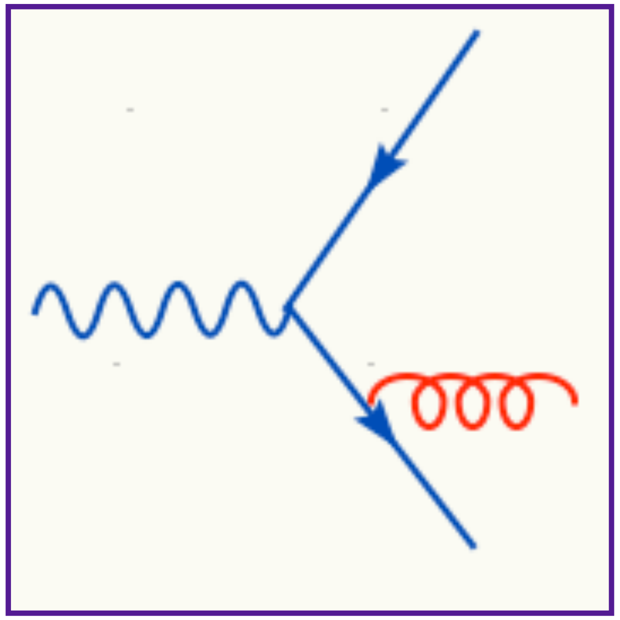
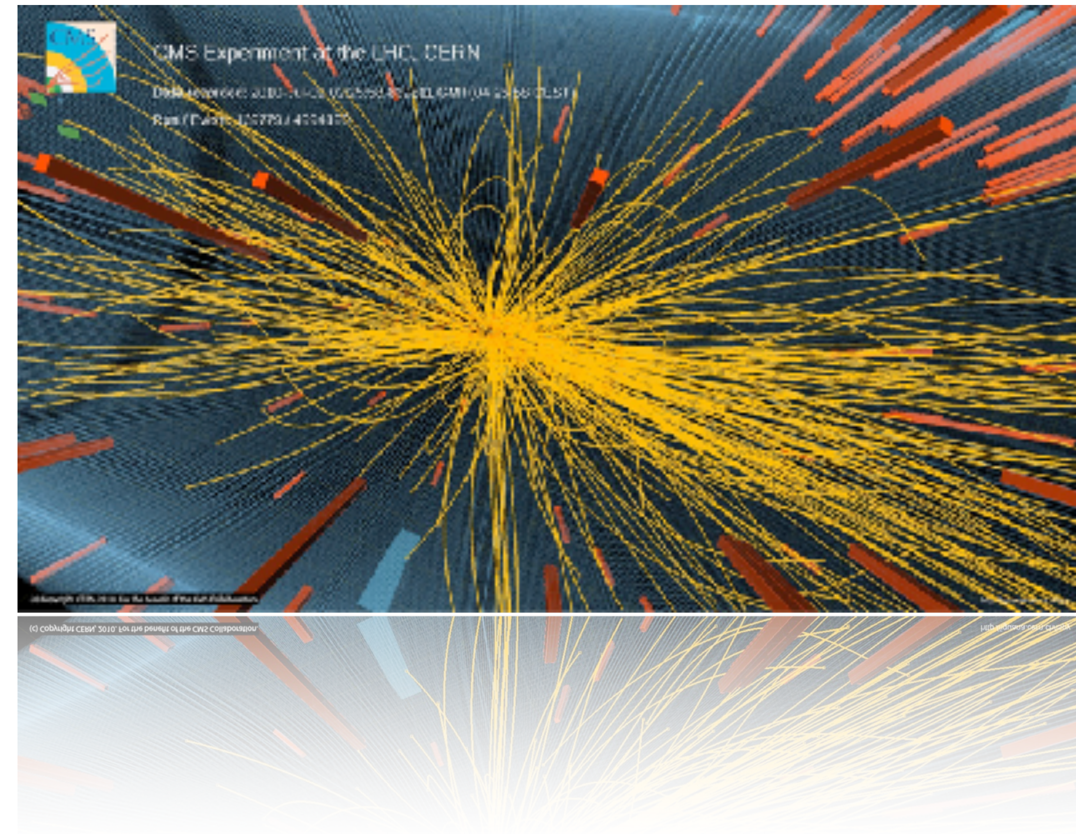
Parton shower and hadronisation

High multiplicity



?



MadGraph5_aMC@NLO
can generate only a couple
of final state particles

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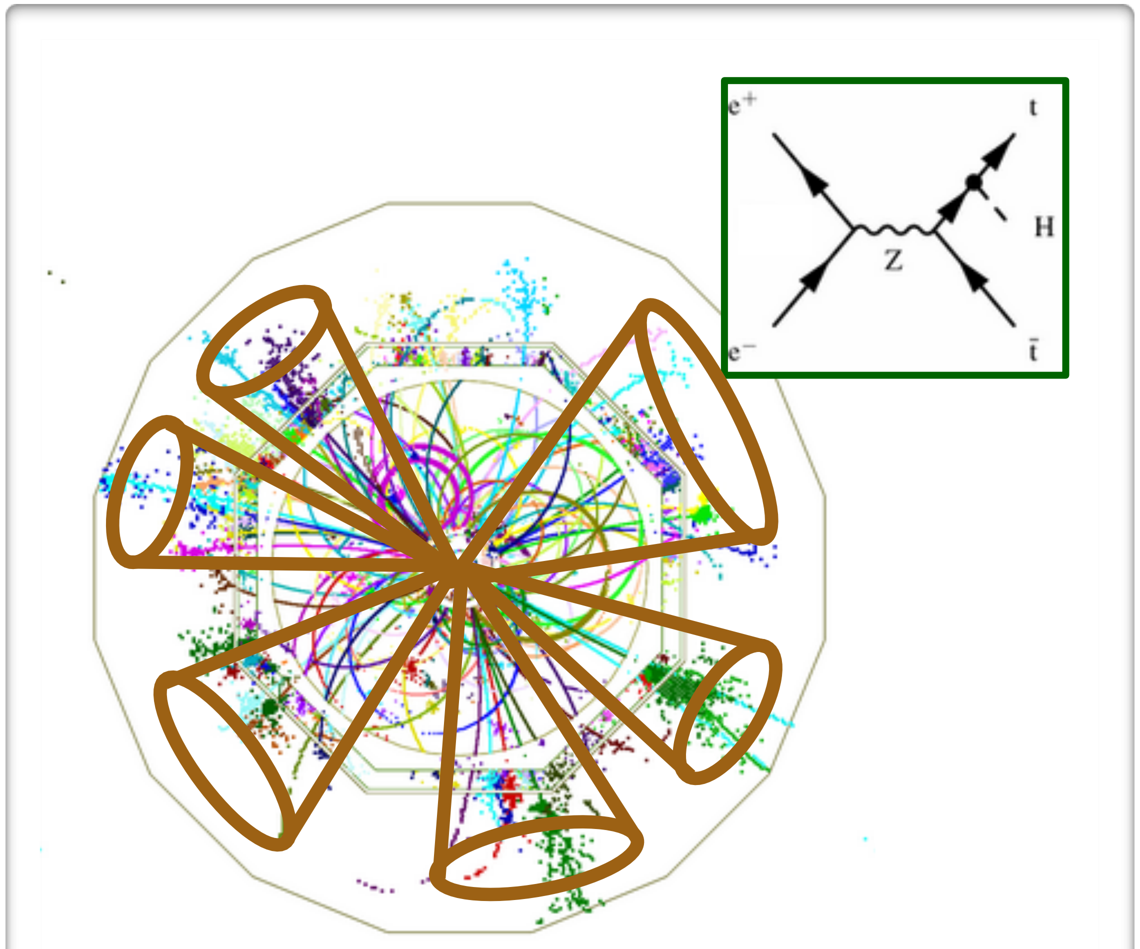
Two options

- Reduce the number of particles in the detected events: jet clustering
- Increase the number of particle in the simulated events: parton shower

Jet clustering



$e^+e^- \rightarrow t\bar{t}h$ [$t\bar{t} \rightarrow 6q$, $h \rightarrow b\bar{b}$]
@ 1000 GeV

- Goal:
  - Cluster particles "that are close in phase-space" into single objects: jets
  - These jets correspond to the quarks (or gluons) generated by MadGraph5_aMC@NLO

Jet clustering

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2} ,$$
$$d_{iB} = k_{ti}^{2p} ,$$

- Sequential algorithm
  - Define all the distances
    - d_{ij} (between particles i and j) and
    - d_{iB} (between particle i and the beam)
  - If d_{ij} is the smallest, replace particles i and j by a new (pseudo) particle
  - If d_{iB} is the smallest, call particle i a jet and remove it from the list
  - Keep going until no particles are left

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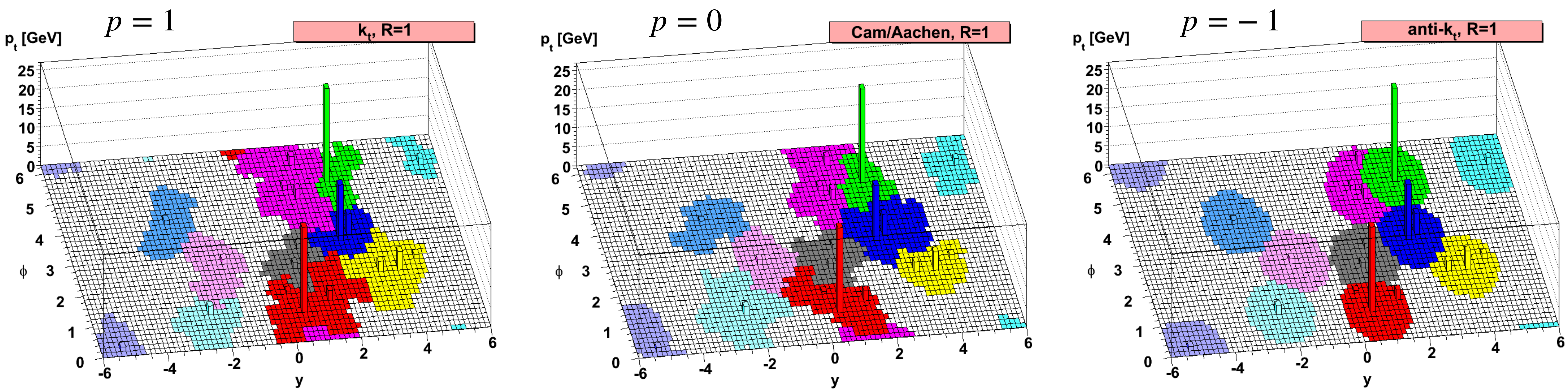
Jet clustering

- Different clustering algorithms exist
- Same event cluster with different algorithms gives slightly different jet(s) and shapes

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2} ,$$

$$d_{iB} = k_{ti}^{2p} ,$$



- Anti-k_T ($p = -1$) algorithm most popular at the LHC
- All implemented in the fastjet package (https://fastjet.fr)

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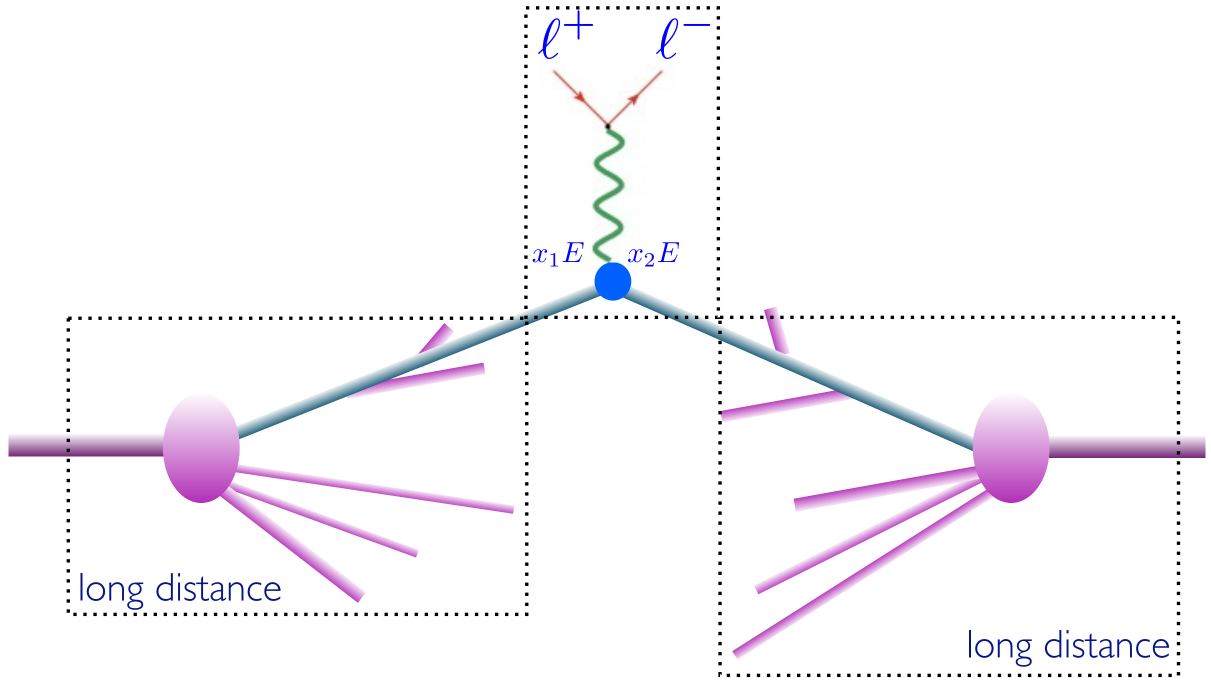
Comments on jet clustering

- The correspondence between quarks/gluons and jets works well
- However:
- The jets that come out of the jet algorithm can be arbitrarily soft (i.e., with a very small energy or transverse momentum)
- For them to correspond to quarks/gluons computed by MadGraph5_aMC@NLO, they need to be "hard"
- Only consider jets above a threshold
- But this is somewhat arbitrary…
- How hard does "hard" need to be to be fine?
- No general rule here… depends on the rest of the event!
- In practice, in your calculation you get a large logarithms that hamper the convergence of perturbation theory

(in the expansion of the strong coupling, each order is larger than the previous)

QCD radiation

- In matrix element calculations in perturbation theory
  - "initial state QCD radiation" is included inclusively ("resummed") in the PDFs (and through strong coupling definition) and
  - "final state QCD radiation" is included through the parton-jet duality (and through strong coupling definition)
- Hence… all is already there! What to do…?
  - "Undo" this resummation and make it explicit



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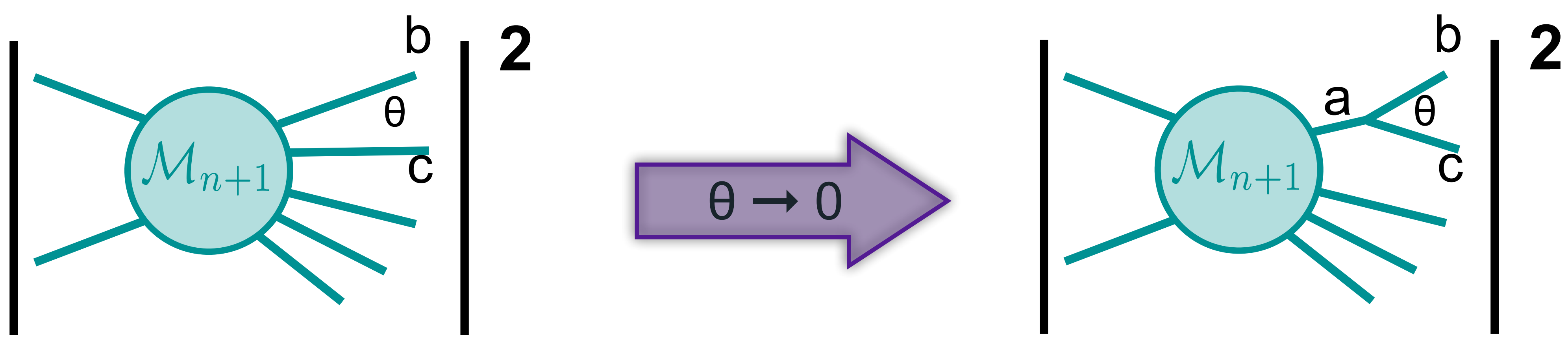
Why do it

- Effects are already included (resummed) in fixed-order perturbation theory
- for many *inclusive* observables do not change much with the shower
- but one would miss an extremely rich variety of observables which may play important roles in experimental analyses.
- When there are large scale differences entering the observables, fixed-order perturbation theory breaks down!
- this does NOT mean that observable is useless/unimportant: it is just that one is not using the right tools to describe it.
- It is better to try and find a way to reorganise the computation in order to take into account emissions close to the singular regions of the phase space, to all orders in perturbation theory.
- "We want to simulate the collisions, hence we want to simulate also the creation of the hadrons, for which we need parton showering"

Collinear factorisation



- A process for which two particles are separated by a small angle θ

- In the limit of θ → 0, the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess

- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability

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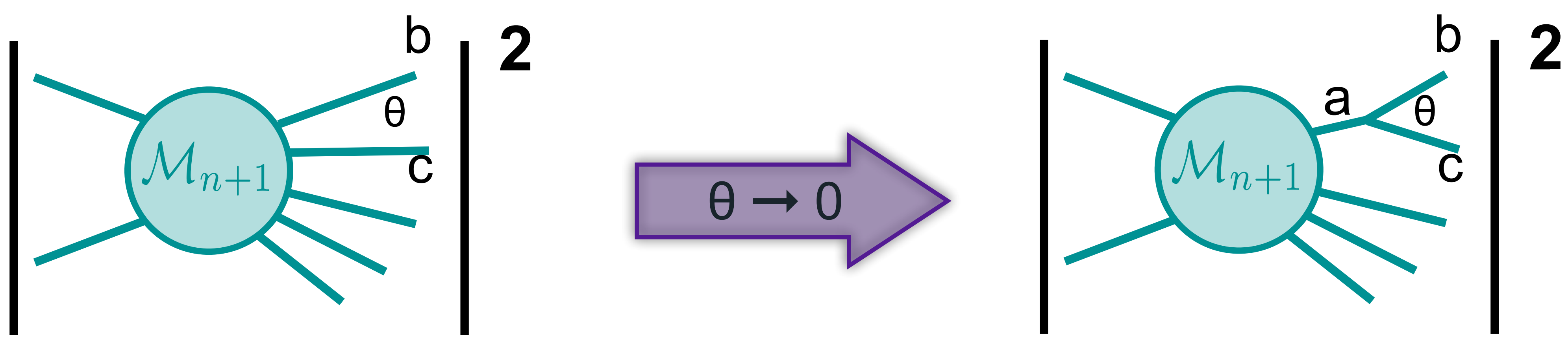
Collinear factorisation



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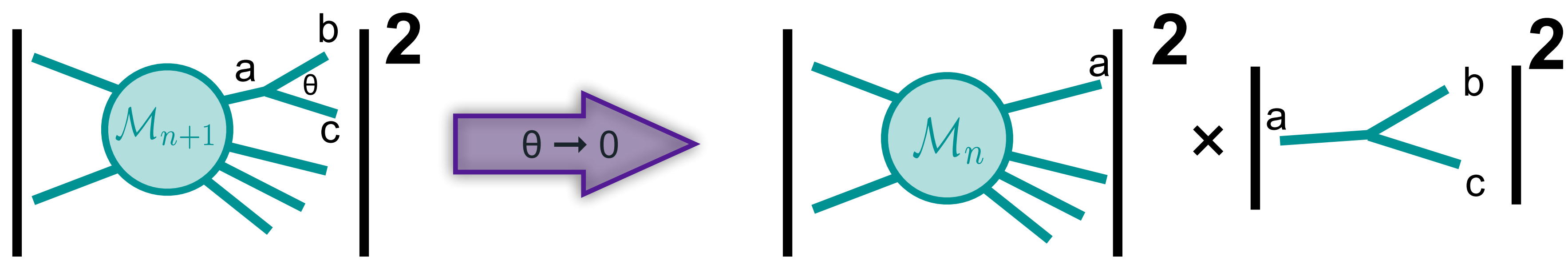
Collinear factorisation



• The process factorises in the collinear limit. This procedure is universal

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

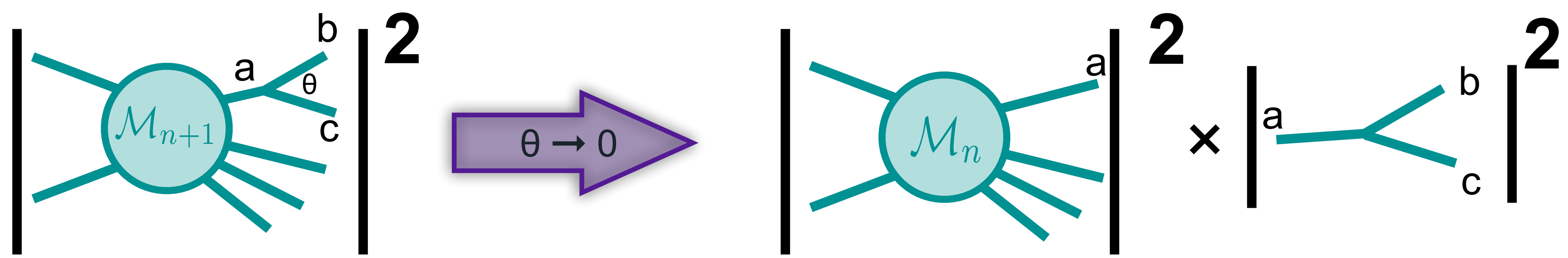
• Notice that what has been roughly called 'branching fraction' is actually a singular factor, so one will need to make sense of this definition.

• At the leading contribution to the (n+1)-body cross section the DGLAP splitting kernels are defined as:

$$P_{g\to qq}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{g\to gg}(z) = C_A\left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right],$$

$$P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{1-z}\right], \qquad P_{q\to gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right].$$

Collinear factorisation



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$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- t can be called the 'evolution variable': it can be the virtuality m^2 of particle a, or its p_T^2, or $E^2\theta^2$ …
  - It represents the hardness of the branching and tends to 0 in the collinear limit.

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$

$$p_T^2 \simeq z m^2$$

- Indeed in the collinear limit one has: so that the factorisation takes place for all these definitions:

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

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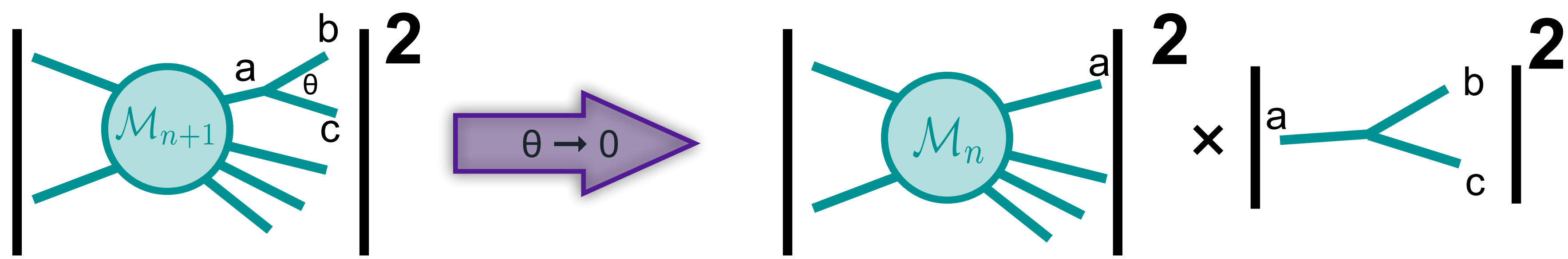
Collinear factorisation



- The process factorises in the collinear limit. This procedure is universal

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- z is defined to be the energy fraction taken by parton b from parton a
- It represents the energy sharing between b and c and tends to 1 in the soft limit (parton c going soft)
- ϕ is the azimuthal angle. It can be chosen to be the angle between the polarisation of a and the plane of the branching

$$P_{g\to qq}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{g\to gg}(z) = C_A\left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right],$$

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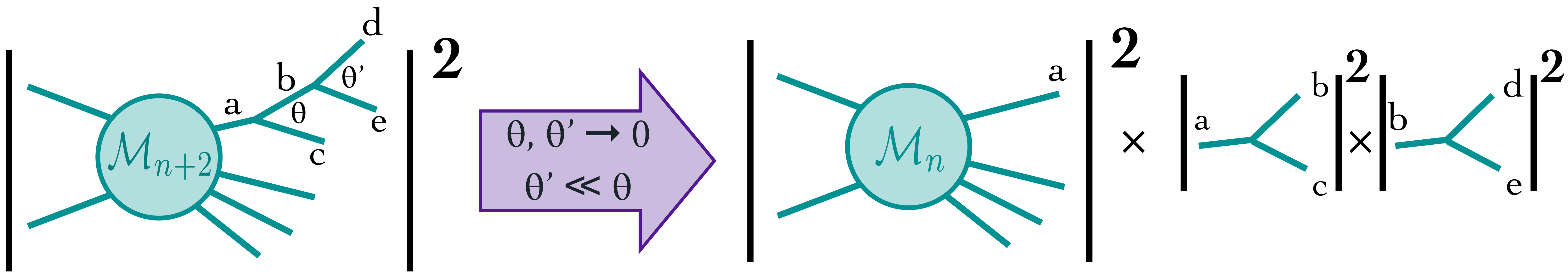
Multiple emission



• Now consider $\mathcal{M}_{n+2}$ as the new process and use the same recipe we used for the first emission and add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) \times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b\to de}(z')$$

• This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a 'Markov chain'.

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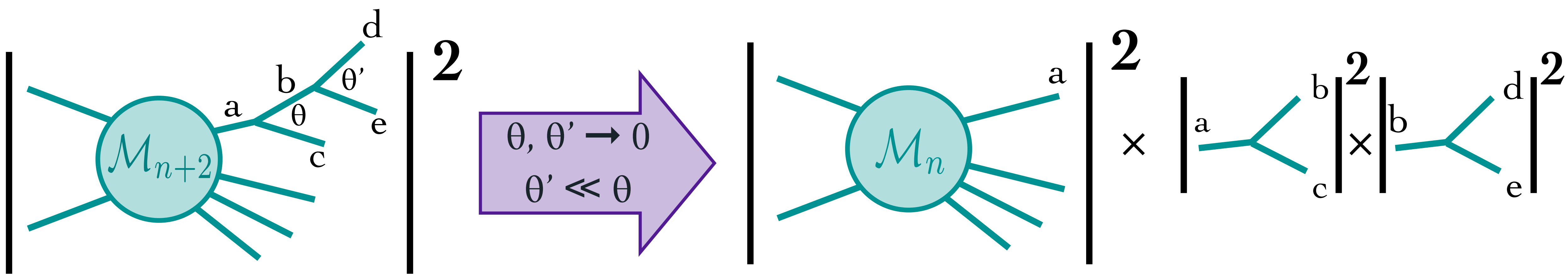
Multiple emission



• The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: θ ≫ θ' ≫ θ''...
The rate for multiple emission is

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \ldots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.
• Each power of α_S comes with a logarithm. The logarithm can easily be large, and therefore we see a breakdown of perturbation theory

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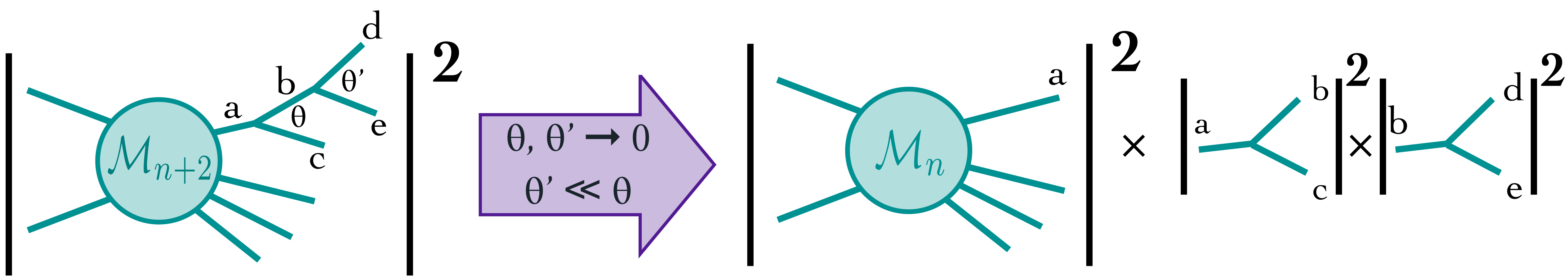
Approximation



- We have an approximation of the matrix elements for multiple emissions
- We know that the $|M_n|^2$ is *inclusive* over all radiation (due to parton/jet duality and PDF evolution)
  - However, in our approximation we multiply $|M_n|^2$ by k "branching fractions" to get $|M_{n+k}|^2$. These branching fractions are actually singular factors
  - How to make sense of this? How to enforce that summing over all branching fractions adds up to one?

We are missing the no emission contributions

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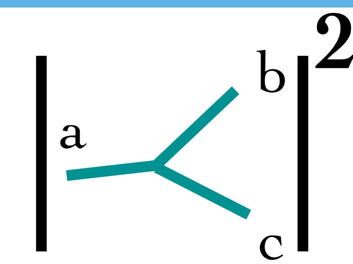
No-/emission probability



- The probability for the branching a $\longrightarrow$ bc between scales t and t+dt is equal to

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- The probability that a parton does NOT split between the scales t and t+dt is given by 1-dp(t)
- Probability that particle a does not emit between scales Q^2 and t

$$\Delta(Q^2,t) = \prod_k \left[1 - \sum_{bc} \frac{dt_k}{t_k} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] =$$

$$\exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] = \exp\left[-\int_t^{Q^2} dp(t')\right]$$

$\Delta(Q^2,t)$ is the Sudakov form factor

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Sudakov factor

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
  *Initial state shower also requires PDF contributions
- This no-emission probability needs to be included to interpret the branchings as probabilities that add up to 1
- Define dP_k as the probability for k ordered splittings from leg a at given scales

$$
\begin{aligned}
dP_1(t_1) &= \Delta(Q^2, t_1)\ dp(t_1)\Delta(t_1, Q_0^2), \\
dP_2(t_1, t_2) &= \Delta(Q^2, t_1)\ dp(t_1)\ \Delta(t_1, t_2)\ dp(t_2)\ \Delta(t_2, Q_0^2)\Theta(t_1 - t_2), \\
\ldots &= \ldots \\
dP_k(t_1, \ldots, t_k) &= \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)
\end{aligned}
$$

- Q_0^2 is the hadronisation scale (~1 GeV_2). Below this scale we do not trust the perturbative description for parton splitting anymore
- This is what is implemented in a parton shower, taking the scales for the splitting t_i randomly but weighted according to the no-emission probability

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly check this by integrating the probability for k splittings

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

- Summing over all number of emissions

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1$$

- Hence, the total probability is conserved

Physics interpretation

- We should **not** interpret the parton shower to be generating an approximation of the $|M_{n+k}|^2$ matrix elements
  - Rather it is an approximation of N^kLO computation of $|M_n|^2$
  - That is, including the (real-)emission contributions, but also virtual (no-)emission corrections

ISR (Initial state radiation)

- To simulate parton radiation from the initial state, we start with the hard scattering, and then “devolve” the DGLAP evolution to get back to the original hadron: backwards evolution!
  - i.e. we undo the analytic resummation and replace it with explicit partons (e.g. in Drell-Yan this gives non-zero p_T to the vector boson)
- In backwards evolution, the Sudakovs include also the PDFs -- this follows from the DGLAP equation and ensures conservation of probability:

$$\Delta_{Ii}(x,t_1,t_2) = \exp\left\{ -\int_{t_1}^{t_2} dt' \sum_j \int_x^1 \frac{dx'}{x'} \frac{\alpha_s(t')}{2\pi} P_{ij}\left(\frac{x}{x'}\right) \frac{f_i(x',t')}{f_j(x,t')} \right\}$$

  This represents the probability that parton i will stay at the same x (no splittings) when evolving from t_1 to t_2.

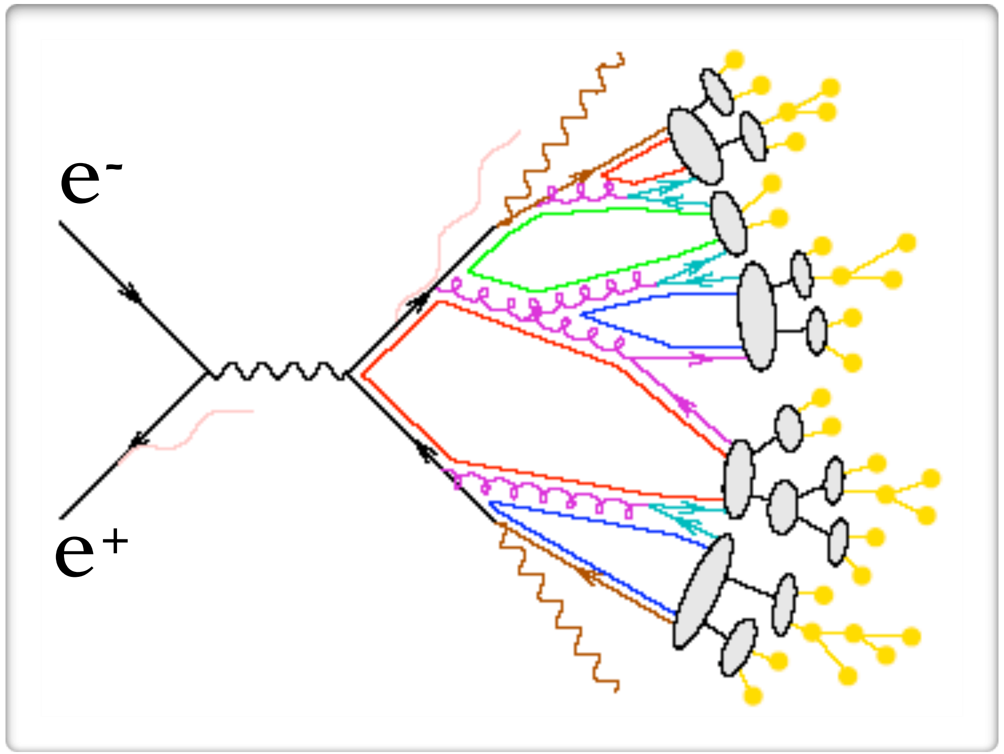
- The shower simulation is now done as in a final state shower

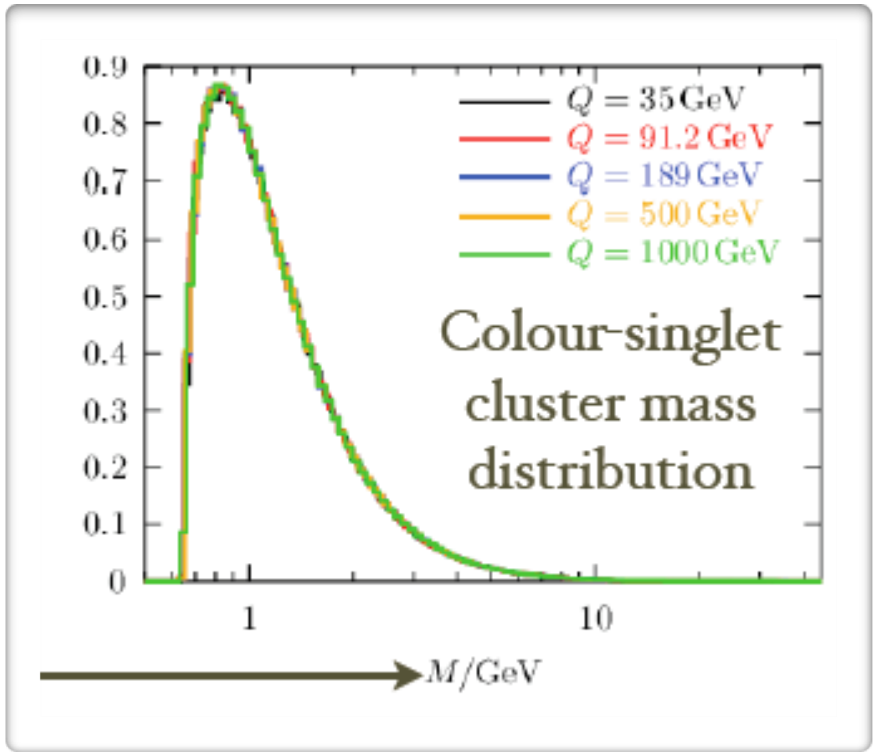
Hadronisation

- The shower stops if all partons are characterised by a scale at the IR cut-off: $Q_0 \sim 1$ GeV
- Physically, we observe hadrons, not (coloured) partons
- We need a non-perturbative model in passing from partons to colourless hadrons
- There are two models, based on physical and phenomenological considerations

The cluster model

The structure of the perturbative evolution including angular ordering, leads naturally to the clustering in phase-space of colour-singlet parton pairs (pre-confinement). Long-range correlations are strongly suppressed. Hadronisation will only act locally, on low-mass colour singlet clusters.





Lund string model

From lattice QCD one sees that the colour confinement potential of a quark-antiquark grows linearly with their distance: $V(r) \sim kr$, with $k \sim 0.2$ GeV, This is modelled with a string with uniform tension (energy per unit length) k that gets stretched between the $q\bar{q}$ pair.

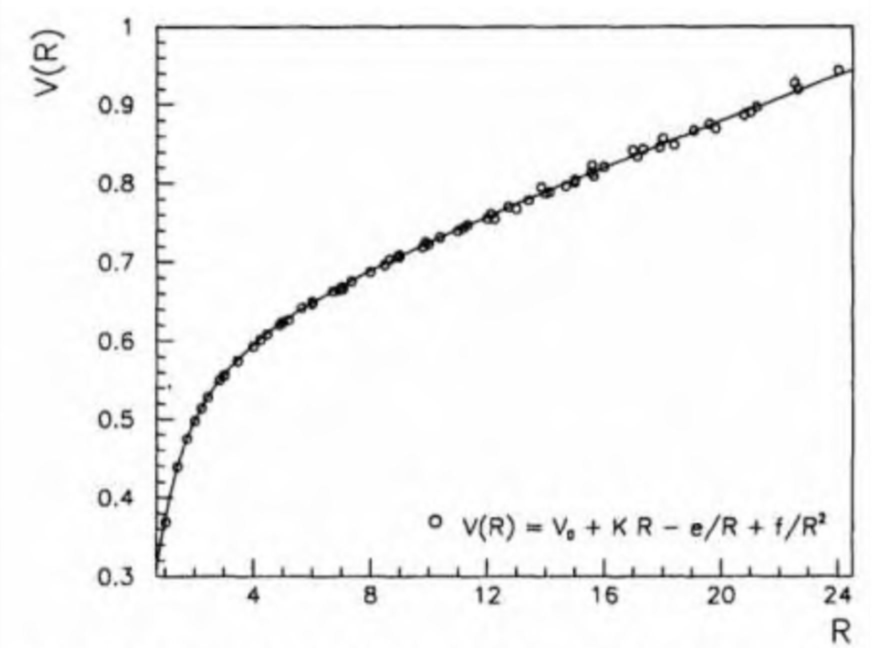
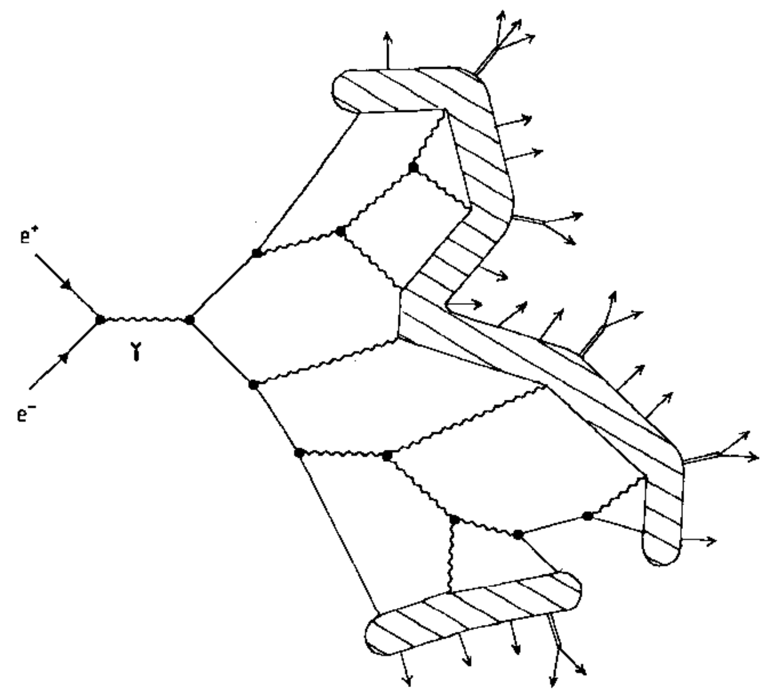
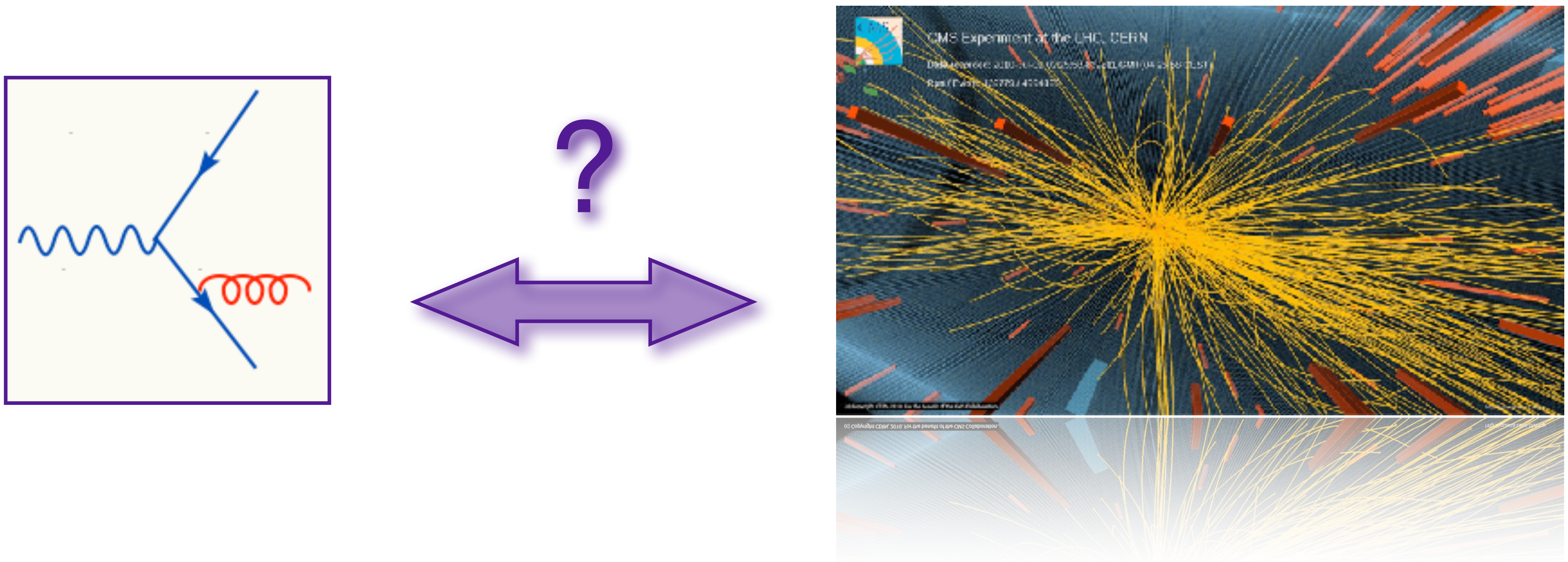


Fig. 2.9. QCD potential *vs.* *R* (in lattice units) from lattice QCD. Figure from ref. [23].



When quark-antiquarks are too far apart, it becomes energetically more favourable to break the string by creating a new $q\bar{q}$ pair in the middle.

Exclusive observables



A parton shower program associates one of the possible histories (and pre-histories in case of pp collisions) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

Pythia, Sherpa, Herwig

- Significant differences between shower implementations (choice of evolution variable and kernel, momentum mappings, phase-space boundaries, massive quarks, photon emissions, etc.)
- All are tuned to data, and describe it reasonably well (typically better than expected from their formal accuracy)
- Some are (formally) more correct than others
  - However, not easy to assess accuracy for a general observable
  - Assessment (and improvement!) of formal accuracy is an active field of research

- General-purpose tools
- Always the first experimental choice
- Reliable and well-tuned tools
- New development in progress

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Ex summary

- ($pp \to t\bar{t}j, pp \to \mu^+\mu^- j$), how large, from which subprocess(es)
- IR divergence (reduce cuts (pt, ΔR) on j)
- Compute $pp \to t\bar{t}$ at NLO, is the cross-section within uncertainties, what about the shape (FO or with shower)
- Same for $pp \to WZ, \ldots$
- change the renormalisation and factorisation scale

at fixed order

Learn the syntax:

- `> tutorial NLO`

Generate the code for $t\bar{t}$ production at NLO

- `> generate p p > t t~ [QCD]`

- Select the analysis `analysis_HwU_pp_ttx` in the `FO_analyse_card` to generate histograms
  - `> launch my_ttbar_nlo`

```
The following switches determine which operations are executed:
 1 Perturbative order of the calculation:                              order=NLO
 2 Fixed order (no event generation and no MC@[N]LO matching):   fixed_order=ON
 3 Shower the generated events:                                       shower=OFF
 4 Decay particles with the MadSpin module:                          madspin=OFF
  Either type the switch number (1 to 4) to change its default setting,
  or set any switch explicitly (e.g. type 'order=LO' at the prompt)
  Type '0', 'auto', 'done' or just press enter when you are done.
 [0, 1, 2, 3, 4, auto, done, order=LO, order=NLO, ... ][60s to answer]
>
INFO: will run in mode: NLO
Do you want to edit a card (press enter to bypass editing)?
  1 / param      : param_card.dat
  2 / run        : run_card.dat
  3 / FO_analyse : FO_analyse_card.dat
 you can also
   - enter the path to a valid card or banner.
   - use the 'set' command to modify a parameter directly.
     The set option works only for param_card and run_card.
```

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at fixed order

```
#####################################################################
#
# This file contains the settings for analyses to be linked to aMC@NLO
# fixed order runs. Analyse files are meant to be put (or linked)
# inside <PROCDIR>/FixedOrderAnalysis/ (<PROCDIR> is the name of the
# exported process directory). See the
# <PROCDIR>/FixedOrderAnalysis/analysis_template.f file for details on
# how to write your own analysis.
#
#####################################################################
#
# Analysis format. Can either be 'topdrawer', 'root', 'HwU' or 'none'.
# When choosing HwU, it comes with a GnuPlot wrapper. When choosing
# topdrawer, the histogramming package 'dbook.f' is included in the
# code, while when choosing root the 'rbook_fe8.f' and 'rbook_be8.cc'
# are included. If 'none' is chosen, all the other entries below have
# to be set empty.
FO_ANALYSIS_FORMAT = HwU
#
# Needed extra-libraries (FastJet is already linked):
FO_EXTRALIBS =
#
# (Absolute) path to the extra libraries. Directory names should be
# separated by white spaces.
FO_EXTRAPATHS =
#
# (Absolute) path to the dirs containing header files needed by the
# libraries (e.g. C++ header files):
FO_INCLUDEPATHS =
#
# User's analysis (to be put in the <PROCDIR>/FixedOrderAnalysis/
# directory). Please use .o as extension and white spaces to separate
# files.
FO_ANALYSE = analysis_HwU_pp_ttx.o
#
#
## When linking with root, the following settings are a working
## example on lxplus (CERN). When using this, comment out the lines
## above and replace <PATH_TO_ROOT> with the physical path to root,
## e.g. /afs/cern.ch/sw/lcg/app/releases/ROOT/5.34.11/x86_64-slc6-gcc46-dbg/root/
#FO_ANALYSIS_FORMAT = root
#FO_EXTRALIBS = Core Cint Hist Matrix MathCore RIO dl Thread
#FO_EXTRAPATHS = <PATH_TO_ROOT>/lib
#FO_INCLUDEPATHS = <PATH_TO_ROOT>/include
#FO_ANALYSE = analysis_root_template.o
```

NLO and LO

```
INFO: ************************************************************
*                                                          *
*          W E L C O M E  to  M A D G R A P H 5            *
*                      a M C @ N L O                       *
*                                                          *
*                *                       *                 *
*                  *        * *        *                   *
*                    * * * * 5 * * * *                     *
*                  *        * *        *                   *
*                *                       *                 *
*                                                          *
*         VERSION 2.2.1                2014-09-25          *
*                                                          *
*    The MadGraph5_aMC@NLO Development Team - Find us at   *
*                 http://amcatnlo.cern.ch                  *
*                                                          *
*               Type 'help' for in-line help.              *
*                                                          *
************************************************************
launch auto
The following switches determine which operations are executed:
 1 Perturbative order of the calculation:                              order=NLO
 2 Fixed order (no event generation and no MC@[N]LO matching):   fixed_order=OFF
 3 Shower the generated events:                                       shower=ON
 4 Decay particles with the MadSpin module:                          madspin=OFF
  Either type the switch number (1 to 4) to change its default setting,
  or set any switch explicitly (e.g. type 'order=LO' at the prompt)
  Type '0', 'auto', 'done' or just press enter when you are done.
 [0, 1, 2, 3, 4, auto, done, order=LO, order=NLO, ... ][60s to answer]
> fixed_order=ON
> order=LO (for LO run)
```

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Results

```
INFO:
      Final results and run summary:
      Process p p > t t~ [QCD]
      Run at p-p collider (6500 + 6500 GeV)
      Total cross-section:      6.871e+02 +- 5.9e+00 pb
      Ren. and fac. scale uncertainty: +9.7% -11.7%

INFO: The results of this run and the HwU and GnuPlot
files with the plots have been saved in /Users/marcozaro/
Physics/MadGraph/2.2.3new/my_tt_nlo_qcd/Events/run_01
```

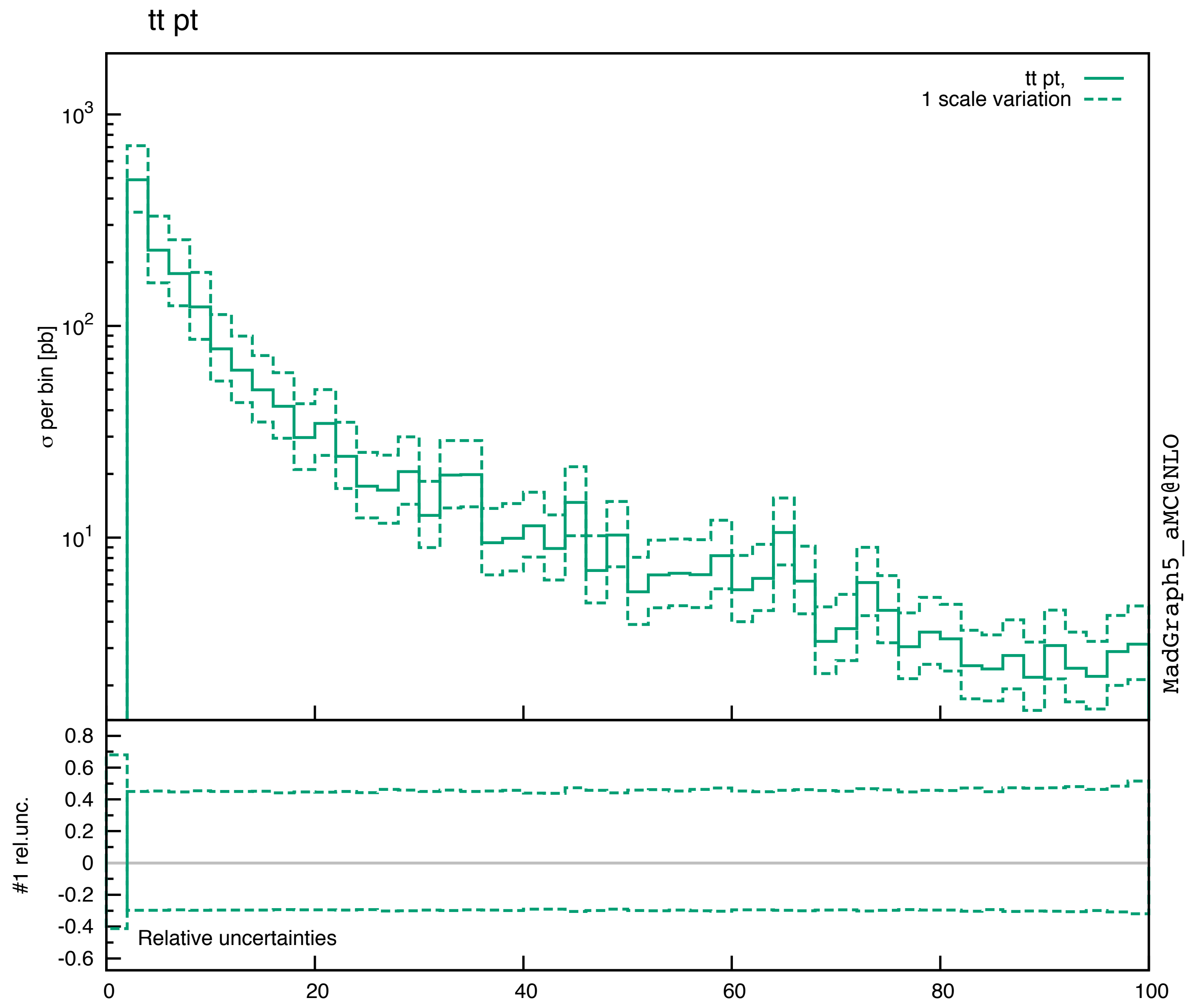
```
INFO:
      Final results and run summary:
      Process p p > t t~ [QCD]
      Run at p-p collider (6500 + 6500 GeV)
      Total cross-section:      4.622e+02 +- 2.2e+00 pb
      Ren. and fac. scale uncertainty: +29.8% -22.3%

INFO: The results of this run and the HwU and GnuPlot
files with the plots have been saved in /Users/marcozaro/
Physics/MadGraph/2.2.3new/my_tt_nlo_qcd/Events/run_02_LO
```

More results

- The HwU (Histogram with Uncertainties) format

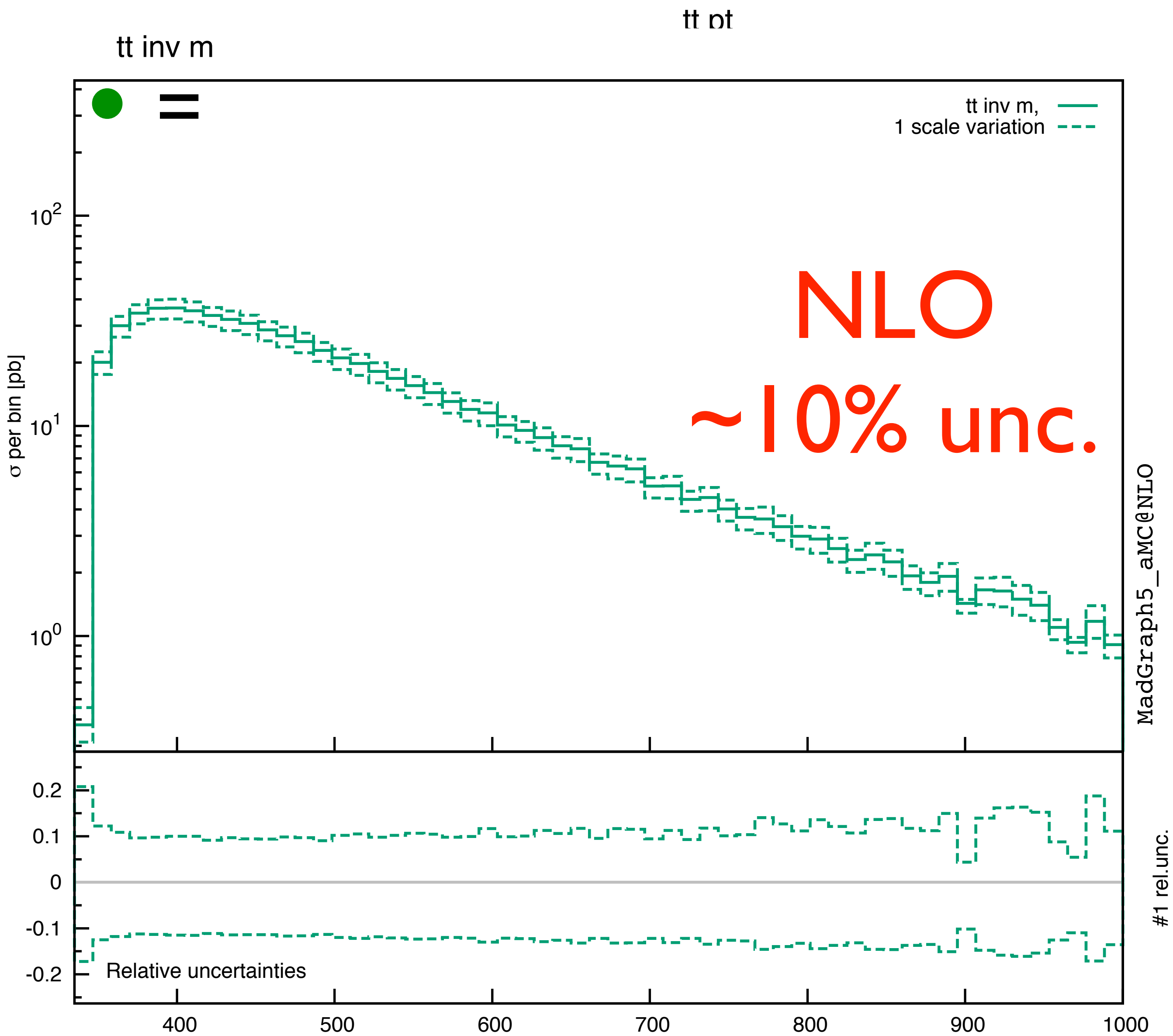
```
 ##& xmin & xmax & central value & dy & delta_mu_min @aux & delta_mu_max @aux & muR=1.00 muF=1.00 & muR=1.00 muF=2.00 &
muR=1.00 muF=0.50 & muR=2.00 muF=1.00 & muR=2.00 muF=2.00 & muR=2.00 muF=0.50 & muR=0.50 muF=1.00 & muR=0.50 muF=2.00 &
muR=0.50 muF=0.50

<histogram> 50 "tt pt |X_AXIS@LIN |Y_AXIS@LOG"
  +0.0000000e+00   +2.0000000e+00   -1.0242367e+03   +2.5047252e+01  -1.7206530e+03   -6.0160203e+02   -1.0242367e+03
-9.0715087e+02   -1.1432407e+03   -6.8421704e+02   -6.0160203e+02   -7.6882229e+02   -1.5496422e+03   -1.3802509e+03
-1.7206530e+03
  +2.0000000e+00   +4.0000000e+00   +4.9088904e+02   +2.0297264e+01  +3.4493531e+02   +7.1188196e+02   +4.9088904e+02
+4.5019210e+02   +5.3086979e+02   +3.7613186e+02   +3.4493531e+02   +4.0679297e+02   +6.5832080e+02   +6.0377117e+02
+7.1188196e+02
  +4.0000000e+00   +6.0000000e+00   +2.2787754e+02   +2.3122314e+01  +1.5999659e+02   +3.3086836e+02   +2.2787754e+02
+2.0857157e+02   +2.4714205e+02   +1.7482611e+02   +1.5999659e+02   +1.8963760e+02   +3.0513912e+02   +2.7932554e+02
+3.3086836e+02
  +6.0000000e+00   +8.0000000e+00   +1.7671803e+02   +9.5392210e+00  +1.2453269e+02   +2.5575724e+02   +1.7671803e+02
+1.6227348e+02   +1.9111959e+02   +1.3562893e+02   +1.2453269e+02   +1.4669918e+02   +2.3651862e+02   +2.1720764e+02
+2.5575724e+02
  +8.0000000e+00   +1.0000000e+01   +1.2311654e+02   +7.1903869e+00  +8.6399100e+01   +1.7898773e+02   +1.2311654e+02
+1.1261446e+02   +1.3369767e+02   +9.4461506e+01   +8.6399100e+01   +1.0258866e+02   +1.6483914e+02   +1.5078780e+02
+1.7898773e+02
  +1.0000000e+01   +1.2000000e+01   +7.8022445e+01   +1.0748137e+01  +5.4873577e+01   +1.1315020e+02   +7.8022445e+01
+7.1570742e+01   +8.4452355e+01   +5.9823787e+01   +5.4873577e+01   +6.4760050e+01   +1.0454718e+02   +9.5909144e+01
+1.1315020e+02
  +1.2000000e+01   +1.4000000e+01   +6.1770611e+01   +3.2903213e+00  +4.3437593e+01   +8.9537046e+01   +6.1770611e+01
```

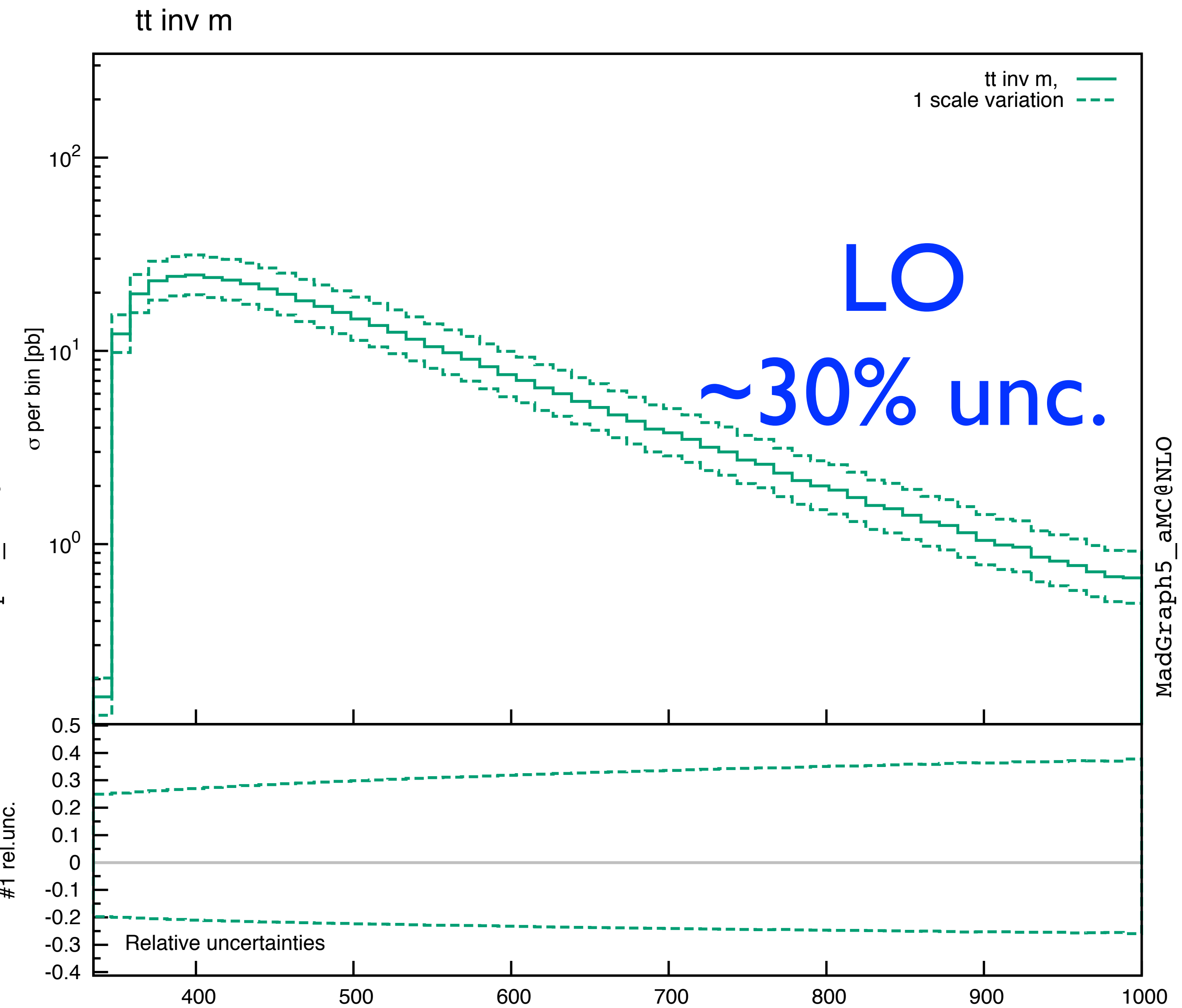
Distribution



C. Degrande

Distribution





C. Degrande

Example (LO and NLO)

LO

- P_T of the full final state (P_T(tt) in $pp \to t\bar{t}$, P_T(Z) in $pp \to Z$, …) by momentum conservation P_T(jets)
- Some angle ΔR(tt) in $pp \to t\bar{t}$
- P_T of the partial final state (P_T(t) in $pp \to t\bar{t}$, $P_T(\mu)$ in $pp > \mu\mu$)
- Invariant mass of subsystem $m(\mu\mu)$ in $pp > \mu\mu$

The OPP

Project the numerator on a basis of propagator denominator

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} [d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)] \prod_{i \neq i_0, i_1, i_2, i_3}^{N_t-1} D_i \\
& + \sum_{i_0<i_1<i_2}^{N_t-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{N_t-1} D_i \\
& + \sum_{i_0<i_1}^{m-1} [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] \prod_{i \neq i_0, i_1}^{N_t-1} D_i \\
& + \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i \neq i_0}^{N_t-1} D_i \\
& + \tilde{P}(q) \prod_{i}^{N_t-1} D_i .
\end{aligned}
$$

$$
\bar{D}_i^{(t)} = [\bar{q} + p_i^{(t)}]^2 - m_i^{(t)2}
$$

Quadruple cuts : complex momenta for which $D_{i=i_0,i_1,i_2,i_3} = 0$

(4 loop propagators on-shell)

$$
\begin{aligned}
N(q^{\pm}) & = [d(0123) + \tilde{d}(q^{\pm}; 0123)] \prod_{i \neq 0,1,2,3}^{N_t-1} D_i(q^{\pm}) \\
& = [d(0123) + \tilde{d}(0123) \epsilon_{\mu\nu\rho\sigma} q^{\pm,\mu}_{i_0 i_1 i_2 i_3} p_1^{\nu} p_2^{\rho} p_3^{\sigma}] \prod_{i \neq 0,1,2,3}^{N_t-1} D_i(q^{\pm})
\end{aligned}
$$

invert and so on

C. Degrande