Lectures on dark matter physics

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Abstract: This is a set of lectures on dark matter physics presented during the 30th VSOP school in Quy Nhon, Vietnam. They are, of course, by no means an exhaustive account of the status of dark matter physics. Their goal is to provide information and/or motivation for further exploration. Some intermediate derivations are explicitly left as exercises, whereas some more qualitative questions are meant to provide material for thought and the development of some intuition on the topic. Comments and/or remarks are welcome!

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1 Introduction: what is dark matter?

Before delving into the main part of these lectures, it's useful to spell out explicitly what their topic will be. Dark matter is an – as of yet – unidentified (in the sense that we do not know what it's made of) form of matter that permeates the Universe and which

- Affects the motion of stars in galaxies.
- Affects the propagation of light emitted by distant objects.
- Affects the formation of structures in our Universe.

These three points constitute, in a sense, the "definition" of dark matter: observations related to these three elements are the reason why dark matter was introduced in the first place.

There are a few more things that we know about dark matter

- It does not emit light (hence, "dark"), and it does not interact with it (at least not too much, in the sense that if it does, it does so extremely weakly).
- Most of it is cold, *i.e.* its velocity is small compared with the speed of light.
- It is quite stable, in the sense that the lifetime of dark matter "particles" is longer than the age of the Universe.
- It accounts for $\sim 27\%$ of the matter-energy content of the Universe.
- It cannot be made up of Standard Model particles. Note that there is one notable caveat to this assertion (here we have in mind primordial black holes) but, even if no exotic elementary particle is added to the Standard Model of particle physics, some form of New Physics must exist.

Hopefully, most of these introductory remarks will be made clear during these lectures.

In order to understand dark matter it is essential to be familiar with a few concepts from Cosmology (which studies why and how the Universe came to have the characteristics that we observe) and particle physics (which studies the fundamental constituents of matter and their interactions). In the first lecture, we will remind some important notions from cosmology. Then, we will sketch the main observational arguments that lead to the introduction of dark matter. Afterwards, we will study in some more detail some of the most popular scenarios which aim at explaining why the Universe appears to contain as much dark matter as we observe. Finally, we will describe two of the most important strategies that have been (and are being) pursued in order to detect dark matter through its non-gravitational interactions.

2 Useful background on cosmology

More often than not, during our undergraduate studies as physicists we solve exercises (explicitly or implicitly) "ignoring gravity". Well, cosmology is one of these cases in which we cannot do this: gravity is completely essential in order to understand the structure and the evolution of most apsects of the Universe at large scales. And, until disproven by experiment, gravity is described by the General theory of Relativity (GR).

In General Relativity gravity is understood as a deformation of spacetime. All objects move and interact within this spacetime (as well as *with* this spacetime), and the way they move is dictated by the geometry of this spacetime (which, in turn, is dictated by its matter/energy content). The fundamental object which describes such a geometry is the metric tensor, $g_{\mu\nu}$, from which we can compute the line element ds^2

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{2.1}$$

where $\mu, \nu \in \{0, 1, 2, 3\}$. It obeys the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \tag{2.2}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and scalar, respectively, Λ is called the cosmological constant and $T_{\mu\nu}$ is the stress-energy tensor.

But what is $g_{\mu\nu}$ for our Universe? In order to constrain its form, we need on the one hand to take into account a few experimental observations. On the other hand, we also need to realize that although at small scales (e.g. at the level of individual solar systems) the Universe does look almost desperately complicated, if we consider scales at which the local fluctuations in the matter/energy density can be averaged out the situation changes quite drastically. In particular, the observations show that, at large enough scales, the Universe is homogeneous (there are translational symmetries from one spatial point to another) and isotropic (the Universe looks the same in all directions). Now, it is very important to note that both of these properties refer to the *spatial* part of spacetime. However, the geometry of spacetime *can* be a function of time. And, indeed, observations also show that the Universe is expanding (and at an accelerated rate).

This situation can be described by the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{2.3}$$

where k can, in principle, take any real value, but can always be chosen (through appropriate redefinitions) to take the values

- $k = 1$ for a closed, finite volume Universe (like a sphere).
- $k = 0$ for a flat, infinite volume Universe[1].
- $k = -1$ for an open, infinite volume one (like a hyperbolic surface).

Observationally, the Universe turns out to be approximately flat, so we will choose $k = 0$ in everything that follows. $a(t)$, on the other hand, is a quantity called the (time-dependent) scale factor, describing the time-evolution of the spatial part of the metric. In particular, and assuming $k = 0$, if an observer at the origin is separated, at $t = 0$, from another observer by a radial distance r, after time t this distance will have evolved to $a(t)r$: the scale factor describes the expansion of the Universe. It is dimensionless and, conventionally, we set (where t_0 is the present and in this context the 0 subscript refers to present-day values)

$$a(t = t_0) \equiv a_0 = 1 \tag{2.4}$$

For completeness, and although we will not use it too much during these lectures, let us also introduce the cosmological redshift z

$$1 + z = \frac{1}{a} \tag{2.5}$$

which implies $z_0 = 0$.

[1] Note that spatial flatness does *not* imply that spacetime is flat! A flat spacetime would require *both* $k = 0$ and $a(t) = \text{const}$.

At a quantitative level, there is another parameter which is highly relevant for the description of the expansion of the Universe, the Hubble parameter

$$H(t) \equiv \dot{a}/a \tag{2.6}$$

where the overdot denotes differentiation with respect to proper time. In cosmology it is assumed that, when averaged over large enough volumes, the matter-energy content of the Universe behaves like a (homogeneous and isotropic) perfect fluid with energy density ρ and pressure p. Under this assumption, the Einstein equations lead to the Friedmann equations

$$H(t) = \frac{1}{M_{Pl}}\sqrt{\frac{8\pi}{3}\rho(t)} \tag{2.7}$$

$$\dot{H} + H^2 = -\frac{8\pi}{6M_{Pl}^2}(\rho + 3p) \tag{2.8}$$

where $M_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass. The present-day value of the Hubble parameter is

$$H_0 = \frac{100\text{km}}{\sec \cdot \text{pc}} \times h = \frac{h}{9.78 \times 10^9\text{yr}}, \quad h \approx 0.7 \tag{2.9}$$

Note also that for perfect fluids, the pressure is related to the energy density through an equation of state $p = w\rho$. By combining entropy conservation with the first law of thermodynamics, $dS = dU + pdV$, we find that

$$a(t) = a_0 t^{\frac{2}{3(1+w)}} \Rightarrow H(t) = \frac{2}{3(1+w)}\frac{1}{t} \tag{2.10}$$

Exercise: Prove equation (2.10).

We can now distinguish three cases:

- Non relativistic matter (ordinary + dark, "dust") is characterized by $w = 0$, which leads to $p_m = 0$ and $\rho_m \sim 1/a^3$
- Relativistic matter ("radiation") is characterized by $w = 1/3$, which leads to $p_r = \rho_r/3$ and $\rho_r \sim 1/a^4$
- Dark energy is characterized by $w = -1$, which leads to $p_\Lambda = -\rho_\Lambda$ and $\rho_\Lambda \sim$ const.

Let us forget about dark energy for now. The scaling behaviour of the dust energy density as a function of the scale factor can be understood intuitively by thinking about the energy density of a number of non-relativistic particles in an expanding box: as long as the total number of particles is constant, if the box doubles in size the density of particles will be divided by a factor two, *i.e.* the number density is inversely proportional to the volume of the box. In the case of radiation (let's say photons), on the other hand, one also needs to consider the fact that all wavelengths are stretched along with the expansion of spacetime: all photons are redshifted. Hence, an additional factor of $1/a$ is needed.

Let us note that the case $k = 0$ (spatially flat Universe) in (2.3) corresponds to a specific ("critical") value for the matter/energy density of the Universe

$$\rho = \rho_c \equiv \frac{3H^2}{8\pi G} \tag{2.11}$$

The Universe is composed of all three components described previously (dust, radiation, dark energy). The energy densities of the three components i are conventionally reported as

$$\Omega_i = \frac{\rho_i}{\rho_c} \tag{2.12}$$

Now, since the energy densities of the three components scale differently with a, the relative contribution of the three components to the total matter-energy content of the Universe changes with time. We distinguish, in particular, three periods: radiation dominiation, matter domination and, eventually, dark energy domination. This is depicted in figure 1.

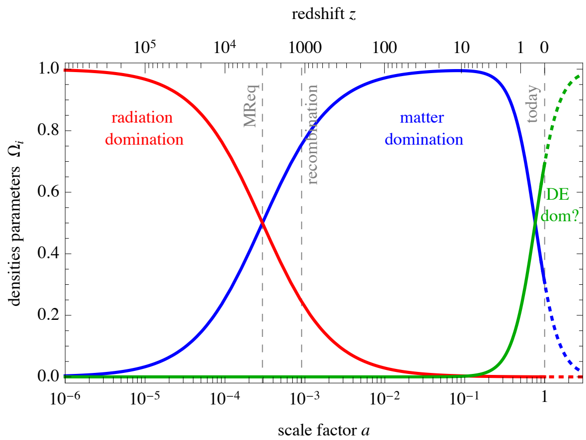

Figure 1: Evolution of the different components of the Universe. Figure taken from [1].

In the majority of dark matter models, most of the action takes place during the radiation domination era. The reason for this is that from CMB observations we know that dark matter must behave as (non-relativistic) matter before matter-radiation equality. This, in turn, leads to most dark matter production mechanisms operating during the radiation domination era.

Let us now gradually start discussing particles. Let's assume a particle species i in the Universe, following a phase space distribution function f_i. For each set of particles in kinetic equilibrium, *i.e.* forming a thermal bath, we can define a temperature T. The number (n) and energy (ρ) density of i per unit volume is given by

$$n_i = \frac{g_i}{(2\pi)^3} \int f_i(\vec{p}) d^3\vec{p} \tag{2.13}$$

$$\rho_i = \frac{g_i}{(2\pi)^3} \int E_i(\vec{p}) f_i(\vec{p}) d^3\vec{p} \tag{2.14}$$

where g_i is the number of internal degrees of freedom of the particle species and $\vec{p}, E$ are the particle 3-momenta and energy respectively, with $E = \sqrt{\vec{p}^2 + m^2}$. Particles in kinetic equilibrium with a thermal bath follow the usual Fermi-Dirac/Bose-Einstein distributions

$$f_i = \frac{1}{\exp(E_i - \mu_i)/T \pm 1} \tag{2.15}$$

where the plus sign applies to fermions and the minus sign to bosons.

Note that in general, if kinetic equilibrium is lost it is much trickier to define a notion of temperature. Moreover, if there exist different sets of particles in kinetic equilibrium within each set but not among sets, we can define multiple thermal baths each with each own temperature.

As we already mentioned, most of what we'll discuss in these lectures will be taking place during the radiation domination epoch. Hence, let's write down a few additional useful relations that hold in such a Universe. In the same manner as we defined the energy density of some component of the Universe, ρ, we can also define an entropy density s. To these, we associate the effective numbers of degree of freedom, $g_{\rm eff}$ and $h_{\rm eff}$, respectively, through

$$\rho_r(T) = \frac{\pi^2}{30} g_{\rm eff}(T) T^4, \quad s_r(T) = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3 \tag{2.16}$$

From total entropy (S) conservation, $dS/dt = 0$, we further obtain

$$\frac{ds}{dt} = -3Hs \tag{2.17}$$

From this equation, we can also deduce a relation between cosmic time and temperature

$$dt = -\frac{dT}{\bar{H}(T) T} \tag{2.18}$$

where

$$\bar{H}(T) = \frac{H(T)}{\left(1 + \frac{1}{3} \frac{d\log(h_{\rm eff})}{d\log(T)}\right)} \tag{2.19}$$

Exercise: Prove equation (2.18).

This relation allows us to compute the time that it takes for the Universe to cool from an initial temperature T_1 to a final temperature T_2

$$t = \int_{T_2}^{T_1} \frac{dT}{\bar{H}(T) T} \tag{2.20}$$

This concludes our small recap of useful notions and relations in cosmology. It goes without saying that there are numerous excellent textbooks and sets lecture notes which one can use in order to get a much more complete picture concerning the early phases of the Universe, including additional effects which we do not have time to even mention, *cf e.g.* [].

3 Evidence for dark matter

Let us now turn to the reasons that lead to the introduction of the notion of dark matter in the first place. There are actually multiple pieces of evidence supporting the existence of dark matter. There have also been competing approaches in order to "explain away" some of the observational anomalies that could be explained through dark matter. One of the most crucial reasons, however, for which dark matter is very widely accepted to exist is due to the fact that the various pieces of evidence concern wildly different astrophysical systems and, in particular, wildly different physical scales.

3.1 Galaxies: Galactic rotation curves

The first class of observations is concerned with spiral galaxies. Spiral galaxies rotate around their vertical axes and, by observing the Doppler shift of the light emitted by their visible components (*e.g.* stars and gas), in many cases it is possible to deduce the circular velocity v_c with which each component rotates as a function of its distance from the galactic centre. This is called a galactic rotation curve.

At the same time, by employing spectroscopic methods it is possible to determine the mass of a galaxy. Or, more accurately, it is possible to determine the part of its mass that emits light.

For simplicity, let us consider a star of mass m living at a distance r from the galactic centre, somewhere in the (visible) outskirts of a spiral galaxy. Let's also assume that the (visible) mass of the galaxy has been measured to be M. Then, we can estimate its circular velocity by equating the centripetal and the gravitational force exerted on the star

$$\frac{mv_c^2(r)}{r} = G\frac{mM}{r^2} \Rightarrow v_c(r) = \sqrt{\frac{GM}{r}} \tag{3.1}$$

In other words, one would expect that stars which are further away from the galactic centre should rotate slower than those which are closer. This, however, is not what is observed.

Indeed, actual rotation curves look much more like the one in figure 2. Contrary to expectation, towards the outskirts of galaxies rotation curves tend to flatten out. We can envisage two possible ways out of this situation

- Either the theory that we used in order to compute v_c is wrong/inadequate (note that even a full-blown GR calculation would lead to a result similar to the naive Newtonian result that we just obtained). This is the approach of modified gravity.

- Or the quantities that we used in these equations are wrong. In other words, this would mean that our estimate of the galaxy's mass based only on luminous matter is

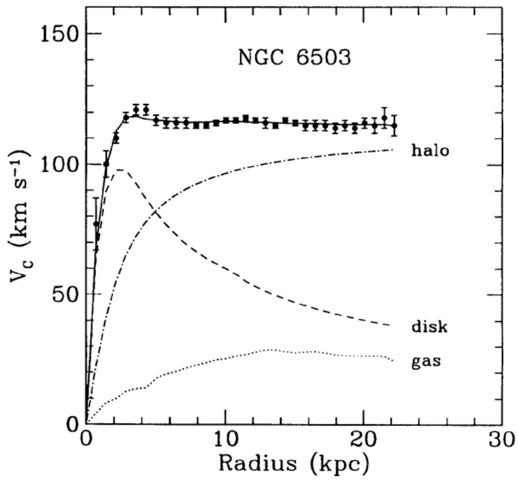

Figure 2: Rotation curve of NGC6503. Figure taken from [].

wrong. Yet in other words, there could be some form of "invisble" or "dark" matter that we haven't accounted for, and which extends well beyond the region in which we observe the galaxy's visible component. This is the approach of dark matter.

It should be noted that at this level, both approaches work equally well (in fact, modified gravity theories tend to work even better at reproducing observations; after all, they are often actually constructed exactly in order to do so). It is also worthwhile noting that, in different contexts, both approaches have worked historically: observations of anomalies in the orbit of Uranus lead astronomers to hypothesize the existence of a new planet in the solar system during the 19th century, which turned out to be Neptune. When this approach was replicated in order to explain away anomalies in the orbit of Mercury, by postulating the existence of a planet named "Vulcan", it actually failed; the solution came with the introduction of General Relativity, a modified theory of gravitation with respect to Newtonian mechanics.

3.2 Galaxy clusters: Weak gravitational lensing vs X-ray spectroscopy

Additional - and perhaps even more compelling - evidence for the existence of dark matter appears once we move to the scale of galaxy clusters. The most characteristic example in this class of observations concerns the "Bullet cluster". The Bullet cluster is located around 3.7 Gyr away from us, and it is the result of the collision of two galaxy clusters. Most of its visible mass is comprised of hot baryonic gas, which can be observed through X-ray spectroscopy. Its spatial distribution is represented by the coloured regions in figure 3.

There is, however, another method which allows the reconstruction of the *total* mass of the cluster: weak gravitational lensing. Gravitational lensing in general relies on the fact that photons move along geodesics, the form of these geodesics is dictated by the geometry of spacetime and the latter is determined by its matter-energy content. If there were no

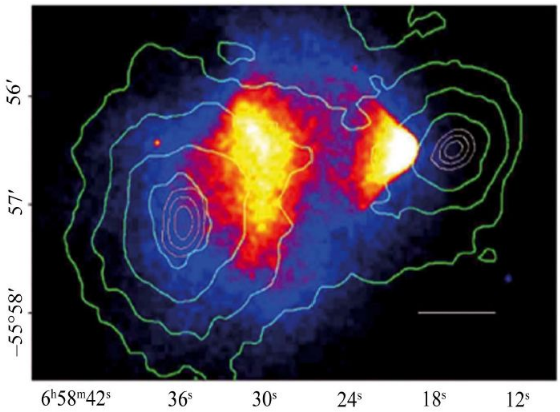

Figure 3: Bullet cluster visible (coloured regions) and total (green contours) mass distributions [2].

matter intervening between us (the observers) and a distant star, then the light emitted by a distant star would travel until us in a straight line. If some amount of matter is interjected between the source and the observer, then spacetime will be curved, and so will the geodesics. This can lead to distant objects appearing as multiple images (in limiting cases, even entire rings, called "Einstein rings"). Such extremely favourable conditions lead to so-called "strong lensing". In most cases, the lensing effect is not strong enough and has to be inferred using statistical methods ("weak lensing"). Long story short, this method can be used in order to reconstruct the mass of (visible or invisible) objects interjected between us and distant light sources. It has also been used for the bullet cluster and its result is represented by the green contours in figure 3.

Clearly, the two measurements tell a different story. The baryonic component of the clusters is mostly located around the brightest regions in figure 3, clearly separated from the bulk of their mass, which lies further away. This behaviour is consistent with the clusters being mostly composed of some non-luminous (dark) form of matter, which is essentially collisionless and, hence, much less impacted by the collision of the two clusters[2]. Their visible components, on the other hand, interact with each other (through the usual interactions that we are aware of) and is substantially slowed down due to dissipation of energy. This leads to a clear separation of the two components which, by the way, is also observed in other galaxy clusters.

This kind of observations constitutes one of the strongest arguments for the existence of dark matter: modified gravity theories cannot really explain this spatial separation since, be it in GR or in alternative theories of gravitation, gravity affects all forms of matter in the same manner.

[2]In passing, note that this type of observations also allows us to set limits on the dark matter self-interaction cross-section!

3.3 The observable Universe: the Cosmic Microwave Background

Perhaps the most celebrated argument for the existence of dark matter comes from observations of the Cosmic Microwave Background (CMB). Properly introducing the CMB would merit a set of lectures on its own. Here, we will contend ourselves with a very brief qualitative description.

As we have already said, at early enough times the Universe was small and hot, with photons, electrons and baryons being in thermal equilibrium. During this period, photons could not travel freely: they scattered on baryons and electrons. However, once the temperature of the Universe dropped sufficiently, electrons started binding with nuclei forming atoms, photons could no longer scatter with such neutral objects and the Universe became transparent.

At the same time, the initial distribution of matter was not completely homogeneous. Small fluctuations did exist, which lead to fluctuations of the total gravitational field felt by cosmic photons. This in turn, lead to the existence of small anisotropies in the temperature of the photon field. After recombination the photons could travel unobstructed throughout the Universe, giving rise to a relic radiation which we call CMB. In this sense, the CMB gives us (among other pieces of information) a snapshot of the Universe at the time around recombination.

The most recent version of the CMB power spectrum can be seen in figure 4 [3].

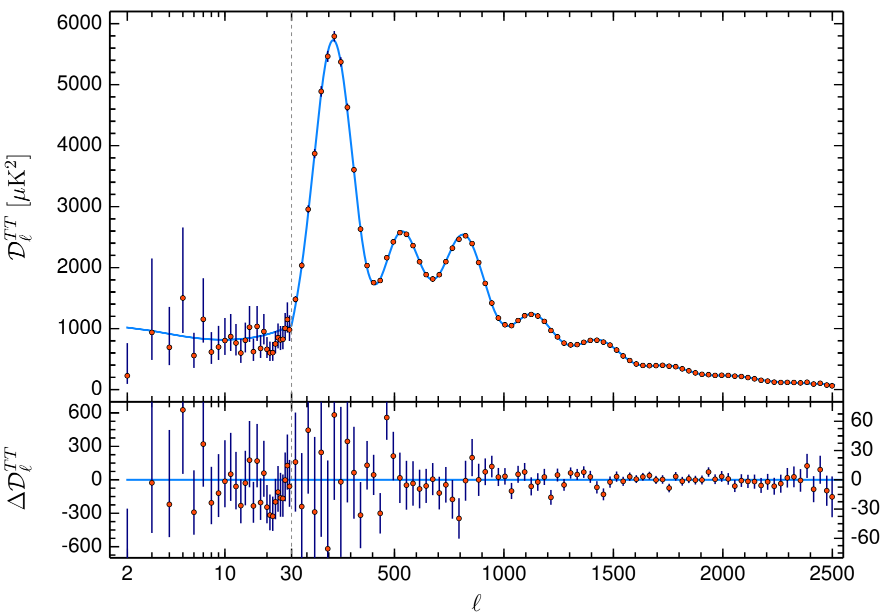

Figure 4: The CMB power spectrum [4].

Very roughly speaking, the scale of the first peak of the CMB power spectrum provides information concerning the curvature of the Universe, whereas the following ones (and, in particular, their mutual relation) are most useful in order to quantify the different components of the Universe[3]. For dark matter, in particular, the Planck 2018 results [3] yield

$$\begin{aligned}
\Omega_{\rm m} h^2 &= 0.1430 \pm 0.0011 \ (68\%) \\
\Omega_{\rm CDM} h^2 &= 0.1200 \pm 0.0012 \ (68\%) \\
\Omega_{\rm b} h^2 &= 0.02237 \pm 0.00015 \ (68\%) \\
\Omega_{\Lambda} h^2 &= 0.3107 \pm 0.008 \ (68\%)
\end{aligned} \tag{3.2}$$

i.e. the Universe contains ~ 5 times more dark matter than ordinary matter.

4 Dark matter candidates

In the previous lecture we reviewed some basic facts about the Universe and its evolution and we presented the main observational arguments that lead to the introduction of dark matter. However, we have not said too much about what dark matter could be.

One of the most important pieces of information that we have concerning dark matter is its cosmic abundance. The reason why it is so important is because it is an actual *measurement* (as opposed, *e.g.*, to an upper/lower limit constraint). In fact, it is so important that, we could say, it sets a defining point for every dark matter model: most existing dark matter models set as their primary goal to explain why there is as much dark matter in the Universe as we observe.

Our goal today is to introduce one of the most popular mechanisms that exist in order to explain the dark matter cosmic abundance, the so-called freeze-out mechanism. Time permitting, we would also like to briefly mention a few alternative dark matter generation mechanisms, knowing that it's impossible to do them justice in such a short time.

4.1 Thermal freeze-out

For reasons of simplicity, let us assume that dark matter is composed of a single type of particle species χ which interact in pairs with pairs of Standard Model particles through reactions of the type

$$\chi\chi \leftrightarrow \text{SM SM} \tag{4.1}$$

Let us also assume that the dark matter particles are heavier than all SM ones.

Let's first try to figure out qualitatively what we expect to happen. If the interactions between dark matter and SM particles are strong enough, it reasonable to expect (and can be verified numerically) that at early enough times, when the Universe was dense and hot, the reactions of the type (4.1) were in equilibrium, *i.e.* that the reaction rate was the same in both directions (annihilation/production). During this period, the dark matter and the SM particles form a common thermal bath with a unique well-defined temperature, whereas the number density of dark matter particles per unit of comoving

[3] In practice, the CMB power spectrum is fitted – along with other observables – with full-blown cosmological models, with multiple parameters varying simultaneously. Once again, in these lecture we can barely scratch the surface of this beautiful topic, which is covered in most Cosmology textbooks.

spacetime volume is expected to stay constant. Note that the temperature provides us with a measure of the mean kinetic energy of particles which, in turn, determines whether a reaction is kinematically allowed or not.

Now, as the Universe expands, its temperature drops and, at some point, it drops below the dark matter mass. Around this time (not exactly, since particle energies follow a distribution with a certain spread), dark matter particles can no longer be produced from annihilations of SM ones, since the reaction becomes kinematically disfavoured: most SM particles are simply not energetic enough to produce a pair of dark matter particles. After this point, dark matter particles start annihilating away and their number density starts dropping exponentially. However, as the Universe continues to expand all particles become more and more dilute. Then, as this process goes on, we can imagine that after a certain point the probability that two dark matter particles meet in order to annihilate becomes so small that, in effect, the reaction stops taking place. After this point, the number density of dark matter particles remains constant. This process is called (thermal) freeze-out.

Let's now see how all this picture works quantitatively. Let us at first keep the discussion quite general, and consider four types of particles, $1, 2, 3$ and 4, in the early Universe, which can interact through reactions of the type $1 + 2 \leftrightarrow 3 + 4$. Let's also assume that each particle species follows a phase space distribution function f_i and let's try to figure out the time(/temperature) evolution of the number density of species 1. The phase space distribution of species 1 obeys the Boltzmann equation which, in full generality, can be written as

$$L[f_1] = C[f_1] \tag{4.2}$$

where L is called the Liouville operator, and C is called the collision operator. Let's examine them one-by-one. This classic computation can, *e.g.*, be found in [5].

The Liouville operator

The Liouville operator describes the time evolution of the phase space distribution function. In General Relativity, it reads

$$L[f] = \frac{df}{d\tau} = \frac{\partial f}{\partial x^\mu}\frac{dx^\mu}{d\tau} + \frac{\partial f}{\partial p^\mu}\frac{dp^\mu}{d\tau} \tag{4.3}$$

By using the geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0 \tag{4.4}$$

and the fact that the canonical momentum is given by $p^\mu = dx^\mu/d\tau$, we arrive at

$$L[f] = \frac{\partial f}{\partial x^\mu}p^\mu - \Gamma^\mu_{\rho\sigma}\, p^\rho p^\sigma \frac{\partial f}{\partial p^\mu} \tag{4.5}$$

where $\Gamma^\mu_{\rho\sigma}$ are the Christoffel symbols. They can be computed by taking derivatives of the metric tensor $g_{\mu\nu}$ as

$$\Gamma^\mu_{\rho\sigma} = \frac{1}{2}g^{\mu\lambda}\left(\partial_\sigma g_{\rho\lambda} + \partial_\rho g_{\sigma\lambda} - \partial_\lambda g_{\sigma\rho}\right) \tag{4.6}$$

Applying this to the FLRW metric with $k = 0$, we obtain

$$L[f] = E\frac{\partial f}{\partial t} - H\,|\vec{p}|^2\,\frac{\partial f}{\partial E} \tag{4.7}$$

Exercise: Prove equation (4.7). If you are unfamiliar with the computation of Christoffel symbols, you can use $\Gamma^0_{00} = \Gamma^0_{0i} = \Gamma^0_{i0} = 0$ and $\Gamma^0_{ij} = \dot{a}a\delta_{ij}$. But it's always a good exercise to compute them yourselves!

Now, the full phase space distribution function of dark matter encodes a lot of useful information, *e.g.* it is essential in order to properly determine the structure formation phenomenology of dark matter. However, in the context of explaining the dark matter abundance in the Universe, we are typically just interested in the time evolution of the total number density of dark matter particles. In this sense, it is enough for our purposes to only consider the integral of the Boltzmann equation over phase space. By integrating the expression that we just found over the entire phase space, after some algebra we obtain

$$g\int L[f]\frac{d^3p}{(2\pi)^3} = \frac{1}{a}\frac{d}{dt}(na^3) = \frac{dn}{dt} + 3Hn \tag{4.8}$$

where we have also used the relation between the number density and the phase space distribution. Note that when there are no reactions taking place, *i.e.* $C[f] = 0$ in (4.2), this equation informs us that na^3 remains constant.

The Collision operator

Let us now turn to the collision operator $C[f]$, which quantifies the number of reactions taking place per unit phase space volume. The latter includes all interactions between our particle species of interest and the other particle species (including itself) that may alter the phase-space density. Depending on the nature of the allowed interactions, it can take a more or less complicated form. Since, as we said, we are interested in the integral of the Boltzmann equation, we can write for the collision term for particle 1, and always referring to interactions of the form $1 + 2 \leftrightarrow 3 + 4$ [5]

$$\begin{aligned} g_1\int C[f_1]\,\frac{d^3p_1}{(2\pi)^3} = -\sum_{\text{spins}}\int &\left[f_1f_2(1\pm f_3)(1\pm f_4)|\mathcal{M}_{12\to 34}|^2 - f_3f_4(1\pm f_1)(1\pm f_2)|\mathcal{M}_{34\to 12}|^2\right] \\ &\times(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\,d\Pi_1\,d\Pi_2\,d\Pi_3\,d\Pi_4\,, \end{aligned} \tag{4.9}$$

where g_i and f_i are the spin degrees of freedom and phase-space densities, respectively, for particle i, and $\mathcal{M}_{x\to y}$ is the matrix element for the reaction $x \to y$. Factors of the form $(1\pm f)$ represent Pauli blocking (minus sign) and Bose enhancement (plus sign). They reflect the fact that it is easier(harder) for a boson(fermion) to transition to a state that already contains a boson(fermion). The delta function imposes four-momentum conservation, and the $d\Pi_i$ factors denote phase-space integration factors

$$d\Pi_i = \frac{d^3p_i}{(2\pi)^3\, 2E_i}.$$

Let us now introduce some simplifying assumptions. Depending on the dark matter model, one (or more) of them may fail. However, in most freeze-out models, they hold.

- All particle species are in kinetic equilibrium and so their phase-space distributions take on the Fermi-Dirac or Bose-Einstein forms.
- The temperature of each species satisfies $T_i \ll E_i - \mu_i$, where μ_i is its chemical potential, so that they follow the Maxwell-Boltzmann distribution. In this case, the statistical mechanical factors in the calculation can be ignored and $(1 \pm f) \sim 1$.
- The Standard Model particles in the interaction are in thermal (kinetic + chemical) equilibrium with the photon bath.

Now, the usual unpolarized cross section σ_{ij} for the reaction $ij \to kl$ is given by

$$\sigma_{ij} = \frac{1}{4\, g_i g_j \sqrt{(p_i \cdot p_j)^2 - (m_i m_j)^2}} \times \sum_{\text{spins}} \int |\mathcal{M}_{ij\to kl}|^2 \times (2\pi)^4 \delta^4(p_i + p_j - p_k - p_l)\, d\Pi_k\, d\Pi_l \tag{4.10}$$

Substituting this back into the collision term, and identifying the Moller velocity

$$(v_{\text{Møl}})_{ij} = \frac{\sqrt{(p_i \cdot p_j)^2 - (m_i\, m_j)^2}}{E_i\, E_j} \tag{4.11}$$

for the $ij \to kl$ process, we obtain

$$g_1 \int C[f_1]\, \frac{d^3p_1}{(2\pi)^3} = -\int \{(\sigma v_{\text{Møl}})_{12}\, dn_1 dn_2 - (\sigma v_{\text{Møl}})_{34}\, dn_3 dn_4\} \tag{4.12}$$

Because $\sigma v_{\text{Møl}}$ varies slowly with changes in the number density of the initial and final-state particles, it can be factored out of the integrand to give

$$\dot{n}_1 + 3Hn_1 = -\langle \sigma v_{\text{Møl}} \rangle_{12} n_1 n_2 + \langle \sigma v_{\text{Møl}} \rangle_{34} n_3 n_4\,. \tag{4.13}$$

where we defined the velocity-averaged cross-section

$$\langle \sigma v \rangle = \frac{\int \sigma v\, dn_1^{\text{eq}}\, dn_2^{\text{eq}}}{\int dn_1^{\text{eq}}\, dn_2^{\text{eq}}} = \frac{\int \sigma v\, e^{-E_1/T}\, e^{-E_2/T}\, d^3p_1\, d^3p_2}{\int e^{-E_1/T}\, e^{-E_2/T}\, d^3p_1\, d^3p_2} \tag{4.14}$$

where $\langle \sigma v \rangle = \langle \sigma v \rangle_{12}$.

Putting everything together

Let us now focus on the specific case in which two dark matter particles with number density n annihilate into two Standard Model ones and vice versa. We assume (and this is calculable) that the latter are in thermal equilibrium with the photon bath. If, moreover, there is no CP violation involved in the reaction, we have

$$|\mathcal{M}_{ij\to kl}|^2 = |\mathcal{M}_{kl\to ij}|^2 \tag{4.15}$$

and

$$f_1 f_2 = f_1^{\rm eq} f_2^{\rm eq} = e^{-\frac{E_1+E_2}{T}} = e^{-\frac{E_3+E_4}{T}} = f_3^{\rm eq} f_4^{\rm eq} \tag{4.16}$$

where we have also used energy conservation between the initial and final states. As long as the dark matter particles are also in thermal equilibrium with the Standard Model bath, the principle of detailed balance leads to

$$\langle\sigma v\rangle_{12}\, n_{\rm eq}^2 = \langle\sigma v\rangle_{34}\, n_3^{\rm eq} n_4^{\rm eq}\,,$$

which can be used to rewrite the second term of (4.13) in terms of the dark matter number density and its annihilation cross-section. Then, our Boltzmann equation acquires the simpler form

$$\dot{n} + 3Hn = \langle\sigma v\rangle \left(n_{\rm eq}^2 - n^2\right)\,, \tag{4.17}$$

Already at this stage, we see that we have a differential equation for the time evolution of the number density n which can be solved, at least numerically. In practical calculations, it is customary to work with slightly different variables. In particular, we define

$$x \equiv m/T \tag{4.18}$$

as well as the abundance

$$Y \equiv n/s \tag{4.19}$$

From these definitions, we have

$$\begin{aligned} \frac{dY}{dt} &= \frac{d}{dt}\left(\frac{n}{s}\right) = \cdots = \frac{1}{s}\left(3Hn + \frac{dn}{dt}\right) \\ \frac{dx}{dt} &= \bar{H}(T)x \\ \frac{dY}{dt} &= \frac{dY}{dx}\frac{dx}{dt} = \frac{dY}{dx}\bar{H}x \end{aligned} \tag{4.20}$$

where in the first equation we used $ds/dt = -3Hs$ and in the second one the time-temperature relation that we wrote down when we discussed cosmology as well as the defition of x. Substituting back in the Boltzmann equation, we obtain

$$\frac{dY}{dt} = \langle\sigma v\rangle s \left(Y_{\rm eq}^2 - Y^2\right) \tag{4.21}$$

which, eventually, leads to

$$\frac{dY}{dx} = -\frac{s\langle\sigma v\rangle}{x\bar{H}}\left(Y^2 - Y_{\rm eq}^2\right)\,. \tag{4.22}$$

Exercise: Starting from Eq.(4.17), derive Eq.(4.22).

Note that quite frequently, the $\bar{H}$ factor in the denominator of the last differential equation is approximated by the Hubble parameter because the entropy effective number of degrees of freedom varies slowly with temperature, at least far from phase transitions. In any case, this is a more precise form of the Boltzmann equation for the dark matter abundance, see also [6]. The result of a numerical resolution of the Boltzmann equation for varying values of $\langle \sigma v \rangle$ can be seen in figure 5.

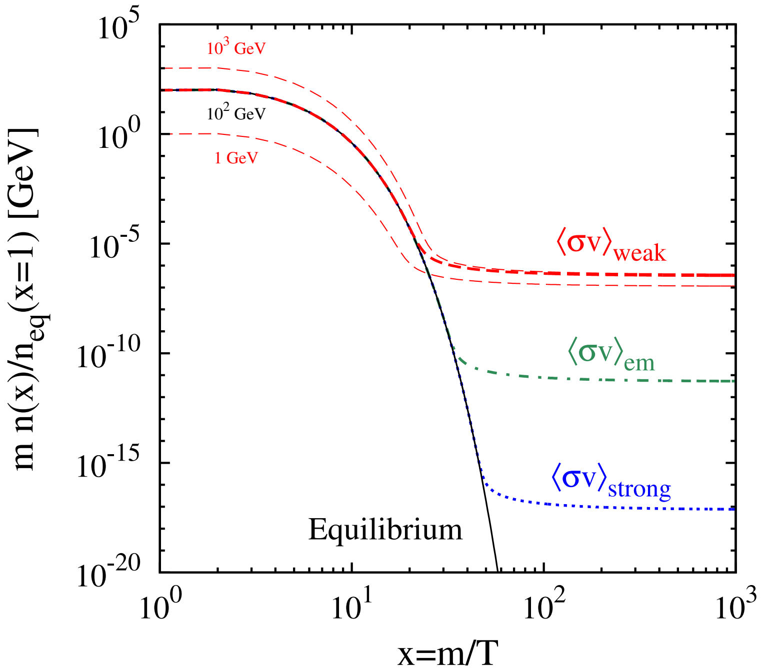

Figure 5: Temperature evolution of the dark matter relic density according to the freeze-out mechanism. Figure taken from [6].

A few further manipulations can be performed in order to render the Boltzmann equation more practical for concrete dark matter models. After some change of variables and some tedious algebra we can, moreover, rewrite $\langle \sigma v \rangle$ as [5]

$$\langle \sigma v \rangle = \frac{1}{8m^4 T K_2^2(m/T)} \int_{4m^2}^{\infty} \sigma(\tilde{s})(\tilde{s} - 4m^2)\sqrt{\tilde{s}}\, K_1(\sqrt{\tilde{s}}/T)\, d\tilde{s} \tag{4.23}$$

where K_i are modified Bessel functions of the i^{th} order and $\tilde{s} = 2m^2 + 2E_1E_2 - 2\vec{p}_1 \cdot \vec{p}_2$.

The cross section can be expanded in x in the non-relativistic limit

$$\langle \sigma v \rangle \approx b_0 + \frac{3}{2} b_1 x^{-1} + \cdots \tag{4.24}$$

where $b_{0,1}$ are calculable coefficients. The case where b_0 dominates is referred to as s-wave annihilation. The case where the second term dominates is called p-wave annihilation.

There is no analytic solution for equations like (4.22); in the general case, it has to be solved numerically. However, using our initial qualitative discussion, there are a few interesting limits that we can consider in order to gain some intuition. As we already mentioned, as long as the interactions of dark matter particles with the Standard Model are strong enough, at early enough times we expect equilibrium to be established. Once the temperature drops below the dark matter mass it still traces its equilibrium distribution and its abundance is exponentially suppressed. Lastly, once Hubble expansion superseeds the annihilation rate its density freezes out. In equation form, we can write

$$Y(x \lesssim x_f) \simeq Y_{\rm eq}(x) \qquad \text{and} \qquad Y(x \gtrsim x_f) \simeq Y_{\rm eq}(x_f)\,,$$

where x_f is the freeze-out time. Note that after freeze-out, the dark matter abundance becomes larger than its equilibrium abundance. Therefore, beyond this point we can ignore the second term in the RHS of the Boltzmann equation and, assuming that $\langle\sigma v\rangle$ is either s- or p-wave dominated, we can pull out all dependence on x and write

$$\frac{dY}{dx} \simeq -\frac{\lambda}{x^{n+2}}Y^2\,, \text{ where } \quad \lambda = \frac{\langle\sigma v\rangle_0 s_0}{\bar{H}}\,.$$

Taking $n = 0$ as an example, we can solve for the DM abundance today:

$$\frac{1}{Y_{\rm today}} - \frac{1}{Y_f} = \frac{\lambda}{x_f} \longrightarrow Y_{\rm today} \simeq \frac{x_f}{\lambda}\,,$$

where the last step uses the fact that the abundance at freeze-out, Y_f, is typically greater than its value today.

Finally, we are in position to plug in some numbers in order to estimate the dark matter density today. Making an educated guess, we will assume that freeze-out occurs around $x \sim 10$. In this case, we have

$$\Omega_\chi = \frac{m\, s_{\rm today}\, Y_{\rm today}}{\rho_{\rm cr}} \longrightarrow \Omega_\chi h^2 \sim \frac{10^{-26}\ {\rm cm}^3/{\rm s}}{\langle\sigma v\rangle} \simeq 0.1 \left(\frac{0.01}{\alpha}\right)^2 \left(\frac{m}{100\ {\rm GeV}}\right)^2 \tag{4.25}$$

taking $\langle\sigma v\rangle \sim \alpha^2/m^2$. Assuming a weakly interacting DM particle with $\alpha \sim 0.01$ and mass $m_\chi \sim 100$ GeV gives the correct abundance today as measured by Planck. This is called the "WIMP miracle" and, during many years (in fact, to this day), it motivated the most widely pursued dark matter detection strategies. However, it should be noted that in (4.25) what is really constrained is the *ratio* of the squared coupling to the mass. It is, then, possible to open up a wider band of allowed masses for thermal DM by taking $\alpha \ll 1$ while keeping α^2/m^2 fixed.

Question: How would the dark matter relic density be impacted if the couplings were weaker/stronger?

Question: Consider a population of WIMPs in the early Universe, with a mass of 100 GeV. Assume also that at a temperature of ~ 100 TeV, an additional population of dark matter

is somehow introduced in the Universe (with the total energy content remaining, again somehow, constant). How would the dark matter abundance today be impacted?
Question: In our discussion we completely ignored reactions of the type $\chi + \text{SM} \leftrightarrow \chi + \text{SM}$. Why?

4.2 Alternative mechanisms

The thermal freeze-out picture that we described previously dominated dark matter physics for more than two decades. The reasons are multiple, ranging from theory-motivated arguments (e.g. a hope that cosmic dark matter production could be related to a resolution of the Hierarchy problem or, at least, be somehow related with the electroweak scale - after all, we know that a lot of interesting physics happens around this scale!) up to the fact that it provides hope that dark matter could be detected non-gravitationally. However, already since the 1980s alternative dark matter genesis frameworks have been proposed. Here we will provide a very short (mostly qualitative) description of a few of them.

4.2.1 Freeze-in

One of the first assumptions (which can, nonetheless, be checked on a model-by-model basis) that we made in our discussion about thermal freeze-out was that dark matter interacts sufficiently strongly with the Standard Model such that the two sectors attained and maintained, at least at high enough temperatures, thermal equilibrium. If the interactions are weaker then, for a given dark matter mass, equilibrium will be lost earlier than it would for WIMPs and the relic abundance would be larger.

But what would happen if these interactions were *really* weak ("feeble")? One might be tempted to answer that the predicted relic abundance would keep increasing. However, if we take such a logic to its extreme, this would mean that a nearly-decoupled dark matter sector would have the maximal abundance. This logic cannot hold.

The answer lies with the fact that, if the interactions are weak enough, then the dark and visible sectors will simply never reach thermal equilibrium. Now, in the standard freeze-out case (*i.e.* when the dark matter - Standard Model interactions are strong enough), the value that we choose for the initial dark matter abundance is somewhat irrelevant: a proper numerical resolution of the Boltzmann equation reveals that very rapidly the dark matter abundance reaches its equilibrium value and, subsequently, follows the usual freeze-out behaviour. In other words, at least in the most standard of cases, equilibrium erases all memory of early Universe physics; standard freeze-out regulates its own initial conditions. This can be viewed as good (because the result does not depend on physics which is, currently, quite poorly understood) or bad (because we do not gain any insight whatsover about the physics of the very early Universe) depending on one's perspective.

Long story short, in the absence of equilibrium, in the case of freeze-in there is a residual dependence on the initial conditions. But then, what was the initial dark matter abundance? A plausible option would be to set it to zero: a feebly interacting massive particle (FIMP) was, likely, absent from the initial plasma[4].

Given the previous discussion, it is reasonable to expect that FIMPs follow a different temperature evolution than the one we saw previously. The important thing to note is that the dark matter annihilation rate scales as $n_\chi^2\langle\sigma v\rangle$. Then, if dark matter is only feebly interacting and with a negligible initial abundance, both of these factors are extremely small. And, in particular, they are much smaller than the rate of the inverse reaction, which scales as $n_{\rm SM}^2\langle\sigma v\rangle$, where the Standard Model particles track their equilibrium distributions. This observation already draws a picture of what will happen: dark matter can be produced from annihilations of SM particles (or, eventually, annihilations and/or decays of any particle species that is in thermal equilibrium with the Standard Model thermal bath), but it does not annihilate back. Note that in this scenario the dark matter production rate is necessarily sub-Hubble, otherwise an enormous amount of dark matter would be produced, eventually rendering dark matter annihilations efficient enough such that the two directions of the reaction would equilibrate and we would fall back to the freeze-out case.

This is the main idea behind the so-called "freeze-in" dark matter production mechanism. Mathematically, we could write in full generality

$$\dot{n}_\chi + 3Hn_\chi = \sum_{A,B}(\xi_B - \xi_A)\mathcal{N}(A \to B) \tag{4.26}$$

where A and B denote generic initial and final states containing $\xi_{A,B}$ particles of type χ respectively and $\mathcal{N}(A \to B)$ is the integrated collision term corresponding to the reaction $A \to B$, *i.e.* the number of $A \to B$ reactions taking place in the thermal bath per unit space-time volume.

Since, in the case of freeze-in, we can ignore backreactions, we can omit the ξ_A term and, eventually, write

$$Y_\chi^0 = \int_{T_0}^{T_R} \frac{dT}{T\overline{H}(T)s(T)}\left(\mathcal{N}(bath \to \chi X) + 2\mathcal{N}(bath \to \chi\chi')\right) \ , \tag{4.27}$$

where X stands for any bath particle and χ' for any dark sector FIMP, which we assume to (eventually) decay into a DM particle along with a visible sector one. We stress that the abundance will depend on the value of the reheating temperature when this temperature is of the same order as the mediator or DM mass, or when the collision term is dominated by high temperatures.

The result can be seen figure 6, taken from reference [7][5]. It is represented by the dashed lines and compared with the corresponding result from freeze-out (solid lines). Note that this behaviour corresponds to a choice of a very high reheating temperature and renormalizable interactions.

[4]Note that the validity of this assumption can, nonetheless, be highly debated.

[5]Incidentally, the term "freeze-in" was first introduced in this reference, even though the mechanism was known quite long before [8].

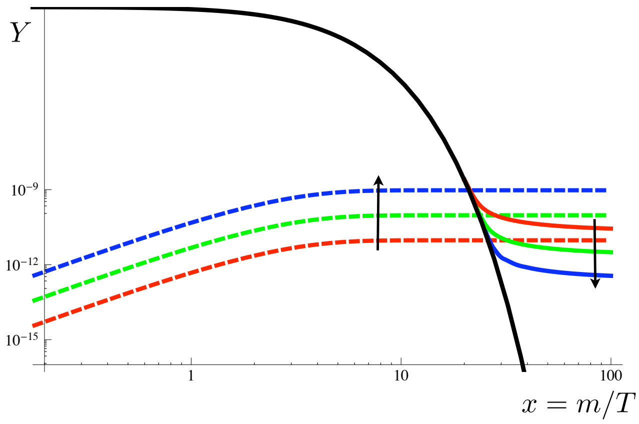

Figure 6: Freeze-out vs freeze-in. Figure taken from [7].

Exercise: Can you write down the integrated collision term for the case of dark matter production through decays of heavier bath particles? In the case of annihilation?

Question: Why do we focus here on "parent" particles from the thermal bath? If the parent particles were themselves FIMPs, can you imagine how dark matter production could proceed?

4.2.2 Asymmetric dark matter

Another assumption that we semi-implicitly made during our discussion on freeze-out is that dark matter is either its own antiparticle or that dark matter particles and antiparticles exist in equal amounts and annihilate with equal rates. However, we know that this doesn't happen in the case of several SM particles. And, indeed, in the Universe's visible we observe much more matter than antimatter. The leading idea about how this asymmetry came to be is that matter and antimatter existed in the Universe in equal amounts and then an asymmetry appeared due to the nature of the underlying interactions. There is a set of celebrated conditions in order to generate such an asymmetry, known as the Sakharov conditions [9], which state that the interactions must violate some symmetries of the theory (B and C/CP) and that they must be out of equilibrium.

So, what if something similar happened with dark matter? This scenario is called asymmetric dark matter. The general logic resembles that of freeze-out: dark matter particles and antiparticles annihilate into Standard Model ones, typically at a rate which is larger than the one required for successful freeze-out. In the symmetric case, this would lead to too small a relic abundance. If, however, some asymmetry is generated during this process, even if the subleading component annihilates away entirely, in the end there will be a net excess of dark matter particles over antiparticles. The sum of the symmetric and the asymmetric contribution determines the final abundance.

From a model-building perspective, in these sxenarios the name of the game is to figure out a *dynamical* mechanism through which such an asymmetry can be generated. Note, also, that quite frequently the asymmetric dark matter framework is also combined with a baryogenesis mechanism. For a review of asymmetric dark matter scenarios *cf* [10].

4.2.3 Parenthesis: stability of dark matter particles

As final remark, let us note that dark matter must be either stable or posess a lifetime which is of the order of, or even many orders of magnitude larger than the age of the Universe[6]. But how can we stabilize dark matter?

The most standard technique in order to ensure dark matter stability is inspired by the reasons leading to the stability of some particles in the Standard Model: the photon is stable because it's the massless gauge boson of an unbroken gauge symmetry, $U(1)_{\rm EM}$. The electron is stable because it's the lightest particle charged under this symmetry whereas the lightest neutrino is stable because it's the lightest fermion (*i.e.* due to Lorentz invariance). Lastly, the proton is stable because the Standard Model turns out to possess an accidental baryon number conservation and the proton is the lightest baryon. In this spirit, in order to ensure that there are no interactions leading to dark matter decay, we can try mimicking the reason why some of the Standard Model particles are stable: dark matter could be the lightest particle charged under some symmetry.

Consider the simplest imaginable extension of the Standard Model, in which we simply add a gauge-singlet real scalar field s, which we hope to play the role of a dark matter candidate, to its particle content. The most general, renormalizable Lagrangian that we can write down reads

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{\mu_s^2}{2}s^2 + \frac{A}{3!}s^3 + \frac{\lambda_s}{4!}s^4 + B(H^\dagger H)s + \lambda_{hs}(H^\dagger H)s^2 \tag{4.28}$$

If this were the end of the story then, clearly, s could not be a dark matter candidate, at least not for general values of the various couplings. Indeed, in such a model s can decay, *e.g.* through processes of the type $s \to h^{(*)}h^{(*)}$, unless the relevant couplings were extremely small.

Consider, now, imposing an additional symmetry to this model, namely a discrete Z_2 symmetry under which all the Standard Model fields transform trivially (Z_2-"even", *i.e.* under Z_2 they transform as $\phi_{\rm SM} \to \phi_{\rm SM}^{'} = \phi_{\rm SM}$) whereas s transforms as $s \to s' = -s$ (Z_2-"odd") and demand that $\mathcal{L}$ remains invariant under this symmetry both classically and

[6]The precise constraint is model-dependent and depends on the potential dark matter decay modes. If, for instance, dark matter can decay into final states producing photons, then indirect detection experiments that will be described in the following impose limits which are indeed several orders of magnitude larger than the age of the Universe.

at the quantum level. Then, the terms proportional to A, B vanish, whereas imposing that Z_2 remains unbroken means that s does not acquire a vaccuum expectation value. Within such a framework, it indeed turns out that s is indeeed exactly stable.

This is the most usual way to ensure dark matter stability in dark matter model-building. Note that much more sophisticated symmetries can be imposed, also depending on the field content of each model, ranging from discrete global symmetries such as Z_N (for $N > 2$ we actually also get additional interesting dark matter annihilation channels), continuous global or, eventually, gauge symmetries. An interesting relavant discussion, which also provides some hints as to the potential UV origin of low-energy stabilizing symmetries can be found in [11].

5 Dark matter detection

What do we mean when we talk about dark matter detection? In a sense one could argue that dark matter has, indeed, been detected: we have "seen" it through its effects in galaxies, galaxy clusters and, eventually, the entire Universe. However, all of these observations share a common characteristic: they rely on the gravitational interactions of dark matter. This has (at least) two consequences: first, it is impossible to exclude alternative explanations such as potential modifications of gravity. Indeed, although essentially all modified gravity theories do fail to explain all the observational evidence that we attribute to dark matter, it is not inconceivable that we have simply "not found the right model". Secondly, gravity treats all forms of matter equally. In this sense, it cannot say much concerning the nature of dark matter[7]. To put it simply, if dark matter possesses non-gravitational interactions, we would like to be able to detect it through their effects.

During the previous we examined in more detail one of the possible dark matter generation mechanisms that have been proposed and briefly sketched a few others. There are still more which we did not have time to present. The diverse character of these different ideas illustrates one point, which will also become clearer in the following: there is no universal dark matter detection strategy. Each dark matter model can feature dark matter interacting in different ways with the Standard Model particles which, in turn, can lead to different ideas about how dark matter could be detected.

But then, is dark matter detection a model-specific endeavour? That would be quite unfortunate, because there are hundreds, if not thousands of dark matter models which have been proposed in the literature. If dark matter detection were entirely model-specific, then for every dark matter model we would need to employ a different dark matter detection technique. Furtunately, the situation is a bit better: different dark matter generation mechanisms (which can accomodate multiple dark matter models) do tend to hint towards some concrete detection strategies. Exceptions can be written down, in the sense that it is frequently possible to write down a model that evades this or that constraint. But i) quite often another "generic" detection strategy might offer complementary constraints

[7]With the potential exception of macroscopic-size dark matter candidates, such as primordial black holes.

and *ii*) these models tend to constitute exceptions, in the sense that they do require a bit of engineering.

With these remarks in mind, we will focus mostly on the three dark matter detection techniques around which most of the existing experimental campaigns have developped: direct detection, indirect detection and collider searches. All of them were mostly motivated by the thermal freeze-out picture, in a relatively simple manner.

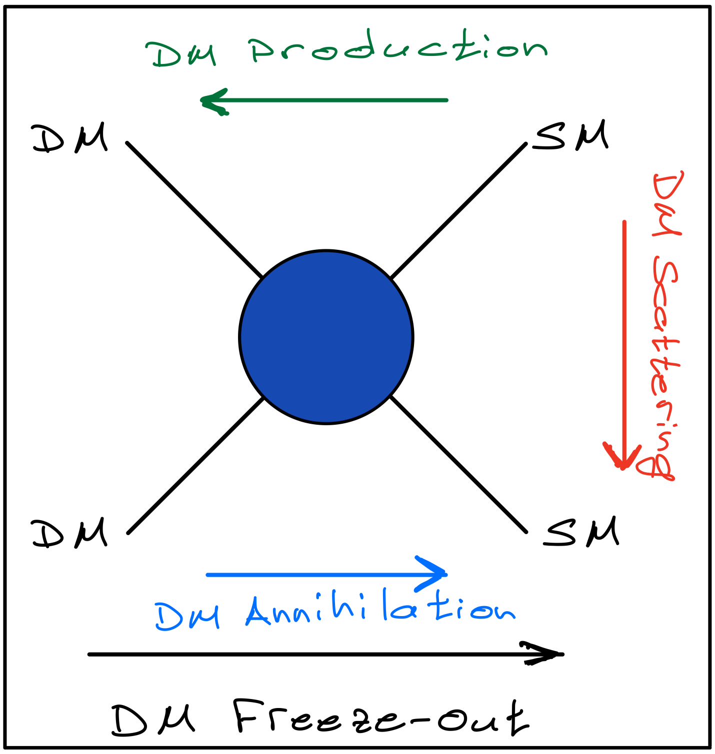

Figure 7: Principle of WIMP-inspired dark matter detection techniques.

In figure 7 we sketch how, keeping all external states the same, modifying the direction of the time arrow leads to different types of processes which can be exploited in order to detect dark matter. Moreover, the fact that in freeze-out scenarios dark matter is required to interact relatively strongly with the Standard Model particles implies that all of these processes could happen at non-negligible rates. Let's see how this fact can be exploited in practice.

5.1 Direct detection

The principle of direct detection was proposed in the mid-80s [12], long before the majority of today's dark matter models were formulated. The general idea is, actually, fairly simple: since our galaxy consists primarily of dark matter, we expect that the dark matter particles constantly reach the Earth. As they are expected to interact quite weakly with ordinary matter, most of the time they should just traverse it[8]. But every now and then, it could be that some of the them actually interact with the Earth's materials. Then, if a large detector were built and exposed for a sufficiently large amount of time to this cosmic flux, some of the dark matter particles might actually interact with the target material.

There is a large number of experiments worldwide that pursue this goal. They are typically built underground in order to reduce the background which can result from the scattering of Standard Model cosmic rays off the detector material. Depending on the specific technology employed by each experiment, a large number of different observables can be measured in order to deduce that it was, indeed, a dark matter particle that scattered off the detector material. In almost all cases however, the basic principle remains the same. Dark matter particles could interact with the nuclei and/or the electrons of the target, causing them to recoil, get excited or ionize and this is an in principle measurable effect.

Let us first consider scattering off nuclei. The differential event rate, *i.e.* the number of scattering events taking place per unit time per unit detector mass per unit of nuclear recoil energy E_R is given by

$$\frac{dR}{dE_R} = \frac{\rho_\chi}{m_\chi} \frac{1}{m_A} \left\langle v \frac{d\sigma}{dE_R} \right\rangle \tag{5.1}$$

where R is the event rate, n_χ is the local dark matter number density, m_A is the nucleus mass, v is the dark matter velocity and the brackets denote averaging (which stems from the fact that not all dark matter particles have the same velocity. In a more explicit form, we write

$$\frac{dR}{dE_R} = \frac{\rho_\chi}{m_\chi m_A} \int_{v_{\min}}^{v_{\max}} d^3v \; v\tilde{f}(\vec{v},t) \; \frac{d\sigma}{dE_R} \tag{5.2}$$

where $\tilde{f}$ is the dark matter velocity distribution in the detector rest (lab) frame. The velocities $v_{\min}$ and $v_{\max}$ correpond to the minimal required velocity that can give rise to a recoil energy E_R and to the Milky Way escape velocity, respectively. The latter is of the order of 550 km/sec whereas the former is given by

$$v_{\min} = \sqrt{\frac{m_A E_R}{2\mu_A^2}} \;, \quad \mu_A = \frac{m_A m_\chi}{m_A + m_\chi} \tag{5.3}$$

What about the dark matter velocity distribution in the lab frame? It can be found from the corresponding velocity in the galactic rest frame, by performing a Gallilean boost of the dark matter velocity distribution in the galactic rest frame, $f(\vec{t})$:

$$\tilde{f}(\vec{v}) = f(\vec{v} + \vec{v}_{\rm obs}) \;, \quad v_{\rm obs}(t) = \vec{v}_\odot + \vec{v}_\oplus(t) \tag{5.4}$$

where $v_\odot$ corresponds to the rotation velocity of the Sun in the galactic rest frame, and $v_\oplus$ to the velocity of the earth in the solar system.

Several comments are in order

[8]One interesting, and slightly counter-intuitive, exception to this rule is actually the case in which dark matter particles interact too strongly with ordinary matter. In this case, the dark matter particles may be stopped at the top of the Earths atmosphere without ever reaching the Earth. Notice, moreover, that

- The dark matter velocity distribution is, in the general case, unknown. It constitutes one of the systematic/theoretical uncertainties in dark matter direct detection. The most commonly adopted assumption (including in the vast majority of experimental papers) is that dark matter folows the so-called "standard halo model".

- Both the escape velocity and the local dark matter density (~ 0.3 GeV/cm^3) are only known with a finite accuracy.

- Typical materials that are used in existing direct detection experiments are Xenon, Argon, Hydrogen, Germanium and different alloys.

- The typical momentum transfer is of the order of 100 MeV. This means that, for a 100 GeV particle scattering off a Xenon nucleus, we can expect nuclear recoil energies of the order of a few keV or higher.

- The cross-section entering the differential event rate refers to dark matter - *nucleus* (not single-nucleon or parton) scattering.

- As the earth moves around the Sun, its velocity changes with respect to the wind of dark matter particles received by the solar system (the galactic disk – which contains the Solar system – rotates within an almost non-rotating dark matter galactic halo).: the wind attains maximum speed around June (as in June the Earth moves faster in the direction of the disk rotation) and minimum speed in December (when the Earth moves fastest opposite to the disk rotation). This leads to a modulation of the expected dark matter scattering rate, called *annual modulation*. The majority of dark matter direct detection experiments aim at detecting the bulk of the signal. However, a number of such experiments aims at detecting exactly this modulation effect.

Without going into details, in order to compute theoretical prediction within concrete dark matter models we initially compute the cross-section in the usual way (*i.e.* between dark matter and quarks/gluons), afterwards mapping the relevant operators to dark matter - nucleon ones. The experimental collaborations, on the other hand, fundamentally constrain the dark matter - nucleus scattering cross-section. The results are, then, translated into constraints on the dark matter - nucleon scattering cross-section, by employing appropriate form factors. Note that not all dark matter models, even some which operate at quite strong coupling, predict sizeable signals in direct detection experiments.

In figure 8 are summarized the leading constraints on the dark matter - nucleon spin-independent scattering cross-section. Note that this quantity is certainly not the end of the story. First, there can be other types of interactions between dark matter particles and nuclei which can be relevant, or even leading, depending on the structure of the underlying theory (*e.g.* the Lorentz structure of the relevant interactions). Secondly, "traditional" direct detection experiments lose sensitivity for dark matter masses below a few GeV (although techniques have been proposed in order to overcome the threshold barriers imposed by the technologies used by the experimental collaborations, *cf e.g.* [13]). In addition, dark

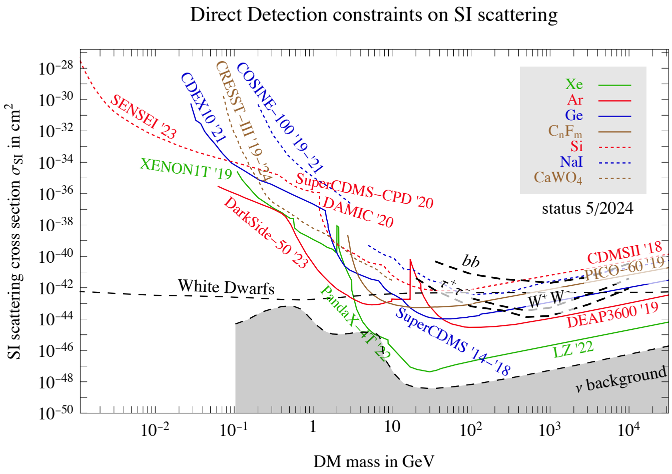

Figure 8: Summary of direct detection constraints on the spin-independent dark matter - nucleon scattering cross-section, circa 05/2024. Figure taken from [1].

matter may even not interact with quarks/gluons at all.

In this spirit, during the last decade there has been substantial effort in order to

- Develop alternative dark matter direct detection strategies, which try to exploit the interactions of dark matter with objects different than the nucleus. These can range from standard scattering off atomic electrons in conventional, existing detectors (for relatively heavy dark matter) up to completely new techniques that rely, *e.g.*, on radiative processes [13], superconducting [14] and/or superfluid [15] materials. In short, in the low (sug-GeV, or even sub-MeV) mass regime the name of the game is to find processes which can take place even if very small momentum transfer is involved (*e.g.* Cooper pair breaking in the case of superconducting detectors).

- Develop the necessary formalism in order to compute the predicted signals in such experimental setups. Note that the problem can become highly non-trivial: even in the case of scattering off atomic electrons, the situation is quite involved.

So, how do these constraints map onto actual dark matter models? The answer is, of course, model-dependent. For illustration, in figure 10 we show a recent result obtained in [16] for the singlet scalar dark matter model. The usual freeze-out Planck-compatible parameter space is depicted by the solid violet line, to be compared with recent results from the XENON1T and LZ experiments. The quasi-vertical lines, on the other hand, correspond to freeze-in scenarios with a low reheating temperature, which require relatively large values of the dark matter - Higgs boson coupling in order to reproduce the observed relic abundance. Interestingly, even though they were first inspired by thermal freeze-out, we observe that direct detection experiments can also constrain freeze-in scenarios under such non-standard cosmological assumptions.

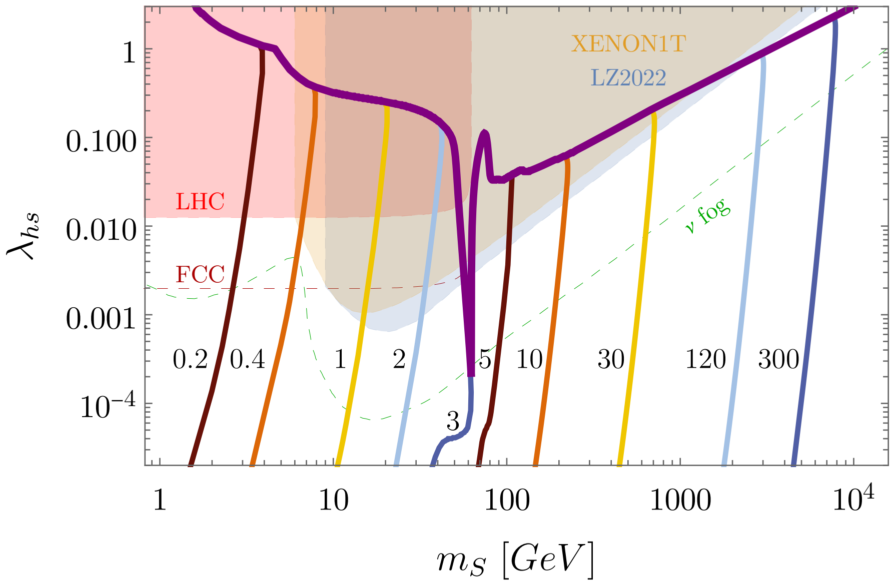

Figure 9: Parameter space of the Higgs portal scalar DM. Along the curves, the correct DM relic abundance is reproduced. The curves are marked by the reheating temperature in GeV, while the purple line corresponds to thermal DM. The shaded areas are excluded by the direct DM detection experiments and the invisible Higgs decay at the LHC. The neutrino background for direct DM detection is represented by the green dashed line "ν fog", while the FCC prospects are shown by the red dashed line. Figure taken from [16].

5.2 Indirect detection

Indirect detection relies on the exact same process that is operational during thermal freeze-out: dark matter annihilation. The idea is that dark matter particles in the Milky Way halo (or, eventually, other galaxies) can annihilate into Standard Model ones which, subsequently, could be detected by earth-bound or airborne telescopes. Besides, as we already mentioned, indirect detection can be extremely efficient in constraining models of decaying dark matter.

There are four main types of "messengers" that are used for indirect detection of dark matter: gamma-rays, positrons, antiprotons and neutrinos. Out of them, in these lectures we will only briefly comment on the former; the dark matter - induced positron and antiproton flux at the Earth is subject to substantial astrophysical uncertainties, stemming from the fact that, since they are charged particles, they are affected by multiple processes such as diffusion, convection, reacceleration etc which, in turn, are affected by considerable uncertainties. Neutrinos, in turn, are typically produced through dark matter annihilations in the Sun and in general provide subleading constraints.

Gamma-rays, on the other hand, travel in straight lines and can be produced abundantly upon dark matter annihilation. This means that they retain the directional information concerning the place where they were produced, allowing telescopes to target specific regions of higher dark matter concentration where annihilation is expected to be much more probable. In principle, the ideal target would be the centre of the Milky Way. Unfortunately, it turns out that the Galactic Centre is a complicated environment, with multiple astrophysical processes contributing to the production of high-energy gamma-rays. One frequently employed solution is to actually mask the centre of the galaxy, along with the galactic plane, and rather focus on the regions around the masked area where the dark matter density remains relatively large but the astrophysical backgrounds are substantially reduced. However, currently the most robust constraints stem from the observation of dwarf spheroidal galaxies. Dwarf spheroidals are galaxies with a very high mass-to-light ratio, since they are believed to have been stripped off their baryons and are mostly composed of dark matter. Moreover, given their larger distance from the Earth, telescopes typically observe them as a whole, *i.e.* the dark matter - induced gamma-ray flux is integrated over the entire galaxy and is, thus, less sensitive to the uncertainties of the dark matter distribution in their inner regions.

The photon flux expected from dark matter annihilation within a cone spanning a solid angle $\Delta\Omega$ reads

$$\frac{d\Phi}{dE_\gamma}(E_\gamma, \psi) = \eta \frac{1}{4\pi} \sum_i \frac{\langle \sigma_i v \rangle}{2m_\chi^2} \frac{dN_i}{dE_\gamma} \int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} d\ell \rho[r(\ell, \psi)]^2 \tag{5.5}$$

where $\eta = 1, 1/2$ depending on whether dark matter is Majorana-like or Dirac-like, respectively, ℓ is the line-of-sight distance oriented at an angle ψ away from the galactic plane and all the astrophysics is encoded in the so-called J-factor

$$J = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} d\ell \rho[r(\ell, \psi)]^2 \tag{5.6}$$

But what is ρ? In other words, how is dark matter distributed in galaxies? The main method that is employed in order to answer this question is by deploying large-scale N-body simulations. To date, most of these simulations essentially include dark matter only, although a few more recent ones try to include the effects of the presence of baryons on the dark matter spatial distribution. Most dark matter - only simulations appear to converge towards the so-called Navarro-Frenk-White profile [17], which is described by

$$\rho_{\text{NFW}} = \frac{\rho_0}{r/r_s(1 + r/r_s)^2} \tag{5.7}$$

where ρ_0 is the local dark matter density and $r_s = 20$ kpc is the "scale radius". Other functional forms have been proposed in the literature, among which *e.g.* the so-called Einasto profile.

$$\rho_{\text{Ein}} = \rho_0 \exp\left[-\frac{2}{\gamma}\left(\left(\frac{r}{r_s}\right)^\gamma - 1\right)\right] \tag{5.8}$$

where $r_s = 20$ kpc and $\gamma = 0.17$. Both of these profiles are quite "cuspy", *i.e.* steep, towards the inner regions of the galaxy. Whether or not they constitute a good description of galactic dark matter haloes is still a matter of debate, with several observations favouring rather “cored” (less steep) profiles. Such modifications could be due to more exotic physics such as dark matter self-interactions, however, standard astrophysical processes such as baryonic feedback can also have an important impact on the structure of dark matter haloes. The inclusion of baryons in N-body simulations is highly non-trivial, but during the last decade substantial efforts have been made in this direction.

A summary of existing constraints from gamma-ray searches can be seen in figure **??**. These constraints assume a specific dark matter annihilation channel, namely annihilation

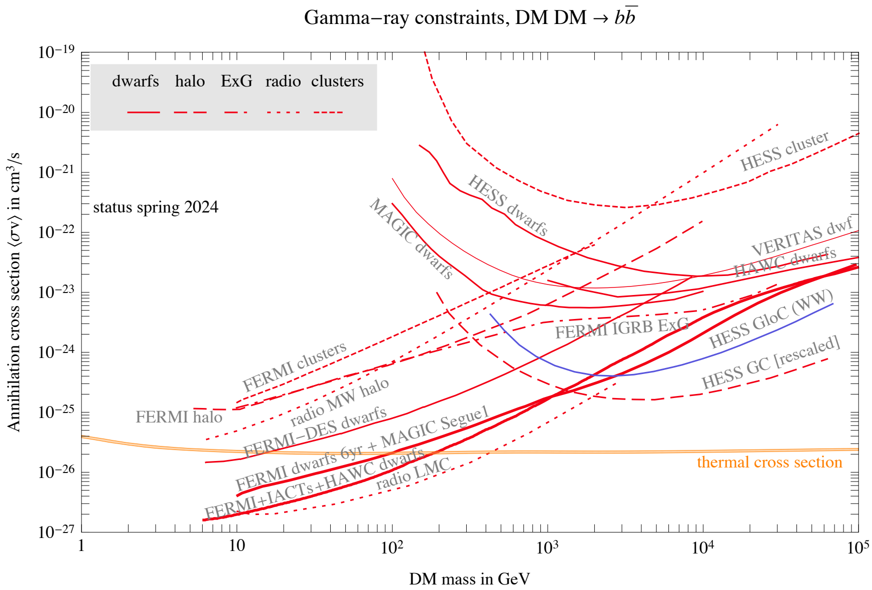

Figure 10: Summary of constraints from searches for dark matter annihilation - induced gamma-rays assuming annihilation into $b\bar{b}$, circa May 2024. Figure taken from [1].

into a $b\bar{b}$ final state.

6 Conclusions

Dark matter is a topic which combines elements from different fields: cosmology, astrophysics, particle physics. In these lectures we tried to give a short introduction to some aspects of dark matter physics. The material presented is by no means complete: the goal was to introduce a few notions and, hopefully, provide motivation for further study. Much more material can be found in different existing reviews, *cf e.g.* [18, 19] or [1], and textbooks [20, 21].

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