



UCLouvain

Institut de recherche en mathématique et physique

Centre de Cosmologie, Physique des Particules et Phénoménologie



Tools and Monte Carlo

Celine Degrande

Plan

- Introduction
- Leading order at fixed order
 - Matrix element
 - Integration
 - errors
- Higher order corrections
 - NLO QCD
 - Beyond NLO QCD
- Parton shower and Hadronisation
 - Jets
 - Parton shower
 - Hadronisation models
- BSM
 - New models and FeynRules
 - Loop corrections

Exercices in
purple by hand or/
and in MadGraph

Questions to know you

- Have you already computed a cross-section?
- With MadGraph or another tool?
- What about loop amplitudes, NLO?
- Have you done it for BSM, with FeynRules?

Questions

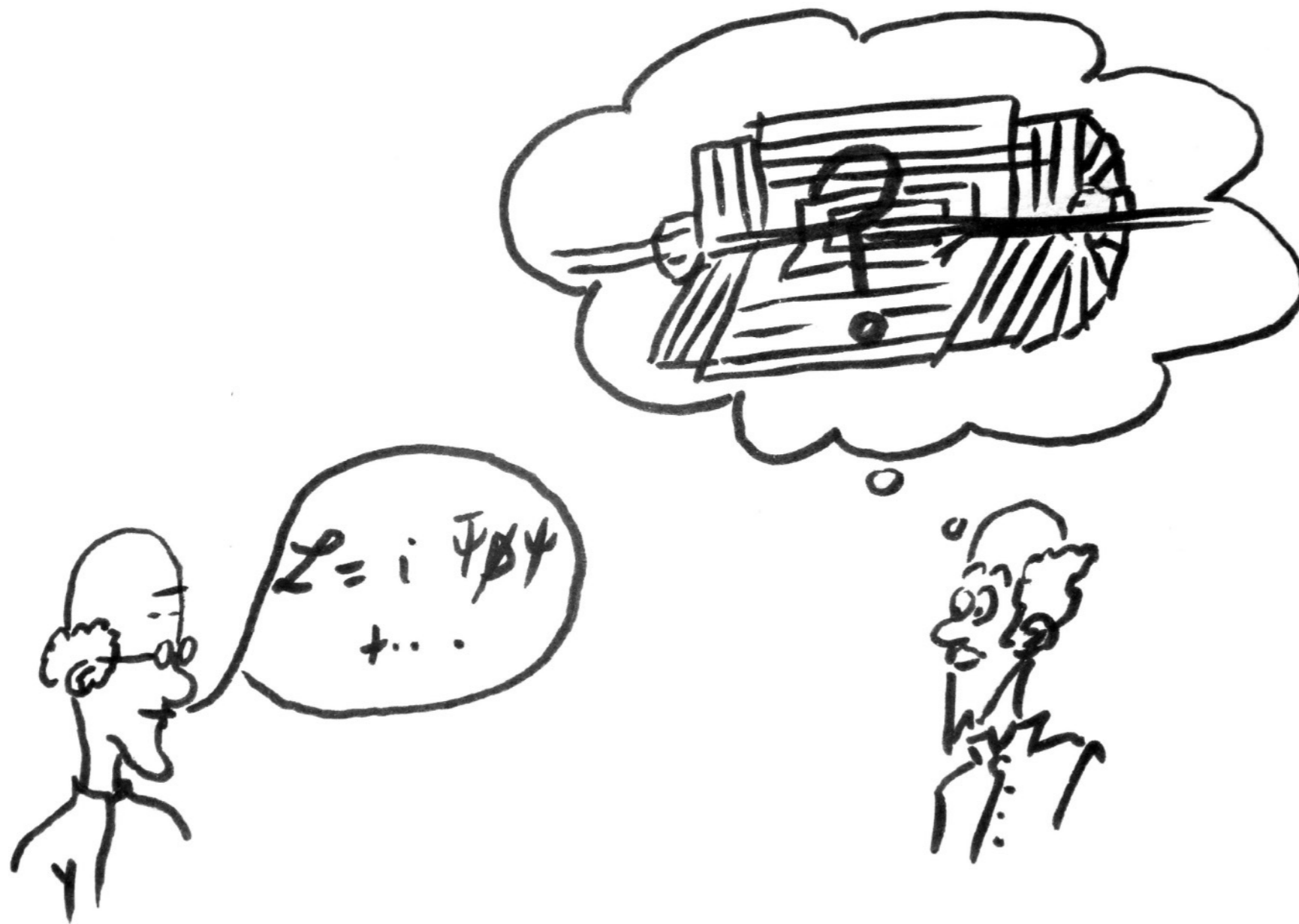
- How many particle in the final state can we predict?
- How accurate are our predictions?
- When do they fail?
- Do we know the shape well?
- How can we check if our results are correct?

Disclaimer

- There many more to know about tools that what I will cover here
- The content reflect my own bias and interest
- I used content by colleagues (O. Mattelaer, M. Zaro, R. Frederic)

Introduction

Why do we need numerical tools?



Why automated tools

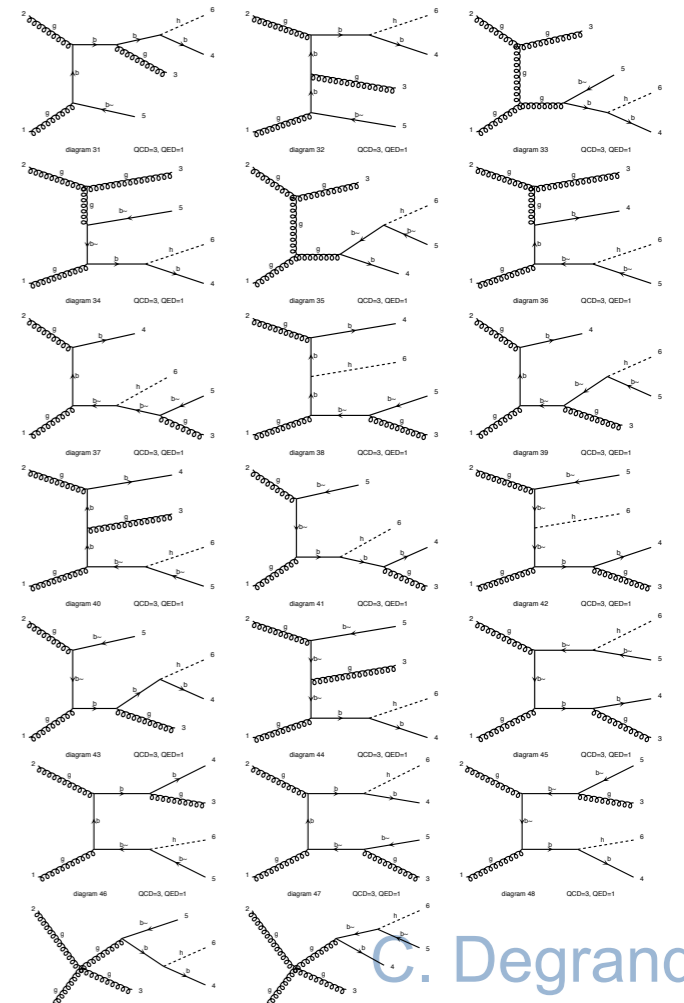
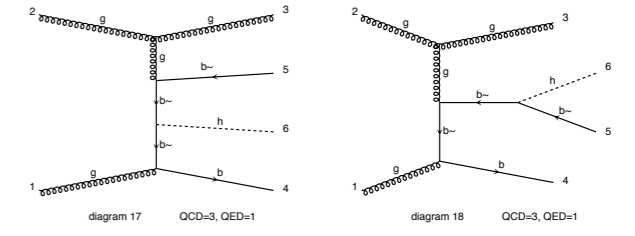
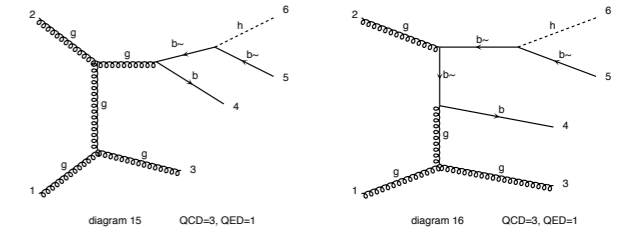
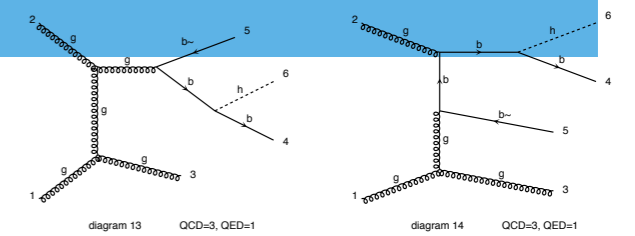
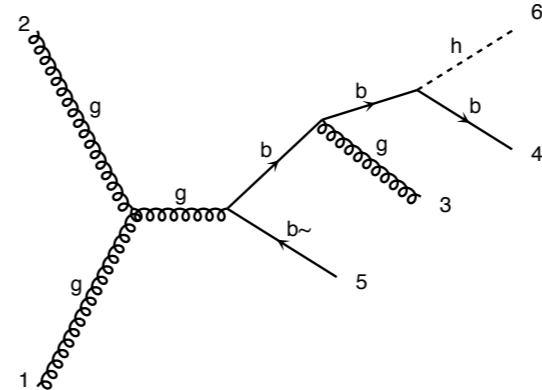
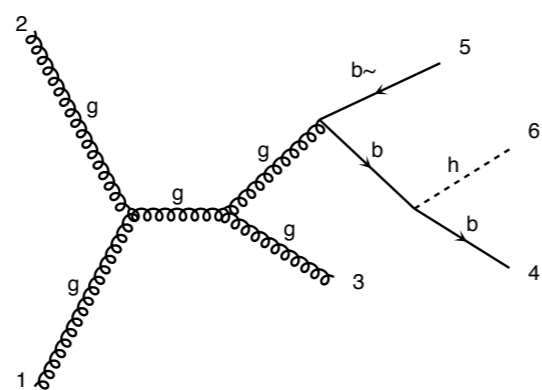
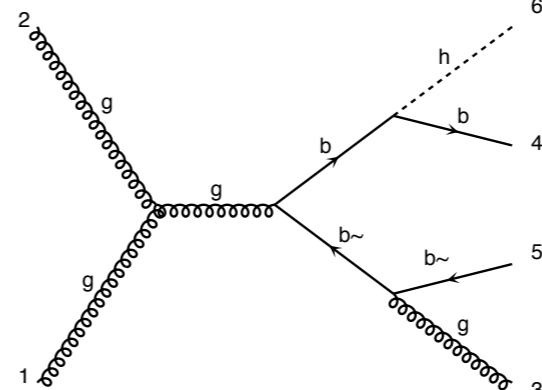
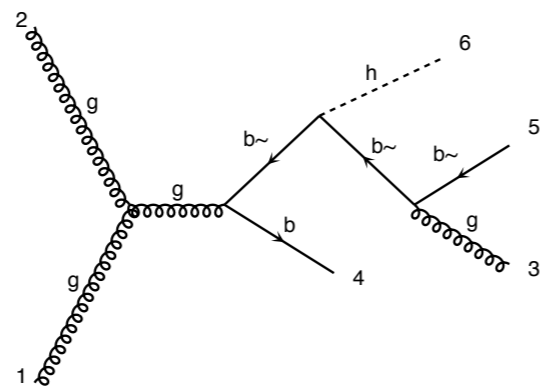
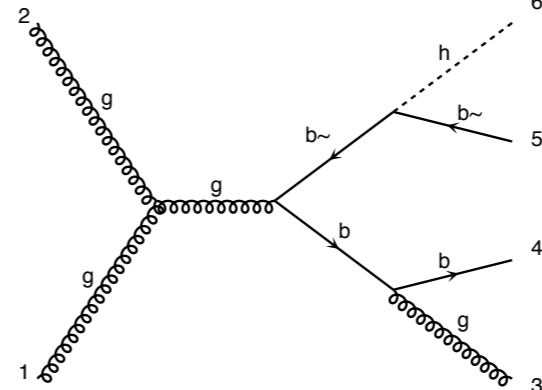
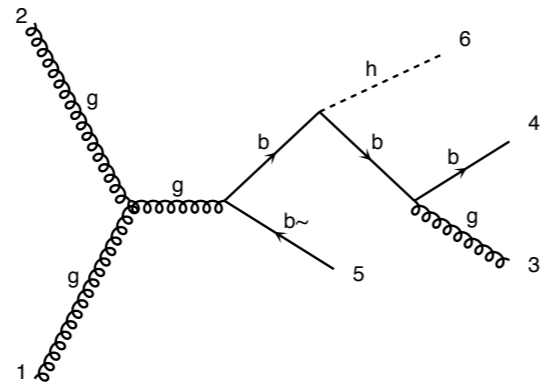
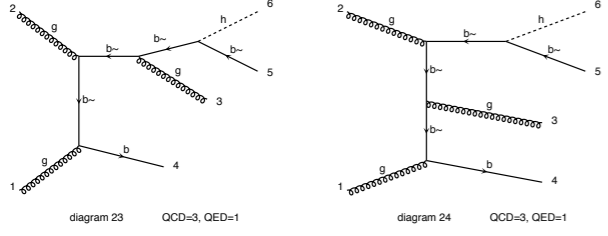
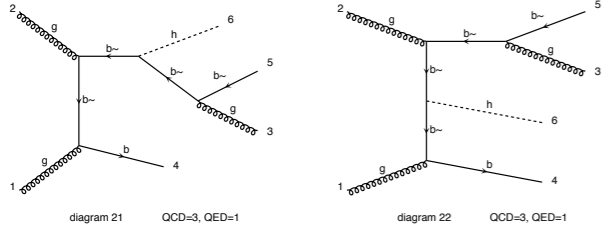
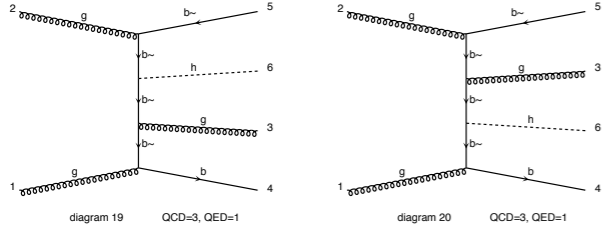
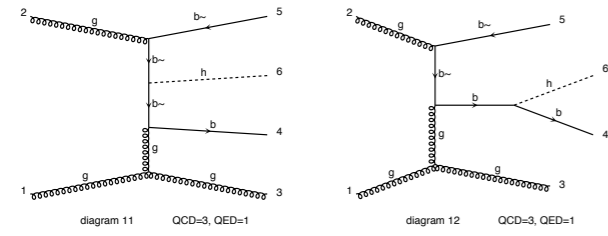
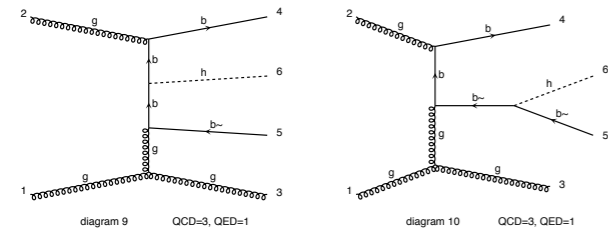
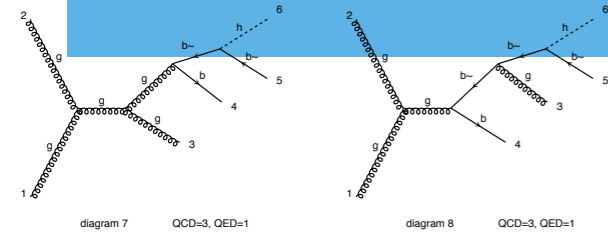
- Algorithmic

- **Less** error prone

- Long $f^{abc} G_{\mu\nu}^a G^{b\nu\rho} G_{\rho}^{c\mu} \ni$ 4 gluons vertex

$$\begin{aligned}
 & 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} p_1^{\mu_4} p_2^{\mu_3} \eta_{\mu_1, \mu_2} - 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} p_1^{\mu_3} p_2^{\mu_4} \eta_{\mu_1, \mu_2} + 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} p_1^{\mu_3} p_3^{\mu_4} \eta_{\mu_1, \mu_2} + \\
 & 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} p_2^{\mu_3} p_3^{\mu_4} \eta_{\mu_1, \mu_2} + 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} p_1^{\mu_4} p_4^{\mu_3} \eta_{\mu_1, \mu_2} + 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} p_2^{\mu_4} p_4^{\mu_3} \eta_{\mu_1, \mu_2} - \\
 & 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} \eta_{\mu_3, \mu_4} p_1 \cdot p_3 \eta_{\mu_1, \mu_2} - 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} \eta_{\mu_3, \mu_4} p_1 \cdot p_4 \eta_{\mu_1, \mu_2} - 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} \eta_{\mu_3, \mu_4} p_2 \cdot p_3 \eta_{\mu_1, \mu_2} - \\
 & 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} \eta_{\mu_3, \mu_4} p_2 \cdot p_4 \eta_{\mu_1, \mu_2} + 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} p_1^{\mu_2} p_2^{\mu_4} \eta_{\mu_1, \mu_3} + 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} p_1^{\mu_4} p_3^{\mu_2} \eta_{\mu_1, \mu_3} - \\
 & 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} p_2^{\mu_4} p_3^{\mu_2} \eta_{\mu_1, \mu_3} - 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} p_1^{\mu_2} p_3^{\mu_4} \eta_{\mu_1, \mu_3} - 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} p_1^{\mu_4} p_4^{\mu_2} \eta_{\mu_1, \mu_3} + \\
 & 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} p_3^{\mu_4} p_4^{\mu_2} \eta_{\mu_1, \mu_3} - 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} p_1^{\mu_2} p_2^{\mu_3} \eta_{\mu_1, \mu_4} - 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} p_1^{\mu_3} p_3^{\mu_2} \eta_{\mu_1, \mu_4} + \\
 & 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} p_1^{\mu_3} p_4^{\mu_2} \eta_{\mu_1, \mu_4} - 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} p_2^{\mu_3} p_4^{\mu_2} \eta_{\mu_1, \mu_4} - 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} p_1^{\mu_2} p_4^{\mu_3} \eta_{\mu_1, \mu_4} - \\
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 & 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} p_3^{\mu_1} p_4^{\mu_2} \eta_{\mu_3, \mu_4} + 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} p_1 \cdot p_2 - 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} p_1 \cdot p_2 + \\
 & 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} p_1 \cdot p_3 + 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} p_1 \cdot p_4 + 6ig_s f_{a_1, a_4, a} f_{a_2, a_3, a} \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} p_2 \cdot p_3 + \\
 & 6ig_s f_{a_1, a_3, a} f_{a_2, a_4, a} \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} p_2 \cdot p_4 + 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} p_3 \cdot p_4 - 6ig_s f_{a_1, a_2, a} f_{a_3, a_4, a} \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} p_3 \cdot p_4
 \end{aligned}$$

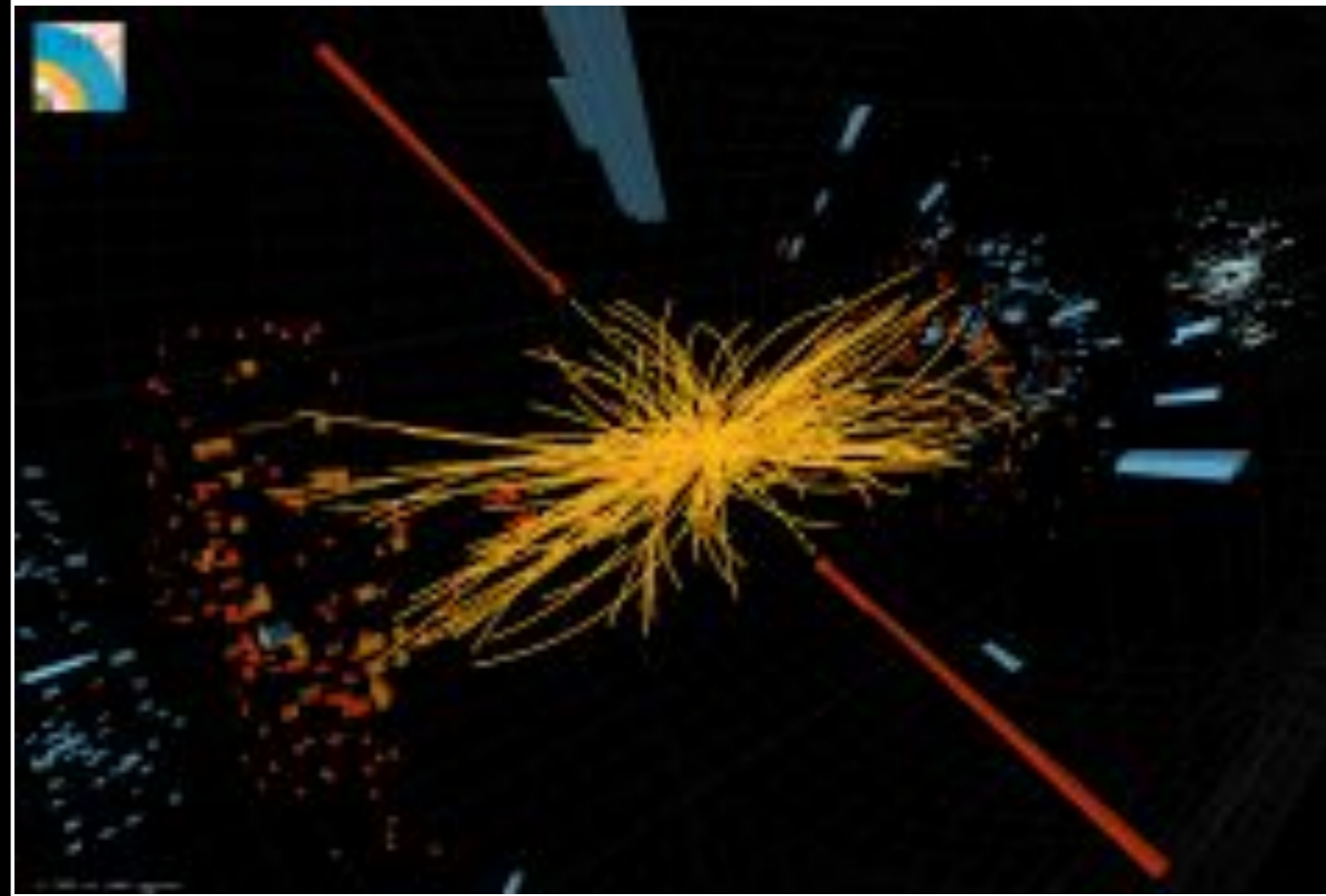
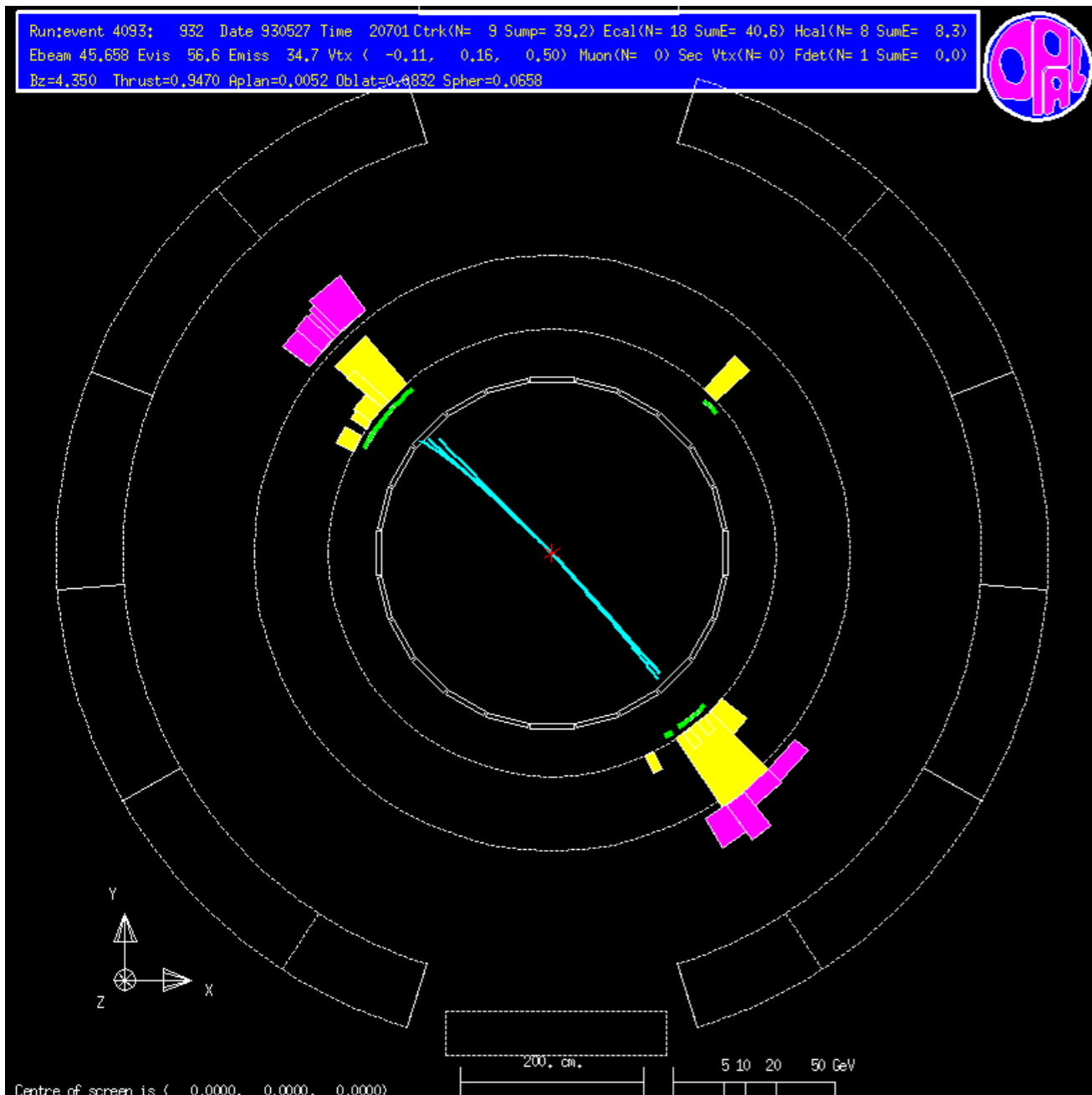
Many diagrams



Hadron colliders

LEP

LHC



Hadron collider event

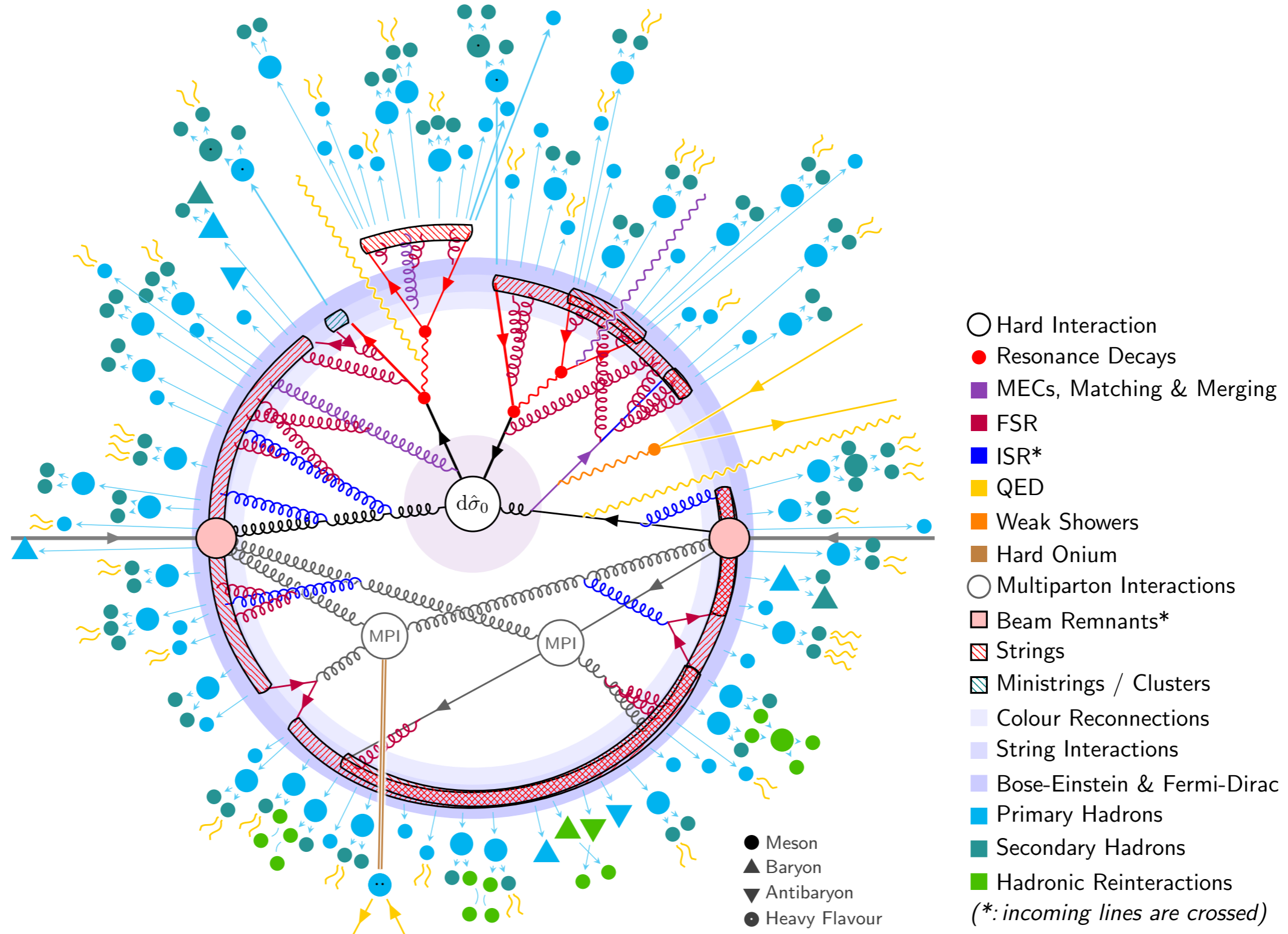


Figure taken from Bierlich et al., 2022 (*Pythia8.3 manual*)

Hadron collider event

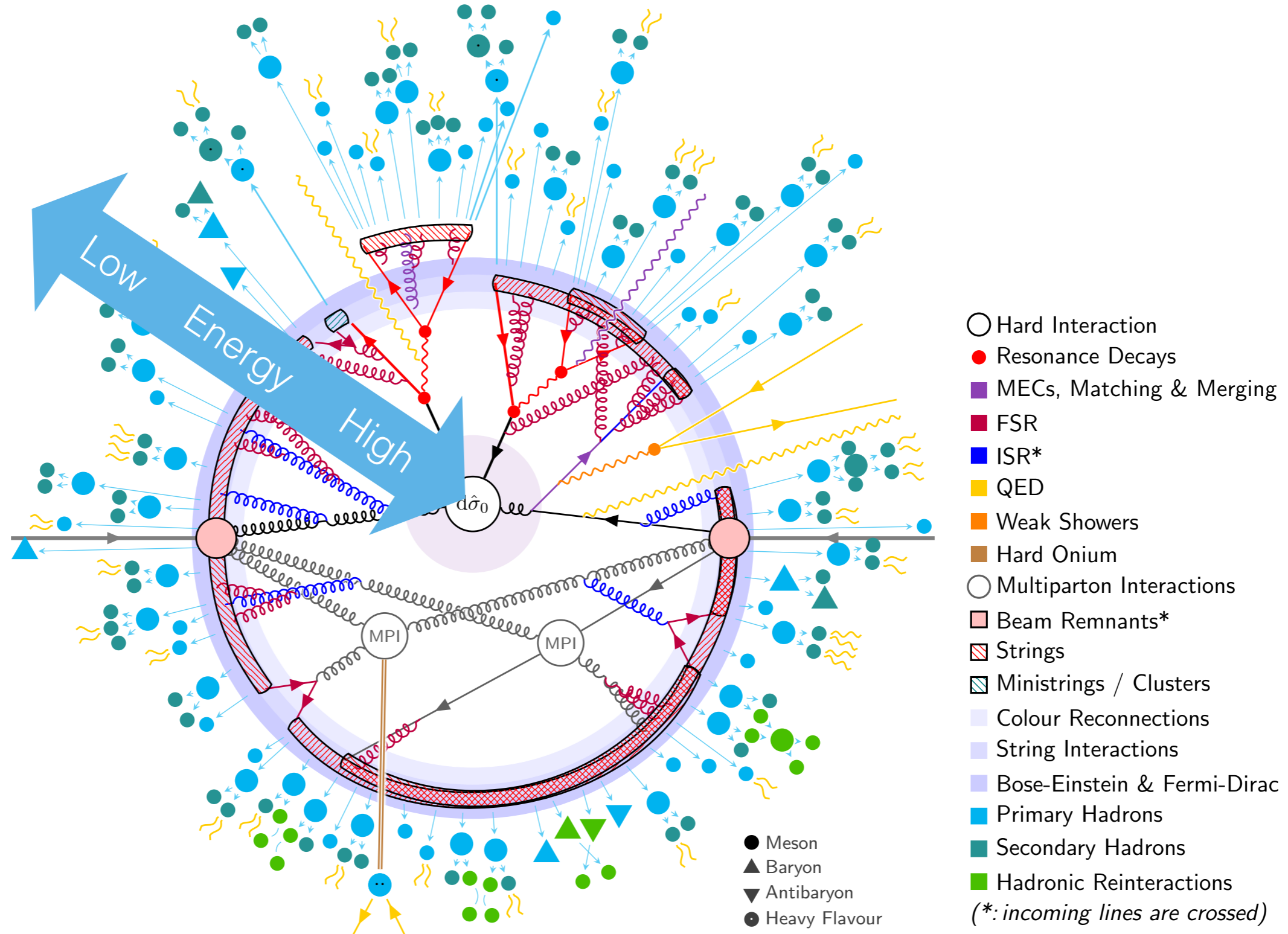


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Hadron collider event

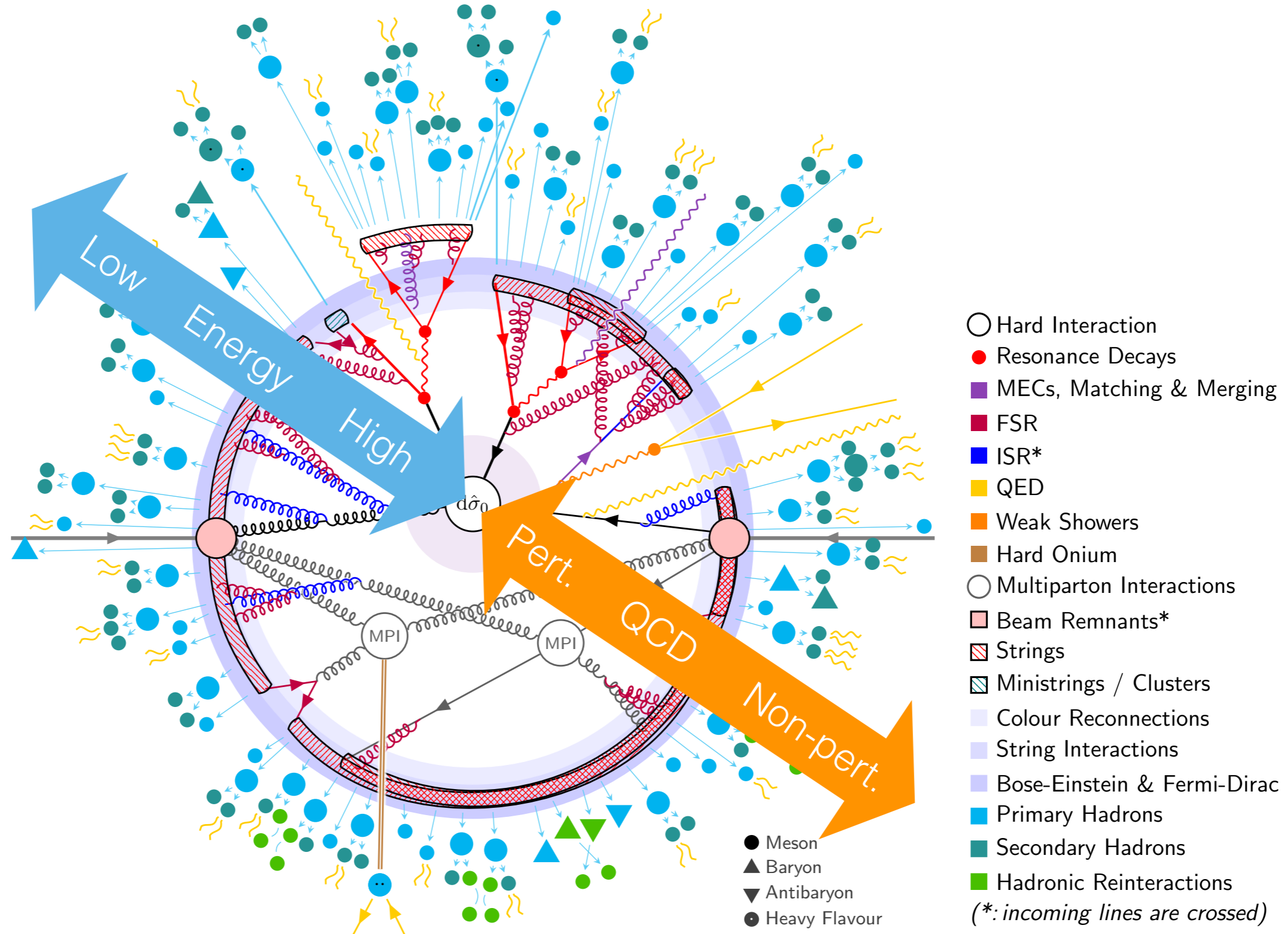
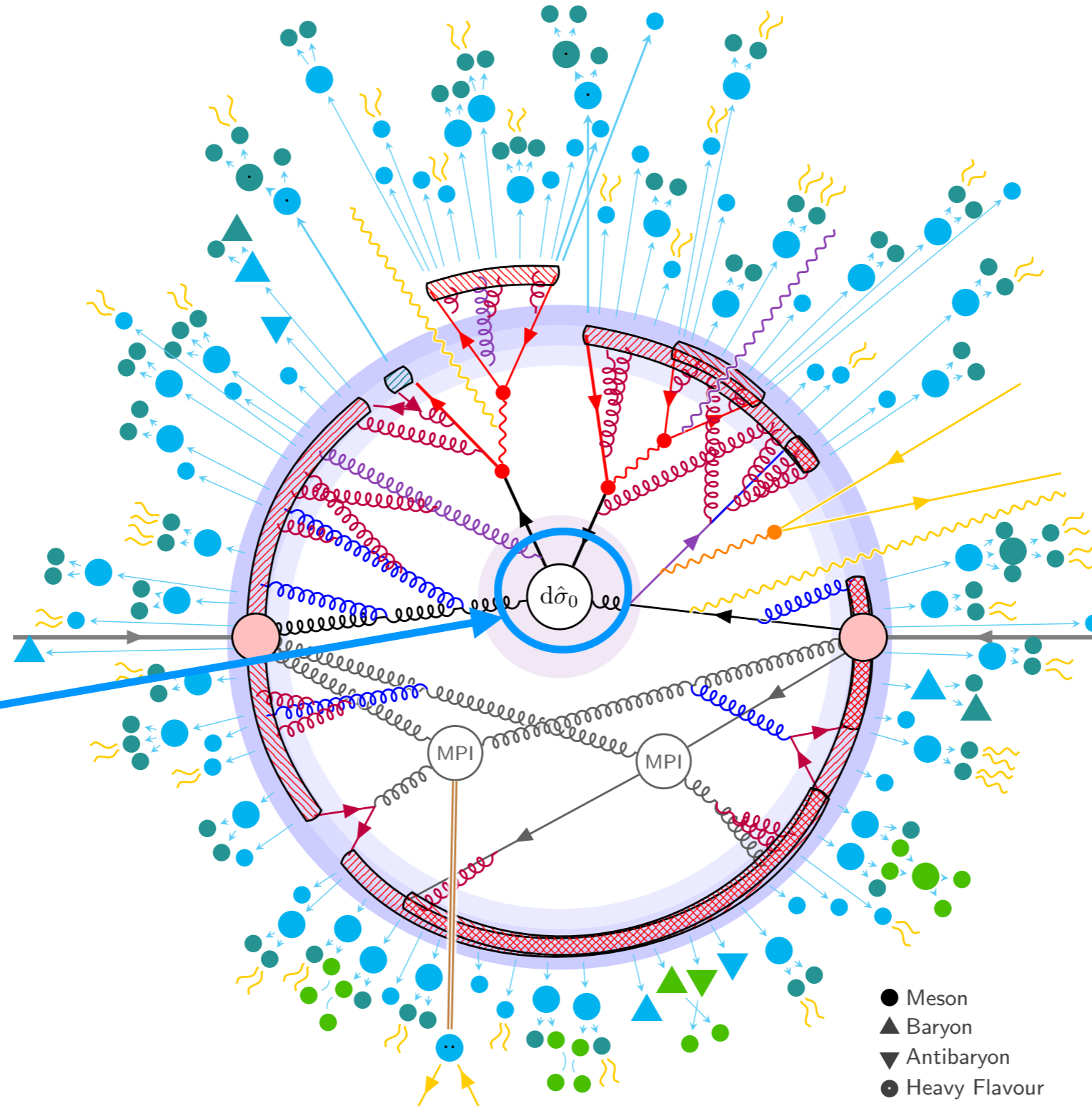


Figure taken from Bierlich et al., 2022 (*Pythia8.3 manual*)

Hadron collider event



- Hard Interaction
 - Resonance Decays
 - MECs, Matching & Merging
 - FSR
 - ISR*
 - QED
 - Weak Showers
 - Hard Onium
 - Multiparton Interactions
 - Beam Remnants*
 - Strings
 - Ministrings / Clusters
 - Colour Reconnections
 - String Interactions
 - Bose-Einstein & Fermi-Dirac
 - Primary Hadrons
 - Secondary Hadrons
 - Hadronic Reinteractions
- (*: incoming lines are crossed)

- Meson
- ▲ Baryon
- ▼ Antibaryon
- Heavy Flavour

Feynman diagrams

Figure taken from Bierlich et al., 2022 (*Pythia8.3 manual*)

Hadron collider event

Approximate
Feynman diagrams

Feynman
diagrams

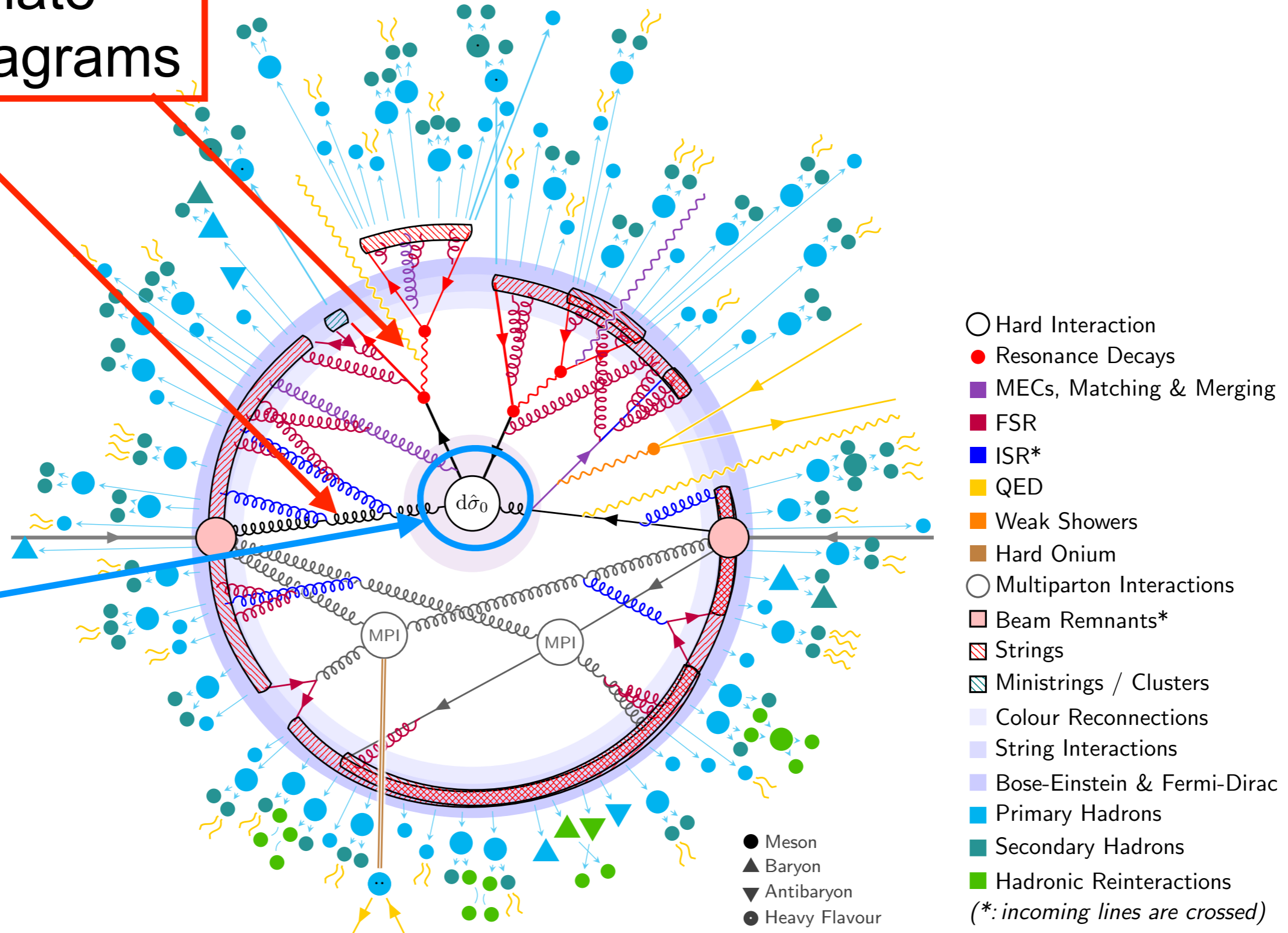


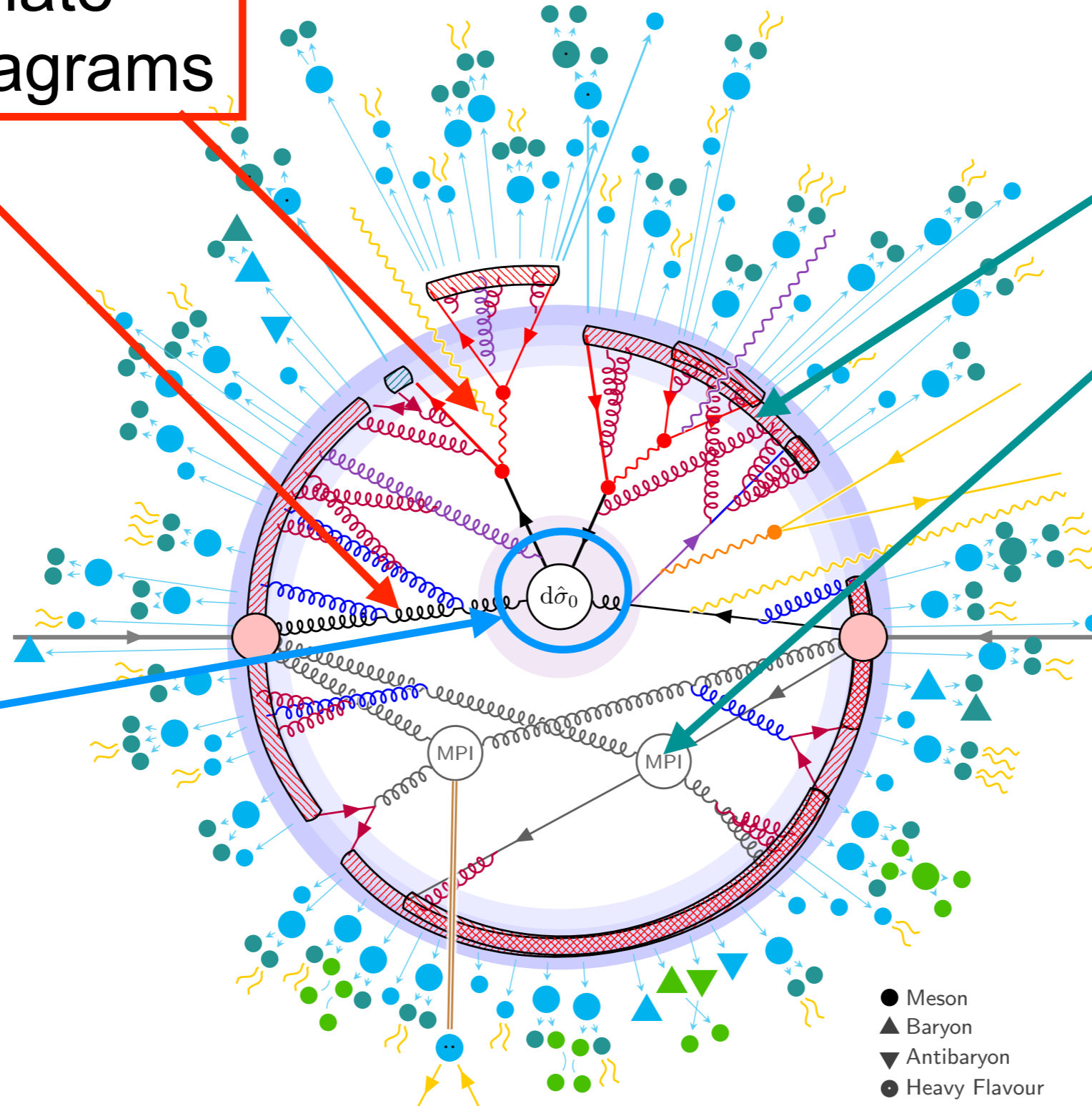
Figure taken from Bierlich et al., 2022 (*Pythia8.3 manual*)

Hadron collider event

Approximate
Feynman diagrams

Models

Feynman
diagrams



- Hard Interaction
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Figure taken from Bierlich et al., 2022 (*Pythia8.3 manual*)

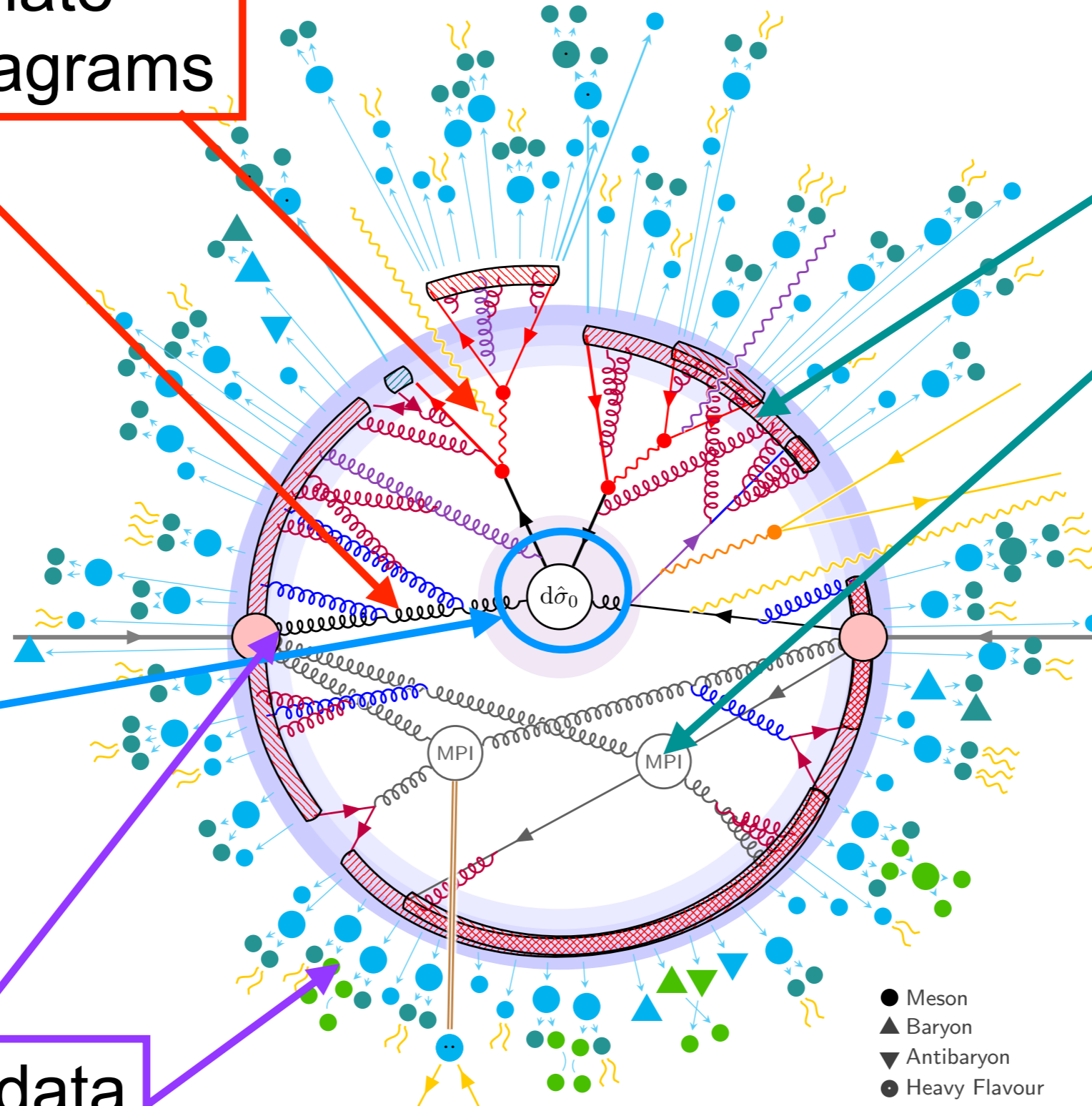
Hadron collider event

Approximate
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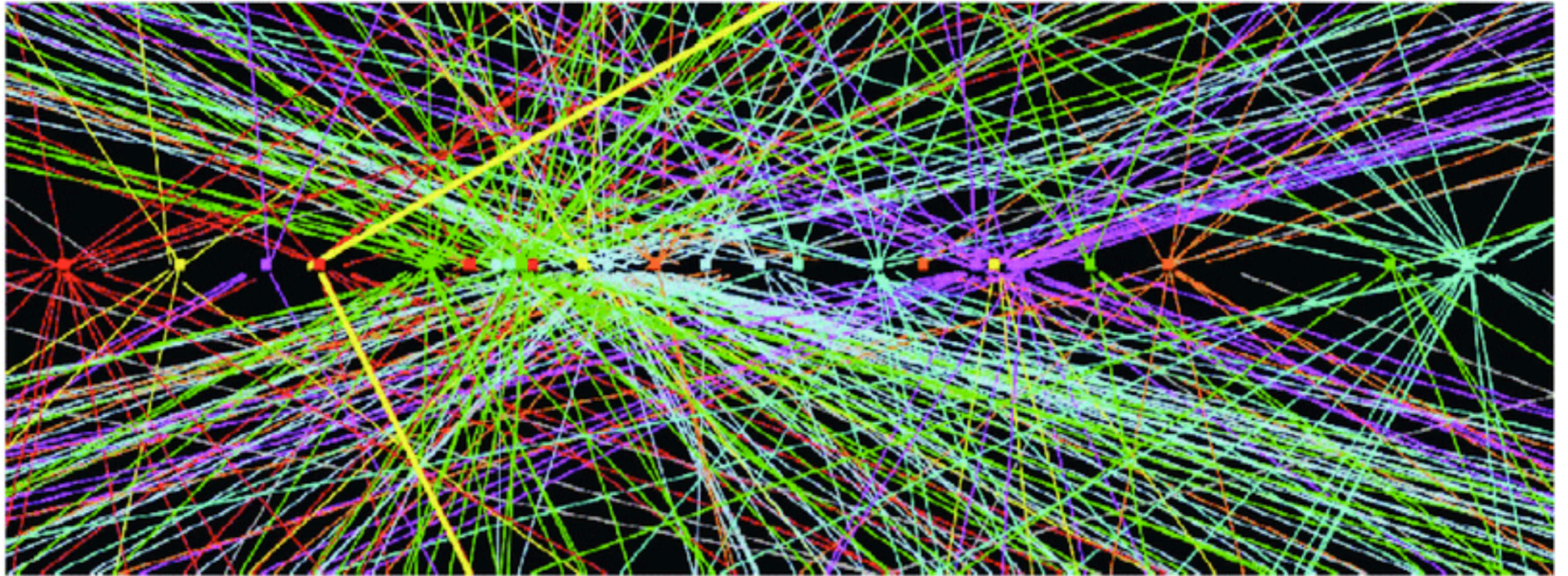
From data



- Hard Interaction
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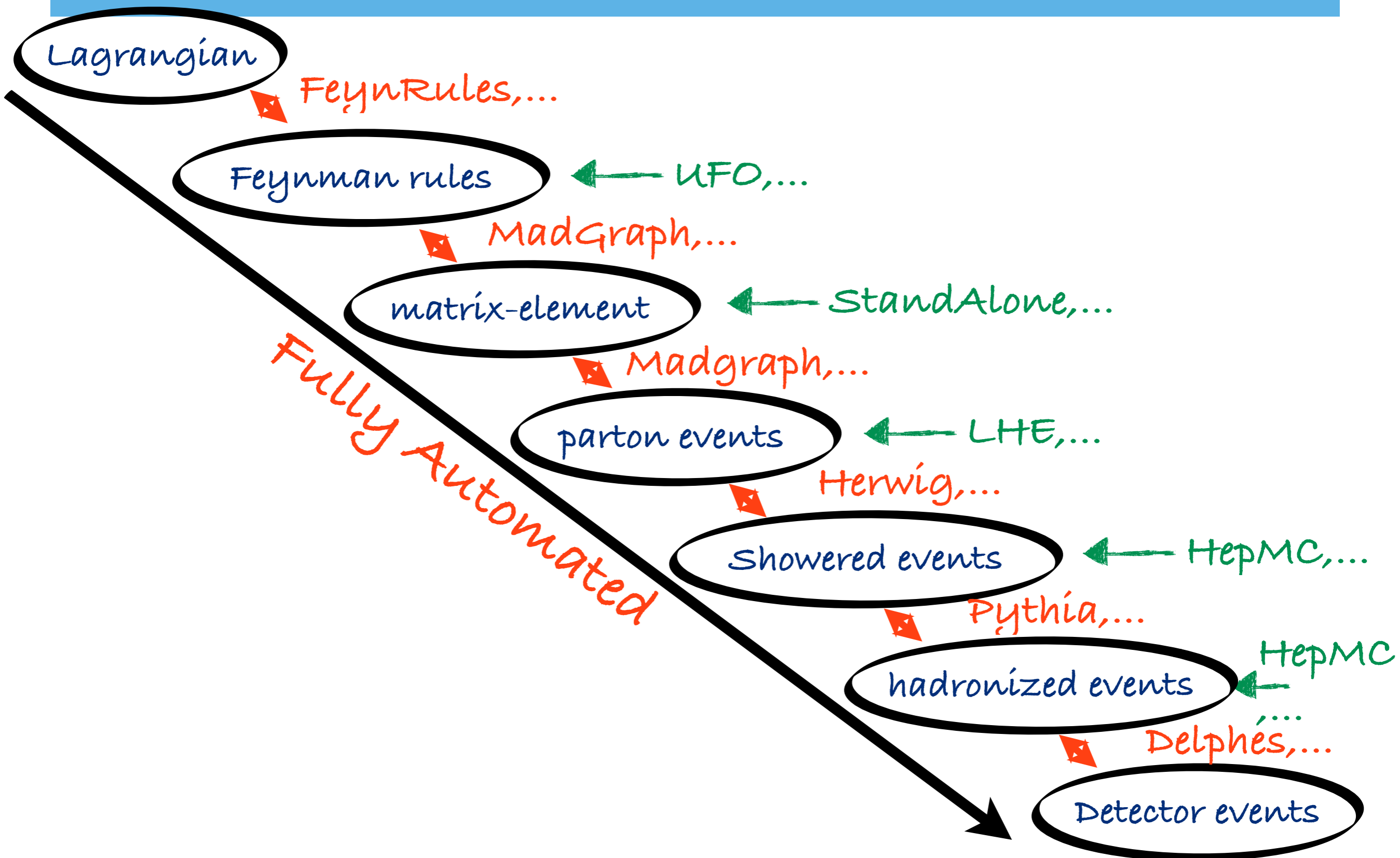
Figure taken from Bierlich et al., 2022 (*Pythia8.3 manual*)

Pile up



A visual example of pile-up in the ATLAS tracker: a Run 1 $Z \rightarrow$ event collected at an instantaneous luminosity $L = 0.5 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ in 8 TeV pp collisions. Two thick yellow lines show muon tracks from the Z final state, triggered among pileup events.

BSM simulation



Where is the new physics?

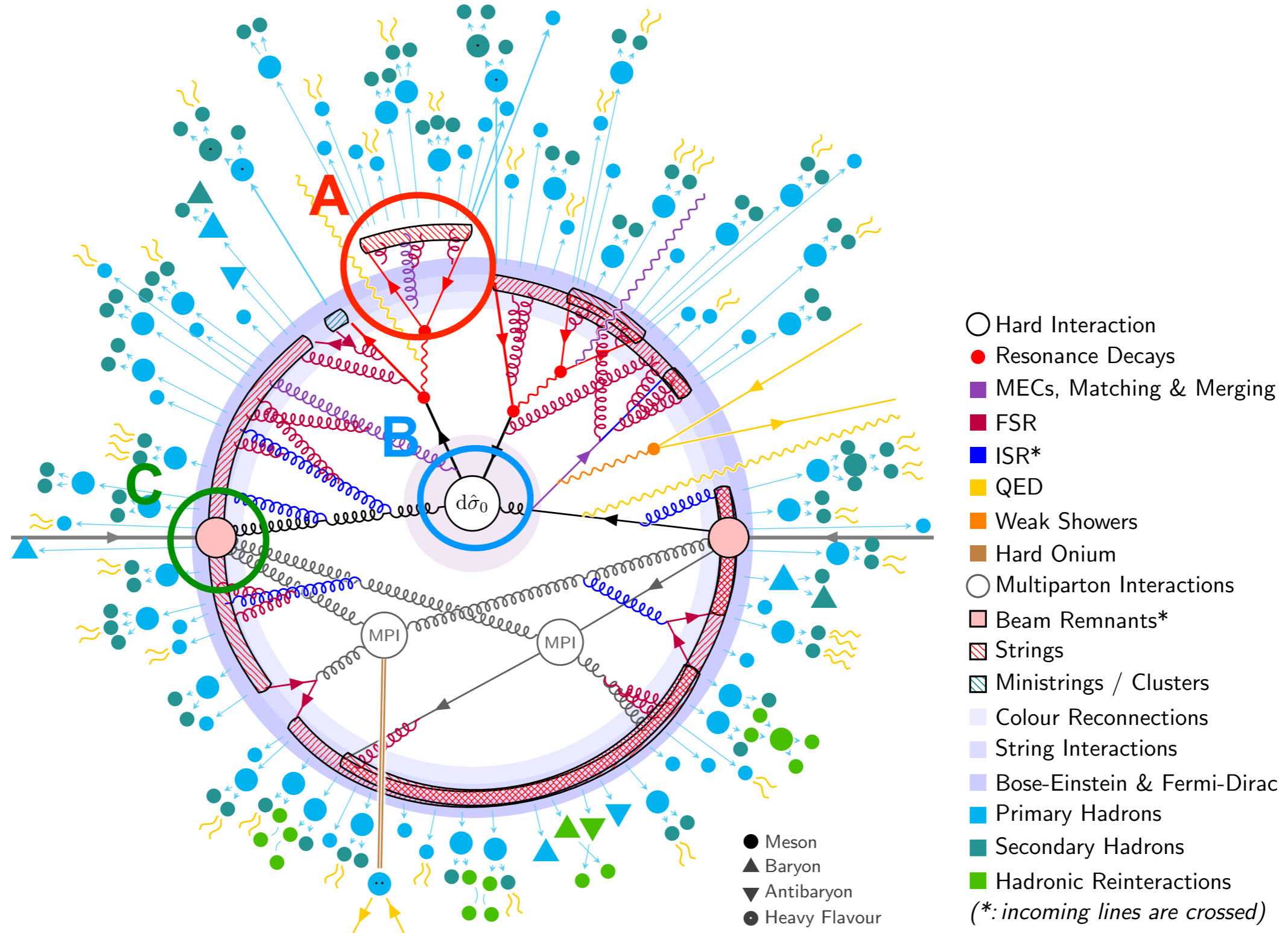


Figure taken from Bierlich et al., 2022 (*Pythia8.3 manual*)

Under which assumptions?

1. The new physics is weakly coupled
2. The new physics is strongly coupled
3. The new physics is heavy
4. The new physics is light

Leading order at fixed order

Which tools



Only if the BSM
model is not available



For the exercices
Today : SM only

Who has Mathematica?

Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

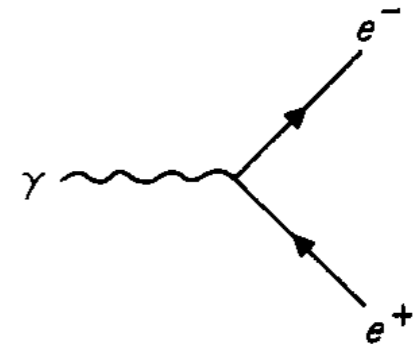
- In momentum space

Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

- In momentum space
- For each vertex

$$-ieQ\gamma^\mu$$

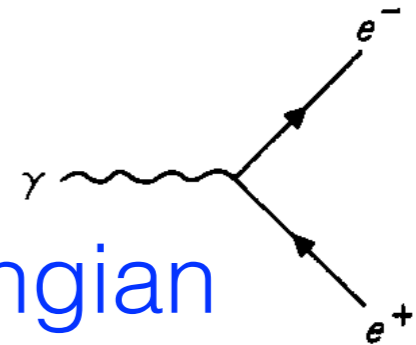


Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

- In momentum space
- For each vertex

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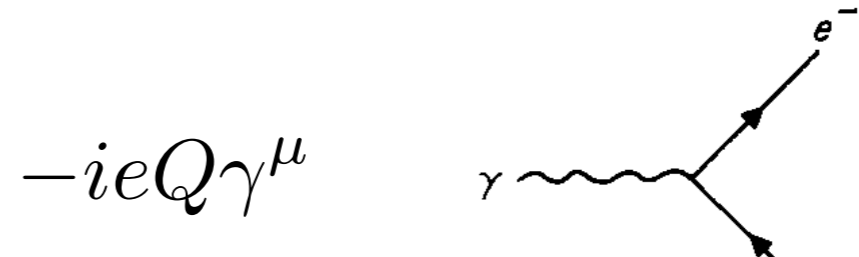
Extracted by FeynRules from the Lagrangian

Used in MadGraph5_aMC@NLO

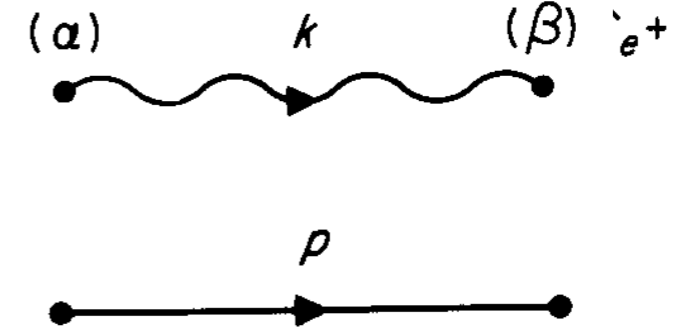
Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

- In momentum space
- For each vertex



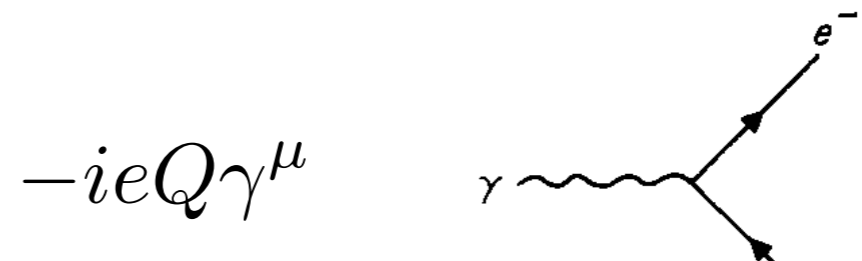
- For each internal line
- $$\frac{-i\eta_{\alpha\beta}}{k^2 + i\epsilon}$$
- $$\frac{i}{\cancel{p} - m + i\epsilon}$$



Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

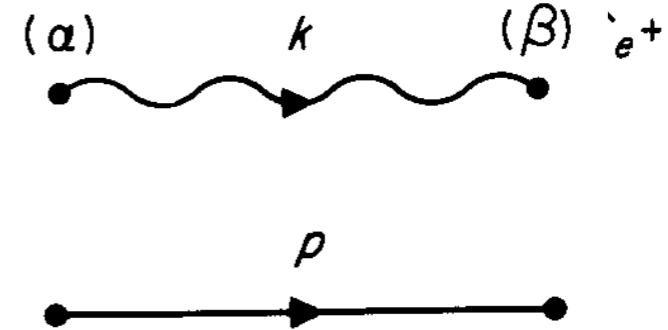
- In momentum space
- For each vertex



- For each internal line

$$\frac{-i\eta_{\alpha\beta}}{k^2 + i\epsilon}$$

$$\frac{i}{\not{p} - m + i\epsilon}$$



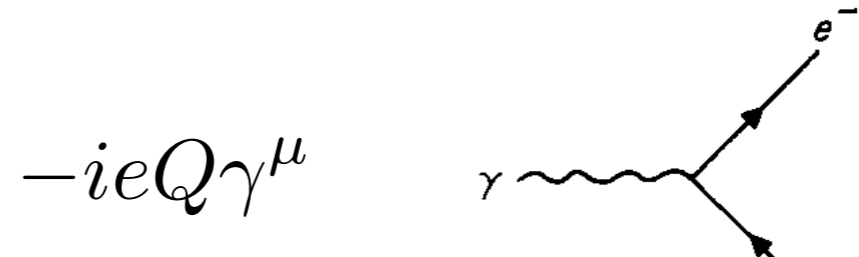
Can be checked in FeynRules

Assumed in MadGraph5_aMC@NLO

Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

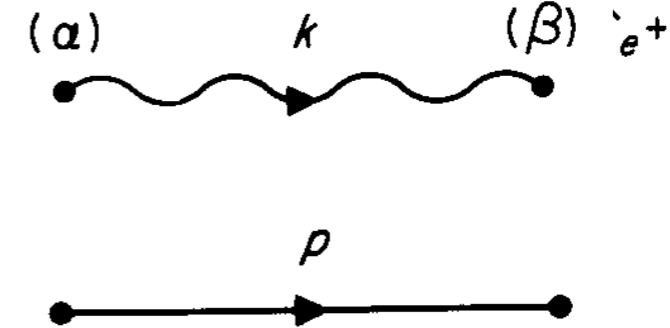
- In momentum space
- For each vertex



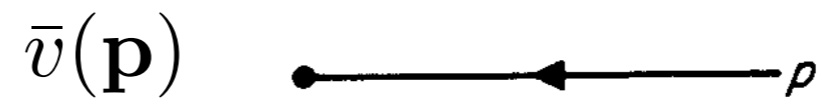
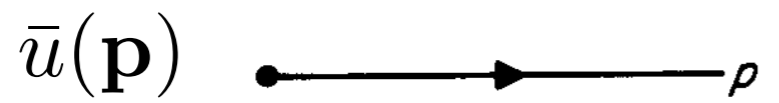
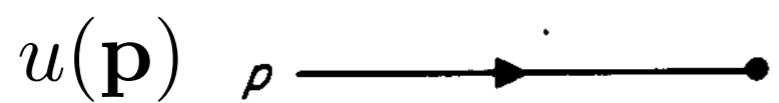
- For each internal line

$$\frac{-i\eta_{\alpha\beta}}{k^2 + i\epsilon}$$

$$\frac{1}{\cancel{p} - m + i\epsilon}$$



- For each external line

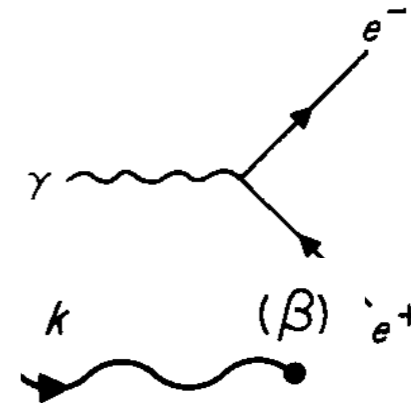


Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

- In momentum space
- For each vertex

$$-ieQ\gamma^\mu$$



- For each internal line $-i\eta_{\alpha\beta}$ (a)
- Can be checked in FeynRules

Assumed in MadGraph5_aMC@NLO

- For each external line

$$u(\mathbf{p}) \quad \rho \longrightarrow \bullet$$

$$v(\mathbf{p}) \quad \rho \longleftarrow \bullet$$

$$\bar{u}(\mathbf{p}) \quad \bullet \longrightarrow \rho$$

$$\bar{v}(\mathbf{p}) \quad \bullet \longleftarrow \rho$$

$$\varepsilon(\mathbf{k}) \quad k \text{ (a)} \text{ wavy line}$$

$$\varepsilon^*(\mathbf{k}) \quad \text{(a)} \text{ wavy line } k$$

Feynman Rules

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta(P_f - P_i) \prod_{\text{ext. F.}} \sqrt{\frac{m}{VE}} \prod_{\text{ext. A.}} \sqrt{\frac{1}{2V\omega}} \mathcal{M}$$

- In momentum space
 - Spinor factor ordered against the fermion flow and relative sign of the amplitude depending on their order
 - Close fermion loop : -1 and Trace
 - For each loop, integration over the momentum not fixed by momentum conservation

$$\int \frac{d^4 p}{(2\pi)^4}$$

- -1 for each exchange of fermions operators

Feynman Rules

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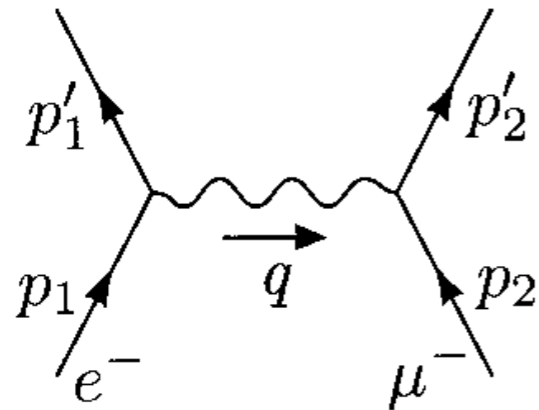
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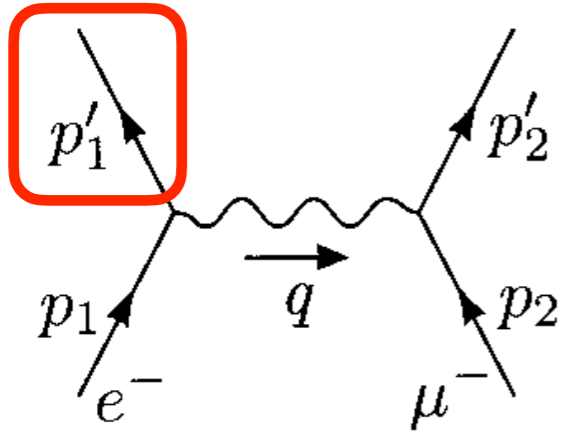
Writing the Feynman amplitude



From Peskin and Schroeder

$$= \frac{ie^2}{q^2} \bar{u}(p'_1) \gamma^\mu u(p_1) \bar{u}(p'_2) \gamma_\mu u(p_2).$$

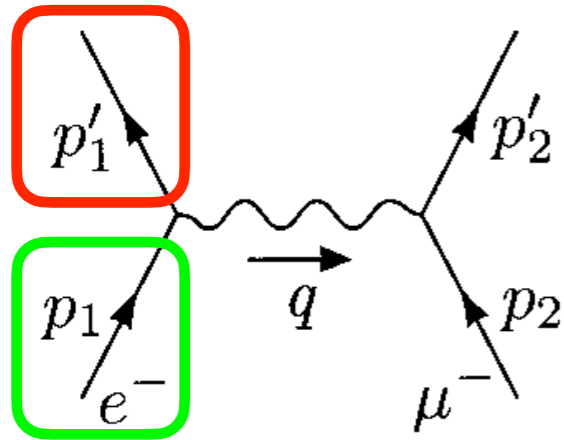
Writing the Feynman amplitude



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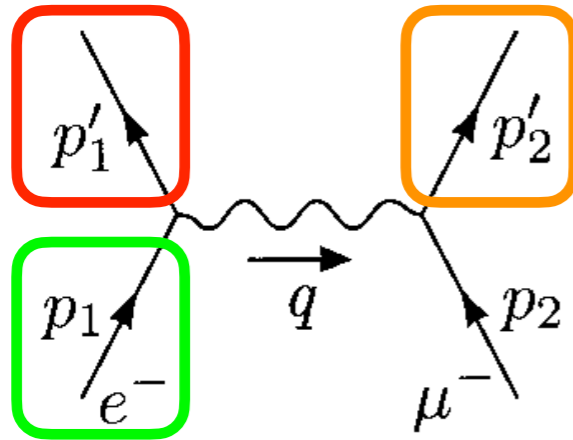
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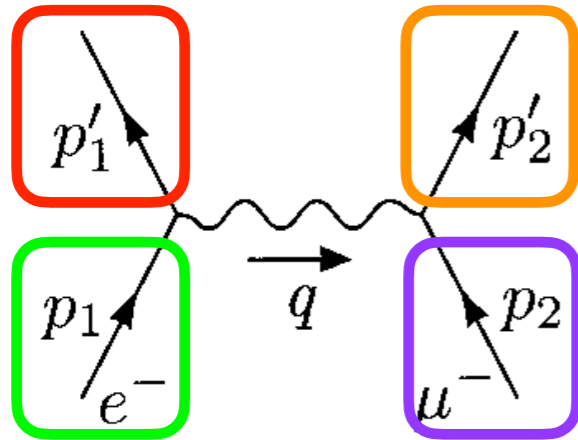
Writing the Feynman amplitude



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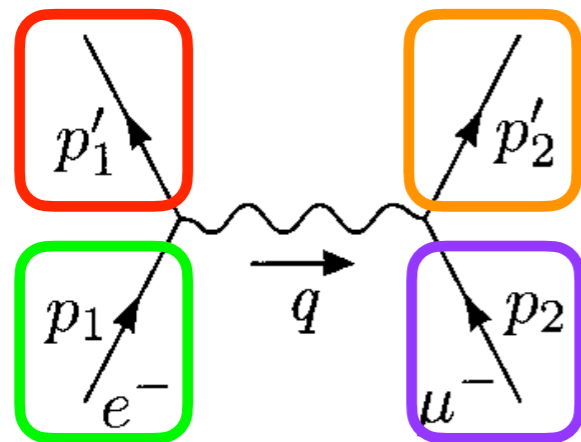
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function of momentum and spin

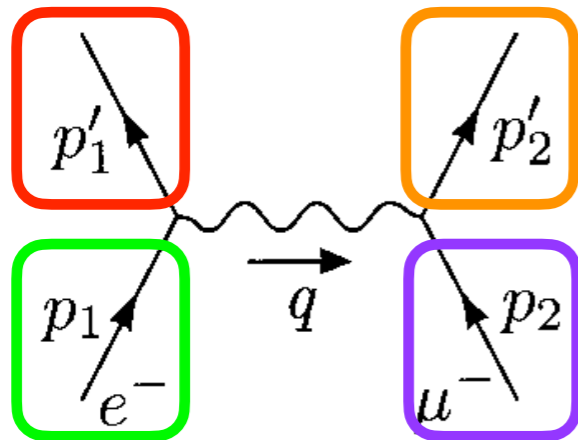
$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p) \chi_\lambda(\vec{p}) \\ \omega_\lambda(p) \chi_\lambda(\vec{p}) \end{pmatrix}$$

$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix},$$

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HELAS : helicity amplitudes

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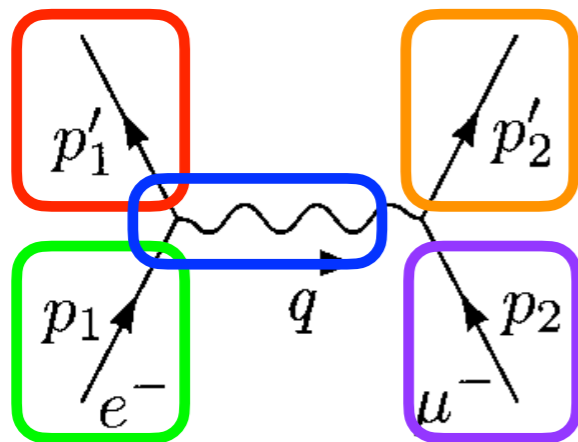
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- Incoming/outgoing fermion
- FFV vertex with/without propagator
- Each output but the last is a spinor/polarisation vector or scalar depending on the particle

Writing the Feynman amplitude



From Peskin and Schroeder

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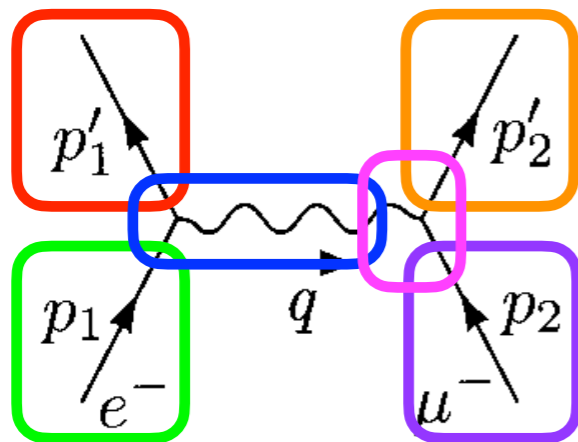
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$$W_a = fct(\bar{v}_1, u_2, e, m_a, \Gamma_a) = e \bar{v}_1 \gamma^\mu u_2 \frac{g_{\mu\nu}}{q^2 - m_a^2 + im_a \Gamma_a}$$

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Spin and co

In textbook :

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m; \quad \sum_s v^s(p)\bar{v}^s(p) = \not{p} - m.$$

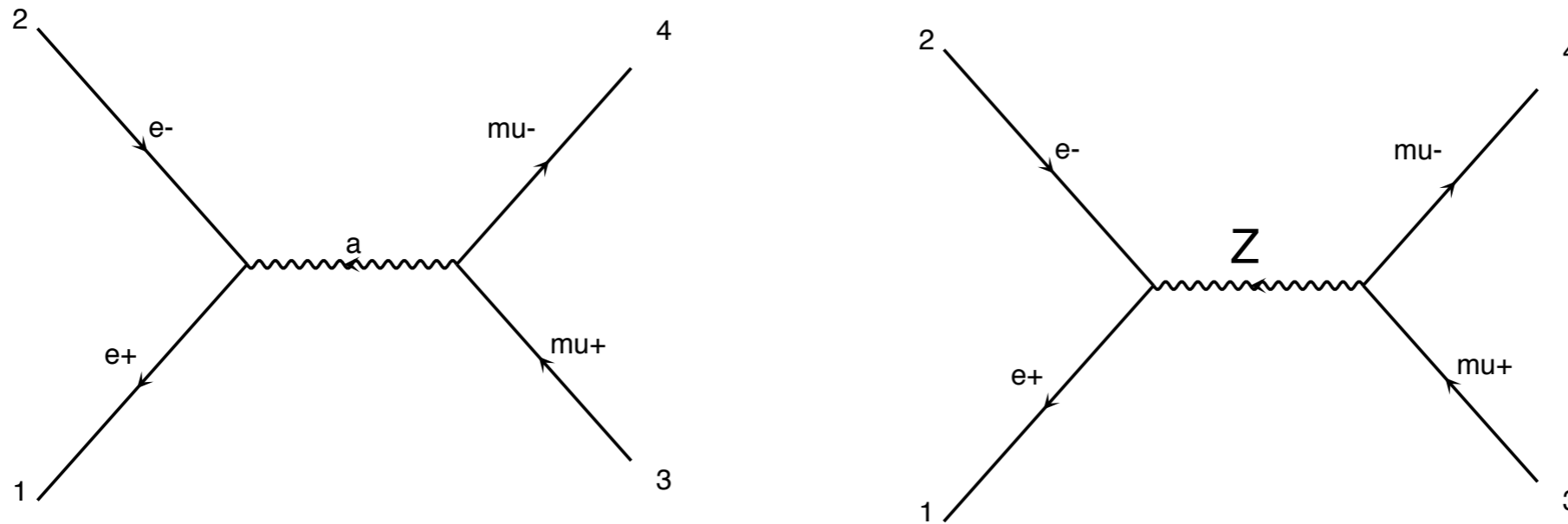
$$\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{4q^4} \text{tr}[(\not{p}'_1 + m_e)\gamma^\mu(\not{p}_1 + m_e)\gamma^\nu] \text{tr}[(\not{p}'_2 + m_\mu)\gamma_\mu(\not{p}_2 + m_\mu)\gamma_\nu]$$

$$\frac{8e^4}{q^4} [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)]$$

In MG : No spin sum

- Faster
- needed to describe the final/initial state
- Evaluate \mathcal{M} for fixed helicity of external particles
 - Multiply \mathcal{M} with \mathcal{M}^* -> $|\mathcal{M}|^2$
 - Loop on Helicity and average the results

Number of term to be computed



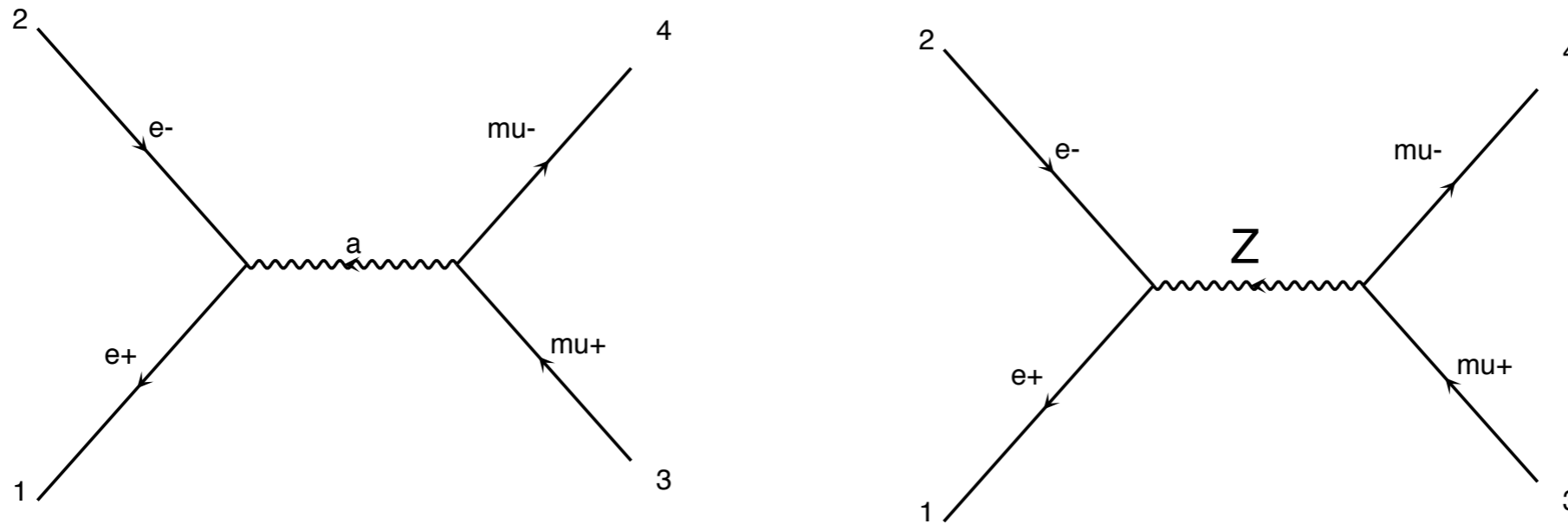
Need to compute $|M_a|^2$ $|M_z|^2$ $2\text{Re}(M_a^*M_z)$

So for M Feynman diagram we need to compute M^2
different term

The number of diagram scales **factorially** with the number
of particle

In practise possible up to $2 > 4$

Number of term to be computed



Need to compute $|M_a|^2$ $|M_z|^2$ $2\text{Re}(M_a^* M_z)$
 $e^+ e^- > \mu^+ \mu^-$, without z/a

So for M Feynman diagram we need to compute M^2
 different term

The number of diagram scales **factorially** with the number
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In practise possible up to $2 > 4$

Comparison

$$|\mathcal{M}_i|^2, \text{Re}(\mathcal{M}_i \mathcal{M}_j)$$

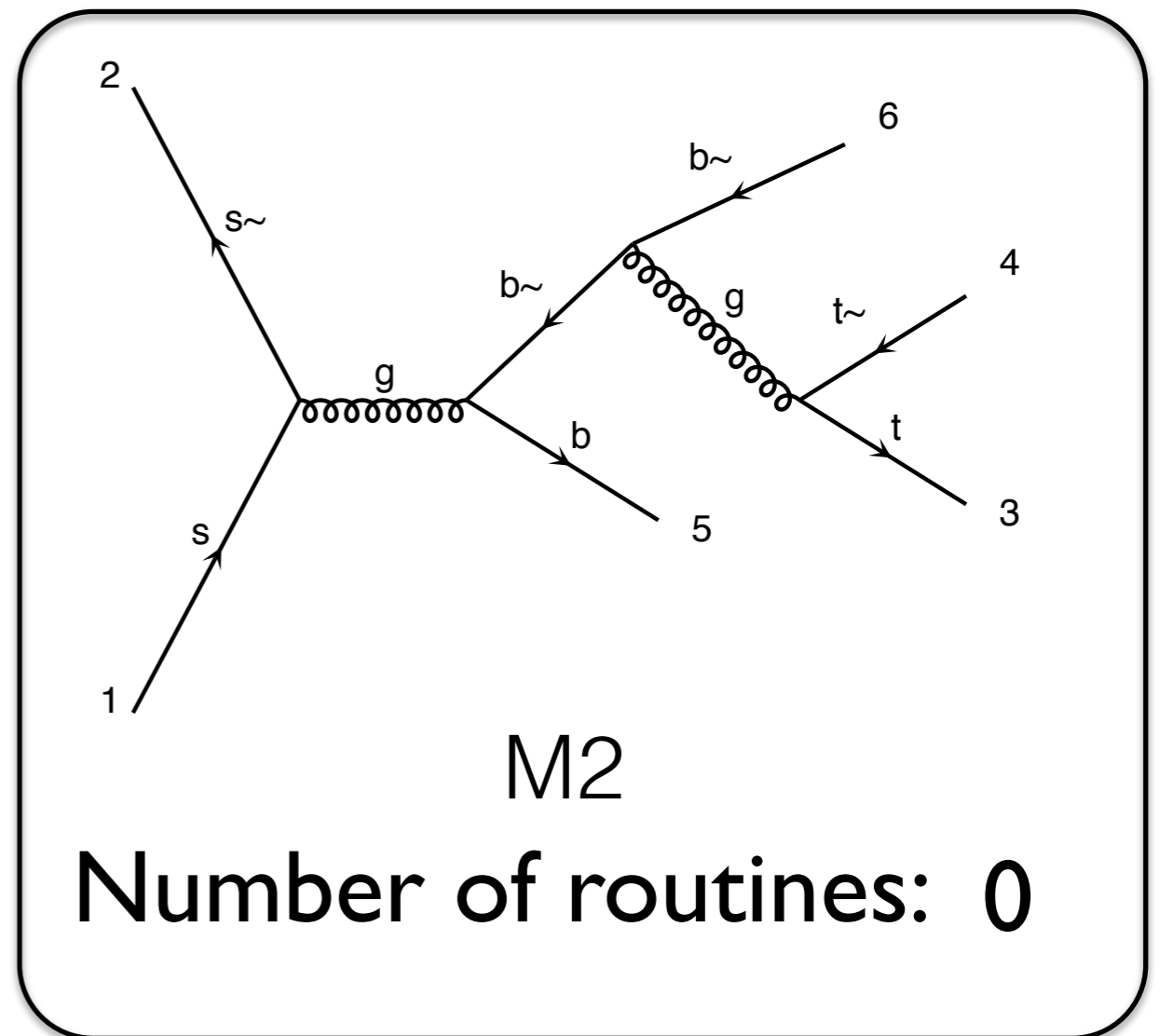
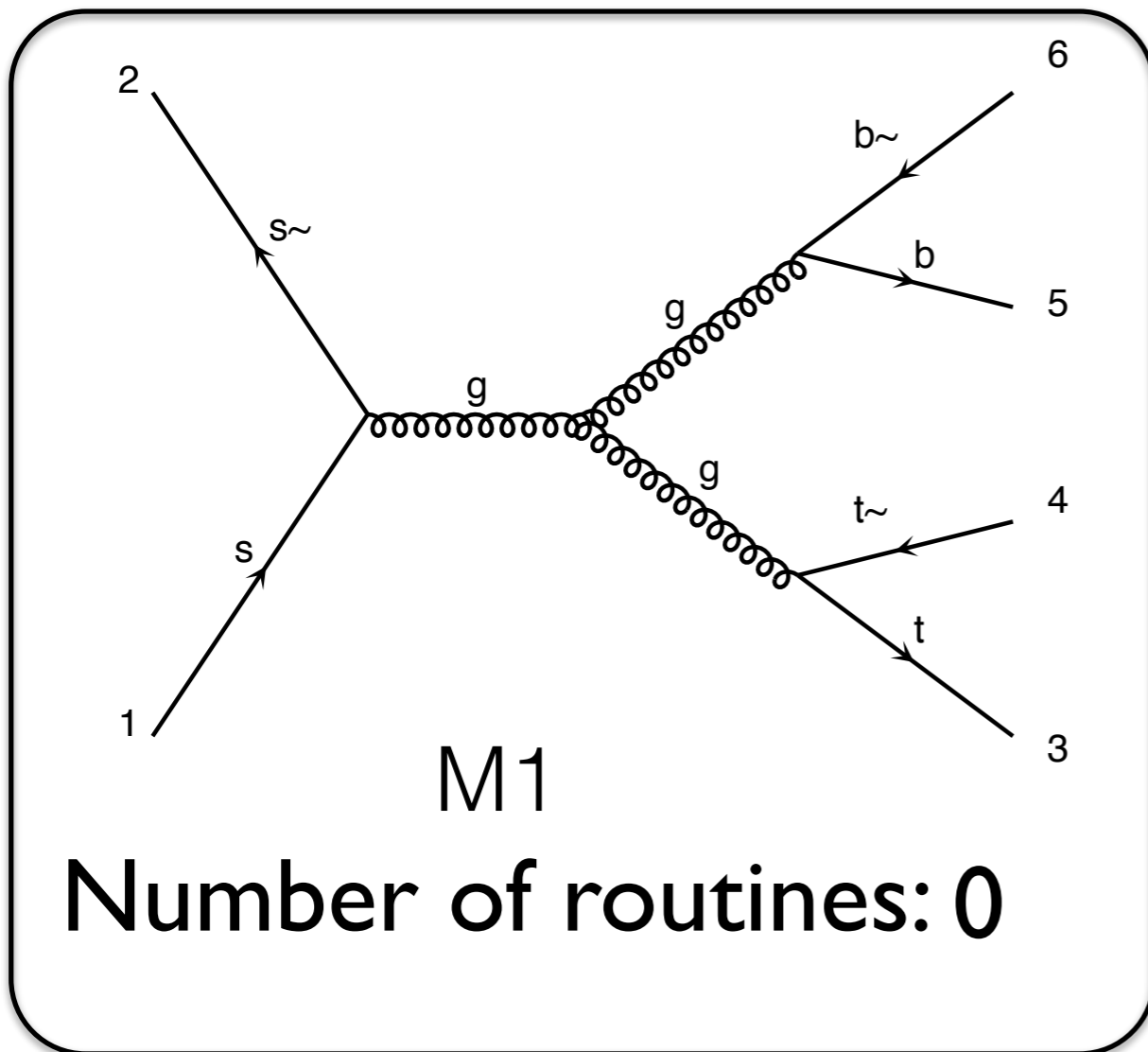
	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!) 2^N$

$$\left(\sum_i \mathcal{M}_i \right)^2$$

$$M \propto N!$$

Recycling

■ Known

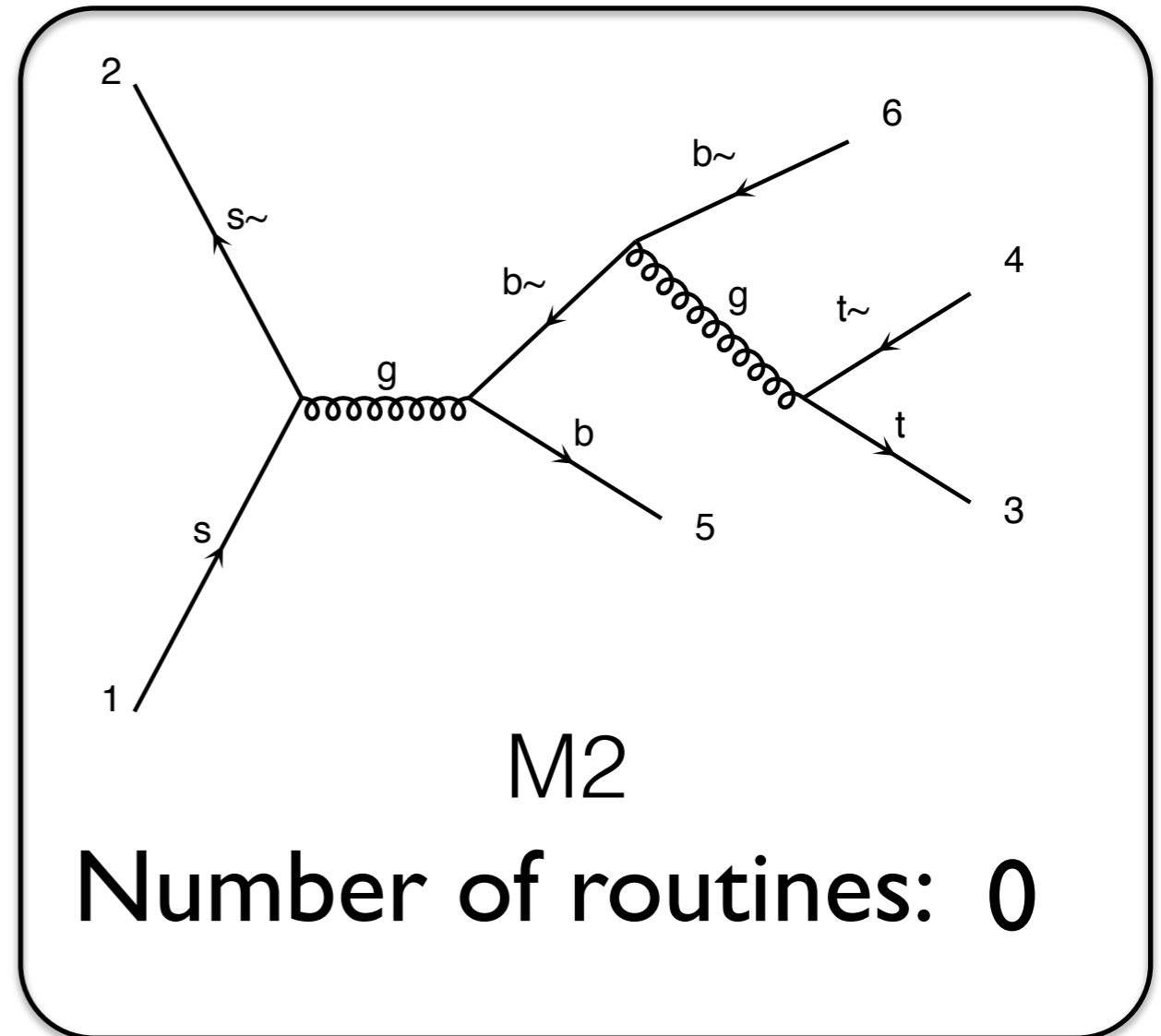
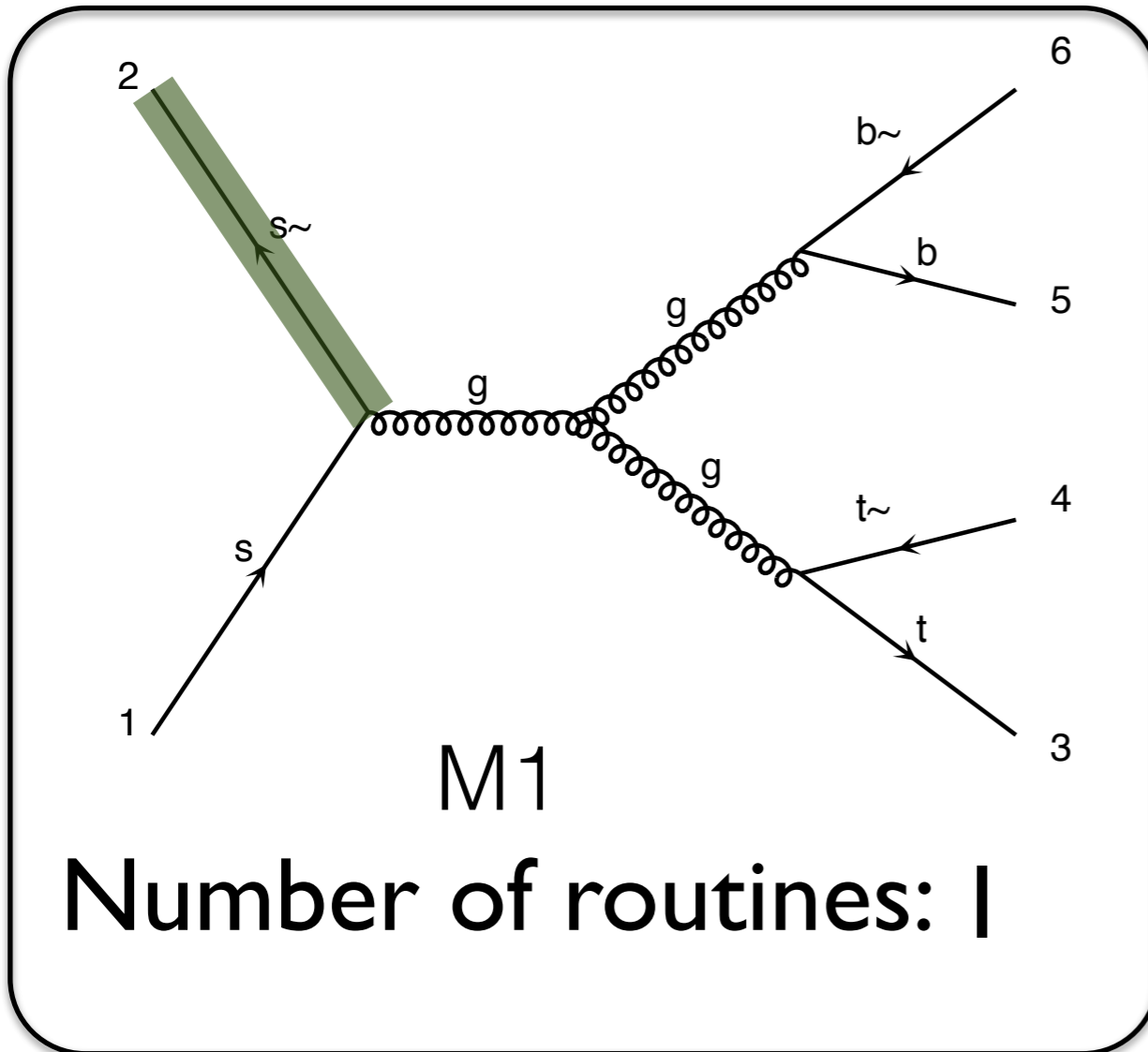


Number of routines for both: 0

$$|M|^2 = |M_1 + M_2|^2$$

Recycling

■ Known



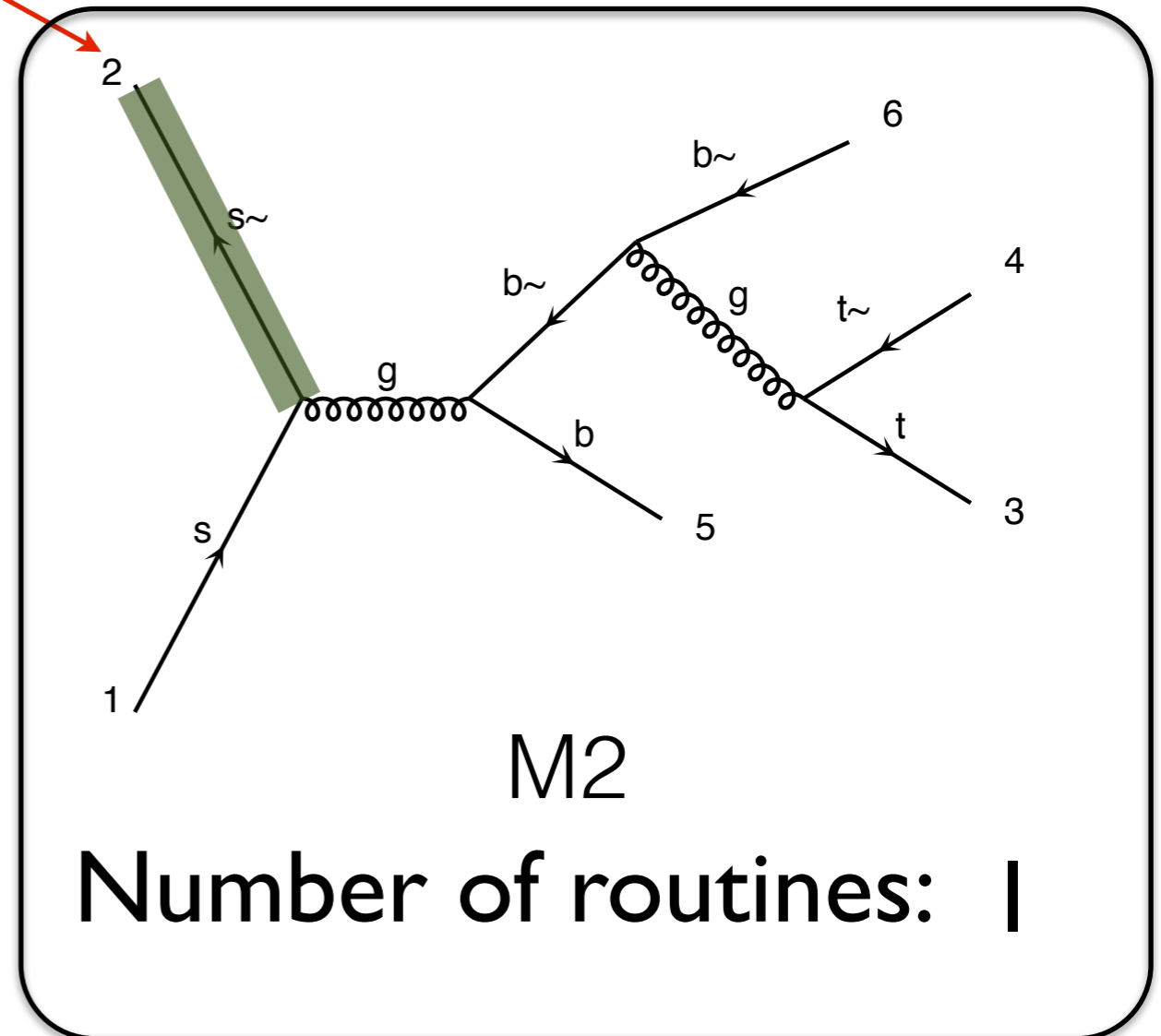
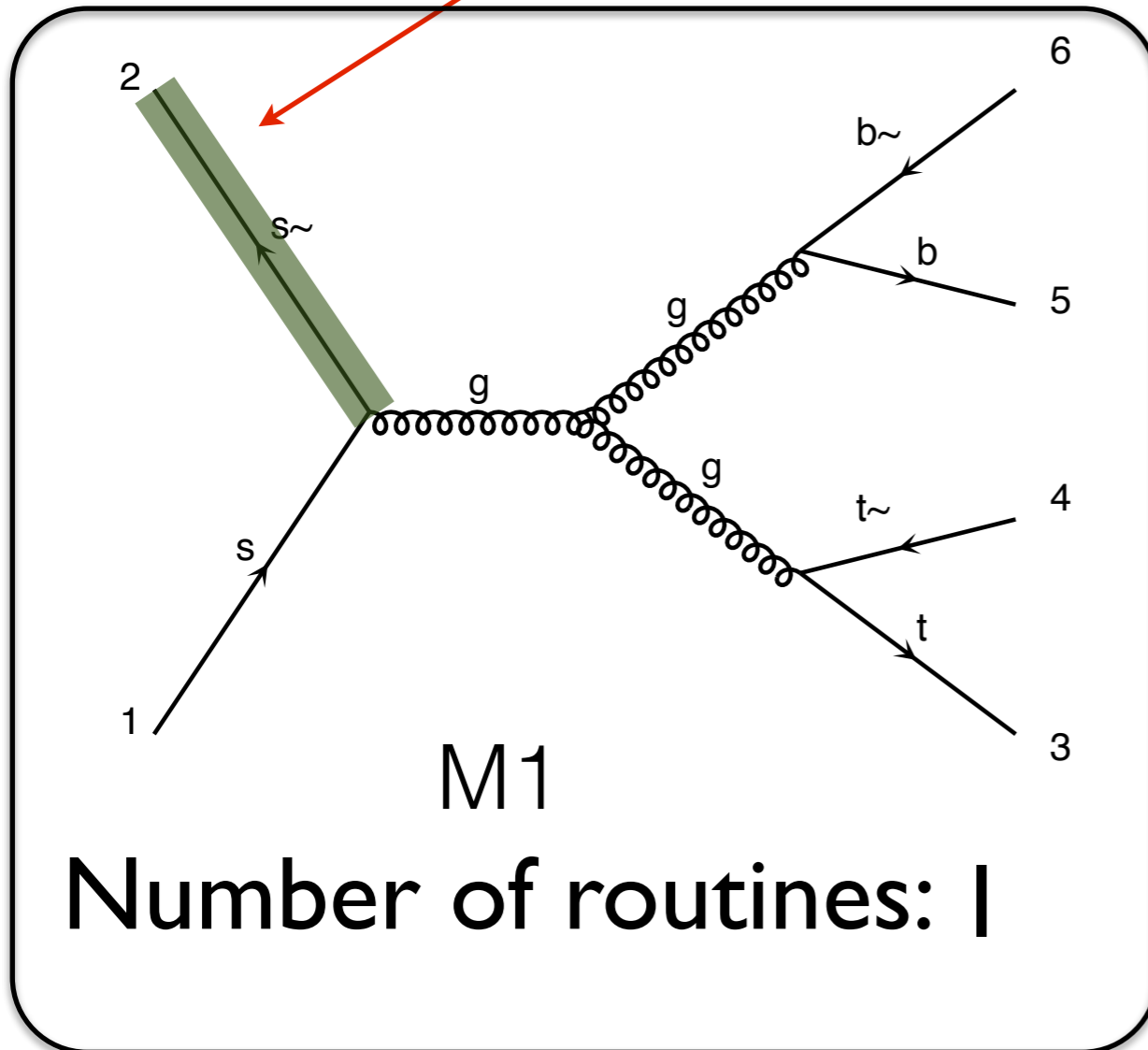
Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

Recycling

Identical

Known

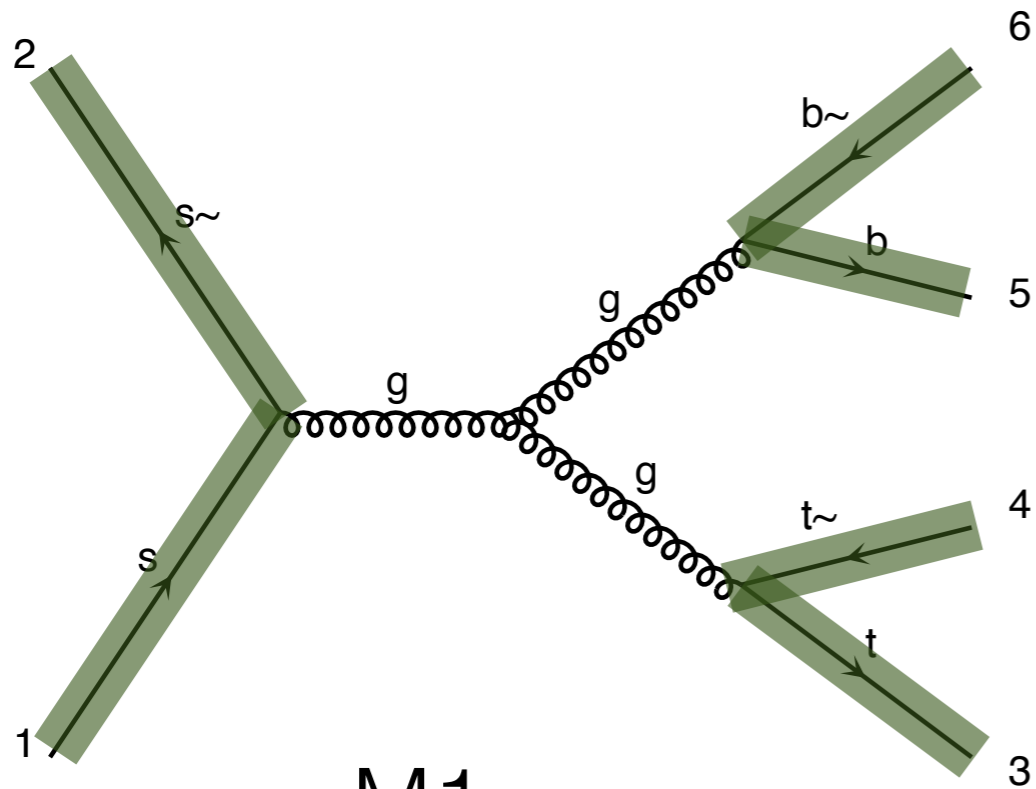


Number of routines for both: 1

$$|M|^2 = |M_1 + M_2|^2$$

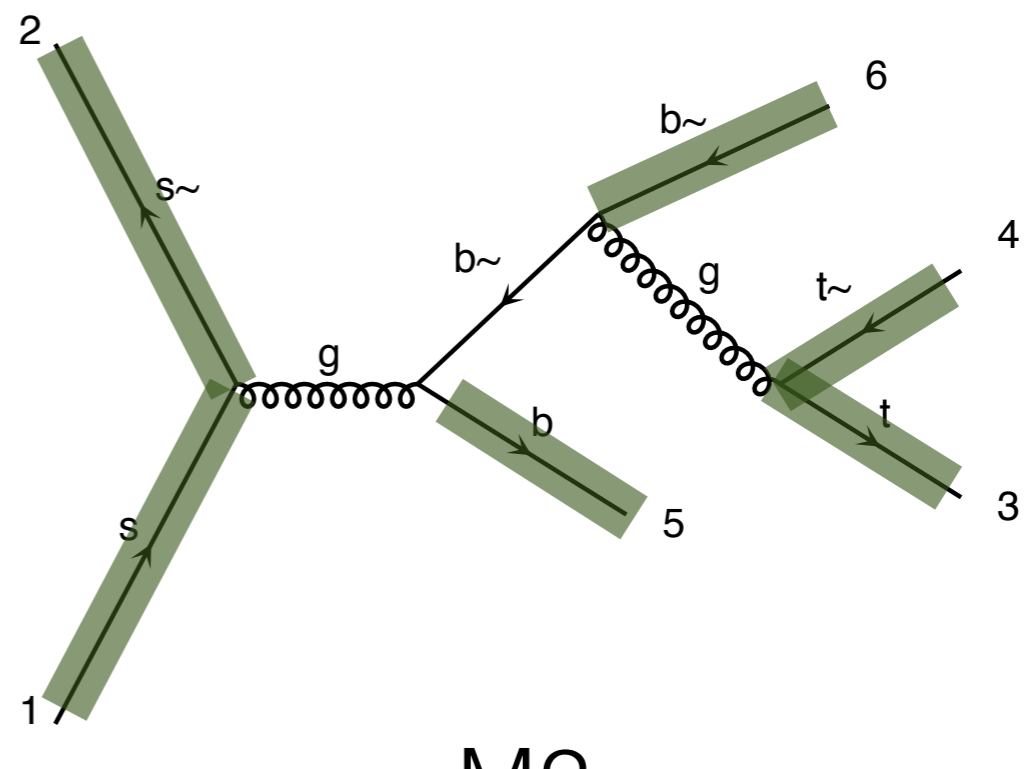
Recycling

 Known



M1

Number of routines: 6



M2

Number of routines: 6

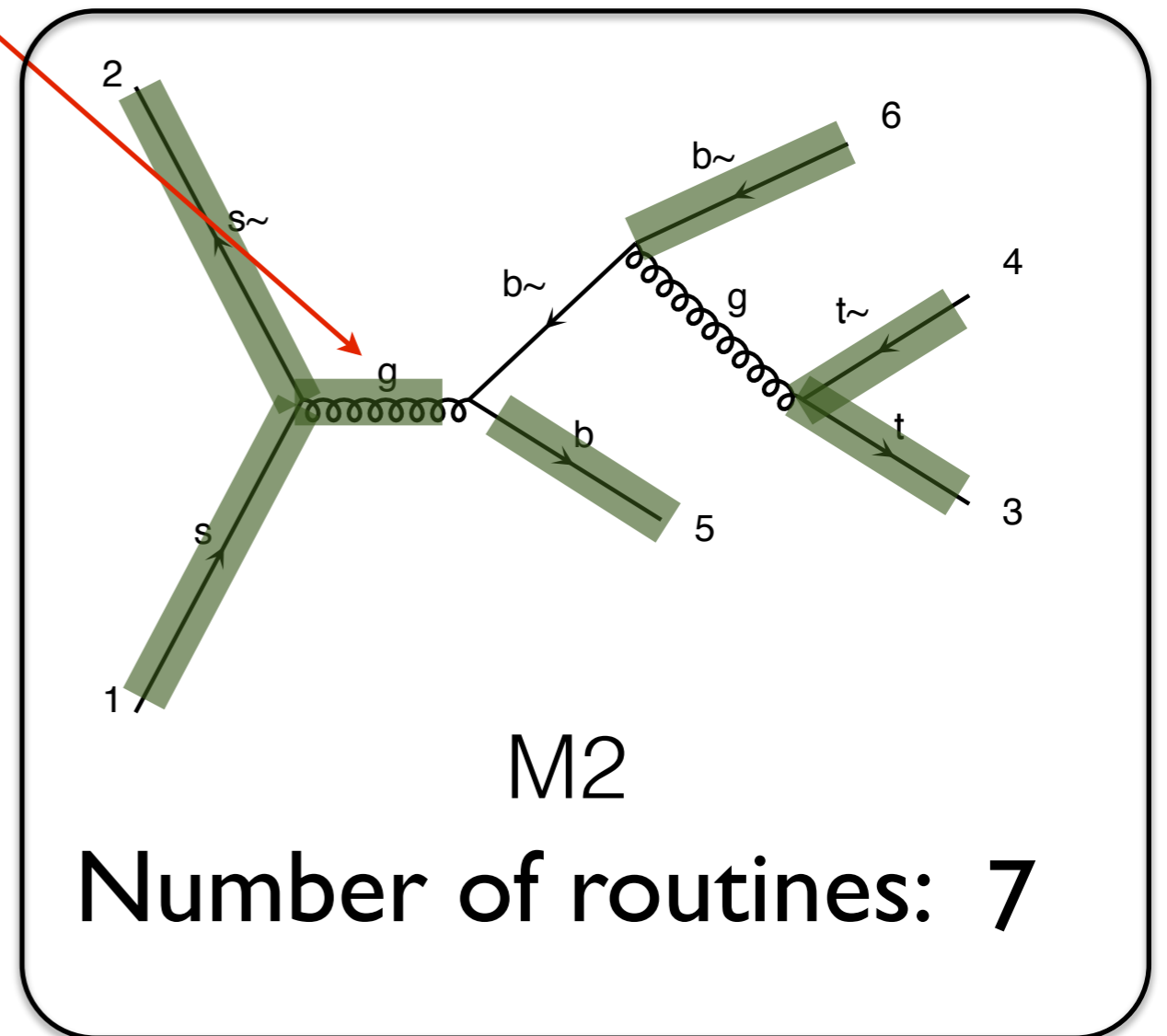
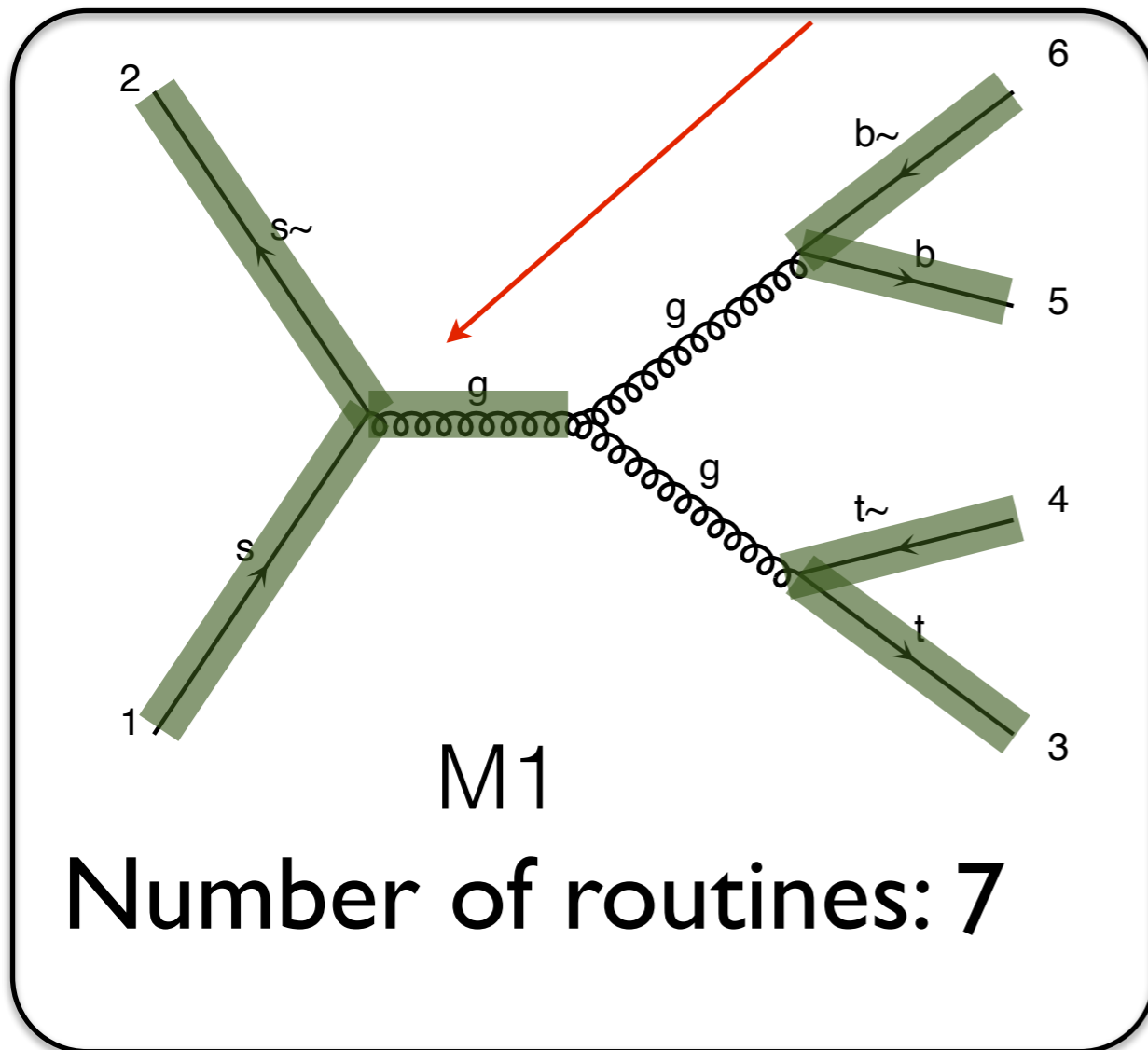
Number of routines for both: 6

$$|M|^2 = |M_1 + M_2|^2$$

Recycling

— Known

Identical

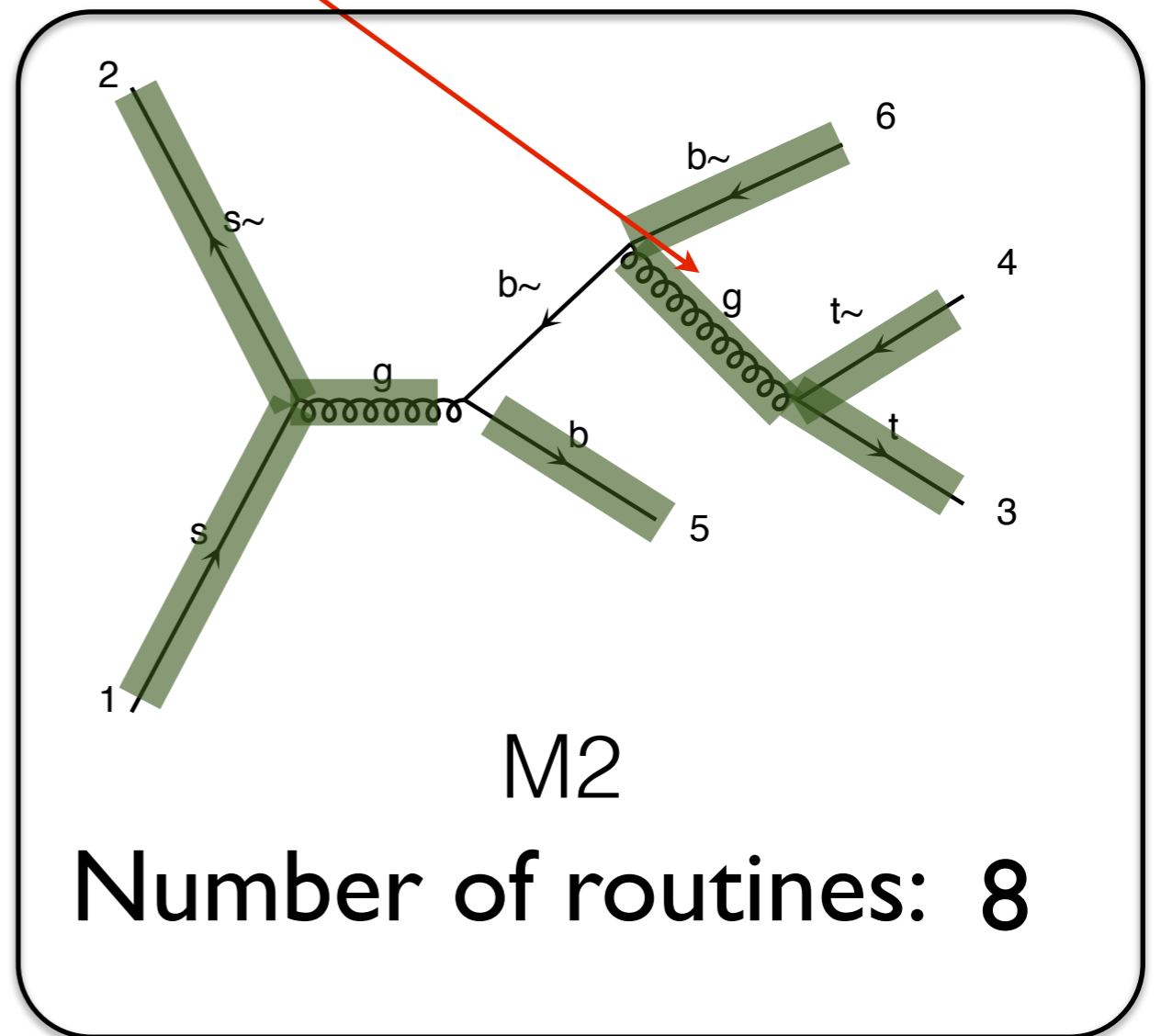
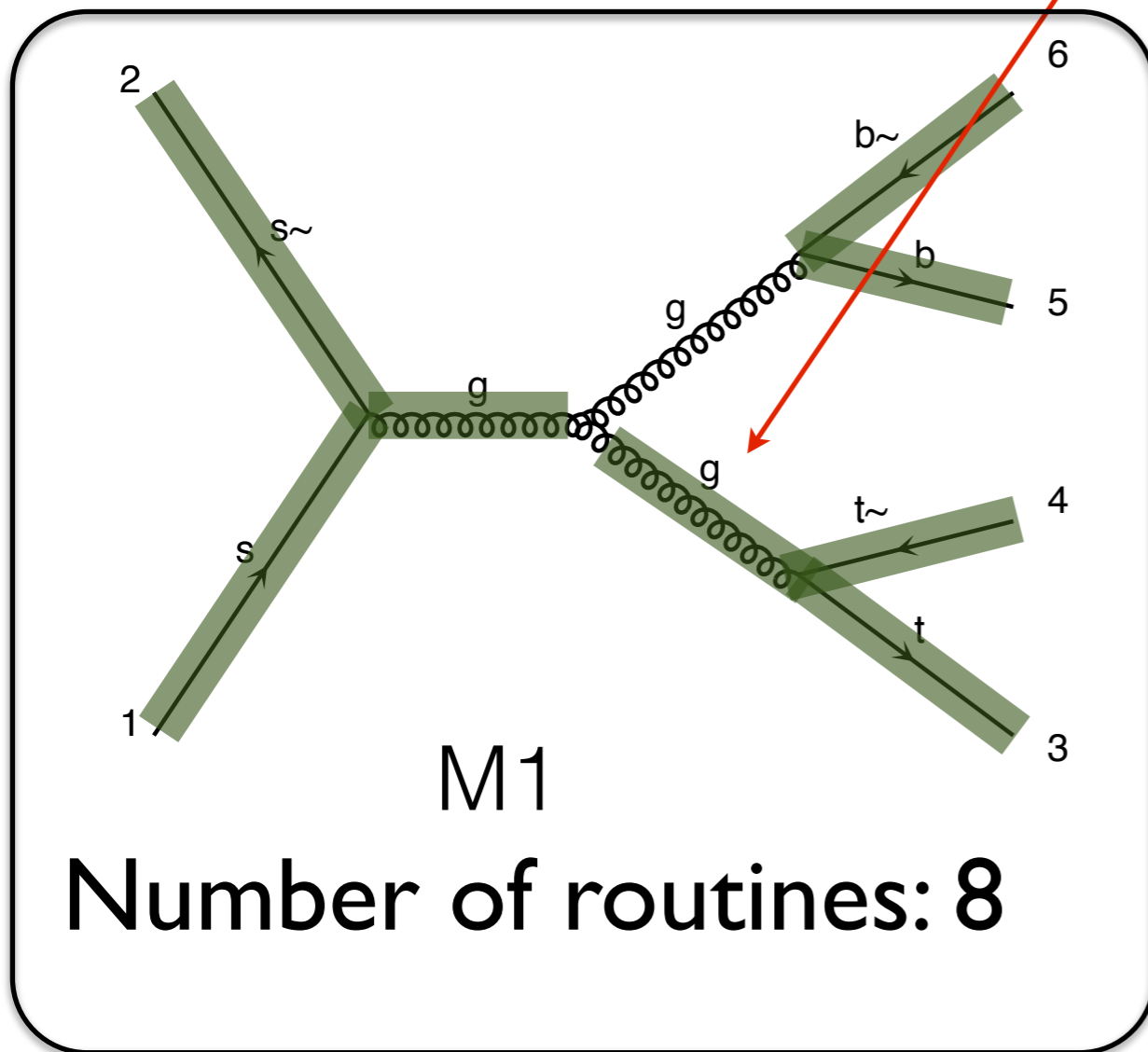


Number of routines for both: 7

$$|M|^2 = |M_1 + M_2|^2$$

Recycling

Identical  Known

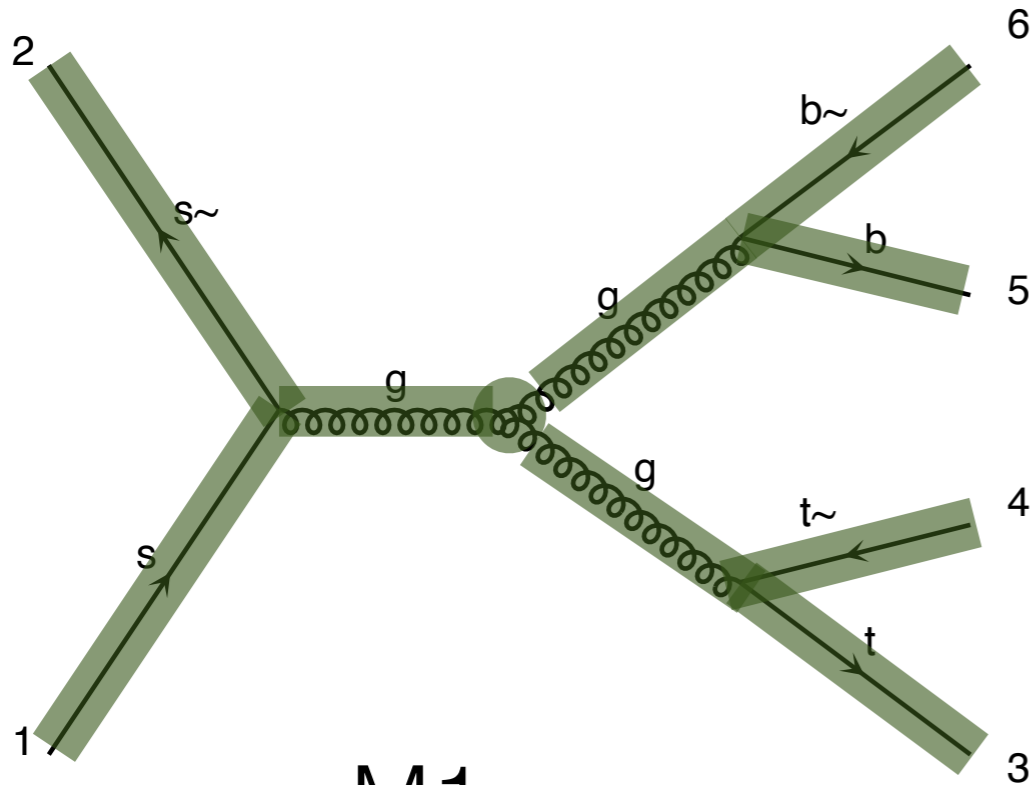


Number of routines for both: 8

$$|M|^2 = |M_1 + M_2|^2$$

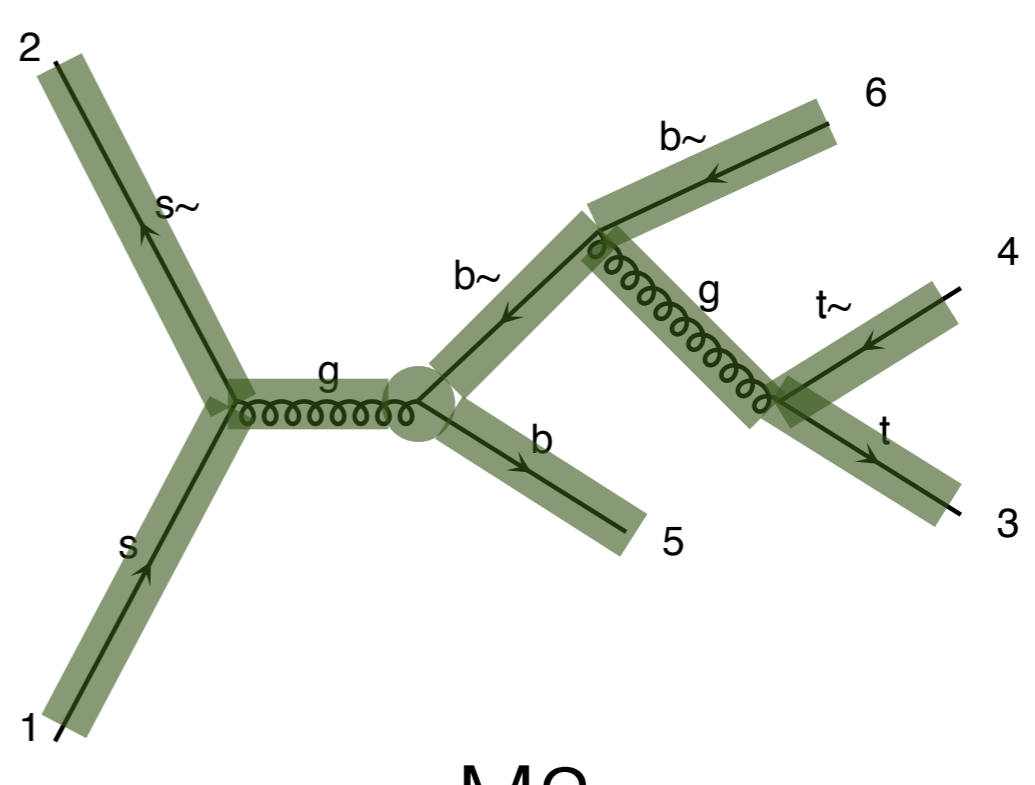
Recycling

— Known



M1

Number of routines: 10
 $2(N+1)$



M2

Number of routines: 10
 $2(N+1)$

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

Recycling

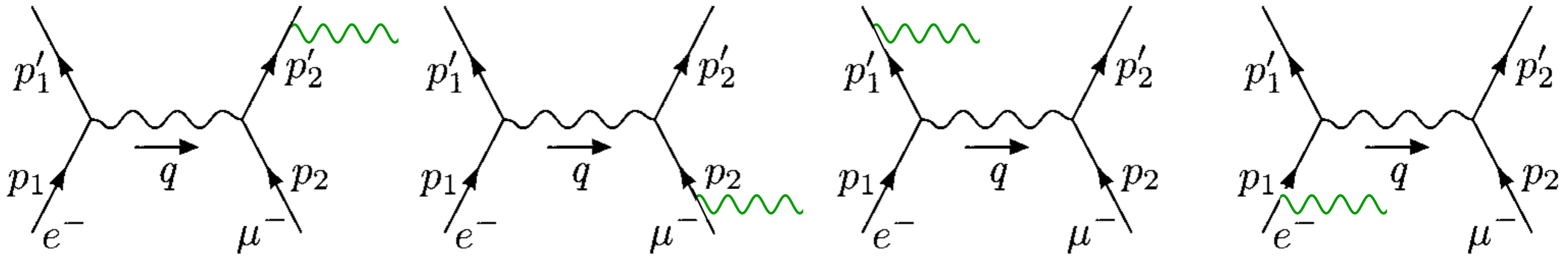
	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!) 2^N$
Recycling	M	$(N - 1)! 2^{(N-1)}$

Number of diagrams

$$e^+e^- \rightarrow \mu^+\mu^-\gamma$$

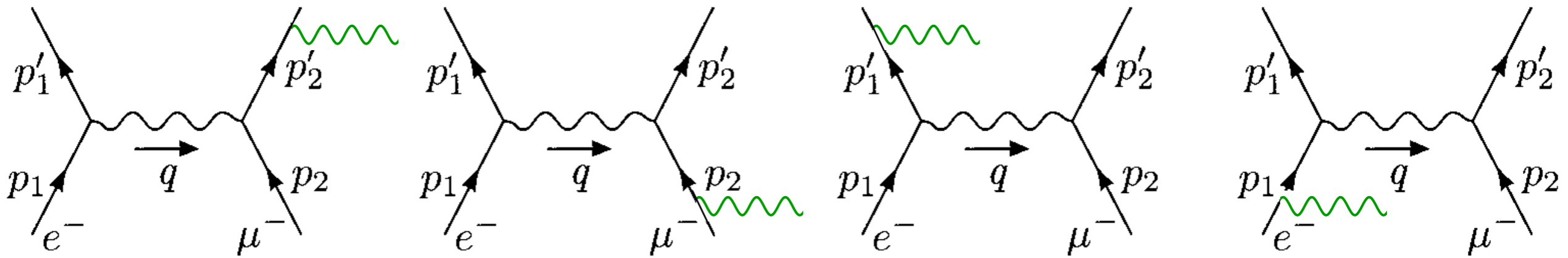
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Number of diagrams

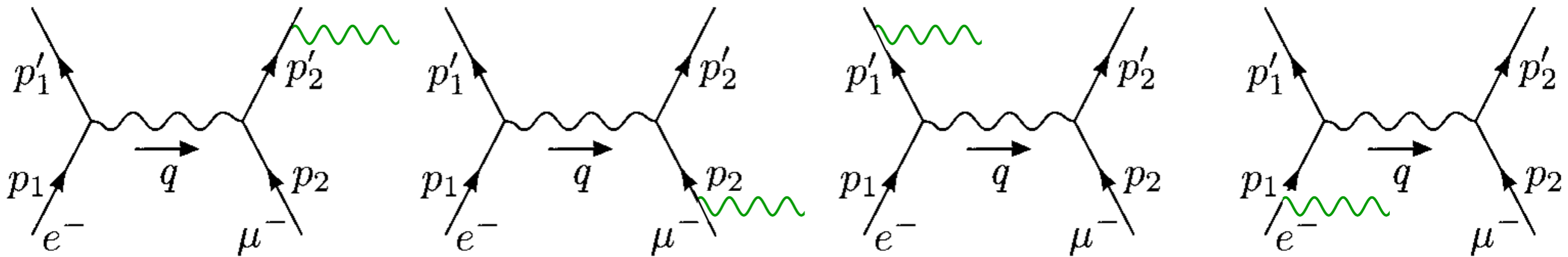
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$$e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$$

Number of diagrams

$$e^+e^- \rightarrow \mu^+\mu^-\gamma$$



$$e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$$

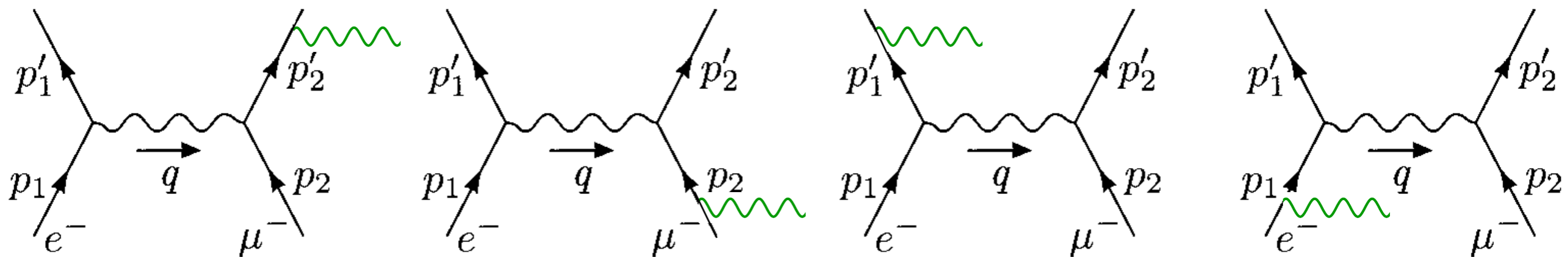
4x3 on 2 different legs

4x2 on the same legs (order of photon)

20 in total

Number of diagrams

$$e^+e^- \rightarrow \mu^+\mu^-\gamma$$



$$e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$$

4x3 on 2 different legs

4x2 on the same legs (order of photon)

20 in total

+3 γ , +4 γ

120, 840

Number of diagrams

- grows factorially with the number of external legs
- only e, μ, γ so far, increases with the number of particles and interactions in the model
- increases with the number of loops
- In MG, generate different processes with increasing number of final state particles
 - generate $e^+ e^- \rightarrow \mu^+ \mu^-$
 - display diagrams

Cross-section

$$d\sigma = \frac{(2\pi)^4 |\mathcal{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \\ \times d\Phi_n(p_1 + p_2; p_3, \dots, p_{n+2})$$

From the PDG

$$d\Phi_n(P; p_1, \dots, p_n) = \delta^4(P - \sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

Phase space integral : how to do it numerically

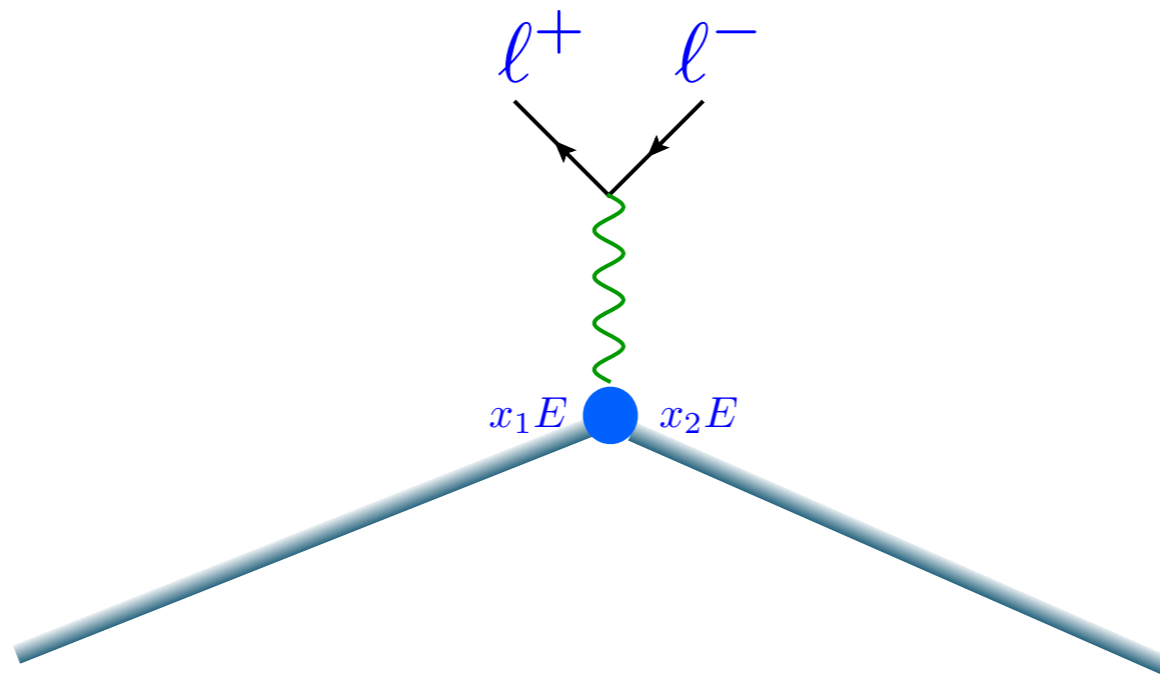
What do we expect :

Ex : check it in MG

- $\sigma \propto \alpha^2$
- At high energy : $\sigma \propto 1/s$, $s = (p_1 + p_2)^2$
- goes to zero at threshold
- lorentz invariant

Who knows s, t, u?

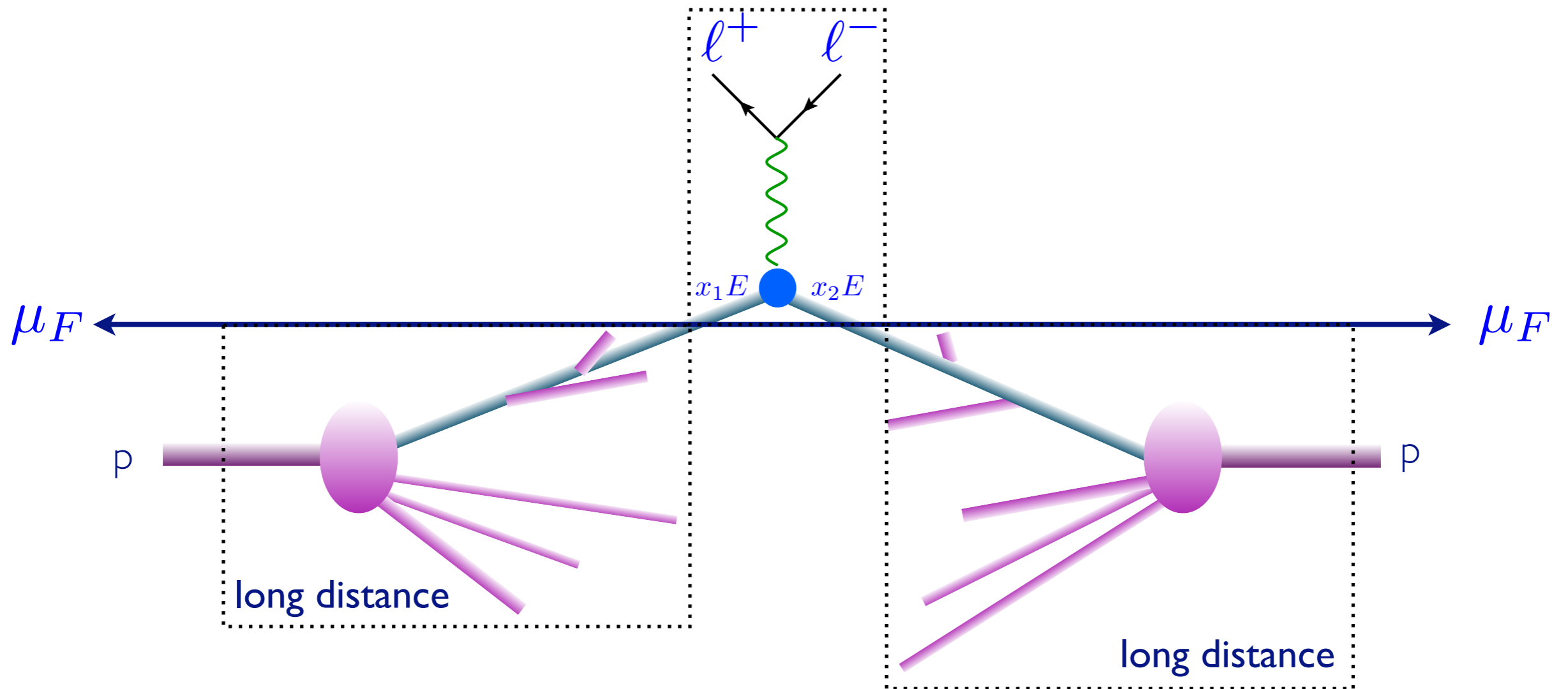
At the LHC



$$\hat{\sigma}_{ab \rightarrow X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross
section

At the LHC

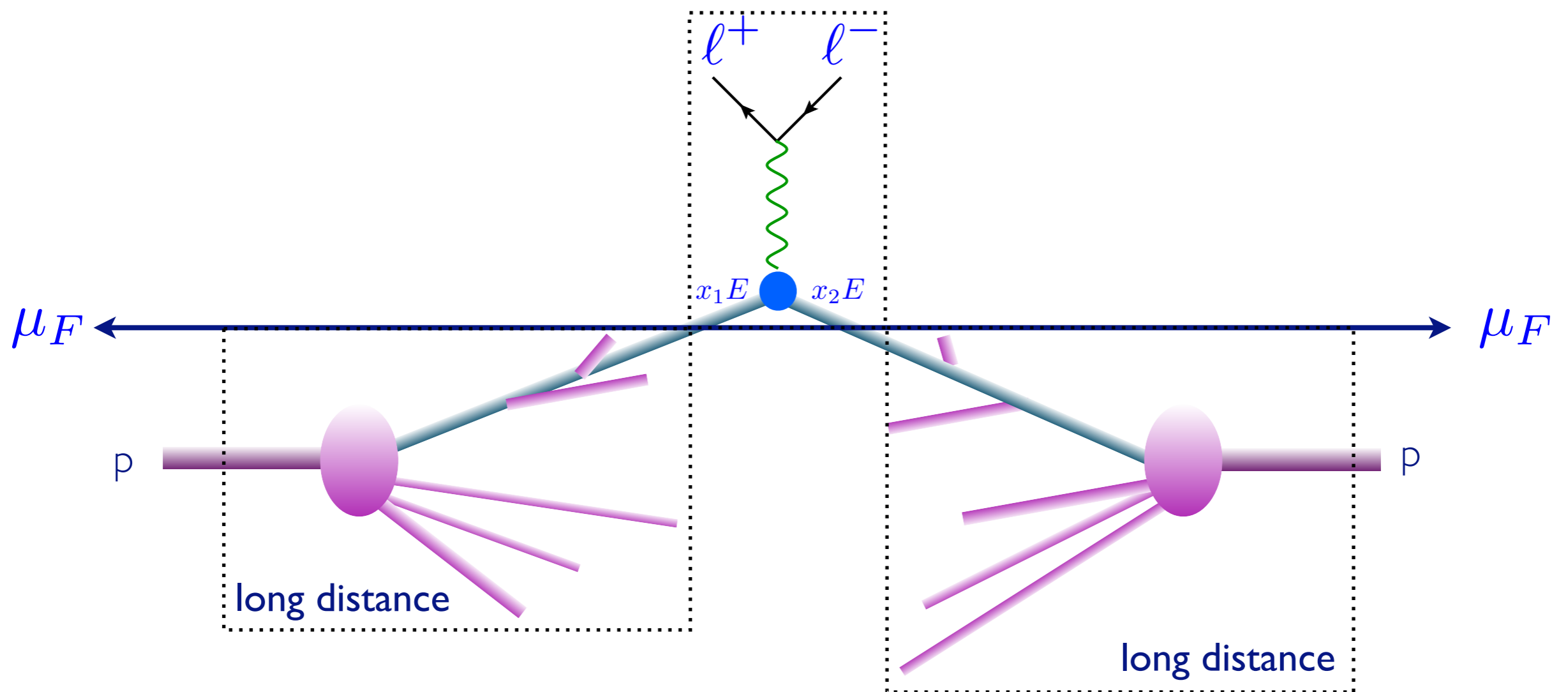


$$f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \rightarrow X}(\hat{S}, \mu_F, \mu_R)$$

Parton density
functions

Parton-level cross
section

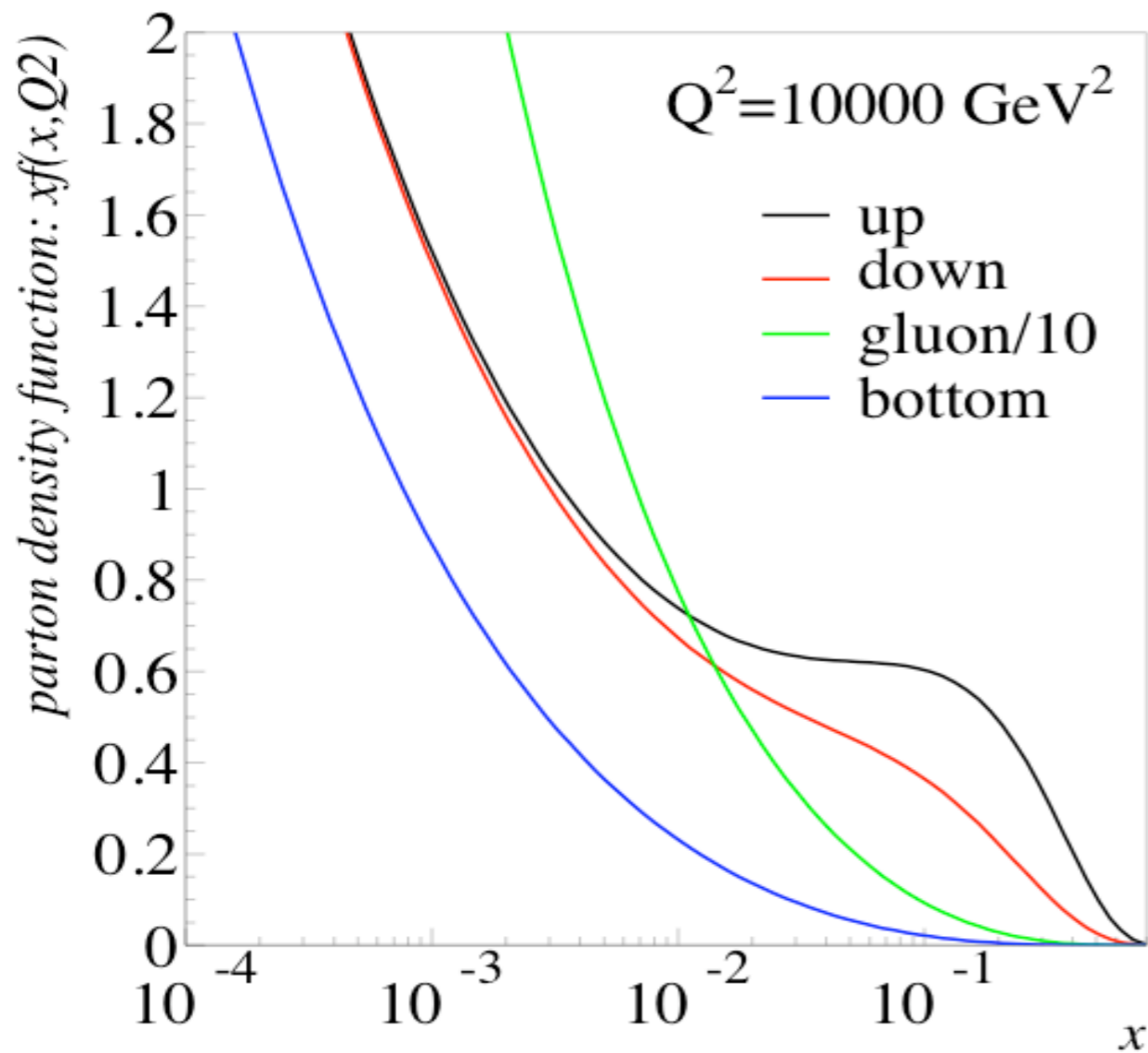
At the LHC



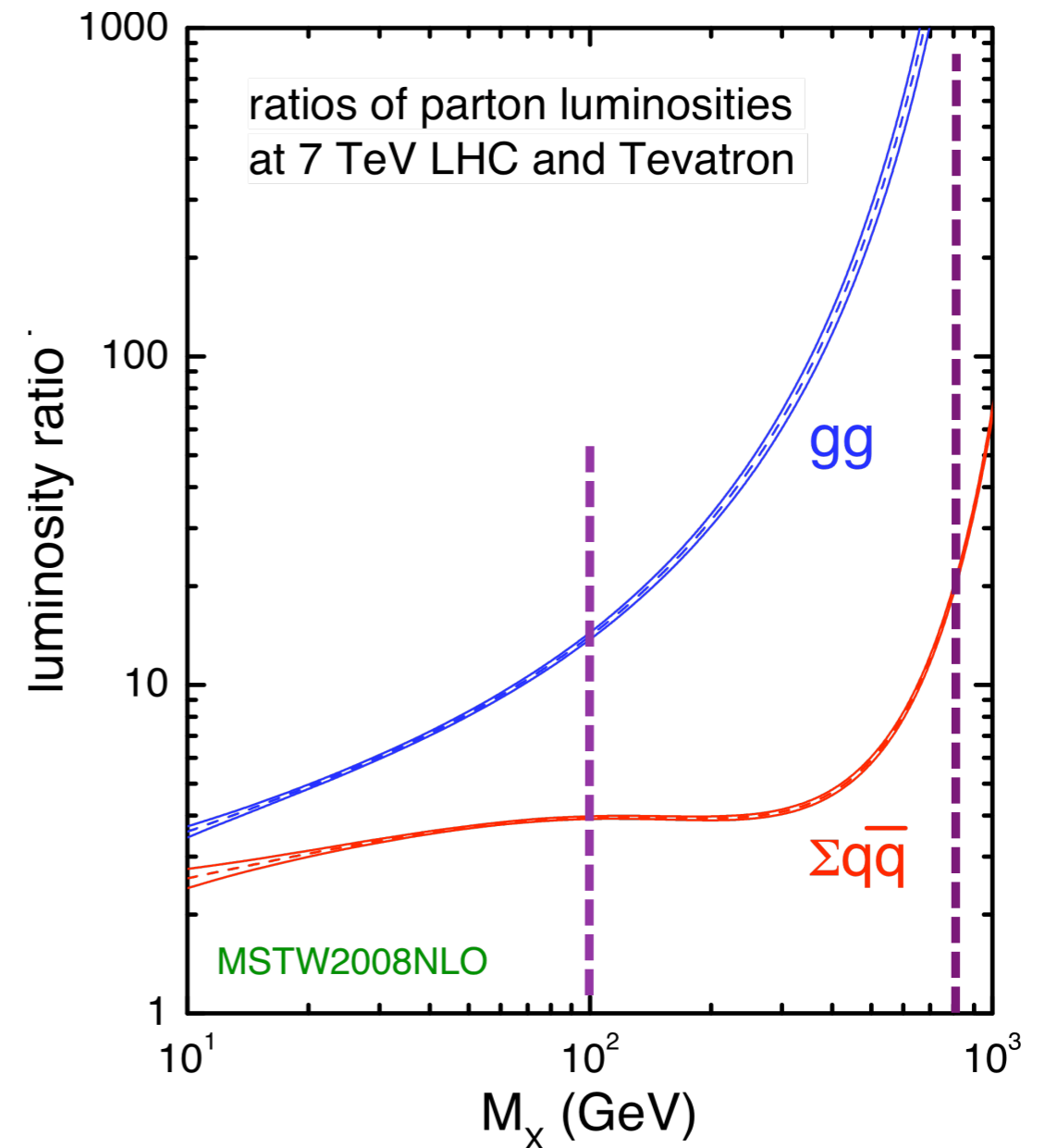
$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \rightarrow X}(\hat{S}, \mu_F, \mu_R)$$

Phase-space integral
Parton density functions
Parton-level cross section

Parton density functions



At small x (small \hat{s}), gluon domination.
At large x valence quarks



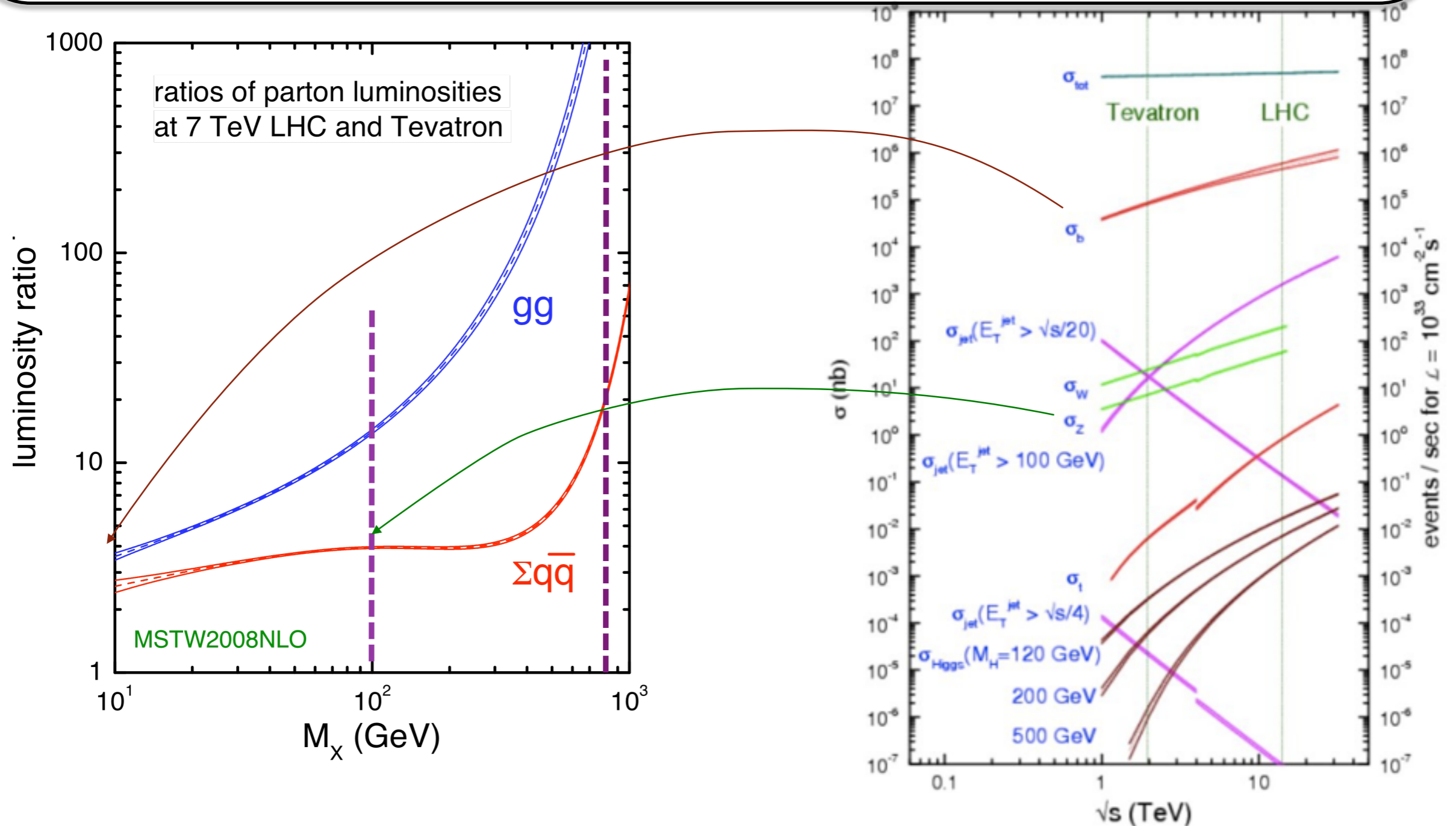
LHC formidable at large mass –
For low mass, Tevatron backgrounds smaller

Energy evolution of the PDF is calculable

Hadron colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\text{FS}} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \rightarrow X}(\hat{s}, \mu_F, \mu_R)$$

proton - (anti)proton cross sections



Hadron collider kinematic

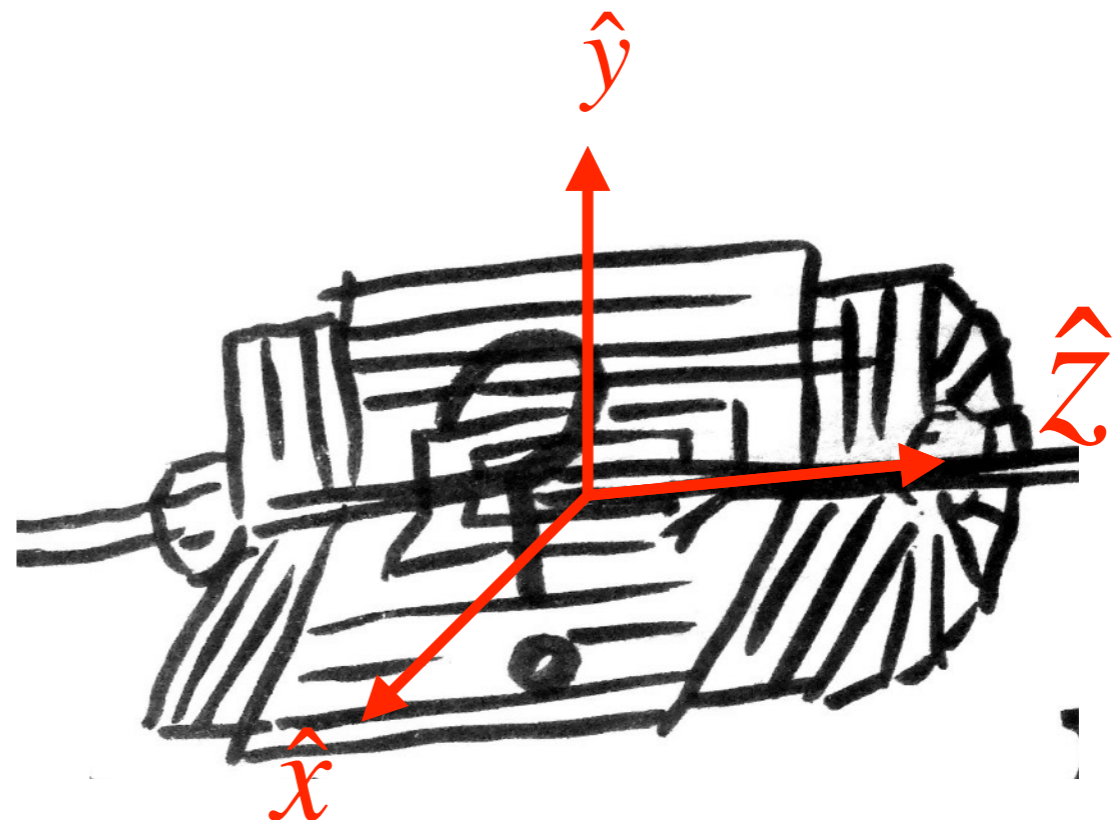
- Partonic frame is boosted along the z direction compared to the lab frame (each parton has a different energy)
- z axis is the beam axis

- scattering angle θ with the beam ($y = \log \left(\frac{E + p_z}{E - p_z} \right)$,

$$\eta = \log \left(\frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} \right)$$

- ϕ is around the beam axis

- $\Delta R = \sqrt{\Delta y^2 + \Delta \phi^2}$



Integration over phase space

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n) \quad \leftarrow \text{Dim}[\Phi(n)] \sim 3n$$

Needs :

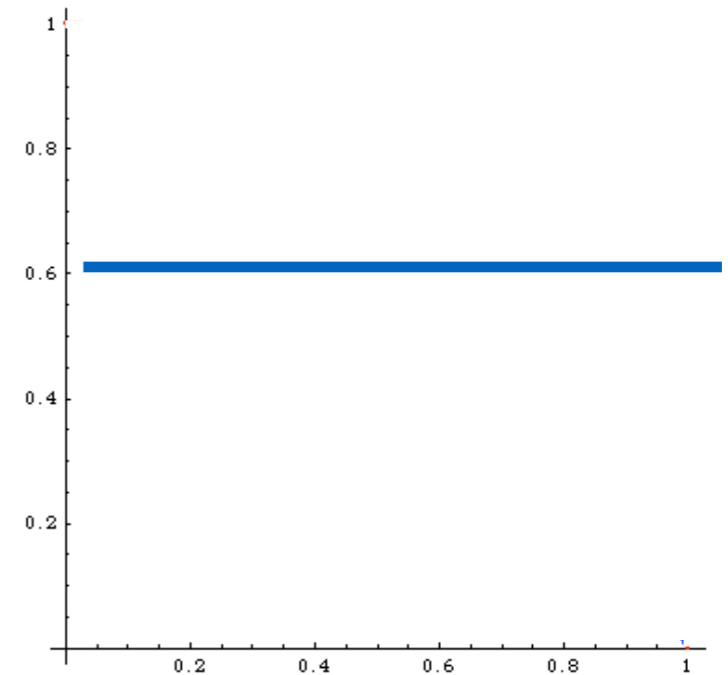
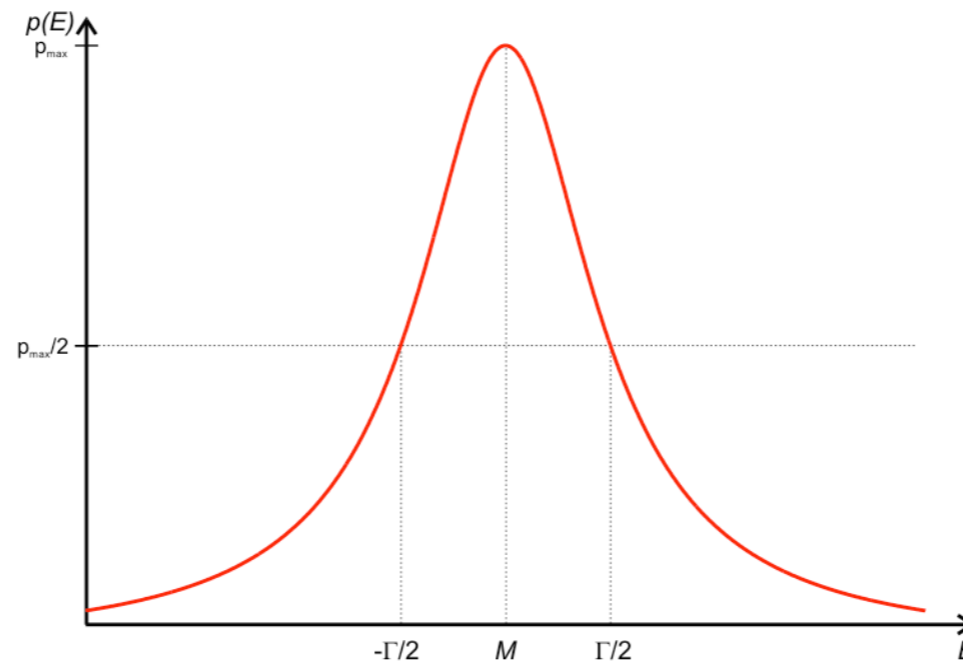
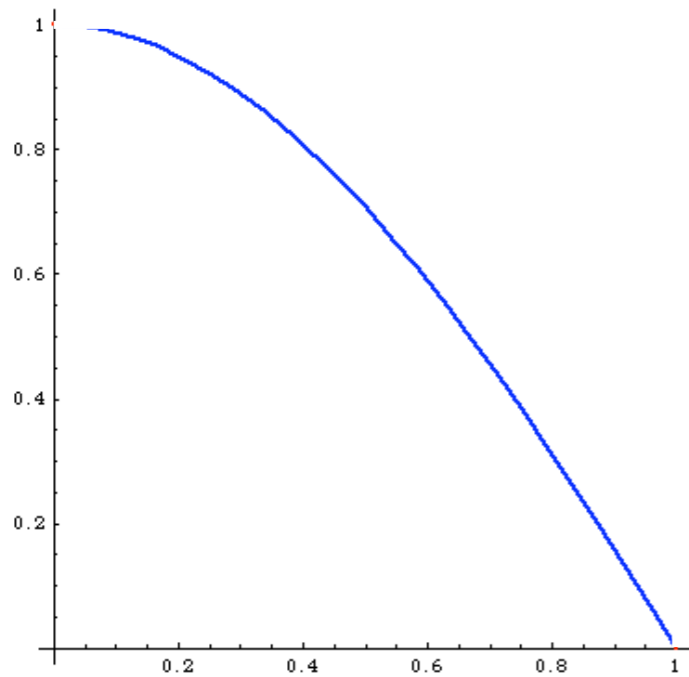
- general and flexible (process/cut independent) method
- generate events

Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx C$$



Method of evaluation

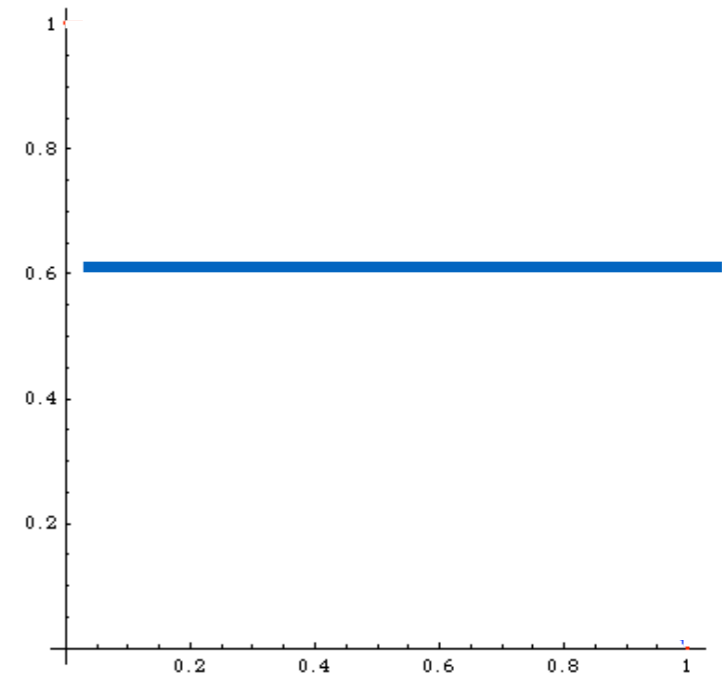
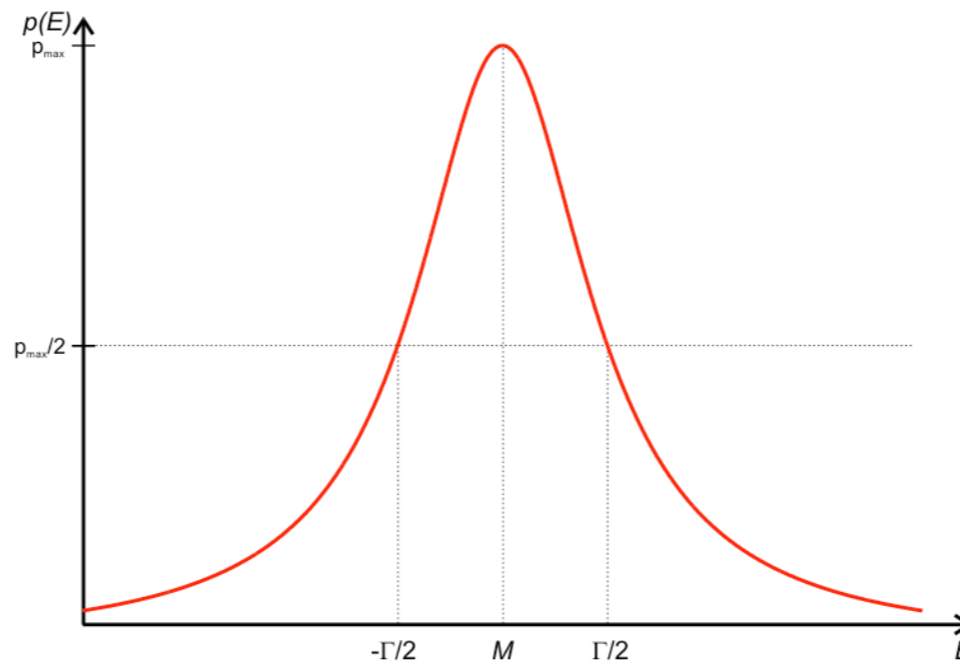
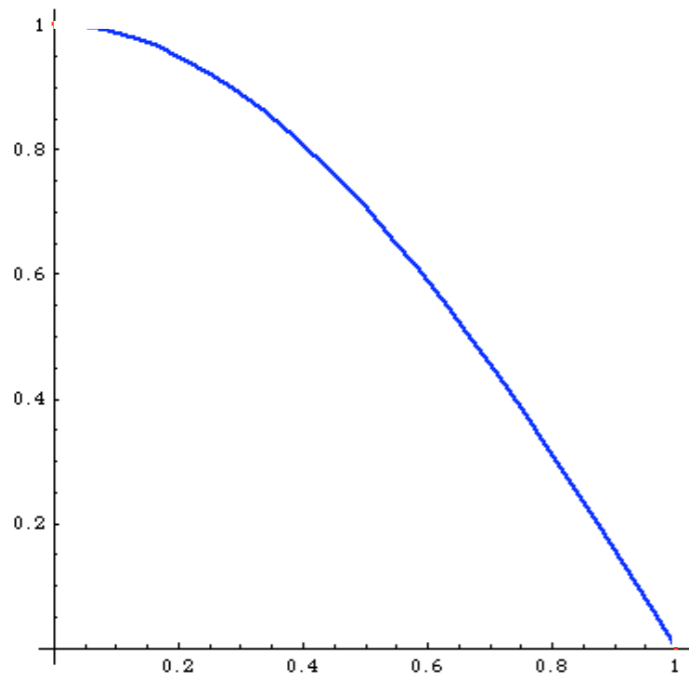
- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx C$$



Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
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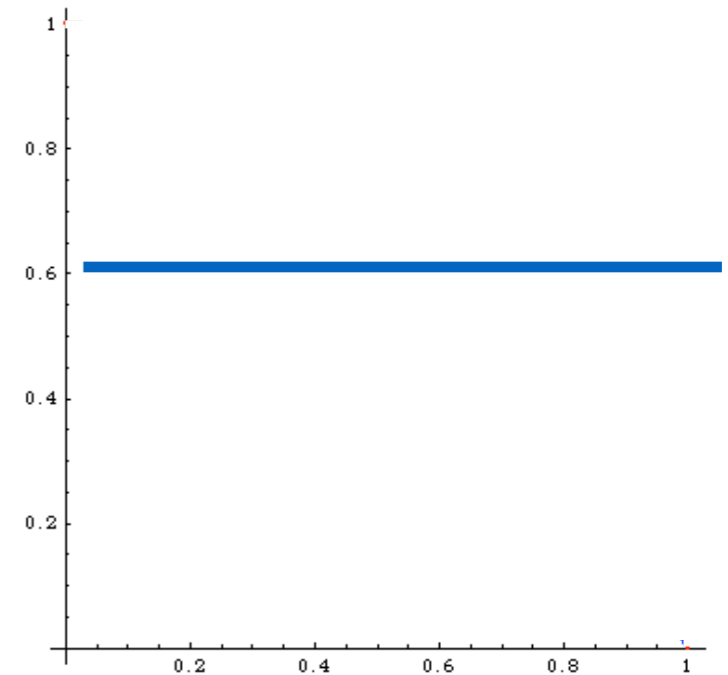
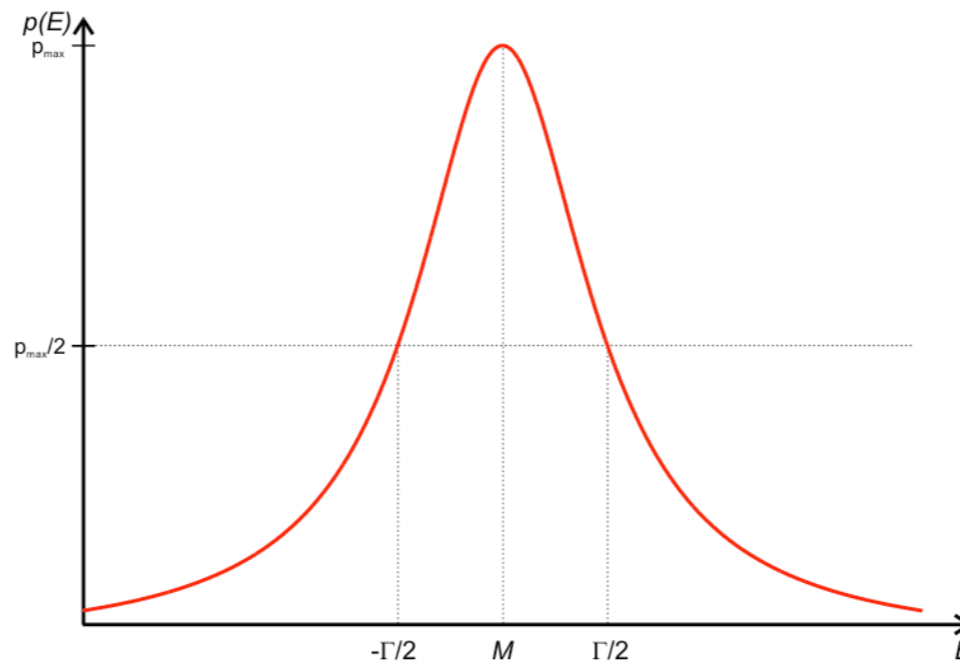
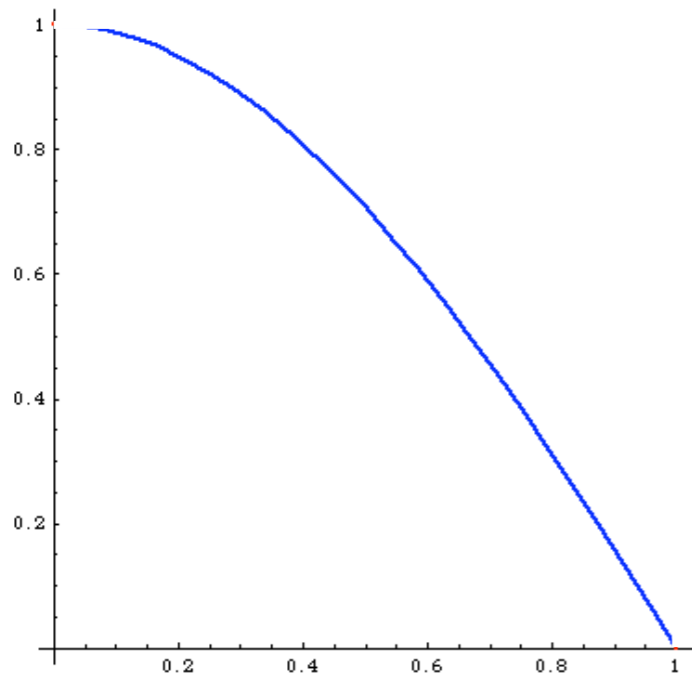
	simpson	MC
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

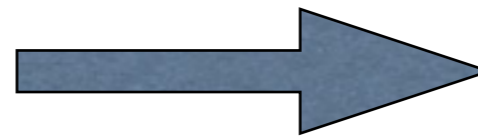
$$\int dx C$$



Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

More Dimension



$$1/\sqrt{N}$$

$$1/N^{2/d}$$

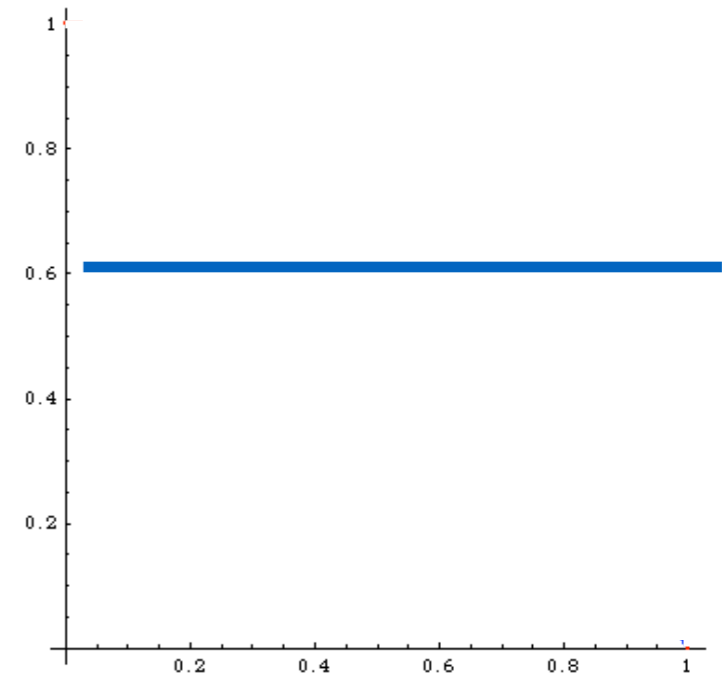
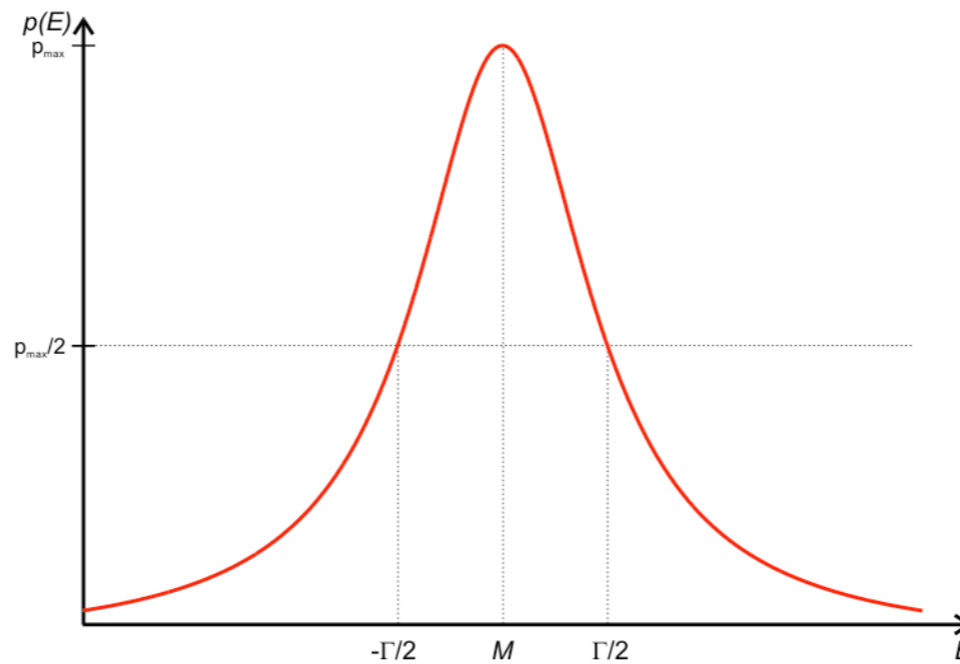
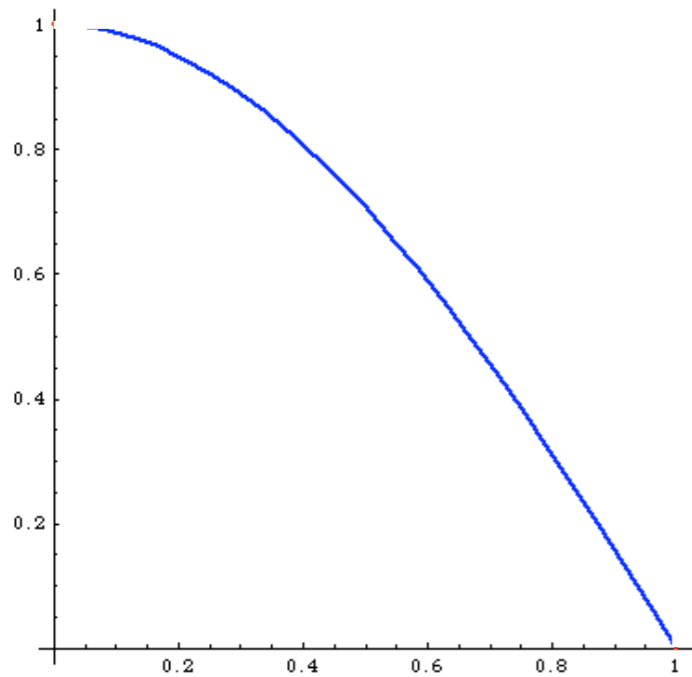
$$1/N^{4/d}$$

Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

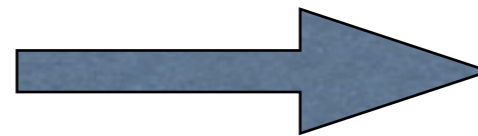
$$\int dx C$$



Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

More Dimension



1

$$1/\sqrt{N}$$

2^d

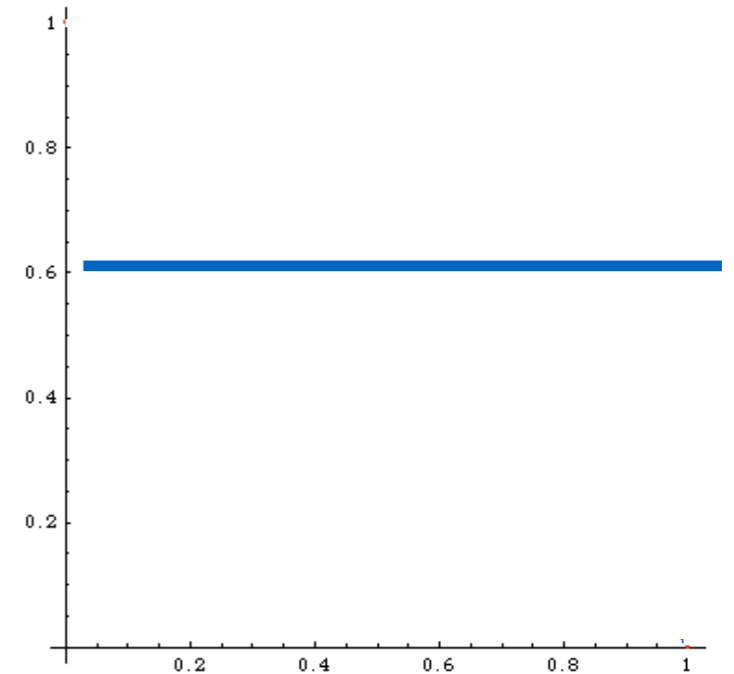
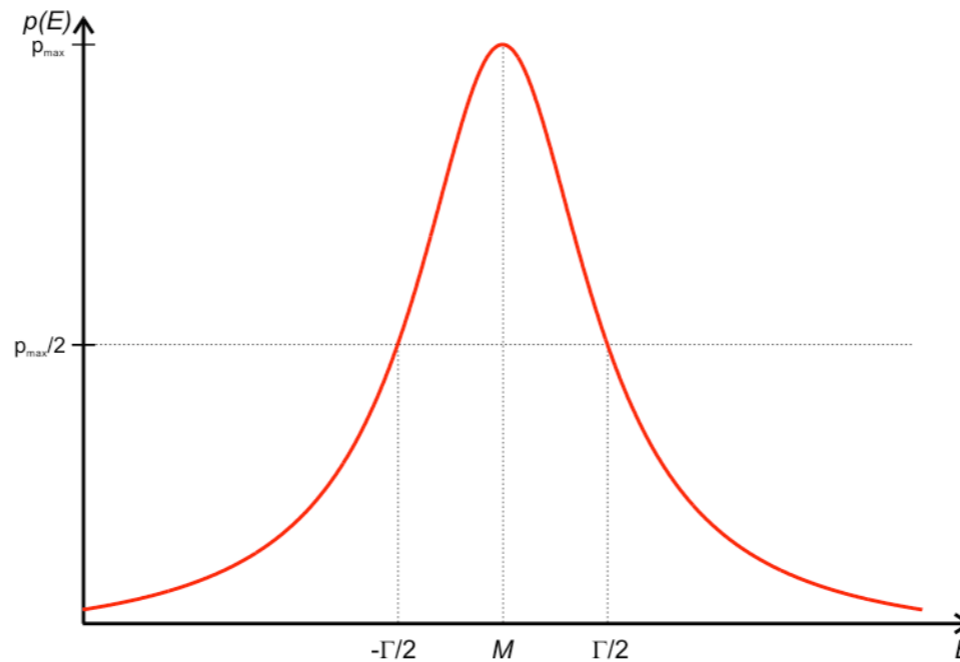
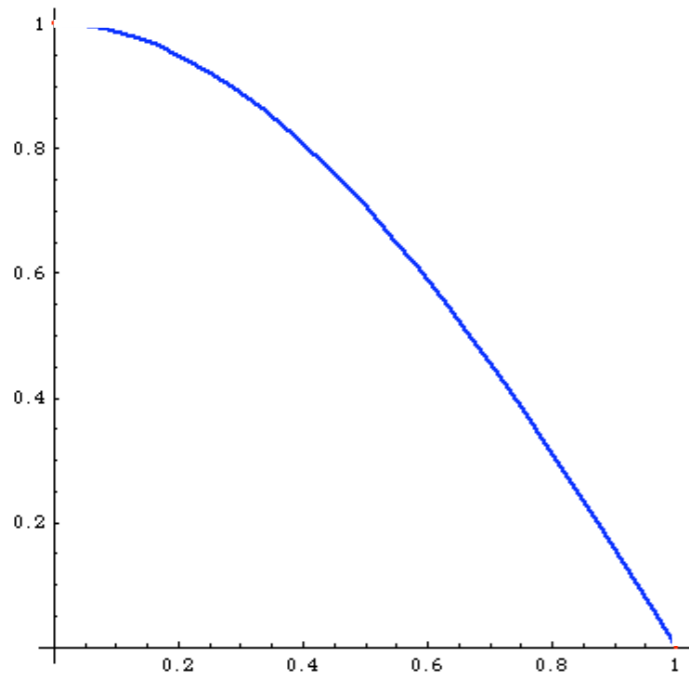
$$1/N^{2/d}$$

3^d

$$1/N^{4/d}$$

Convergence

$$I = \int_0^1 dx \cos \frac{\pi}{2} x \quad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \quad \int dx C$$



$$I = \int_{x_1}^{x_2} f(x) dx \quad \Rightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x) \quad \text{🎲 🎲}$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x)]^2 - I_N^2$$

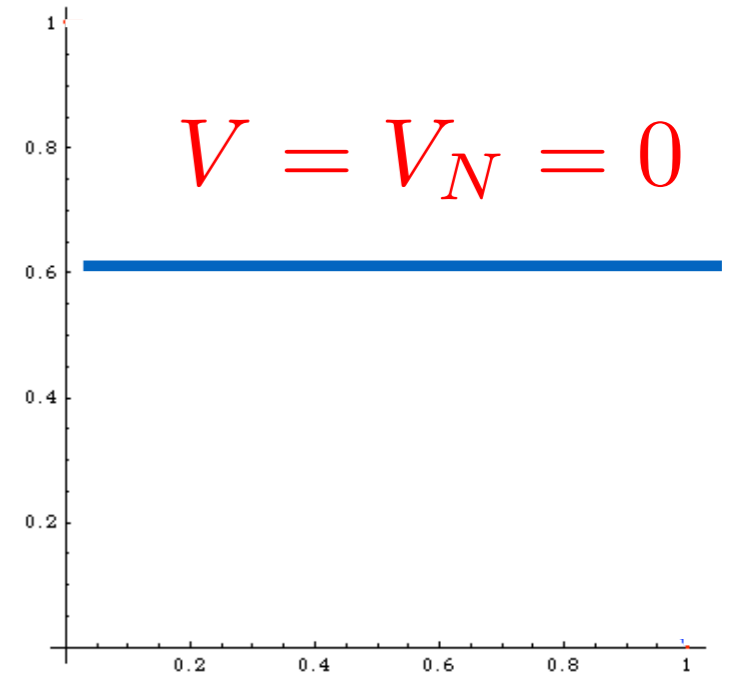
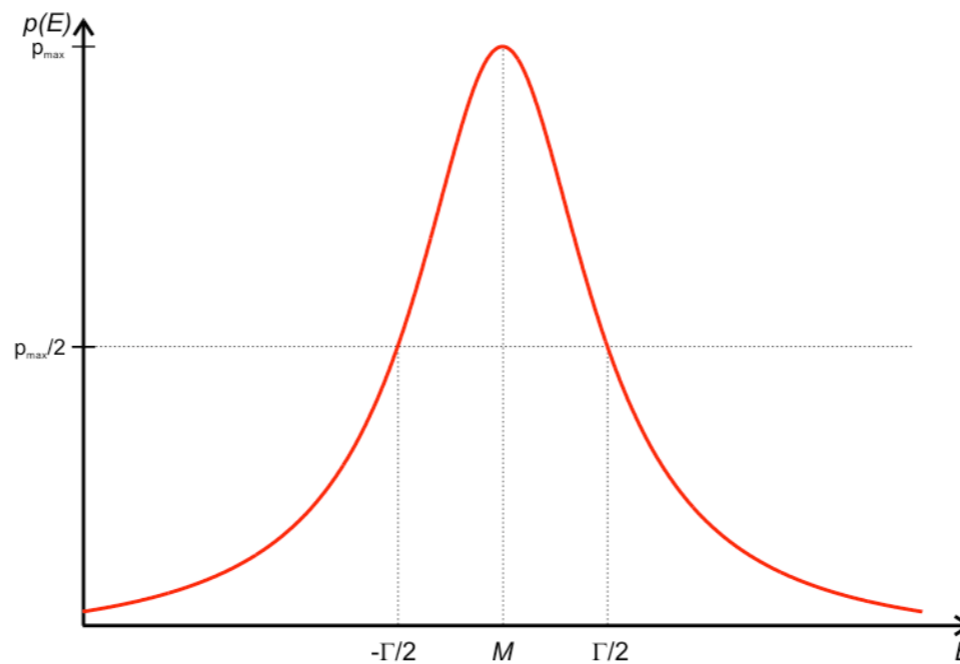
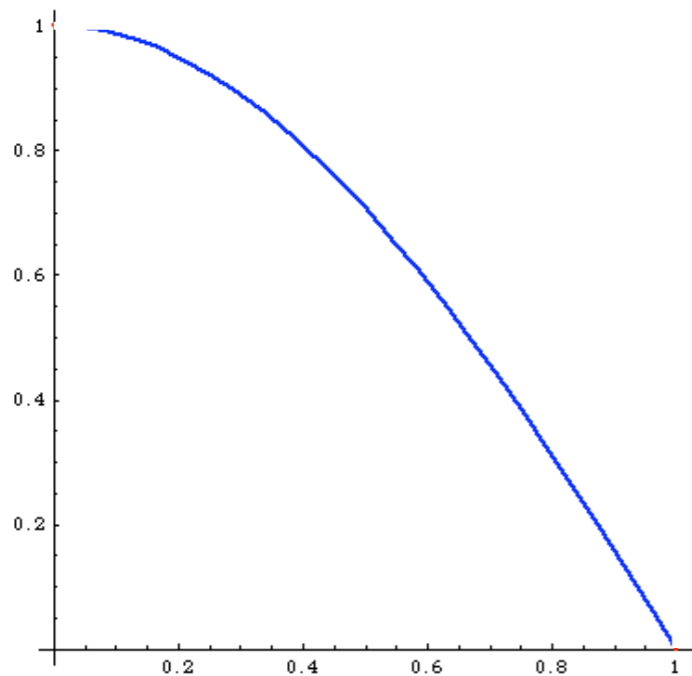
$$I = I_N \pm \sqrt{V_N/N}$$

Convergence

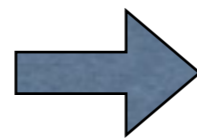
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx C$$

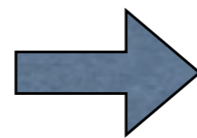


$$I = \int_{x_1}^{x_2} f(x) dx$$



$$I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x)$$

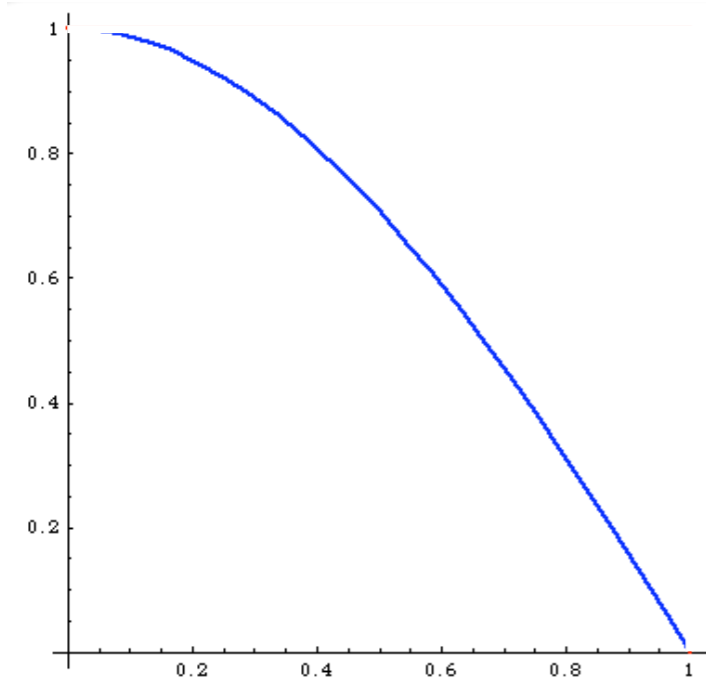
$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2$$



$$V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x)]^2 - I_N^2$$

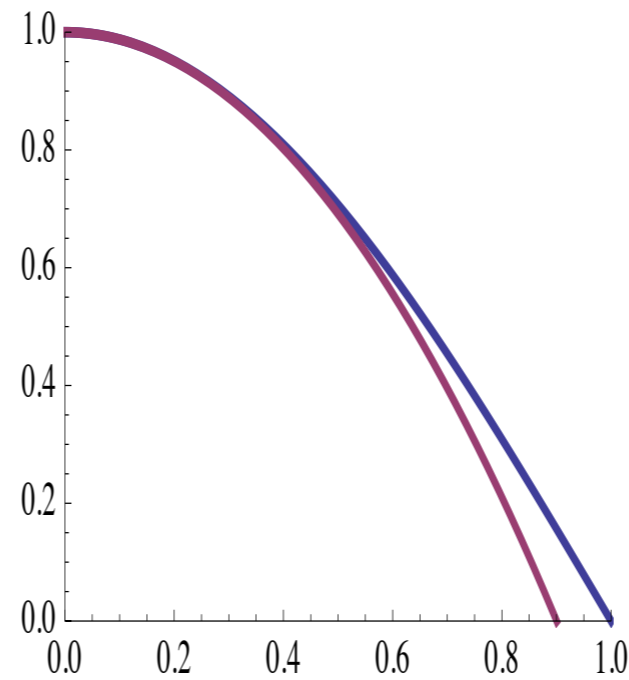
$$I = I_N \pm \sqrt{V_N/N} \leftarrow \text{Minimize!}$$

Importance sampling

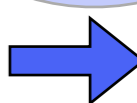


$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

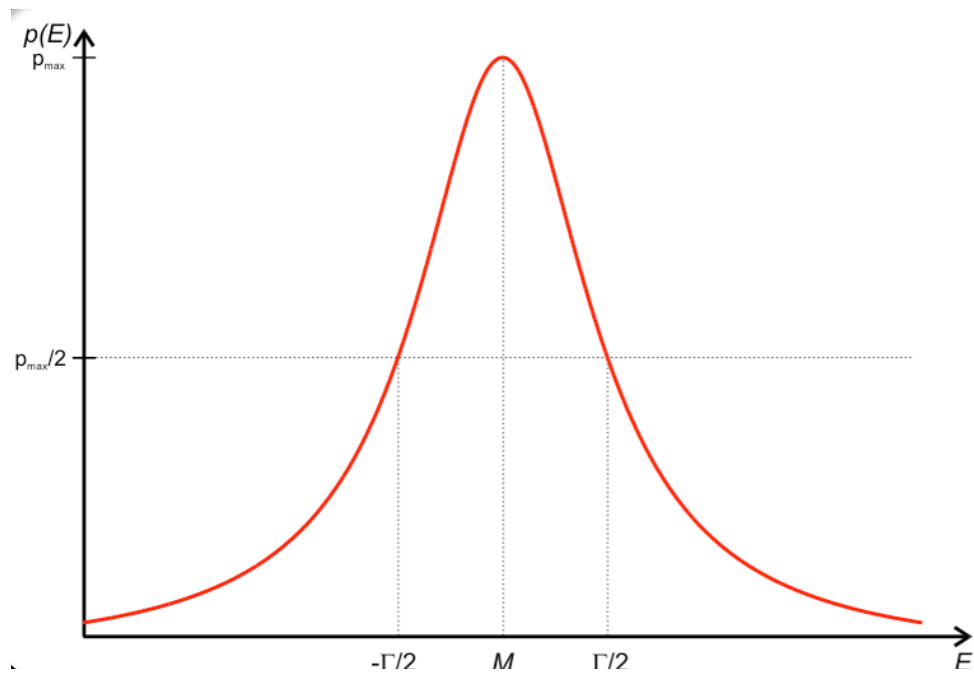


$$I = \int_0^1 dx (1 - cx^2) \frac{\cos(\frac{\pi}{2} x)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

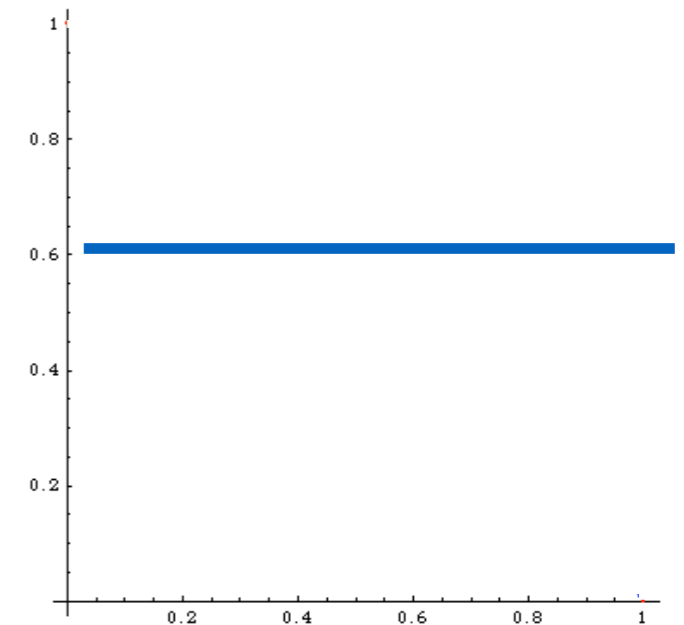
 $\simeq 1$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

Importance sampling

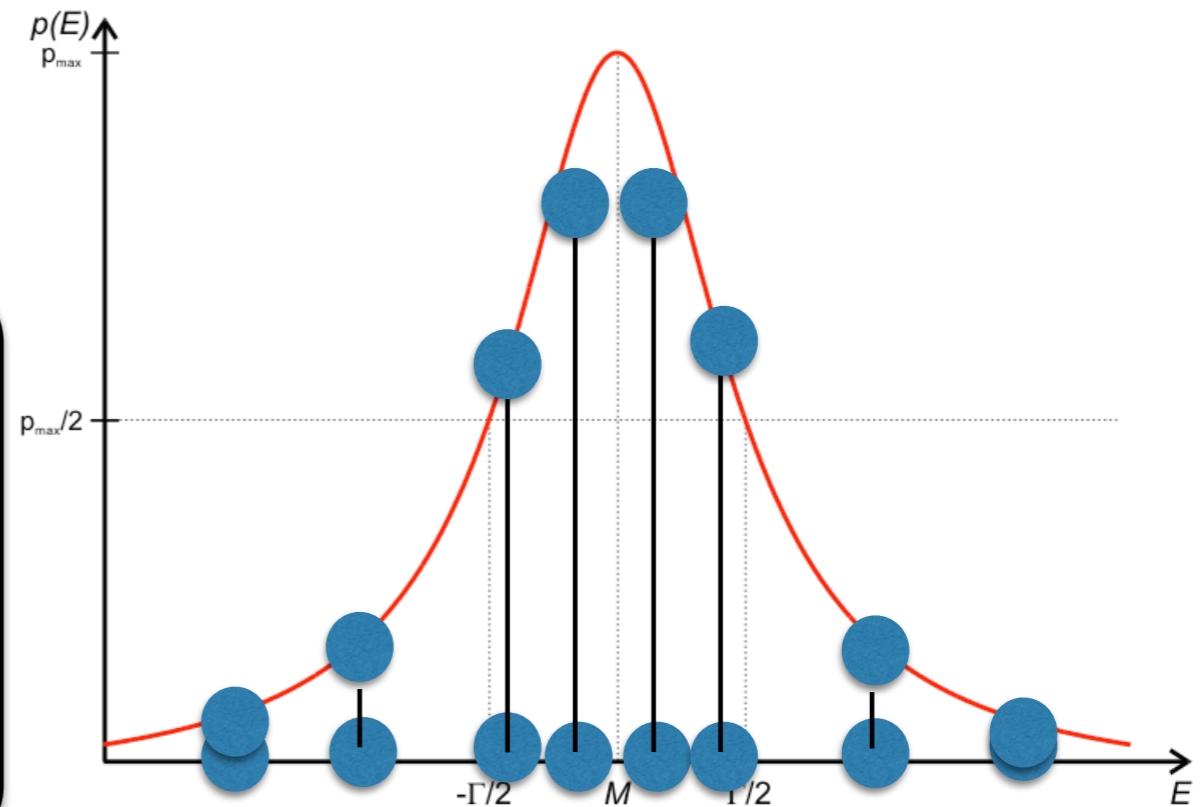
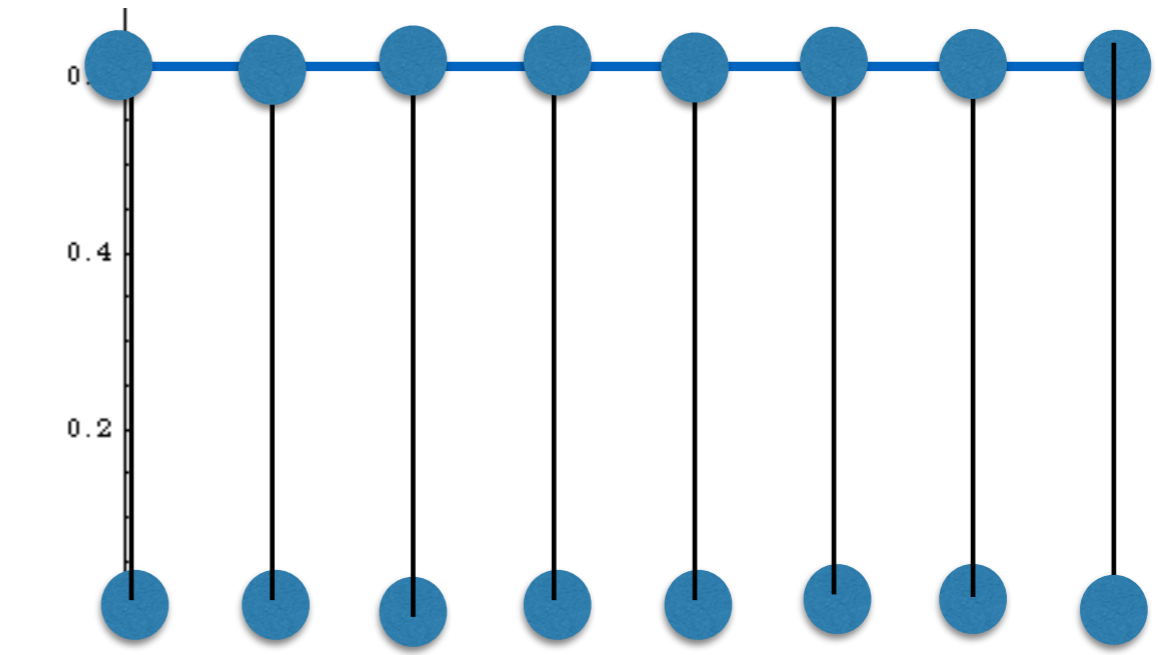
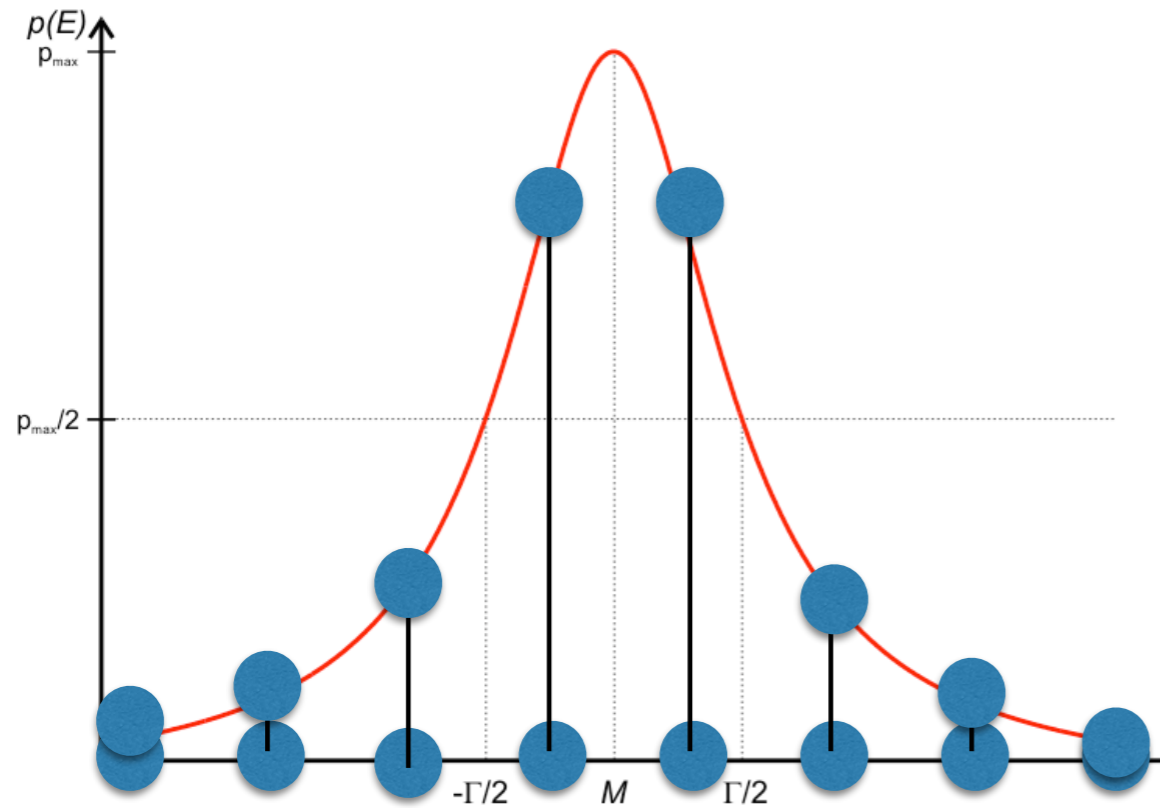


$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$
$$\xi = \arctan \left(\frac{q^2 - M^2}{\Gamma M} \right)$$



Phase space parametrisation matter for efficiency
smaller numerical error/faster computation

Importance



Why Importance Sampling?

We probe more often the region where the function is high!

Importance sampling

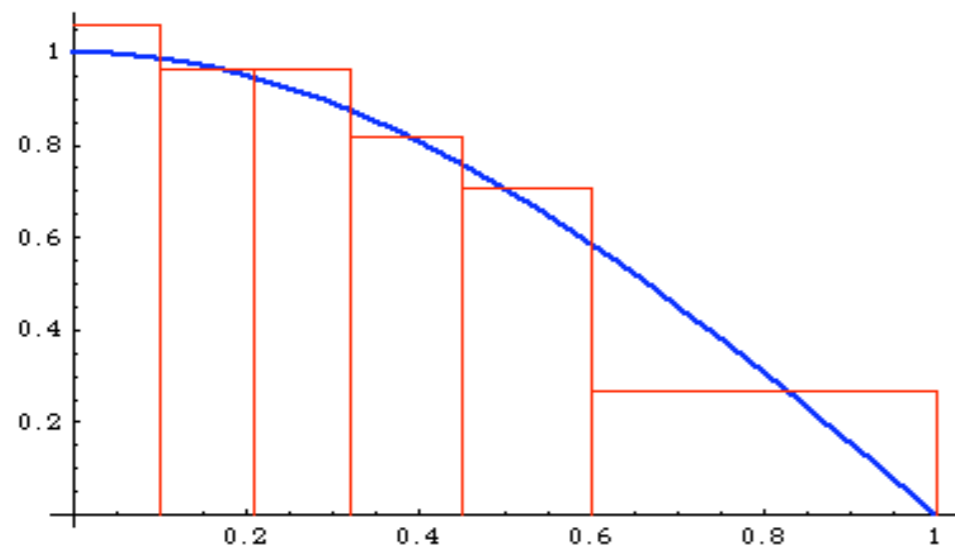
Summary

- Generate the **random points** in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is **flatter** in this new variable.
- Needs to know an **approximate function**.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Vegas



1. Creates bin such that each of them have the same contribution.

➡ Many bins where the function is large

2. Use the approximate for the importance sampling method.

Vegas

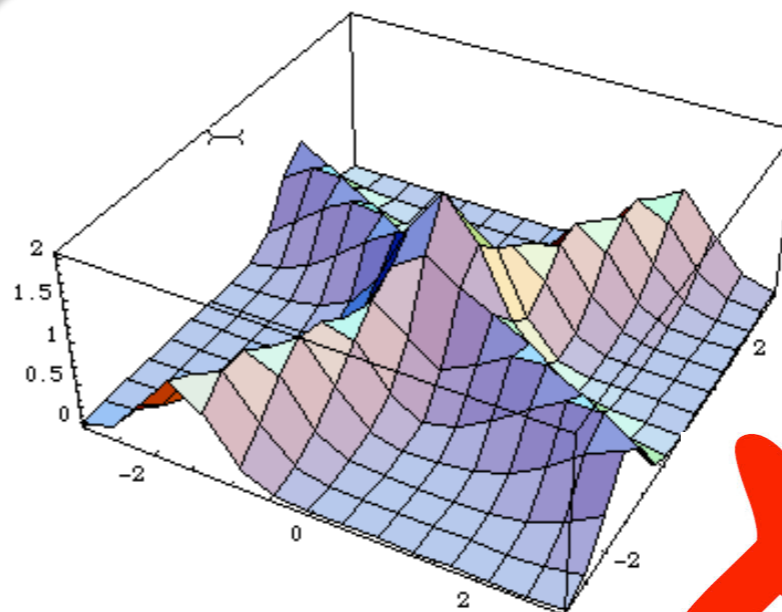
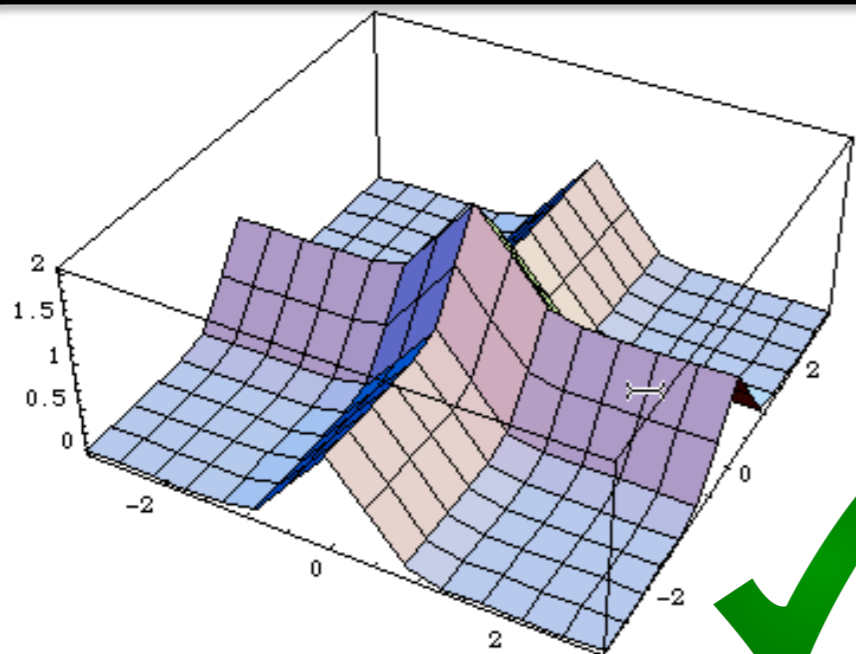
More than one Dimension

- VEGAS works only with 1 (few) dimension
➔ memory problem

Solution

- Use projection on the axis

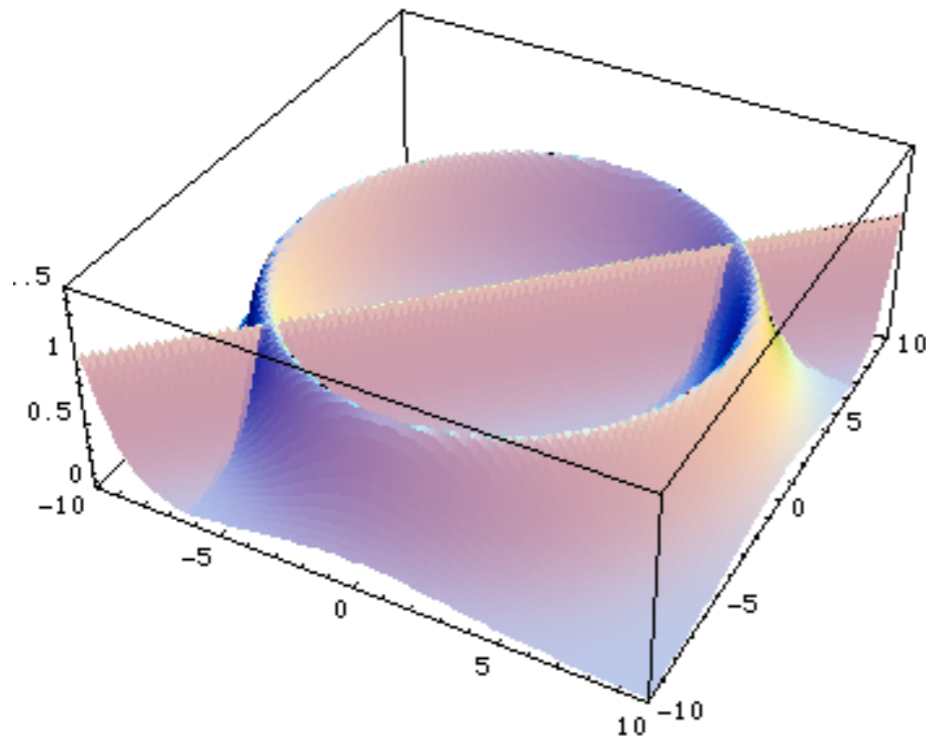
$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \dots$$



- We need to ensure the factorization !

➔ Additional change of variable

Multiple amplitude



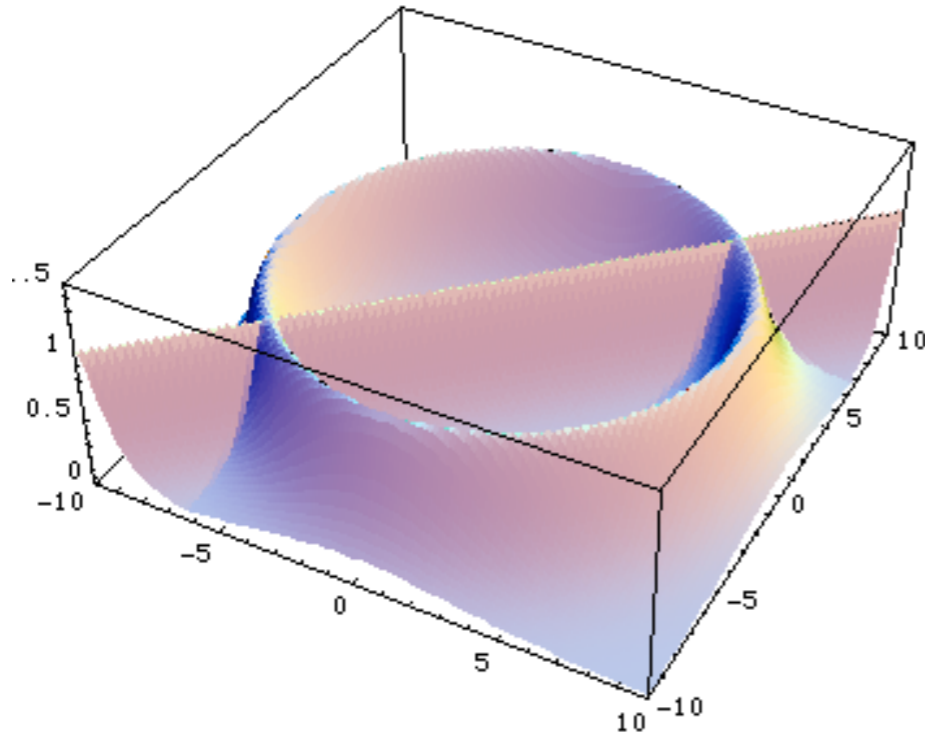
What do we do if there is no transformation that aligns all integrand peaks to the chosen axes?
Vegas is bound to fail!

Solution: use different transformations = channels

$$p(x) = \sum_{i=1}^n \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^n \alpha_i = 1$$

with each $p_i(x)$ taking care of one “peak” at the time

Multiple amplitude

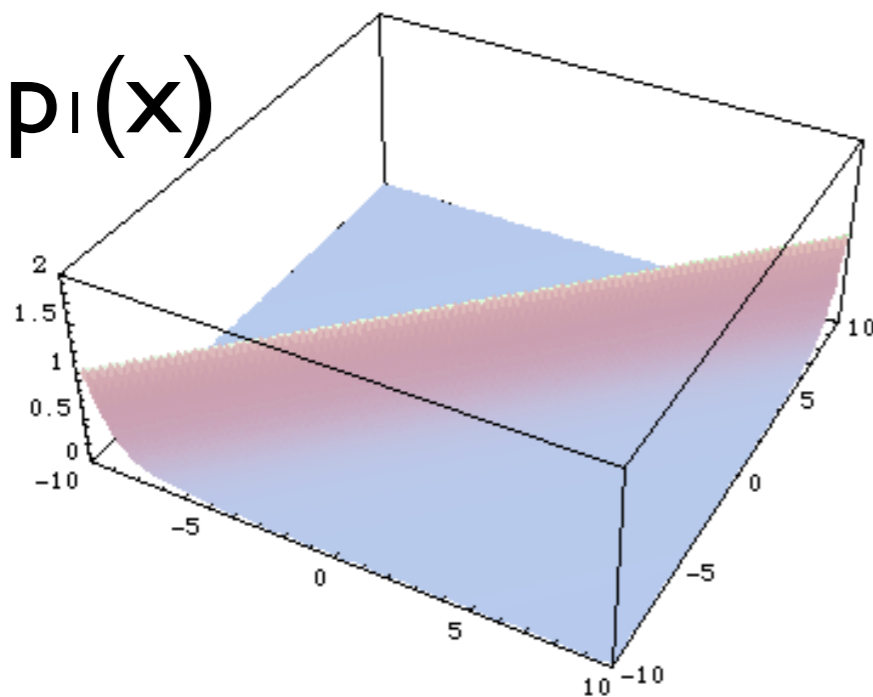


$$p(x) = \sum_{i=1}^n \alpha_i p_i(x)$$

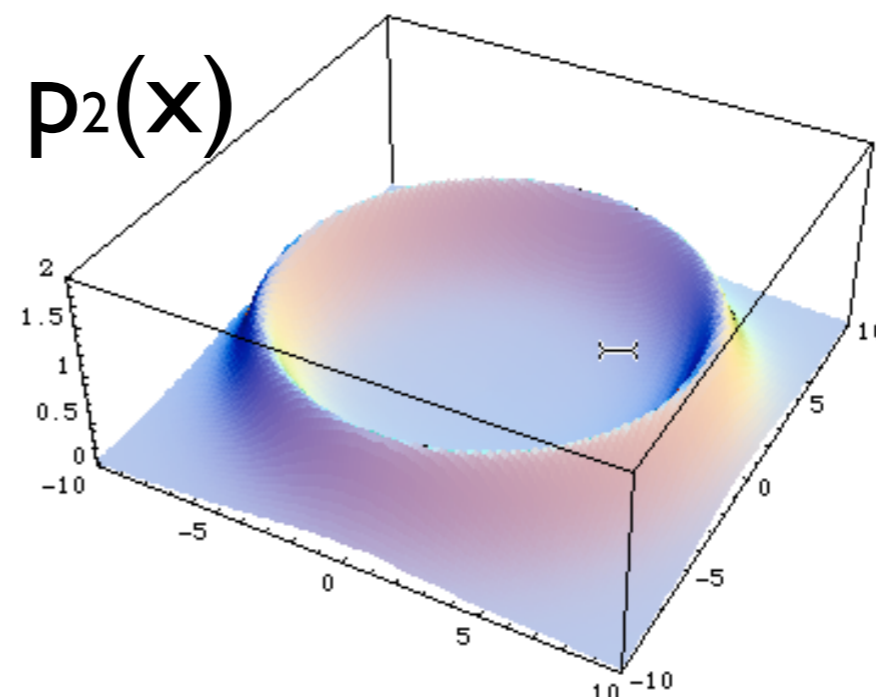
with

$$\sum_{i=1}^n \alpha_i = 1$$

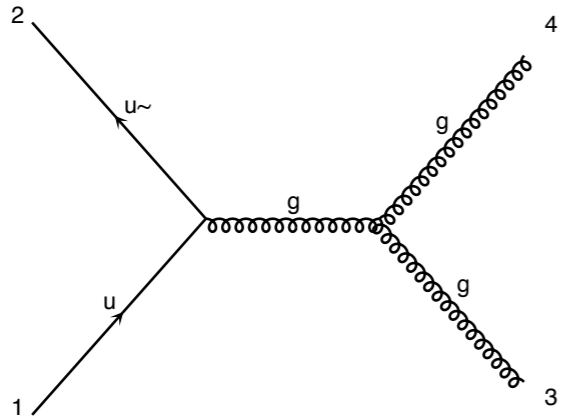
$p_1(x)$



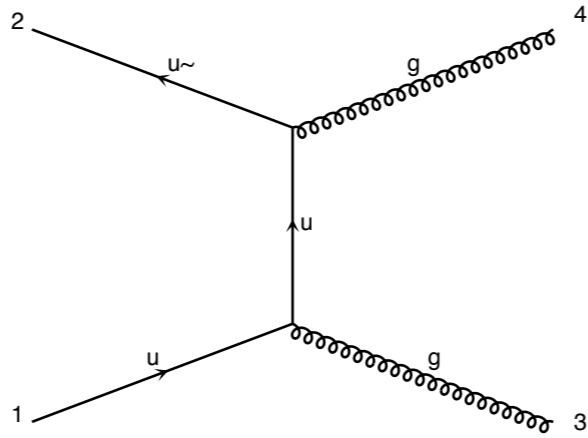
$p_2(x)$



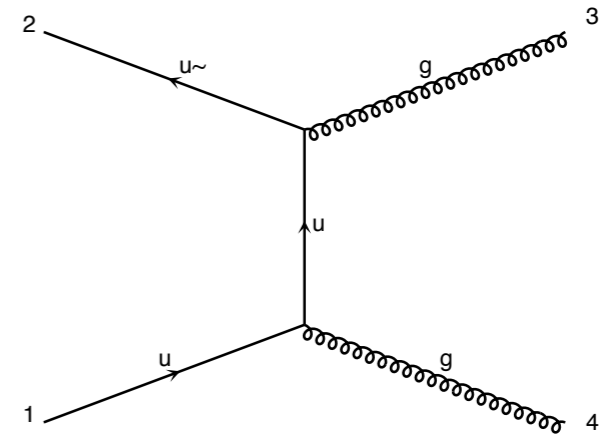
Example



$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2}$$



$$\propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2}$$



$$\propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Example

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

- Any single diagram is “easy” to integrate (pole structures/suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

≈ 1

N Integral

- Errors add in quadrature so no extra cost
- “Weight” functions already calculated during $|M|^2$ calculation
- Parallel in nature

Example

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

[P1 qq wpwm](#)

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

[P1 gg wpwm](#)

s= 20.714 ± 0.332 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

term of the above sum.

each term might not be gauge invariant

Previous exercises

- In MG, generate different processes with increasing number of final state particles
 - generate $e^+ e^- \rightarrow \mu^+ \mu^-$
 - display diagrams

What do we expect :

- $\sigma \propto \alpha^2$
- At high energy : $\sigma \propto 1/s$, $s = (p_1 + p_2)^2$
- goes to zero at threshold
- lorentz invariant

Ex : check it in MG

Previous exercises

- In MG, generate different processes with increasing number of final state particles
 - generate $e^+ e^- \rightarrow \mu^+ \mu^-$
 - display diagrams

$$e^+ e^- \rightarrow \mu^+ \mu^-, \text{ without } z/a$$

What do we expect :

- $\sigma \propto \alpha^2$
- At high energy : $\sigma \propto 1/s$, $s = (p_1 + p_2)^2$
- goes to zero at threshold
- lorentz invariant

Ex : check it in MG

Exercises

- Compute the cross section for
 - $e^+e^- \rightarrow \mu^+\mu^-$
 - $e^+e^- \rightarrow t\bar{t}$
 - $pp \rightarrow \mu^+\mu^-$
 - $pp \rightarrow t\bar{t}$
- Change beam energy/ top mass
- Understand the uncertainties

How to start with MadGraph

- Download the MadGraph folder from <https://launchpad.net/mg5amcnlo>
- untar the folder
- open a terminal and run `./bin/mg5_aMC`
- run `tutorial`
- and follow the instructions
- run `install madanalysis5`

Who has
windows only?

Download MadGraph



MadGraph5_aMC@NLO

Overview

Code

Bugs

Blueprints

Translations

Answers

Registered 2009-09-15 by  Michel Herquet

MadGraph5_aMC@NLO is a framework that aims at providing all the elements necessary for SM and BSM phenomenology, such as the computations of cross sections, the generation of hard events and their matching with event generators, and the use of a variety of tools relevant to event manipulation and analysis. Processes can be simulated to LO accuracy for any user-defined Lagrangian, and the NLO accuracy in the case of models that support this kind of calculations -- prominent among these are QCD and EW corrections to SM processes. Matrix elements at the tree- and one-loop-level can also be obtained.

MadGraph5_aMC@NLO is the new version of both MadGraph5 and aMC@NLO that unifies the LO and NLO lines of development of automated tools within the MadGraph family. It therefore supersedes all the MadGraph5 1.5.x versions and all the beta versions of aMC@NLO. As such, the code allows one to simulate processes in virtually all configurations of interest, in particular for hadronic and e+e- colliders; starting from version 3.2.0, the latter include Initial State Radiation and beamstrahlung effects.

The standard reference for the use of the code is: J. Alwall et al, "The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations", arXiv:1405.0301 [hep-ph]. In addition to that, computations in mixed-coupling expansions and/or of NLO corrections in theories other than QCD (eg NLO EW) require the citation of: R. Frederix et al, "The automation of next-to-leading order electroweak calculations", arXiv:1804.10017 [hep-ph]. A more complete list of references can be found here: http://amcatnlo.web.cern.ch/amcatnlo/list_refs.htm

Download:

The latest stable release can be downloaded as a tar.gz package (see the right of this page), or through the git versioning system, using git clone

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released on 2023-05-12

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