Scaling Analysis of Proton Cumulants and the QCD Critical Point

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w/ Paul Sorensen

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arxiv:2405.10278

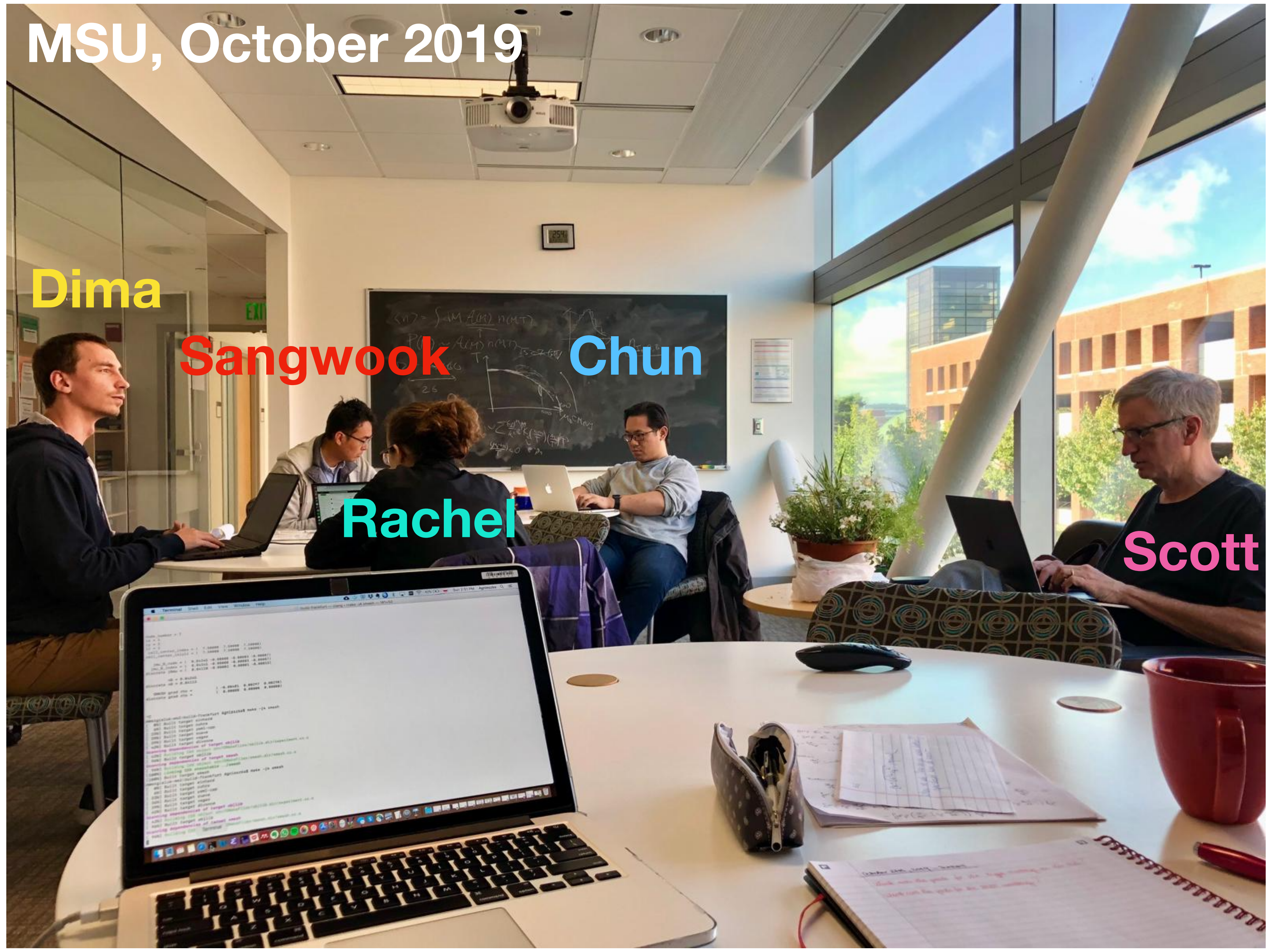
What does this have to do with Scott Pratt? At first sight, not much...

Beam Energy Scan Theory collaboration: BEST COLLABORATION

"... will construct and provide a theoretical framework for interpreting the results from the ongoing BES program at RHIC..."

— 3D hydrodynamics (MUSIC)
— particlization: 3 different choices of samplers (microcanonical, iSS w/ out-of-equilibrium corrections, MSUsampler)
— hadronic afterburner (SMASH) with potentials (VDF)

Task: create a unified framework to take a hydro event and run different samplers and the afterburner
Who: the BEST Interfaces group



- the BEST Interfaces group continued to meet on Zoom, weekly, for 4 years
- extremely helpful for early-career researchers
- beyond BEST, 1 paper as a result of the meetings (arXiv:2210.03877)

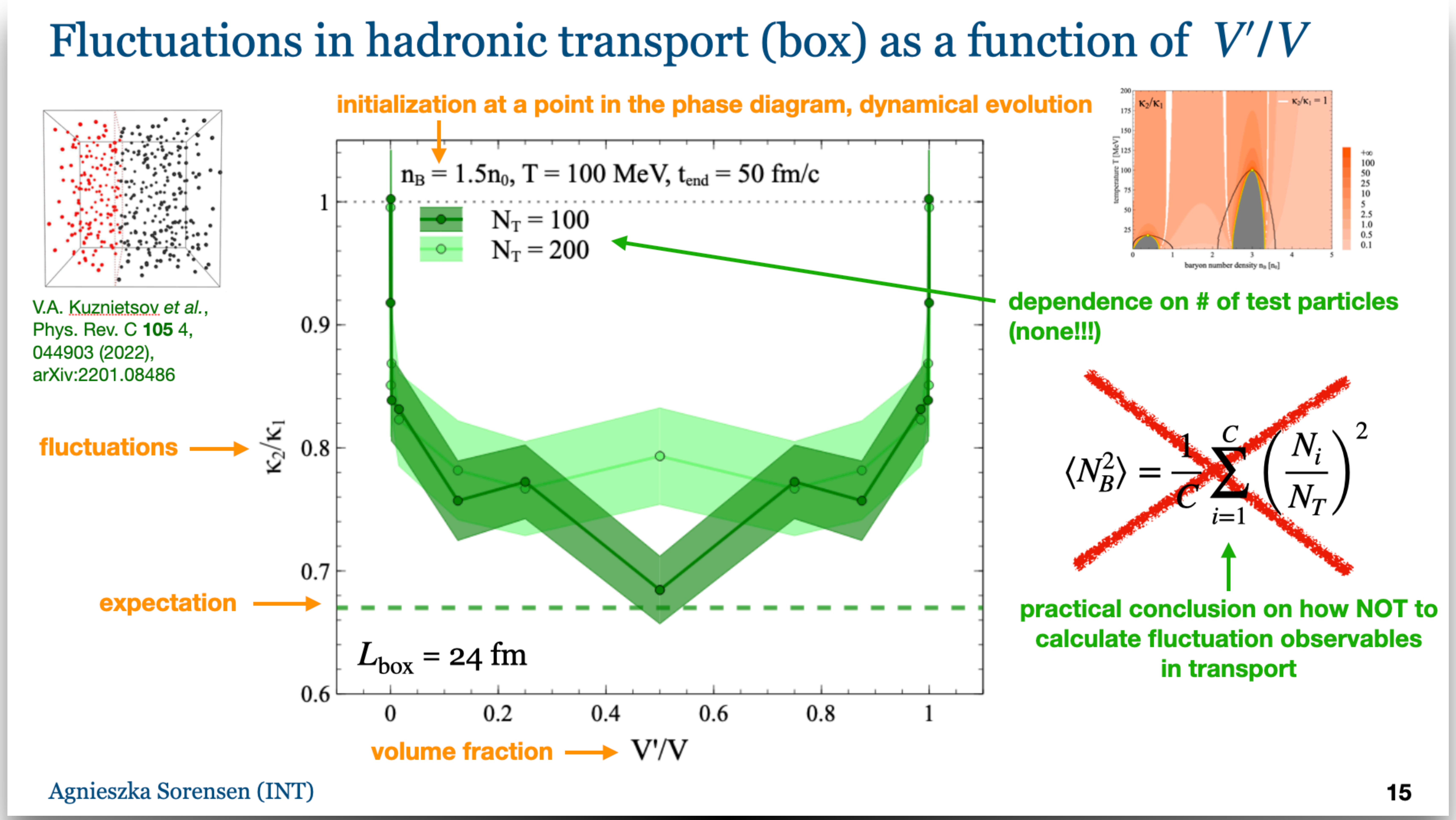
From this you can infer two things:
1) Scott really likes to meet, if needed online, and chat about physics (BEST Interfaces, HBT Camp, ...)
2) Recent upgrade from BEST Interfaces to HBT Camp includes
   - improved documentation of the activities of the group (Maria!)
   - doubling the rate for writing papers together (arXiv:2410.13983)

Scott is a community builder and an enthusiastic mentor

What does this have to do with Scott Pratt?

At a time when I did not obtain much direction or encouragement, Scott was a constant positive voice

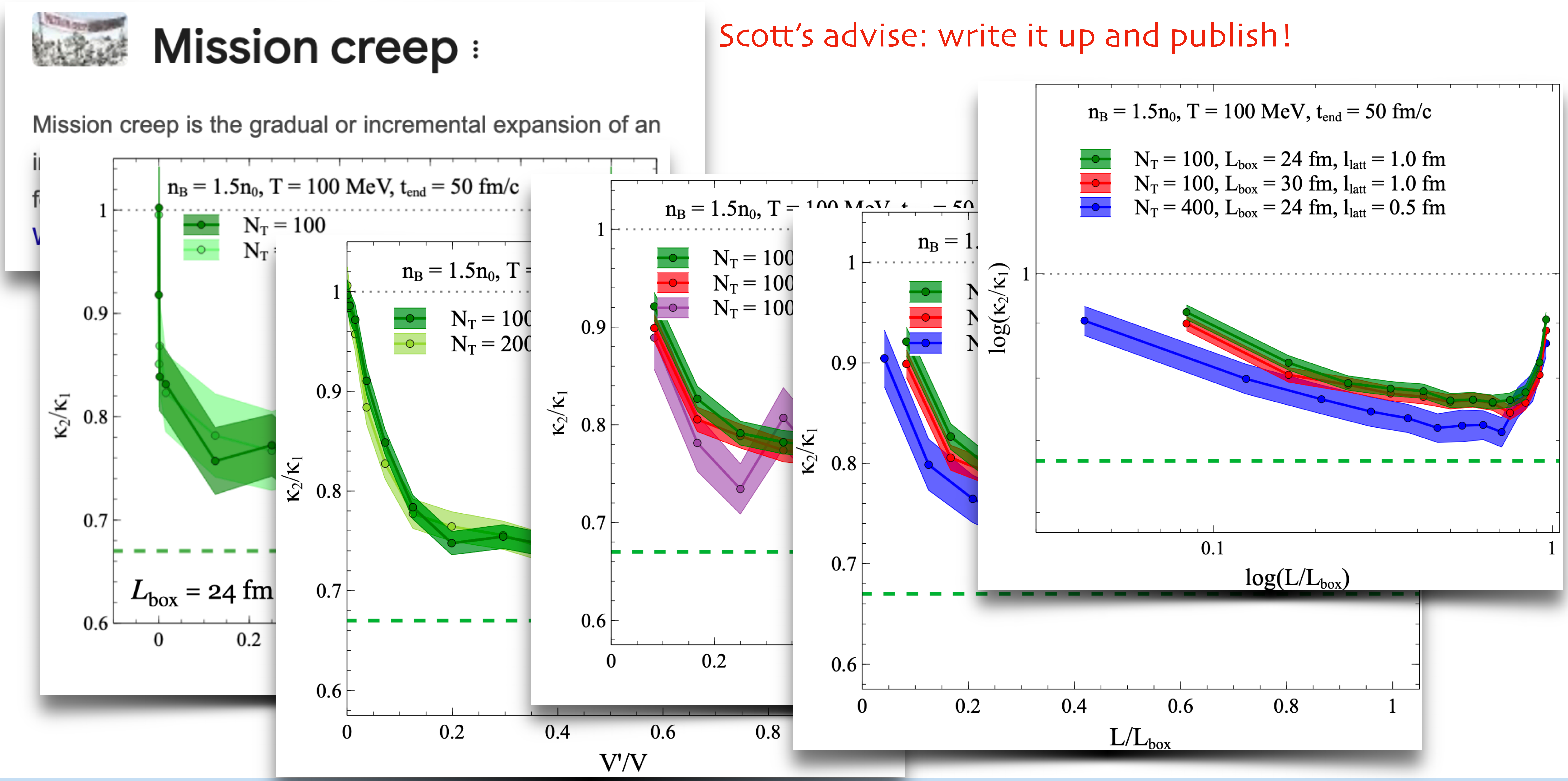
Support beyond just encouragement: invited talk at WPCF 2022 in East Lansing, MI



- somewhat niche subject, but important for BES
- encouragement from Scott
- at WPCF 2022, Scott warns me of mission creep

What does this have to do with Scott Pratt?

Scott's advise: write it up and publish!



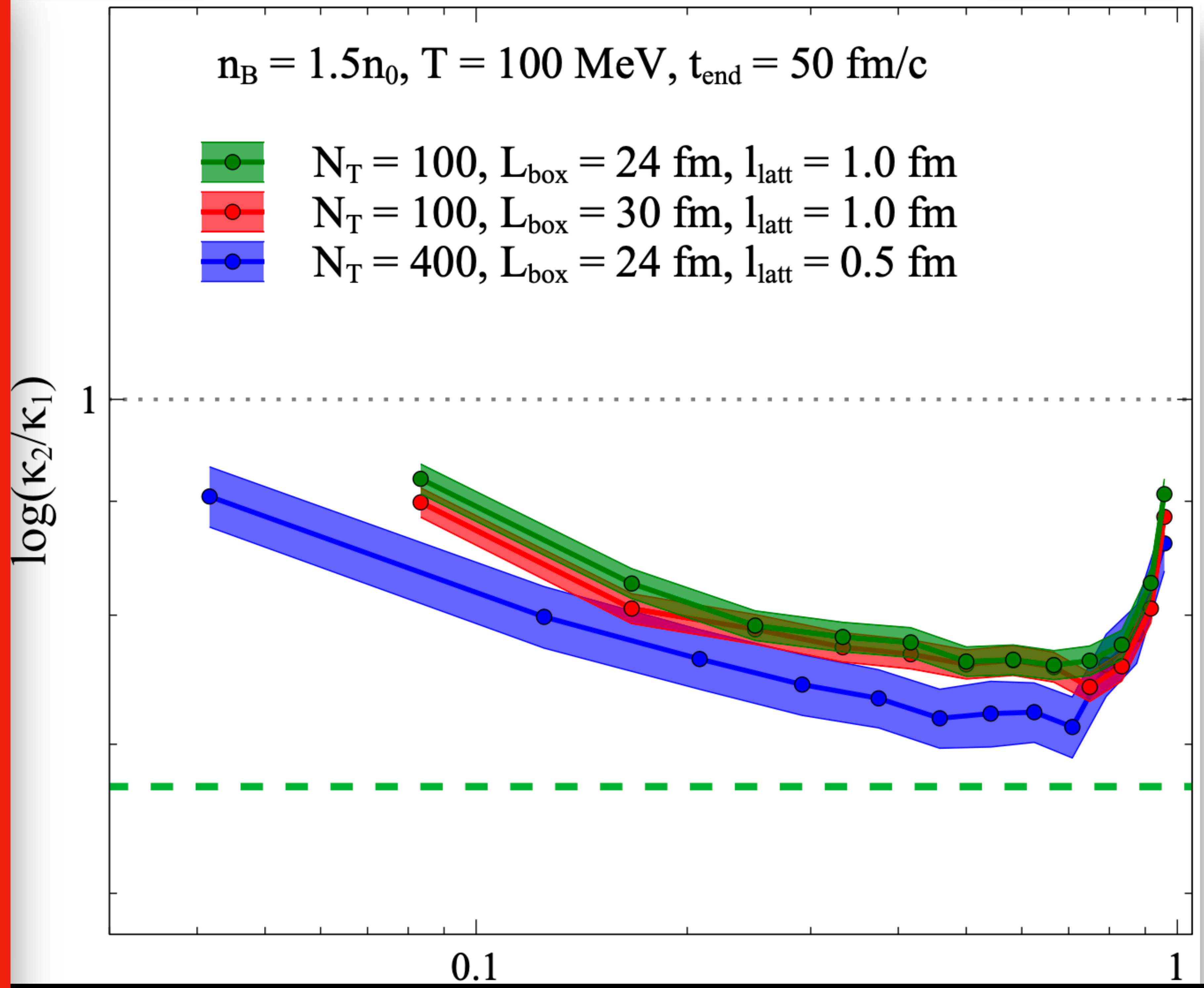
What does this have to do with Scott Pratt?

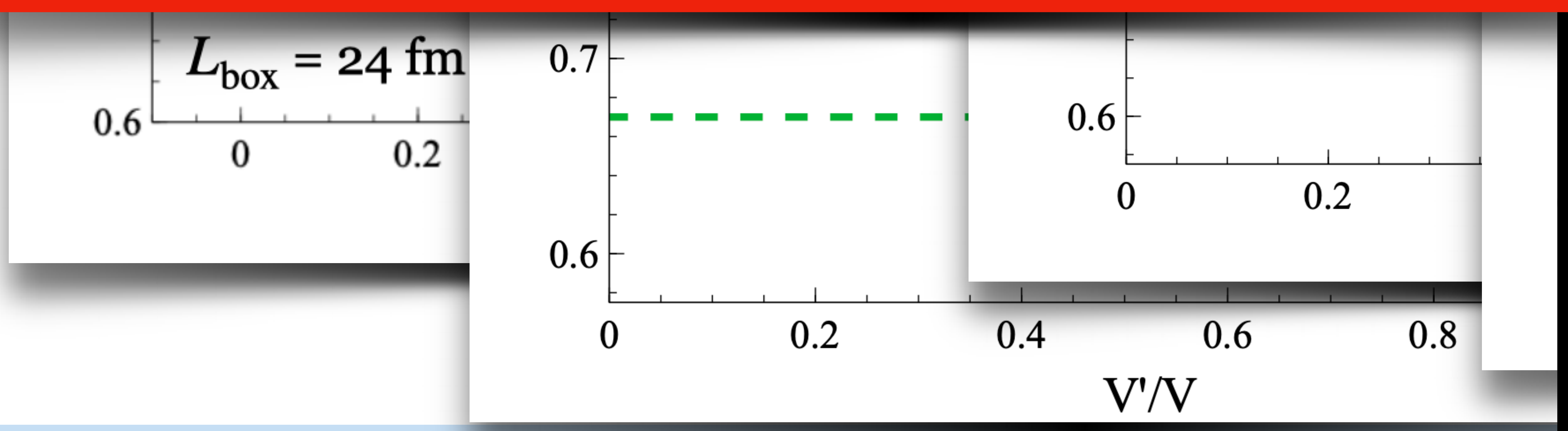
Mission creep

Scott's advise: write it up and publish!

The following research is a result of completely ignoring what other people think I should do

and that is also something you can learn from Scott :-)





- two scales: macro & micro
- I discuss this with Paul Sorensen: finite-size scaling
- this is quite far from the CP!
- can we see the same in experimental data?

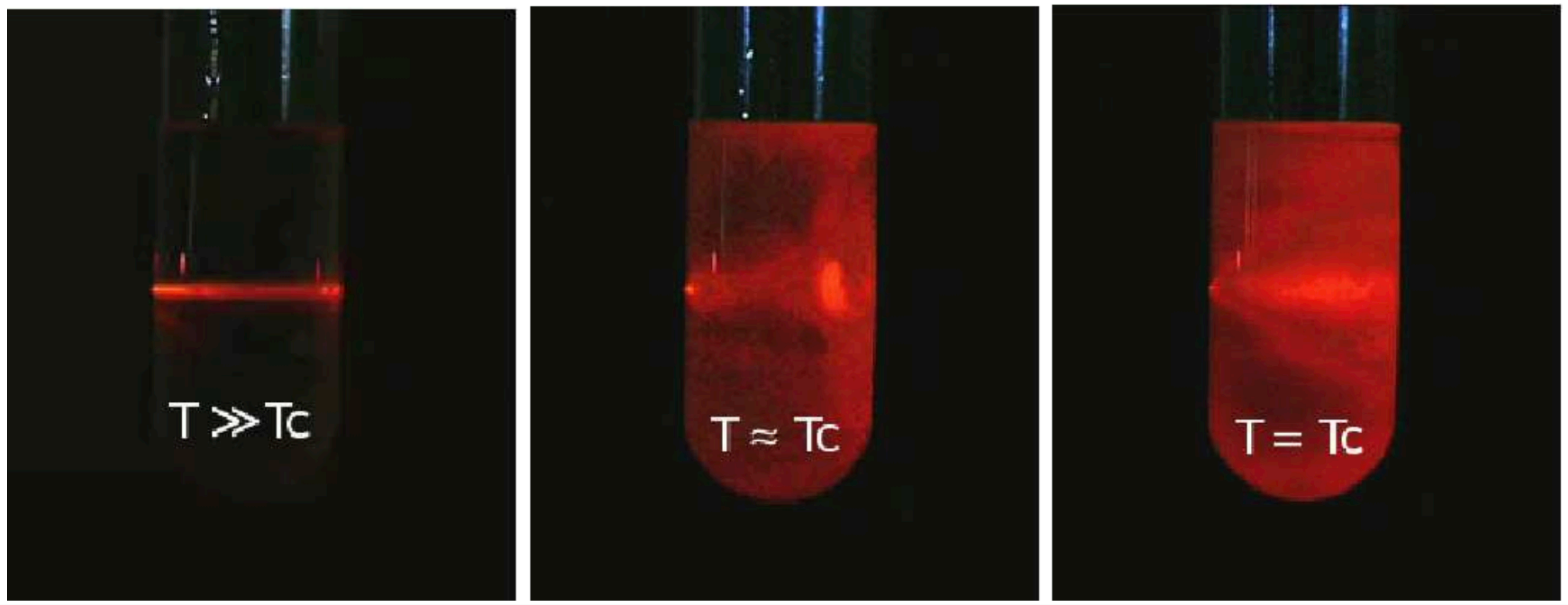
Introduction

Behavior near a critical point

- Critical point (CP):
  a single point in the phase diagram where change from an ordered to disordered phase occurs

- The endpoint of a 1st order phase transition

  As systems approach the CP, latent heat decreases
  $\Rightarrow$ it costs little energy for components of one phase to form a local "bubble" of the other phase
  $\Rightarrow$ as CP is approached, correlation length ξ increases = large fluctuations (large bubbles)
  $\Rightarrow$ critical opalescence phenomenon:
  $\rightarrow$ "bubbles" grow to sizes comparable with visible light wavelengths ($\xi \approx \lambda$)
  $\rightarrow$ light can be scattered and a translucent system becomes cloudy (like fog)
  $\Rightarrow$ at CP, correlation length formally diverges;
  system experiences correlations of all sizes
  (proof: critical opalescence in methanol+cyclohexane persists at CP where ξ ~ 1 cm)



Universal behavior

Near CP:

$$c_\infty(t,0) \sim |t|^{-\alpha}$$

$$\tilde{n}_\infty(t,0) \sim (-t)^{\beta}$$

$$\tilde{n}_\infty(0,m) \sim m^{\frac{1}{\delta}}$$

$$\chi_\infty(t,0) \sim |t|^{-\gamma}$$

$$\xi_\infty(t,0) \sim |t|^{-\nu}$$

$$\xi_\infty(0,m) \sim |m|^{-\nu_c}$$

$$t \equiv \frac{T - T_c}{T_c}$$

$$m \equiv \frac{\mu - \mu_c}{\mu_c}$$

For a thermodynamic quantity X: $$X_\infty(t) \sim |t|^{-\sigma} \sim \left[\xi_\infty(t)\right]^{\frac{\sigma}{\nu}}$$

Scaling is not unique to critical phenomena, e.g., Kepler's third law!
The orbital period of a planet scales as the cube of the semi-major axis of its orbit:

$$P^2 = a^3$$

The important question for scaling is: what is the scale relevant to the problem?

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CP: infinite volume concept

In real world ξ does not go to infinity = thermodynamic functions do not exhibit singularities

ξ is bound by the size of the system L $\Rightarrow$ $X_L(t_L) \sim L^{\frac{\sigma}{\nu}}$

$$\Rightarrow \quad X_L(t_L) = L^{\frac{\sigma}{\nu}}\phi(t, L) = L^{\frac{\sigma}{\nu}}\phi(tL^{\frac{1}{\nu}})$$

$$\Rightarrow \quad X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

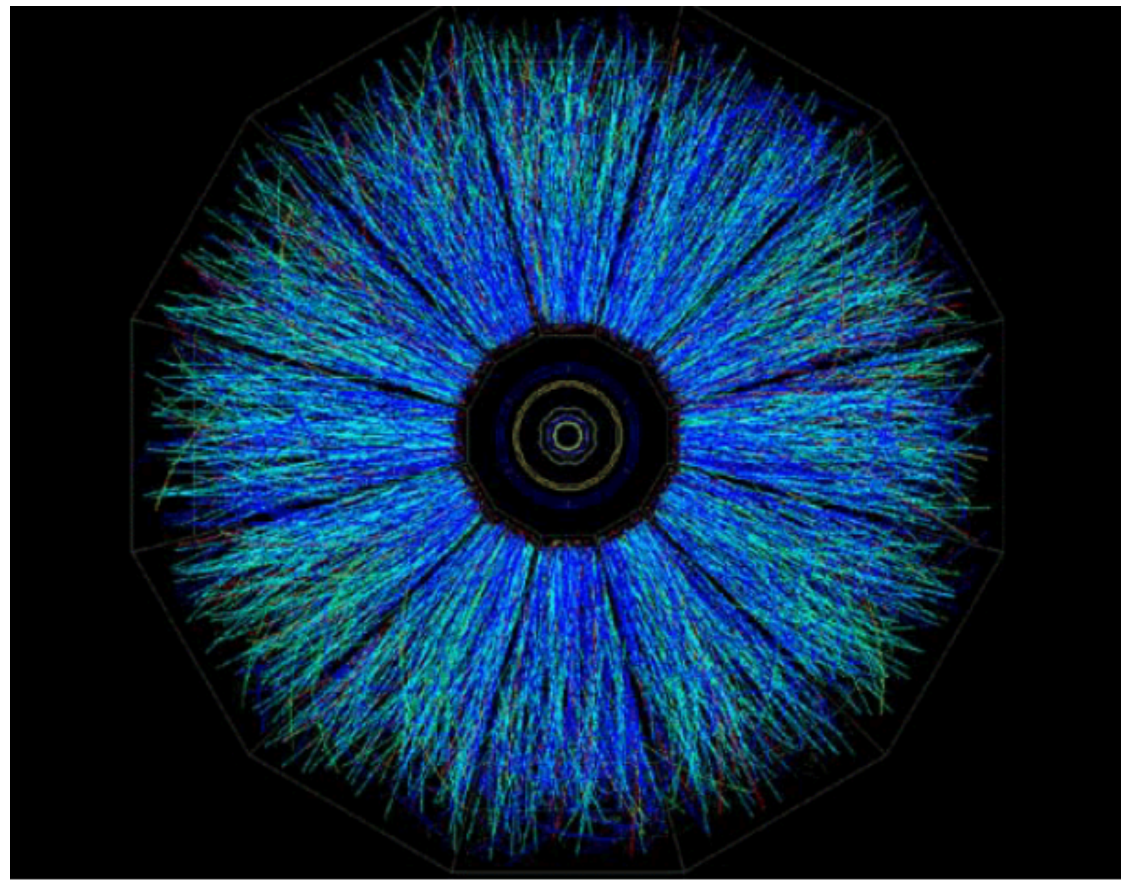
one can find CP by plotting

Finite size vs. window size

$$X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$$

Finite-size scaling (original): change the size of the system, calculate $X_L(t_L)$, repeat

However, changing SIZE is not always possible or doesn't probe the same system:
bird flocks, heavy-ion collisions, …



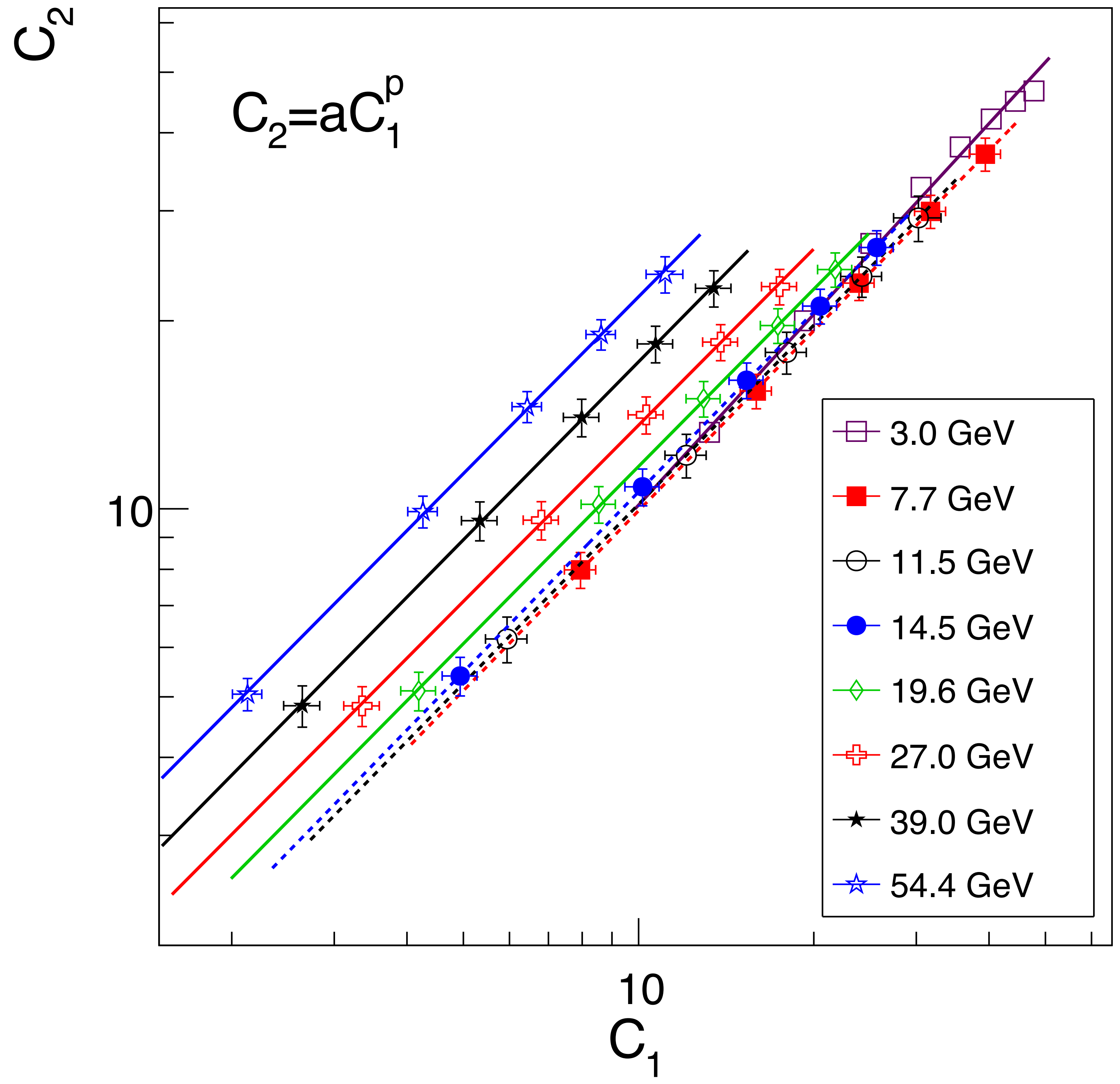
Solution: study the dependence of X on the size of the *subsystem* that is considered

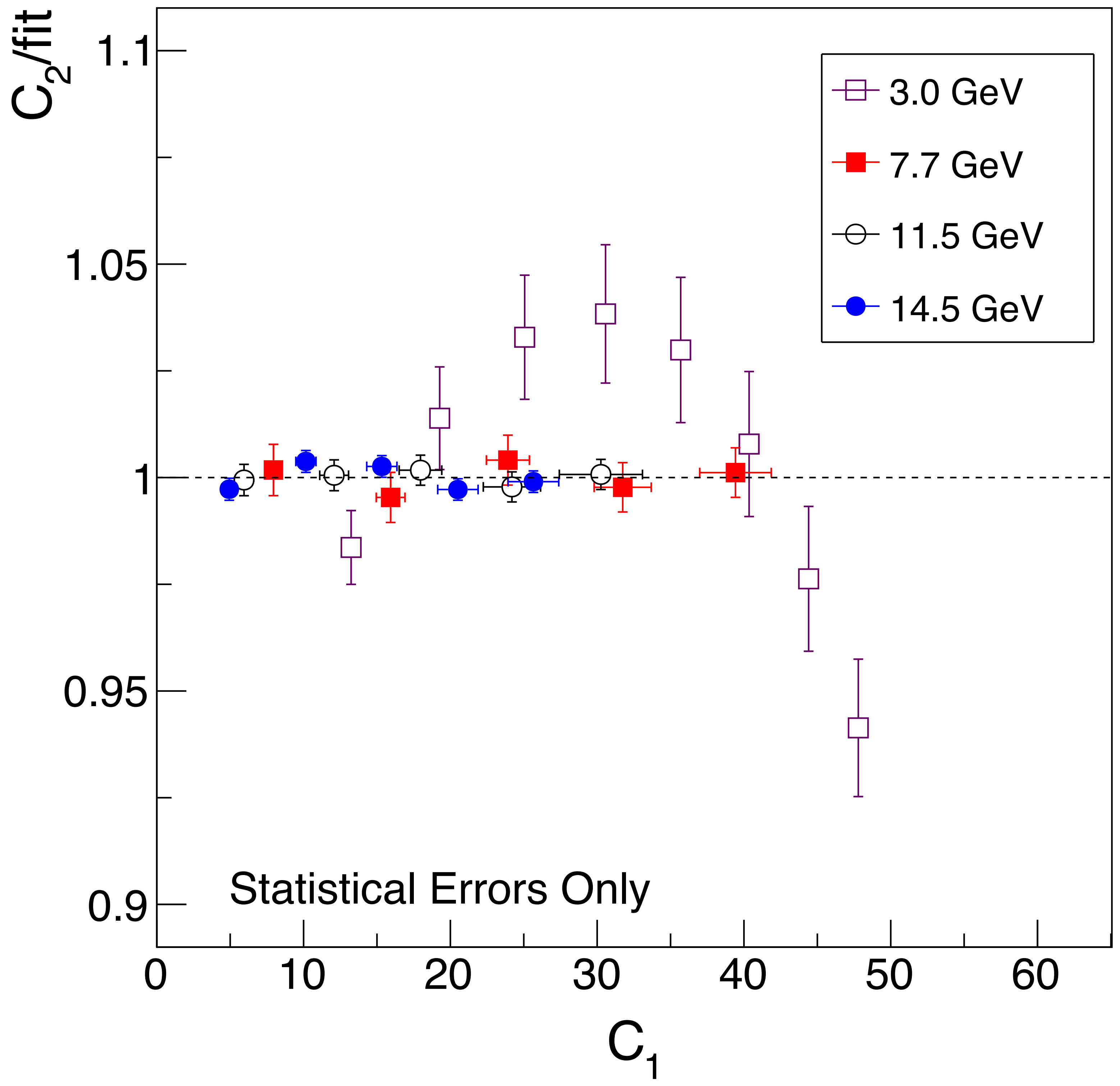
D. Martin, T. Ribeiro, S. Cannas, *et al.*, Box scaling as a proxy of finite size correlations, Sci Rep 11, 15937 (2021)

Where can we expect scaling behavior?

- For fluids far from the critical region, a mean-field treatment is good enough.
  The transition between the critical scaling region, intermediate scaling region, and extended scaling region has been studied: for fluids, the extended scaling region essentially covers the entire phase diagram where fluctuation contributions are small but finite.
  M.A. Anisimov, S.B. Kiselev, J.V. Sengers, S.Tang, Crossover approach to global critical phenomena in fluids, Physica A 188, 4 (1992)

- In the region of the phase diagram where the bulk of the evolution is well described by hydrodynamics (a scale free theory), the data follows Taylor's Law (is scale free): $\sigma^2 = a\lambda^p$





$$C_2 = aW^p$$
$$C_2 = a(xW)^p = ax^pW^p = a'W^p$$
where $C_1 \propto W$ in this energy range

Scale invariance supports the applicability of FSS
(but not in at $\sqrt{s_{NN}} = 3$ GeV!)

Results using data from BES-I

Thermal model

$$\chi_2 = \frac{C_2}{T^3 V} \quad \Rightarrow \quad \chi_2(W, \mu_{\rm fo}) = \frac{C_2(W, \mu_{B,\rm fo})}{T_{\rm fo}^3 W dV_{\rm fo}/dy}$$

- We use rapidity bin width W as the subsystem size
- We use published thermal model fits for $T_{\rm fo}$ and $\mu_{B,\rm fo}$
- We parameterize $dV_{\rm fo}/dy$ from several publications
  (for 2.4 GeV, $T_{\rm fo}^3 V$ is highly uncertain, ranging from about 65 to 650)
- Experiments can improve results by publishing
  $dV_{\rm fo}/dy$, $T_{\rm fo}$, $\mu_{B,\rm fo}$
  from thermal model fits for specific W

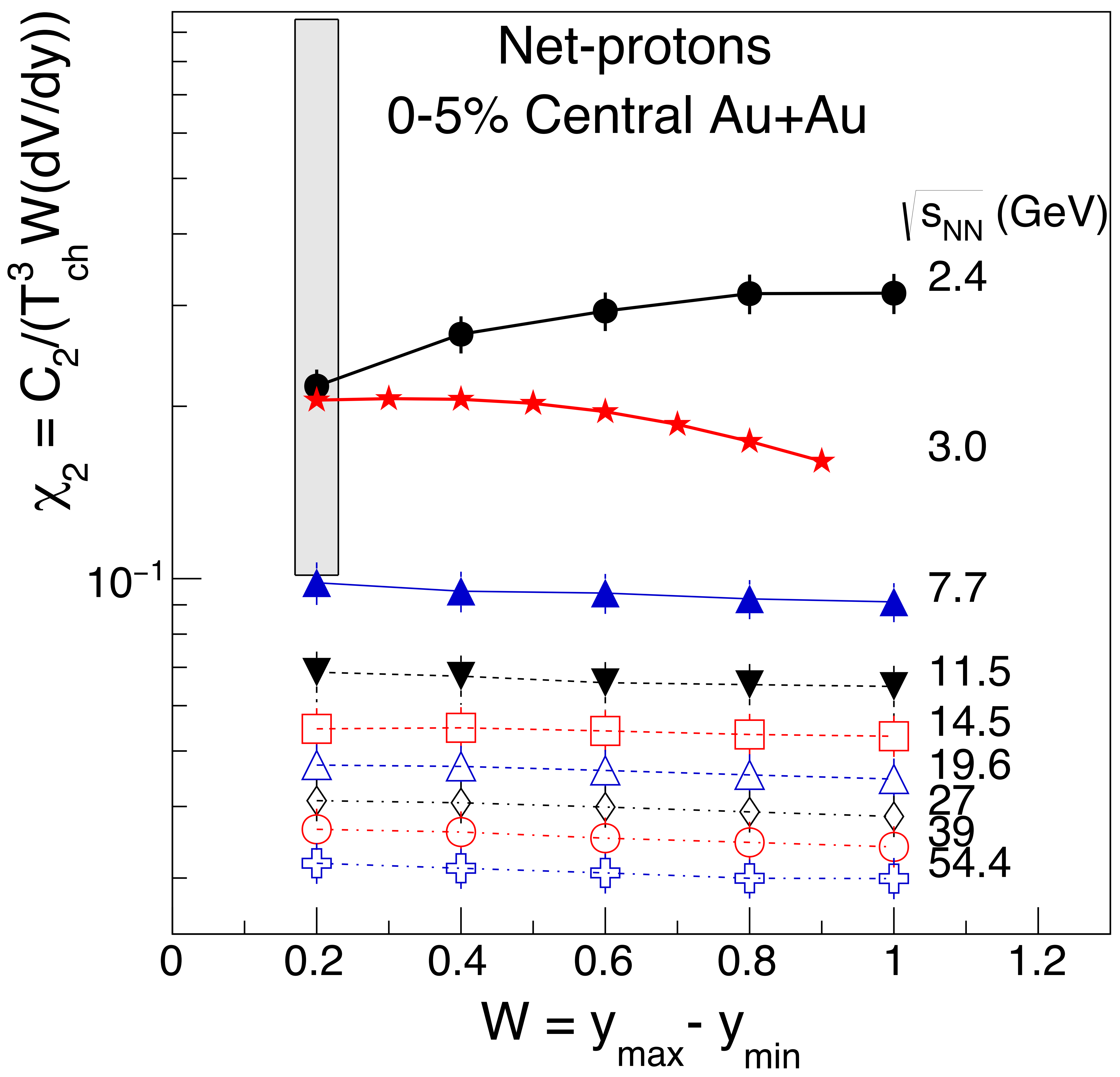
$\sqrt{s_{\rm NN}}$ (GeV)	$y_{\rm beam}$	$\mu_{\rm fo}$ (GeV)	$T_{\rm fo}$ (GeV)	$dV_{\rm fo}/dy$ (fm^3)
2.4	0.73	0.776	0.050	17157
3.0	1.05	0.720	0.080	4850
7.7	2.09	0.398	0.144	1044
11.5	2.50	0.287	0.149	1047
14.5	2.73	0.264	0.152	1080
19.6	3.04	0.188	0.154	1137
27	3.36	0.144	0.155	1218
39	3.73	0.103	0.156	1341
54.4	4.06	0.083	0.160	1487

J. Adamczewski-Musch et al. (HADES), Phys. Rev. C 102, 024914 (2020)
M. Abdallah *et al.* (STAR), Phys. Rev. C 104, 024902 (2021)
M. Abdallah *et al.* (STAR), Phys. Rev. C 107, 024908 (2023)
A. Andronic, P. Braun-Munzinger, J. Stachel, Acta Phys. Polon. B 40, 1005-1012 (2009)
A. Motornenko *et al.*, Phys. Lett. B 822, 136703 (2021)
S. Chatterjee *et al.*, Adv. High Energy Phys. 2015, 349013 (2015)

Susceptibility

$$\chi_2(W, \mu_{\rm fo}) = \frac{C_2(W, \mu_{B,\rm fo})}{T^3_{\rm fo} W dV_{\rm fo}/dy}$$

- Grey band shows uncertainty from freeze-out ambiguities for the 2.4 GeV data.
  Uncertainty precludes any conclusion about observing a maximum in χ_2
- Data do indicate a change in slope at higher μ_B and at small W:
  χ_2 decreases with increasing W for 7.7–54.4 GeV but changes slope at 2.4 GeV
  (3.0 GeV is ~flat)



Scaled susceptibility: 2D fit w/ mean-field exponents

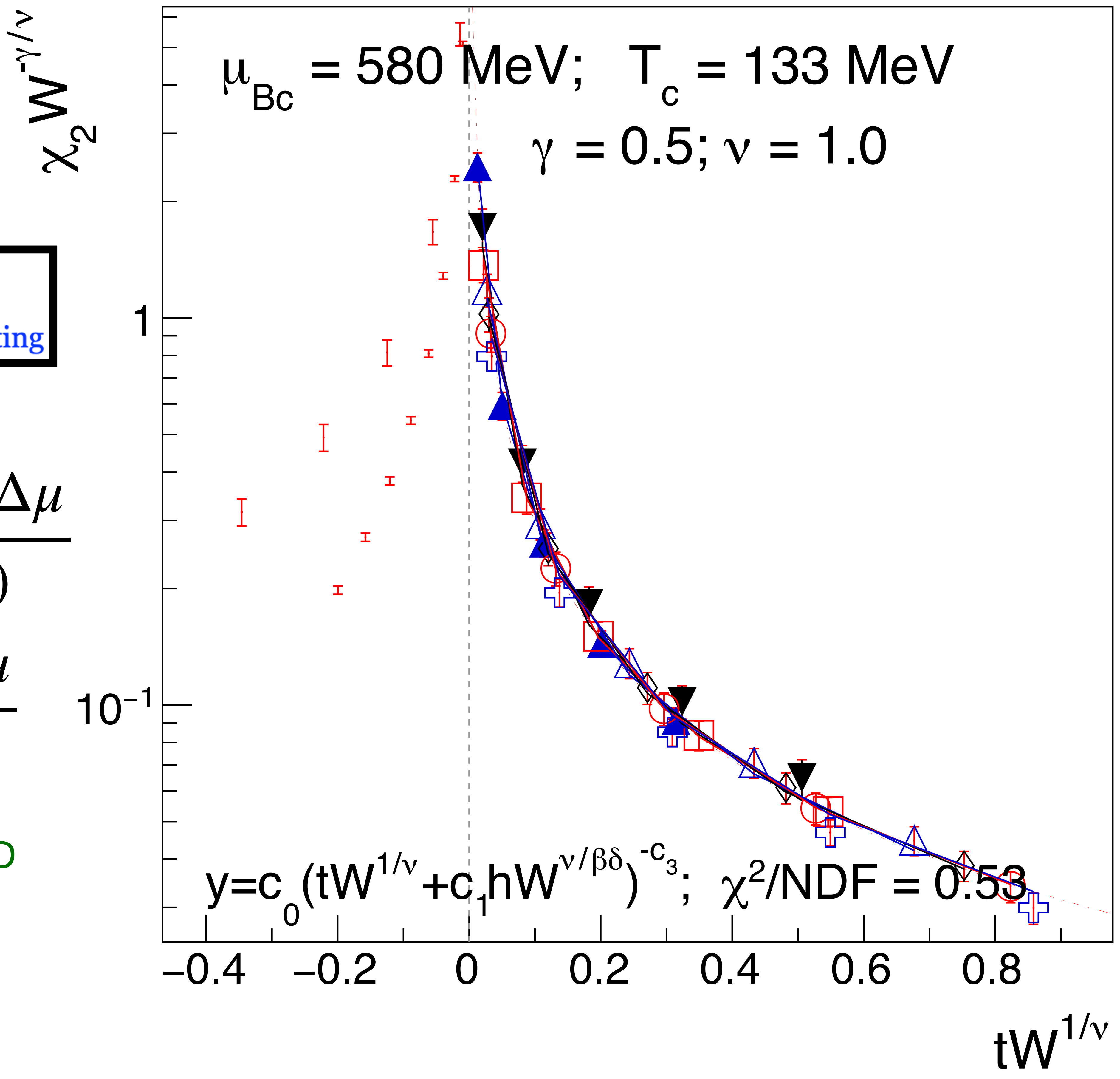
$X_L(t_L)L^{-\frac{\sigma}{\nu}} = \phi(tL^{\frac{1}{\nu}})$

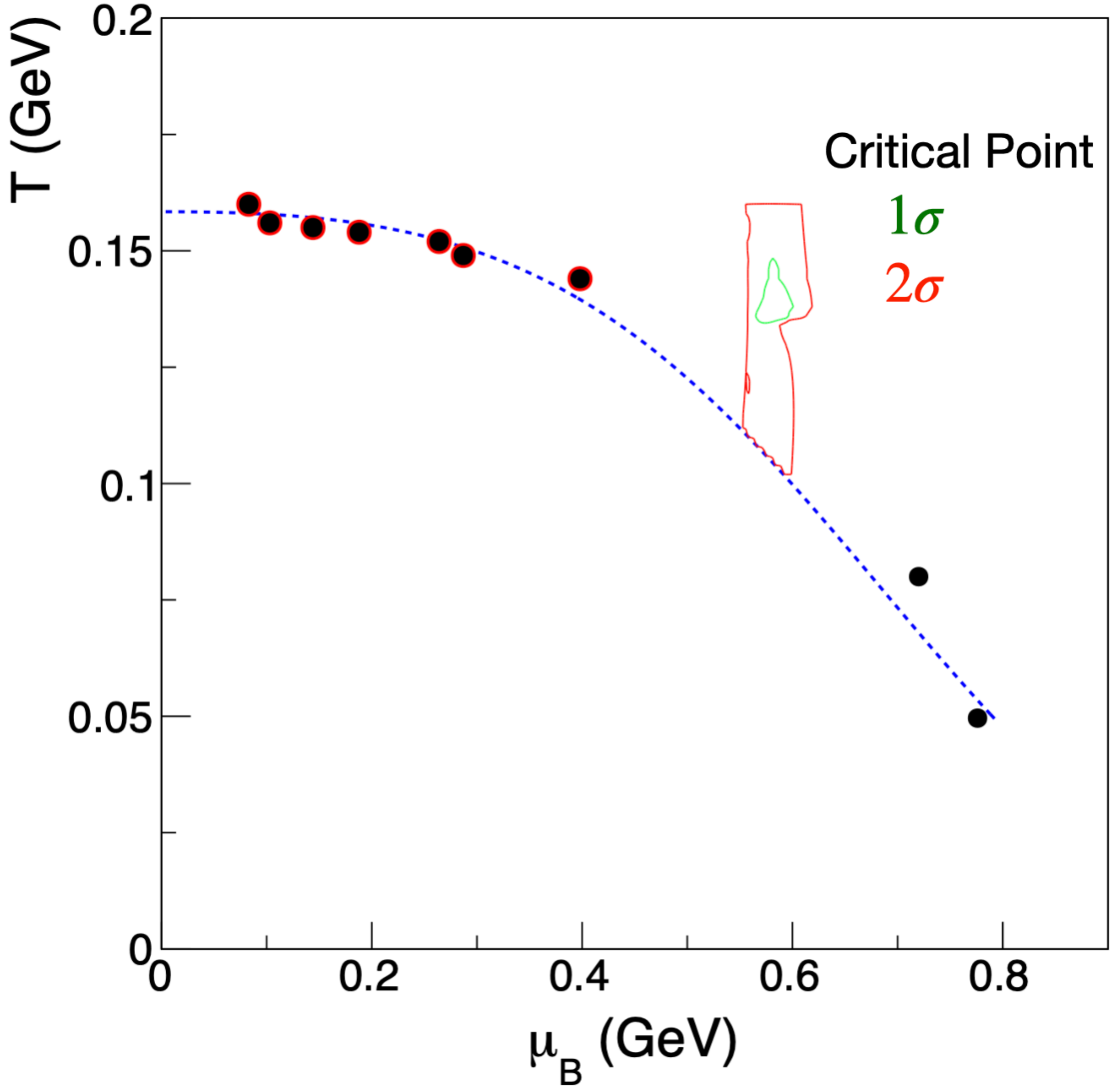
one can find CP by plotting

$$h(\mu, T) = -\frac{\cos\alpha_1\Delta T + \sin\alpha_1\Delta\mu}{wT_c\sin(\alpha_1 - \alpha_2)}$$

$$t(\mu, T) = \frac{\cos\alpha_2\Delta T + \sin\alpha_2\Delta\mu}{\rho wT_c\sin(\alpha_1 - \alpha_2)}$$

M.S. Pradeep, M. Stephanov, Phys. Rev. D **100** 5, 056003 (2019) arXiv:1905.13247





With mean-field exponents, we find scaling for $555 < \mu_{B,c} < 610$ MeV;

T_c only constrained by "plausibility" (below $T_{pc,\mu_B=0}$ and above T_{fo})

Chi-square contours identify an allowed region in the phase diagram: $\mu_{B,c} = 580 \pm 30$ MeV

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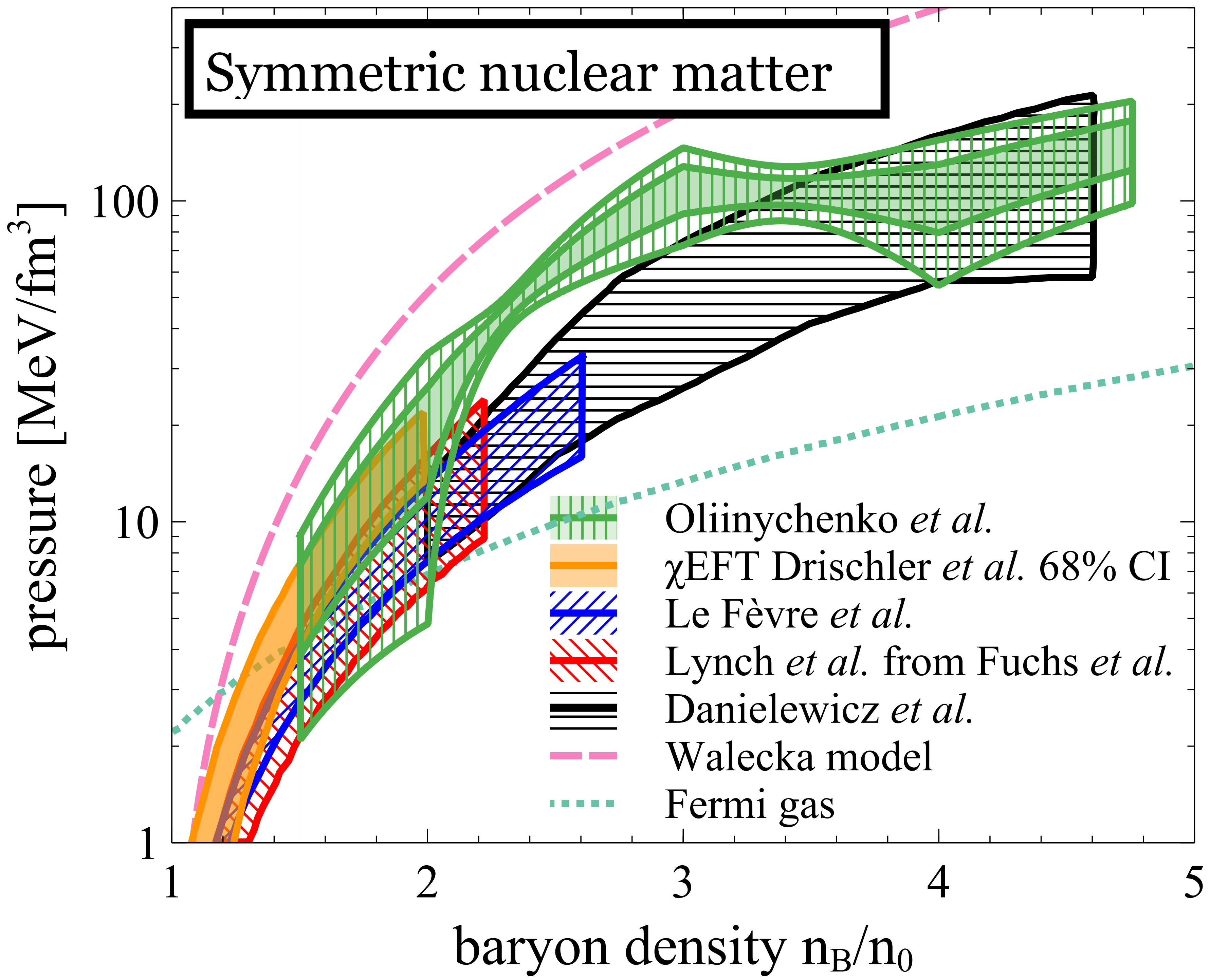
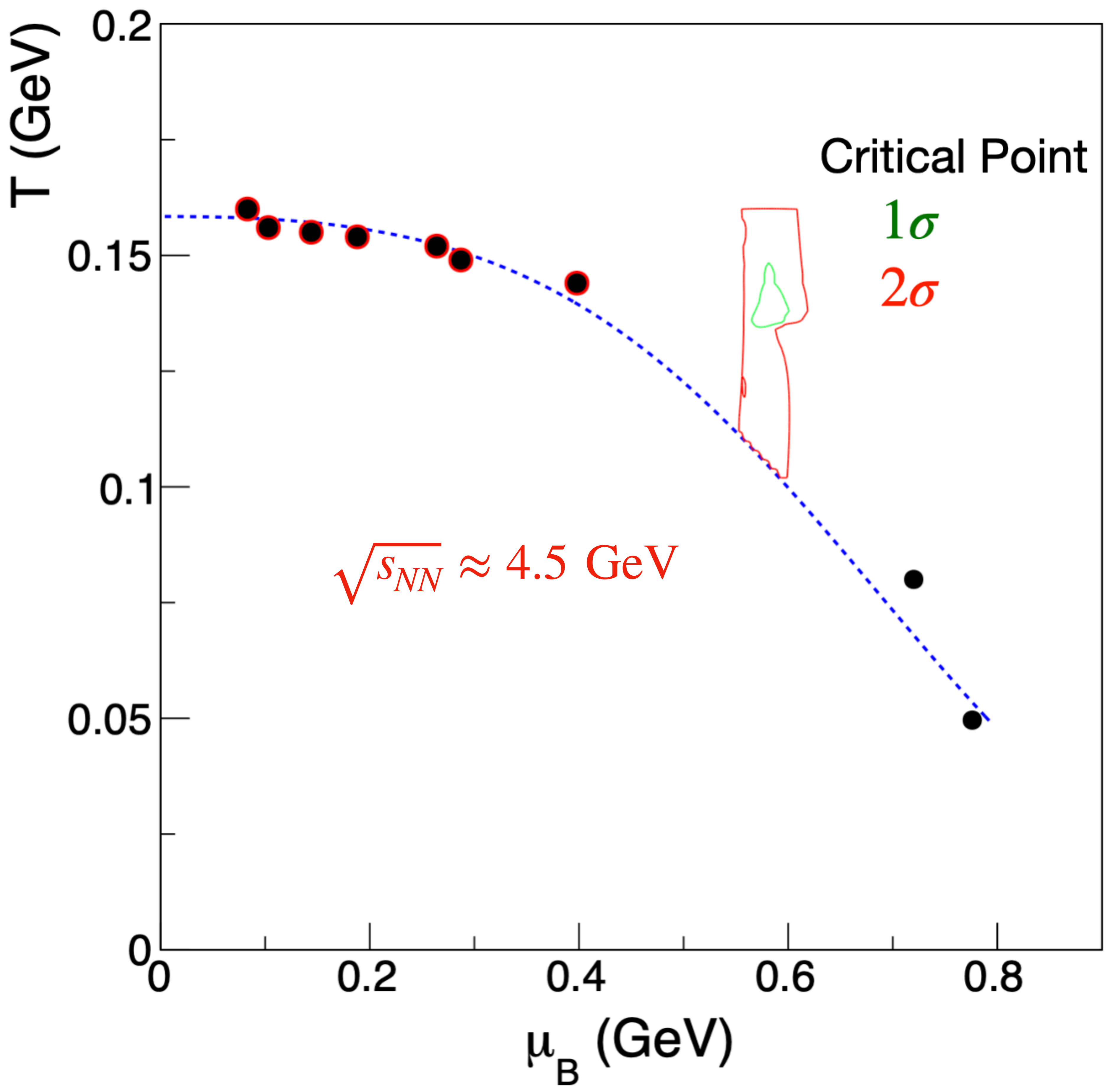
M.S. Pradeep, M. Stephanov,
100 5, 056003 (2019) arXiv:190



A. Sorensen *et al.*, Prog. Part. Nucl. Phys. **134**, 104080 (2024) arXiv:2301.13253



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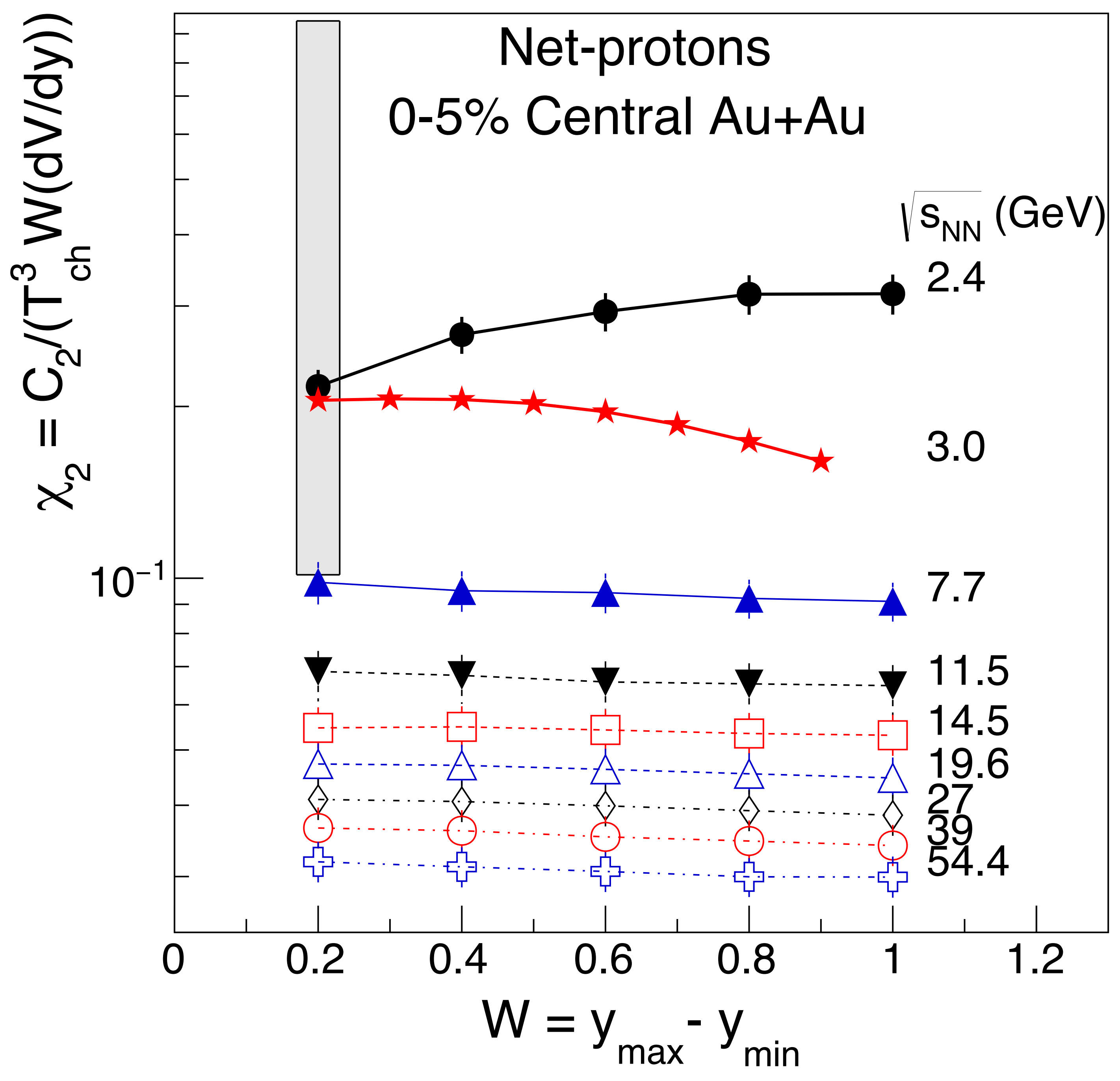
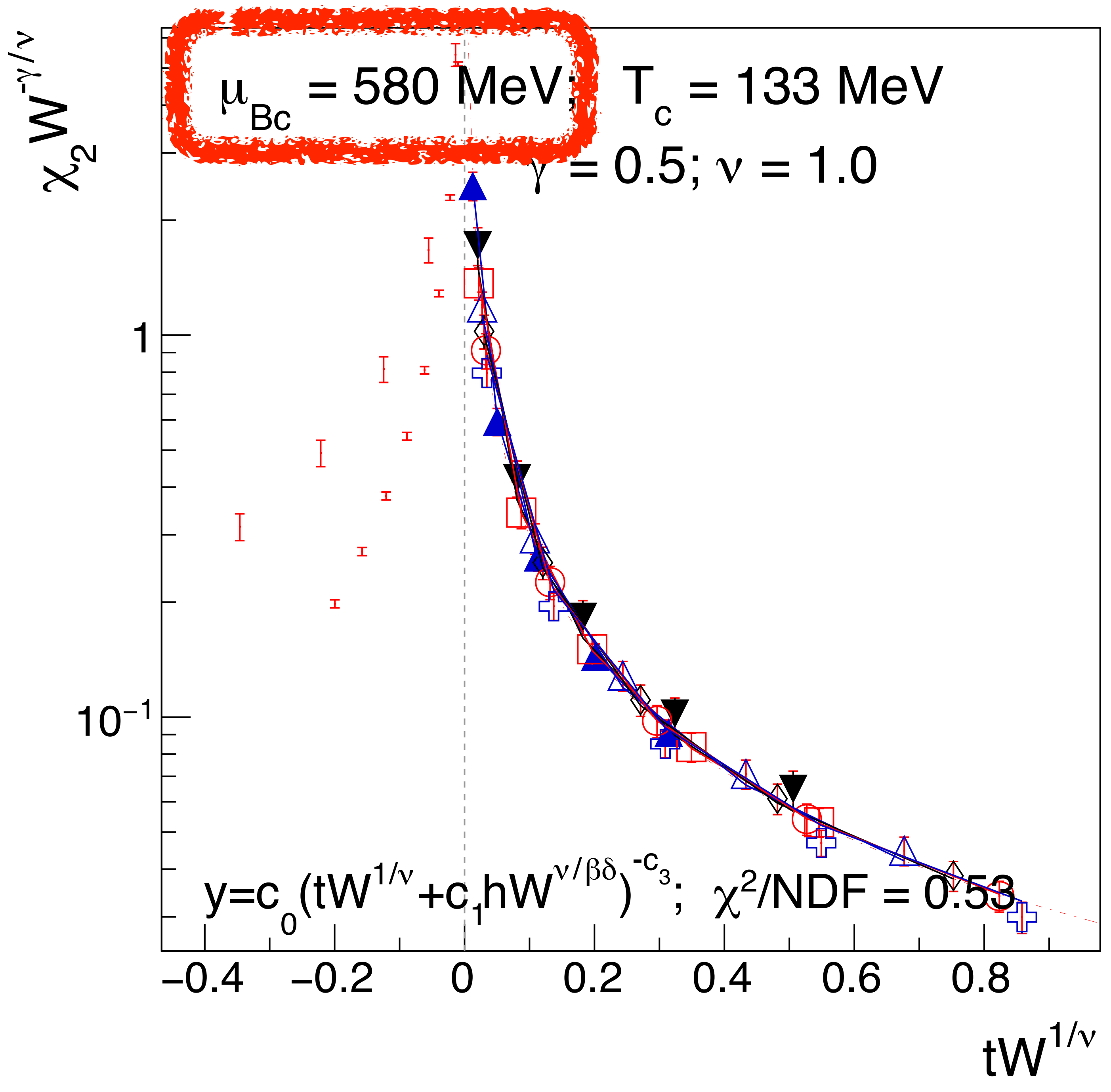
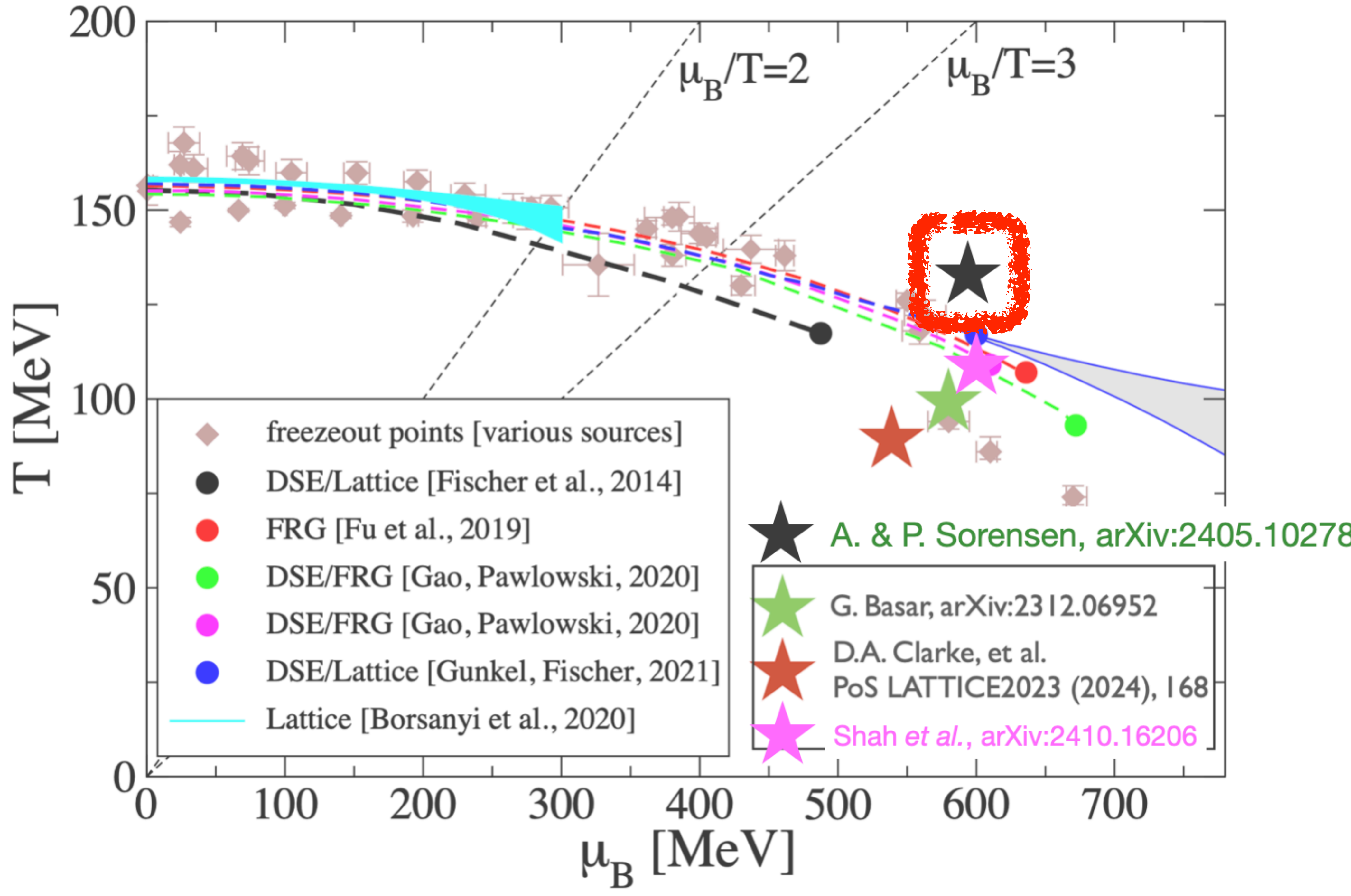
Summary

- Window-size analysis allows studying the influence of probed scale (dynamics changes if size is studied *via*, e.g., centrality-dependence)
- Data follows Taylor's law ($C_2 \propto C_1^p$) = exhibits scale-free behavior for 7.7-54.4 GeV data
- In a 2-D fit, we extract $\mu_B \approx 580 \pm 30$ MeV: a first extraction from data!







Thank you for your attention, and thank you Scott for your support over the years!