

# Loop computation

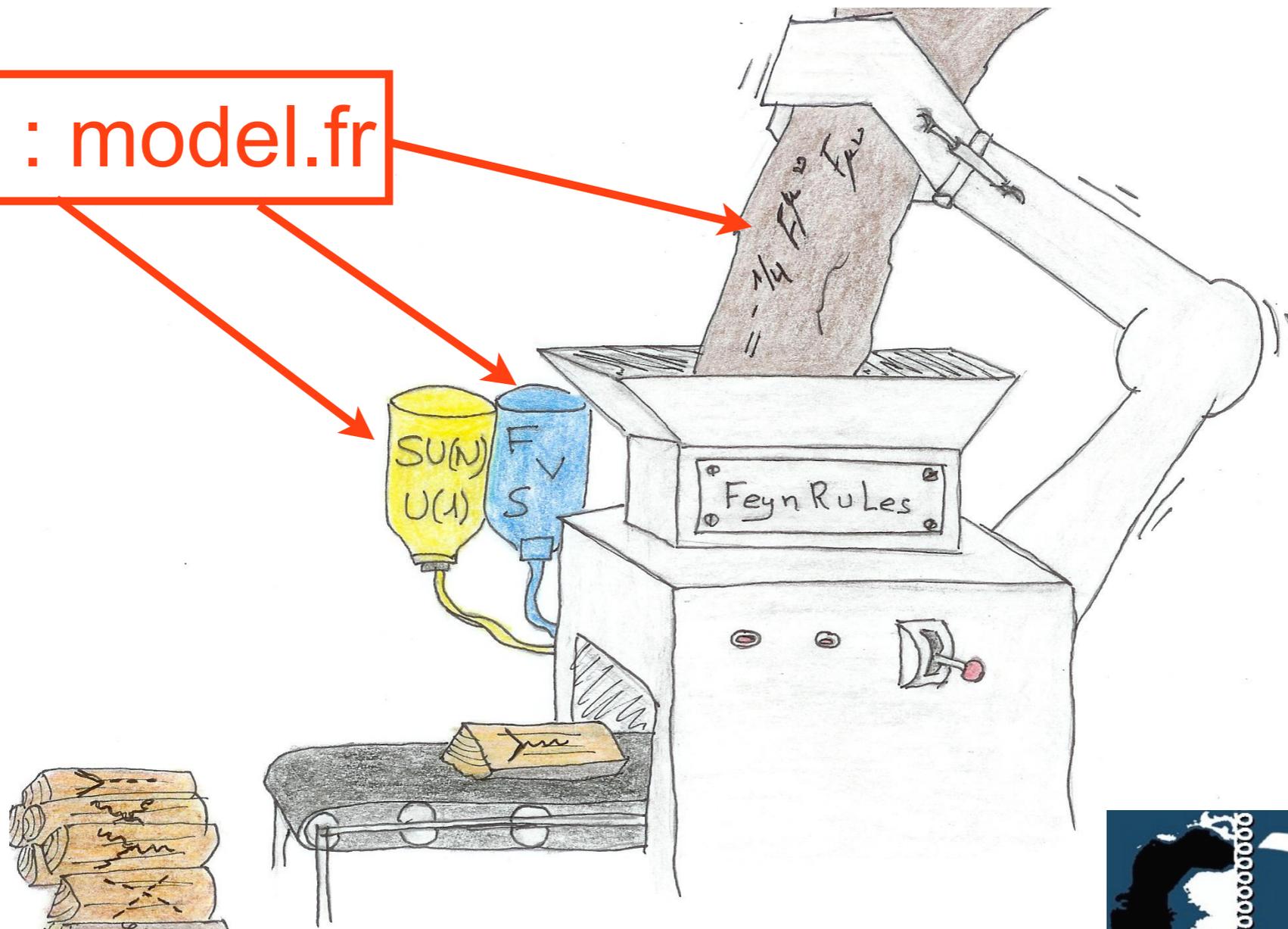
$$\mathcal{A}^{1-loop} = \sum_i d_i \text{Box}_i + \sum_i c_i \text{Triangle}_i + \sum_i b_i \text{Bubble}_i + \sum_i a_i \text{Tadpole}_i + R$$

- Box, Triangle, Bubble and Tadpole are known scalar integrals
- Loop computation = find the coefficients
  - Unitarity
  - Multiple cuts
  - Tensor reduction (OPP)

# Automated computation for BSM

# FeynRules in a nutshell

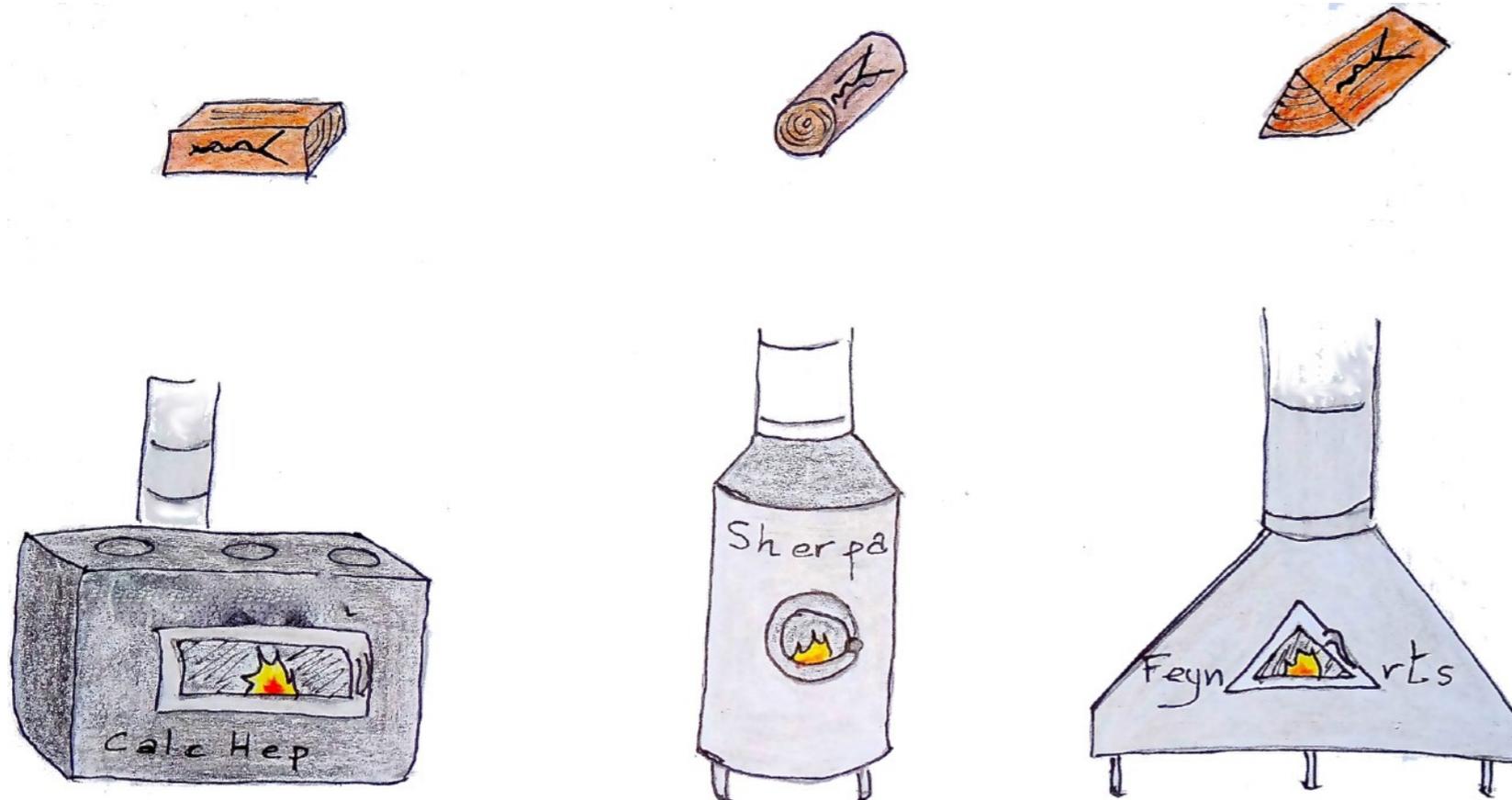
Input : model.fr



Output : vertices



# Feynman rules outputs



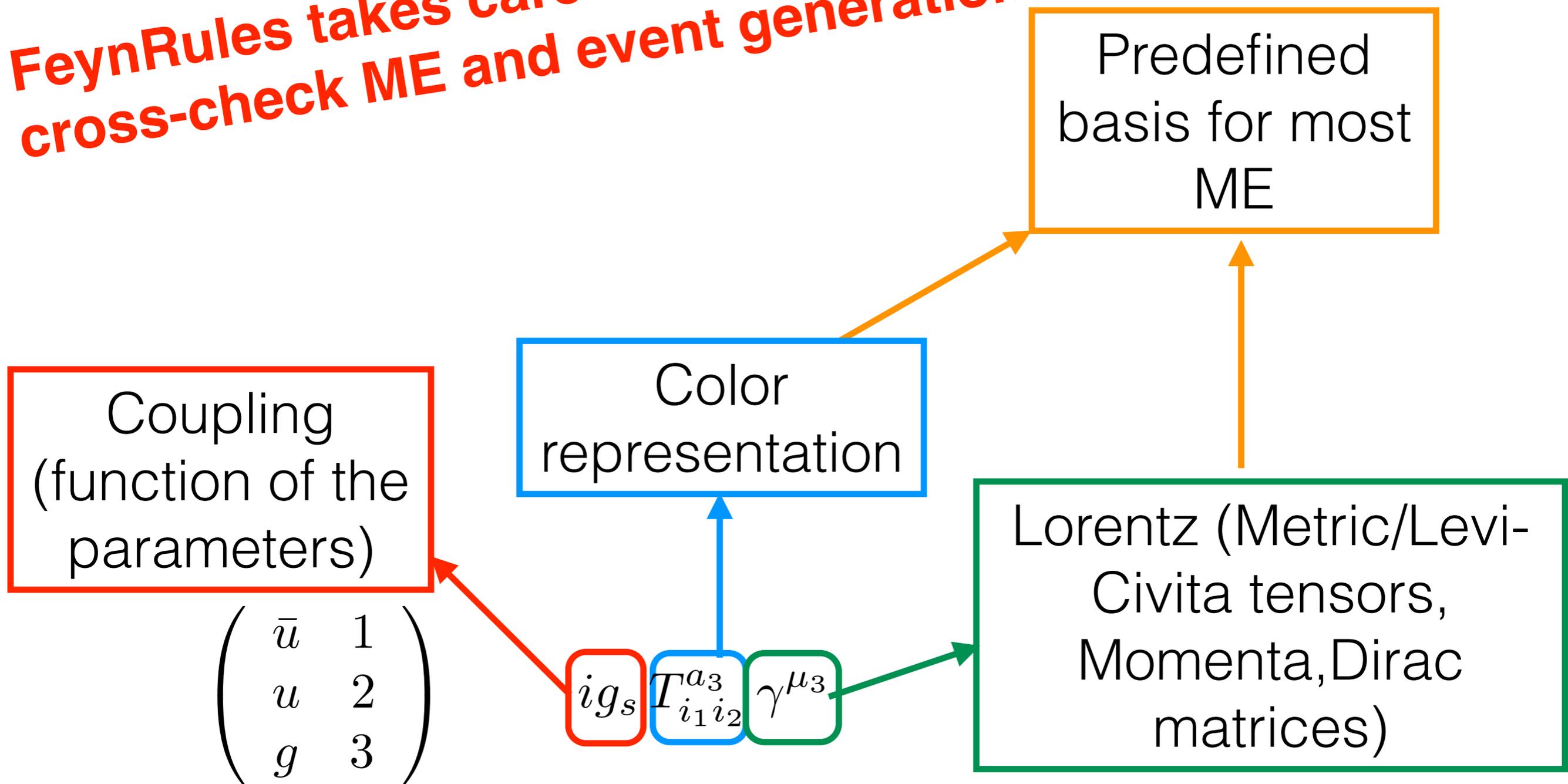
FeynRules  
outputs can be  
used directly by  
event generators

UFO : output with  
the full information  
used by several  
generators

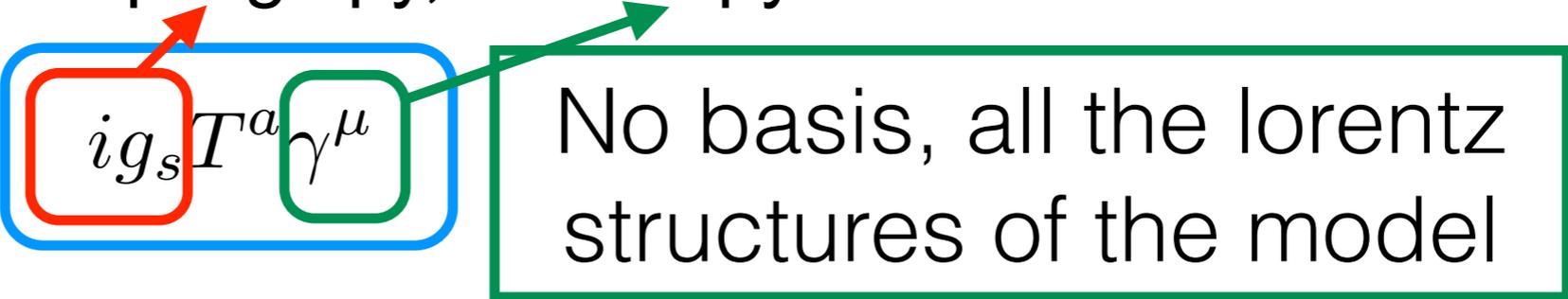


# Feynman Rules

**FeynRules takes care of all the conventions  
cross-check ME and event generation**



# UFO

- Generic output with the **full** model information
  - coupling\_orders.py, parameters.py, particles.py, write\_param\_card.py, \_\_init\_\_.py,
  - vertices.py, couplings.py, lorentz.py
- 

No basis, all the lorentz structures of the model
- decays.py
- CT\_vertices.py, CT\_couplings.py (For NLO)
- Python module used in MadGraph, Herwig, Gosam, Sherpa

# model file

```
(***** This is a template model file for FeynRules *****)
```

```
(***** Index definition *****)
```

```
IndexRange[ Index[Generation] ] = Range[3]
```

```
IndexFormat[Generation, f]
```

```
(***** Parameter list *****)
```

```
M$Parameters = {  
}
```

```
(***** Gauge group list *****)
```

```
M$GaugeGroups = {  
}
```

```
(***** Particle classes list *****)
```

```
M$ClassesDescription = {  
}
```

**Definition of variables  
in Mathematica syntaxe**

# Model information

```
M$ModelName = "my_new_model";
```

```
M$Information = {  
  Authors      -> {"Mr. X", "Ms. Y"},  
  Institutions -> {"UC Louvain"},  
  Emails       -> {"X@uclouvain.be", "Y@uclouvain.be"},  
  Date         -> "01.03.2013",  
  References   -> {"reference 1", "reference 2"},  
  URLs         -> {"http://feynrules.irmp.ucl.ac.be"},  
  Version      -> "1.0"  
};
```

**Good practice for credit, issue(s) tracking**

# Indices definition

Used in parameters, gauge groups  
and fields

```
IndexRange[ Index[Colour] ] = Range[3];  
IndexRange[ Index[SU2W] ] = Unfold[ Range[3] ];  
IndexRange[ Index[Gluon] ] = NoUnfold[ Range[8] ];
```

Tells FR to remplace  
summed indices by  
the explicite sum

Tells FA/FC **not** to  
remplace summed  
indices by the  
explicite sum

# Indices definition

Used in parameters, gauge groups  
and fields

```
IndexRange[ Index[Colour] ] = Range[3];  
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```

Tells FR to remplace  
summed indices by  
the explicite sum

Tells FA/FC **not** to  
remplace summed  
indices by the  
explicite sum

## Format:

```
IndexStyle[ Colour, i ];  
IndexStyle[ Gluon, a ];
```

# Indices definition

**Used in parameters, gauge groups and fields**

```
IndexRange[ Index[Colour] ] = Range[3];  
IndexRange[ Index[SU2W] ] = Unfold[ Range[3] ];  
IndexRange[ Index[Gluon] ] = NoUnfold[ Range[8] ];
```

Tells FR to remplace summed indices by the explicite sum

Tells FA/FC **not** to remplace summed indices by the explicite sum

## Format:

```
IndexStyle[ Colour, i ];  
IndexStyle[ Gluon, a ];
```

**Predefined indices:** Lorentz, Spin, Spin1, Spin2

# Parameters definition

```
M$Parameters = {  
    param1 == { options1 },  
    param2 == { options2 },  
    ...  
};
```

```
aEWM1 == {  
    ParameterType -> External,  
    BlockName     -> SMINPUTS,  
    OrderBlock    -> 1,  
    Value         -> 127.9,  
    InteractionOrder -> {QED,-2},  
    Description    -> "Inverse of the EW coupling constant at the Z  
pole"  
},
```

**Compulsory!**

Numerical value

# Parameters definition

```
M$Parameters = {  
  param1 == { options1 },  
  param2 == { options2 },  
  ...  
};
```

```
MW == {  
  ParameterType -> Internal,  
  Value -> Sqrt[MZ^2/2+Sqrt[MZ^4/4-Pi/Sqrt[2]*aEW/  
Gf*MZ^2]],  
  TeX -> Subscript[M,W],  
  Description -> "W mass"  
},
```

Expression

# Parameters definition

```
M$Parameters = {  
    param1 == { options1 },  
    param2 == { options2 },  
    ...  
};
```

```
aEWM1 == {  
    ParameterType -> External,  
    BlockName     -> SMINPUTS,  
    OrderBlock    -> 1,  
    Value         -> 127.9,  
    InteractionOrder -> {QED,-2},  
    Description    -> "Inverse of the EW coupling constant at the Z  
pole"  
},
```

For the LHA cards

Dependence in the expansion parameters

# Parameters definition

```
M$Parameters = {  
    param1 == { options1 },  
    param2 == { options2 },  
    ...  
};
```

```
aEWM1 == {  
    ParameterType -> External,  
    BlockName     -> SMINPUTS,  
    OrderBlock    -> 1,  
    Value         -> 127.9,  
    InteractionOrder -> {QED,-2},  
    Description    -> "Inverse of the EW coupling constant at the Z  
pole"  
},
```

For the LHA cards

Dependence in the expansion parameters

# Interaction order

**In the SM :** QCD the power of  $g_s$   
QED the power of  $e$

```
aEWMI == { ...  
  InteractionOrder -> {QED,-2},  
  Description      -> "Inverse of the EW coupling constant at the Z pole"  
},
```

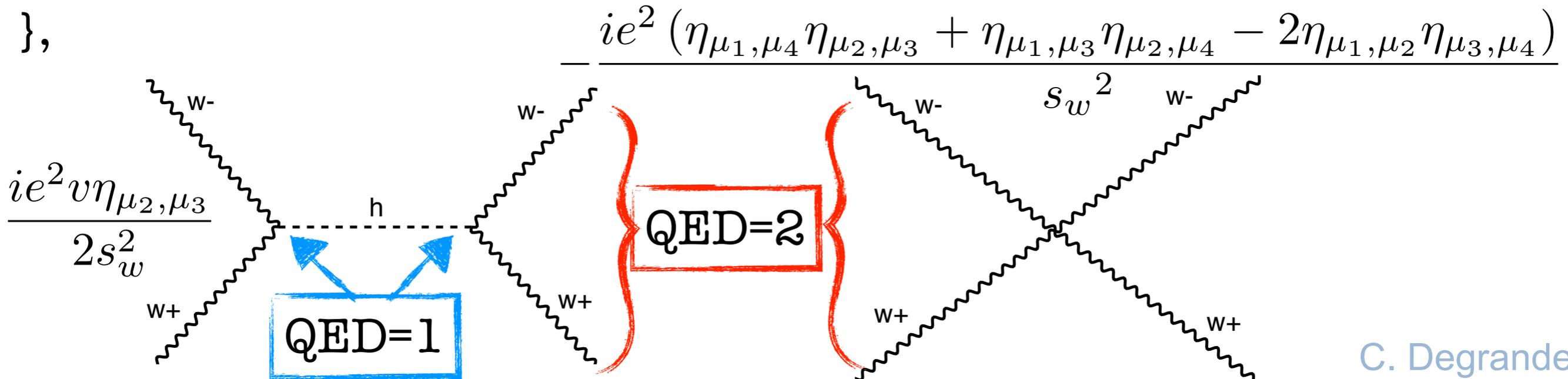
```
vev == {...  
  InteractionOrder -> {QED,-1},  
  Description      -> "Higgs vacuum expectation value"  
},
```

# Interaction order

**In the SM :** QCD the power of  $g_s$   
 QED the power of  $e$

aEWM1 == { ...  
 InteractionOrder -> {QED,-2},  
 Description -> "Inverse of the EW coupling constant at the Z pole"  
 },

vev == { ...  
 InteractionOrder -> {QED,-1},  
 Description -> "Higgs vacuum expectation value"  
 },



# Interaction order

**In the SM :** QCD the power of  $g_s$   
QED the power of  $e$

```
vev == { ...  
  InteractionOrder -> { QED, -1 },  
  Description      -> "Higgs vacuum expectation value"  
},
```

```
yu == { ...  
  InteractionOrder -> { QED, 1 },  
  Description      -> "Up-type Yukawa couplings"  
},
```

Such that masses have QED=0

However  $y_t$  is not a small parameter!

# Interaction order

$\text{InteractionOrderHierarchy} = \{ \{ \text{QCD}, 1 \}, \{ \text{QED}, 2 \} \};$

$$g_s \sim e^2$$

# Interaction order

InteractionOrderHierarchy = { {QCD, 1},  
{QED, 2}};

$$g_s \sim e^2$$

$$\mathcal{L} = \mathcal{L} + \sum_i \frac{1}{\Lambda^2} \mathcal{O}_i + \mathcal{O}(\Lambda^{-4})$$

NP the power of  $\Lambda^{-2}$

,{NP, 2}



# Interaction order

$\text{InteractionOrderHierarchy} = \{ \{ \text{QCD}, 1 \}, \{ \text{QED}, 2 \} \};$

$$g_s \sim e^2$$

$$\mathcal{L} = \mathcal{L} + \sum_i \frac{1}{\Lambda^2} \mathcal{O}_i + \mathcal{O}(\Lambda^{-4})$$

NP the power of  $\Lambda^{-2}$

, {NP, 2}

$\text{InteractionOrderLimit} = \{ \{ \text{NP}, 1 \} \};$

Max power per diagram of  $\Lambda^{-2}$  is 1

# Fields definition I

```
M$ClassesDescription = {  
  spin1[1] == { options1 },  
  spin1[2] == { options2 },  
  spin2[1] == { options3 },  
  ...}
```

# Fields definition I

|      |     |
|------|-----|
| S    | 0   |
| W,F  | 1/2 |
| V    | 1   |
| RW,R | 3/2 |
| T    | 2   |
| U    | -1  |

```
M$ClassesDescription = {  
  spin1[1] == { options1 },  
  spin1[2] == { options2 },  
  spin2[1] == { options3 },  
  ... }
```

# Fields definition I

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| S    | 0   |
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| U    | -1  |

```
M$ClassesDescription = {  
  spin1[1] == { options1 },  
  spin1[2] == { options2 },  
  spin2[1] == { options3 },  
  ... }
```

Unique Id

# Fields definition I

ClassName->..., SelfConjugate->...,  
 Indices->..., QuantumNumbers->...,  
 FlavorIndex->..., ClassMembers,  
 Mass->..., Width->..., PDG->...,  
 Definitions->..., Unphysical->...,  
 Chirality->..., MajoranaPhase->...,  
 WeylComponents->...,  
 Goldstone->..., Ghost->..., ...(Format)

|      |     |
|------|-----|
| S    | 0   |
| W,F  | 1/2 |
| V    | 1   |
| RW,R | 3/2 |
| T    | 2   |
| U    | -1  |

`M$ClassesDescription = {`  
`spin1[1] == { options1 },`  
`spin1[2] == { options2 },`  
`spin2[1] == { options3 },`  
`... }`

Unique Id

# Fields definition II

```
F[3] == {  ClassName      -> uq,  
          ClassMembers  -> {u, c, t},  
          Indices        -> {Index[Generation], Index[Colour]},  
          FlavorIndex    -> Generation,  
          SelfConjugate  -> False,  
          Mass           -> {Mu, {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},  
          Width          -> {0, 0, {WT,1.50833649}},  
          QuantumNumbers -> {Q -> 2/3},  
          PDG            -> {2, 4, 6},  
          ...  
}
```

# Fields definition II

Spin index

```
F[3] == {  ClassName      -> uq,  
  ClassMembers    -> {u, c, t},  
  Indices         -> {Index[Generation], Index[Colour]},  
  FlavorIndex     -> Generation,  
  SelfConjugate   -> False,  
  Mass            -> {Mu, {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},  
  Width          -> {0, 0, {WT,1.50833649}},  
  QuantumNumbers -> {Q -> 2/3},  
  PDG            -> {2, 4, 6},  
  ...  
}
```

# Fields definition II

Generation index distinguishes  
the class members

```
F[3] == {  ClassName      -> uq,  
  ClassMembers -> {u, c, t},  
  Indices       -> {Index[Generation], Index[Colour]},  
  FlavorIndex  -> Generation,  
  SelfConjugate -> False,  
  Mass         -> {Mu, {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},  
  Width        -> {0, 0, {WT,1.50833649}},  
  QuantumNumbers -> {Q -> 2/3},  
  PDG          -> {2, 4, 6},  
  ...  
}
```

# Fields definition II

```
F[3] == {  ClassName      -> uq,  
          ClassMembers  -> {u, c, t},  
          Indices       -> {Index[Generation], Index[Colour]},  
          FlavorIndex   -> Generation,  
          SelfConjugate -> False,  
          Mass          -> {Mu, {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},  
          Width         -> {0, 0, {WT,1.50833649}},  
          QuantumNumbers -> {Q -> 2/3},  
          PDG           -> {2, 4, 6},  
          ...  
}
```

Same representation

# Fields definition II

```
F[3] == {  ClassName      -> uq,  
          ClassMembers  -> {u, c, t},  
          Indices        -> {Index[Generation], Index[Colour]},  
          FlavorIndex    -> Generation,  
          SelfConjugate  -> False, External parameters  
          Mass           -> {Mu, {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},  
          Width          -> {0, 0, {WT,1.50833649}},  
          QuantumNumbers -> {Q -> 2/3},  
          PDG            -> {2, 4, 6},  
          ...  
}
```

# Fields definition II

```
F[3] == {  ClassName      -> uq,
  ClassMembers   -> {u, c, t},
  Indices        -> {Index[Generation], Index[Colour]},
  FlavorIndex    -> Generation,
  SelfConjugate  -> False,
  Mass           -> {Mu {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},
  Width         -> {0, 0, {WT,1.50833649}},
  QuantumNumbers -> {Q -> 2/3},
  PDG           -> {2, 4, 6},
  ...
}
```

External parameters

Generic label

```
Mass -> {MW, Internal}
Mass -> {MZ, 91.188}
Mass -> {{MU,0}, {MC,0}, {MT, 174.3}}
Mass -> {Mu, {MU, 0}, {MC, 0}, {MT, 174.3}}
```

# Fields definition II

```
F[3] == {  ClassName      -> uq,  
  ClassMembers   -> {u, c, t},  
  Indices        -> {Index[Generation], Index[Colour]},  
  FlavorIndex    -> Generation,  
  SelfConjugate  -> False,  
  Mass           -> {Mu, {MU, 2.55*^-3}, {MC,1.27}, {MT,172}},  
  Width          -> {0, 0, {WT,1.50833649}},  
  QuantumNumbers -> {Q -> 2/3},  
  PDG            -> {2, 4, 6},  
  ...  
}
```

Not used in FR but by  
following codes

# Fields definition III

## Interaction eigenstates

```
V[12] == {  
  ClassName    -> Wi,  
  Unphysical   -> True,  
  SelfConjugate -> True,  
  Indices      -> {Index[SU2W]},  
  FlavorIndex  -> SU2W,  
  Definitions  -> { Wi[mu_,1] -> (Wbar[mu]+W[mu])/Sqrt[2],  
    Wi[mu_,2] -> (Wbar[mu]-W[mu])/(I*Sqrt[2]), Wi[mu_,3] -> cw  
    Z[mu] + swA[mu]}  
}
```

FR does not export  
them to matrix  
element code

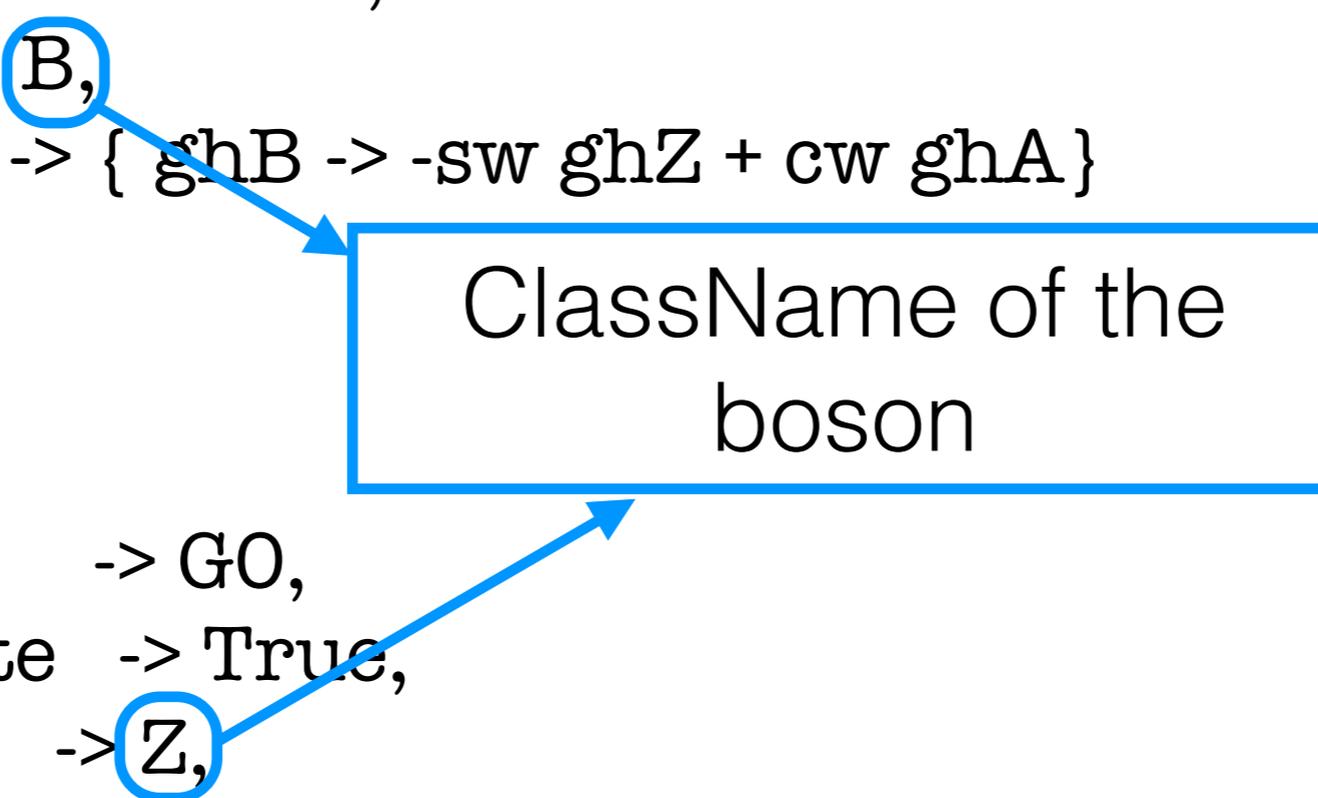
Physical fields

# Fields definition IV

```
U[11] == {  
  ClassName    -> ghB,  
  Unphysical   -> True,  
  SelfConjugate -> False,  
  Ghost        -> B,  
  Definitions  -> { ghB -> -sw ghZ + cw ghA }  
},
```

```
S[2] == {  
  ClassName    -> GO,  
  SelfConjugate -> True,  
  Goldstone    -> Z,  
  ...  
},
```

ClassName of the  
boson



# Gauge Groups

```
M$GaugeGroups = {
  U1Y == {
    Abelian      -> True,
    CouplingConstant -> g1,
    GaugeBoson   -> B,
    Charge       -> Y
  },...
  SU3C == {
    Abelian      -> False,
    CouplingConstant -> gs,
    GaugeBoson   -> G,
    StructureConstant -> f,
    Representations -> {T,Colour},
    SymmetricTensor -> dSUN
  }
};
```

# Gauge Groups

```
M$GaugeGroups = {  
  U1Y == {  
    Abelian      -> True,  
    CouplingConstant -> g1,  
    GaugeBoson   -> B,  
    Charge       -> Y  
  },...  
  SU3C == {  
    Abelian      -> False,  
    CouplingConstant -> gs,  
    GaugeBoson   -> G,  
    StructureConstant -> f,  
    Representations -> {T,Colour},  
    SymmetricTensor -> dSUN  
  }  
};
```

# Gauge Groups

```
M$GaugeGroups = {  
  U1Y == {  
    Abelian      -> True,  
    CouplingConstant -> g1,  
    GaugeBoson   -> B,  
    Charge       -> Y  
  },...  
  SU3C == {  
    Abelian      -> False,  
    CouplingConstant -> gs,  
    GaugeBoson   -> G,  
    StructureConstant -> f,  
    Representations -> {T,Colour},  
    SymmetricTensor -> dSUN  
  }  
};
```

# Gauge Groups

```
M$GaugeGroups = {  
  U1Y == {  
    Abelian      -> True,  
    CouplingConstant -> g1,  
    GaugeBoson   -> B,  
    Charge       -> Y  
  },...  
  SU3C == {  
    Abelian      -> False,  
    CouplingConstant -> gs,  
    GaugeBoson   -> G,  
    StructureConstant -> f,  
    Representations -> {T,Colour}  
    SymmetricTensor -> dSUN  
  }  
};
```

Generator label

Associated index

# Gauge groups

$$\text{FS}[A, \mu, \nu, a] \xrightarrow{\text{abelian}} F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^a_{bc} A_\mu^b A_\nu^c$$

$$\text{DC}[\phi, \mu] \xrightarrow{\quad} D_\mu \phi = \partial_\mu \phi - ig A_\mu^a T_a \phi$$

# Lagrangian

$$\mathcal{L}^{QCD} \equiv -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + i\bar{d}\not{D}d$$

$$L = -1/4 \text{FS}[G, \text{mu}, \text{nu}, a] \text{FS}[G, \text{mu}, \text{nu}, a] \\ + \text{I} \text{dqbar} \cdot \text{Ga}[\text{mu}] \cdot \text{DC}[\text{dq}, \text{mu}]$$

FeynRules creates the “anti”-particle name

Dot to avoid commuting the fermions

$\text{dqbar} \cdot \text{Ga}[\text{mu}] \cdot \text{T}[a] \cdot \text{dq}$

$\rightarrow \text{Ga}[\text{mu}, \text{s}, \text{r}] \text{T}[a, \text{i}, \text{j}] \text{dqbar}[\text{s}, \text{f}, \text{i}] \cdot \text{dq}[\text{r}, \text{f}, \text{j}]$

FeynRules restores the indices internally

# Running FeynRules

## In Mathematica :

### Loading Feynrules

```
$FeynRulesPath = SetDirectory[ <the address of the package> ];  
<< FeynRules`
```

### Loading the model

```
LoadModel[ < file.fr >, < file2.fr >, ... ]
```

### Extracting the Feynman rules

```
vertsQCD = FeynmanRules[ LQCD ];
```

### Checking the Lagrangian

```
CheckKineticTermNormalisation[ L ]  
CheckMassSpectrum[ L ]
```

### Outputting the Lagrangian

```
WriteUFO[ L ]
```

# Running FeynRules

## In Mathematica :

### Loading Feynrules

```
$FeynRulesPath = SetDirectory[ <the address of the package> ];  
<< FeynRules`
```

### Loading the model

```
LoadModel[ < file.fr >, < file2.fr >, ... ]
```

All the model files should  
be loaded at once

### Extracting the Feynman rules

```
vertsQCD = FeynmanRules[ LQCD ];
```

### Checking the Lagrangian

```
CheckKineticTermNormalisation[ L ]  
CheckMassSpectrum[ L ]
```

### Outputting the Lagrangian

```
WriteUFO[ L ]
```

# Running FeynRules

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### Extracting the Feynman rules

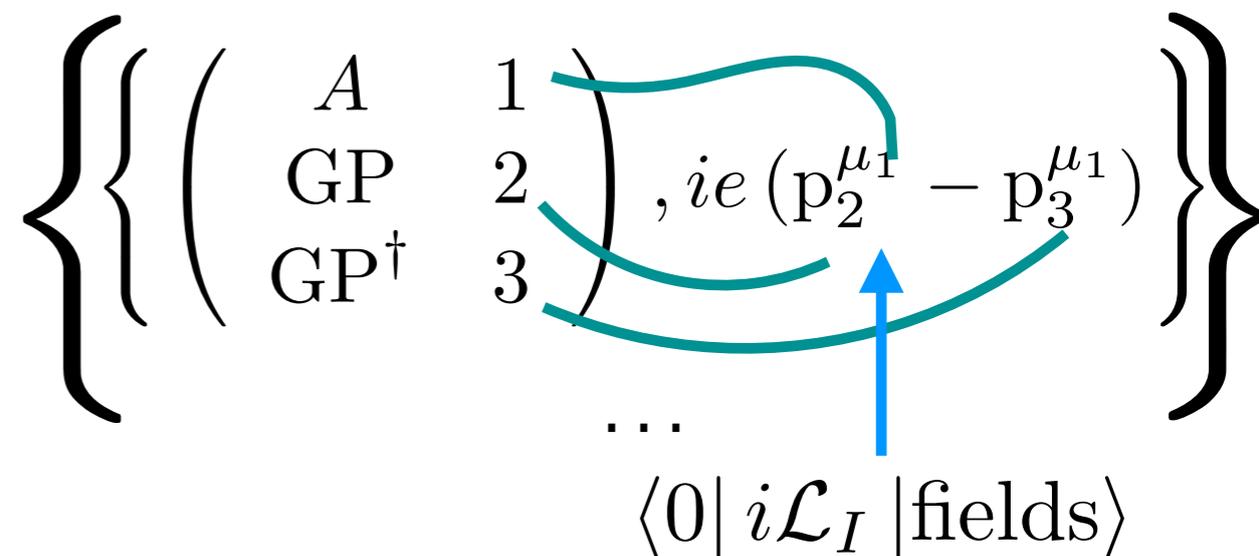
```
vertsQCD = FeynmanRules[ LQCD ];
```

### Checking the Lagrangian

```
CheckKineticTermNormalisation[ L ]  
CheckMassSpectrum[ L ]
```

### Outputting the Lagrangian

```
WriteUFO[ L ]
```



All momenta are incoming

# Running FeynRules

## In Mathematica :

### Loading Feynrules

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$FeynRulesPath = SetDirectory[ <the address of the package> ];  
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```

### Loading the model

```
LoadModel[ < file.fr >, < file2.fr >, ... ]
```

### Extracting the Feynman rules

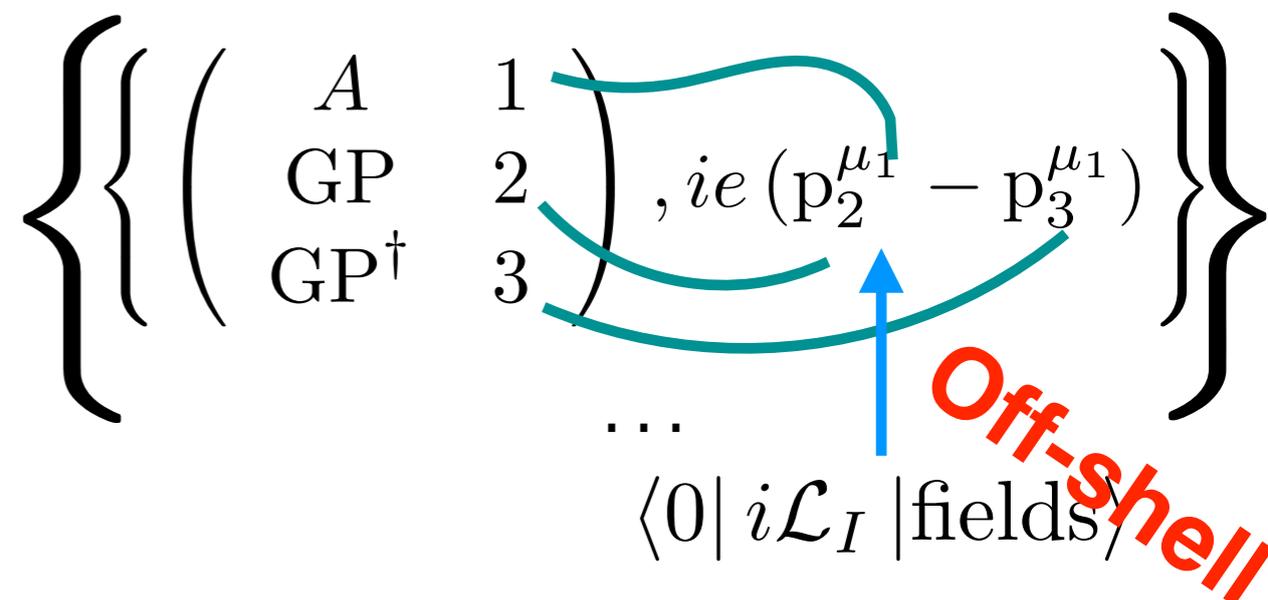
```
vertsQCD = FeynmanRules[ LQCD ];
```

### Checking the Lagrangian

```
CheckKineticTermNormalisation[ L ]  
CheckMassSpectrum[ L ]
```

### Outputting the Lagrangian

```
WriteUFO[ L ]
```



All momenta are incoming

# Checks

CheckHermiticity[ L, options ]

CheckDiagonalKineticTerms[ L, options ]

CheckDiagonalMassTerms[ L, options ]

CheckDiagonalQuadraticTerms[ L, options ]

CheckKineticTermNormalisation[ L, options ]

$$\begin{array}{lll} \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 & \frac{1}{2} \bar{\lambda} i \not{\partial} \lambda - \frac{1}{2} m \bar{\lambda} \lambda & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu \\ \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi & \bar{\psi} i \not{\partial} \psi - m \bar{\psi} \psi & - \frac{1}{2} F_{\mu\nu}^\dagger F^{\mu\nu} - m^2 A_\mu^\dagger A^\mu \end{array}$$

CheckMassSpectrum[ L, options ]

# Toolbox

`ExpandIndices[L, options]`

`GetKineticTerms[L, options]`

`GetMassTerms[L, options]`

`GetQuadraticTerms[L, options]`

`GetInteractionTerms[L, options]`

`SelectFieldContent[L, list]`

# Check your model in MG

check gauge/ permutation/ Lorentz + your process

play with coupling order, check the diagrams

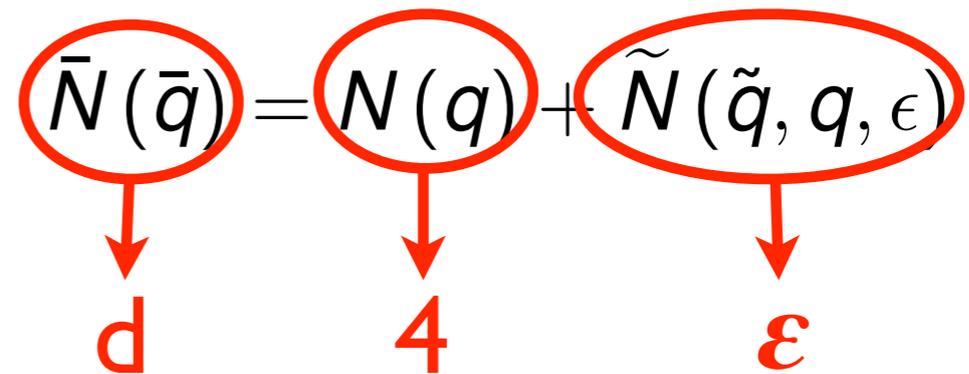
check the behaviour of your model, high energy,  
resonance

# BSM at one loop

- Why
  - LHC : QCD correction are larges
  - loop induced processes may dominate (DM DM to 2 photons, scalar production 'a la Higgs')
- How :
  - Automated based on development done in FeynRules and MadGraph5\_aMC@NLO

# $R_2$

$$\bar{A}(\bar{q}) = \frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}}, \quad \bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}, q, \epsilon)$$


$$R_2 \equiv \lim_{\epsilon \rightarrow 0} \frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{\tilde{N}(\tilde{q}, q, \epsilon)}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}}$$

Finite set of vertices that can be computed once  
for all

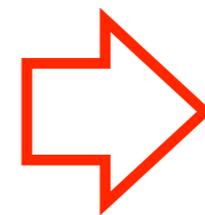
# R<sub>1</sub>

Due to the  $\epsilon$  dimensional parts of the denominators

Like for the 4 dimensional part but with a different set of integrals

$$\int d^n \bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j} = -\frac{i\pi^2}{2} \left[ m_i^2 + m_j^2 - \frac{(p_i - p_j)^2}{3} \right] + \mathcal{O}(\epsilon),$$
$$\int d^n \bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon),$$
$$\int d^n \bar{q} \frac{\tilde{q}^4}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon).$$

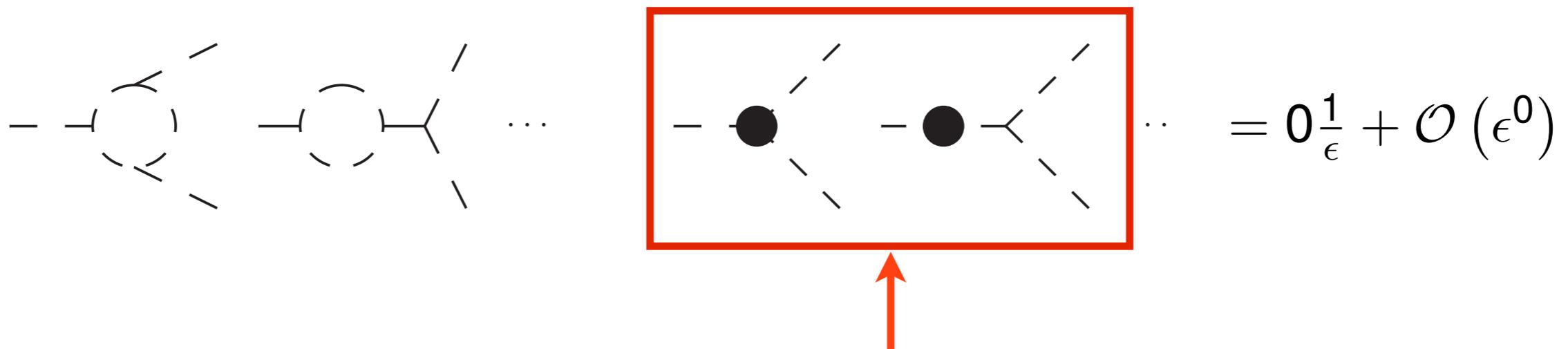
Only  $R = R_1 + R_2$  is gauge invariant



Check

# UV

$$\bar{A}(\bar{q}) = \frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}} = K \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)$$



Relations fixed by the Lagrangian (finite part)

Finite set of vertices that can be computed once for all

# Renormalization

External parameters

$$\begin{aligned}x_0 &\rightarrow x + \delta x, \\ \phi_0 &\rightarrow \left(1 + \frac{1}{2}\delta Z_{\phi\phi}\right)\phi + \sum_{\chi} \frac{1}{2}\delta Z_{\phi\chi}\chi.\end{aligned}$$

Same for the conjugate field

Internal parameters are renormalised by replacing the external parameters in their expressions

# How does it work?

**FeynRules**  
Renormalize the Lagrangian

model.mod  
model.gen

**FeynArts**  
Write the amplitudes

**NLOCT.m**  
Compute the NLO vertices

model.nlo



# Restrictions/Assumptions

- Renormalizable Lagrangian, maximum dimension of the operators is 4
- Feynman Gauge  
 $\{\gamma_\mu, \gamma_5\} = 0$
- 't Hooft-Veltman scheme
- On-shell/complex mass scheme for the masses and wave functions
- $\overline{MS}$  by default for everything else (zero-momentum possible for fermion gauge boson interaction)

## R2 : Validation

- tested\* on the SM (QCD:P. Draggiotis et al. +QED:M.V. Garzelli et al)
- tested\* on MSSM (QCD:H.-S. Shao, Y.-J. Zhang) :  
test the Majorana

\*Analytic comparison of the expressions

# UV Validation

- SM QCD : tested\* (W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper)
- SM EW : tested\* (expressions given by H.-S. Shao from A. Denner)

\*Analytic comparison of the expressions

# Tests in event generators

- aMC@NLO
- The SM QCD has been tested by V. Hirschi (Comparison with the built-in version)
- The MSSM QCD and SM EW are tested by H.-S. Shao and V. Hirschi
- 2HDM QCD is currently tested ( $p p \rightarrow S, H^\pm t$ )
  - gauge invariance
  - pole cancelation

# SM tests

```
==== Finite ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d d~ > w+ w- g  -1.2565695610e+01 -1.2565705416e+01 -1.2565696276e+01  3.9018817097e-07  Pass

==== Born ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d d~ > w+ w- g  1.8518318521e-06  1.8518318521e-06  1.8518318521e-06  8.0617231411e-15  Pass

==== Single pole ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d d~ > w+ w- g  -1.9397426502e+01 -1.9397426502e+01 -1.9397426504e+01  5.5894073017e-11  Pass

==== Double pole ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d d~ > w+ w- g  -5.6666666667e+00 -5.6666666667e+00 -5.6666666667e+00  3.0015206007e-14  Pass

==== Summary ====
I/I passed, 0/I failed==== Finite ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  -5.3971186943e+01 -5.3971193753e+01 -5.3971189940e+01  6.3091071914e-08  Pass

==== Born ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  6.4168774056e-05  6.4168764370e-05  6.4168764370e-05  7.5467680882e-08  Pass

==== Single pole ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  -3.7439549398e+01 -3.7439549398e+01 -3.7439549397e+01  6.8122965983e-12  Pass

==== Double pole ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  -8.6666666667e+00 -8.6666666667e+00 -8.6666666667e+00  2.2443585452e-14  Pass

==== Summary ====
I/I passed, 0/I failed==== Finite ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z g g  -5.3769573669e+01 -5.3769573347e+01 -5.3769566412e+01  6.7475496780e-08  Pass
```

# SM tests

```
==== Born ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z g g  3.1531233900e-04  3.1531235770e-04  3.1531235770e-04  2.9654886777e-08  Pass

==== Single pole ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z g g  -3.7464897007e+01 -3.7464897007e+01 -3.7464897007e+01  4.2333025503e-12  Pass

==== Double pole ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z g g  -8.6666666667e+00 -8.6666666667e+00 -8.6666666667e+00  2.1316282073e-14  Pass

==== Summary ====
I/I passed, 0/I failed==== Finite ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z z g  -5.9990384275e+00 -5.9990511729e+00 -5.9990379587e+00  1.1013604745e-06  Pass

==== Born ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z z g  2.2616997126e-06  2.2617000449e-06  2.2617000449e-06  7.3450366526e-08  Pass

==== Single pole ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z z g  -1.5469587040e+01 -1.5469587040e+01 -1.5469587040e+01  1.5226666708e-11  Pass

==== Double pole ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z z g  -5.6666666667e+00 -5.6666666667e+00 -5.6666666667e+00  2.6645352591e-15  Pass

==== Summary ====
I/I passed, 0/I failed==== Finite ====
Process      Stored MadLoop v4  ML5 opt      ML5 default  Relative diff.  Result
g g > h t t~  2.9740187004e+01  2.9740187005e+01  2.9740187036e+01  5.3265970697e-10  Pass
```

# SM tests

```

                                     === Born ===
Process      Stored MadLoop v4 ML5 opt      ML5 default      Relative diff.  Result
g g > h t t~ 1.1079653971e-07  1.1079653974e-07  1.1079653974e-07  1.3190849004e-10  Pass

                                     === Single pole ===
Process      Stored MadLoop v4 ML5 opt      ML5 default      Relative diff.  Result
g g > h t t~ -7.0825709000e+00 -7.0825709000e+00 -7.0825709000e+00  5.0901237085e-13  Pass

                                     === Double pole ===
Process      Stored MadLoop v4 ML5 opt      ML5 default      Relative diff.  Result
g g > h t t~ -6.0000000000e+00 -6.0000000000e+00 -6.0000000000e+00  1.7023419711e-15  Pass

                                     === Summary ===
                                     I/I passed, 0/I failed=== Finite ===
Process      Stored MadLoop v4 ML5 opt      ML5 default      Relative diff.  Result
g g > z t t~ 3.6409017466e+01  3.6409021125e+01  3.6409021117e+01  5.0242920154e-08  Pass

                                     === Born ===
Process      Stored MadLoop v4 ML5 opt      ML5 default      Relative diff.  Result
g g > z t t~ 7.0723041711e-07  7.0723046101e-07  7.0723046101e-07  3.1039274206e-08  Pass

                                     === Single pole ===
Process      Stored MadLoop v4 ML5 opt      ML5 default      Relative diff.  Result
g g > z t t~ -7.1948086812e+00 -7.1948086773e+00 -7.1948086773e+00  2.7349789963e-10  Pass

                                     === Double pole ===
Process      Stored MadLoop v4 ML5 opt      ML5 default      Relative diff.  Result
g g > z t t~ -6.0000000000e+00 -6.0000000000e+00 -6.0000000000e+00  2.5165055225e-15  Pass

                                     === Summary ===
                                     I/I passed, 0/I failed=== Finite ===
Process      Stored ML5 opt  ML5 opt      ML5 default      Relative diff.  Result
d d~ > w+ w- g -1.2565695610e+01 -1.2565705416e+01 -1.2565696276e+01  3.9018817097e-07  Pass

```

# SM tests

```
==== Born ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d d~ > w+ w- g  1.8518318521e-06  1.8518318521e-06  1.8518318521e-06  8.0617231411e-15  Pass

==== Single pole ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d d~ > w+ w- g -1.9397426502e+01 -1.9397426502e+01 -1.9397426504e+01  5.5894073017e-11  Pass

==== Double pole ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d d~ > w+ w- g -5.6666666667e+00 -5.6666666667e+00 -5.6666666667e+00  3.0015206007e-14  Pass

==== Summary ====
I/I passed, 0/I failed==== Finite ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  -1.1504816412e+01 -1.1504816557e+01 -1.1504815497e+01  4.6089385415e-08  Pass

==== Born ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  2.3138920858e-06  2.3138920858e-06  2.3138920858e-06  4.3012538015e-15  Pass

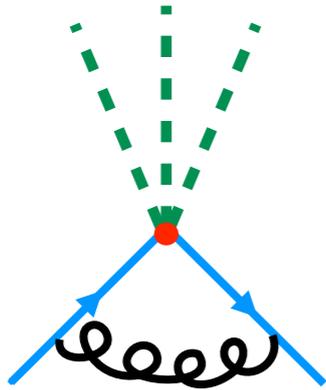
==== Single pole ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  -2.8637049838e+01 -2.8637049838e+01 -2.8637049838e+01  1.5718407645e-13  Pass

==== Double pole ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d~ d > a g g  -8.6666666667e+00 -8.6666666667e+00 -8.6666666667e+00  1.7421961310e-15  Pass

==== Summary ====
I/I passed, 0/I failed==== Finite ====
Process      Stored ML5 opt  ML5 opt      ML5 default  Relative diff.  Result
d~ d > z g g  -1.0306105482e+01 -1.0306105654e+01 -1.0306102645e+01  1.4600800434e-07  Pass
```

+2/3

# EFT at NLO



In the loop: same as SM  
 +axial anomaly  $(\varphi^\dagger \varphi) (\bar{q}_p u_r \tilde{\varphi})$   
 $(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{q}_p \gamma^\mu q_r)$

$$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$$

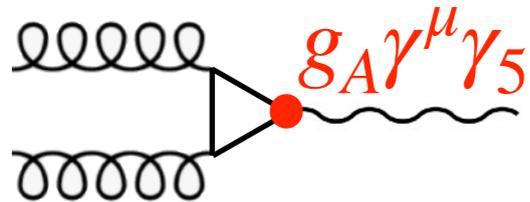
$$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$$

More momenta: higher rank of the integral numerator

Additional gamma and colour algebra

$$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$$

# Axial anomaly



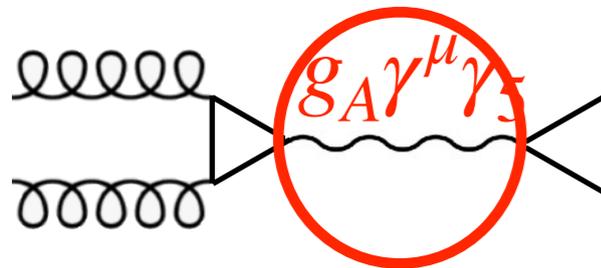
SM:

$$g_A^u = g_A^c = g_A^t = -g_A^d = -g_A^s = -g_A^b$$

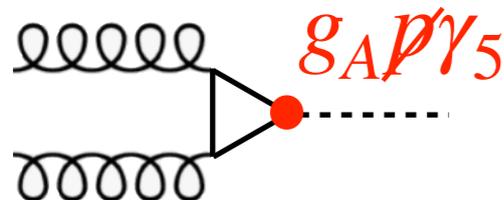
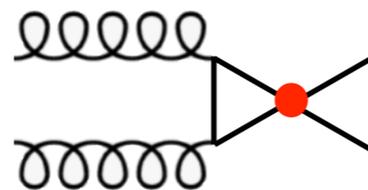
SMEFT:

$$g_A^u \neq g_A^c \neq g_A^t \neq -g_A^d \neq -g_A^s \neq -g_A^b$$

+ modification of quarks-gluon vertex (chromo)

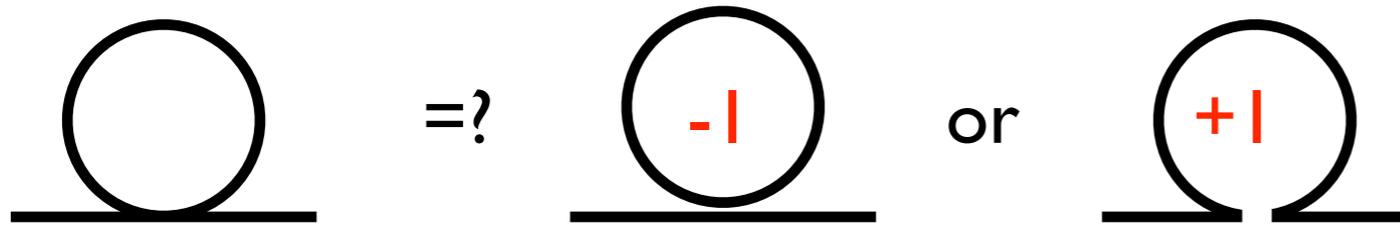


=



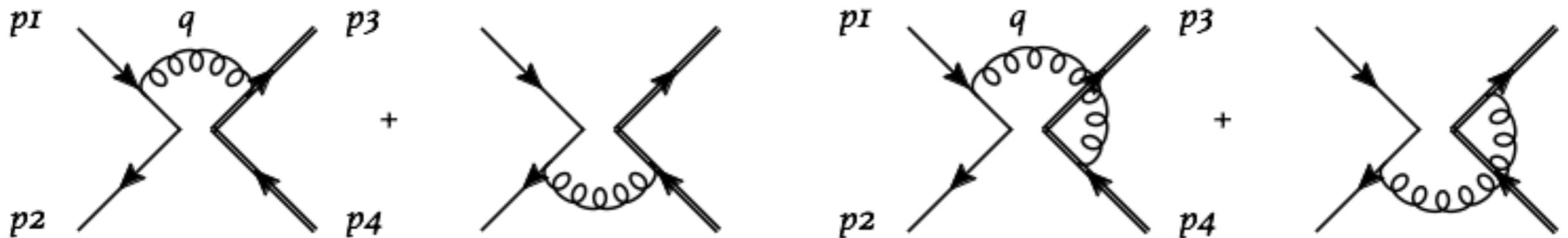
$$\propto \epsilon^{p_1 p_2 \mu_1 \mu_2}$$

# EFT at NLO



Evanescent operators:

$$O_{ut}^{(8)} = (\bar{u}\gamma^\mu T^A u) (\bar{t}\gamma_\mu T^A t)$$



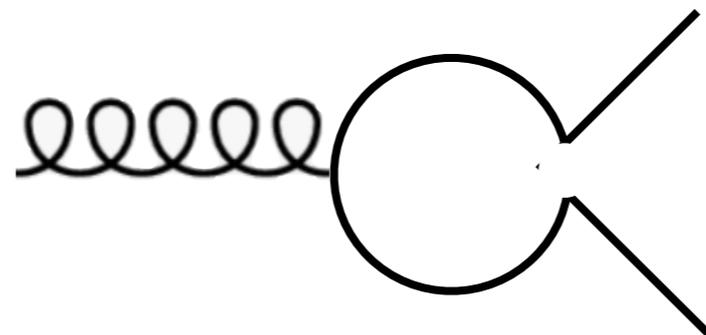
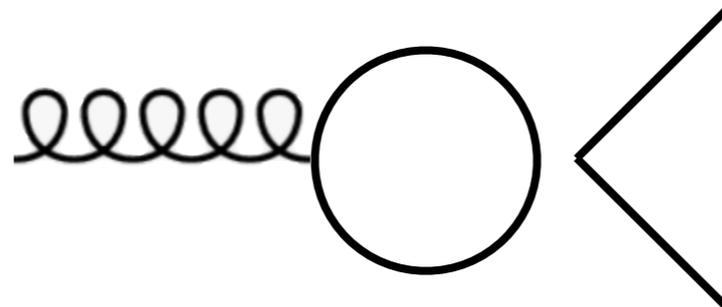
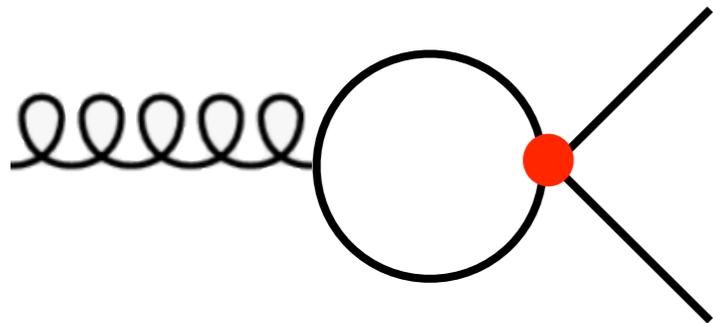
$$\gamma^\mu \gamma^\nu \gamma^\rho P_R \otimes \gamma_\mu \gamma_\nu \gamma_\rho P_R = E + (16 - 4a\varepsilon) \gamma^\mu P_R \otimes \gamma_\mu P_R$$

$$\gamma^\mu \gamma^\nu \gamma^\rho P_R \otimes \gamma_\rho \gamma_\nu \gamma_\mu P_R = -E + [4 - (12 - 4a)\varepsilon] \gamma^\mu P_R \otimes \gamma_\mu P_R$$

**Extra R2 (gauge invariant)  
Change the UV matching**

# 4F to 2F R2

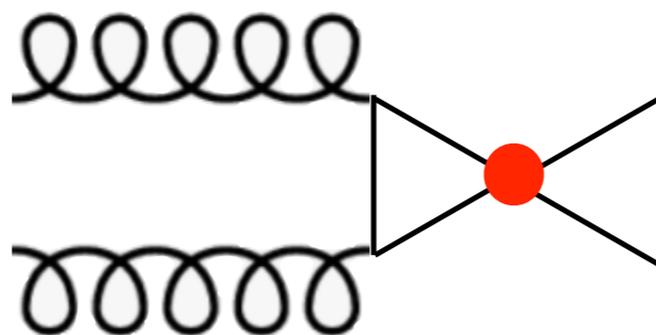
$$\mathcal{O}^1 = \bar{t}_R \gamma^\mu t_R \bar{t}_R \gamma^\mu t_R \sim \bar{t}_R \gamma^\mu T^a t_R \bar{t}_R \gamma^\mu T^a t_R = \mathcal{O}^8$$



R2 difference :

- gauge inv.
- basis dependent

Connected by gauge invariance and operator to



# SMEFT@NLO



- C.D., G. Durieux, F. Maltoni, K. Mimasu, E Vryonidou, C. Zhang, 2008.11743

# Tutorial

| Notation | Spin | Mass  | SU(3) | SU(2) | U(1) |
|----------|------|-------|-------|-------|------|
| $\Phi_1$ | 0    | $M_1$ | 1     | 1     | 0    |
| $\Phi_2$ | 0    | $M_2$ | 1     | 1     | 0    |
| $U$      | 1/2  | $M_U$ | 3     | 1     | 2/3  |
| $E$      | 1/2  | $M_E$ | 1     | 1     | -1   |

$$M_U > M_2 > M_L > M_1$$

$$\mathcal{L}_{\text{s.m.}} = -\frac{m_1^2}{2}\phi_1^2 - \frac{m_2^2}{2}\phi_2^2 - m_{12}^2\phi_1\phi_2$$

**+Gauge kinetic term**

$$\mathcal{L}_{\text{f.m.}} = M_U\bar{U}U + M_E\bar{E}E$$

$$\mathcal{L}_{\text{Yuk}} = \lambda_1\phi_1\bar{U}P_Ru + \lambda_2\phi_2\bar{U}P_Ru + \lambda'_1\phi_1\bar{E}P_Re + \lambda'_2\phi_2\bar{E}P_Re$$

$$pp \rightarrow \bar{U}U$$

$$U \rightarrow u\Phi_1,$$

$$U \rightarrow u\Phi_2, \quad \Phi_2 \rightarrow eE, \quad E \rightarrow e\Phi_1$$

# Step 0

- Download FeynRules 2.0 from
  - <https://feynrules.irmp.ucl.ac.be>
- Copy the SM directory in feynrules/models and rename it Tutorial
- Create a model file Tutorial.fr (text file)

# Step 1 : Model information

```
M$ModelName = "Tutorial";  
M$Information = {Authors -> {"C.  
Degrande"},  
Version -> "1.0",  
Date -> "21. 07. 2014",  
Institutions -> {"UCLouvain"},  
Emails ->  
{"celine.degrande@uclouvain.be"}  
};
```

## Step 2 : parameters

- 9 new external parameters :
  - $m_1, m_2, m_{12}, M_U, M_E, \lambda_1, \lambda_2, \lambda_1', \lambda_2'$   
See  
Step 3

```
M$Parameters = {  
  ...  
  MM1 == {  
    ParameterType -> External,  
    Value -> 200},  
  ...  
};
```

## Step 2 : parameters

- 9 new external parameters :

- $m_1, m_2, m_{12}, M_U, M_E, \lambda_1, \lambda_2, \lambda_1', \lambda_2'$

See  
Step 3

```
M$Parameters = {
```

```
...
```

```
MM1 == {
```

```
ParameterType -> External,
```

```
Value -> 200},
```

```
...
```

```
};
```

InteractionOrder -> {NP, 1},

## Step 2 : parameters

- 3 internal parameters :  $M_1, M_2, \vartheta$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} -\sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

Interaction  
eigenstates

Mass  
eigenstates

- `ParameterType` is `Internal`
- `Value` is a Mathematica expression

## Step3 : fields

|     |     |          |          |          |          |
|-----|-----|----------|----------|----------|----------|
| $U$ | $E$ | $\phi_1$ | $\phi_2$ | $\Phi_1$ | $\Phi_2$ |
| uv  | ev  | pi1      | pi2      | p1       | p2       |

```
S M$ClassesDescription = {  
  ...  
  F[100] == {  
    ClassName -> uv,  
    SelfConjugate -> False,  
    Indices -> {Index[Colour]},  
    QuantumNumbers -> {Y -> 2/3, Q -> 2/3},  
    Mass -> {Muv, 500},  
    Width -> {Wuv, 1}  
  },  
  ...  
}
```

Unique ID (check in SM.fr)

Defined in SM

{Mpe1, Internal}

## Step 3 : fields

|     |     |          |          |          |          |
|-----|-----|----------|----------|----------|----------|
| $U$ | $E$ | $\phi_1$ | $\phi_2$ | $\Phi_1$ | $\Phi_2$ |
| uv  | ev  | pi1      | pi2      | p1       | p2       |

```
S[100] == {  
  ClassName -> pi1,  
  SelfConjugate -> True,  
  Indices -> {},  
  Unphysical -> True,  
  Definitions -> {pi1 -> - Sin[th] p1 +  
  Cos[th] p2}  
},
```

## Step 4 : Lagrangian

```
$FeynRulesPath =  
SetDirectory["~/feynrules"];  
<< FeynRules`  
  
SetDirectory[ $FeynRulesPath <> "/Models/  
Tutorial"]  
  
LoadModel["SM.fr", "Tutorial.fr"]  
LoadRestriction["DiagonalCKM.rst",  
"Massless.rst"]
```

## Step 4 : Lagrangian

```
1/2 del[pi1, mu]del[pi1, mu] - 1/2 MM1^2 pi1^2  
Lint:=lam1 pi1 uvbar.ProjP.t  
HC[Lint]
```

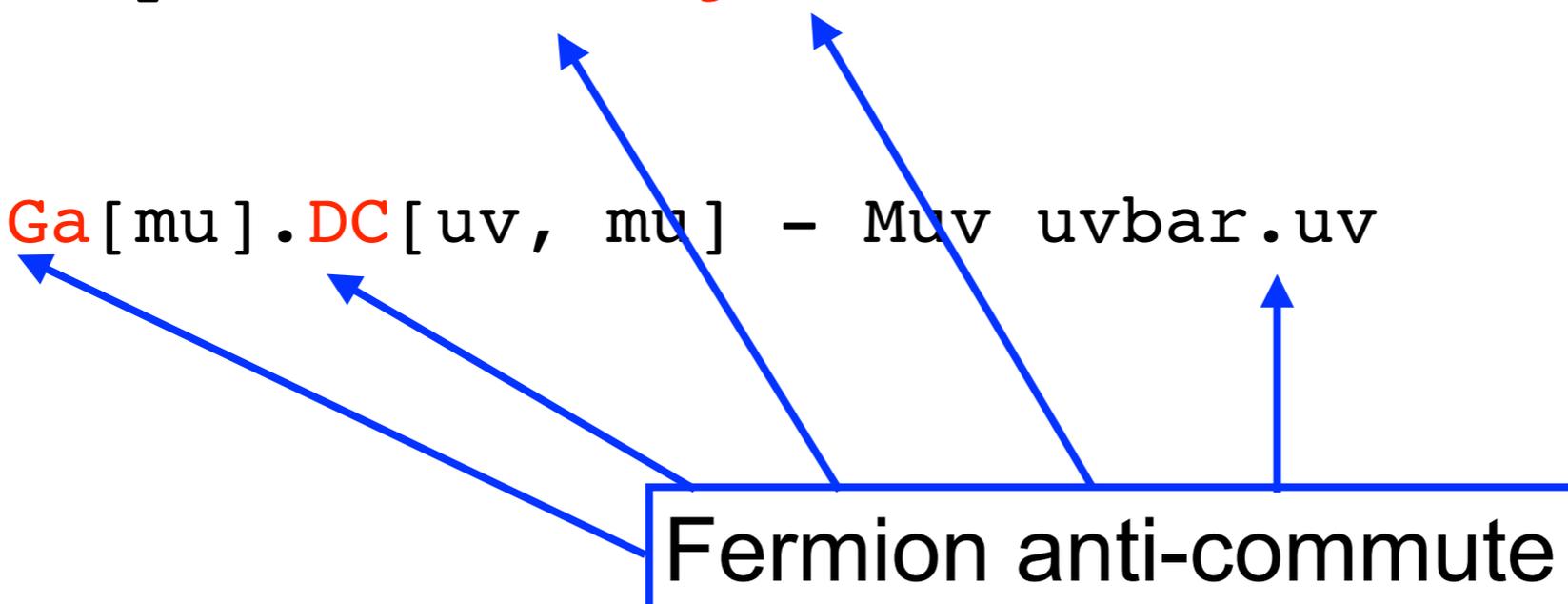
```
I uvbar.Ga[mu].DC[uv, mu] - Muv uvbar.uv
```

# Step 4 : Lagrangian

$\frac{1}{2} \text{del}[\text{pi1}, \text{mu}] \text{del}[\text{pi1}, \text{mu}] - \frac{1}{2} \text{MM1}^2 \text{pi1}^2$   
 $\text{Lint} := \text{lam1 pi1 uvbar} \cdot \text{ProjP} \cdot t$   
 $\text{HC}[\text{Lint}]$

$I \text{ uvbar} \cdot \text{Ga}[\text{mu}] \cdot \text{DC}[\text{uv}, \text{mu}] - \text{Muv uvbar} \cdot \text{uv}$

Fermion anti-commute



## Step 5 : run FeynRules

```
vertices = FeynmanRules[ LNew ];  
CheckMassSpectrum[ LNew ]  
ComputeWidths[vertices];  
PartialWidth[ {uv, t, p1} ]  
TotWidth[ uv ]  
BranchingRatio[ {uv, t, p1}]  
SetDirectory["~/mg5amcnlo/models"];  
WriteUFO[ LSM + LNew ];
```

# Step 6 : run in MadGraph

run checks!

Run your process as in the SM

Check diagrams, distributions, ...

## Step 7 : NLO

```
Lren = OnShellRenormalization[LSM + LNew, QCDOnly -> True,  
FlavorMixing -> False]; // Timing
```

```
SetDirectory["~/FeynArts-3.8/Models"];  
WriteFeynArtsOutput[Lren, GenericFile -> False,  
Output -> "Tutorial"]; // Timing
```

```
Quit[]
```

```
SetDirectory["~/FeynArts-3.xx"];  
<< FeynArts`  
SetDirectory["~/feynrules-current"]  
<< NLOCT`
```

```
SetDirectory["~/feynrules-current/Models/tutorial"]  
WriteCT["Tutorial/Tutorial", "Lorentz", QCDOnly -> True,  
Exclude4ScalarsCT -> True,  
ZeroMom -> {{aS, {F[14], V[4], -F[14]}}} // Timing
```

## Step 7 : NLO

```
Quit[]
```

```
SetDirectory["~/feynrules-current/Models/tutorial"]  
Get["Tutorial.nlo"]
```

```
SetDirectory["~/mg5amcnlo/models"];  
WriteUFO[LSM + LNew, Output -> "Tutorial_NLO",  
R2Vertices -> R2$vertlist, UVCounterterms -> UV$vertlist]
```

# More information

Tutorial of MC4BSM 1209.0297v1

FeynRules manual

NLOCT manual : 1406.3030