\xi \left[\tanh^2\left(\sqrt{\frac{d_2}{8}} v_S z\right) - 1\right] = \frac{2}{3}\sqrt{\frac{d_2}{8}} v_S^3 \approx \frac{2}{3} m_{h_2} v_S^2. \tag{17}$$

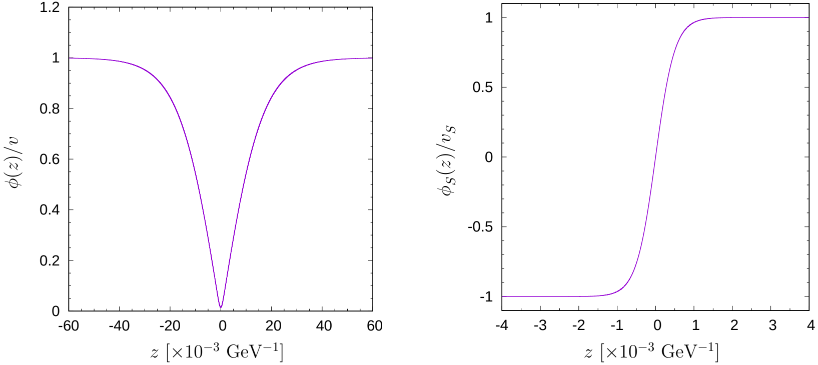


Figure 4: The DW profiles of $\phi(z)$ (left) and $\phi_S(z)$ (right), respectivley. We take $m_{h_2} = 4.0$ TeV, $m_\chi = 2.0$ TeV, $v_S = 100$ TeV, and $\alpha = 0.10^\circ$ ($= 1.7 \times 10^{-3}$ radians). This parameter set gives $\sigma_{\mathrm{DW}} = 2.7 \times 10^{13}$ [GeV$^3$].

The DW collapse when the bias enough large:

$$\Delta V = C_{\rm ann} \frac{\mathcal{A}\sigma_{\rm DW}}{t_{\rm ann}}. \tag{18}$$

Consider the bias term $a_1$ breaking the $\mathcal{Z}_2$. We have the degeneracy of the two vacua:

$$\Delta V \equiv |V(v, v_S) - V(v, v_S)| = 2\sqrt{2}|a_1|v_S. \tag{19}$$

The bound for the DW annihilation:

- Big Bang nucleosynthesis (BBN) $t_{\rm ann} < 0.01$s, constrain to the $|a_1|$

$$|a_1| > 2.3 \times 10^{-15}\ {\rm GeV}^3 \left(\frac{m_{h_2}}{10^3\ {\rm GeV}}\right)\left(\frac{v_S}{10^5\ {\rm GeV}}\right) C_{\rm ann}\mathcal{A}\hat{\sigma}_{\rm DW}. \tag{20}$$

- The DWs should not dominate the universe:

$$|a_1| > 8.0 \times 10^{-18}\ {\rm GeV}^3 \left(\frac{m_{h_2}}{10^3\ {\rm GeV}}\right)^2\left(\frac{v_S}{10^5\ {\rm GeV}}\right)^3 C_{\rm ann}\mathcal{A}^2\hat{\sigma}_{\rm DW}^2. \tag{21}$$

with

$$\hat{\sigma}_{\rm DW} \equiv \frac{\sigma}{m_{h_2} v_S^2}. \tag{22}$$

$C_{\rm ann} = 2, \mathcal{A} = 0.8$ given by Chen, Li, and Wu, "The gravitational waves from the collapsing domain walls in the complex singlet model"

The peak frequency given by[1]:

$$f_{\text{peak}} = 1.1 \times 10^{-9} \text{ Hz} \left( \frac{g_*(T_{\text{ann}})}{10} \right)^{1/2} \left( \frac{g_{*s}(T_{\text{ann}})}{10} \right)^{-1/3} \left( \frac{T_{\text{ann}}}{10^{-2} \text{ GeV}} \right), \tag{23}$$

$$\Omega_{\text{GW}} h^2(f_{\text{peak}}) = 7.2 \times 10^{-10} \, \tilde{\epsilon}_{\text{GW}} \mathcal{A}^2 \left( \frac{g_{*s}(T_{\text{ann}})}{10} \right)^{-4/3} \left( \frac{T_{\text{ann}}}{10^{-2} \text{ GeV}} \right)^{-4} \times \left( \frac{m_{h_2}}{10^3 \text{ GeV}} \right)^2 \left( \frac{v_S}{10^5 \text{ GeV}} \right)^4 \hat{\sigma}^2_{\text{DW}}, \tag{24}$$

where $\tilde{\epsilon}_{\text{GW}} = 0.7 \pm 0.4$. And from the simulation[2]:

$$\Omega_{\text{GW}} h^2(f < f_{\text{peak}}) = \Omega_{\text{GW}} h^2(f_{\text{peak}}) \left( f/f_{\text{peak}} \right)^3, \tag{25}$$

$$\Omega_{\text{GW}} h^2(f > f_{\text{peak}}) = \Omega_{\text{GW}} h^2(f_{\text{peak}}) \left( f_{\text{peak}}/f \right). \tag{26}$$

The signal-to-noise ratio $\text{SNR} = \sqrt{t_{\text{dur}} \int_{f_{\text{min}}}^{f_{\text{max}}} df \left( \frac{\Omega_{\text{GW}} h^2}{\Omega_{\text{exp}} h^2} \right)^2}$, with $t_{\text{dur}}$ denotes the duration of the mission. The nano-Hz scale future experiment SKA plan with $t_{\text{dur}} = 20$, we assuming SNR$= 20$.

[1] Saikawa, "A Review of Gravitational Waves from Cosmic Domain Walls".
[2] Hiramatsu, Kawasaki, and Saikawa, "On the estimation of gravitational wave spectrum from cosmic domain walls".

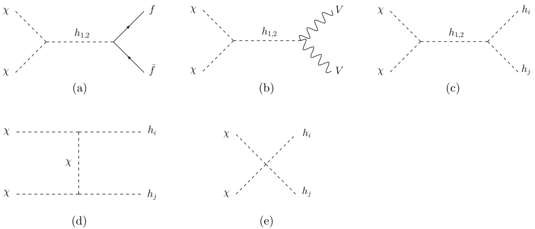


Figure 5: The DM annihilation processes, where $f$ denote the SM fermions, while $V$ represent $W^{\pm}$ and $Z$.

The most relevant diagram in the TeV scale of the DM is diagram (c).

The Spin-independent (SI) cross section[3]:

$$\sigma_{\mathrm{SI}}^{N} = \frac{1}{8\pi v^2} \frac{m_N^4}{(m_\chi + m_N)^2} \frac{s_{2\alpha}^2 (m_{h_1}^2 - m_{h_2}^2)^2 a_1^2}{m_{h_1}^4 m_{h_2}^4 v_S^4} \left| \sum_{q=u,d,s} f_{T_q} + \frac{2}{9} f_{T_G} \right|^2 . \tag{27}$$

In our study, we use `micrOMEGAs`[4] to calculate $\Omega_\chi h^2$ and $\sigma_{\mathrm{SI}}^{p}$.

[3] Chiang, Ramsey-Musolf, and Senaha, "Standard Model with a Complex Scalar Singlet: Cosmological Implications and Theoretical Considerations".
[4] **BARDUCCI2018327**.

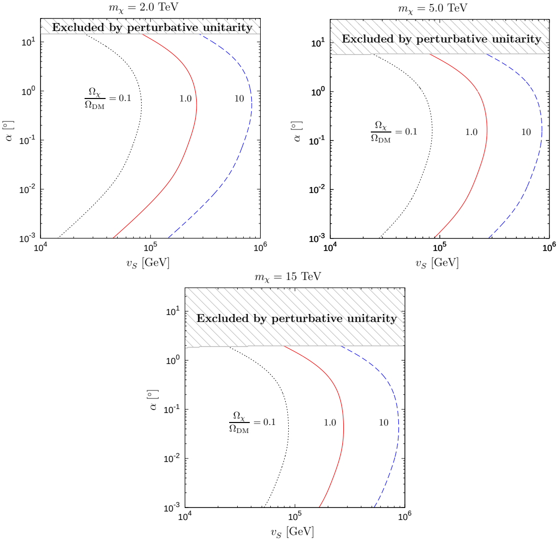


Figure 6: Contours of the DM relic abundance with $m_\chi = 2.0$ TeV, 5.0 TeV, and 15 TeV, respectively. In each panel, the three lines denote $\Omega_\chi/\Omega_{DM}$ =0.1 (black, dotted line), 1.0 (red, solid line), and 10 (blue, dashed line), with $\Omega_{DM}$ representing the observed value of the DM relic abundance.

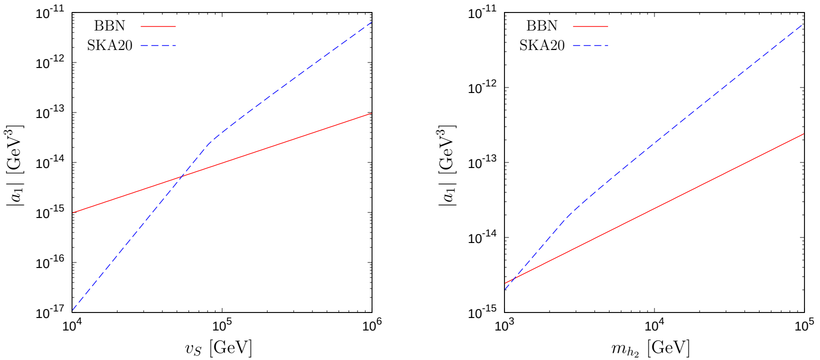


Figure 7: Constraints on the biased term $|a_1|$ with $m_{h_2} = 4.0$ TeV in the left panel and $v_S = 100$ TeV in the right panel, repspectively, and $m_{h_1} = 125$ GeV, $m_\chi = 2.0$ TeV, and $\alpha = 0.10^\circ$.

The solid line in red represents the BBN bound which yields the lower bound on $|a_1|$. On the other hand, the dashed line in blue (SKA20) denotes the discovery potential case with SNR $= 20$ which sets the upper bound on $|a_1|$.

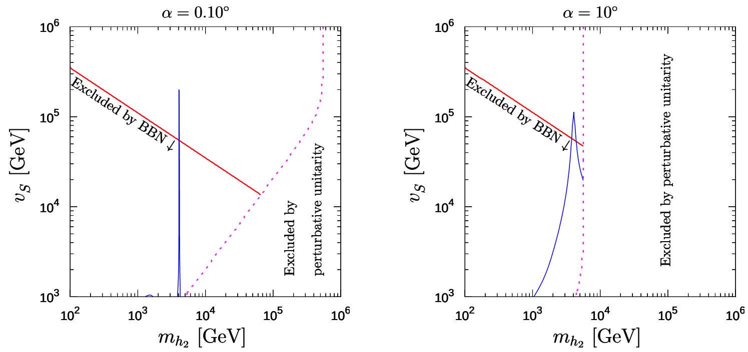


Figure 8: Discovery potential at SKA as a function of $m_{h_2}$ and $v_S$. The lower region of the solid line is excluded by the BBN bound. The solid curve in blue shows the observed DM relic density $\Omega_\chi h^2 = 0.12$, and the narrower region rounded by the curve, $\Omega_\chi h^2 < 0.12$. $\alpha = 0.10°$, and $10°$, the DM mass is fixed to $m_\chi = 2.0$ TeV.

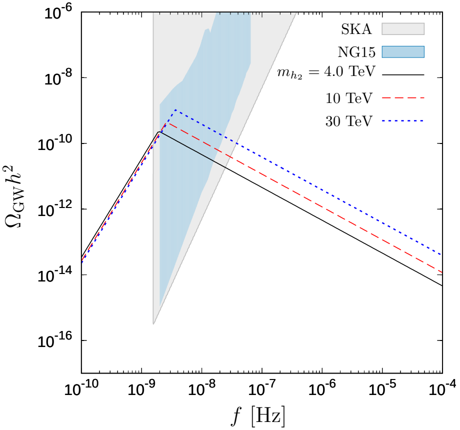


Figure 9: $\Omega_{\rm GW} h^2$ as a function of frequency $f$ with $v_S = 100$ TeV and $\alpha = 0.10^\circ$, DM mass is fixed to $m_\chi = m_{h_2}/2$ in order to satisfy $\Omega_\chi h^2 \leq \Omega_{\rm DM} h^2 = 0.12$. The grey-shaded region represents the SKA sensitivity, while the light-blue region is indicated by the NANOGrav 15-year data.

DW interpretation is not favored since the best-fit low-frequency slope of GWspectrum reported by the NG15 data is $\Omega_{\rm GW} \propto f^{1.2-2.4}$ **PhysRevD.108.123529**, while $\Omega_{\rm GW} \propto f^3$ in our case.

We check some mark point in out alowed region ($m_{h_2} = 4.0, 10, 30$ TeV), that's all those cases lie outside the 95% CL NG15-favored region, they are not ruled out.

Considering DW annihilation can alteration the DM relic density:

- The collapse of DW can generate the entropy given by **physletb.2005.05.022**. But, in our parameters space, the energy density of DW is subdominant.
- DM could be nonthermally produced after collapse of DW (**JCAP01(2013)001**). If DW, annihilate to $h_2$, with our allow region $m_\chi \approxeq m_{h_2}/2$, the $h_2 \to \chi\chi$ produce is suppressed.

- The bias term $|a_1|$ must be greater than $\mathcal{O}(10^{-15})$ GeV$^3$ (BBN bound). Such a small value of a1 results in $\sigma_{\rm SI}^N \propto a_1^2$ being far below thelatest LZ bound.
- With future SKA experiment, we should take 10 TeV $\lesssim v_S \lesssim$ 200 TeV and 1 TeV $\lesssim m_{h_2} \lesssim$ 100 TeV for a relatively small mixing angle $\alpha$, such as $\alpha = 0.1^\circ$.
- Allowed region can be marginally found if $m_\chi \simeq m_{h_2}/2$. If we take $\alpha$ to be larger, the region where $\Omega_\chi h^2 < 0.12$ gets broadened to some extent. However, the upper limit of $m_{h_2}$ becomes smaller owing to the perturbative unitarity constraint, diminishing the parameter space that gives detectable GW signatures.

# References I


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# References II

Saikawa, Kenichi. "A Review of Gravitational Waves from Cosmic Domain Walls". In: *Universe* 3.2 (2017). ISSN: 2218-1997. DOI: 10.3390/universe3020040. URL: https://www.mdpi.com/2218-1997/3/2/40.

# Thank you for your attention!

# Backup slides

# The perturbative unitarity

Quadratic term:

$$V(\Phi, \mathbb{S}) \propto \frac{\lambda}{4}|\Phi|^4 + \frac{\delta_2}{2}|\Phi|^2|\mathbb{S}|^2 + \frac{d_2}{4}|\mathbb{S}|^4$$

with

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} G^+ \\ h + iG^0 \end{pmatrix}, \quad \mathbb{S} = \frac{1}{\sqrt{2}}(s + i\chi),$$

so we have:

$$\begin{aligned} V(G^\pm, G^0, h, S, \chi) \propto & \frac{\lambda}{4}\left(\frac{1}{2}h^2 + \frac{1}{2}(G^0)^2 + G^+G^-\right)^2 \\ & + \frac{\delta_2}{4}\left(S^2 + \chi^2\right)\left(\frac{1}{2}h^2 + \frac{1}{2}(G^0)^2 + G^+G^-\right) + \frac{d_2}{16}\left(S^2 + \chi^2\right)^2 \end{aligned}$$

## The perturbative unitarity

Taking the neutral states of $|G^+G^-\rangle$ , $\frac{1}{\sqrt{2}}|G^0G^0\rangle$ , $\frac{1}{\sqrt{2}}|hh\rangle$ , $\frac{1}{\sqrt{2}}|SS\rangle$ , $\frac{1}{\sqrt{2}}|\chi\chi\rangle$, we have the s-wave matrix:

$$a_0^+ = \frac{1}{16\pi}\begin{pmatrix} \lambda & \frac{\lambda}{2\sqrt{2}} & \frac{\lambda}{2\sqrt{2}} & \frac{\delta_2}{2\sqrt{2}} & \frac{\delta_2}{2\sqrt{2}} \\ \frac{\lambda}{2\sqrt{2}} & \frac{3\lambda}{4} & \frac{\lambda}{4} & \frac{\delta_2}{4} & \frac{\delta_2}{4} \\ \frac{\lambda}{2\sqrt{2}} & \frac{\lambda}{4} & \frac{3\lambda}{4} & \frac{\delta_2}{4} & \frac{\delta_2}{4} \\ \frac{\delta_2}{2\sqrt{2}} & \frac{\delta_2}{4} & \frac{\delta_2}{4} & \frac{3d_2}{4} & \frac{d_2}{4} \\ \frac{\delta_2}{2\sqrt{2}} & \frac{\delta_2}{4} & \frac{\delta_2}{4} & \frac{d_2}{4} & \frac{3d_2}{4} \end{pmatrix},$$

and for and $|hG^0\rangle$ , $|SG^0\rangle$ , $|\chi G^0\rangle$ , $|\chi S\rangle$:

$$a_0^- = \frac{1}{16\pi}\mathrm{diag}\left(\frac{\lambda}{2}, \frac{\delta_2}{2}, \frac{\delta_2}{2}, \frac{d_2}{2}\right).$$

the s-wave matrix for the charged states $|h\pi^\pm\rangle$ , $|\pi^0\pi^\pm\rangle$ , $|S\pi^\pm\rangle$ $|\chi\pi^\pm\rangle$:

$$a_\pm = \frac{1}{16\pi}\mathrm{diag}\left(\frac{\lambda}{2}, \frac{\lambda}{2}, \frac{\delta_2}{2}, \frac{\delta_2}{2}\right)$$

# Dynamics of the DW

The tension force, defined by the tension in a unit area $p_T \sim \sigma / R_{\rm wall}$; The friction force, which appears when the particles interact with the DW. With a DW moving with the velocity $v$, the momentum transfer per collision is $\Delta p \sim Tv$; we can estimate the friction force as:

$$p_F \sim \Delta p n \sim v T^4,$$

When these two forces are balanced, we can obtain the following:

$$v \sim \frac{\sigma}{T^4 R_{\rm wall}} \sim \frac{\sigma t^2}{m_{\rm pl}^2 R_{\rm wall}},$$

we also have $R_{\rm wall} \sim vt$, so:

$$v \sim \frac{\sigma^{1/2} t^{1/2}}{m_{\rm pl}},$$

$$R_{\rm wall} \sim \frac{\sigma^{1/2} t^{3/2}}{m_{\rm pl}}.$$

DW reaches relativity speed as $t \sim m_{\rm pl}^2 \sigma^{-1}$, recently, $R_{\rm wall} \sim t$.

$$\rho_{\rm wall} \sim \frac{\sigma}{t}.$$

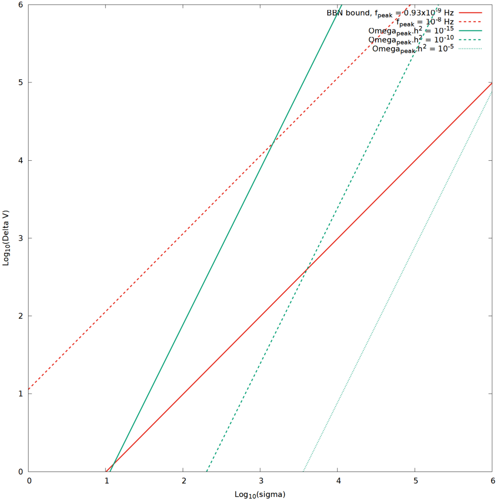
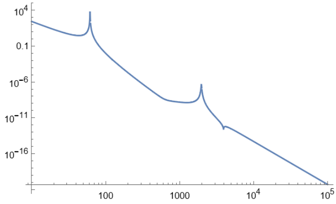


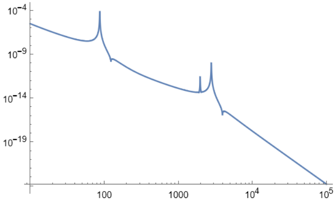

Figure 10: The density spectrum of the GWs $\Omega_{\mathrm{gw,peak}} h^2$

$$(\sigma v)_{\chi\chi\to h_{i,j}\to h_i h_j} = \mathcal{S}\frac{\beta_{h_{ij}}}{8\pi s}\left|\frac{\lambda_{\chi\chi h_1}\lambda_{h_1 h_i h_j}}{s - m_{h_1}^2 + i m_{h_1}\Gamma_{h_1}} + \frac{\lambda_{\chi\chi h_2}\lambda_{h_2 h_i h_j}}{s - m_{h_2}^2 + i m_{h_2}\Gamma_{h_2}}\right|^2,$$

$$(\sigma v)_{\chi\chi\to\chi\to h_i h_j} = \mathcal{S}\frac{\beta_{h_{ij}}}{\pi s}\frac{\lambda_{\chi\chi h_i}^2\lambda_{\chi\chi h_j}^2}{\left(s - m_{h_i}^2 - m_{h_j}^2\right)^2},$$




(a) $\chi\chi \to h_{i,j} \to h_i h_j$




(b) $\chi\chi \to \chi \to h_1 h_j$

Figure 11: Cross section of the DM annihilation in some domination processes

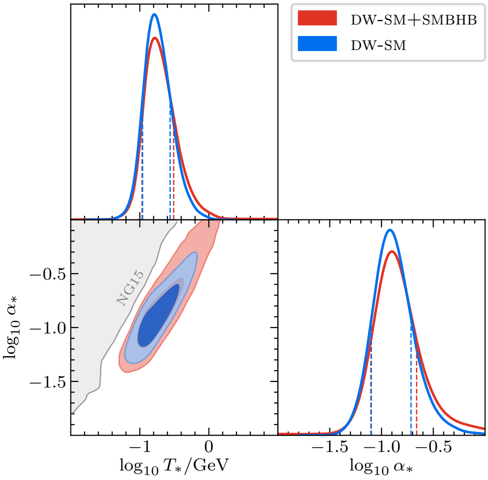



Figure 12: favored region and exeption region from Nanograv15