Charge Balance Functions

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S.P., J. Kim & C.Plumberg PRC(2018)
S.P. & C.Plumberg PRC(2019)
S.P. PRC (2020)
S.P. & R.Steinhorst, PRC (2020)
S.P. & C.Plumberg, PRC (2020)
S.P. & K.Martirosova, PRC (2022)
S.P., D.Oliinchenko
& C.Plumberg, PRC (2022)
O.Savchuk and S.P., PRC (2024)

Why Charge Balance Functions are the Most Important Measurement in High-Energy Heavy-Ion Collisions

Properties of the QGP

1. Eq. of State
2. Chemistry (charge fluctuations)
3. Chiral Symmetry Restoration

Transport Coefficients

4. Viscosity (shear & bulk)
5. Diffusivity & Conductivity (light / heavy quark)
6. Electromagnetic Opacity & Emissivity
7. Gluonic Opacity and Emissivity (jet quenching)

Charge balance functions are principal tool

Also important for:

- CME background
- Background for fluctuations for phase transitions

Charge Susceptibility

Because $\langle Q_a \rangle = 0$, chemistry (at $\mu = 0$), you must look at fluctuations

$$\chi_{ab} = \frac{\langle \delta Q_a \delta Q_b \rangle}{V}$$

For non-interacting

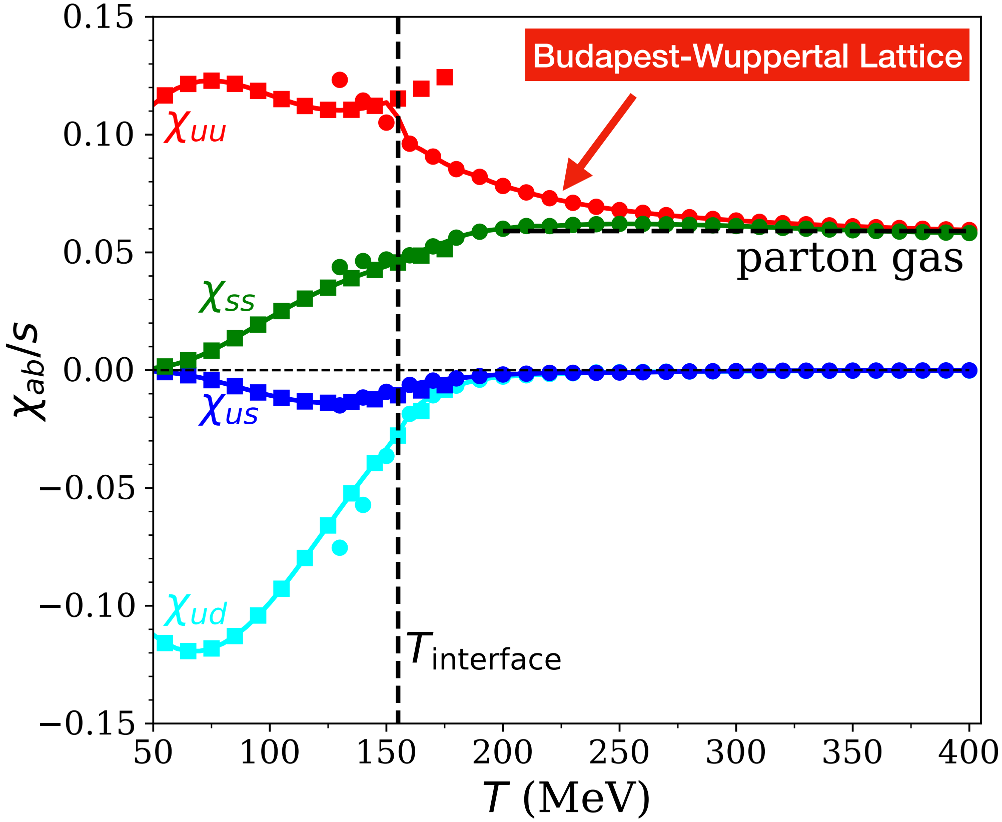
Quark Gas:

$$\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$$

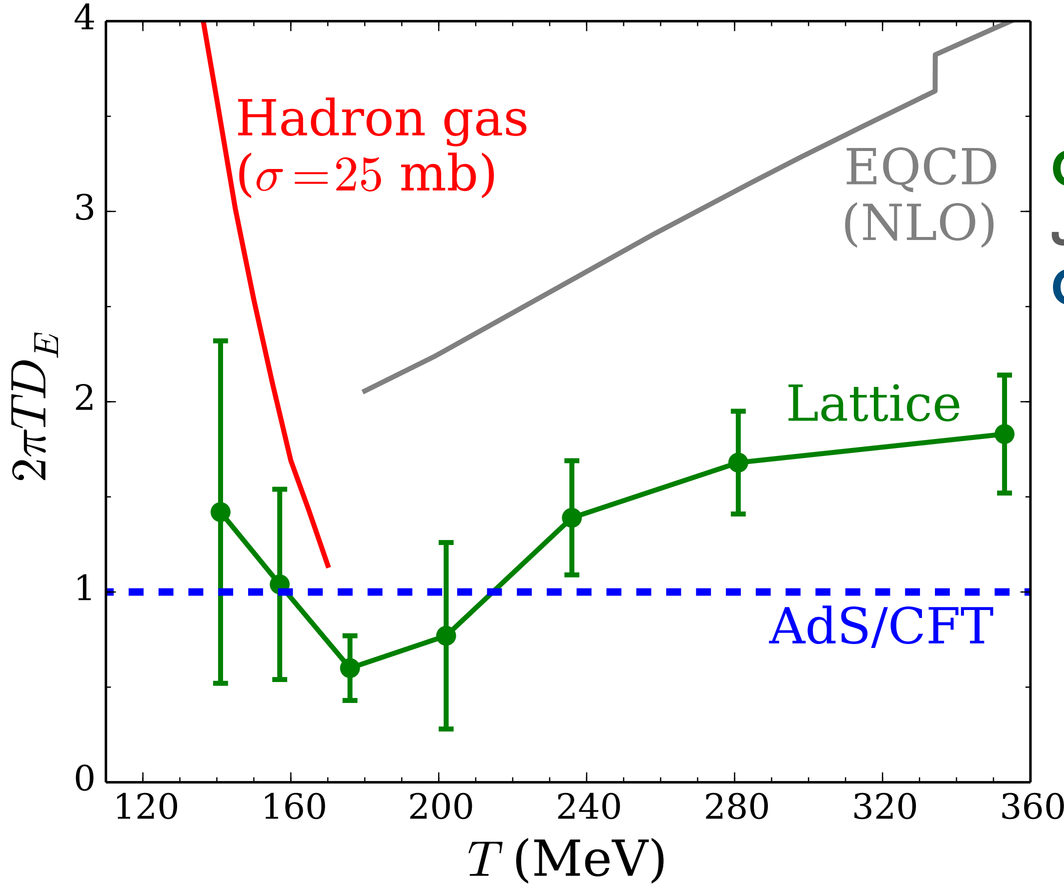
Hadron Gas:

$$\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$$

Susceptibility represents chemistry!!!!



Diffusion/Conductivity (light quark)



G.Aarts et al, JHEP (2015)

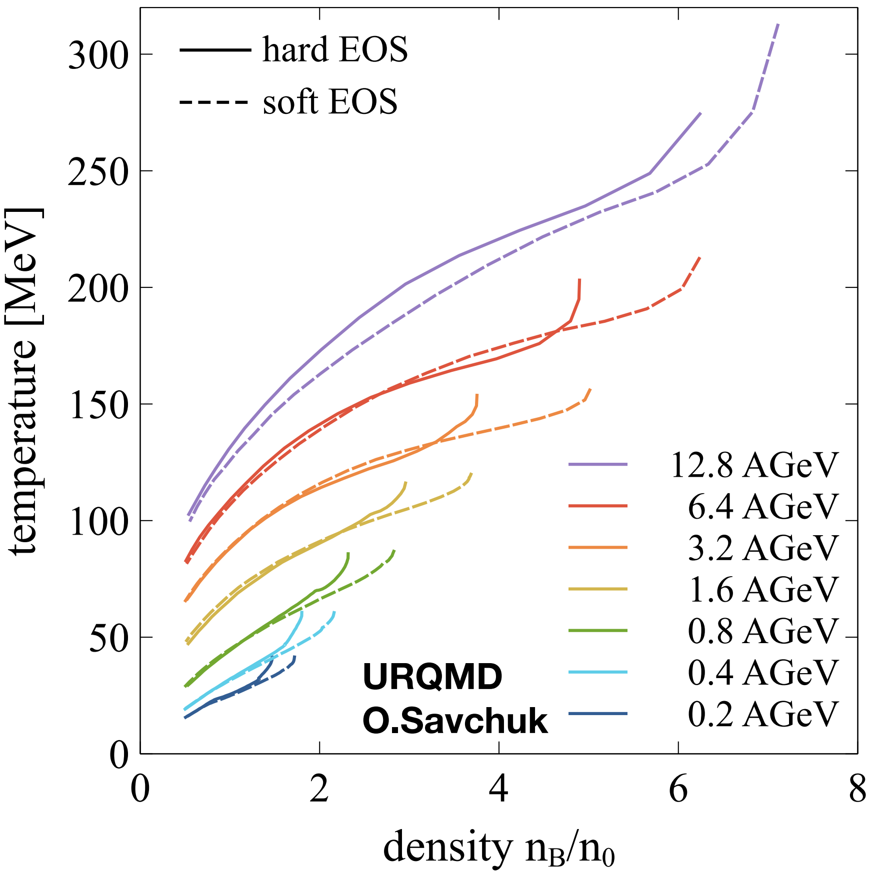
J.Ghiglieri et al, JHEP (2018)

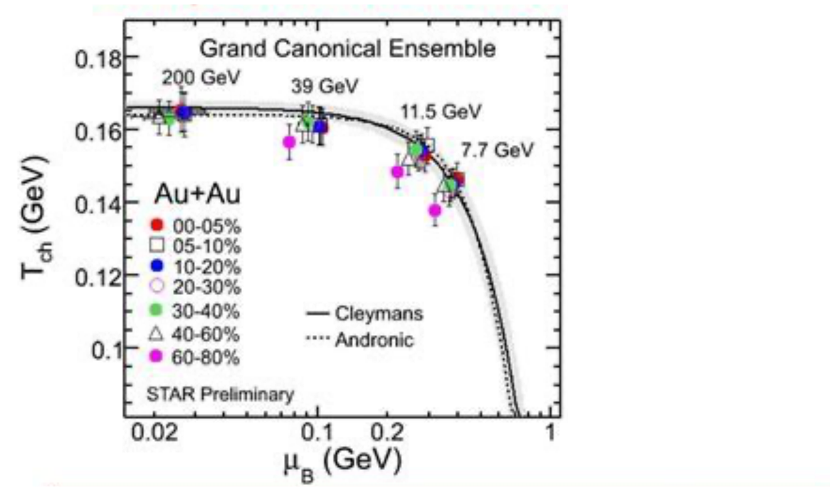
G.Policastro et al, JHEP (2002)

Transport Coefficient (like viscosity)

$$j_a = -D_{ab}\nabla\rho_b$$
$$= -\sigma_{ab}\nabla\mu_b$$
$$\sigma_{ab} = \chi_{ac}D_{cd}$$
$$\sigma_{ab} = \frac{1}{2T}\int d^4x\langle\{j_a(0), j_b(x)\}\rangle$$

Charge Susceptibilities





At what part of trajectory do you *measure* χ_{ab}?

Answer: BFs sensitive to ENTIRE trajectory!!!

$$B_{hh'}(\Delta y) \leftrightarrow \chi_{ab}(\tau)$$

Theory of Correlations and Balance Functions

Charge Correlations
(Equilibrated System)

$$C_{ab}(\vec{r}_1,\vec{r}_2) \equiv \langle \delta\rho_a(\vec{r}_1)\delta\rho_b(\vec{r}_1)\rangle = \chi_{ab}\delta(\vec{r}_1-\vec{r}_2),$$

$$\boxed{\chi_{ab}} = \frac{1}{V}\langle \delta Q_a \delta Q_b\rangle = \frac{T^2}{Z}\frac{\partial^2}{\partial\mu_a\partial\mu_b}\ln Z,$$

3x3 matrix

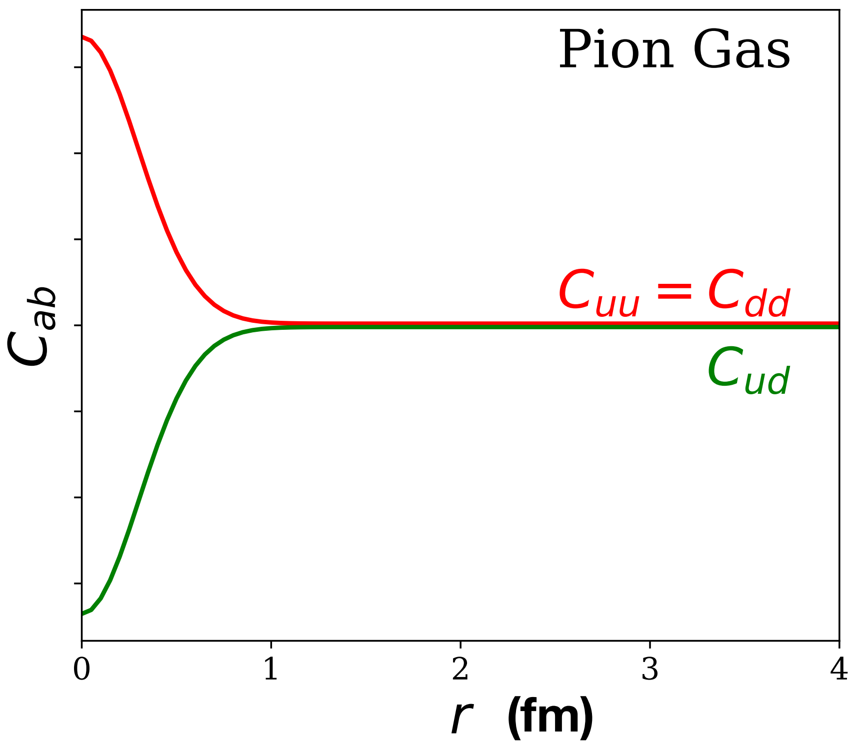
$$= \int d^3r\; C_{ab}(0,\vec{r}).$$

Quark Gas:

$$\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$$

Hadron Gas:

$$\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$$

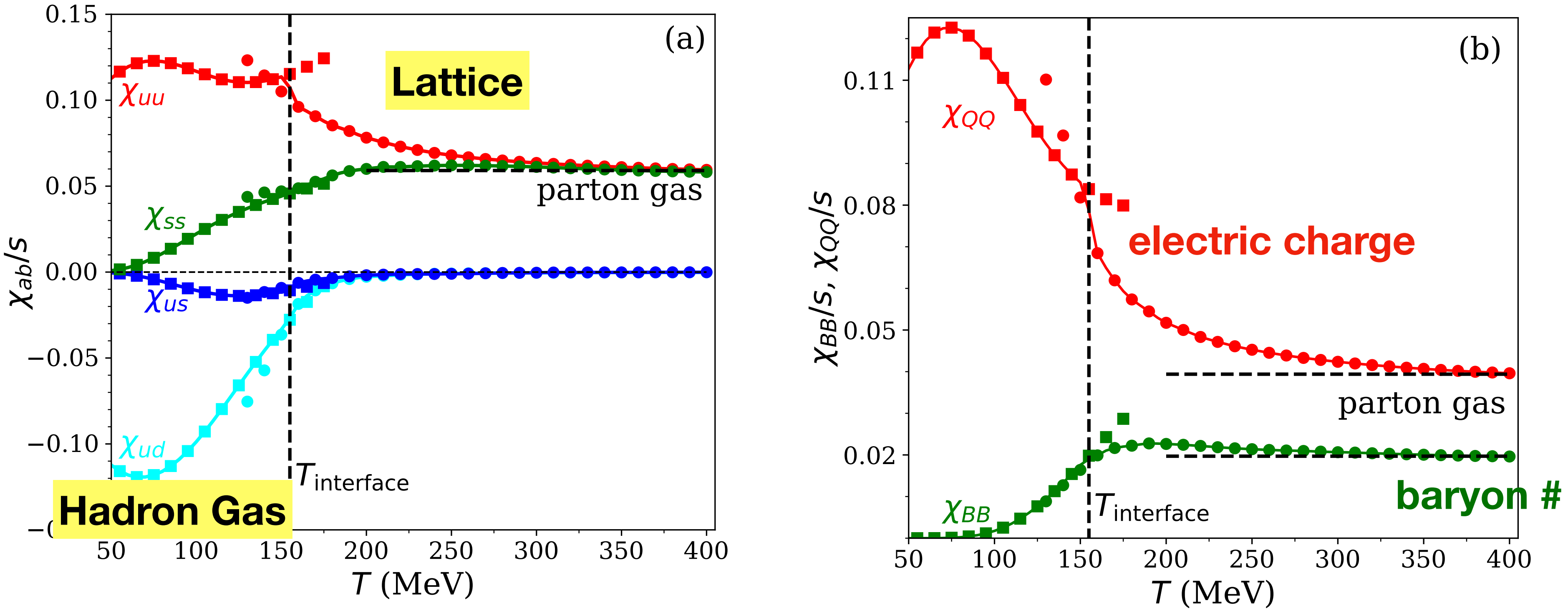


Susceptibility
(Lattice, BW-Claudia Ratti)

For hadron gas: $\chi_{ab} = \sum_h n_h q_{ha} q_{hb}$

a=(u,d,s)

For parton gas: $\chi_{ab} = \sum_a (n_a + n_{\bar{a}})\delta_{ab}$



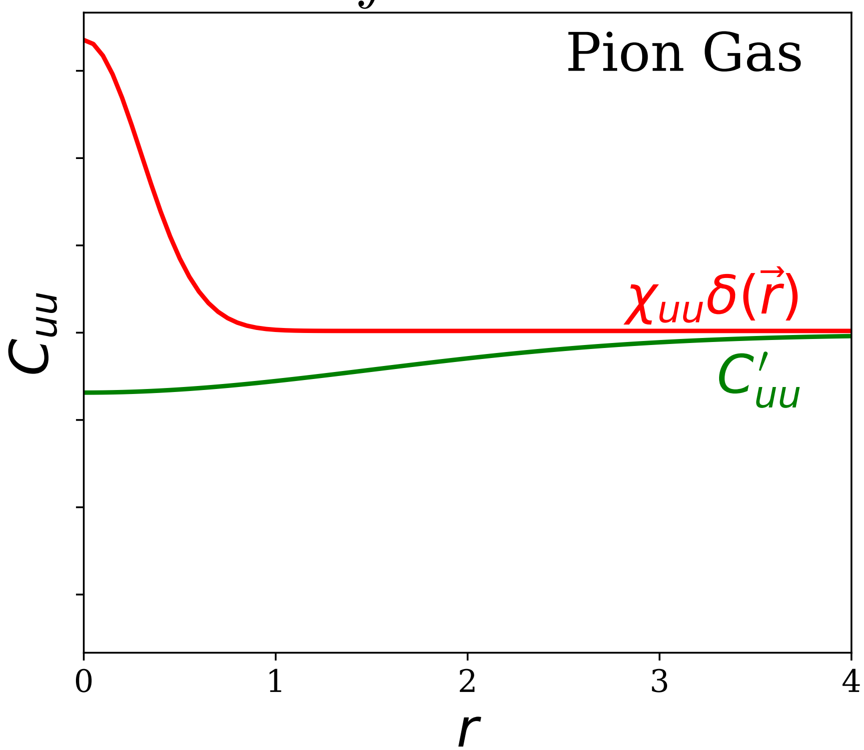
Charge Correlations

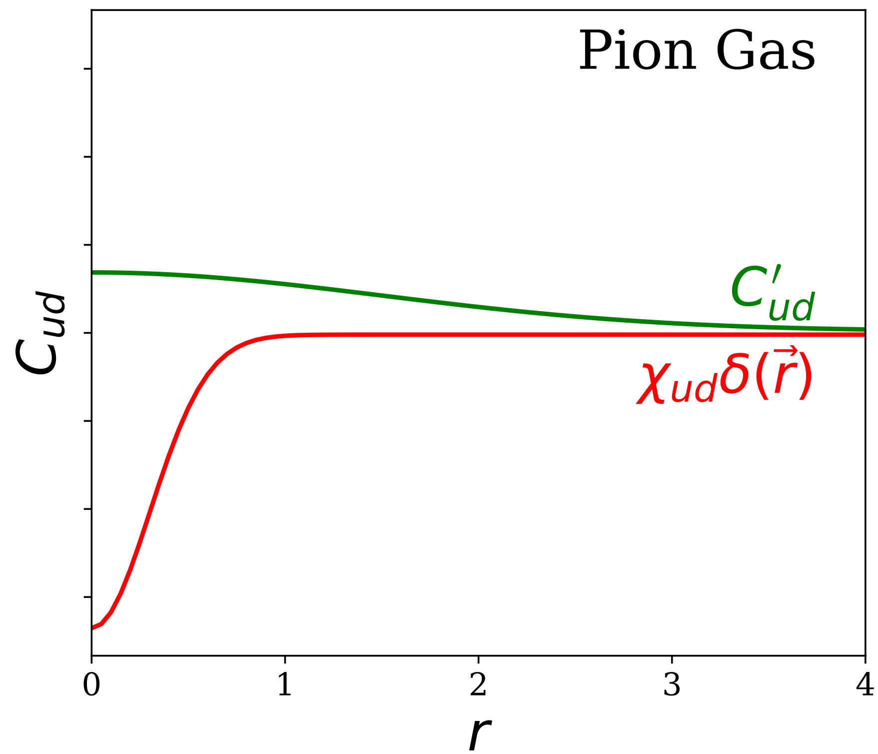
(Dynamic System with Charge Conservation)

$$C_{ab}(\vec{r}_1, \vec{r}_2) = \chi_{ab}\delta(\vec{r}_1 - \vec{r}_2) + \boxed{C'_{ab}(\vec{r}_1, \vec{r}_2)}$$

***Balancing* correlation**

$$\int d^3r C'_{ab}(\vec{r}) = -\chi_{ab}$$





Eq.s of motion

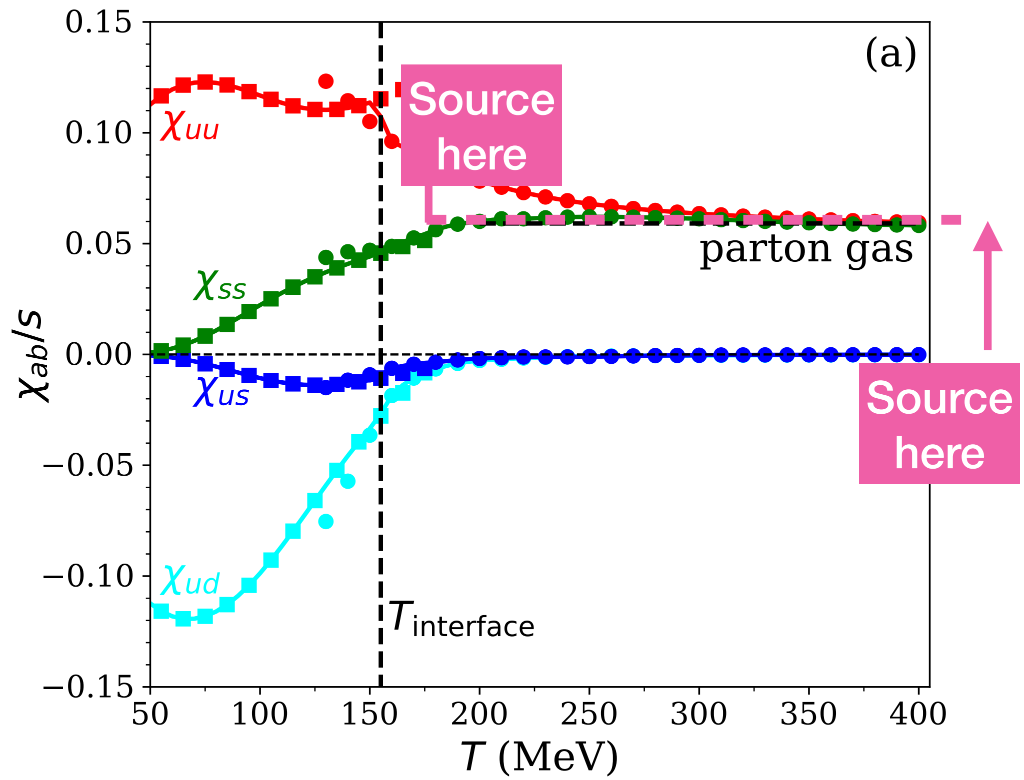
Diffusivity

$$\partial_t C'_{ab}(\vec{r}_1 - \vec{r}_2) - D_{ab}\nabla_1^2 C'_{ab}(\vec{r}_1, \vec{r}_2) - D_{ab}\nabla_2^2 C'_{ab}(\vec{r}_1, \vec{r}_2) = -S_{ab}(\vec{r}_1, t)\delta(\vec{r}_1 - \vec{r}_2),$$

$$S_{ab}(\vec{r}, t) = \left[\partial_t + \vec{v}\cdot\vec{\nabla} + (\nabla\cdot\vec{v})\right]\chi_{ab}(\vec{r}, t) \approx s(\vec{r}, t)\left[\partial_t + \vec{v}\cdot\vec{\nabla}\right]\frac{\chi_{ab}(\vec{r}, t)}{s(\vec{r}, t)}$$

Source **term**

When $\chi_{ab}V$ changes,
$C'_{ab}(\Delta\vec{r})$ gets sourced at $\Delta\vec{r} = 0$.



Eq.s of motion

$$\partial_t C'_{ab}(\vec{r}_1 - \vec{r}_2) - D_{ab}\nabla_1^2 C'_{ab}(\vec{r}_1, \vec{r}_2) - D_{ab}\nabla_2^2 C'_{ab}(\vec{r}_1, \vec{r}_2) = -S_{ab}(\vec{r}_1, t)\delta(\vec{r}_1 - \vec{r}_2),$$

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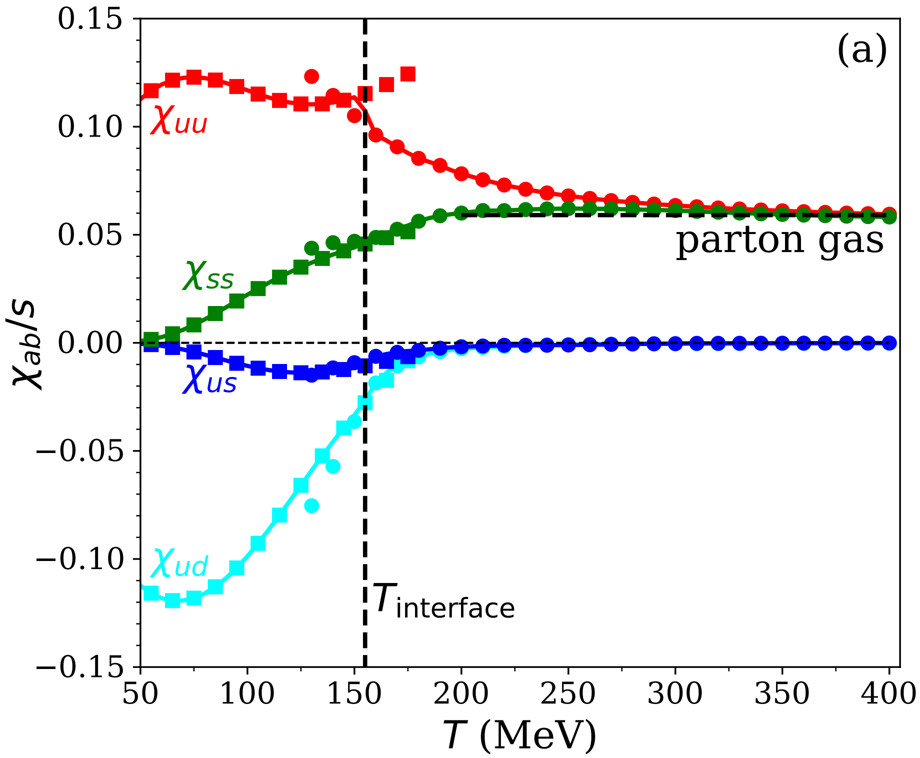
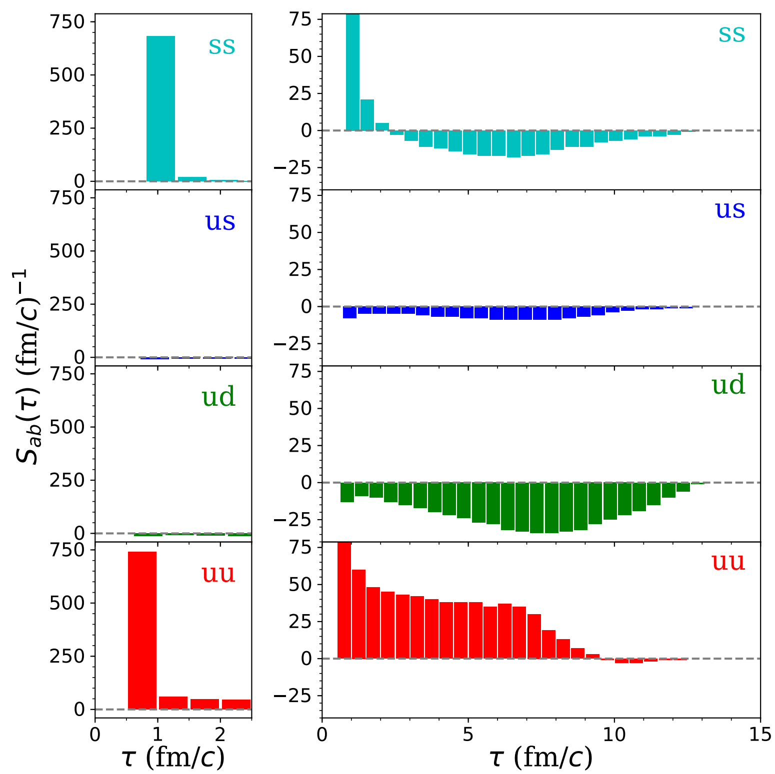
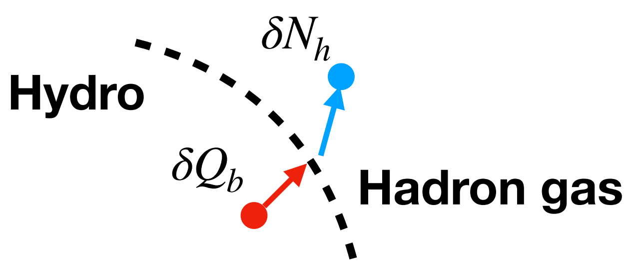




Translate C_{ab} into $C_{hh'}$

$$\delta N_h = \chi^{-1}_{ab}(T_{\rm interface}) q_{ha} n_h \delta Q_b$$



Charge Balance Function

$$C_{hh'}(\vec{r}_1, \vec{r}_2) = \langle [\delta n_h(\vec{r}_1) - \delta n_{\bar{h}}(\vec{r}_1)][\delta n_{\bar{h}'}(\vec{r}_2) - \delta n_{h'}(\vec{r}_2)] \rangle,$$

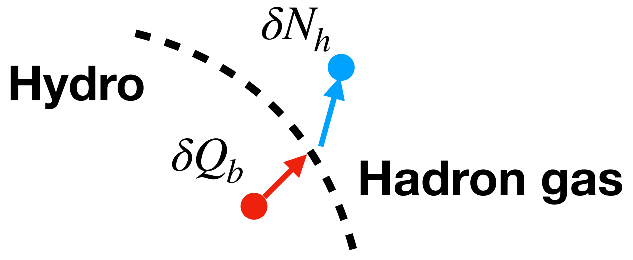
$$B_{h|h'}(\vec{p}_1|\vec{p}_2) = \frac{1}{N_{h'}(\vec{p}_2) + N_{\bar{h}'}(\vec{p}_2)} \langle [\delta N_h(\vec{p}_1) - \delta N_{\bar{h}}(\vec{p}_1)][\delta N_{\bar{h}'}(\vec{p}_2) - \delta N_{h'}(\vec{p}_2)] \rangle$$

If you see a charged particle at $\vec{p}_2$,

chance of finding balancing (opposite charge) at $\vec{p}_1$

Translate C_{ab} into $C_{hh'}$

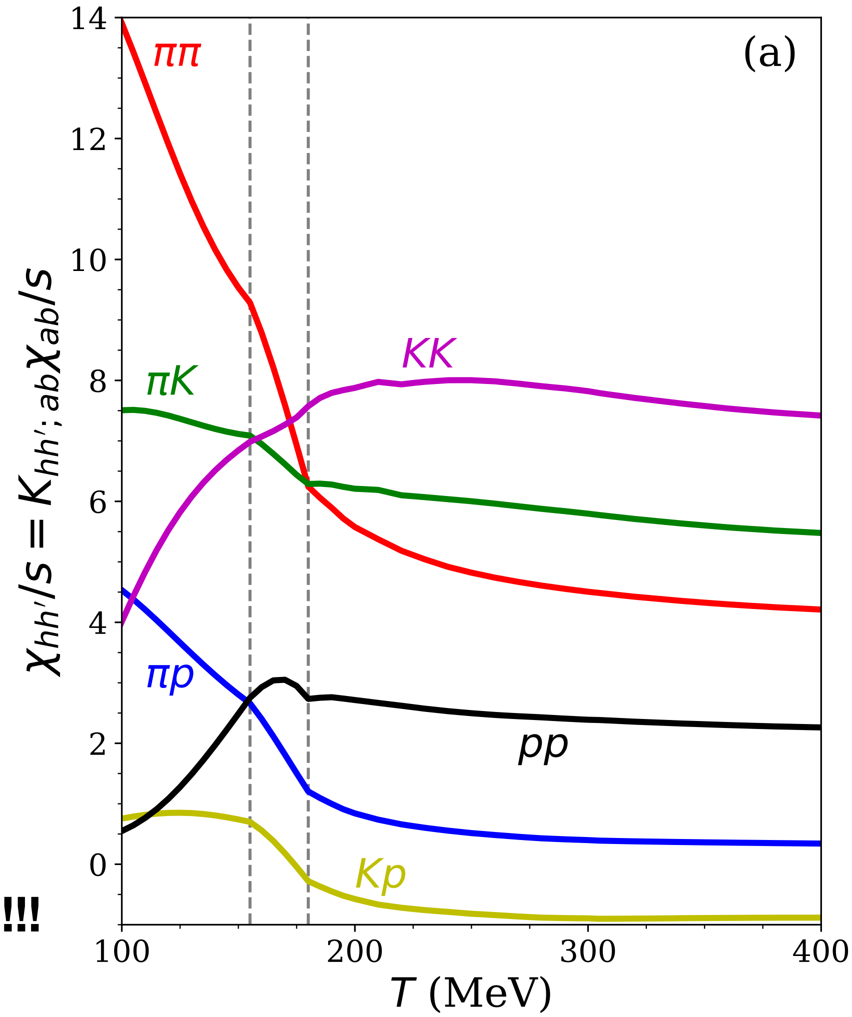
$$\delta N_h = \chi^{-1}_{ab}(T_{\text{interface}}) q_{ha} n_h \delta Q_b$$



$B_{\pi\pi}$ **has contribution from hadronization stage**

B_{pp} **and** B_{KK} **are sourced at thermalization**

$B_{\pi\pi}(\Delta y)$ **should be narrower than** $B_{KK}(\Delta y)$ **or** $B_{pp}(\Delta y)$**!!!**



Diffusivity

$$\vec{j}_a = -\boxed{D_{ab}}\nabla\rho_b,$$

3x3 matrix (colors)

$$= -\sigma_{ab}\nabla(\mu_b/T),$$

$$\sigma = \chi D,$$

$$\boxed{\chi_{ab}} = \langle\delta Q_a \delta Q_b\rangle/V = \partial\rho_a/\partial(\mu_b/T)$$

susceptibility

Kubo Relation

$$\sigma_{ab} = \frac{1}{2T}\int d^4x \, \langle\{j_a(0), j_b(x)\}\rangle$$

difficult (not impossible) for lattice gauge theory

The Calculation

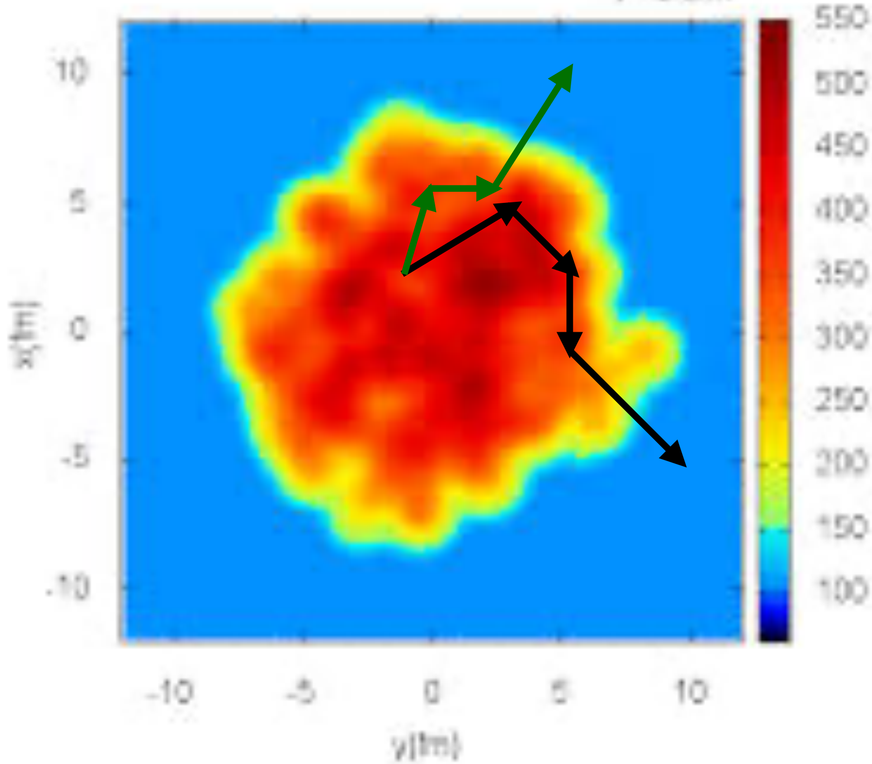
Diffusion = Random walk

Monte Carlo procedure:

A) Overlay with hydro evolution to create $S_{ab}(t, \vec{r})$
B) Generate partners (uu,dd,ss,ud,us,ss) proportional to $S_{ab}(t, \vec{r})$ with weights
C) Move particles in random directions punctuated by re-directioning according to $\tau_{\rm coll}$
D) Translate δQ_a to δN_h at hyper surface
E) Collide (fixed σ) and decay particles
F) Combine decay products with those from partner
G) Correlations created during hadronic phase: create uncorrelated hadrons, run through cascade, combine ALL particles to create BF
H) Add contributions from (E) and (F)
I) Fold with acceptance/efficiency
J) Test sum rules

ALGORITHM

$$\tau_{\rm coll} = 6D$$



ALGORITHM

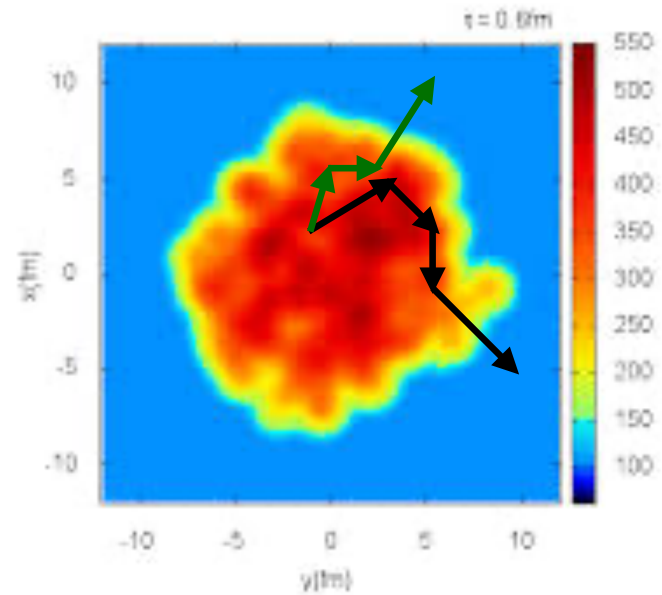
<u>TYPE I:</u>

Correlations from Hydro:

— Depends of D and σ_0

— Only a few hours of CPU

— track charges from same source point



<u>TYPE 2:</u>

Correlations from Cascade

— Weeks of CPU

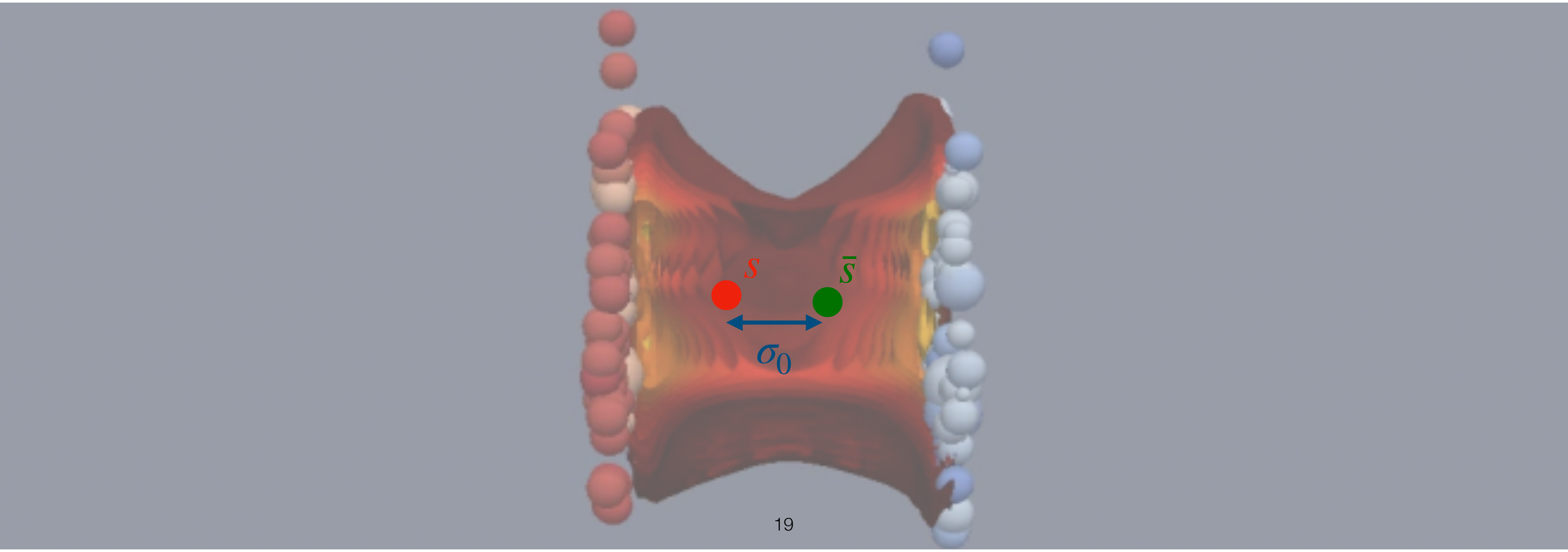
— One hydro event (independent of D, σ_0)

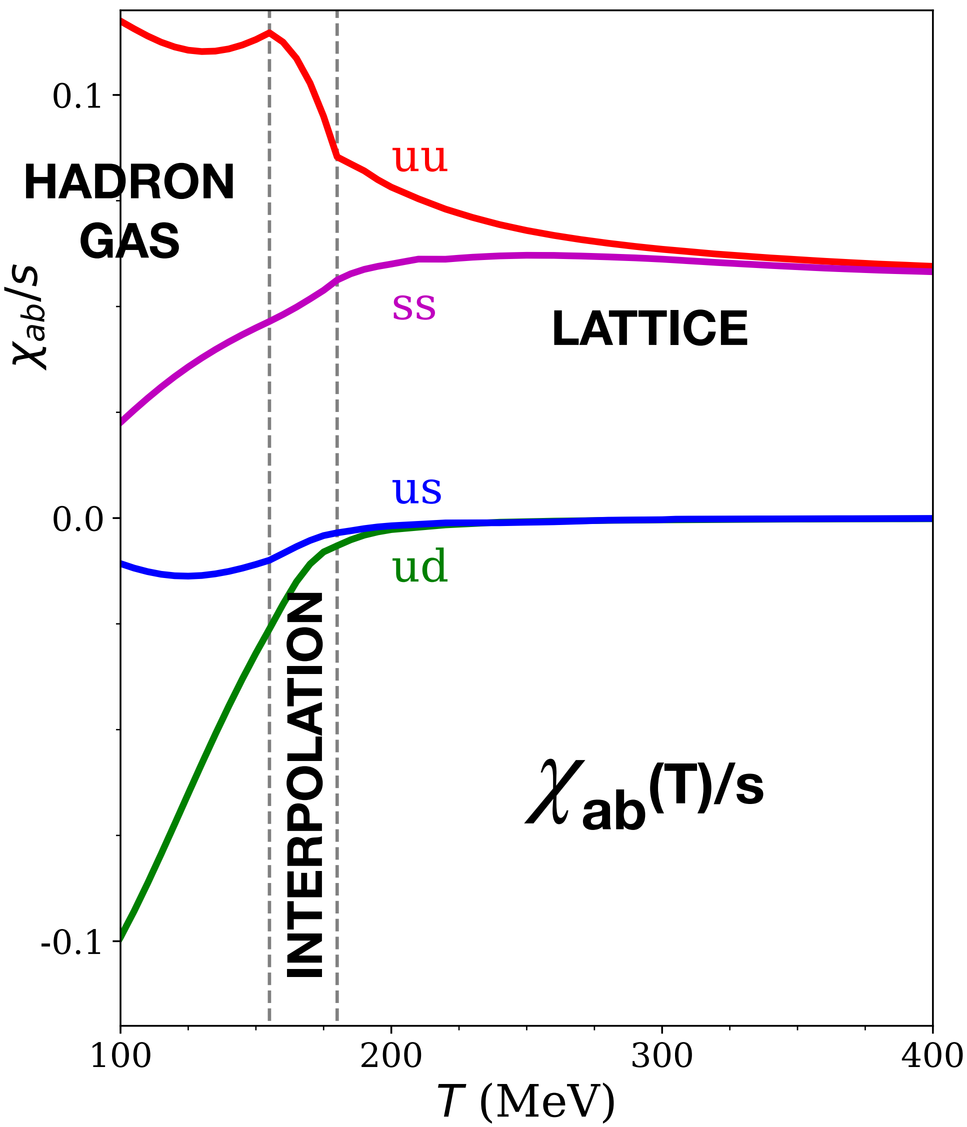
— Millions of cascade events

Adjustable Parameters

1. **Diffusion Constant $D(T)$ (multiples of lattice values)**
2. **T_h = 155 MeV**
3. **σ_0 = spread in spatial rapidity at τ_0 = 0.6 fm/c**

   — creation before τ_0 or tunneling (flux tubes)

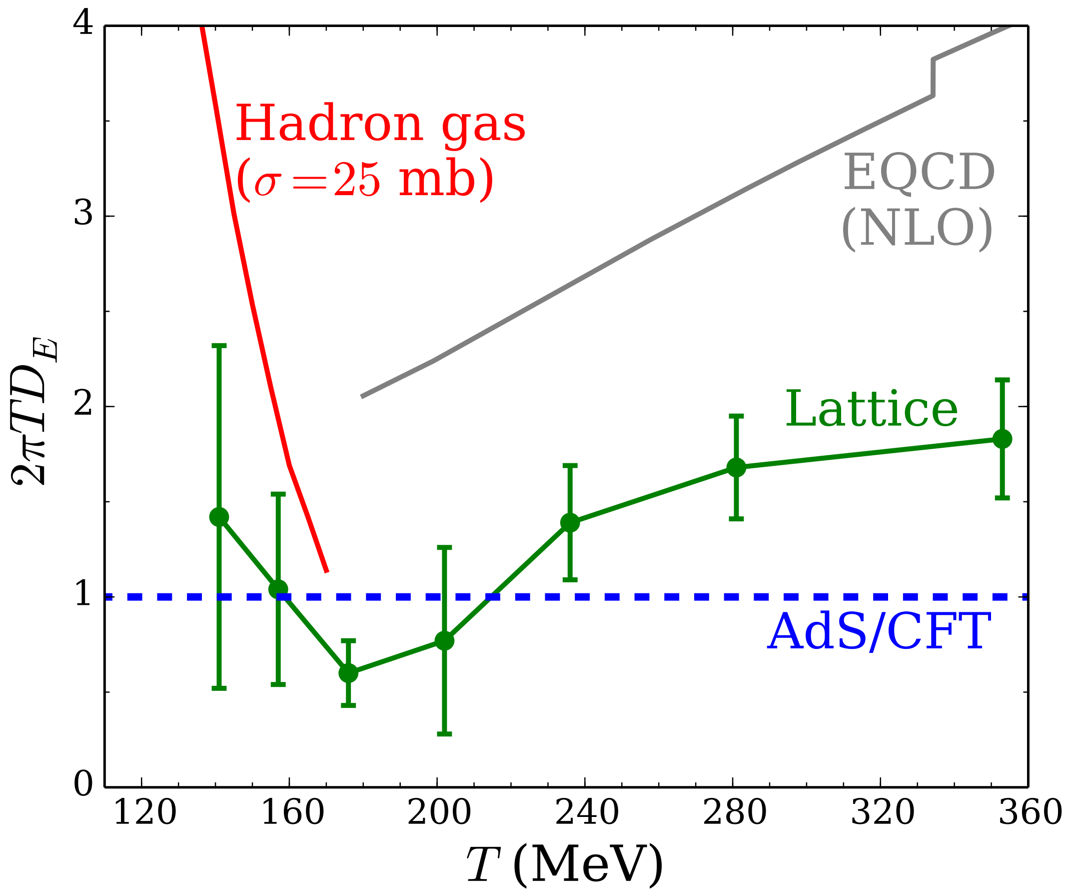




Model input Susceptibility

Claudia Ratti
BW Collaboration

$D(T)$ — No Clear Consensus



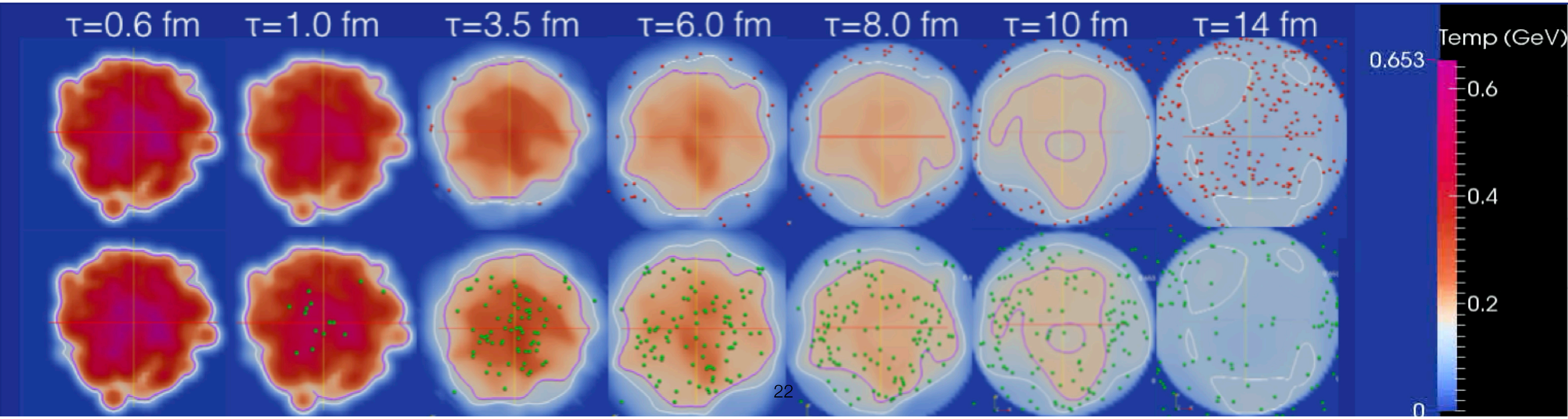
G.Aarts et al, JHEP (2015)
J.Ghiglieri et al, JHEP (2018)
G.Policastro et al, JHEP (2002)

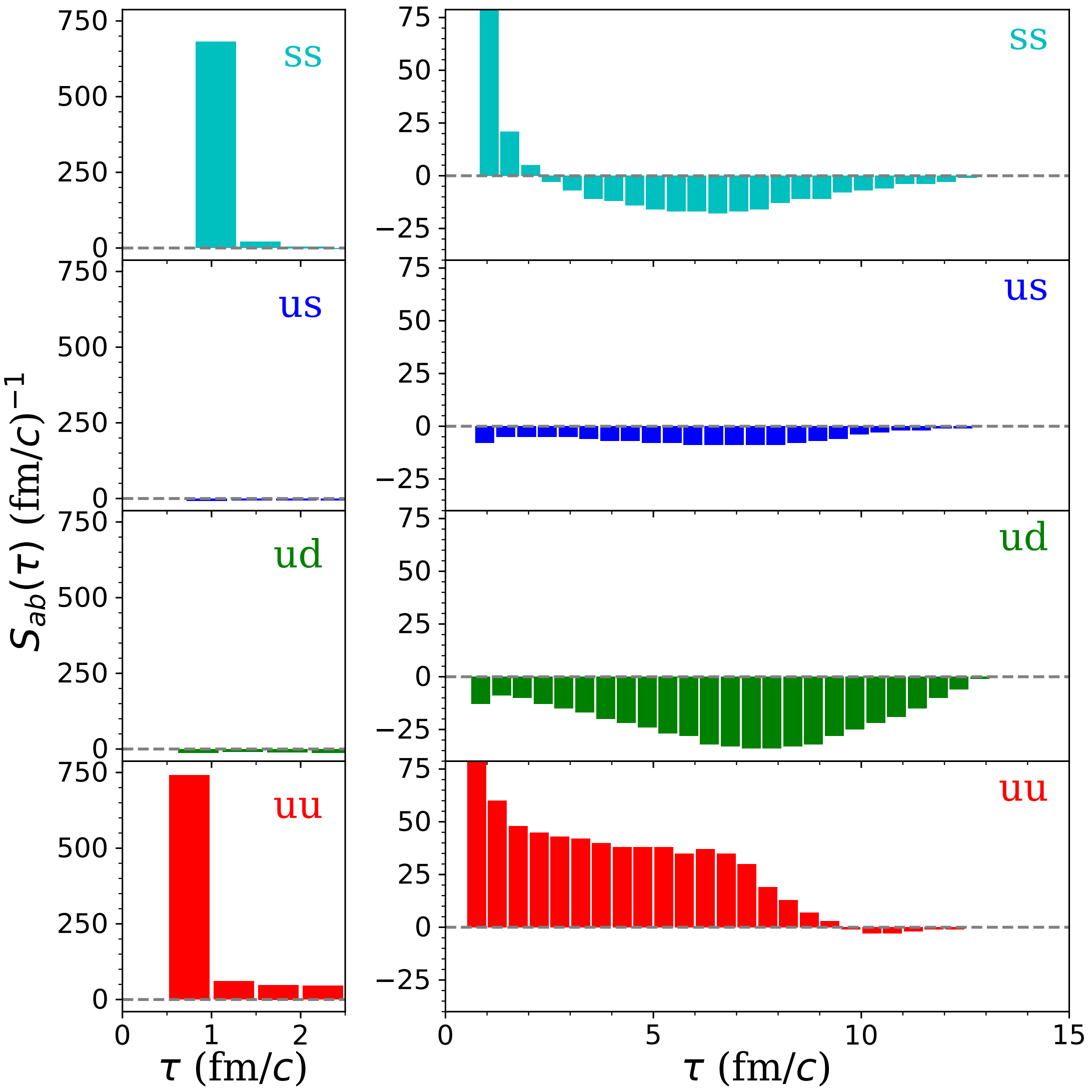
Model input
Hydro history

Andrew Gordeev

Chris Plumberg

VISHNU Hydro, Au+Au (200A GeV)

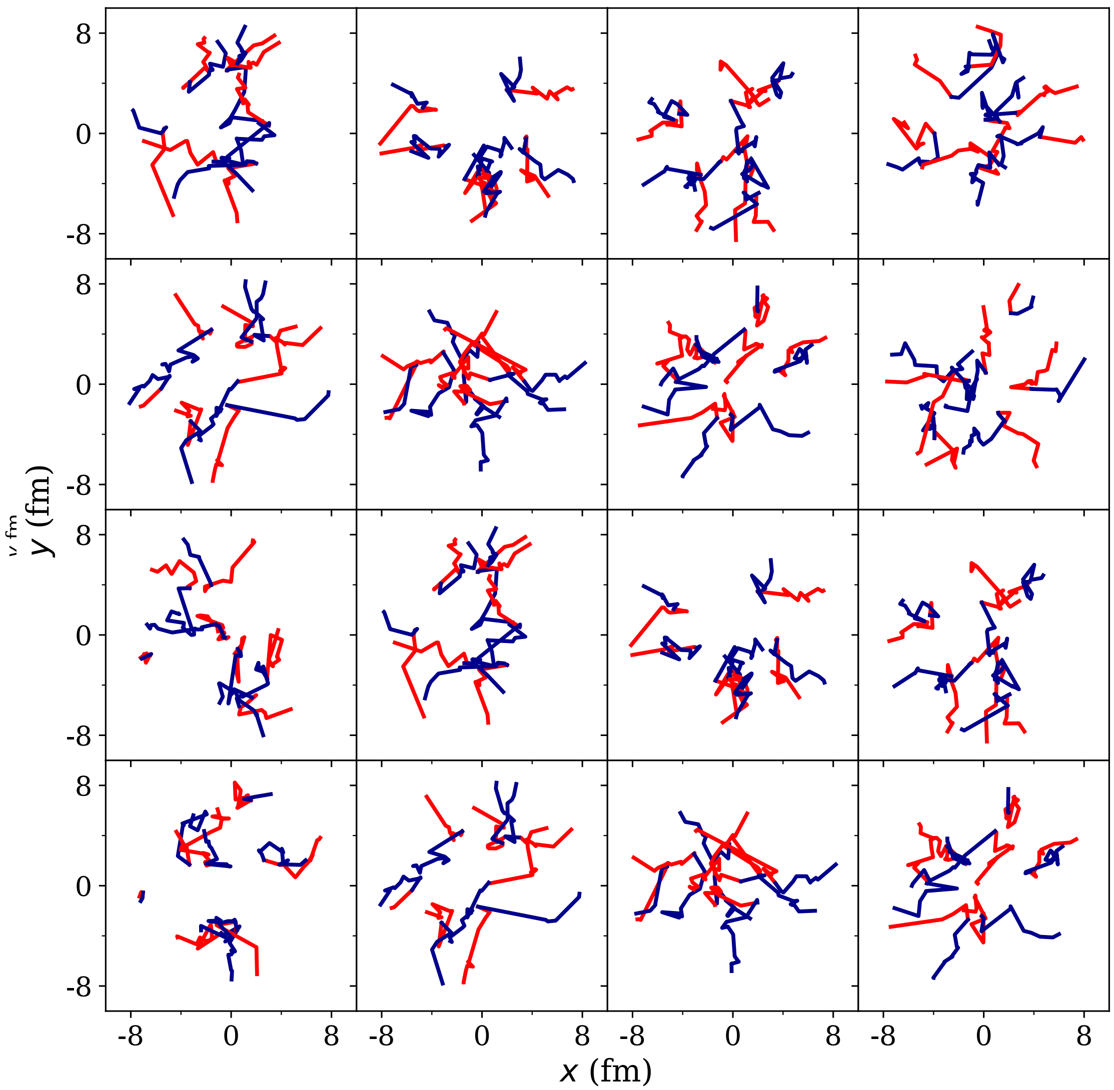




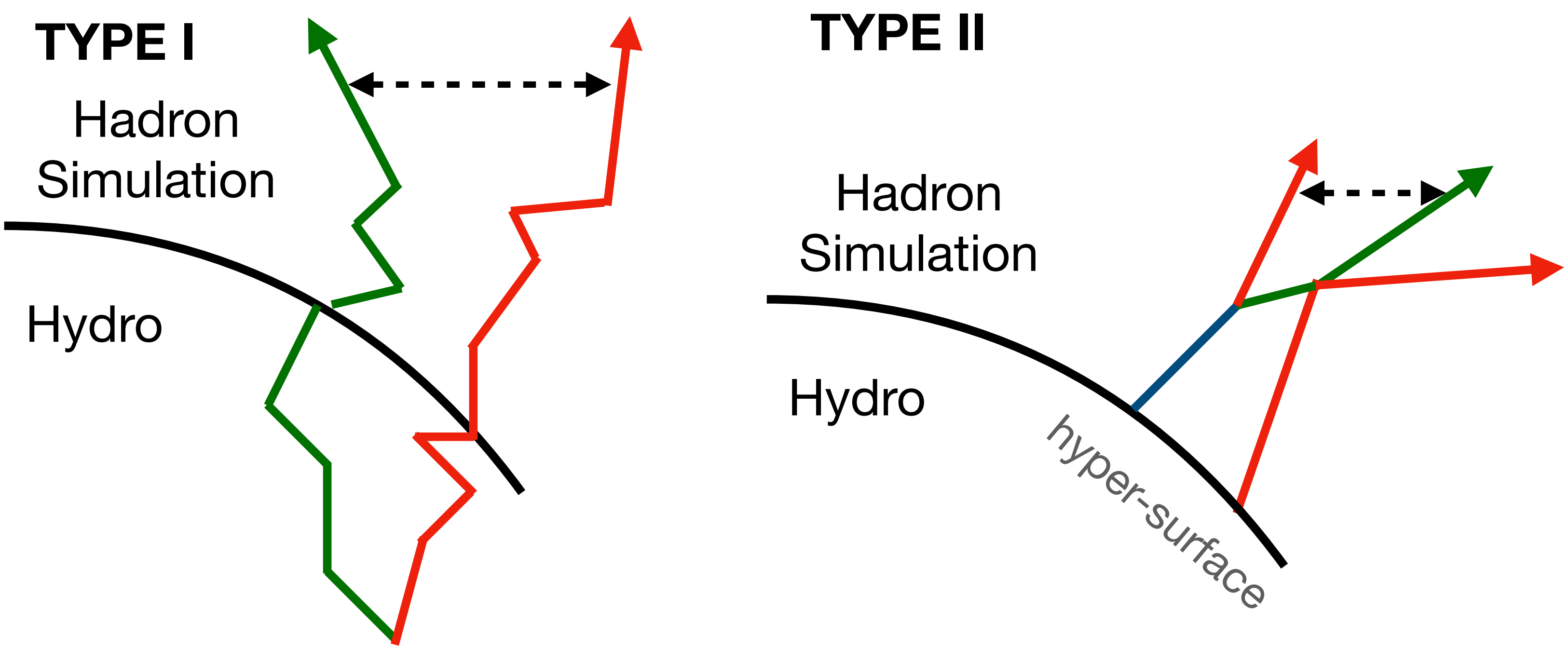
Source Function

- First surge when QGP is created
- uu,dd continuously created
- ss nearly steady
- ud,us,ds at hadronization

Diffusive Trajectories

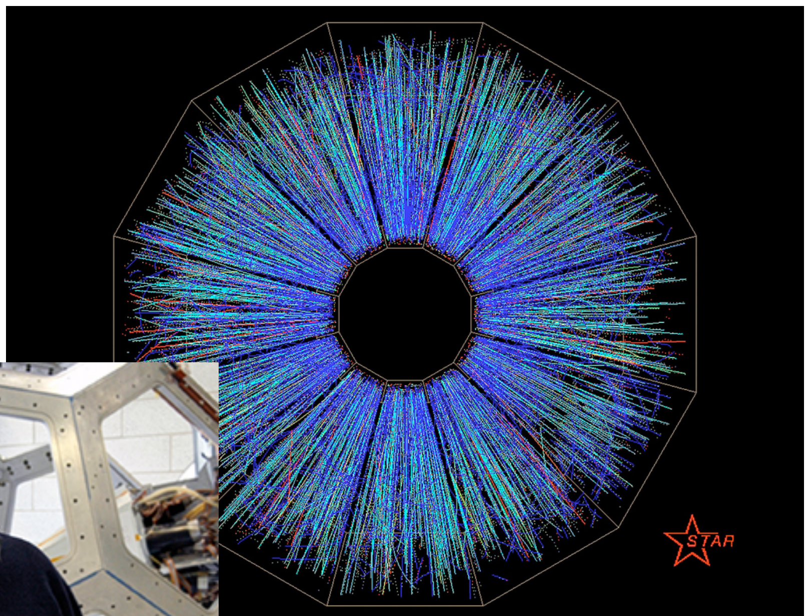


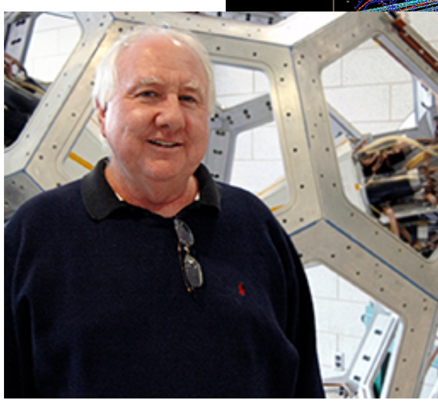
ALGORITHM: Add in Type II



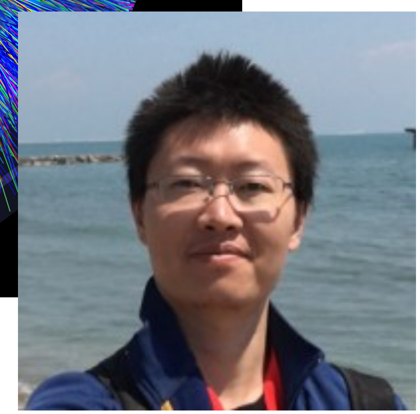
Type II done by brute force (lots of combinatoric noise)

Results vs. Data

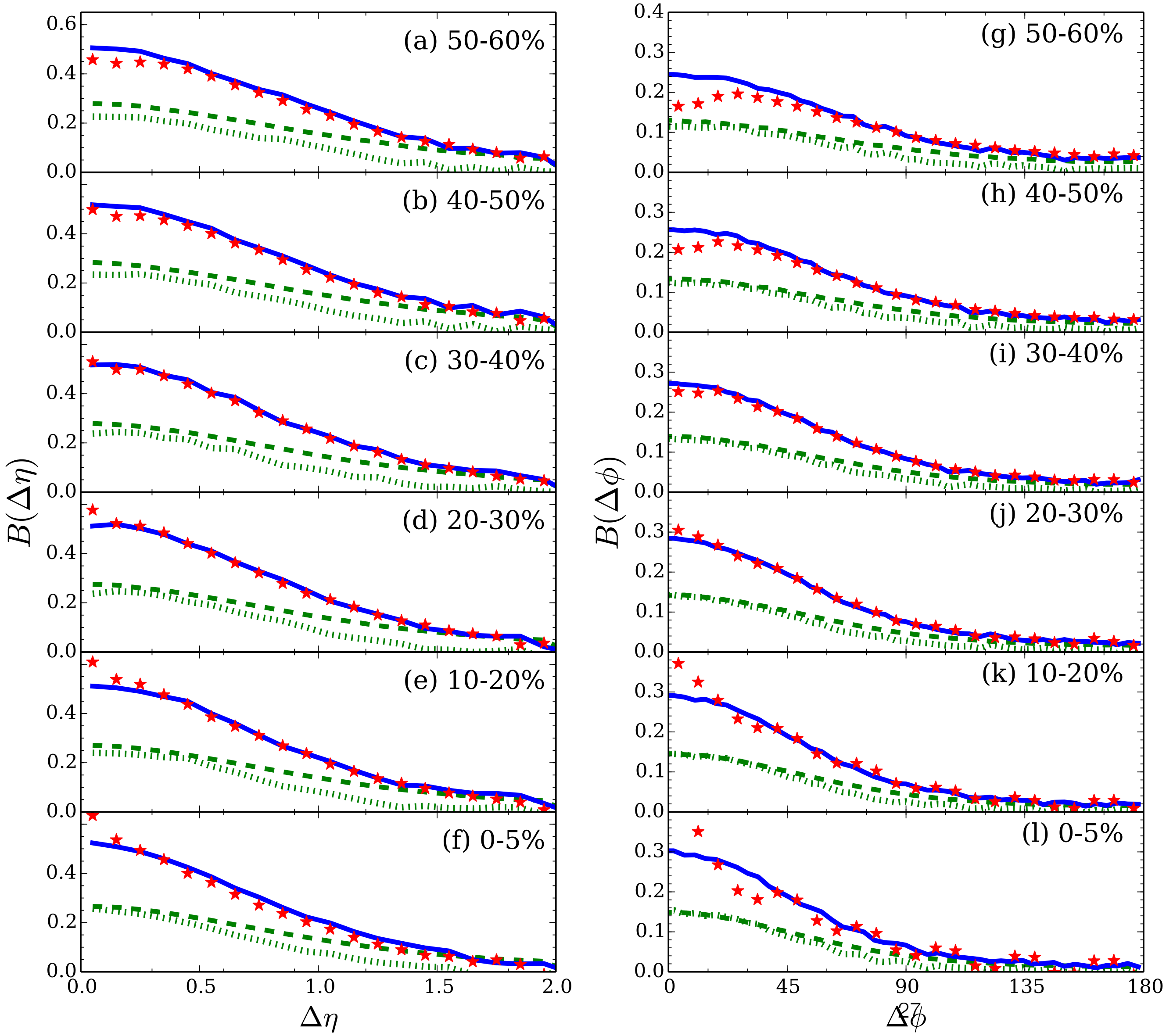




Gary Westfall
MSU



Jinjin Pan
Wayne State



Model vs. STAR

Unidentified Particles

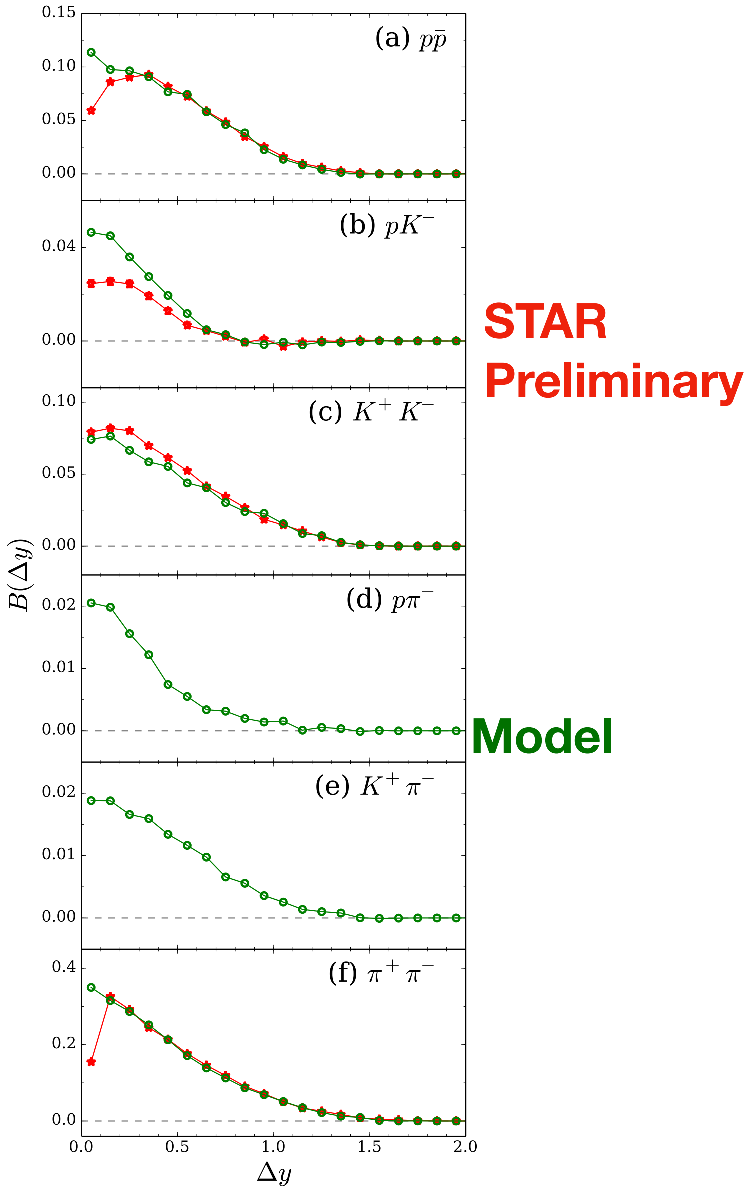
★ **STAR Preliminary**

Model, Type 1 + Type 2

Type 1 (dashes, hydro)

Type 2 (dots, cascade)

Model vs. STAR

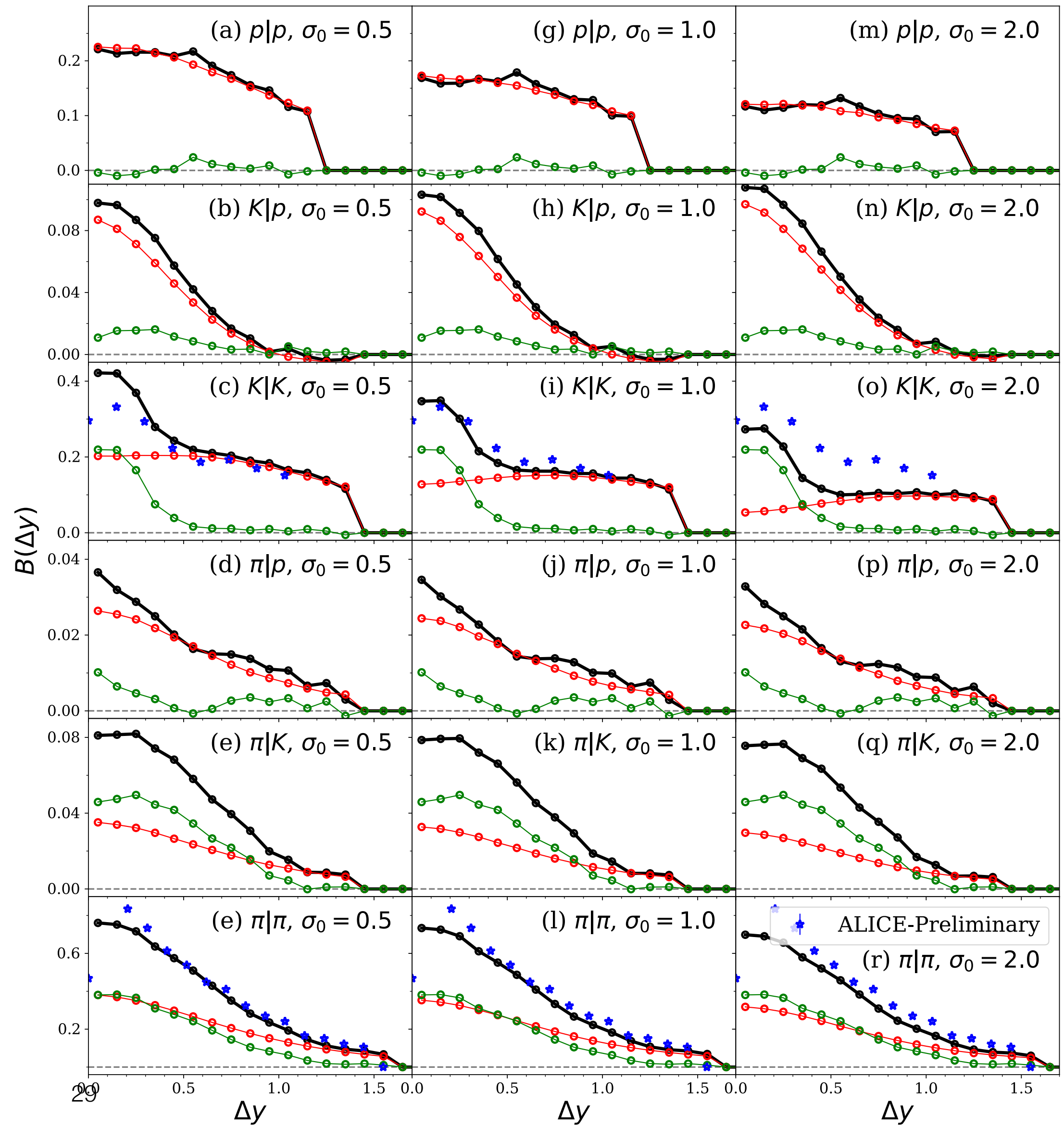


- Identified particles (vs. Δy)
- pK is off
- pp is off (annihilation missing)

Model vs ALICE

Thesis of Jin-Jin Pan

Binned by Δy



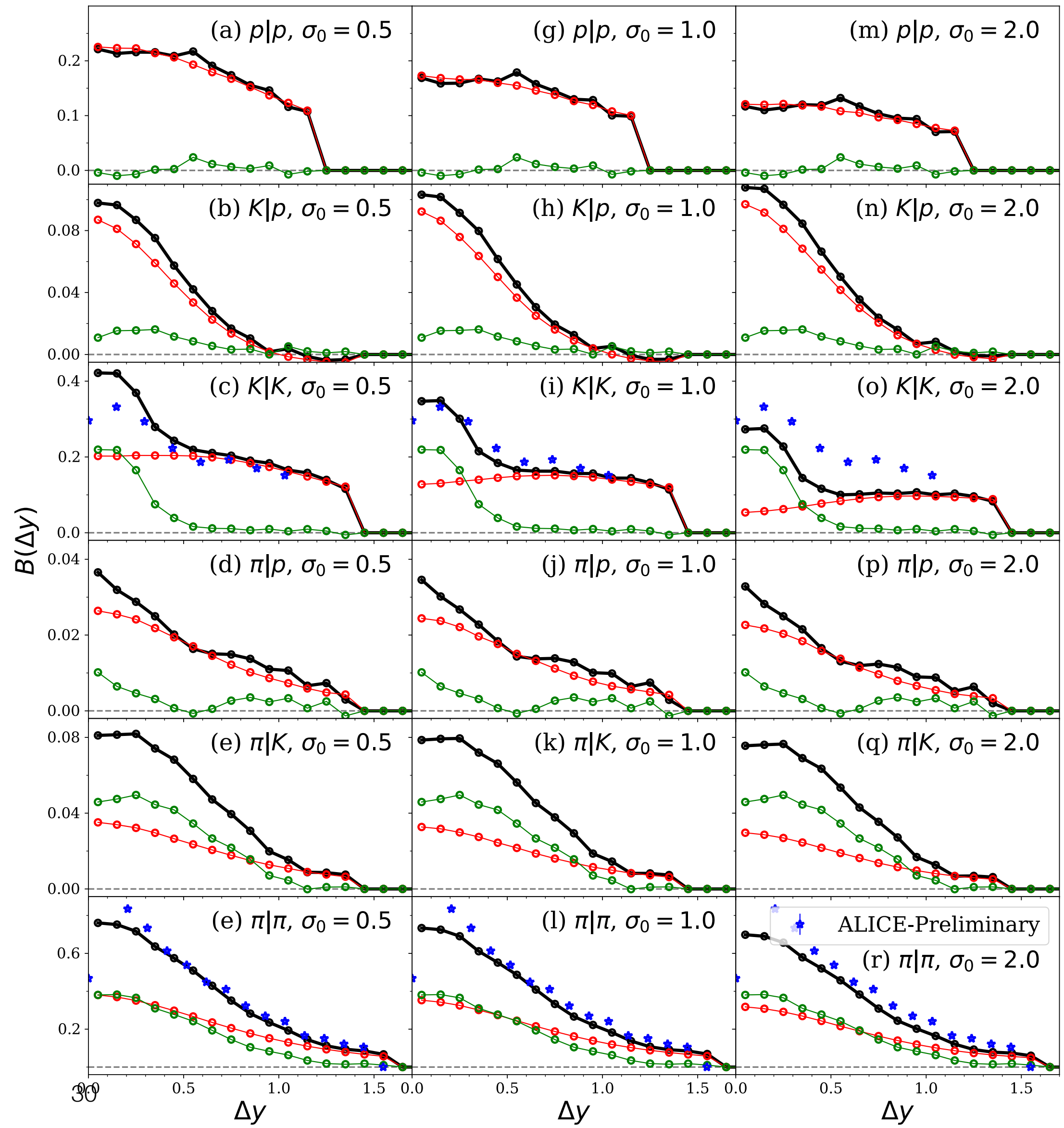
Type 1 + Type 2

Type 1 (hydro)

Type 2 (cascade)

Evidence of Early Chemical Equilibrium

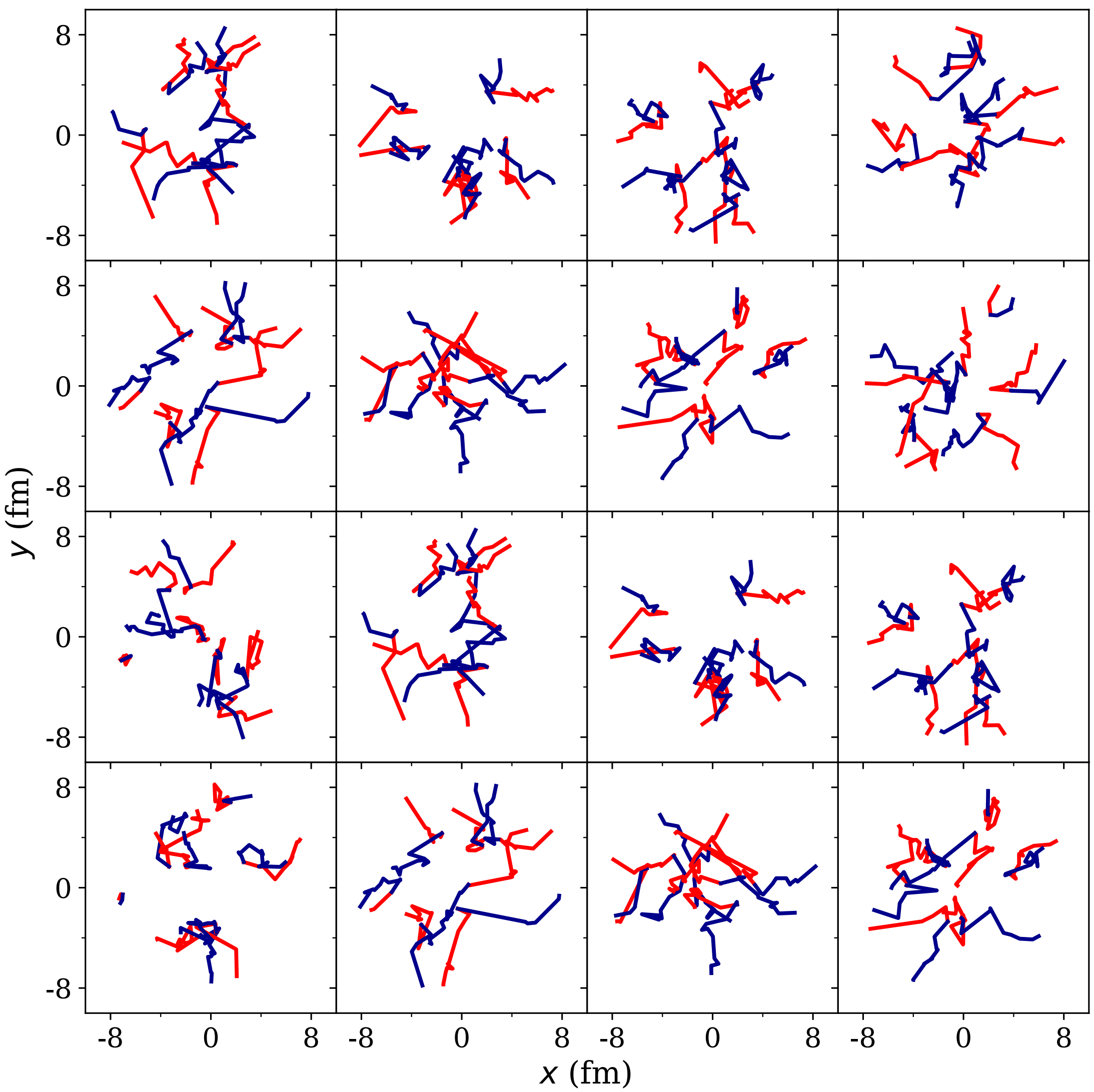
- $p\bar{p}$, K^+K^-
  BFs broader than $\pi^+\pi^-$ BFs!!
- $\sigma_0 > 0$**! charge already created at separated at** τ_0



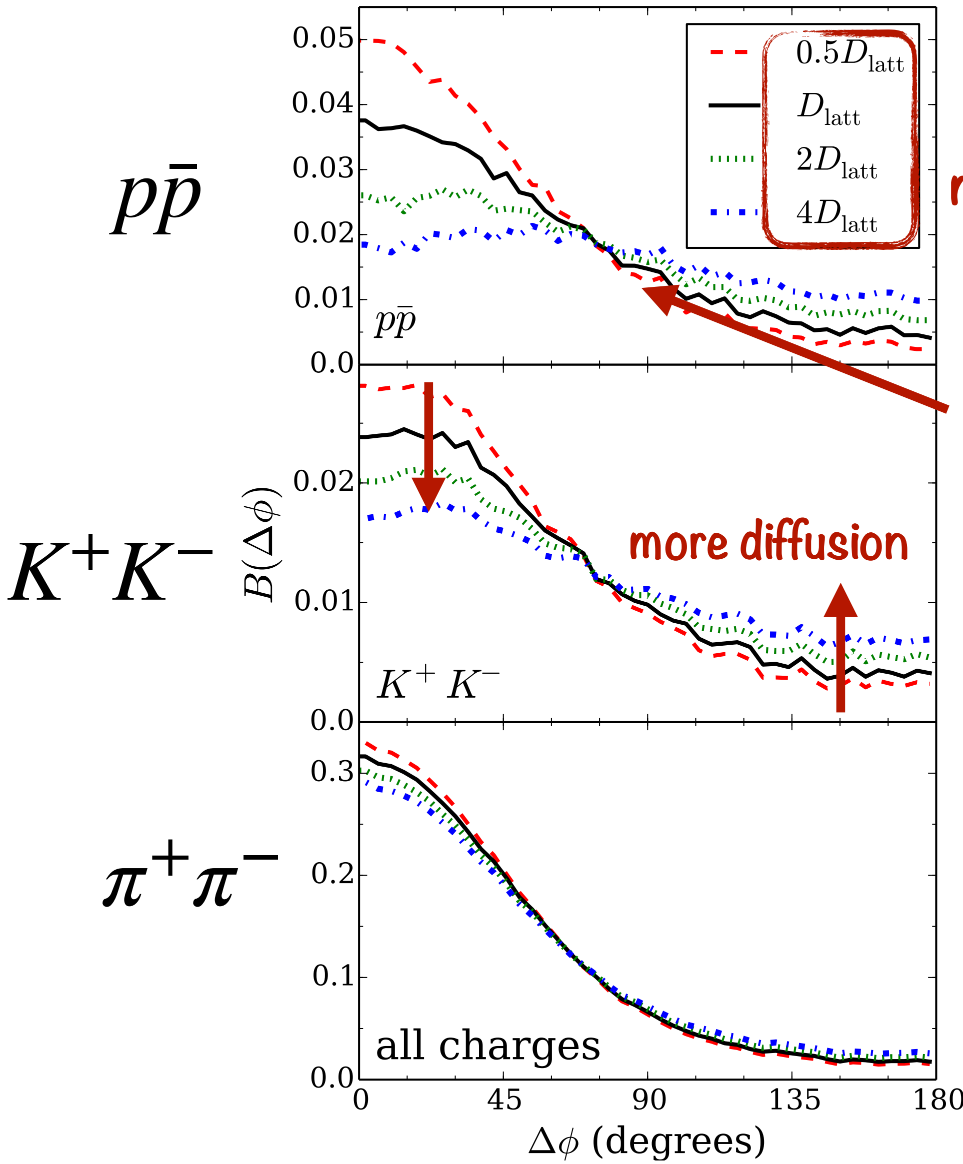
Look at $B(\Delta\phi)$

- **Width:**
  Insensitive to pre-thermal separation

- **Eliminate sensitivity to late-production**
  —pp or KK BFs
  — Only consider $\Delta y \gtrsim 1.0$



Sensitivity to Diffusivity

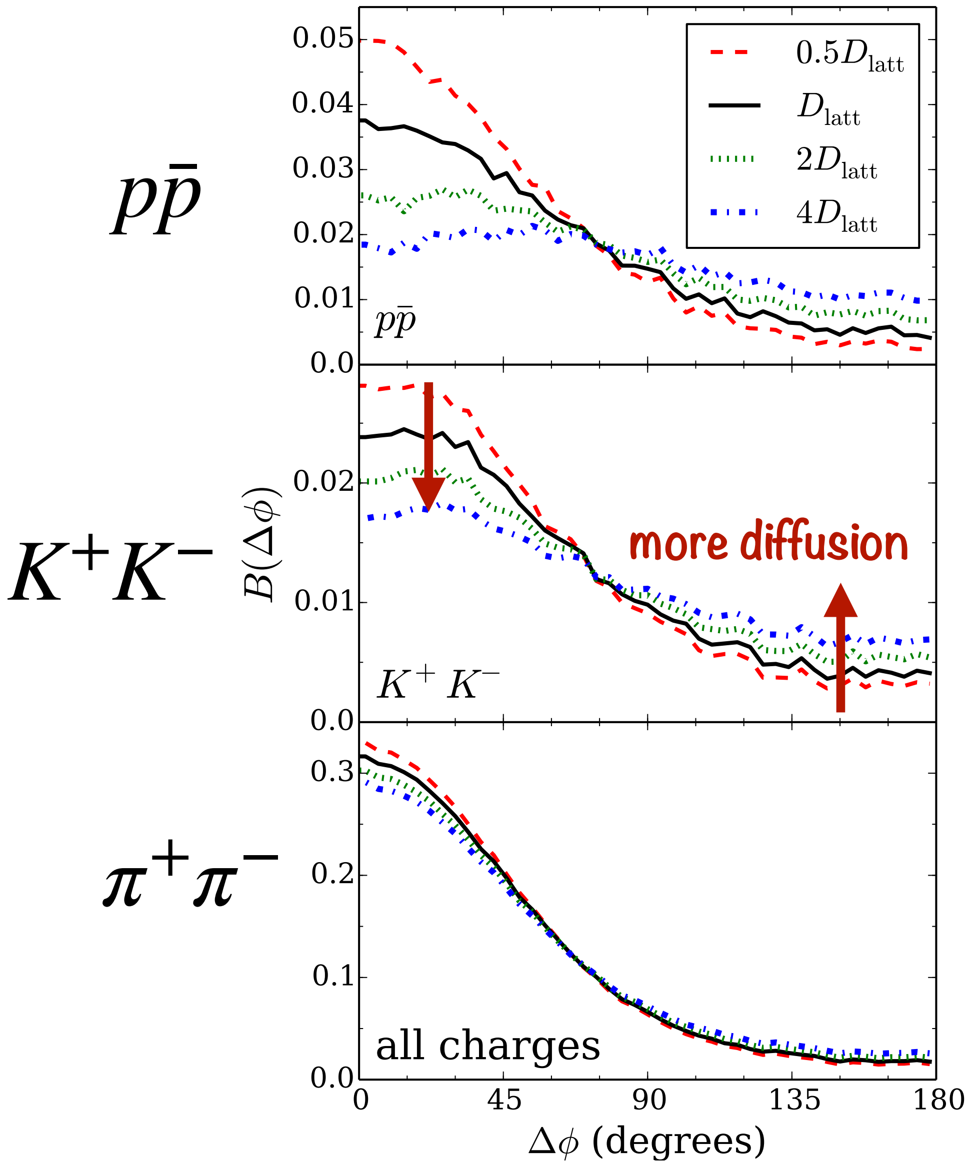


multiples of Lattice D(T)

annihilation affects results

0-5% centrality, Au+Au (200A GeV)

simulated STAR acceptance

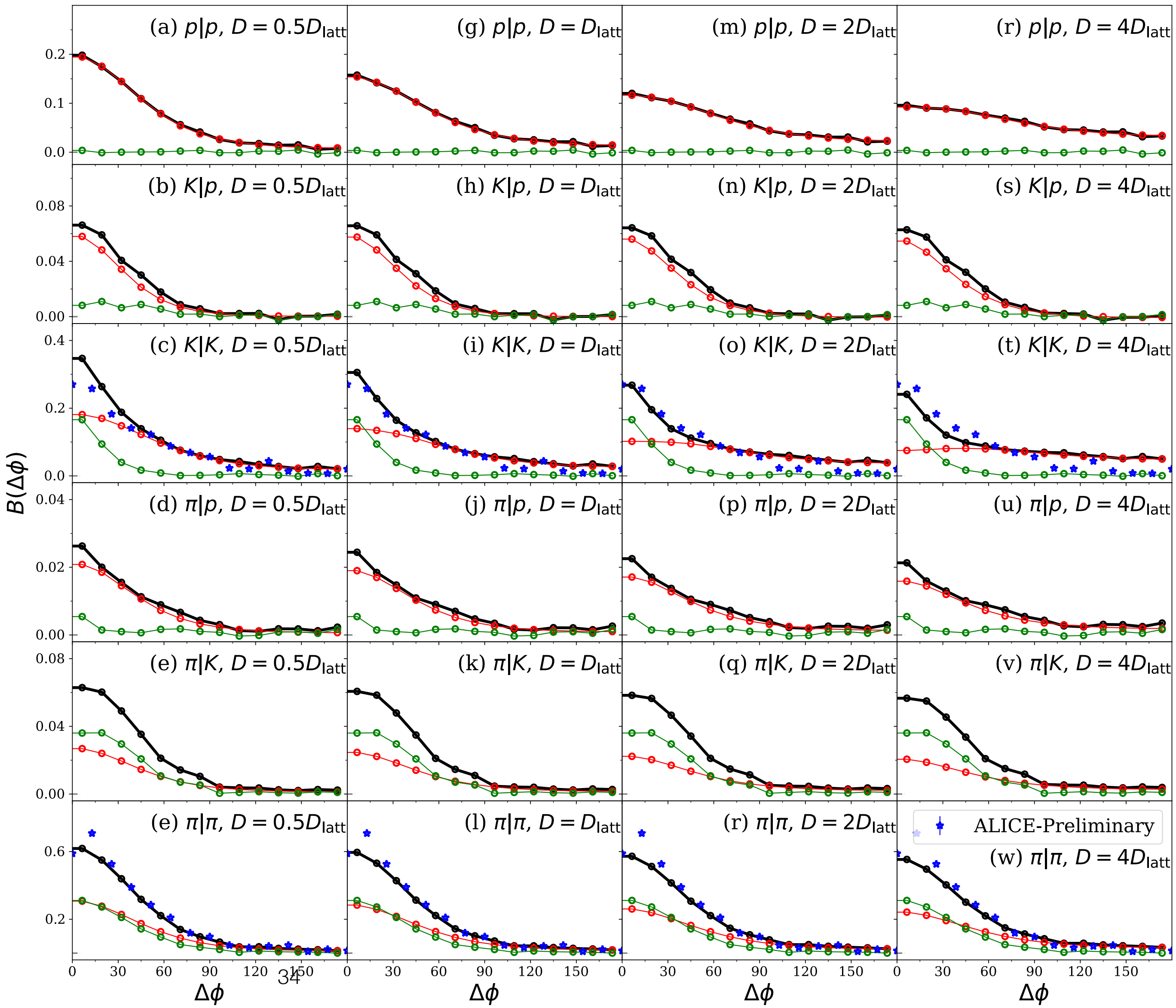


Sensitivity to Diffusivity

- $\Delta\phi$ binning reduces dependence on σ_0
- kaons or protons best suited:
- χ_{ss}/s roughly constant
  $\approx$ only phi contributes from final state
- χ_{BB}/s roughly constant
  annihilation an issue

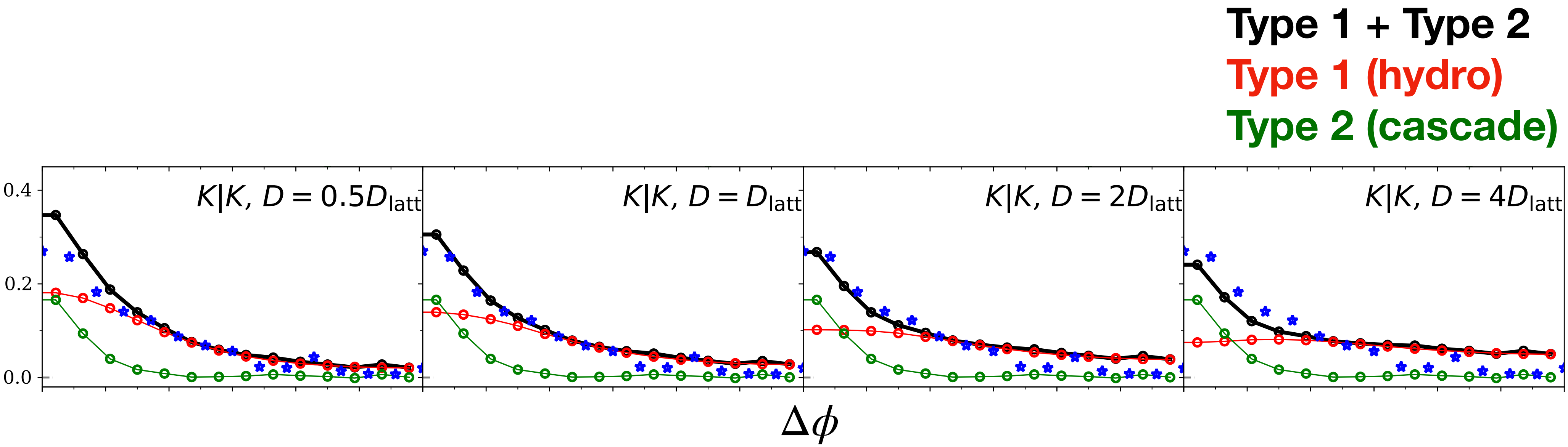
Model vs. ALICE

Binned by $\Delta\phi$



Type 1 + Type 2
Type 1 (hydro)
Type 2 (cascade)

Model vs. ALICE

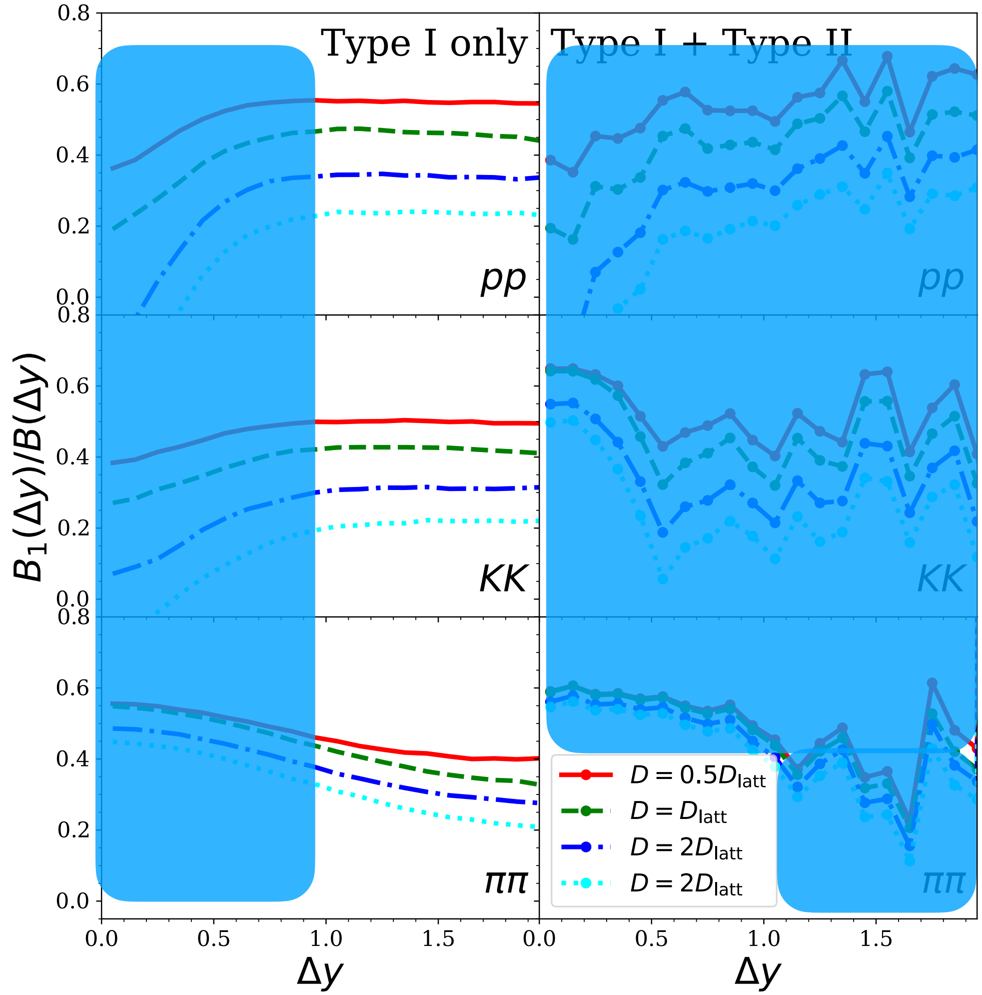


Lattice Diffusivity looks good

Analyze $B(\Delta\phi)$
Cutting on large Δy

$$B_1(\Delta y) \equiv \int d\Delta\phi B(\Delta y, \Delta\phi)\cos(\Delta\phi),$$

$$B(\Delta y) = \int d\Delta\phi B(\Delta y, \Delta\phi)$$

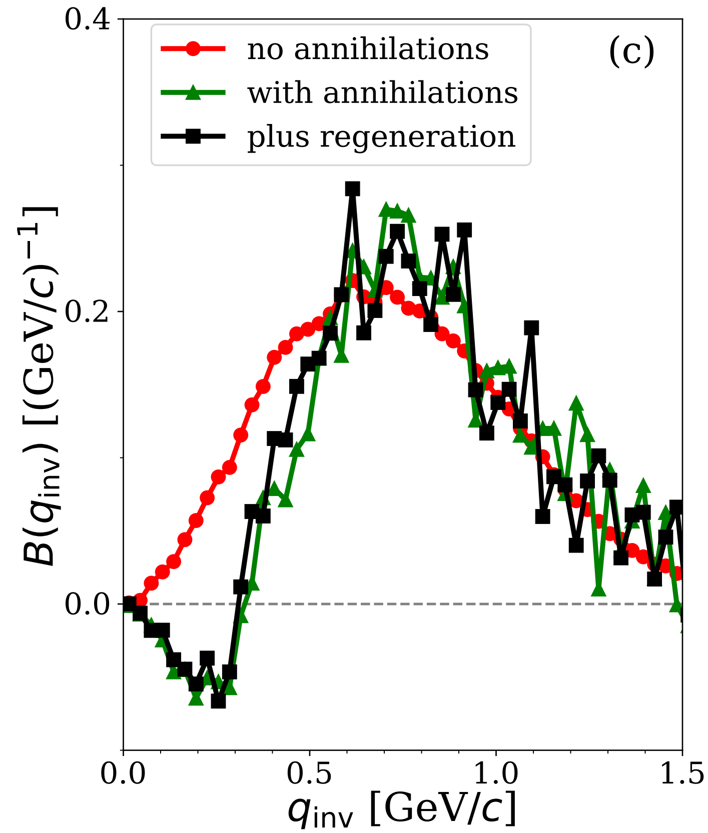


Type II only provides noise for $\Delta y \gtrsim 1$

Robust extraction of diffusivity

Eliminate HBT, annihilation, decays..

Related Progress



- Theory for Charge Balance of Higher-Order BFs
  S.P., PRC 2020
- HBT / Charge Balance Interference
  S.P. and K.Martirosova, PRC 2022
- Using BFs to Understand Baryon Annihilation
  S.P., D.Olliinchenko and C.Plumberg, PRC 2022
- Analagous Treatment of Energy-Momentum Correlations
  O. Savchuk and S.P. PRC 2024

In Progress

- Bayesian Analysis
  - New emulator built (BAND Collaboration, Oleh Savchuk)
    Based on Taylor expansion

$$E(x, y, \cdots) = \sum_{\langle n_x, n_y \cdots \rangle} \frac{1}{\sqrt{n_x! n_y! \cdots}} A_{\langle n_x, n_y \cdots \rangle} \left(\frac{x}{\Lambda}\right)^{n_x} \left(\frac{y}{\Lambda}\right)^{n_y} \cdots$$

  - Non-equilibrium chemistry (Steffen Bass and Andrew Gordeev)
  - Rigorously constrain initial chemistry!
  - Will better constrain diffusivity if we can get more data!

Summary

- Charge Balance correlations calculated in "standard model"
- STAR/ALICE BFs vs Δy suggest early chemical equilibration
  $K^+K^-, p\bar{p}, \pi^+\pi^-$ systematics reproduced (STAR pK normalization off)
- Diffusivity can be extracted from BFs binned by $\Delta\phi$ cut on large Δy
  High statistics STAR & ALICE data coming !?!?
- MANY opportunities for progress
  — Both theoretical and experimental
  — Both for diffusivity and for chemistry
  — Similar to femtoscopy

Bonus Slides

- CME background
- Skewness/kurtosis background
- Theory for higher-order charge fluctuations

U.S. DEPARTMENT OF ENERGY
Office of Science

Bonus Slides

- **CME background**
- **Baryon Annihilation**
- **Higher Moments**

Model vs. STAR

Effect of Elliptic Flow

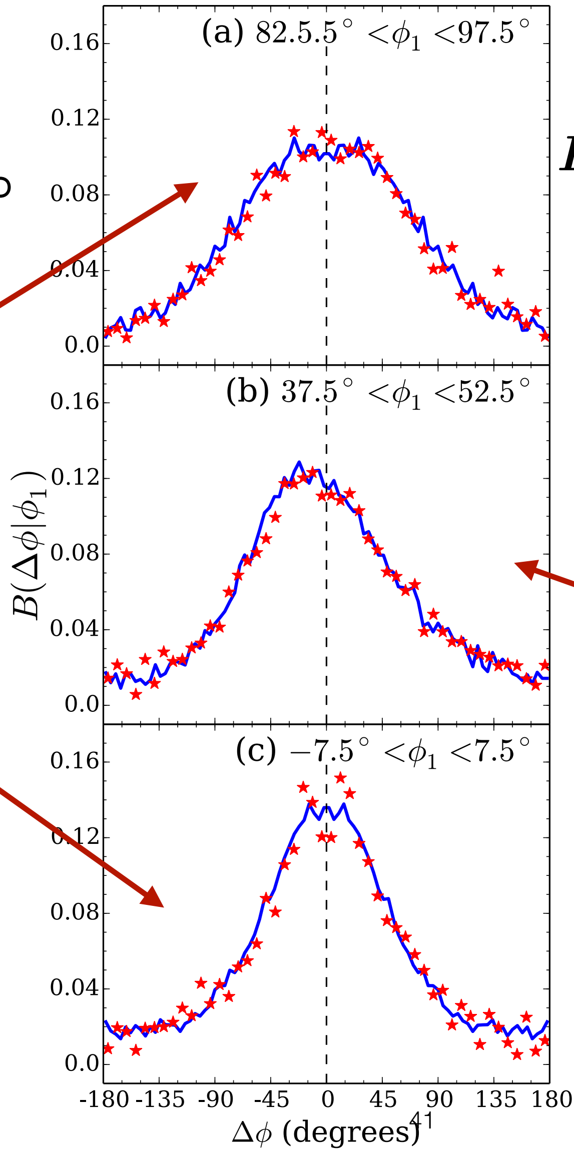
$$B(\Delta\phi|\phi_1) = \int d\phi_2 \; B(\phi_1,\phi_2)\delta(\Delta\phi - \phi_1 + \phi_2)$$

$\phi_1 \sim 90^\circ$

correlation
tighter in-plane
due to elliptic flow

$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$

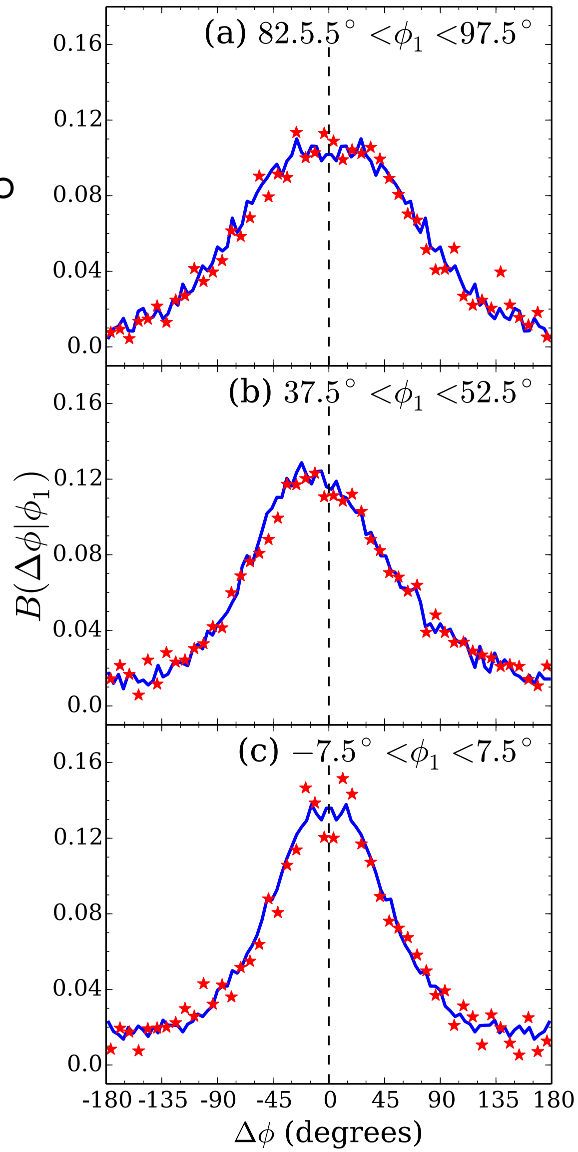


balancing charge
more likely to be
in-plane

$\phi_1 \sim 90^\circ$

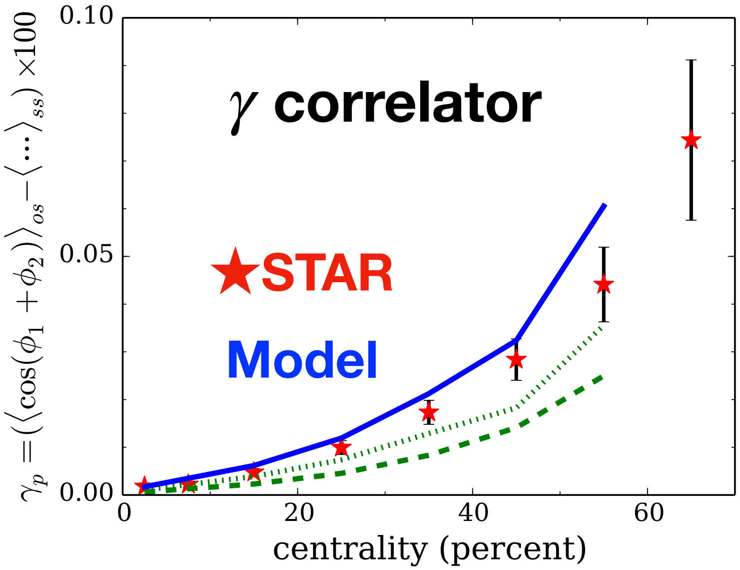
$\phi_1 \sim 45^\circ$

$\phi_1 \sim 0^\circ$



ASIDE: CME correlator

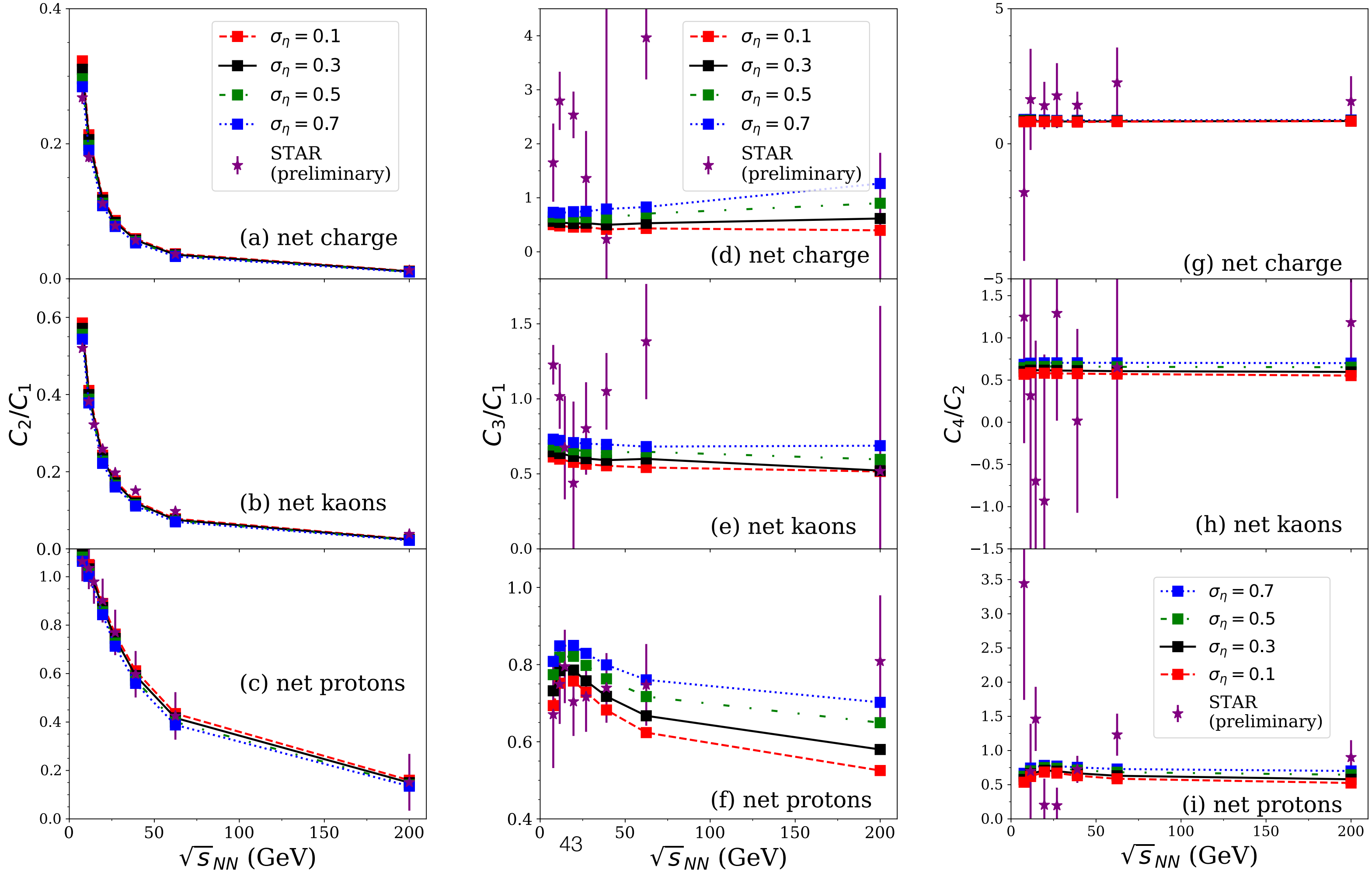
$$\gamma = \frac{1}{2}\{\langle\cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\text{opp.sign}}\}$$
$$- \frac{1}{2}\{\langle\cos(\phi_1)\cos(\phi_2) - \sin(\phi_1)\sin(\phi_2)\rangle_{\text{same.sign}}\}$$
$$= \frac{1}{M^2}\int d\phi_1 d\Delta\phi\ \frac{dM}{d\phi_1} B(\Delta\phi|\phi_1)\cos(2\phi_1 + \Delta\phi)$$



Model predicts ~ 115% of signal

Charge Conservation and Q^3, Q^4 correlations

b) Perform canonical ensemble on sub-volumes & superimpose on blast wave (crude)

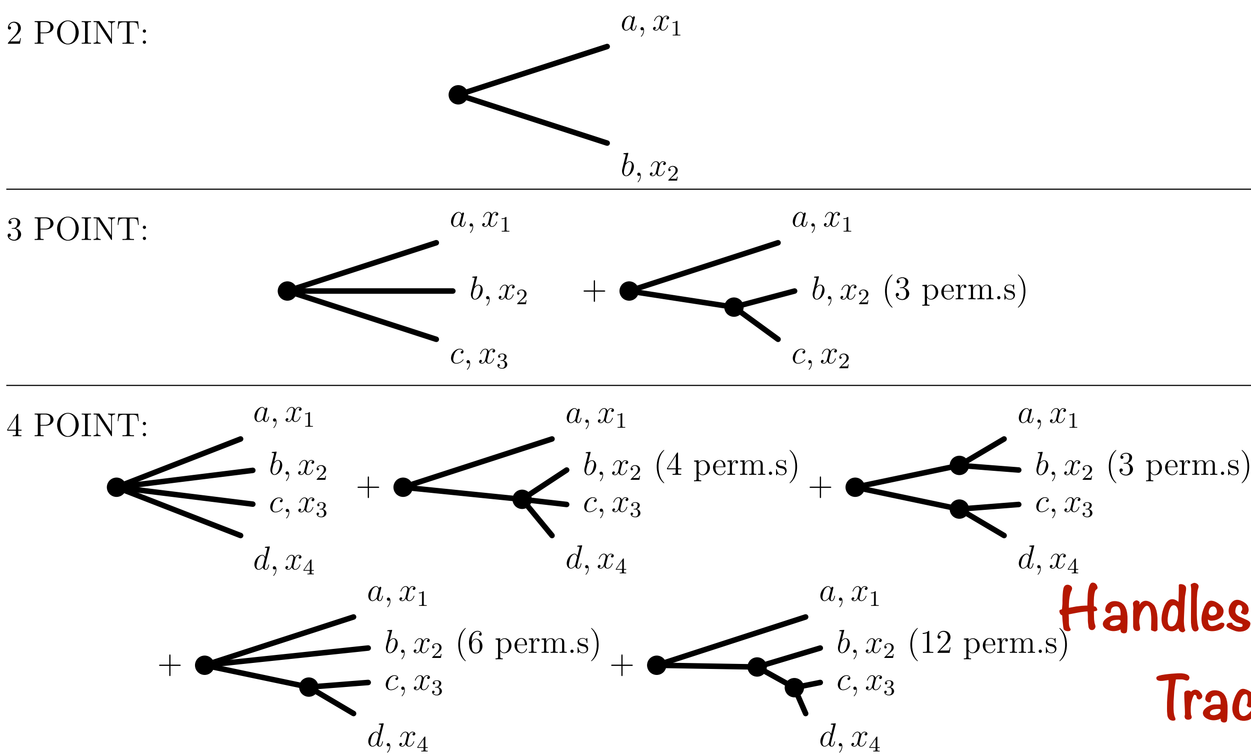


S.P. and R.Steinhorst PRC (2020)

BONUS: Charge conservation and Q^3, Q^4 correlations (formalism)

a) Integrate n-point correlations to obtain skewness & kurtosis

S.P., PRC (2020)



Handles full 3x3 flavor dynamics

Tractable, but DIFFICULT

BONUS: baryon annihilation

- $p - \bar{p}$ **BF vs** q_{inv}
- **Includes recombination**
- **Great way to constrain baryon annihilation in final-state**