# QCD Lectures

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  - Lecture III : Renormalisation
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- For each lecture, there will be some slides as well as some more detailed notes. They can be downloaded from this URL :
  `https://mydrive.lapth.cnrs.fr/s/rK9Qb6Nggfo78aT`
  They are labelled, for lecture X : note_cX.pdf and slide_cX.pdf where $X \in [I, II, III, IV]$

# Outline

1. Lecture I : The naive parton model
   - The hadrons are not elementary particles
   - The quark model
   - The parton model
   - The parton model in the deep inelastic
     - The electron–parton cross section
     - The electron–proton cross section
     - Partons $\equiv$ quarks + ...

# Status of strong interaction

**Hadrons**

particles sensitive to the strong interaction, ex. proton, neutron, $\cdots$
mesons : hadrons having integer spin
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**Particle accelerators**

Before the 50', people thought that hadrons were elementary particles
With the coming of accelerators, hundred of hadrons have been discovered! (cf. Particle Data Book)
Clearly not elementary...

# Isospin symmetry

**A first attempt to classify**

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$$\begin{array}{lll} p \equiv (I = 1/2 \; ; \; I_3 = +1/2) & \text{isospin "up",} & \text{proton} \\ n \equiv (I = 1/2 \; ; \; I_3 = -1/2) & \text{isospin "down",} & \text{neutron} \end{array}$$

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Only for the couple proton–neutron!

# Strangeness

**Strange properties**
Some hadrons have "extremely" long lifetime

$$\Sigma^- \to n\pi^- \quad \tau \simeq 10^{-10}s$$
$$\Delta^- \to n\pi^- \quad \tau \simeq 10^{-23}s$$

with $m_\Sigma \simeq m_\Delta$ or

$$K(m_K \simeq 500\text{MeV}) \quad \tau \simeq 10^{-8}s$$
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**Strangeness**

New quantum number : strangeness, conserved by strong and E.M. interactions

# The quark model of Gell-Mann and Zweig

| quark | saveur | spin | I | $I_3$ | S | B | Y | Q |
|---|---|---|---|---|---|---|---|---|
| u | up | $1/2$ | $1/2$ | $1/2$ | 0 | $1/3$ | $1/3$ | $2/3$ |
| d | down | $1/2$ | $1/2$ | $-1/2$ | 0 | $1/3$ | $1/3$ | $-1/3$ |
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- $B = 0$ for mesons (hadrons with integer spin),
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The electric charge is related to the other quantum numbers by

$$Q = I_3 + \frac{Y}{2}.$$

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All the knowns hadrons (at that time!) were arranged in the irreducible **representations** of $SU(3)_{\text{flavour}}$

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- Other quarks have been discovered : $c$, $b$ and $t$. The symmetry group has to be extended to $SU(6)$ but huge mass difference $m_t/m_u \sim 10^4$
- No information on the dynamic! How the quarks interact between themselves?

# Colour

Let us consider, the hadron $\Delta^{++} = (uuu)$ in a spin state $s_z = \frac{3}{2}$ (each quarks has its spin up)

$$\Delta^{++}(s_z = \frac{3}{2}) = (u^{\uparrow} u^{\uparrow} u^{\uparrow})$$

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and the $\Delta^{++}$ wave function is **symmetric** when exchanging two quarks in contraction with the Fermi-Dirac statistic which requires that the wave function is **antisymmetric**! To solve this problem, a new quantum number is introduced : the colour.

$$u = \begin{pmatrix} u_R \\ u_G \\ u_B \end{pmatrix} \quad d = \begin{pmatrix} d_R \\ d_G \\ d_B \end{pmatrix} \quad s = \begin{pmatrix} s_R \\ s_G \\ s_B \end{pmatrix}$$

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To this new quantum number is associated a **colour symmetry group** SU(3) (to be distinguished from $SU(3)_{\text{flavour}}$). Each quark is a colour triplet and the hadrons are colour singlets (their wave functions are invariant under this group of transformation).

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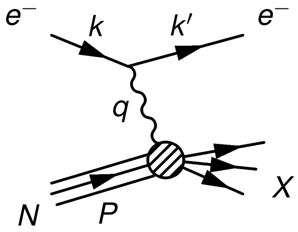
In this way, the $\Delta^{++}$ wave function is

$$\Delta^{++} = \frac{1}{\sqrt{6}} \epsilon_{ijk} \, u_i^{\uparrow} u_j^{\uparrow} u_k^{\uparrow}$$

which is **antisymmetric** under the permutation of two elements.

# The electron–nucleus scattering



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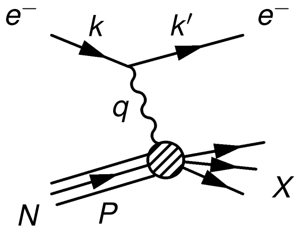


Kinematics $\omega \gg m$

$$\begin{aligned} P &= (M, \vec{0}) \\ k &= (\omega, 0, 0, \omega) \\ k' &= (\omega', \omega' \sin\theta, 0, \omega' \cos\theta) \end{aligned}$$

$$Q^2 = -q^2 = -(k - k')^2 = 4\,\omega\,\omega'\,\sin^2\frac{\theta}{2} > 0$$

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Let us introduce some new variables :

$$\nu \equiv \omega - \omega'$$

$$y \equiv \frac{2\,P\cdot q}{2\,P\cdot k} = \frac{\nu}{\omega}$$

$$x \equiv \frac{Q^2}{2\,P\cdot q} = \frac{Q^2}{2\,M\,\nu}$$

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The invariant mass of the hadronic final state

$$M_X^2 \equiv (P+q)^2 = M^2 + Q^2\,\frac{1-x}{x} \quad 0 \le x \le 1 \text{ and } y \in [0,1]$$

# The photon–nucleon coupling

It is **unknown**! The nucleon is not **point like**, it has a size! need to be parametrised

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$L^{\mu\nu}$ : the **leptonic** tensor; and $W_{\mu\nu}$ : the **hadronic** one. The most general parametrisation in terms of $P$ and $q$

$$W_{\mu\nu} = V_1 \, g_{\mu\nu} + V_2 \, P_\mu P_\nu + V_3 \, (q_\mu P_\nu + q_\nu P_\mu) + V_4 \, (q_\mu P_\nu - q_\nu P_\mu) + V_5 \, q_\mu q_\nu + V_6 \, \epsilon_{\mu\nu\rho\sigma} P^\rho q^\sigma$$

$V_i$, $i = 1, \cdots, 6$ : functions of $Q^2$, $x$ and $M^2$.

# Constraints on the $V_i$ parameters

But this a **QED interaction** thus the hadronic tensor is expected to be **transverse**

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$$W_{\mu\nu} = V_1 \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + V_2 \left( P_\mu - q_\mu \, \frac{P \cdot q}{q^2} \right) \left( P_\nu - q_\nu \, \frac{P \cdot q}{q^2} \right)$$

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It is more convenient to introduce **two other parameters** $W_1$ and $W_2$ such that $W_1 = -V_2/(2\,M)$ and $W_2 = M/2\,V_2$.

# The amplitude squared

The **leptonic tensor** can be easily obtained using the standard QED Feynman rules yielding

$$L^{\mu\nu} = 2\,(k^{\mu} k'^{\,\nu} + k^{\nu} k'^{\,\mu} - k \cdot k'\, g^{\mu\nu})$$

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The **contraction** of the leptonic and the hadronic tensors gives

$$L^{\mu\nu}\, W_{\mu\nu} = 2\,M\,\left[2\,Q^2\,W_1 + W_2\,\left(4\,\frac{k\cdot P\,k'\cdot P}{M^2} - Q^2\right)\right]$$

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In the **laboratory frame** :

$$L^{\mu\nu}\, W_{\mu\nu} = 8\,M\,\omega\,\omega' \left[ 2\,W_1\, \sin^2\frac{\theta}{2} + W_2\, \cos^2\frac{\theta}{2} \right]$$

# The cross section

The **cross section** is given by

$$\sigma = \frac{1}{4 P \cdot k} \int \frac{d^3 k'}{(2\,\pi)^3\, 2\,\omega'}\, \frac{d^4 P_X}{(2\,\pi)^3}\, (2\pi)^4\, \delta^4(k + P - k' - P_X) \overline{\sum} |M|^2$$

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The **differential cross section** is given by

$$\frac{d\sigma}{d\omega'\, d\cos\theta} = \frac{\pi\, \alpha^2}{2\,\omega^2\, sin^4 \frac{\theta}{2}} \left[ 2\, W_1\, \sin^2 \frac{\theta}{2} + W_2\, \cos^2 \frac{\theta}{2} \right]$$

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One can use also the variables $Q^2$ and $\nu$ instead of $\omega$ and $\omega'$, the new differential cross section becomes

$$\frac{d\sigma}{dQ^2\,d\nu} = \frac{4\,\pi\,\alpha^2}{Q^4}\,\frac{\omega-\nu}{\omega}\left[2\,W_1\,\sin^2\frac{\theta}{2} + W_2\,\cos^2\frac{\theta}{2}\right]$$

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- at fixed initial energy ($\omega$), modifying $\omega'$, $\theta$, the variables $x$ and $Q^2$ varies and $W_1(x, Q^2, M^2), \ W_2(x, Q^2, M^2)$ can be extracted from experiment.

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- the functions $W_1$, $W_2$ have the dimension of the **inverse of an energy**

$$MW_1\left(x, \frac{M^2}{Q^2}\right) = \mathcal{F}_1(x, \frac{M^2}{Q^2}), \quad \nu W_2\left(x, \frac{M^2}{Q^2}\right) = \mathcal{F}_2(x, \frac{M^2}{Q^2}).$$

# Results of the experiment

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1. $$\nu W_2\left(x, \frac{M^2}{Q^2}\right) \equiv \nu W_2(x)$$

   This is the property of **scale invariance**
2. The relation

$$2MW_1(x) \equiv \frac{\nu W_2(x)}{x} = \frac{P.q\; W_2(x)}{Mx}$$

is satisfied experimentally (**Callan-Gross relation**).

# The SLAC experiment

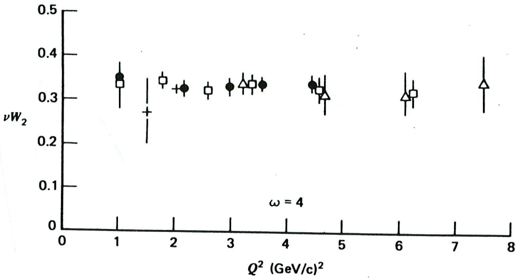


Figure: *Results of the SLAC experiment in 1968, showing the scale invariance of the function $\nu W_2$ at the value of $x = 1/\omega = 0,25$.*

Figure 2.1: *Résultats expérimentaux de SLAC montrant l'invariance d'échelle de la fonction* $\nu W_2$.

# Lorentz invariant form

**"Mandelstam variables"** $s, \; t, \; u$

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$$\frac{\omega' d\sigma}{d^3k'} = \frac{\alpha^2}{s}\frac{2}{Q^4}\left\{Q^2\,(2\,M\,W_1 - \frac{W_2}{M}\,\frac{P.q}{x}) + \frac{W_2}{2M}\,(s^2+u^2)\right\}. \tag{1}$$

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coefficient of the term in $Q^2$ : **invariant form** of $2\,M\,W_1 - \nu\,W_2/x$ which is **zero** (an experimental fact)

$$\frac{\omega' d\sigma^{\text{exp}}}{d^3k'} = \frac{\alpha^2}{s}\,\frac{(s^2+u^2)}{Q^4}\,\frac{W_2}{M}$$

# The parton model

The **proton** (or nucleon) is made of **partons** which are **point like objects** whose quantum numbers are a priori unknown (charge, spin, etc). If it is made of **parton of type** $i$ carrying a **4-momentum** $p_i$.

$$p_i = y_i P \quad \text{with} \sum_i y_i = 1 \text{ and } P = (E, 0, 0, E)$$

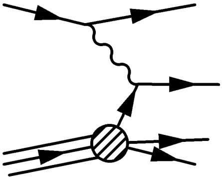
Frame where **the components of** $P \to \infty$ (the mass of the proton and the partons are neglected).

# The parton model

The **proton** (or nucleon) is made of **partons** which are **point like objects** whose quantum numbers are a priori unknown (charge, spin, etc). If it is made of **parton of type** $i$ carrying a **4-momentum** $p_i$.

$$p_i = y_i P \quad \text{with} \sum_i y_i = 1 \text{ and } P = (E, 0, 0, E)$$

Frame where **the components of** $P \to \infty$ (the mass of the proton and the partons are neglected).
Describing the $\gamma^*$-hadron interaction in terms of $\gamma^*$-parton interaction



# A very intuitive reasoning

- The **"life time of the virtual photon"** in the centre of mass frame photon-proton

$$\Delta\tau_{\rm em} \sim \frac{1}{\sqrt{Q^2}}\frac{q_0}{\sqrt{Q^2}} \sim \frac{1}{\sqrt{Q^2}}, \quad \Delta\tau_{\rm em} \to 0, \text{ when } \sqrt{Q^2} \to \infty$$

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- The **characteristic time of the strong interaction** which binds the partons inside the proton in the frame $\gamma^*-$proton

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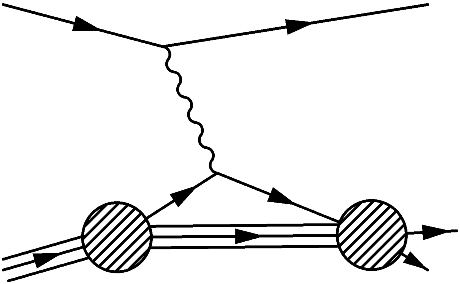
If the characteristic times are compared, we get that

$$\Delta\tau_{\rm em} \sim \frac{1}{\sqrt{Q^2}} \ll \Delta\tau_{\text{strong int.}} \sim \frac{\sqrt{Q^2}}{M^2}.$$

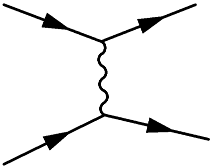
# A very intuitive reasoning

During the time $\Delta\tau_{\rm em}$ that the $\gamma^* p_i$ interaction lasts, one can neglect the hadronic interaction which lasts on a much larger time scale



# A very intuitive reasoning

The **confinement interactions** do not affect the **interaction $\gamma^*$–parton**, we thus have to compute



and add in a **incoherent manner** the cross sections electron–parton to form the cross section electron–proton.

# Squared amplitude $\gamma^*$–parton

$$| \mathcal{M} |^2_{ep_i} = \frac{q_i^2\, e^4}{Q^4}\, L^{\mu\nu} \underbrace{\widehat{W}_{\mu\nu}}_{\gamma^*\text{-parton int.}} \quad .$$

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$$\widehat{W}_{\mu\nu} = 2 \left( p_{i\mu} \, p'_{i\nu} + p_{i\nu} \, p'_{i\mu} - p_i.p'_i \, g_{\mu\nu} \right),$$

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$$| \mathcal{M} |^2_{ep_i} = 8 \, \frac{e^4 \, q_i^2}{Q^4} \left( (p_i.k)^2 + (p_i.k')^2 \right) = 2 \, \frac{e^4 \, q_i^2}{Q^4} \left( \hat{s}^2 + \hat{u}^2 \right),$$

with the partonic invariants $\hat{s} = (p_i + k)^2$ et $\hat{u} = (p_i - k')^2$.

# The cross section $\gamma^*$–parton

The cross section will be

$$\hat{\sigma} = \frac{1}{2\hat{s}} \frac{1}{(2\pi)^2} \int \frac{d^3k'}{2\omega'} \frac{d^3p_i'}{2p_i'^0} \, \delta^{(4)}(k + p_i - k' - p_i') \ | \mathcal{M} |^2_{ep_i} \qquad (2)$$

$$= 2 \, \frac{\alpha^2 q_i^2}{Q^4} \int \frac{d^3k'}{\omega'} \, \delta\left(2p_i.q - Q^2\right) \, \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}}. \qquad (3)$$

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At the **partonic level**, the differential cross section will have the following form

$$\frac{\omega' d\hat{\sigma}}{d^3k'} = 2 \frac{\alpha^2 q_i^2}{Q^4} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}} \delta(2p_i.q - Q^2). \quad (4)$$

# The cross section $\gamma^*$–proton

The hadronic cross section : **incoherent sum** of the partonic cross sections

$$\frac{\omega' d\sigma}{d^3 k'} = \sum_i \int_0^1 dy\, F_i(y)\, \frac{\omega' d\hat{\sigma}}{d^3 k'}\bigg|_{p_i = yP}, \tag{5}$$

$\hat{s} = ys$ et $\hat{u} = yu$, $2p_i \cdot q = y\, 2P \cdot q$. The quantities $F_i(y)$ are the **number of partons** of type $i$ carrying a **4-momentum fraction** $y$ of the proton one.

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$$\frac{\omega' d\sigma}{d^3k'} = \frac{\alpha^2}{s}\frac{s^2+u^2}{Q^4}\sum_i e_i^2 \frac{x}{P.q} F_i(x) \tag{6}$$

with $y = Q^2/2Pq = x$.

# Comparison

Comparing with the formula given the differential cross section electron–proton, one can identify

$$\frac{W_2}{M}(x, \frac{M^2}{Q^2}) = \sum_i q_i^2 \frac{x}{P.q} F_i(x) \tag{7}$$

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and since there is no term in $Q^2$ in $\dfrac{\omega' d\sigma}{d^3k'}$ (see eq. (1)), we also recover

$$2\, M\, W_1(x, \frac{M^2}{Q^2}) = \frac{1}{x} \, \nu W_2(x). \tag{9}$$

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Exercice : Show that for spin 0 partons (coupling to the $\gamma$ given by $q_i\,(p_i + p_i')^\mu$) one has $W_1 \equiv 0$.

# Feynman partons I

Feynman partons $\stackrel{?}{=}$ the Gell-Mann and Zweig quarks $\Rightarrow$

$$\text{proton} = (uud)$$
$$\text{neutron} = (udd).$$

This is the **"valence" quarks** : $u_v(x)$ and $d_v(x)$. The nucleon quantum number are carried by the "valence" quarks.

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The experimental results : proton and neutron more complex than this "3 quarks" model, they contain also antiquarks,... : **"sea" quarks** $u_m(x) = \bar{u}_m(x), \ d_m(x) = \bar{d}_m(x)$.

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The sum of the quantum number carried by this quarks is **zero**!

# Feynman partons II

$$\begin{aligned} u(x) &= u_v(x) + u_m(x) \\ d(x) &= d_v(x) + d_m(x). \end{aligned}$$

**Neglecting** the role of $s$, $c$ and $b$ quarks

$$\begin{aligned} \frac{1}{x}\nu W_2^{ep} &= \frac{4}{9}(u(x)+\bar{u}(x)) + \frac{1}{9}(d(x)+\bar{d}(x)) \\ \frac{1}{x}\nu W_2^{en} &= \frac{1}{9}(u(x)+\bar{u}(x)) + \frac{4}{9}(d(x)+\bar{d}(x)). \end{aligned} \tag{10}$$

where $u_m = \bar{u}_m = \bar{u}$, $d_m = \bar{d}_m = \bar{d}$.

# Feynman partons III

**Experimental measurement** on an target of deuterium $= p + n$

$$\frac{1}{x}\nu W_2^{ep+en} = \frac{5}{9}\left(u(x) + \bar{u}(x) + d(x) + \bar{d}(x)\right)$$

and thus to compute the integral

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which measure the **total momentum** carried by all the quarks $u$, $d$, $\bar{u}$ and $\bar{d}$,should be **equal to 1**!

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The quarks carry half of the proton momentum, the other half is carried by **neutral partons.**

# The parton model: general formulation I

- A hadron is **made of partons**, frame : infinite momentum frame

$H = \{p_i\} \quad i = 1, \infty$

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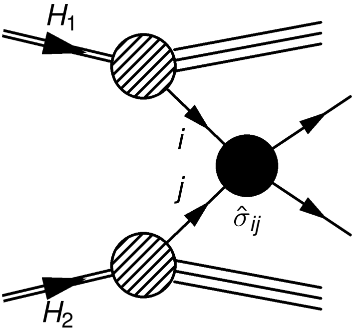
Partons are **point like** and their interactions is **ignored** inside the hadron

# The parton model: general formulation II

• **Interactions** between hadrons reduce to interactions between partons following the diagram



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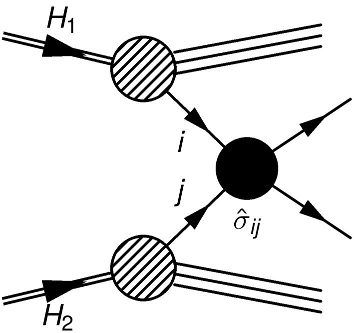


$\hat{\sigma}_{ij}$ is the **"hard" cross section** describing the interactions between partons. The hadronic cross section is a **incoherent superposition** of partonic cross sections, the probabilities are added, not the amplitudes!

# The parton model: general formulation III

We can the write

$$\sigma^{H_1 H_2} = \sum_{i,j} \int dx_1 dx_2 \; F_i^{H_1}(x_1) \; F_j^{H_2}(x_2) \; \alpha_s^p \; \hat{\sigma}_{ij}(x_1, x_2, s).$$

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• The parton model is a valid postulate when all the dimensional variables $s, t, u$ are **large** compared to the proton mass ($\sim$ 1 GeV$^2$).

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- No information on the dynamics between partons