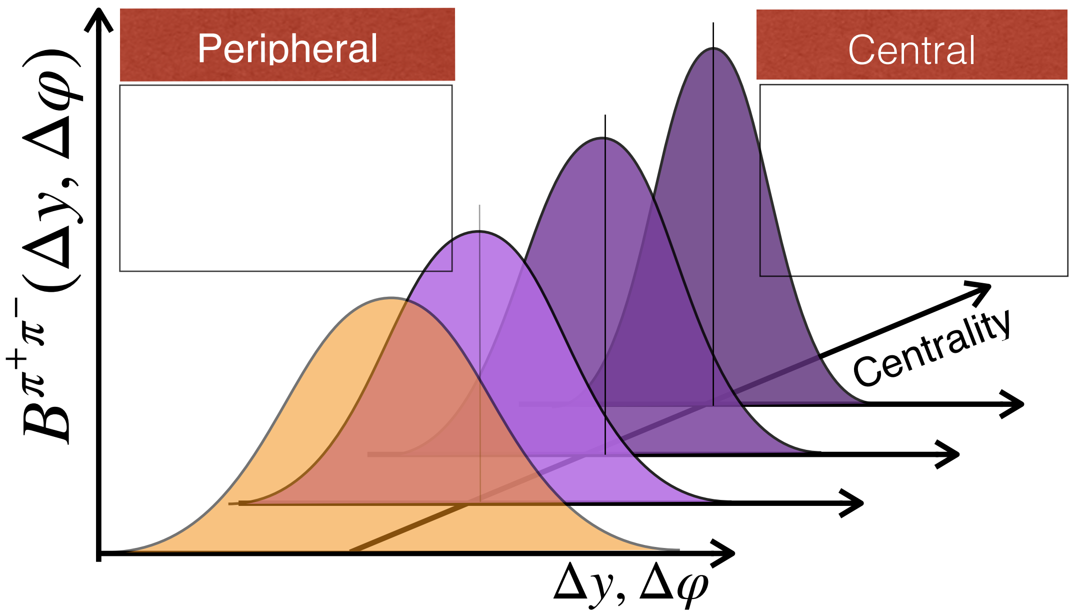
WPCF 2024 — Toulouse, France
Advances in Balance Function Measurements
Claude A. Pruneau, Wayne State University

Special Session:
Celebrating the career of Professor Scott Pratt



- Collaborators:
  - S. Basu, S. Dash, A. Dobrin, V. Gonzalez, B. Hanley, A. Manea, A. Marin, B. Nandi, Y. Patley. O. Sheibani
- Recent Papers:
  - *Quark Flavor Balancing in Nuclear Collisions*, 2408.09923 [hep-ph]
  - *Mixed Species Charge and Baryon Balance Functions Studies with PYTHIA*, PRC 109 (2024) 6, 064913,
  - *Multi-particle Integral and Differential Correlation Functions*, Phys.Rev.C 109 (2024) 4, 044904,
  - *Accounting for non vanishing net-charge with unified balance functions*, PRC 107 (2023) 1, 014902,
  - *Effects of non vanishing net charge on balance functions and their integrals*, PRC 107 (2023) 5, 054915
  - *Role of baryon number conservation in measurements of fluctuations*, PRC 100 (2019) 3, 034905,
- Related Talks at this conference:
  - *Clocking the particle production and tracking quantum numbers balance and radial flow effects at top*

Balance Functions

The beginning!!!!

VOLUME 85, NUMBER 13 PHYSICAL REVIEW LETTERS 25 SEPTEMBER 2000

Clocking Hadronization in Relativistic Heavy-Ion Collisions with Balance Functions

Steffen A. Bass, Pawel Danielewicz, and Scott Pratt
Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824
(Received 15 May 2000)

A novel state of matter has been hypothesized to exist during the early stage of relativistic heavy-ion collisions, with normal hadrons not appearing until several fm/c after the start of the reaction. To test this hypothesis, correlations between charges and their associated anticharges are evaluated with the use of balance functions. It is shown that late-stage hadronization is characterized by tightly correlated charge-anticharge pairs when measured as a function of relative rapidity.

Balance Function Definition:

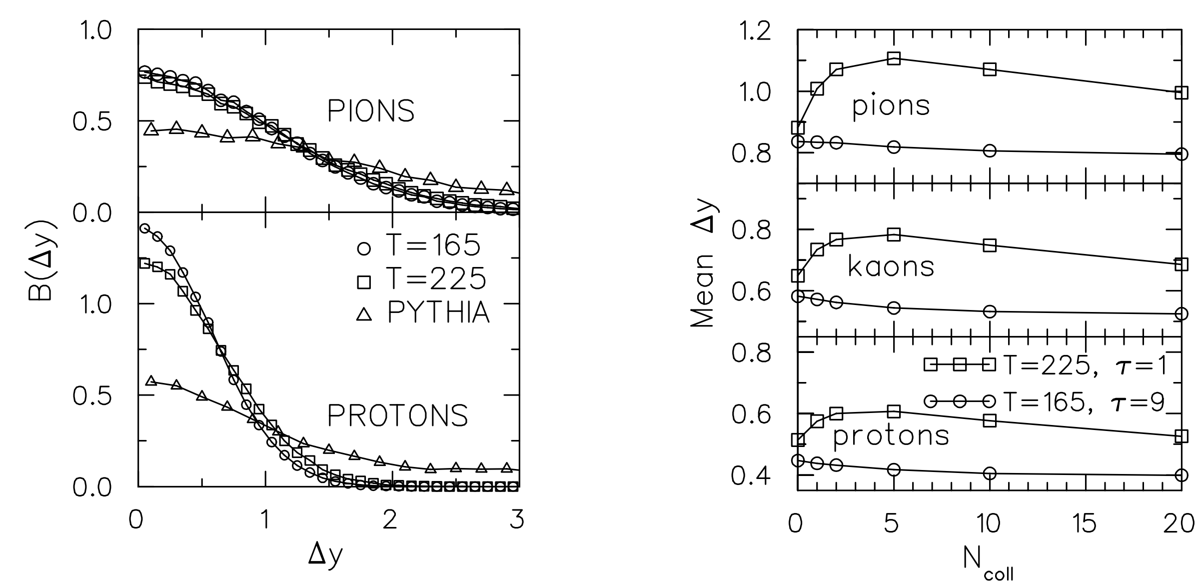
$$B(p_2|p_1) \equiv \frac{1}{2}\{\rho(b,p_2|a,p_1) - \rho(b,p_2|b,p_1) + \rho(a,p_2|b,p_1) - \rho(a,p_2|a,p_1)\},$$

Conditional Densities

$$\rho(b,p_2,a,p_1) = \frac{N(b,p_2|a,p_1)}{N(a,p_1)}$$

Balance Function "Sum Rule":

$$\sum_{p_2} B(p_2|p_1) = \frac{1}{2}\{M_b - (M_b - 1) + M_a - (M_a - 1)\} = 1$$



Narrowing of Balance Functions in the presence of isentropic expansion of QGP medium !

Pratt *et al.* on Balance Functions

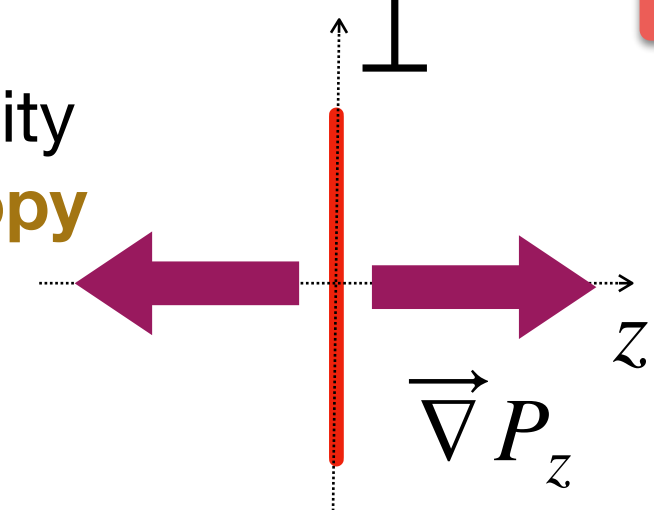
A Very Prolific Era (Balance Function Related Works)

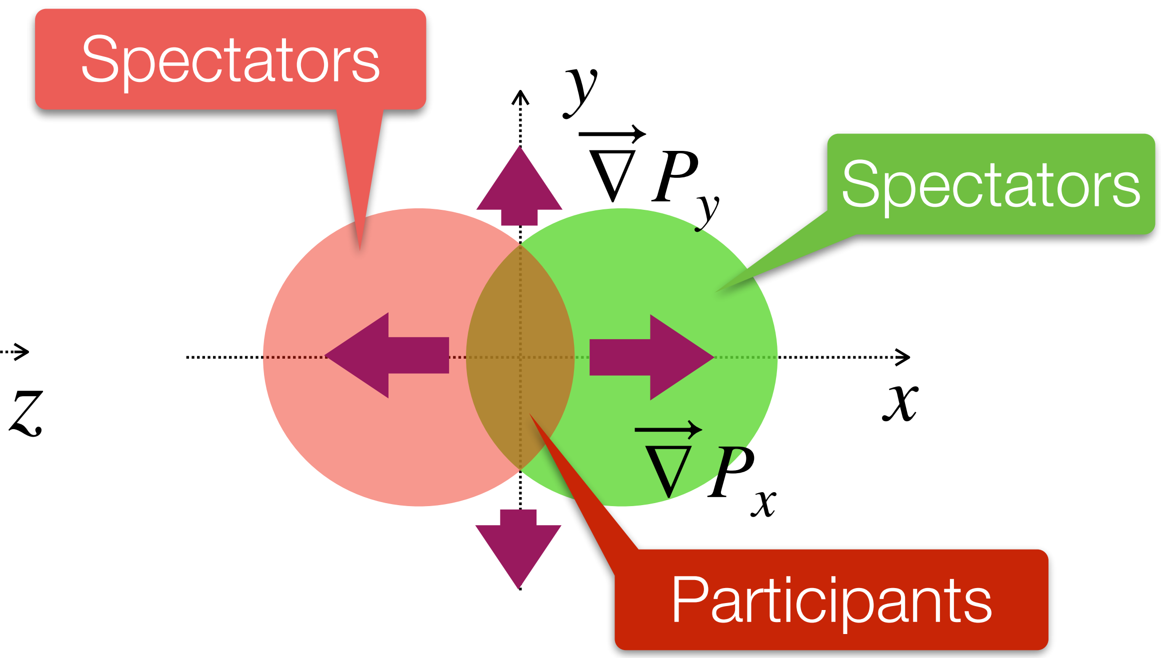
- *Correlations of conserved quantities at finite baryon density*, O. Savchuk, S. Pratt, PRC 109 (2024) 2, 024910,
- *Using baryonic charge balance functions to resolve questions about the baryo-chemistry of the quark gluon plasma,* S. Pratt, et al., PRC 106 (2022) 6, 064911,
- *Interplay of femtoscopic and charge-balance correlations*, S. Pratt, K. Martirosova, PRC 105 (2022) 5, 054906,
- ***Determining the Diffusivity for Light Quarks from Experiment*, S. Pratt, C. Plumberg, PRC 102 (2020) 4, 044909**,
- ***Calculating n-Point Charge Correlations in Evolving Systems*, S. Pratt, PRC 101 (2020) 1, 014914**,
- *Charge Conservation and Higher Moments of Charge Fluctuations*, S. Pratt, R. Steinhorst, PRC 102 (2020) 064906,
- *Charge balance functions for heavy-ion collisions at energies available at the CERN Large Hadron Collider*, S. Pratt, C. Plumberg, PRC 104 (2021) 1, 014906,
- *Determining the Diffusivity for Light Quarks from Experiment*, S. Pratt, C. Plumberg, PRC 102 (2020) 4, 044909,
- *Evolving Charge Correlations in a Hybrid Model with both Hydrodynamics and Hadronic Boltzmann Descriptions*, S. Pratt, C. Plumberg, PRC 99 (2019) 4, 044916,
- *Evolution of Charge Fluctuations and Correlations in the Hydrodynamic Stage of Heavy Ion Collisions*, S. Pratt, J. Kim, C. Plumberg, PRC 98 (2018) 1, 014904,
- *Relating Measurable Correlations in Heavy Ion Collisions to Bulk Properties of Equilibrated QCD Matter*, S. Pratt, C. Young, PRC 95 (2017) 5, 054901,
- *Identifying the Charge Carriers of the Quark-Gluon Plasma*, S. Pratt, PRL 108 (2012) 212301,
- ***Production of Charge in Heavy Ion Collisions*, S. Pratt, et al., PRC 92 (2015) 064905**,
- *Viewing the Chemical Evolution of the Quark-Gluon Plasma with Charge Balance Functions*, S. Pratt, *PoS* CPOD2013 (2013) 023,
- *General Charge Balance Functions, A Tool for Studying the Chemical Evolution of the Quark-Gluon Plasma*, S. Pratt, PRC 85 (2012) 014904,
- *Statistical and dynamic models of charge balance functions*, Cheng, et al , PRC 69, 054906 (2004),
- ***Removing distortions from charge balance functions*, S. Pratt, S. Cheng, PRC 68 (2003) 014907**,
- *Chemical properties of super-hadronic matter created in relativistic heavy ion collisions,* S. Pratt, C. Ratti, W.P. McCormack, 1409.2164 [nucl-th]
- ***Clocking hadronization in relativistic heavy ion collisions with balance functions*, S. Bass, P. Danielewicz, S. Pratt, PRL 85 (2000) 2689-2692**.
- And more…

Geometry, Thermalization, Expansion

Fast Equilibration/Thermalization (w/ quark/gluon dof):

- Enormous Energy Density
- **Large Spatial Anisotropy**

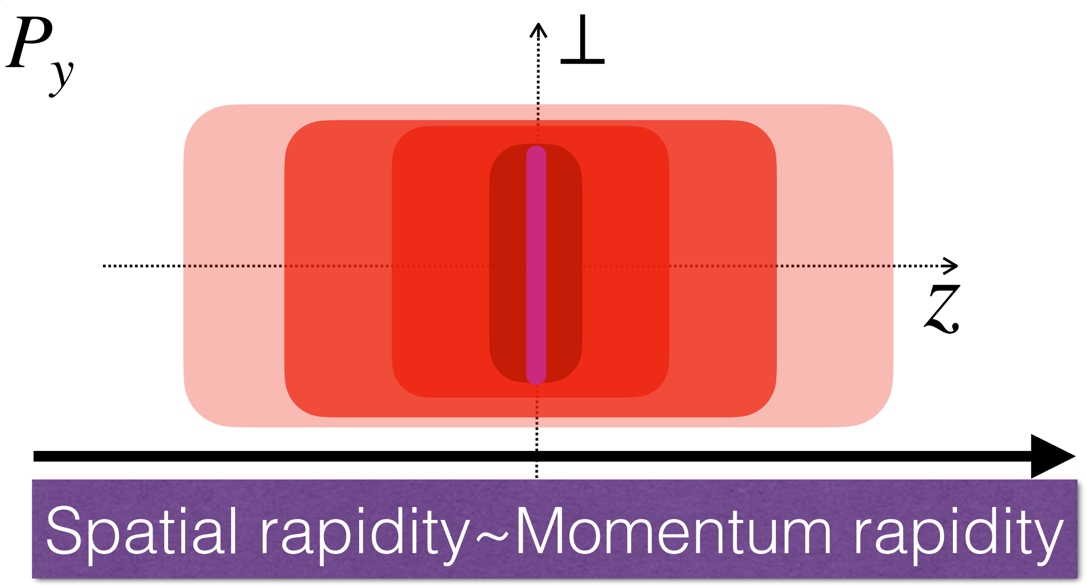




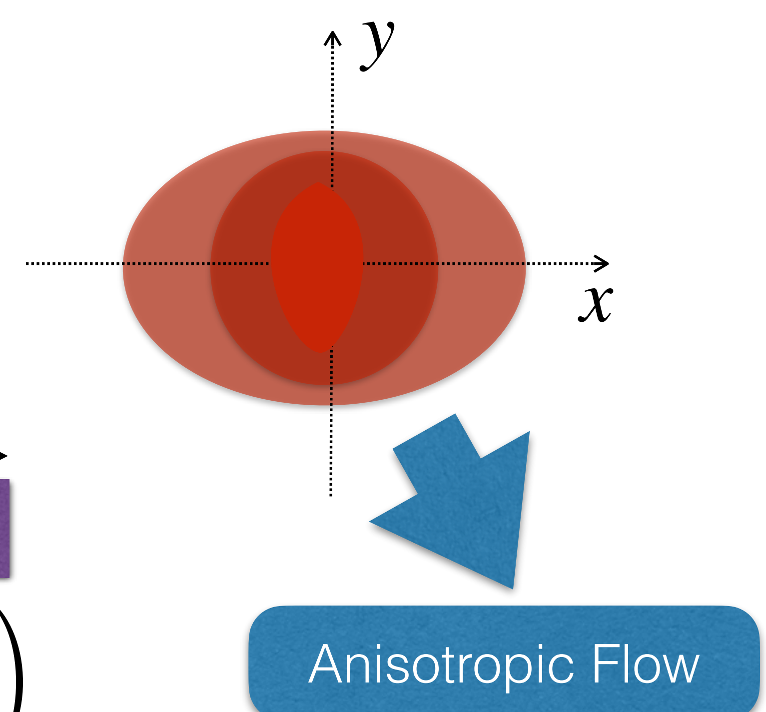
Anisotropic Pressure Gradients

$$\vec{\nabla} P_z \gg \vec{\nabla} P_x \gg \vec{\nabla} P_y$$

- Longitudinal/Isentropic Expansion
- Anisotropic Transverse Expansion
  - Anisotropic Flow

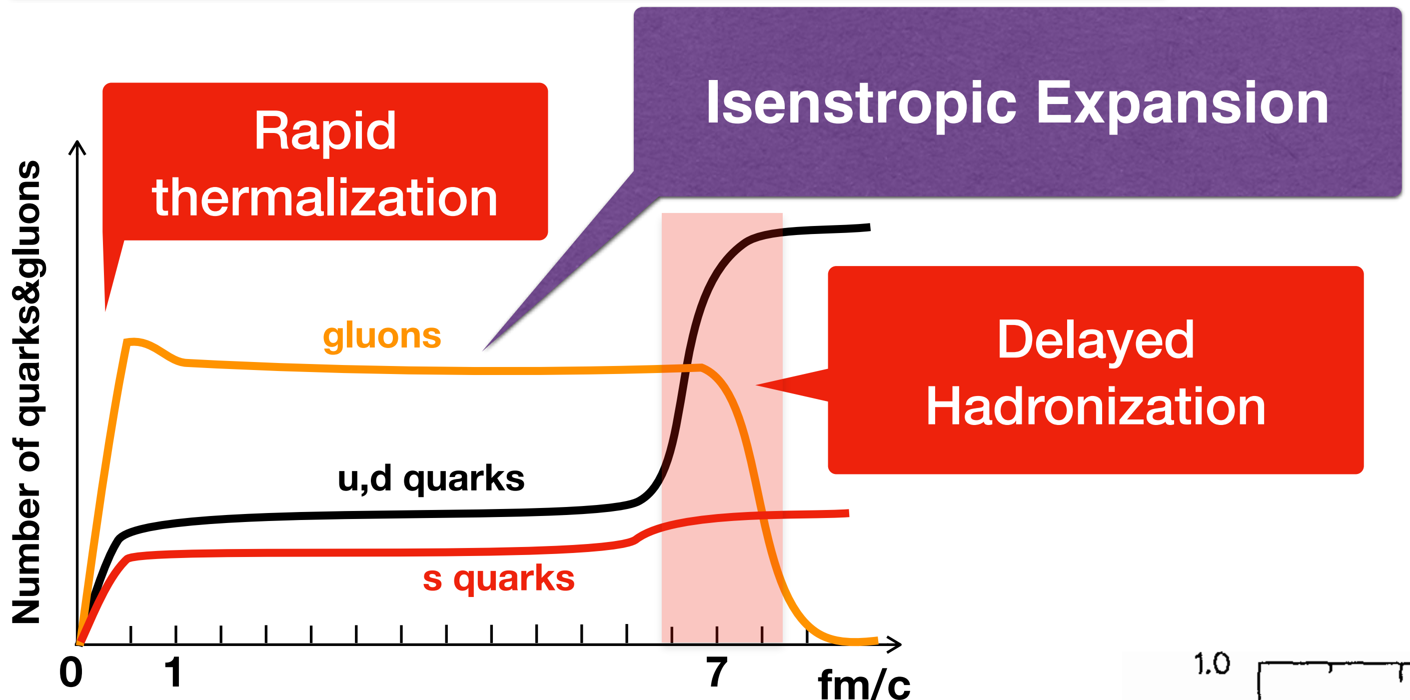


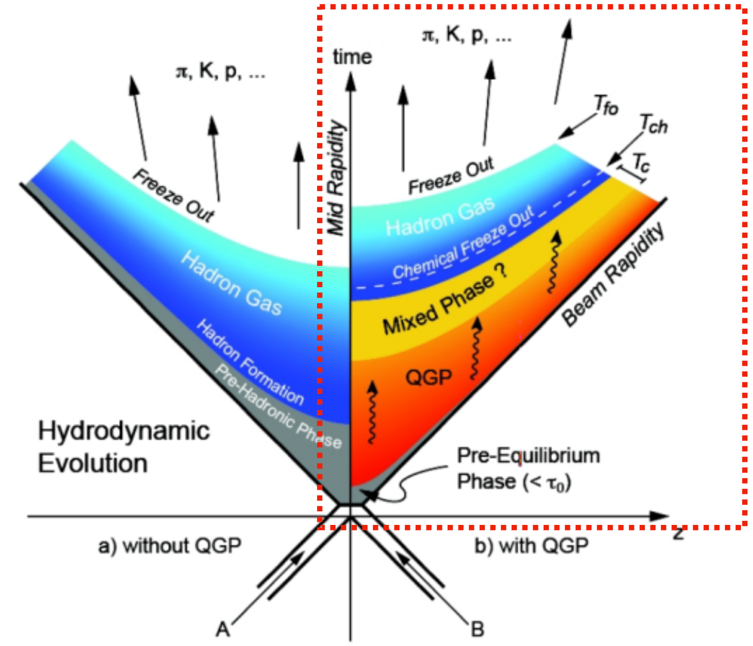
$$\eta_s = \frac{1}{2} \ln\left(\frac{t+z}{t-z}\right) \rightarrow y = \frac{1}{2} \ln\left(\frac{E+p_z}{E-p_z}\right)$$

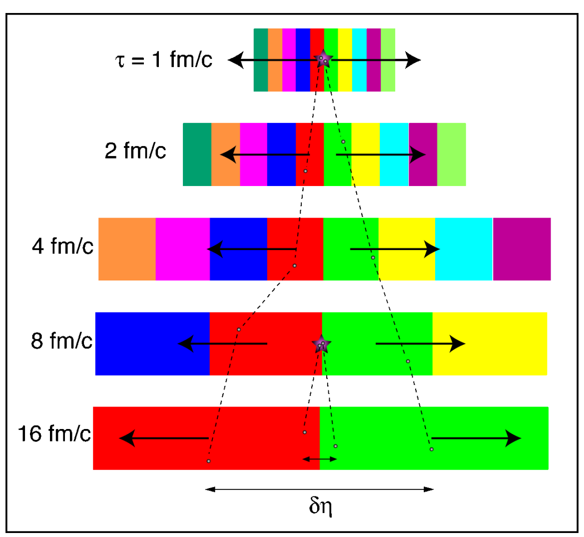


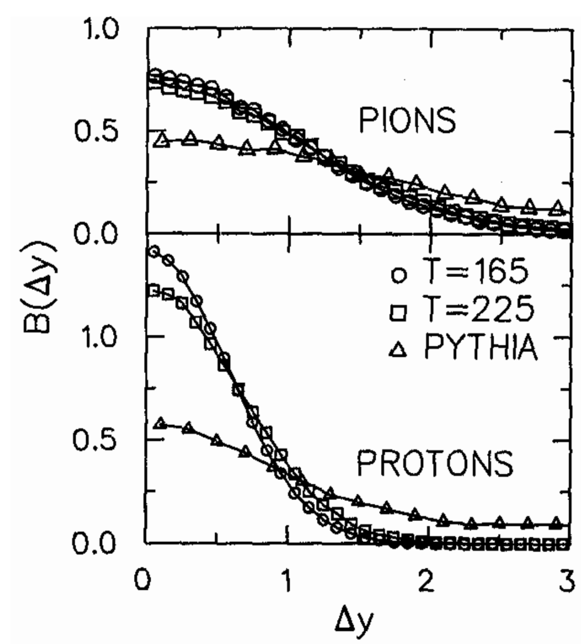
QGP Hypothesis, Thermalization, *Isentropic Expansion*

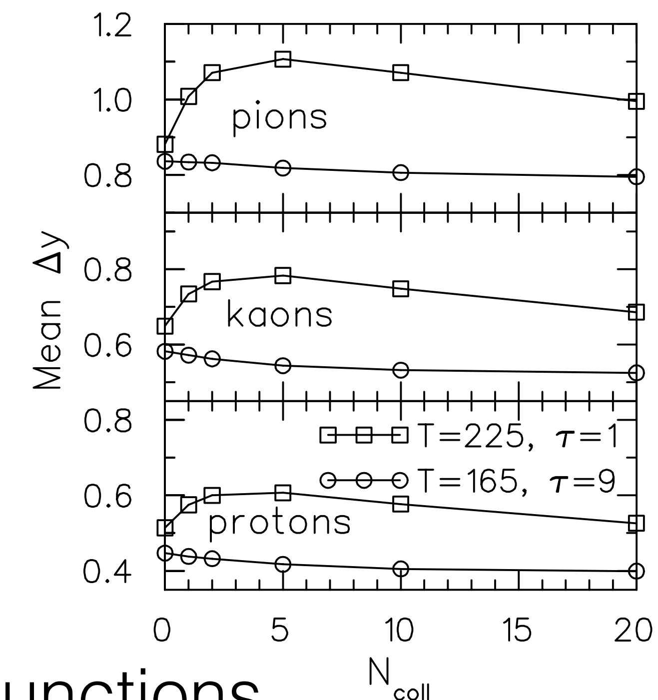
Bass, Danielewicz, Pratt, PRL 85 (2000) 2689











Early vs. Late Emission →Narrowing of Balance Functions

Have we fully vetted/exploited this scenario/information ?

C.P. et al., *Phys.Rev.C* 107 (2023) 5, 054915

A Technical Detail: Redefining BFs

Why a new definition?

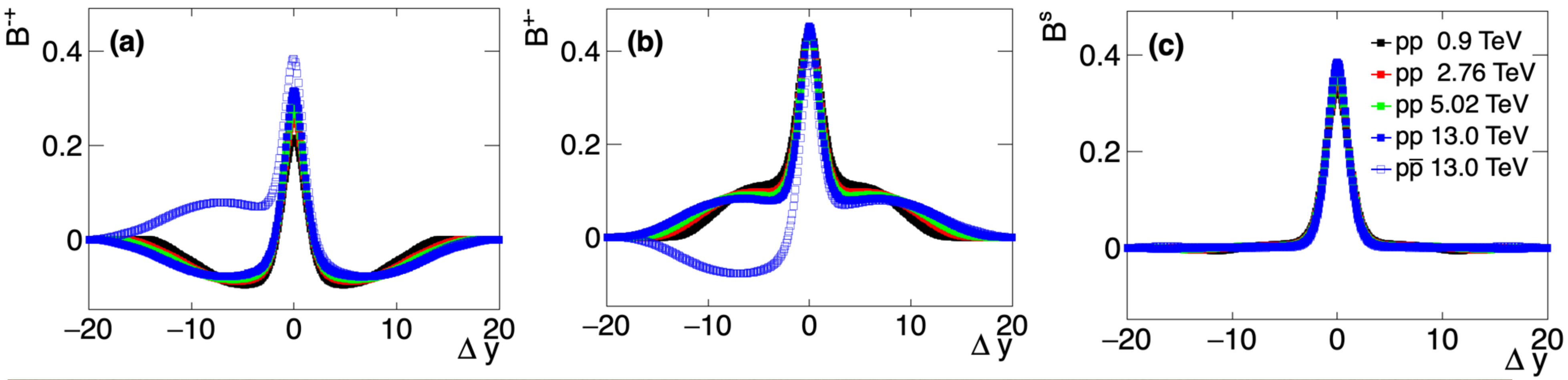
Original Balance Function Definition [1]
Difference of Conditional Densities
Simulations: pp collisions w/ PYTHIA-8 Monash, w/ CR

(a) $B^{-|+}(\Delta y) \equiv \rho_2^{-|+}(\Delta y) - \rho_2^{+|+}(\Delta y)$

(b) $B^{+|-}(\Delta y) \equiv \rho_2^{+|-}(\Delta y) - \rho_2^{-|-}(\Delta y)$

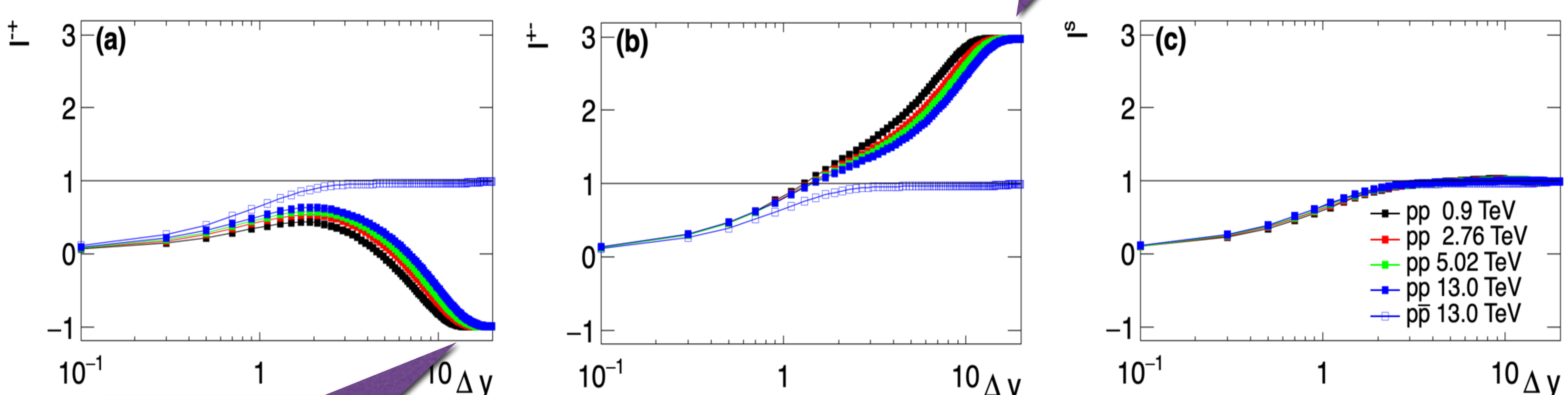
(c) $B^{s}(\Delta y) \equiv \left[B^{-|+}(\Delta y) + B^{+|-}(\Delta y)\right]/2$

Definition used by STAR



Cumulative Integrals: $I = \int_{\mathrm{Acc}} B(\Delta y')\mathrm{d}\Delta y'$

I: 1+2=3



Net charge of the system impacts BFs and their integrals

I: 1-2=-1

[1] Clocking hadronization … with balance functions, S. Bass, P. Danielewicz, S. Pratt, PRL 85 (2000) 2689-2692

Technical Details: Redefining BFs

C.P. et al., *Phys.Rev.C* 107 (2023) 5, 054915

New Definition Based on Cumulants

Balance Functions defined w/ **Cumulants** pp collisions w/ PYTHIA-8 Monash

(a) $$B^{+-}(y_1, y_2 | y_0) = \frac{1}{\langle N_1^- \rangle} \left[C_2^{+-}(y_1, y_2) - C_2^{--}(y_1, y_2) \right]$$

(b) $$B^{-+}(y_1, y_2 | y_0) = \frac{1}{\langle N_1^+ \rangle} \left[C_2^{-+}(y_1, y_2) - C_2^{++}(y_1, y_2) \right]$$

(c) $$B^s(\Delta y) \equiv \left[B^{-|+}(\Delta y) + B^{+|-}(\Delta y) \right] / 2$$

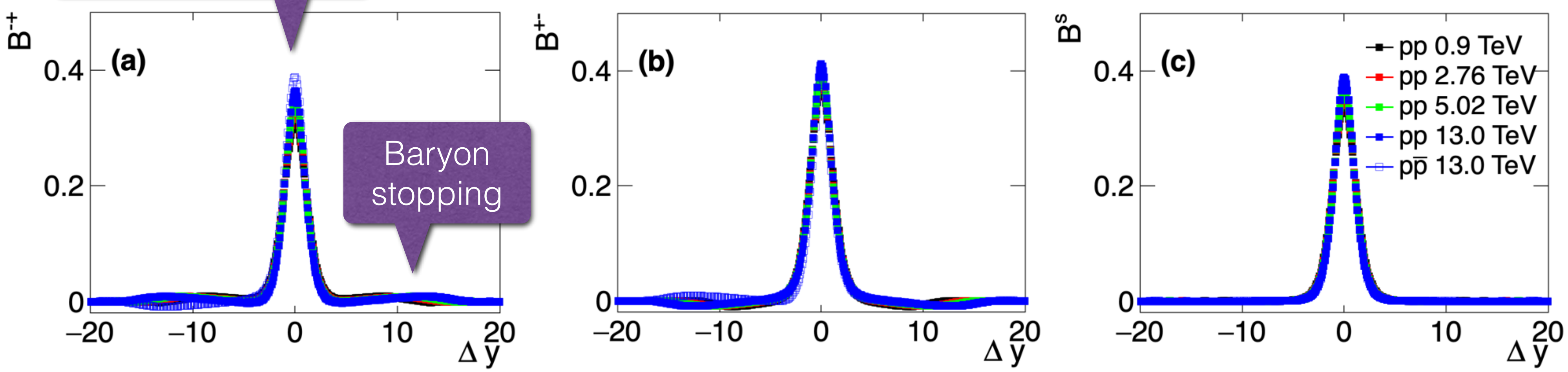
Definition used by ALICE

$$C_2^{\alpha\beta}(y_1, y_2) = \rho_2^{\alpha\beta}(y_1, y_2) - \rho_1^{\alpha}(y_1)\rho_1^{\beta}(y_2) .$$

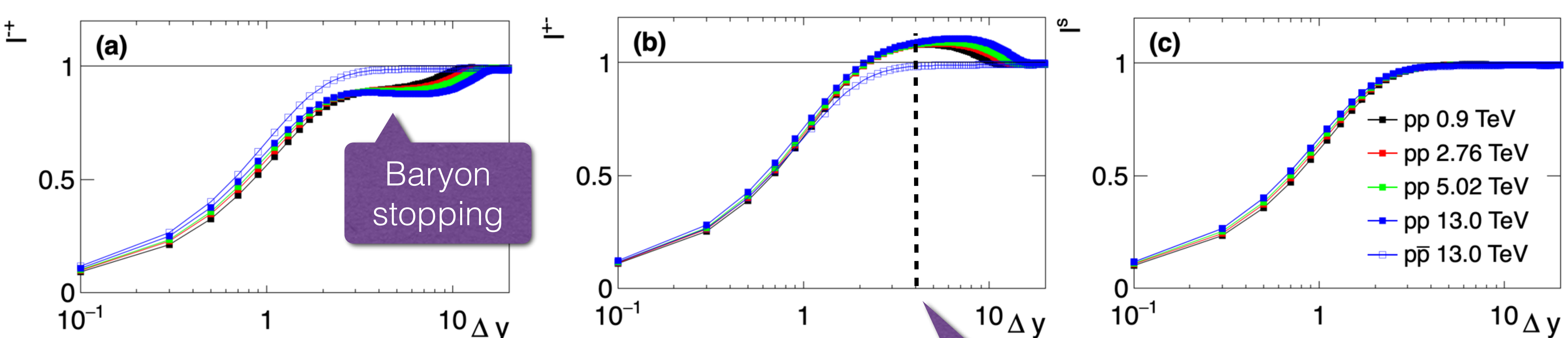
Two-particle Cumulant

"Pair Creation"

Baryon stopping



Baryon stopping



All Integrals converge to unity

Accessible to ALICE 3, CMS

Balance Function Measurements

STAR Balance Function Results

First Measurement [1]

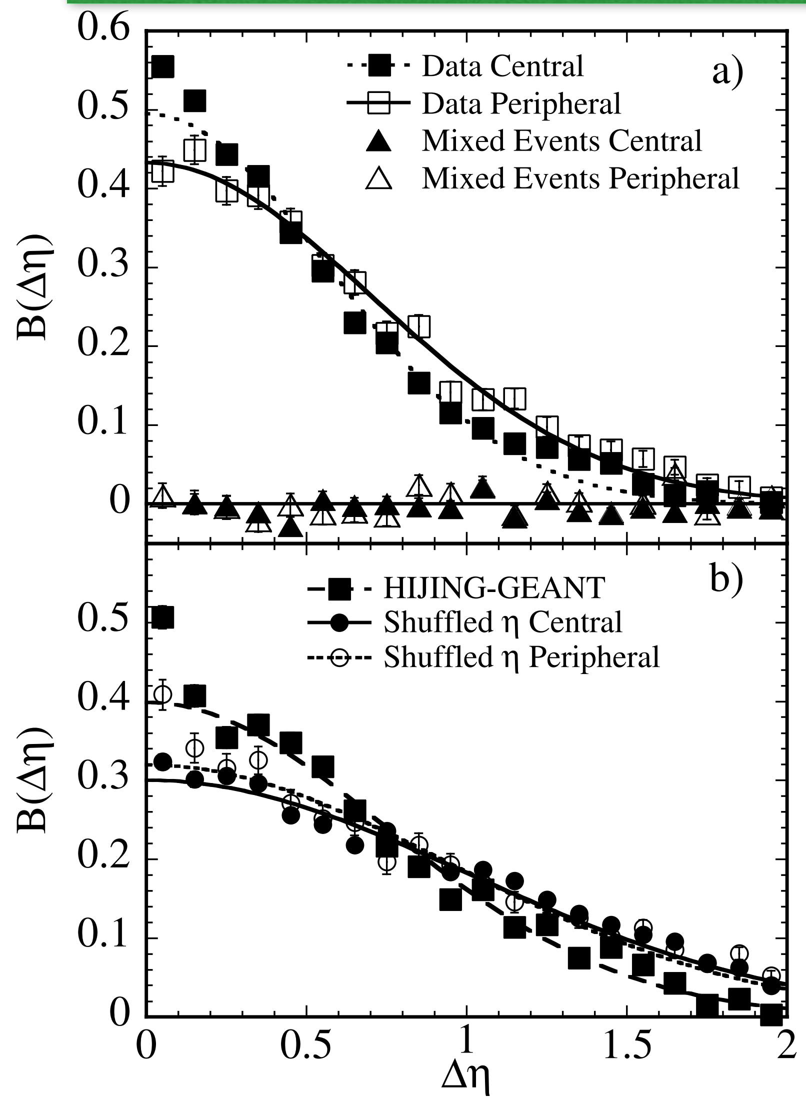
Au-Au @ $\sqrt{s_{NN}} = 130$ GeV

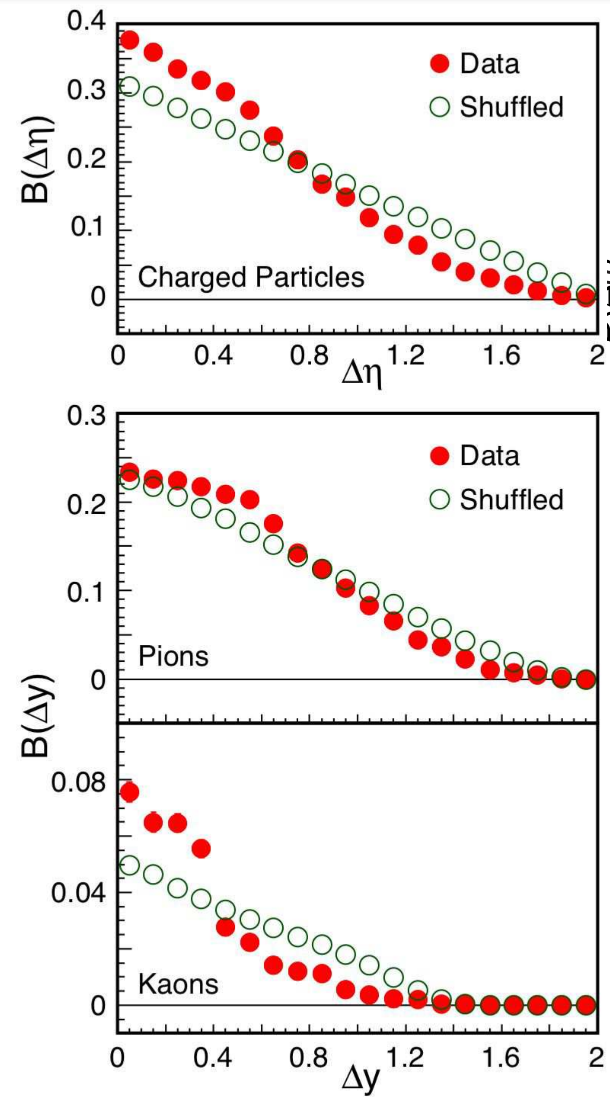
Identified Particles [4]

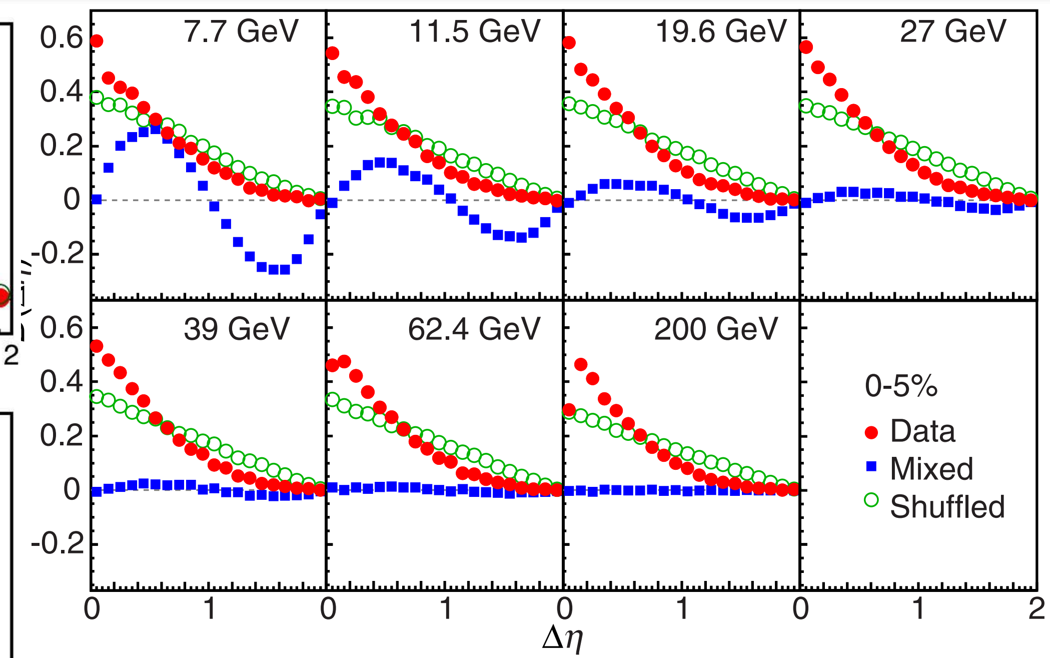
Au-Au @ $\sqrt{s_{NN}} = 200$ GeV

Beam Energy Scan [2]

Measurements based on original BF definition







1. STAR, PRL 90 (2003) 172301,
2. STAR, J. Phys. G: Nucl. Part. Phys. 35 (2008) 104031
3. STAR, PLB B 690 (2010) 239–244,
4. STAR, PRC 8(2010) 024905
5. STAR, Phys. Rev. C 94 (2016) 024909

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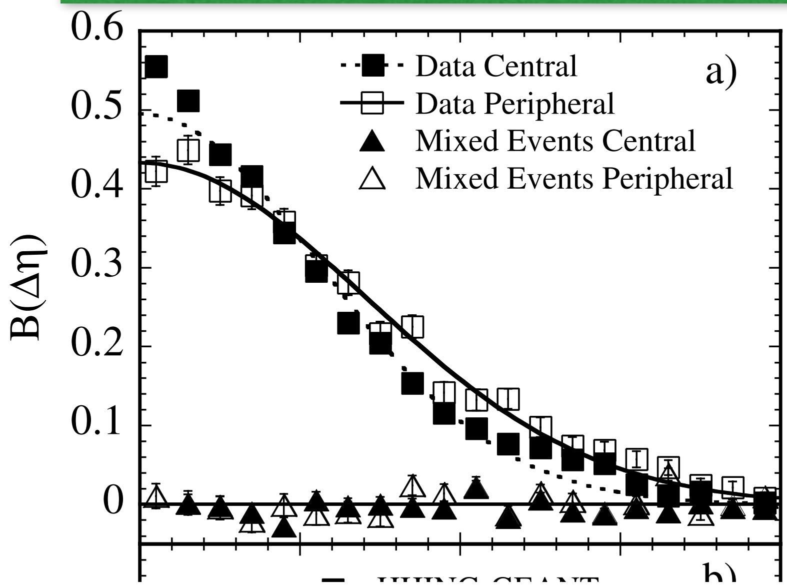
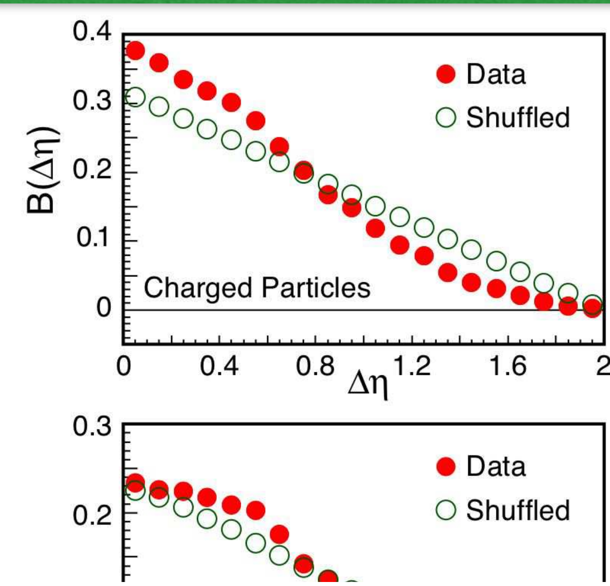
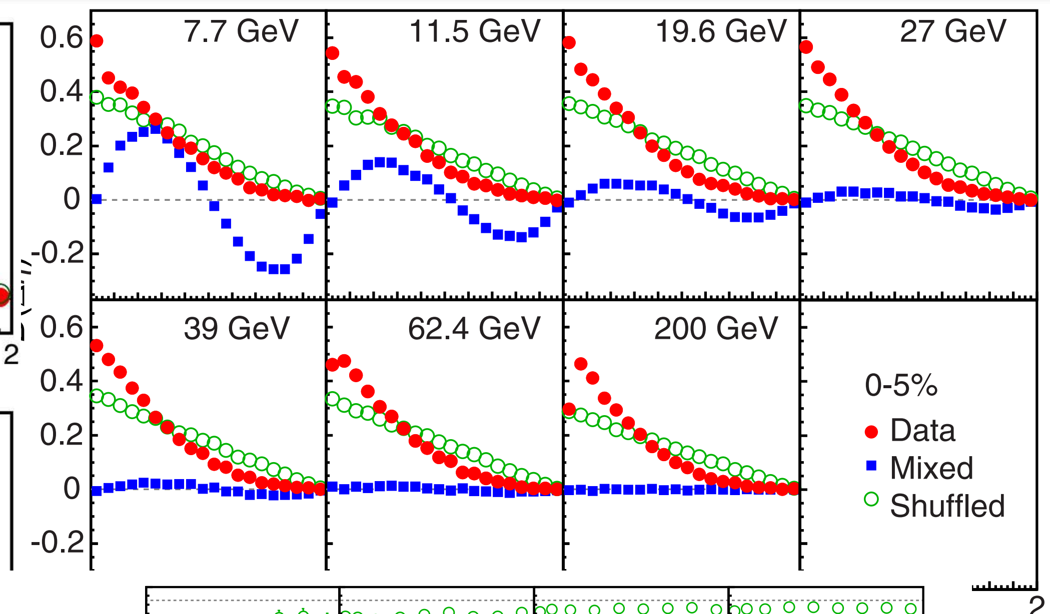
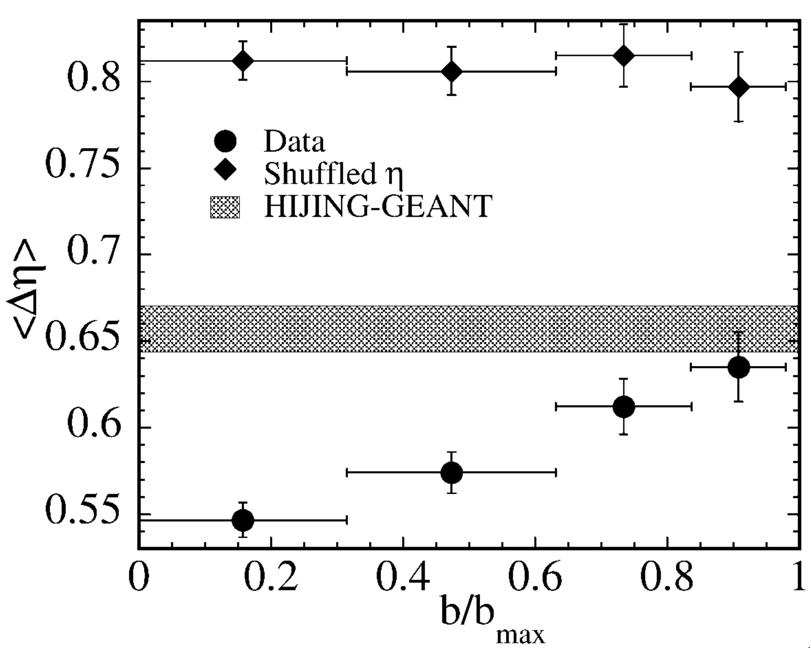
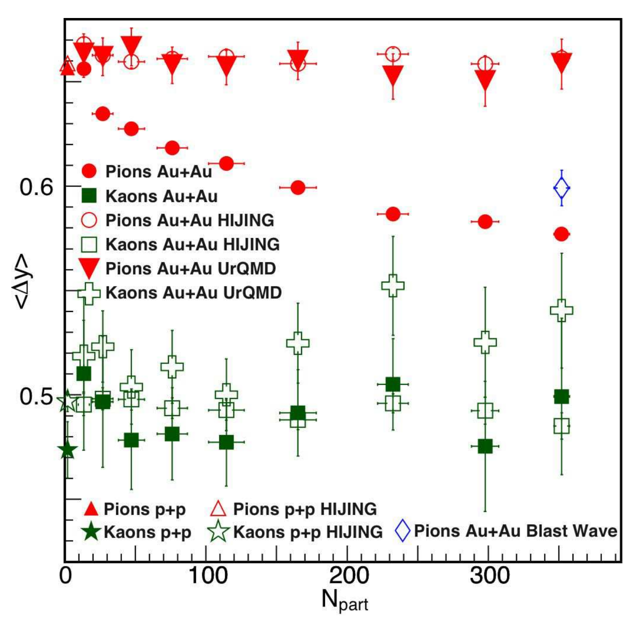
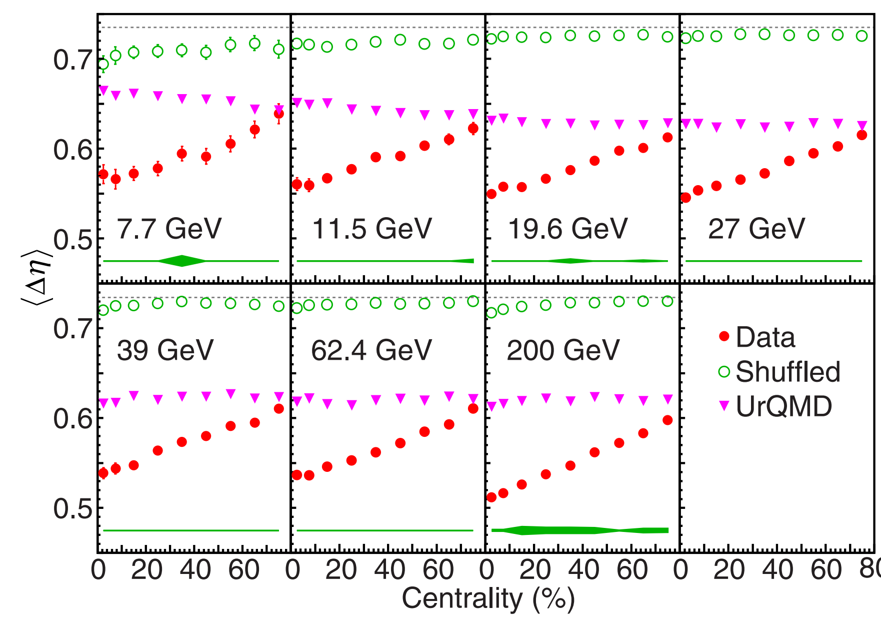








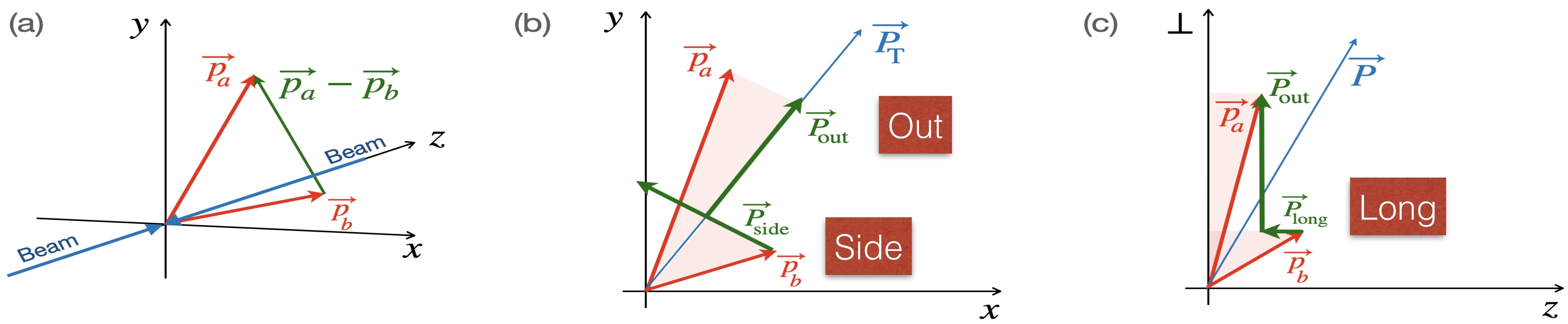




3. STAR, PLB B 690 (2010) 239–244,
4. STAR, PRC 8(2010) 024905
5. STAR, Phys. Rev. C 94 (2016) 024909

Narrowing at ALL Beam Energies

Balance Functions in invariant relative momentum [1]



- Pratt proposed a measurement on the invariant relative momentum of identified particle pairs.
  - $q^\mu = (p_a^\mu - p_b^\mu) - P^\mu \dfrac{P \cdot (p_a - p_b)}{P^2}$
  - $q^\mu = (p_a^\mu - p_b^\mu) - P^\mu \dfrac{m_a^2 - m_b^2}{s}$
  - $Q_{\rm inv}^2 = -q^2 = -(p_a^\mu - p_b^\mu)^2 + \dfrac{m_a^2 - m_b^2}{P^2}$

- And a 3D analysis w/ coordinates such that the total spatial momentum of the pair is zero
  - $Q_{\rm inv}^2 = Q_{\rm long}^2 + Q_{\rm side}^2 + Q_{out}^2$
  - $Q_{\rm long} = \dfrac{1}{\sqrt{s + P_T}}\left(P_0 q_z - P_z q_0\right)$
  - $Q_{\rm side} = \dfrac{1}{P_T}\left(P_x q_y - P_y q_x\right)$
  - $Q_{\rm out} = \sqrt{\dfrac{s}{s + P_T}}\dfrac{1}{P_T}\left(P_x q_x - P_y q_y\right)$

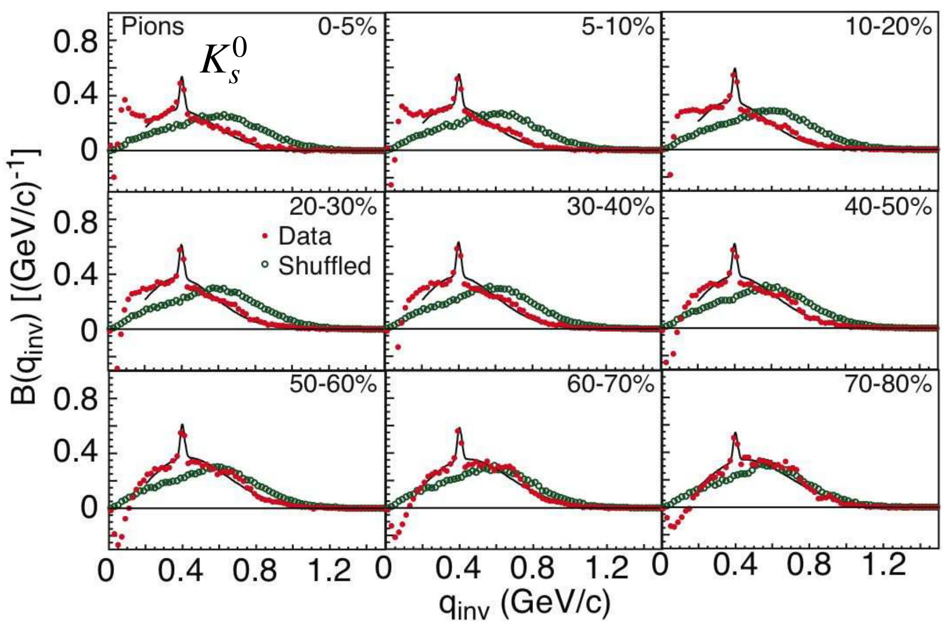
- Purpose: Simplify interpretation of BF with thermal models
  - Reduce the sensitivity to collective flow.
  - Collective flow affects spectra, but leaves $\vec{Q}_{\rm inv}$ unchanged if particles originate from the same space-time point of the blast wave.
  - Thus minimizes the confusion associated with the collective flow as the width would only depend on the local thermal properties of the individual sources.

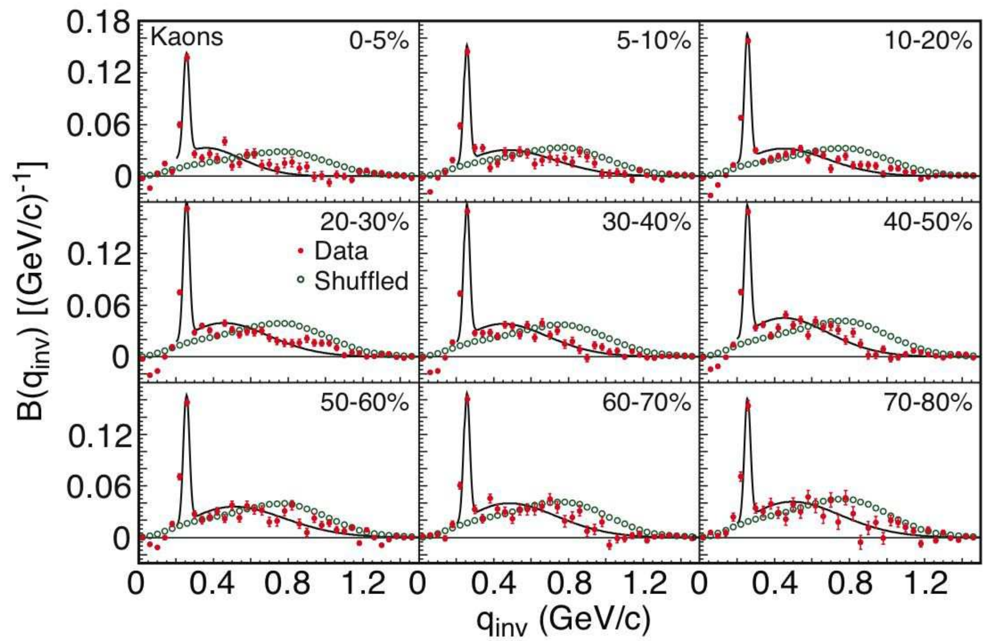
[1] S. Pratt, S. Cheng, PRC 68, 014907 (2003)

STAR, PRC82:024905,2010

Balance Functions in invariant relative momentum

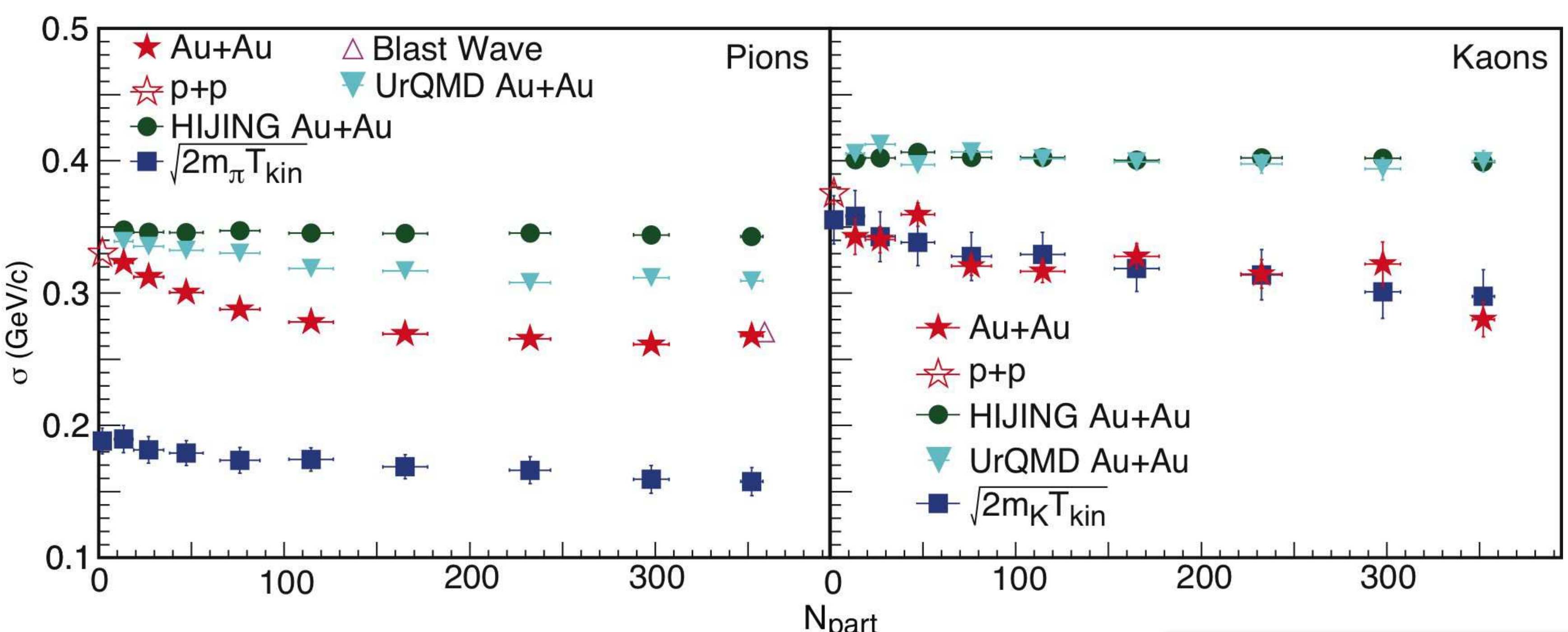
Au+Au at √sNN = 200 GeV





These mixed events are not zero for all ... Corrected by subtracting the balance functions calculated using mixed ever...

Note that no peak from the decay ... collisions around q_{inv} = 0.718 Ge... observed in the ... HBT/Coulomb effects are visible ... 8. Figure 9 shows the balance fu... q_{inv} < 0.2 GeV/c for the most cen... most peripheral bin (70%–80%)... opposite charges closer together... apart, leading to an enhancemen... suppression of same-sign pairs a... to a rise in the balance function a... in central collisions, where the lo... affects more particles [30].



STAR's statement:
The centrality evolution in freeze-out temperature may explain much of the narrowing of the balance function in terms of qinv for pions, as well as for kaons. However, firm conclusions require more complete calculations including **all detector effects**.

ALICE: Analysis In progress

STAR, PRC82:024905,2010

Balance Functions in invariant relative momentum

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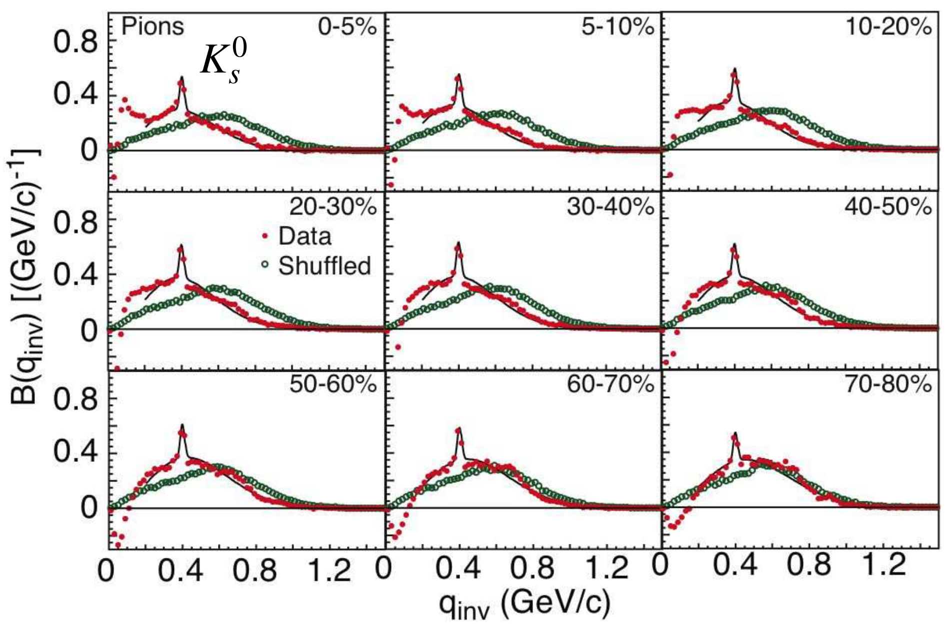

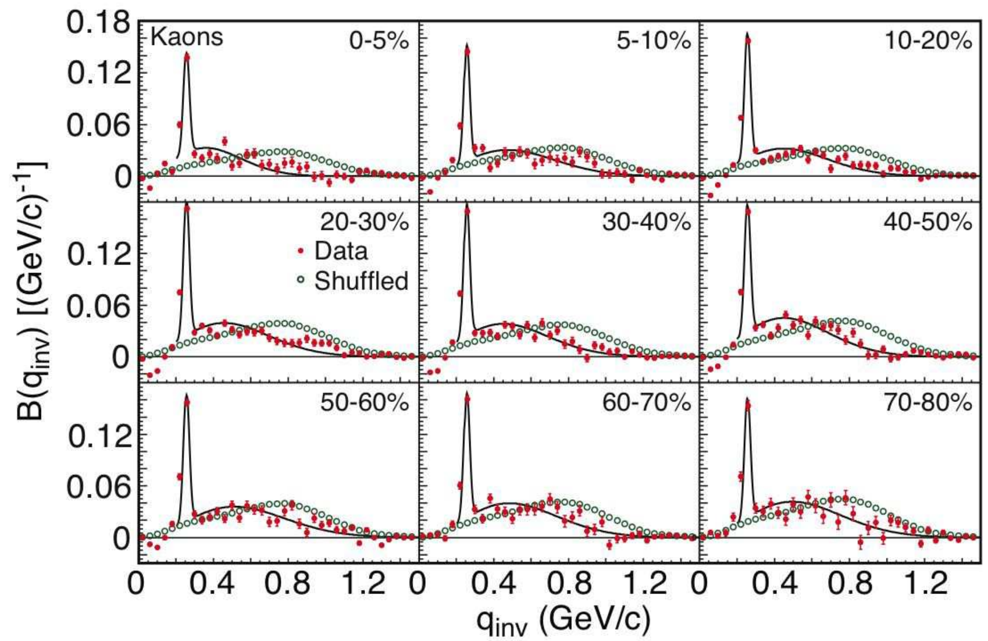

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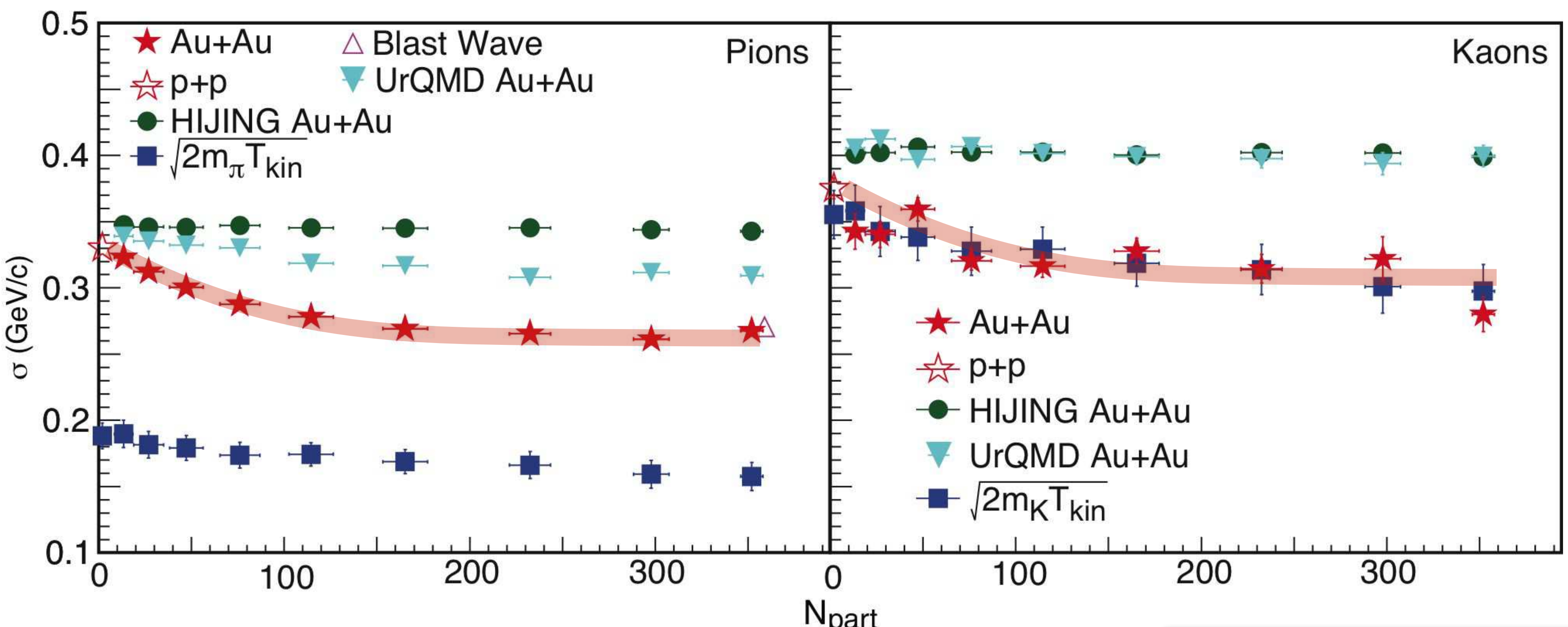

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ALICE: Analysis In progress

STAR, PRC82:024905,2010

Balance Functions in invariant relative momentum

Au+Au at √sNN = 200 GeV

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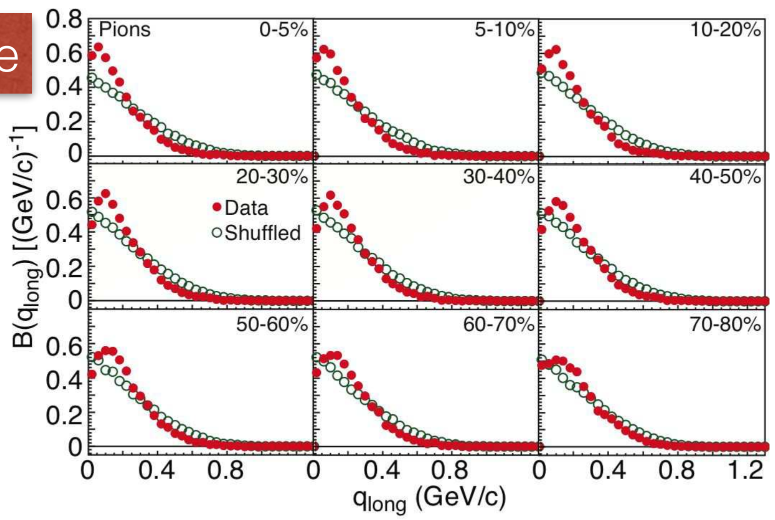
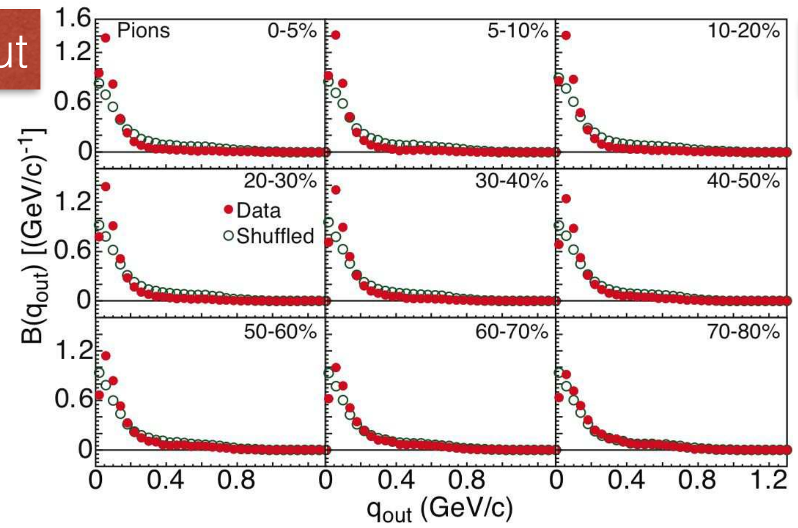
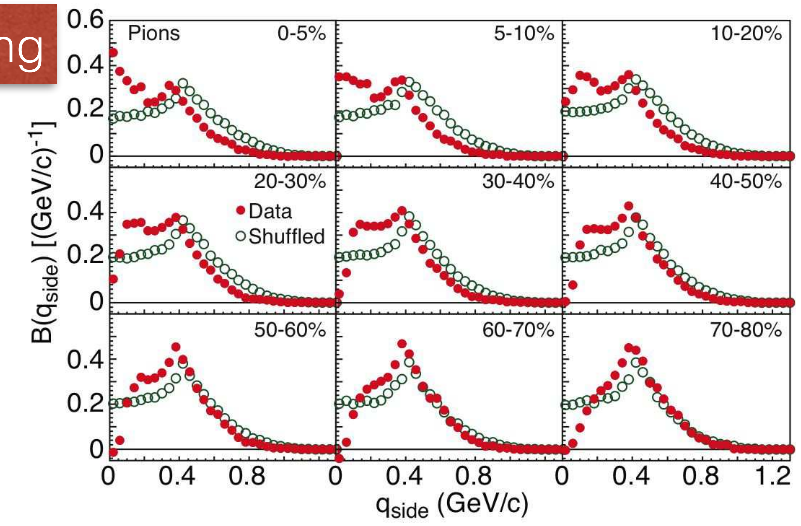
Side

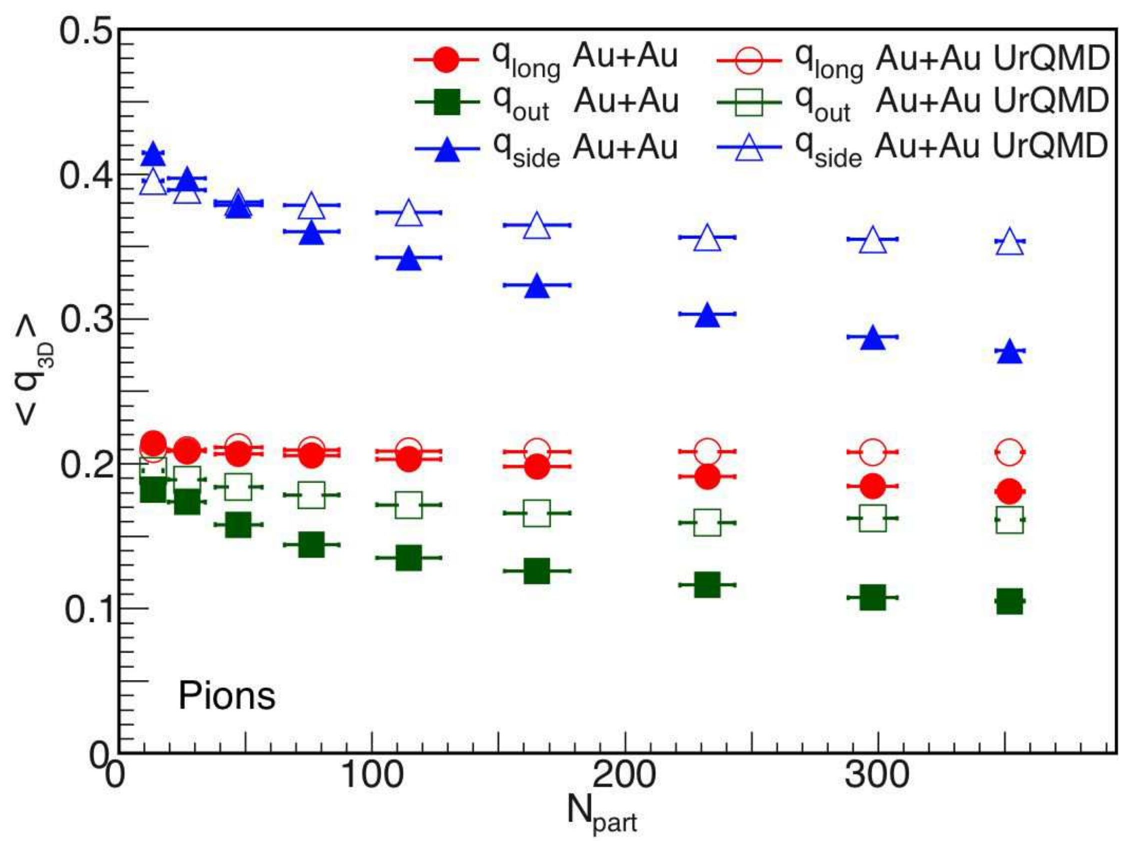


Out



Long





STAR's conclusion:

- This may imply that string dynamics and diffusion owing to longitudinal expansion may keep 〈qlong〉 from decreasing as much in more central collisions [1].
- The decrease in the transverse widths is consistent with the decrease in T_{kin} as the collisions become more central — a final state effect…

[1] S. Cheng,et al., PR **69**, 054906 (2004).

Note: ALICE is working on a similar analysis. Stay tune for results next year.

Wayne State University
College of Liberal Arts & Sciences
Department of Physics and Astronomy

STAR, PRC82:024905,2010

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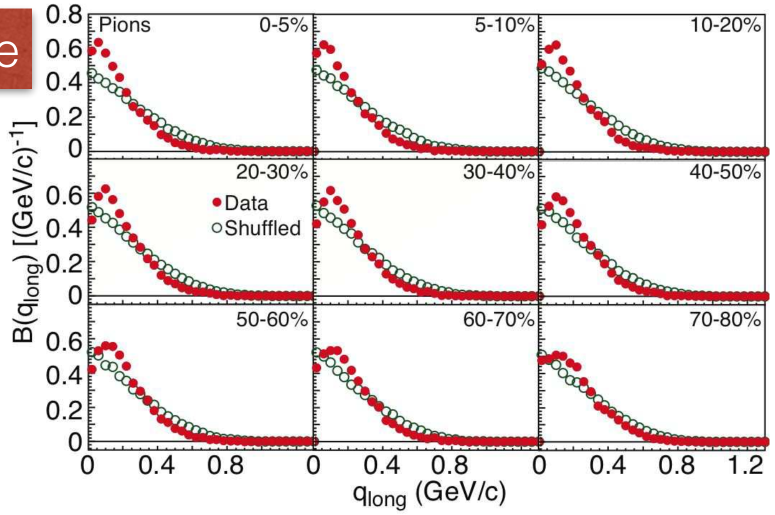

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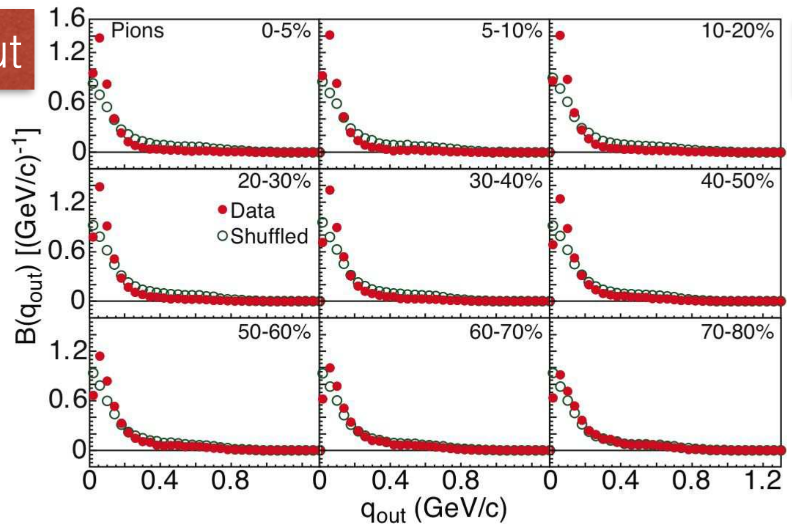

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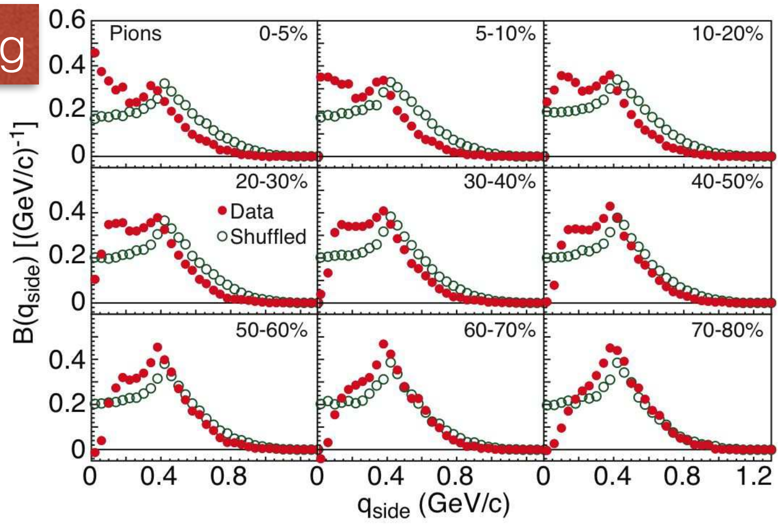

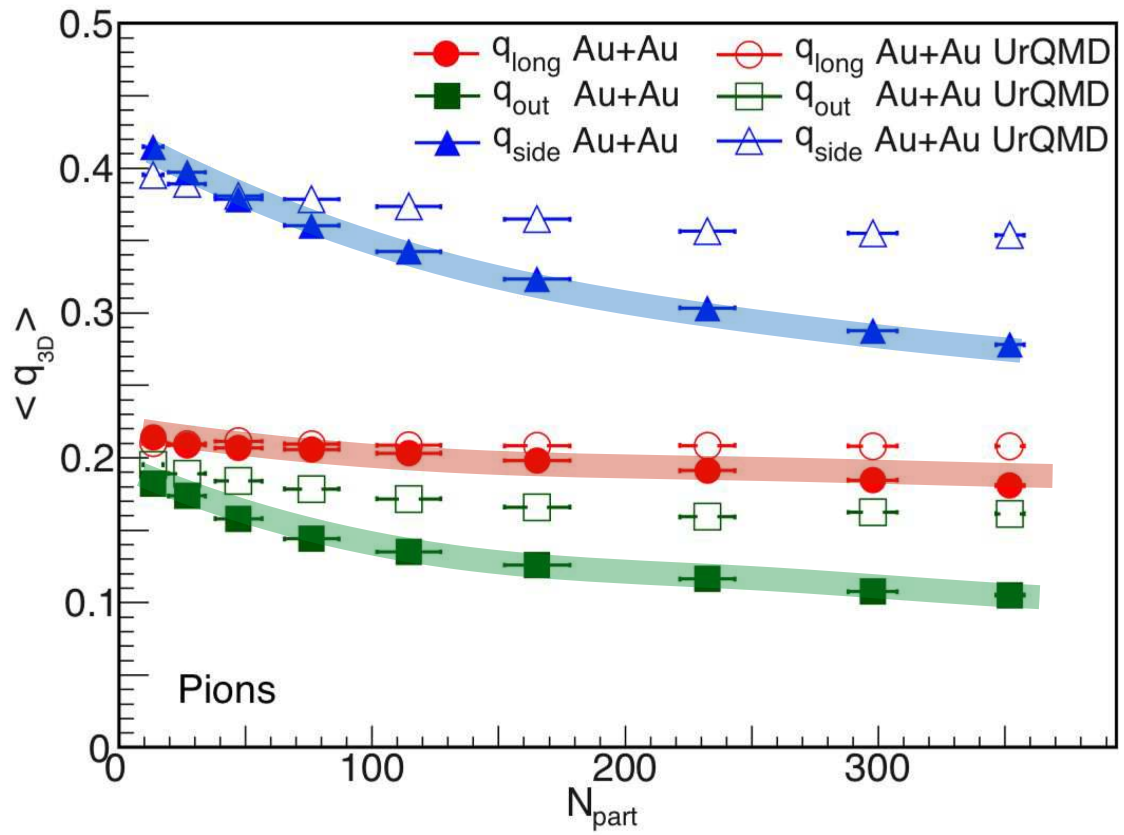

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Note: ALICE is working on a similar analysis. Stay tune for results next year.

ALICE Measurements

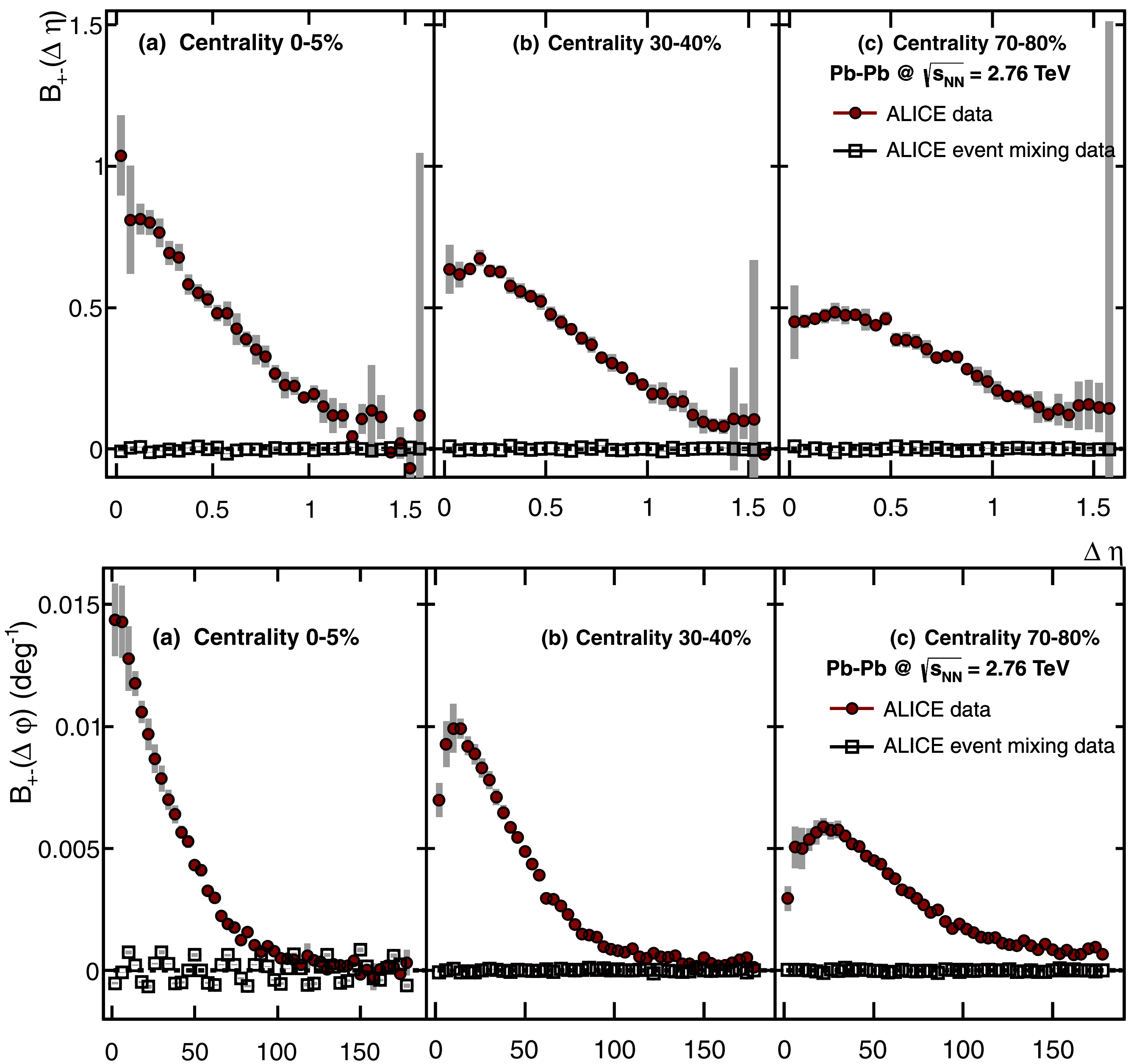
ALICE, PLB 723 (2013) 267.
ALICE, Eur. Phys. J. C (2016) 76

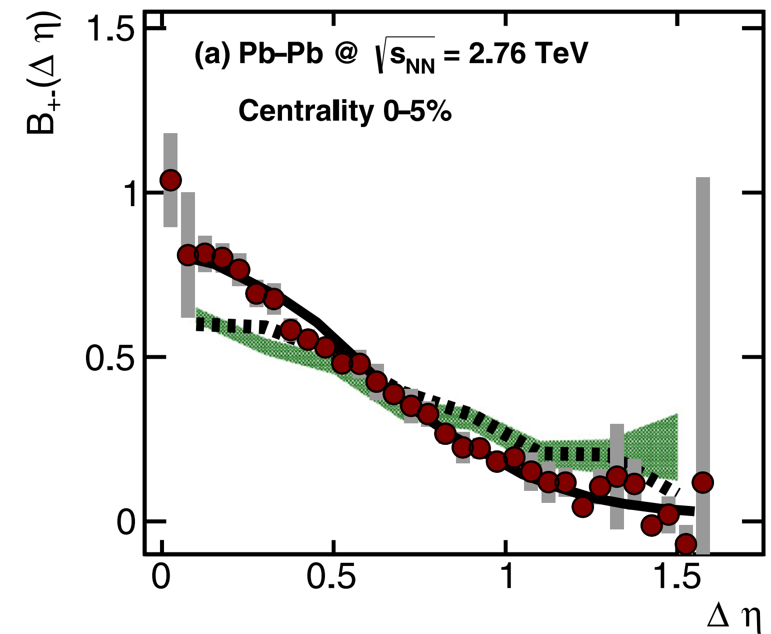
Balance Functions at

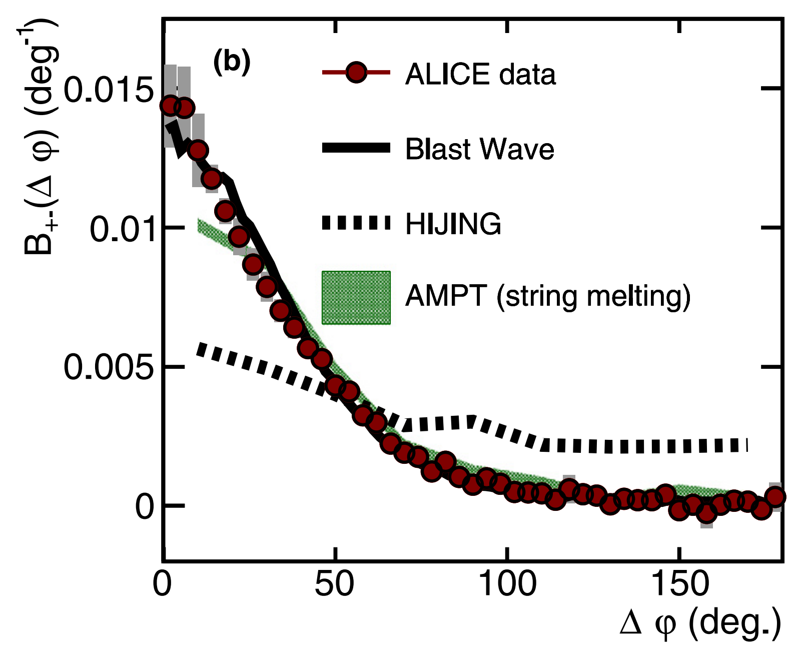
$$B_{+-}(\Delta\eta) = \frac{1}{2}\left(C_{+-}(\Delta\eta) + C_{-+}(\Delta\eta) - C_{--}(\Delta\eta) - C_{++}(\Delta\eta)\right). \qquad C_{ab} = (N_{ab}/N_b)/f_{ab}$$

Pb-Pb @ $\sqrt{s_{NN}} = 2.76$ TeV

Data corrected for efficiency and acceptance

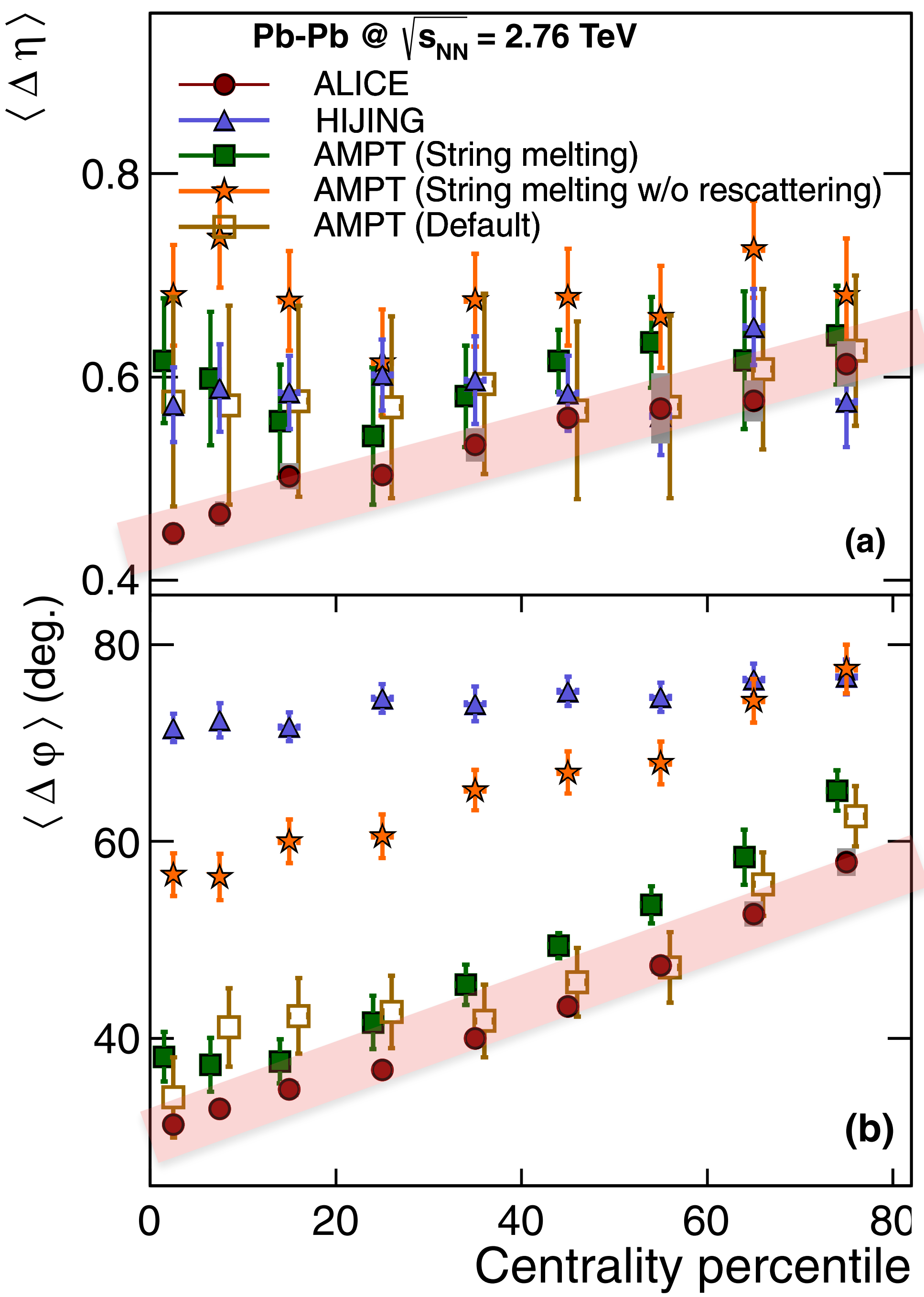


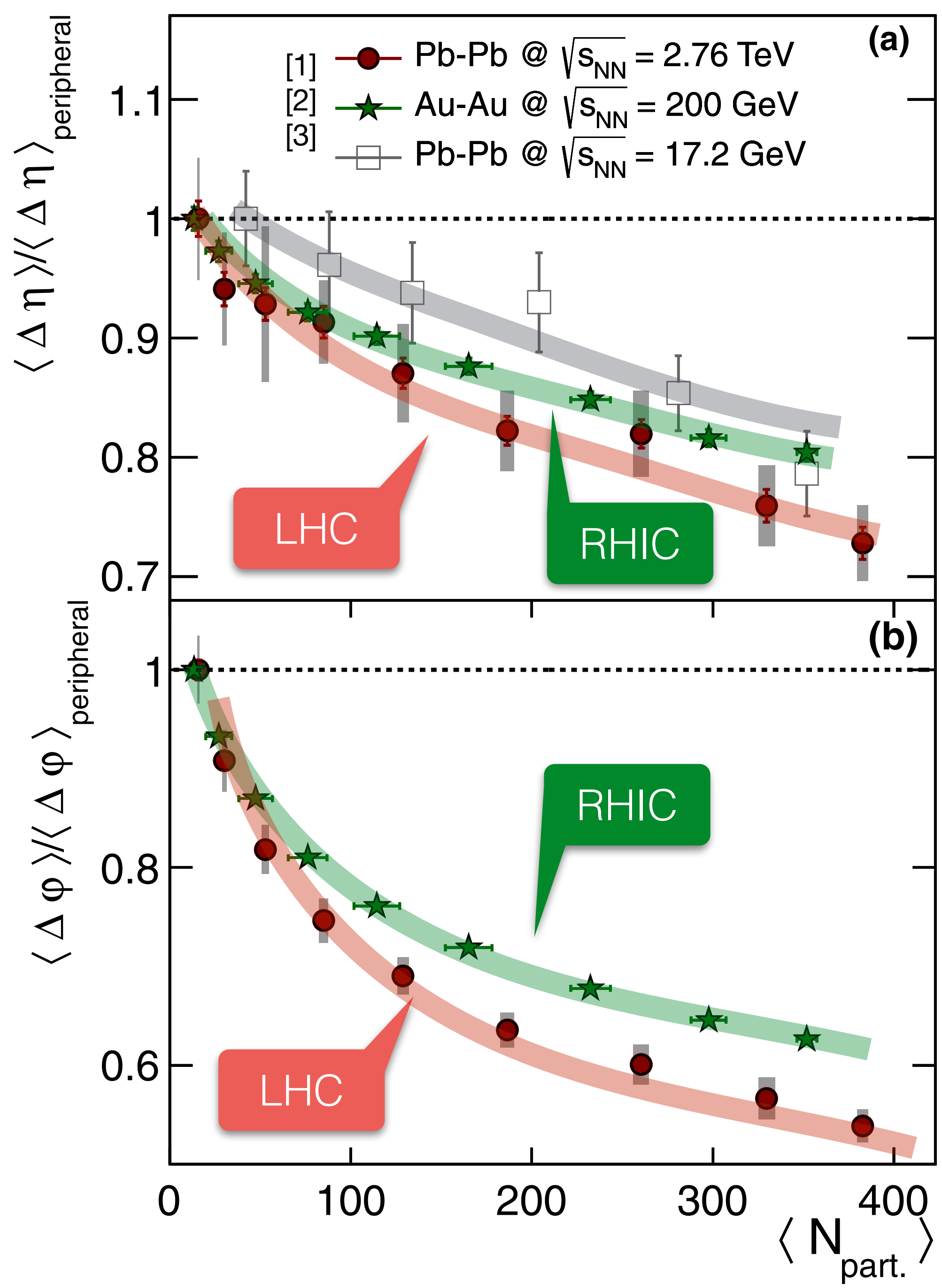




Results in qualitative agreement with those obtained by STAR — PRL 90 (2003)172301; PRC 82 (2010); PRC 94 (2016) 024909

Balance Functions: NA49, STAR, & ALICE





Somewhat stronger reduction of the $\Delta\eta$ width in Pb-Pb

Final state T_{kin} effect?

Stronger radial flow effects at LHC

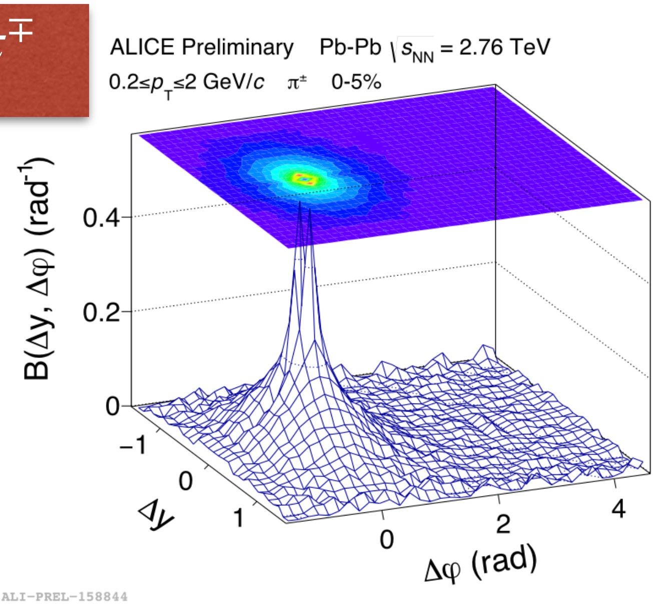
[1] ALICE, PLB 723 (2013) 267.
[2] STAR, Phys. Rev. C **82**, 024905
[3] NA49, Phys. Rev. C **76**, 024914

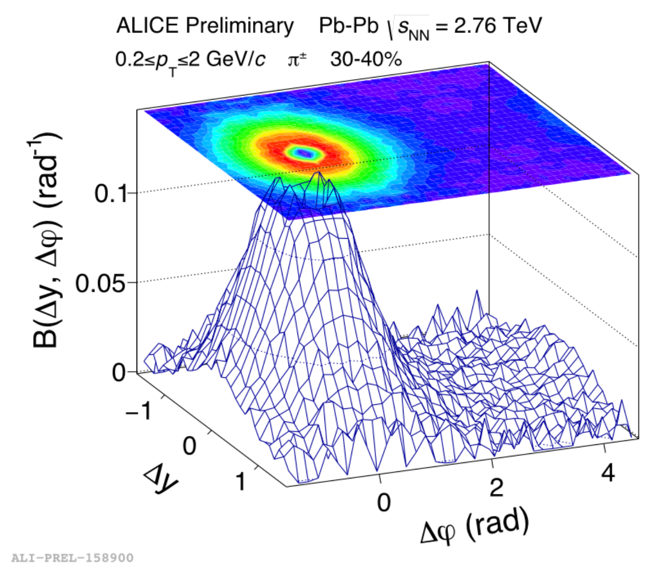
J. Pan, PhD, Wayne State (2019)
ALICE PLB 833 (2022) 137338

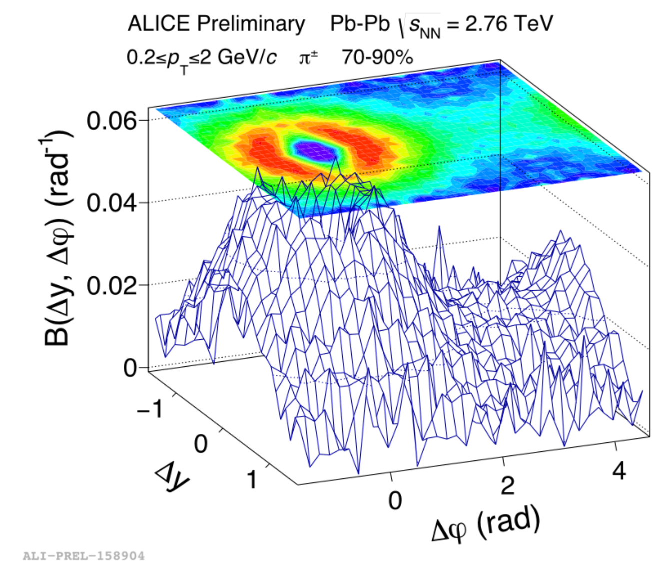
Pion, Kaon Balance Functions

Identified Cross Species Balance Functions $(\pi, K, p) \otimes (\pi, K, p)$

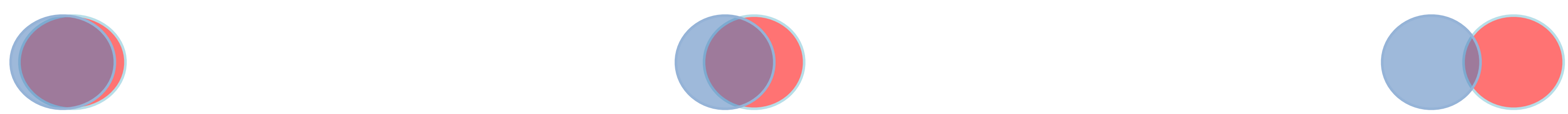
$B^{\pi^{\pm}\pi^{\mp}}$



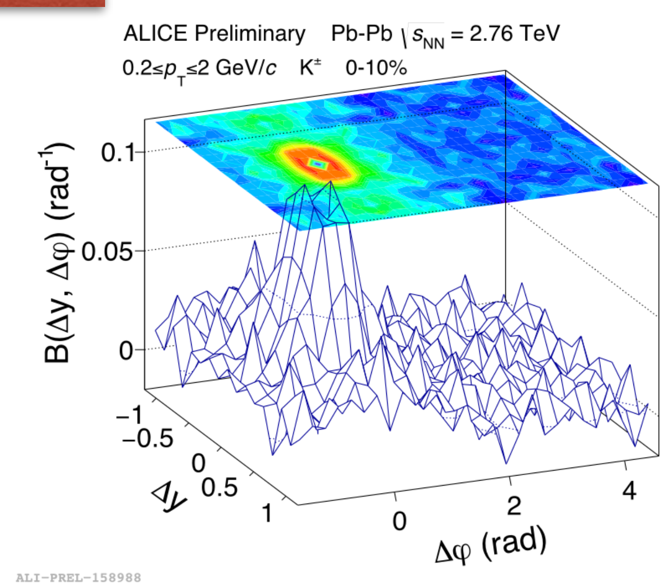


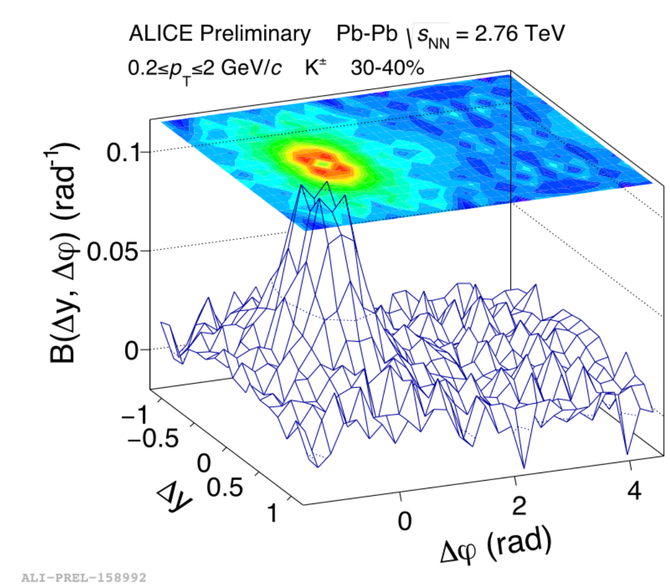


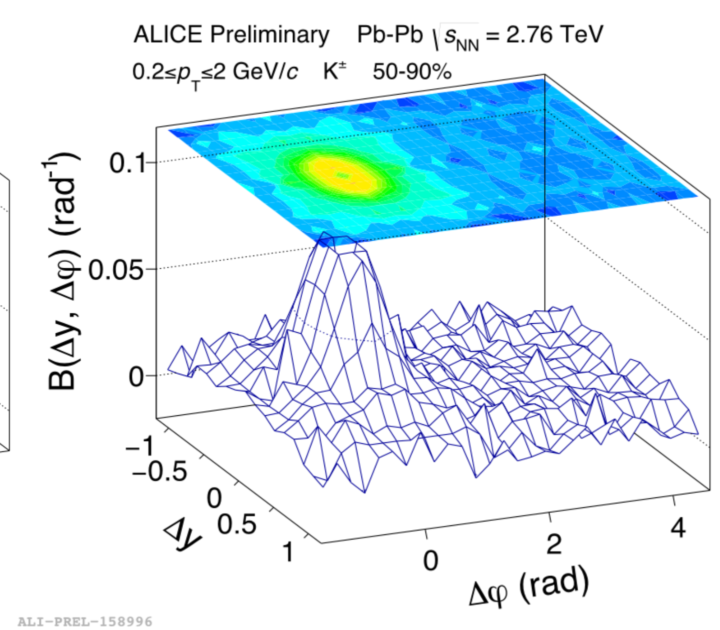
Considerable shape dependence on collision centrality



$B^{K^{\pm}K^{\mp}}$







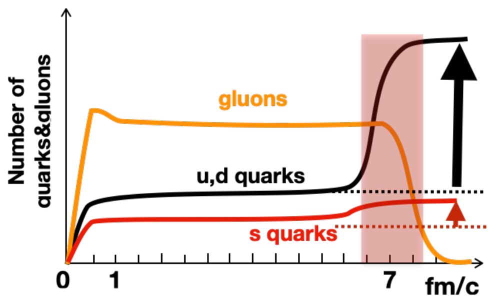
Very modest shape dependence on collision centrality

PID Balance Functions in Pb—Pb

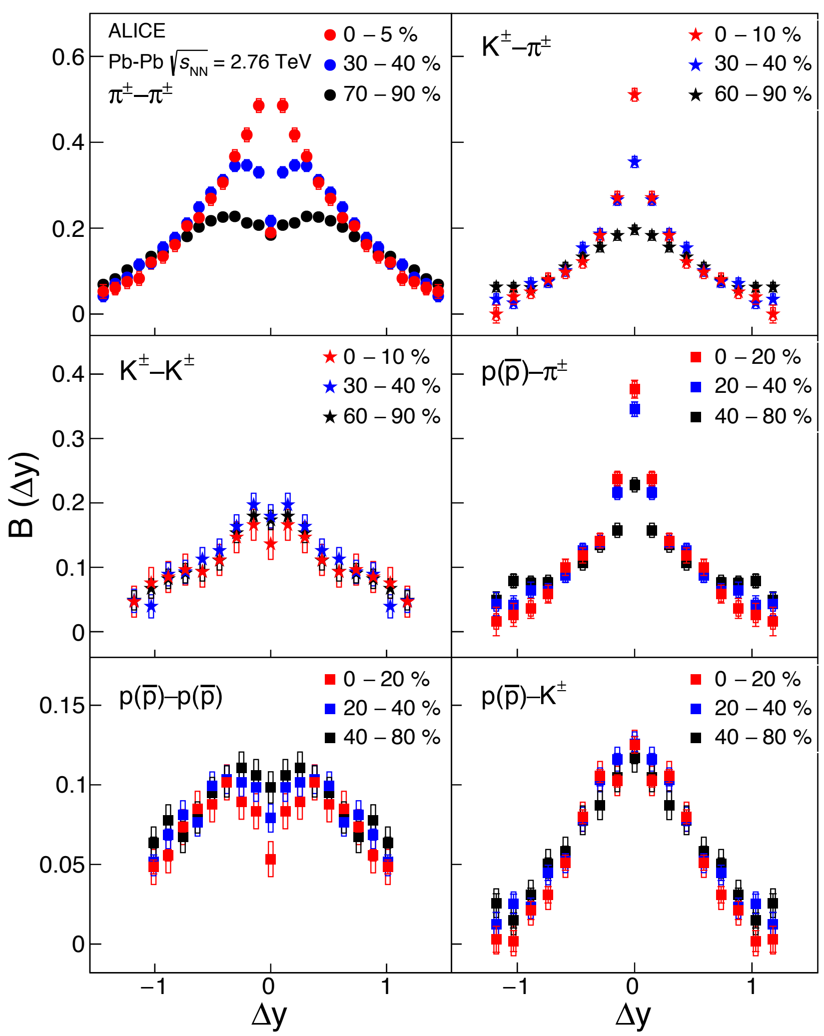
J. Pan, PhD, Wayne State (2019)
ALICE PLB 833 (2022) 137338

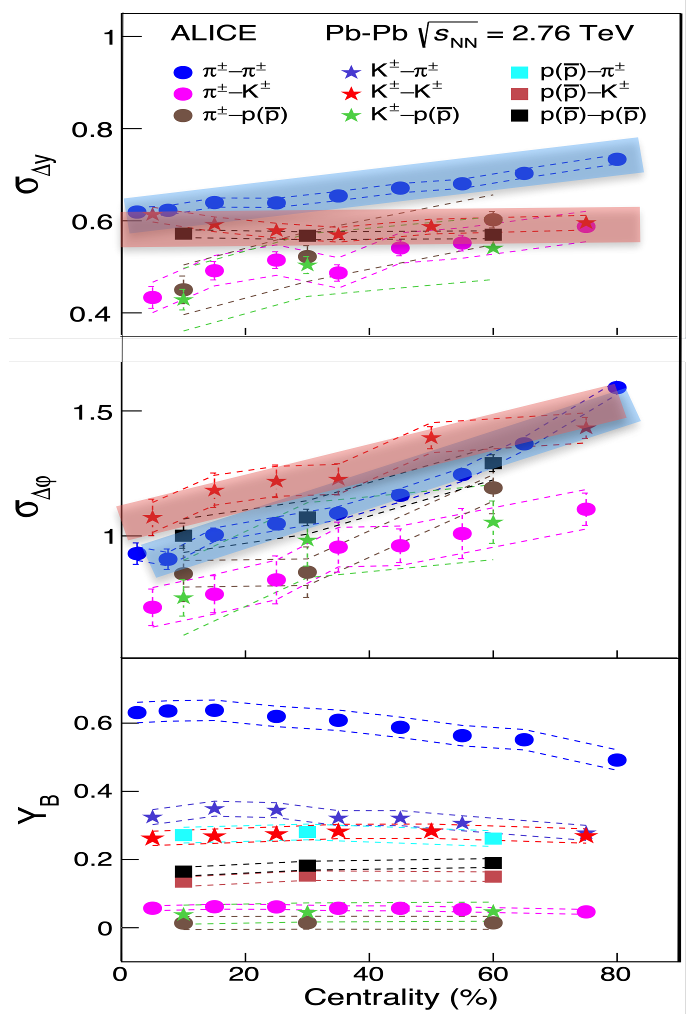
$(\pi, K, p) \otimes (\pi, K, p)$

ALICE



S. Pratt, NPA 698 (2002) 531c: Pions and Kaons BFs to have different collision centrality dependence.





- **1st BF measurement of "full" cross-species matrix** of $\pi^{\pm}$, $K^{\pm}$ and $p(\bar{p})$
  - Differential B(Δy) profile.
  - Better constraints on models.
- $B_{\pi^{\pm}\pi^{\pm}}$ **: clear centrality dependence,**
- $B_{K^{\pm}K^{\pm}}$ **: no centrality dependence,**
- $B_{p\bar{p}}$ and cross-species pairs : moderate centrality dependence.
- Differences in BF shape, magnitude, centrality evolution between different species pairs
- **Different pair production mechanisms/times** for $\pi^{\pm}$ (up/down quark meson), $K^{\pm}$ (strangeness meson) and $p(\bar{p})$ (baryon).

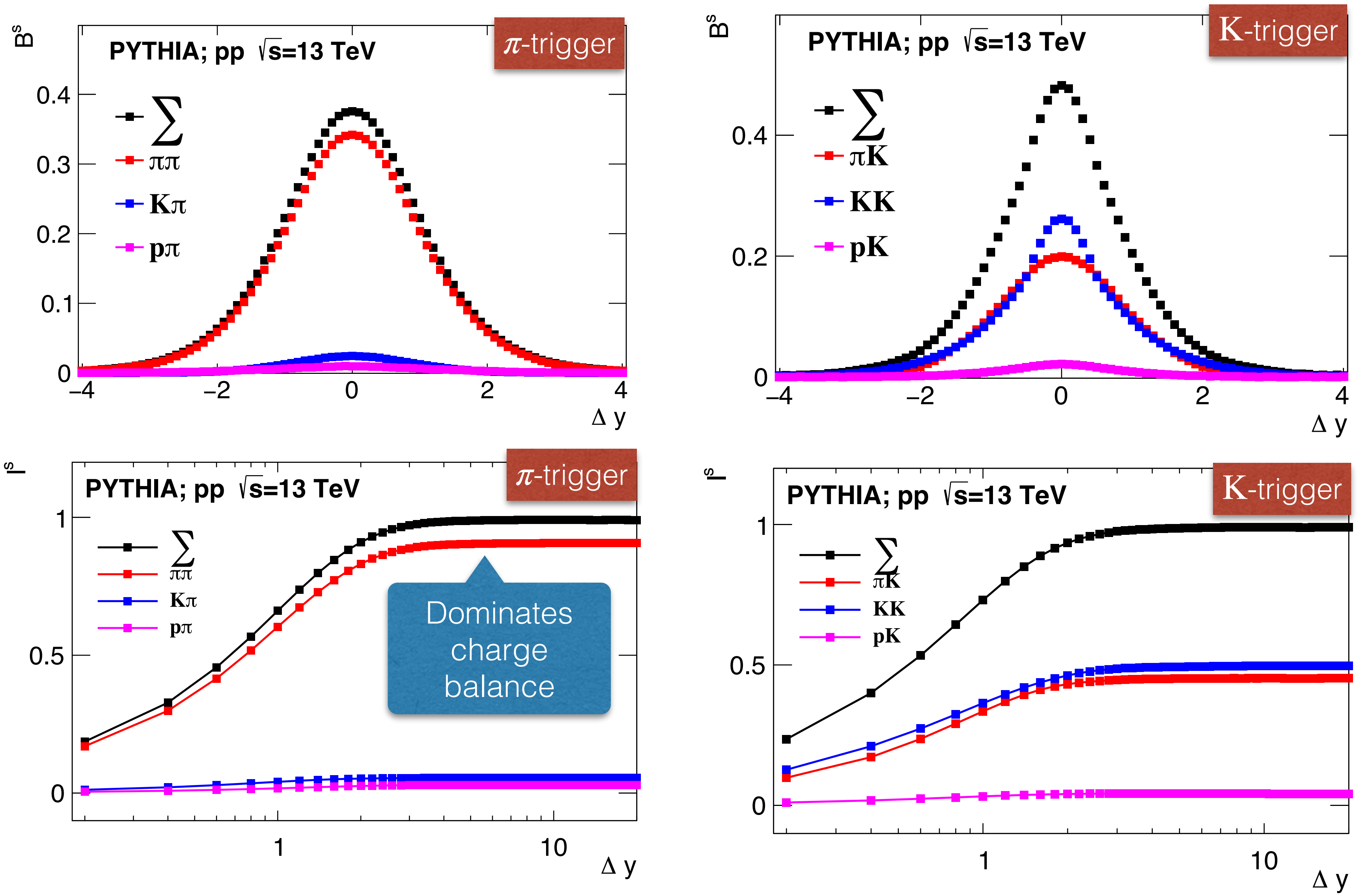
Corroborates/Qualitative Agreement with K BFs by STAR PRC 82 (2010) 024905.

Supports two stage emission scenario (delayed hadronization)
Interpretation requires proper models

CP et al., PRC 109 (2024) 6, 064913

Charge BF w/ Mixed Species: $\pi^{\pm}$, $K^{\pm}$, $p(\bar{p})$

$$B^{\alpha\bar{\beta},s}(\Delta y) = \frac{1}{\langle N_1^{\bar{\beta}} \rangle}\left[C_2^{\alpha\bar{\beta}}(\Delta y) - C_2^{\bar{\alpha}\bar{\beta}}(\Delta y)\right] + \frac{1}{\langle N_1^{\beta} \rangle}\left[C_2^{\bar{\alpha}\beta}(\Delta y) - C_2^{\alpha\beta}(\Delta y)\right]$$



Charge balance shared…

PYTHIA8 Simulations
pp @ $\sqrt{s} = 13$ TeV

$\pi^{\pm}$, $K^{\pm}$, $p(\bar{p})$
pT>0, |y|<10

Charge balancing determined by particle production dynamics and hadron chemistry
Similar work to be done for AA w/ EPOS etc, w/ micro-canonical and charge conservation

CP et al., e-Print: 2403.13007 [hep-ph]

Baryon Balance Functions

Simulations w/ PYTHIA8

pp @ $\sqrt{s} = 2.76, 5.02, 13$ TeV

Species	$c\tau$ (m)	Observation Method
p	long lived	spectrometer
n	$\tau = 877.8$ s	hadronic calorimeter
Λ^0	0.079	$\Lambda^0 \to p + \pi^-$
Σ^-	0.045	$\Sigma^- \to n + \pi^-$
Σ^0	0.022 nm	$\Sigma^0 \to \Lambda^0 + \gamma$
Σ^+	0.024	$\Sigma^+ \to p + \pi^0$
Ξ^-	0.049	$\Xi^- \to \Lambda^0 + \pi^-$
Ξ^0	0.087	$\Xi^0 \to \Lambda^0 + \pi^0$
Ω^-	0.024	$\Omega^- \to \Lambda^0 + K^-$

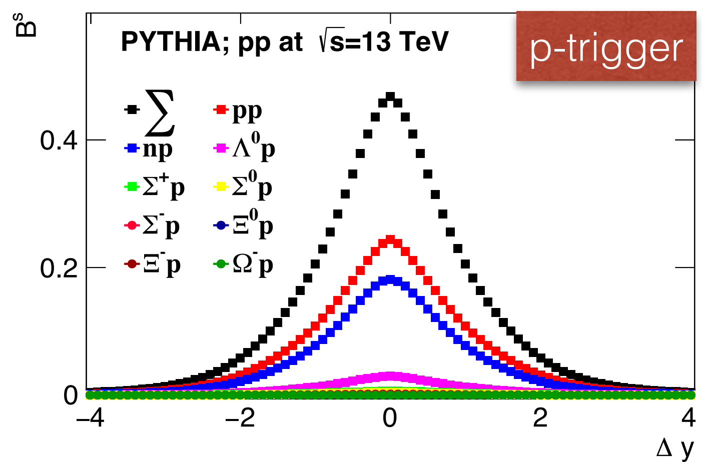
Charge balancing determined by particle production dynamics:

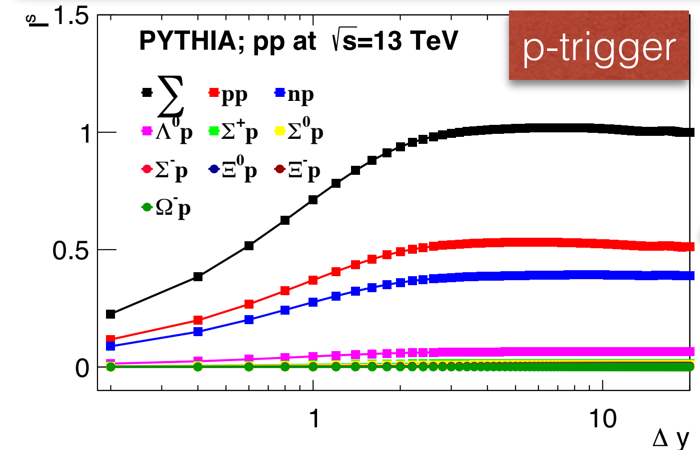
BF & Integral have great potential to constrain models…

In pp & AA collisions

Note: ALICE is working on this analysis. Stay tune for results “soon”.

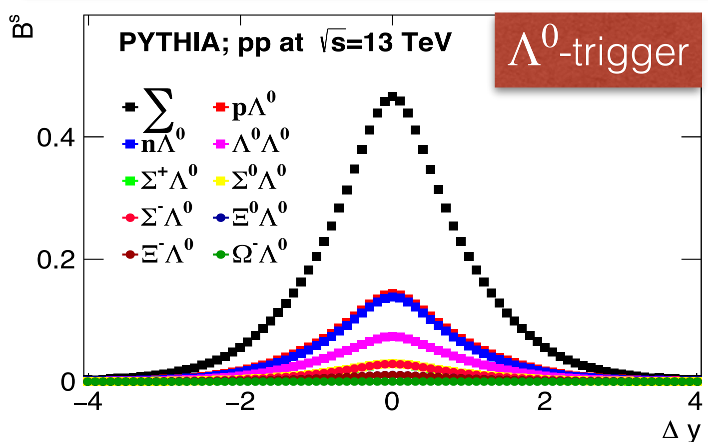
Cumulative Integrals

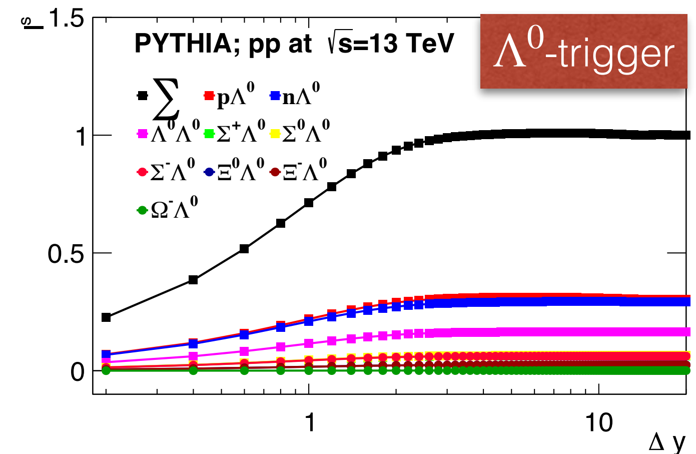




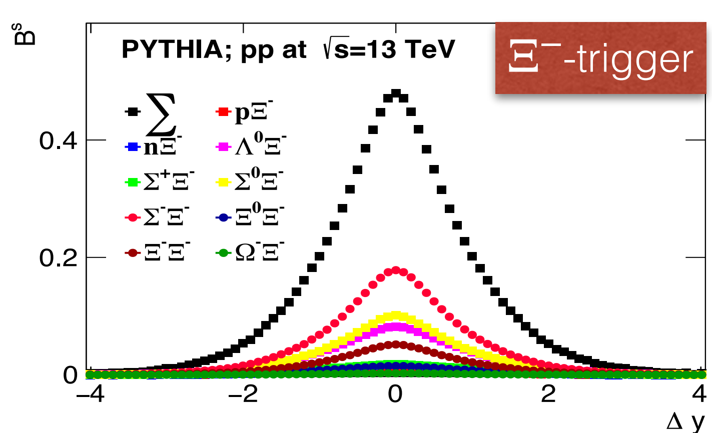
Dominates baryon balance

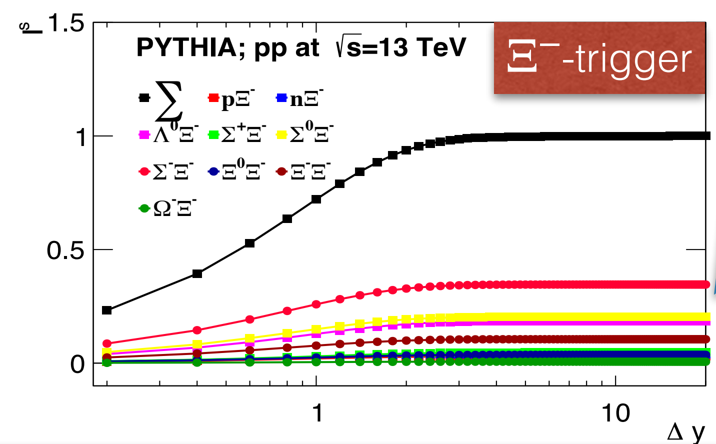
$\bar{\mathrm{p}}$, $\bar{\mathrm{n}}$ balance p





$\bar{\mathrm{p}}$, $\bar{\mathrm{n}}$, $\bar{\Lambda}$ balance Λ





Shared baryon balance

$\bar{\Sigma}^+$, $\bar{\Sigma}^0$, $\bar{\Lambda}^0$ balance Ξ^-

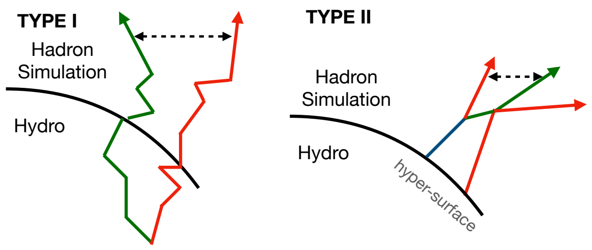
PID Balance Functions

J. Pan, PhD, Wayne State (2019)
ALICE PLB 833 (2022) 137338

Light Quark Diffusivity

Pratt & Plumberg, PRC 104 (2021) 014906

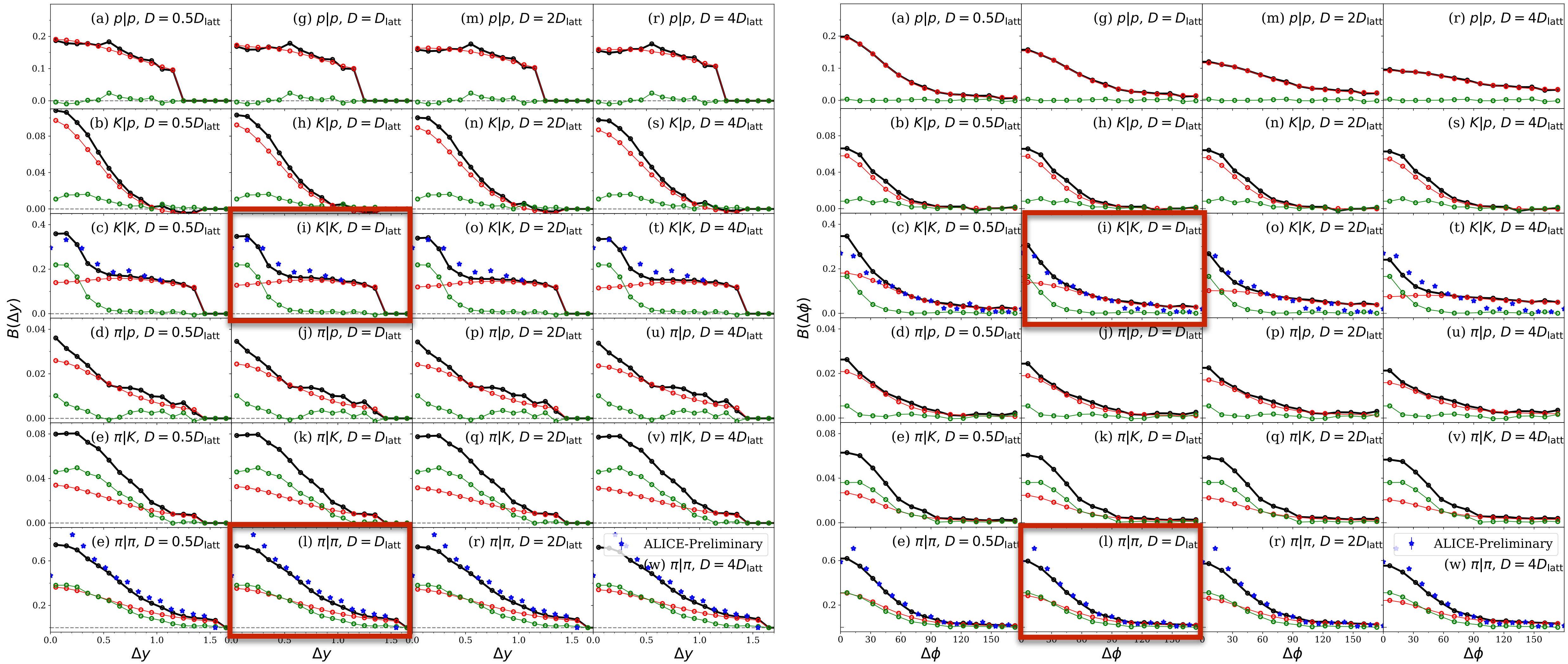
Comparison of theoretical predictions with ALICE data.
QGP evolution with diffusion and lattice susceptibilities, hadronic simulations, resonance cocktail…



$$\sigma_y^2 = 2\sigma_0^2 + 2T_b/M_\perp + 4\beta\ln(\tau_f/\tau_0).$$

$$D = \beta\tau$$ from lattice calculations

Type-I Contributions · Type-II Contributions · Total · ALICE preliminary



ALICE BFs sensitive to light quark diffusivity
ALICE results favor LQCD values!

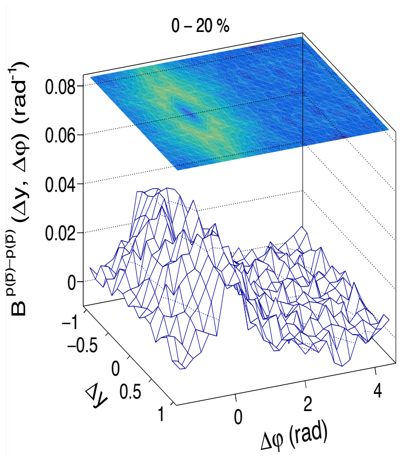
CP et al., 2310.07618 [hep-ex]

Suppressing Hadron (Strong) Decays

Role of decays depends on types of BF considered:

Very few particles decay into a baryon and anti-baryon

$p\bar{p}$

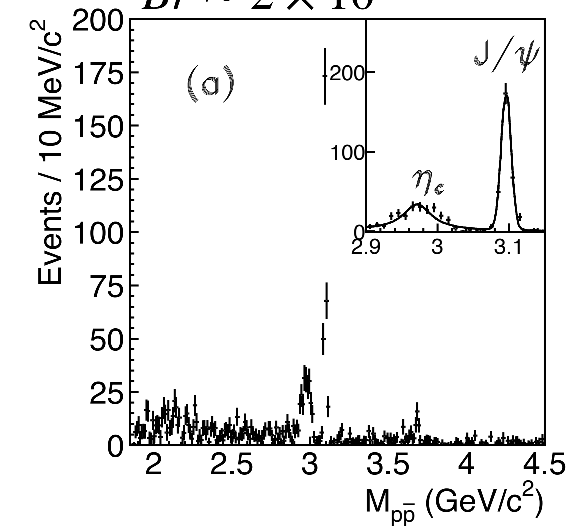


Correlation not from decays!*

*but decays may contribute e.g., $\Delta^+ \to \mathrm{p} + X$

Longitudinally broad
Azimuthally narrow.

$J/\psi \to \mathrm{p} + \bar{\mathrm{p}}$
$Br \sim 2 \times 10^{-3}$



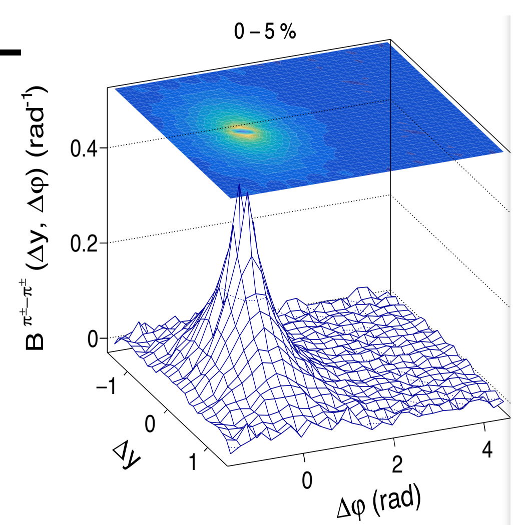
But resonances can decay into (Examples)

+Q & -Q: $\rho^0 \to \pi^+ + \pi^-$

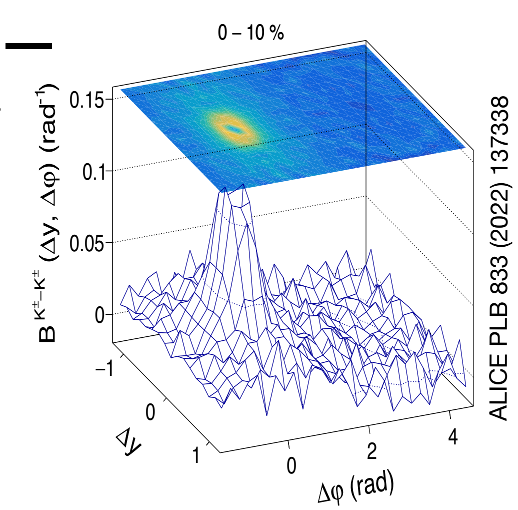
+S & -S: $\phi^0 \to \mathrm{K}^+ + \mathrm{K}^-$

Baryon + Meson: $N^* \to p + \pi^-$

$\pi^+\pi^-$



$\mathrm{K}^+\mathrm{K}^-$



Longitudinally &
Azimuthally narrow.

Can we suppress or eliminate correlations originating from decays?

A New Method

Multi-particle Balance Functions [1]

General Balance Function (Unified)

$$B_2^{+-}(\vec{p}_1,\vec{p}_2) = \frac{C_2^{+-}(\vec{p}_1,\vec{p}_2) - C_2^{--}(\vec{p}_1,\vec{p}_2)}{\langle N^- \rangle}$$

Two particles

Measure Charge Balance while Suppressing Decays and Jets: Use n-cumulants!!!!

$$B_4^{+-}(\vec{p}_1,\ldots,\vec{p}_4) = \frac{1}{6} \times \frac{3C_4^{2+2-} - 4C_4^{1+3-} - C_4^{4-}}{\langle N^-(N^- - 1)\rangle}$$

Four particles

$$B_6^{+-}(\vec{p}_1,\ldots,\vec{p}_6) = \frac{1}{60} \times \frac{10C_6^{3+3-} - 15C_4^{2+4-} + 6C_4^{1+5-} + C_4^{6-}}{\langle N^-(N^- - 1)(N^- - 2)\rangle}$$

Six particles

$$B_8^{+-}(\vec{p}_1,\ldots,\vec{p}_8) = \frac{1}{840} \times \frac{35C_8^{4+4-} - 56C_8^{3+5-} + 25C_8^{2+6-} - 8C_8^{1+7-} + C_8^{8-}}{\langle N^-(N^- - 1)(N^- - 2)(N^- - 3)(N^- - 3)\rangle}$$

Eight particles

$$B_{10}^{+-}(\vec{p}_1,\ldots,\vec{p}_{10}) = \frac{1}{15120} \times \frac{126C_{10}^{5+5-} - 210C_{10}^{4+6-} + 120C_{10}^{3+7-} - 45C_{10}^{2+8-} + 10C_{10}^{1+9-} - C_{10}^{10-}}{\langle N^-(N^- - 1)(N^- - 2)(N^- - 3)(N^- - 3)(N^- - 4)\rangle}$$

Ten particles

- Mixed differential/integral correlation cumulants
- **Suppress two/three body correlations**.
- **Use rapidity gaps to suppress jet contributions**

[1] Multi-particle integral and differential correlation functions, CP et al., PRC 109 (2024) 4, 044904

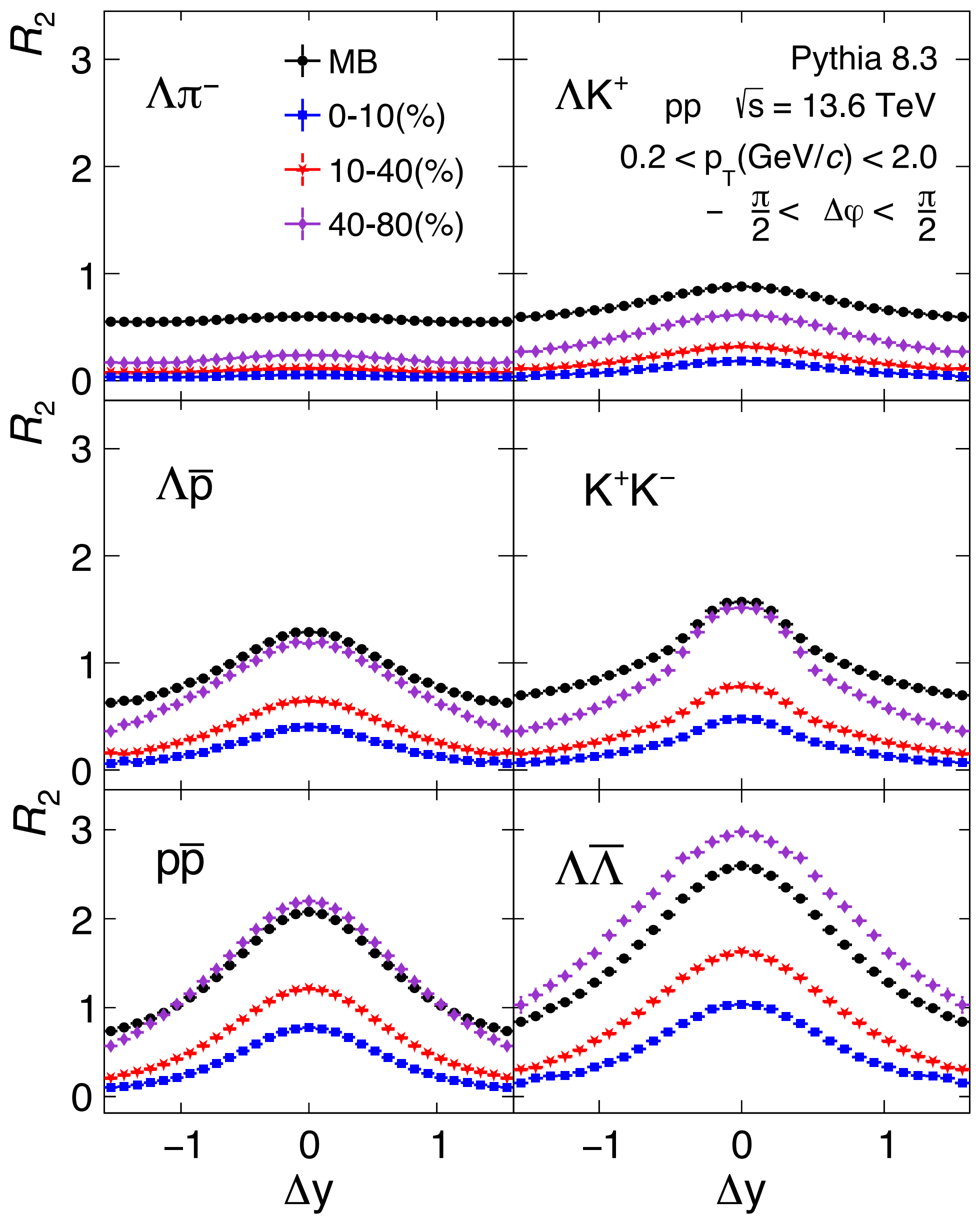
[2] Also see: Calculating n-Point Charge Correlations in Evolving Systems, S. Pratt, PRC 101 (2020) 1, 014914

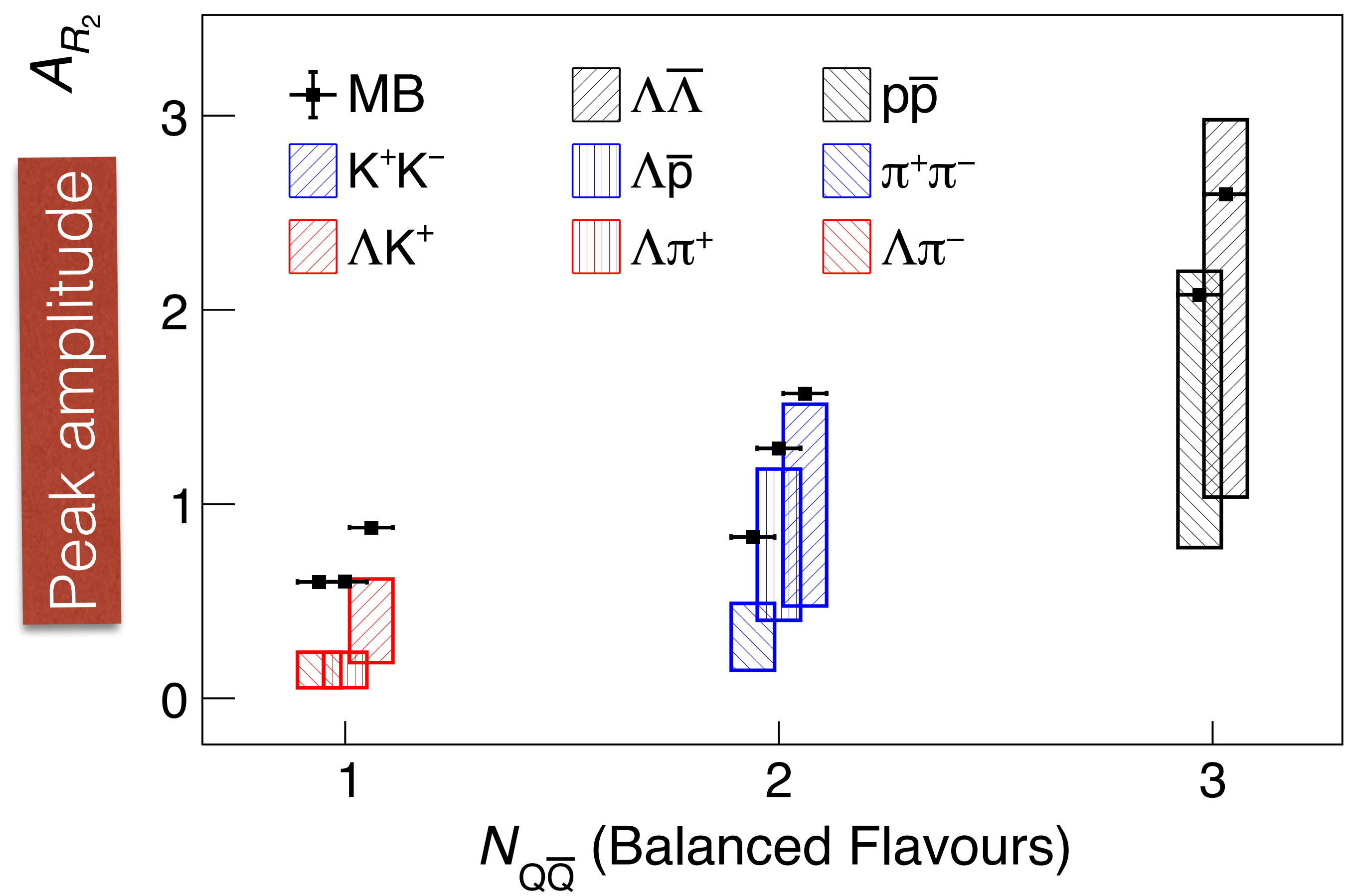
Yash Patley, et al., e-Print: 2408.09923 [hep-ph]

Feasibility Study Based on PYTHIA

Analysis/Plots by Yash Patley, IIT Mumbai

- Flavor Balancing as an extension of Charge Balance
- Strength and shape of correlations depends on the number of balance flavors.





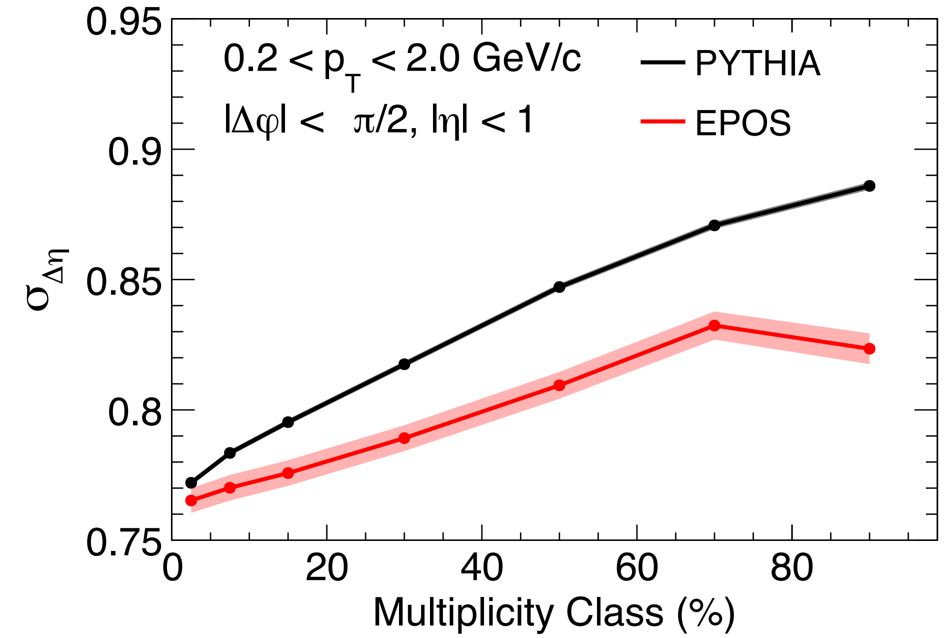
Balance Function Modeling

Analysis/Plots by Alex Manea, ISS, Romania

Success and Challenges

- As for flow and jet studies, a plurality of models (and modelers) is needed to achieve a reliable interpretation of balance functions.
- Minimum requirement:
    - Model particle production while enforcing “local” energy/moment and quantum number conservation.
    - Ready accomplisher in models such as PYTHIA and HERWIG
    - More difficult with models involving a hydrodynamic phase and particlization.
        - Implemented in the latest version of EPOS 4.
- Full description: include flavor currents

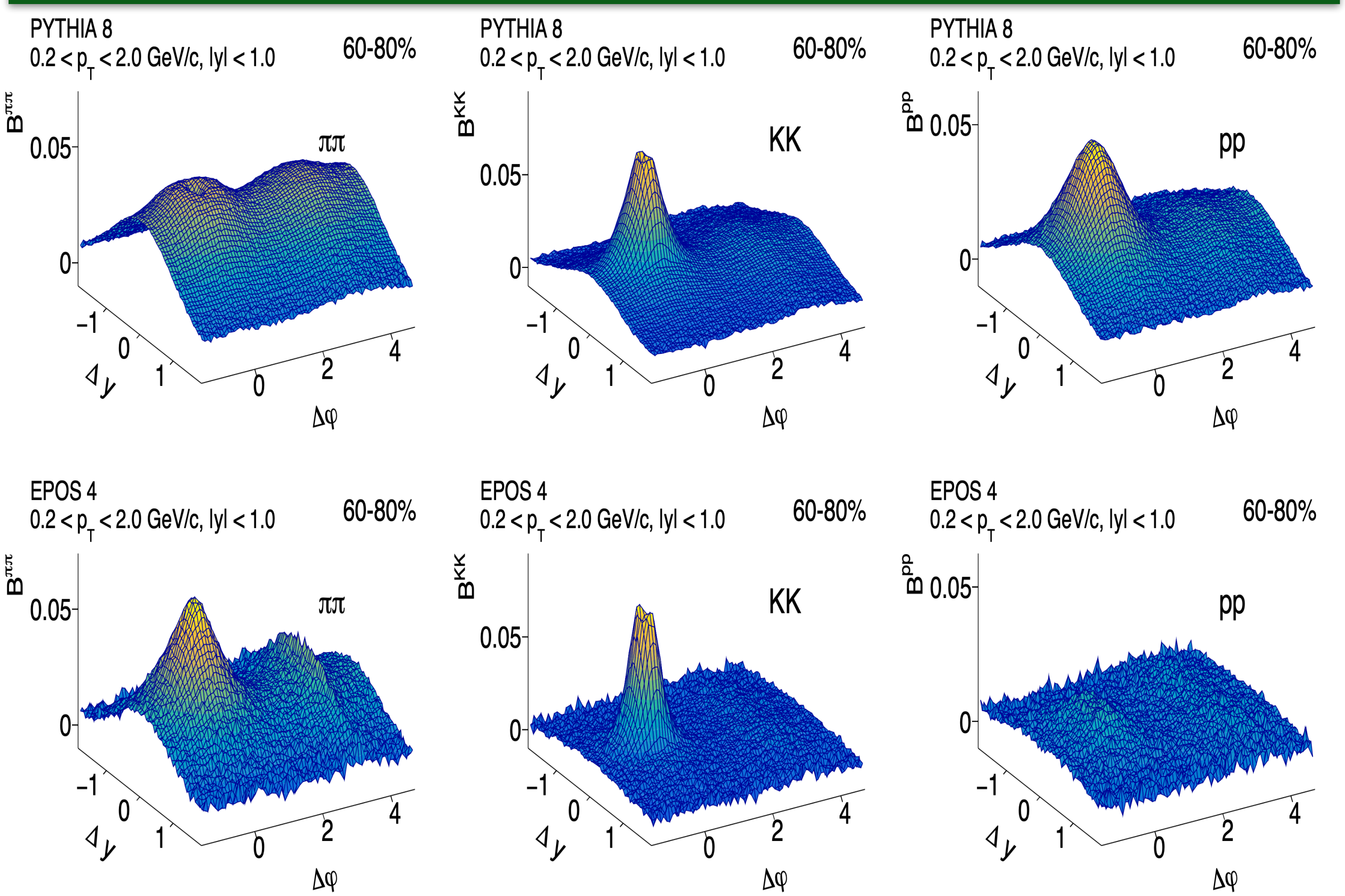
Near-side Peak Width (Azimuth)



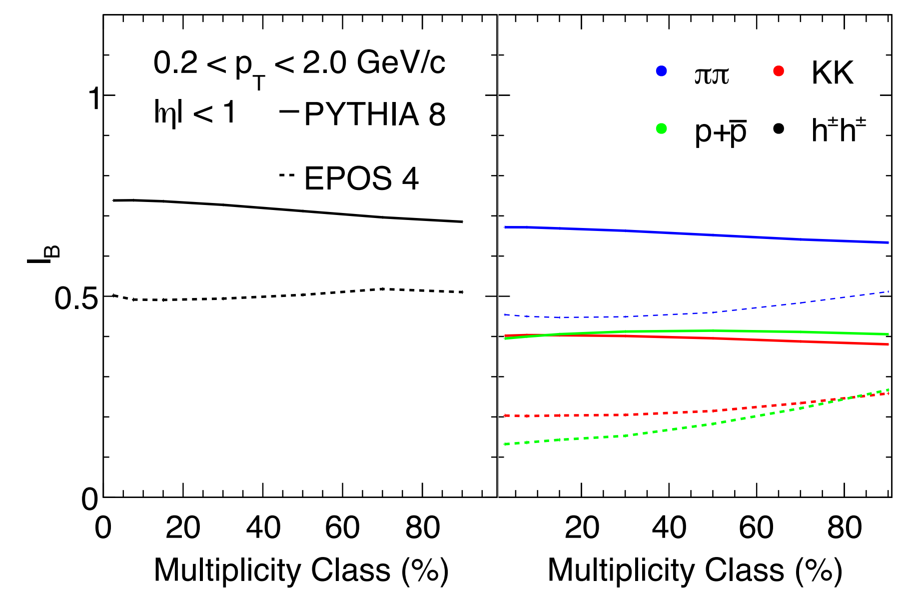
p-p collisions \sqrt{s}=13 TeV (Balance Functions)

PYHTHIA

EPOS 4



Balance Function Integral vs Multiplicity



Summary

- **Pratt et al have created the subfield of Balance Functions (BF)**
  - Initially to probe delayed hadronization/Two-stage Quark Production
- But BFs can also …
  - Test thermalization and production models, **Provide support for net quantum number fluctuation analyses** (QGP Susceptibilities).
  - Understanding baryon production and transport (Junctions?)
  - Measuring quark anti-quark correlation length in A-A vs. p-p
  - Testing charm-charm recombination models
  - Determine the onset of thermalization by contrasting $c\bar{c}$ and $b\bar{b}$ BFs in A-A vs. p-p.
  - Better characterization of the expansion dynamics in A-A collisions.
  - And more…

Thank you Scott!!!! Happy Retirement!!!

Additional Materials

Identified BFs w/ PYTHIA8

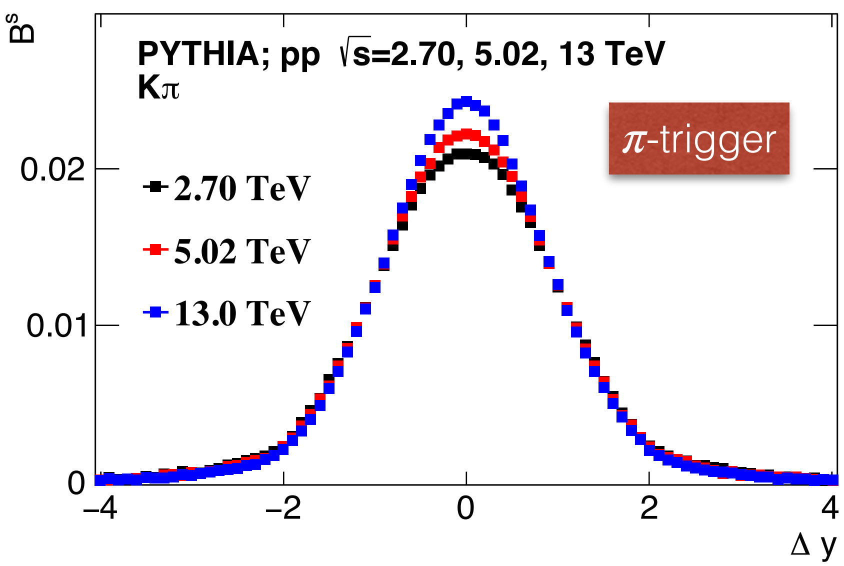
CP et al., e-Print: 2403.13007 [hep-ph]

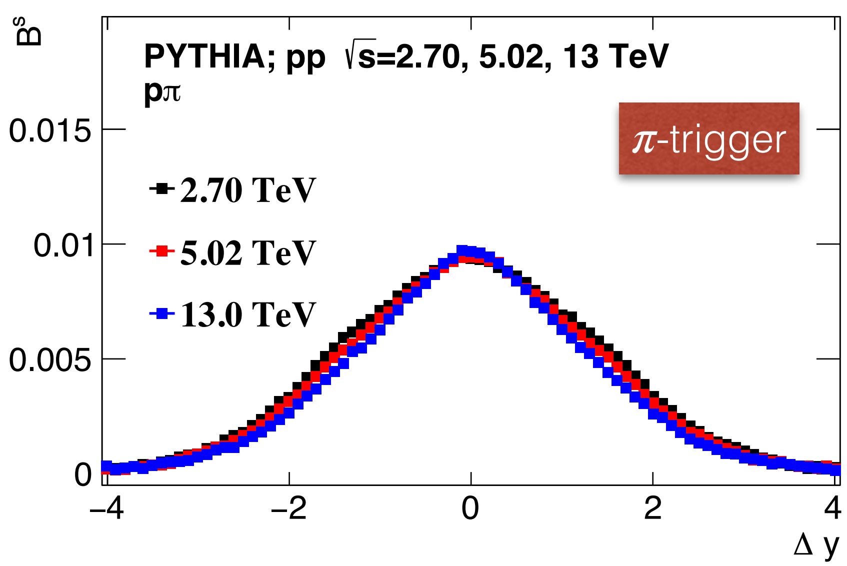
Charge BF w/ Mixed Species: $\pi^{\pm}$, $K^{\pm}$, $p(\bar{p})$

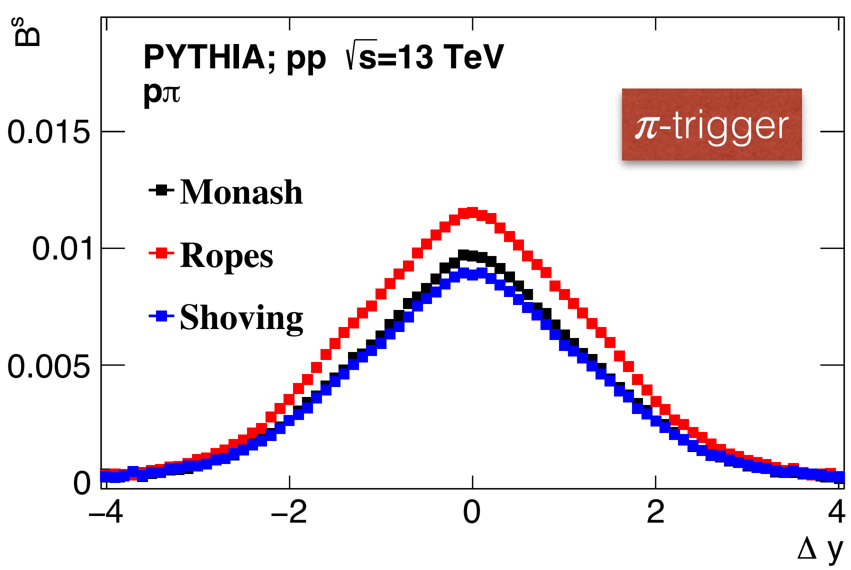
Simulations w/ PYTHIA8

pp @ $\sqrt{s} = 2.76, 5.02, 13$ TeV

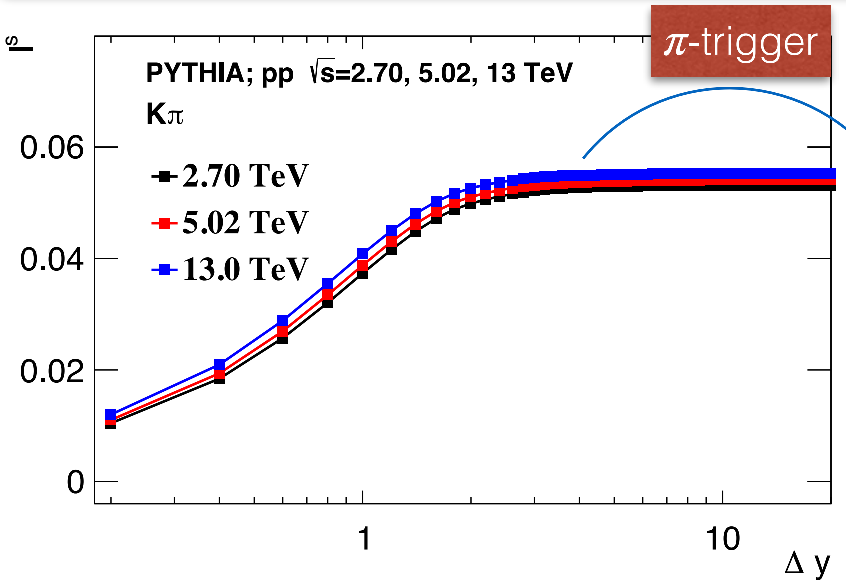
$\pi^{\pm}$, $K^{\pm}$, $p(\bar{p})$ pT>0, |y|<10

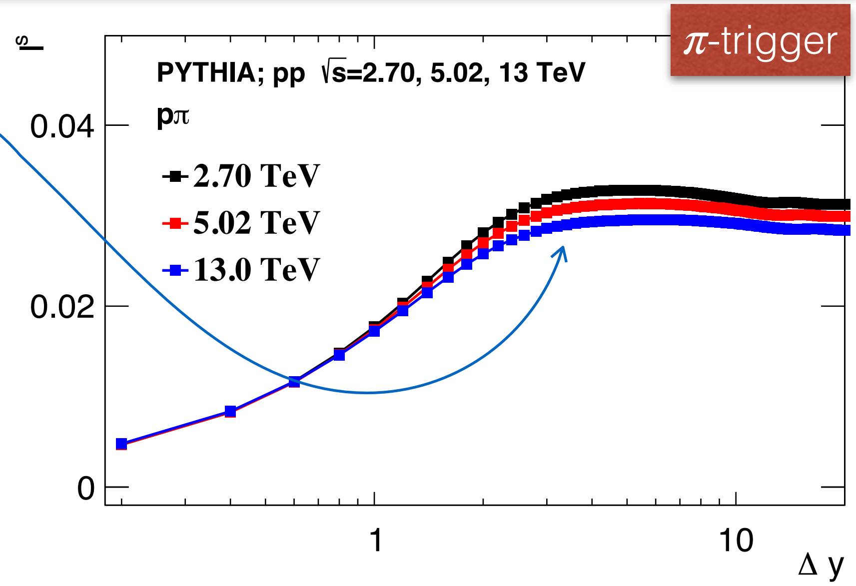






Cumulative Integrals





Charge balancing determined by particle production mechanisms:

- $\sqrt{s}$ dependence
- Model dependence
- BF have great potential to further constrain models in pp & AA collisions

Charm/Bottom vs. Light Flavor Balance Functions

- QCD lagrangian conserves flavor
- May then use flavor balancing in addition to charge, strangeness, and baryon balance functions.
- Charm&Bottom quarks are heavy
  - Require large $\sqrt{s}$ processes to be produced efficiently.
  - Charm & Bottom production thus "limited" to early collision times - event at LHC energy scale.
- Use BF to examine the evolution of $c\bar{c}$, $b\bar{b}$ production in pp, p-A, and A-A collisions.
  - Expect relatively smaller changes of BF vs. system size compared to light flavors.
  - Might expect a quantitative hierarchy in the width and strength of BFs from $b\bar{b}$ to $c\bar{c}$ to $s\bar{s}$ to $u\bar{u}$, $d\bar{d}$

C.P., Phys.Rev.C 100 (2019) 3, 034905

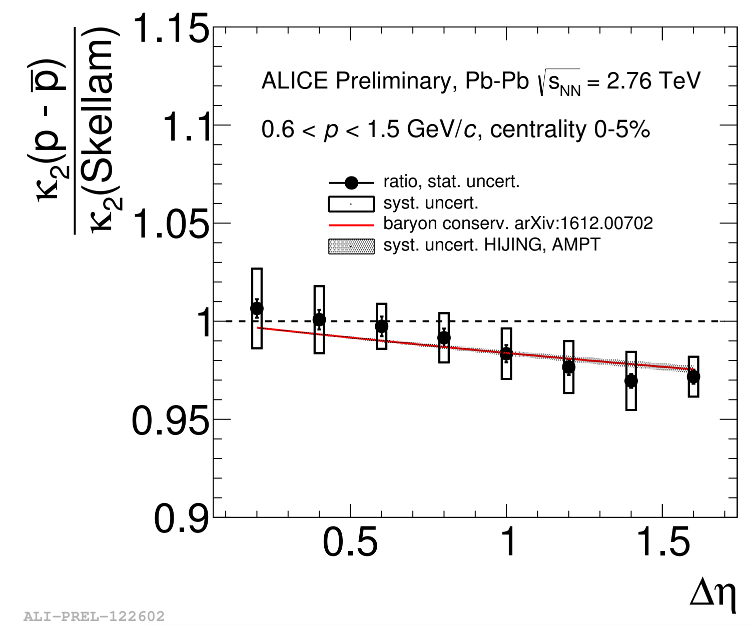
Baryon Number Conservation

Second order cumulant : $\kappa_2(\Delta N_p) = F_1^{p} + F_1^{\bar{p}} + F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}$

Poisson limit (Skellam) : $\kappa_2^{Skellam}(\Delta N_p) = F_1^{p} + F_1^{\bar{p}} = \langle N_p \rangle + \langle N_{\bar{p}} \rangle$

Ratio of $\kappa_2(\Delta N_p)$ to Skellam :

$$r_{\Delta N_p} \equiv \frac{\kappa_2(\Delta N_p)}{\kappa_2^{\text{Skellam}}(\Delta N_p)} = 1 + \frac{F_2^{p,p} + F_2^{\bar{p},\bar{p}} - 2F_2^{p,\bar{p}}}{F_1^{p} + F_1^{\bar{p}}}.$$



LHC: $\langle N_p \rangle \approx \langle N_{\bar{p}} \rangle$:

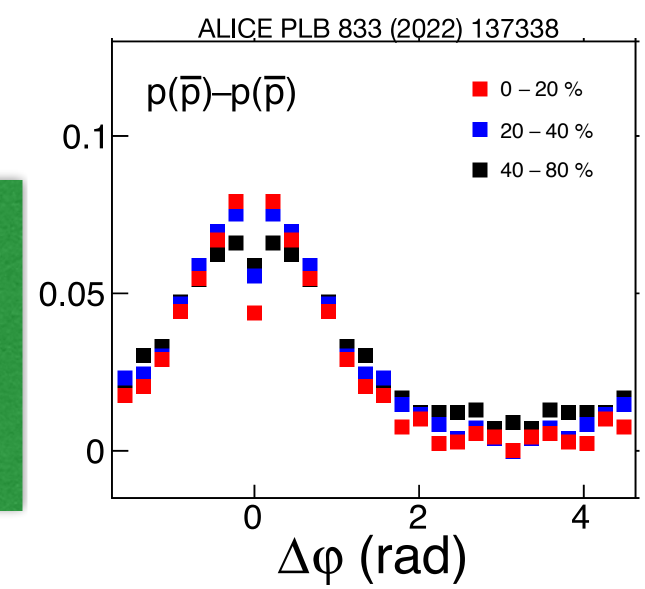
$$r_{\Delta N_p} = 1 + \frac{F_1^{p}}{2}\left[R_2^{p,p} + R_2^{\bar{p},\bar{p}} - 2R_2^{p,\bar{p}}\right],$$

Proportional to Integral of Balance Function

Consequently :

$$r_{\Delta N_p} = 1 + \frac{1}{4}\frac{dN_T}{d\eta}\Delta\eta\,\nu_{\text{dyn}}^{p,\bar{p}},$$

(Local) Baryon Number conservation



Strong correlations exist: non Poisson behavior obtained from ν_{dyn} vs. $\Delta\eta$...

Connection to QGP Susceptibilities??

CP et al., Phys.Rev.C 109 (2024) 4, 044904

n-Particle BFs vs. Net Charge Cumulants

RHIC BES: **Search for critical point**…
LHC/ALICE: **Study of QGP Susceptibilities**

$$\kappa_2^Q = F_1^+ + F_1^- + \underbrace{F_2^{++} - 2F_2^{+-} + F_2^{--}}_{B_2^{+-}},$$

$$\kappa_4^Q = F_1^+ + F_1^- + \cdots + \underbrace{F_4^{4+} - 4F_4^{3+1-} + 6F_4^{2+2-} - 4F_4^{1+3-} + F_4^{4-}}_{B_4^{+-}}$$

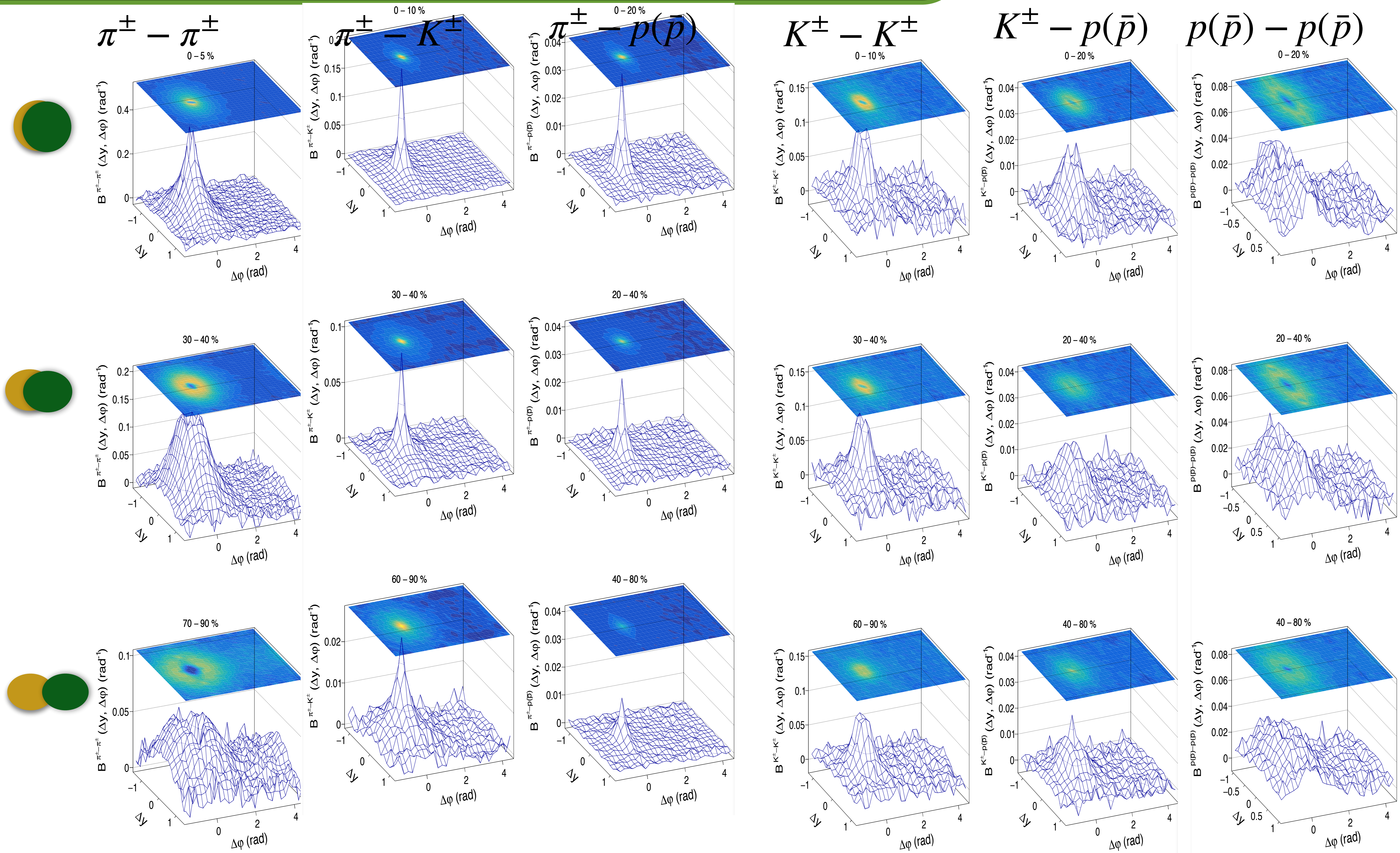
$$\kappa_6^Q = F_1^+ + F_1^- + \cdots + \underbrace{F_6^{6+} - 6F_6^{5+1-} + 15F_6^{4+2-} - 20F_6^{3+3-} + 15F_6^{2+4-} - 6F_6^{1+5-} + F_6^{6-}}_{B_6^{+-}}$$

Order "n" Net Charge Cumulants determined by order "n" balance functions!
What is the role of collision dynamics?
What is the role of susceptibilities?

General Balance Functions

J. Pan, PhD, Wayne State (2019)
ALICE PLB 833 (2022) 137338

$(\pi, K, p) \otimes (\pi, K, p)$ Pb+Pb @ 2.76 TeV



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