

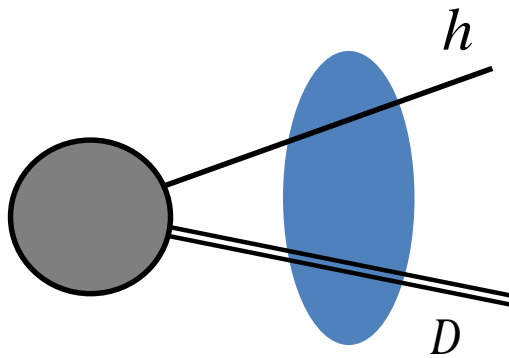
# Femtoscopic correlations of light nuclei

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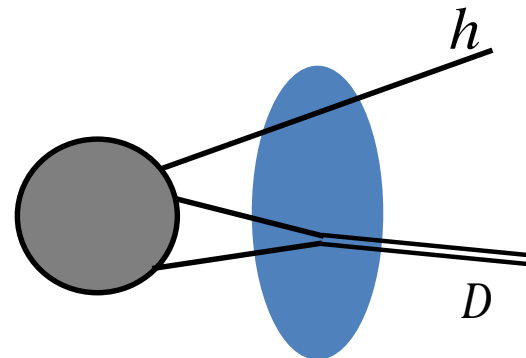
# Hadron-deuteron correlations

Hadron-deuteron correlations carry information about a mechanism of deuteron production.



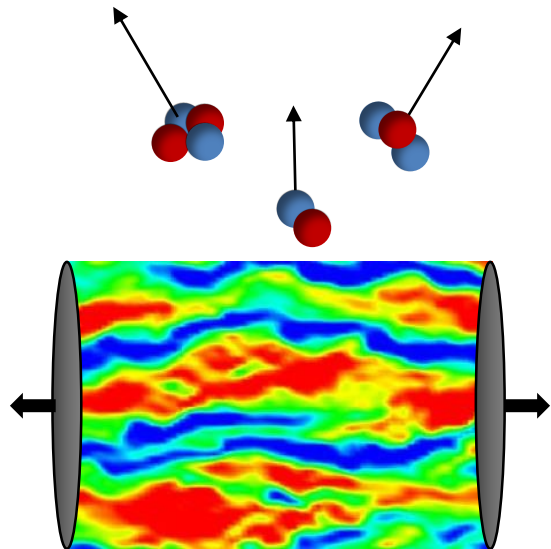
direct production

or



final state interaction

# Production of light nuclei at RHIC & LHC



baryonless matter

${}^2\text{H}$ ,  ${}^2\bar{\text{H}}$ ,  ${}^3\text{H}$ ,  ${}^3\bar{\text{H}}$ ,  ${}^3\text{He}$ ,  ${}^3\bar{\text{He}}$ ,  ${}^4\text{He}$ ,  ${}^4\bar{\text{He}}$ ,  ${}^3_{\Lambda}\text{H}$ ,  ${}^3_{\Lambda}\bar{\text{H}}$ ,  ${}^4_{\Lambda}\text{H}$ ,  ${}^4_{\Lambda}\bar{\text{H}}$

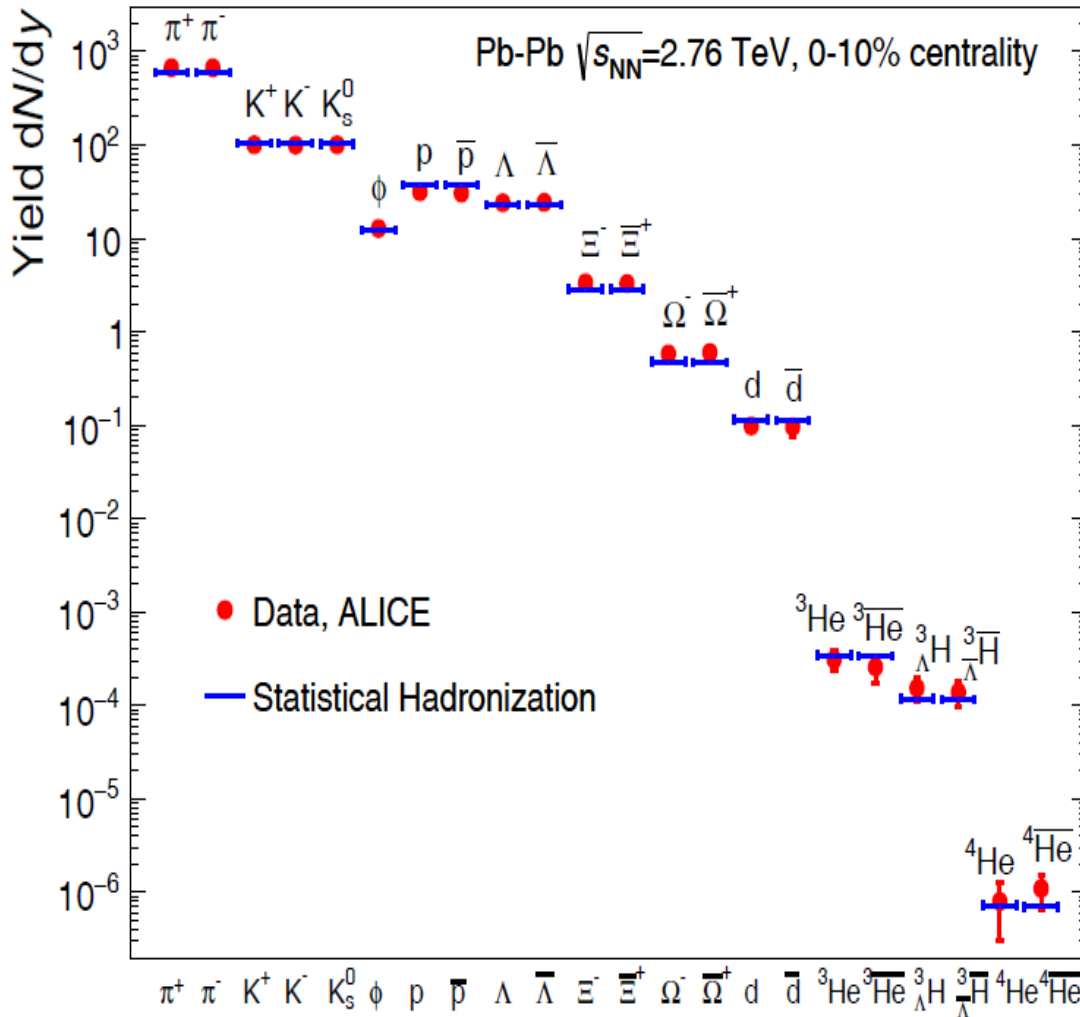
Genuine production!

Matter-antimatter symmetry!

## Two approaches to production of light nuclei

- ▶ Thermal model – direct production from thermalized hadron matter
- ▶ Coalescence model – final state interactions of nucleons

# Thermal model prediction



baryonless fireball

$$\text{Yield} \sim g e^{-\frac{m}{T}}$$

$$T = 156 \text{ MeV}$$

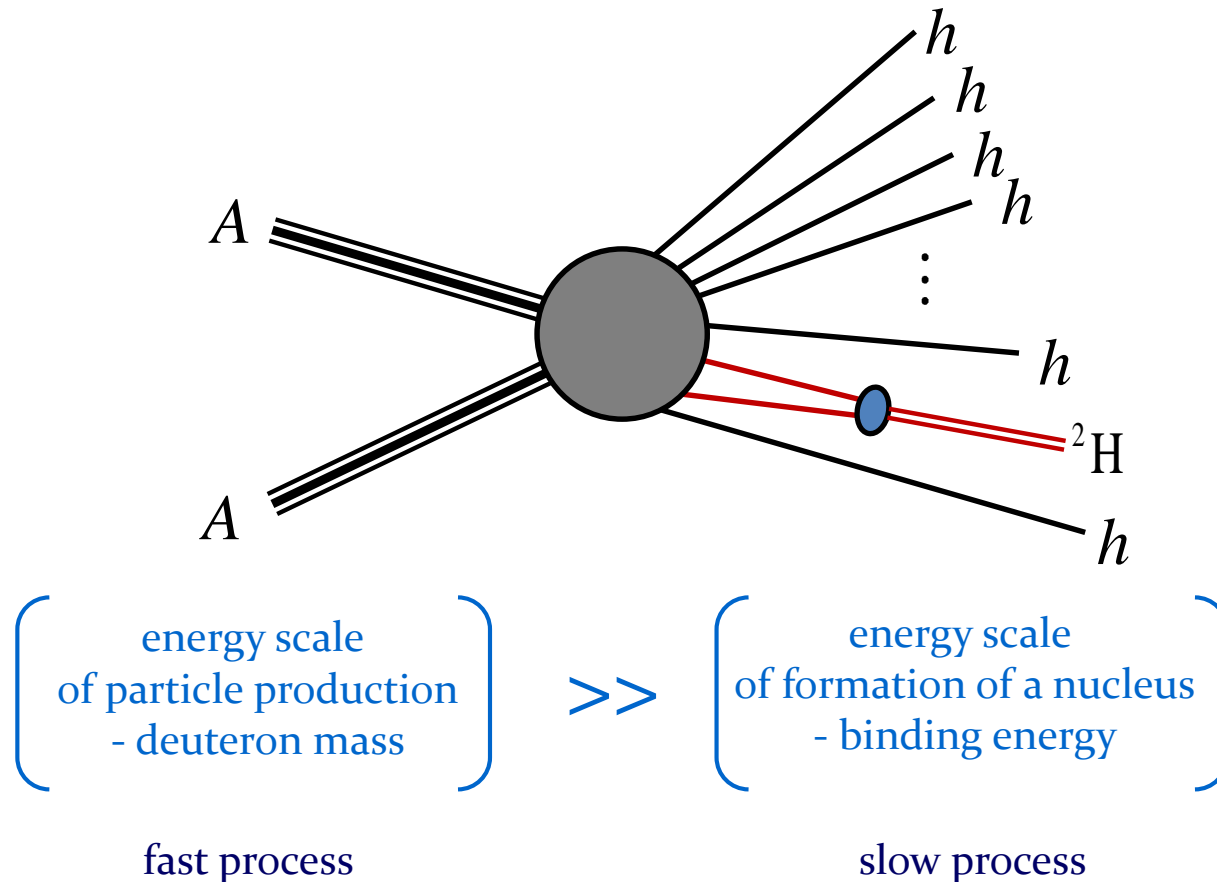
# Can light nuclei exist in a fireball?

- ▶ Interparticle spacing in a hadron gas is about 1.5 fm at  $T = 156$  MeV.
- ▶ Root mean square radius of a deuteron is 2.0 fm.
- ▶ Binding energy of a deuteron is  $\varepsilon_B = 2.2$  MeV.
- ▶ A characteristic time of deuteron formation  $t$  is longer than 2 fm/c.
- ▶ A hadron gas at  $T = 156$  MeV is essentially a classical system.

*Snowflakes in hell ?  
or  
Snowflakes from hell ?*



# Final state interaction



S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)

A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

# Factorization of production of nucleons and formation of a deuteron

Deuteron yield

$$\frac{dN^D}{d^3\mathbf{P}_D} = \frac{1}{2} A_D \frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \quad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

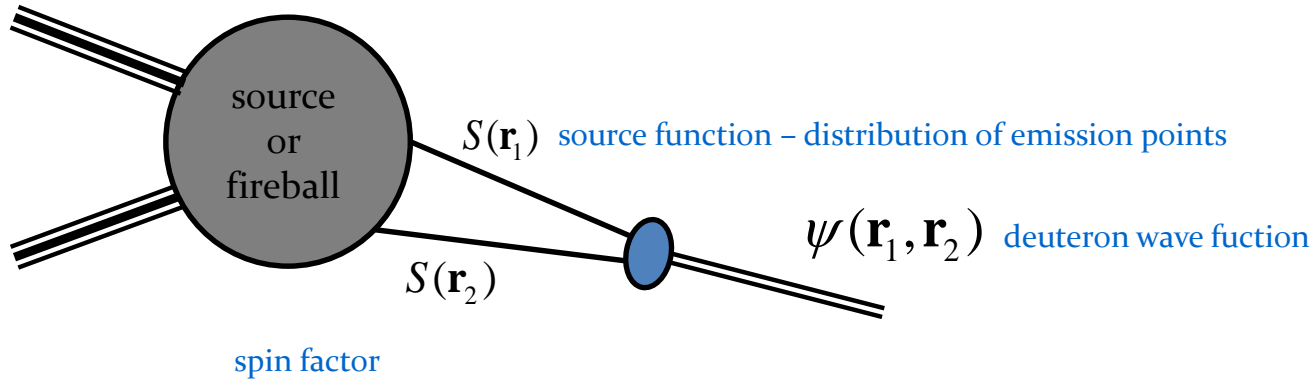
isospin factor
yield of np pairs

deuteron formation rate

$$\frac{1}{2} \frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \approx \frac{dN^{pp}}{d^3\mathbf{p}_p d^3\mathbf{p}_p} \approx \left( \frac{dN^p}{d^3\mathbf{p}_p} \right)^2$$

$$\frac{dN^D}{d^3\mathbf{P}_D} = A_D \left( \frac{dN^p}{d^3\mathbf{p}_p} \right)^2$$

# Deuteron formation rate



$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 S(\mathbf{r}_1) S(\mathbf{r}_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}} \varphi_D(\mathbf{r})$$

$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r} S_r(\mathbf{r}) |\varphi_D(\mathbf{r})|^2$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R} S\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right) \quad \text{distribution of relative distance of } n \text{ and } p$$



# Quantum-mechanical meaning of the formation rate formula

Sudden approximation

$$E\Delta t \ll 1$$

Transition matrix element

$$M = \left| \int d^3\mathbf{r} \psi^*(\mathbf{r}) \varphi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r} d^3\mathbf{r}' \varphi^*(\mathbf{r}') \underbrace{\psi(\mathbf{r}') \psi^*(\mathbf{r})}_{\text{density matrix}} \varphi(\mathbf{r})$$

$$M = \int d^3\mathbf{r} d^3\mathbf{r}' \varphi^*(\mathbf{r}') \rho(\mathbf{r}', \mathbf{r}) \varphi(\mathbf{r})$$

If density matrix is diagonal

$$\rho(\mathbf{r}', \mathbf{r}) = S(\mathbf{r}) \delta^{(3)}(\mathbf{r}' - \mathbf{r}) \quad \Rightarrow \quad M = \int d^3\mathbf{r} S(\mathbf{r}) |\varphi(\mathbf{r})|^2$$

# Diagonal density matrix

$$\langle \psi | \hat{A} | \psi \rangle = \sum_{i,j} c_i^* c_j \langle \alpha_i | \hat{A} | \alpha_j \rangle = \sum_{i,j} \rho_{ji} A_{ij}$$

$$|\psi\rangle = \sum_i c_i |\alpha_i\rangle$$

$$\rho_{ji} \equiv c_i^* c_j$$

$$A_{ij} \equiv \langle \alpha_i | \hat{A} | \alpha_j \rangle$$

density matrix

..... - averaging over time or events

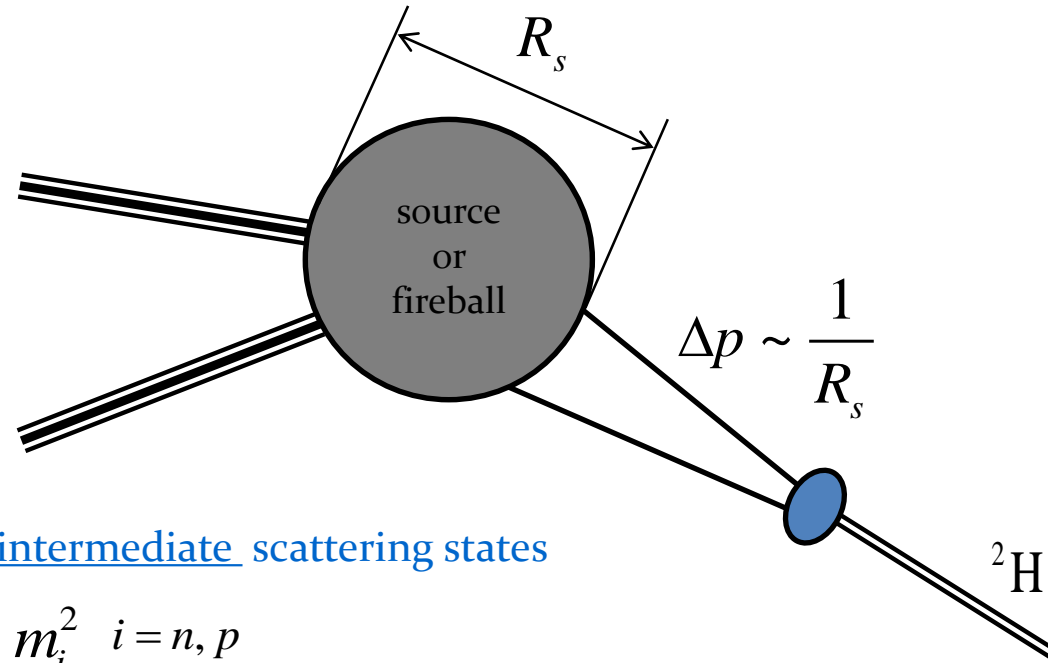
$$\overline{\langle \psi | \hat{A} | \psi \rangle} = \sum_{i,j} \overline{c_i^* c_j} \langle \alpha_i | \hat{A} | \alpha_j \rangle = \sum_i |c_i|^2 A_{ii}$$

$$\overline{\rho_{ji}} = \overline{c_i^* c_j} = \delta^{ij} |c_i|^2$$

random phase approximation

diagonal density matrix

# Energy-momentum conservation



Nucleons are intermediate scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

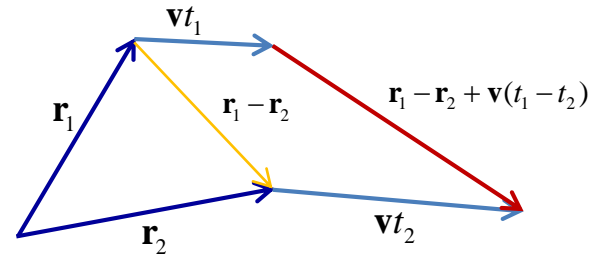
Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

# Emission time

## Instantaneous emission

$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 S(\mathbf{r}_1) S(\mathbf{r}_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$



## Emission extended in time

$$A_D = \frac{3}{4} (2\pi)^3 \int dt_1 d^3\mathbf{r}_1 dt_2 d^3\mathbf{r}_2 S(t_1, \mathbf{r}_1) S(t_2, \mathbf{r}_2) |\psi(\mathbf{r}_1 + \mathbf{v}t_1, \mathbf{r}_2 + \mathbf{v}t_2)|^2$$

$$\int dt d^3\mathbf{r} S(t, \mathbf{r}) = 1 \quad \mathbf{v} = \frac{\mathbf{P}_D}{E_D} \quad \left\{ \begin{array}{l} \mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2 \\ T \equiv \frac{1}{2}(t_1 + t_2), \quad t \equiv t_1 - t_2 \end{array} \right. \quad \psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P} \cdot \mathbf{R}} \varphi(\mathbf{r})$$

$$S_r(t, \mathbf{r}) \equiv \int dT d^3\mathbf{R} S\left(T - \frac{1}{2}t, \mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(T + \frac{1}{2}t, \mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

$$S_r(\mathbf{r}) \equiv \int dt S_r(t, \mathbf{r} - \mathbf{v}t)$$

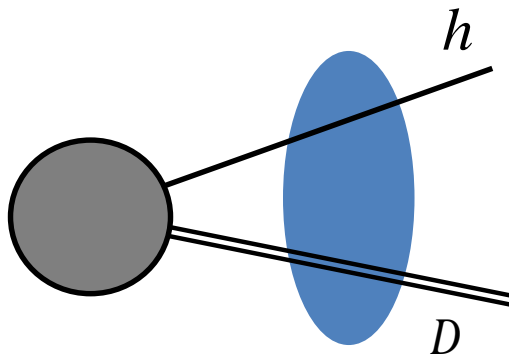
$$S(t, \mathbf{r}) = \left(\frac{1}{2\pi\tau^2}\right)^{1/2} \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{t^2}{2\tau^2}\right) \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r} S_r(\mathbf{r}) |\varphi(\mathbf{r})|^2$$

$$R_s \rightarrow \sqrt{R_s^2 + v^2\tau^2}$$

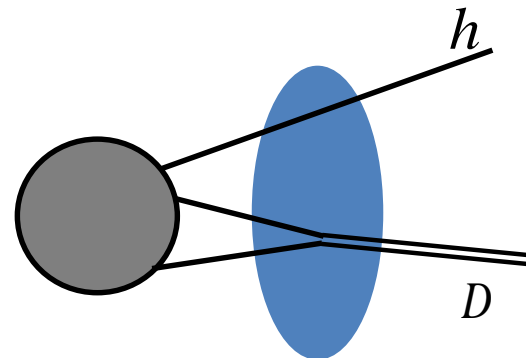
# Hadron-deuteron correlations

Hadron-deuteron correlations carry information about a mechanism of deuteron production.



direct production

or



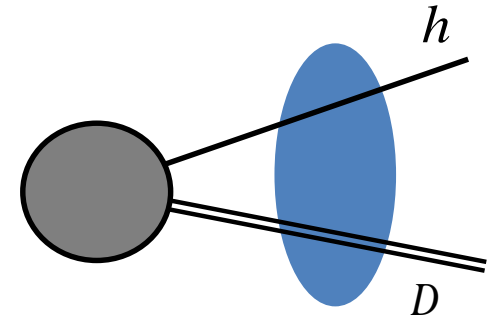
final state interaction

# Hadron-deuteron correlation function

## 1) Deuteron is treated as an elementary particle

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D) \frac{dN_h}{d\mathbf{p}_h} \frac{dN_D}{d\mathbf{p}_D}$$



Theoretical formula

$$C(\mathbf{p}_h, \mathbf{p}_D) = \int d^3r_h d^3r_D S(\mathbf{r}_h) S(\mathbf{r}_D) |\psi(\mathbf{r}_h, \mathbf{r}_D)|^2$$

↑ ↑  
distribution  
of emission points

←  
 $h$ - $D$  wave function

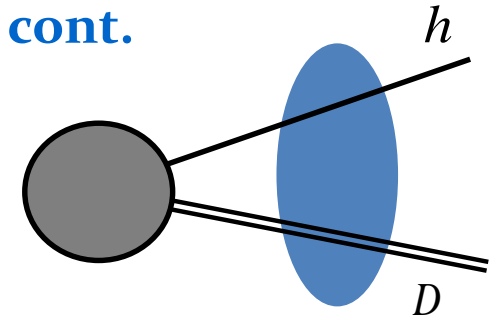
S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

# Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.

Separation of CM and relative motion



$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{array} \right. \quad \psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$C(\mathbf{q}) = \int d^3 r S_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

„Relative” source function

$$S_r(\mathbf{r}) \equiv \int d^3 R S\left(\mathbf{R} - \frac{m_D}{m_D + m_h} \mathbf{r}\right) S\left(\mathbf{R} + \frac{m_h}{m_D + m_h} \mathbf{r}\right) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2}\right)$$

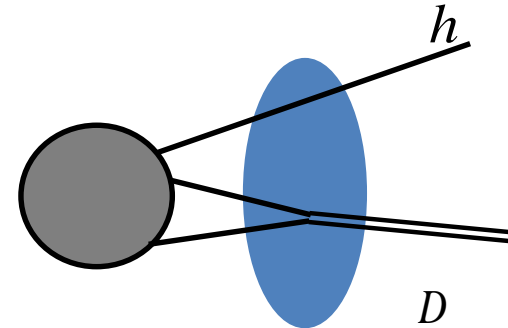
$$S(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

# Hadron-deuteron correlation function

## 2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D) A_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$C(\mathbf{p}_h, \mathbf{p}_D) A_D = \int d^3 r_h d^3 r_n d^3 r_p S(\mathbf{r}_h) S(\mathbf{r}_n) S(\mathbf{r}_p) |\psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p)|^2$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = A_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \quad \frac{1}{2} \mathbf{p}_D = \mathbf{p}_n = \mathbf{p}_p$$

$$A_D = \frac{3}{8} (2\pi)^3 \int d^3 \mathbf{r}_n d^3 \mathbf{r}_p S(\mathbf{r}_n) S(\mathbf{r}_p) |\psi_D(\mathbf{r}_n, \mathbf{r}_p)|^2 = \frac{3}{8} (2\pi)^3 \int d^3 r_{np} S_r(\mathbf{r}_{np}) |\phi_D(\mathbf{r}_{np})|^2$$

spin-isospin factor

$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_D(\mathbf{r}_{np})$$

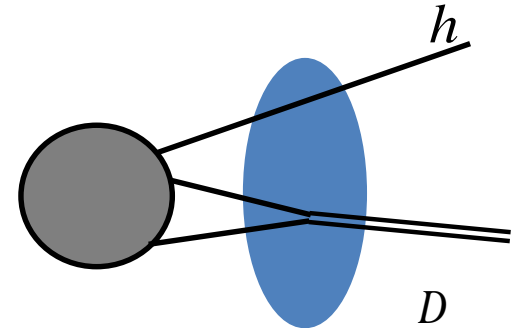


# Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion

$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{array} \right.$$



assumed factorization

$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3 R d^3 r_{np} d^3 r S(\mathbf{r}_h) S(\mathbf{r}_n) S(\mathbf{r}_p) |\phi_{\mathbf{q}}(\mathbf{r})|^2 |\varphi_D(\mathbf{r}_{np})|^2$$

For Gaussian source

$$C(\mathbf{q}) = \int d^3 r S_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

$$S_{3r}(\mathbf{r}) = \left( \frac{1}{3\pi R^2} \right)^{3/2} \exp\left( -\frac{\mathbf{r}^2}{3R^2} \right)$$

For a non-Gaussian source,  $A_D$  remains in the correlation function!

# Direct vs. final state interaction

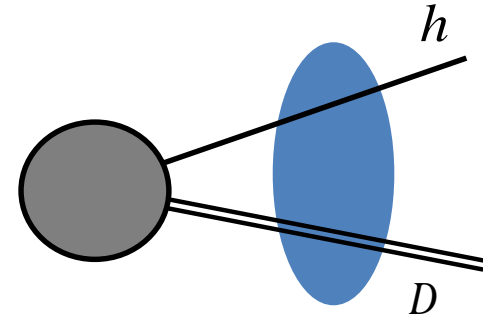
Direct production

$$C(\mathbf{q}) = \int d^3r S_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

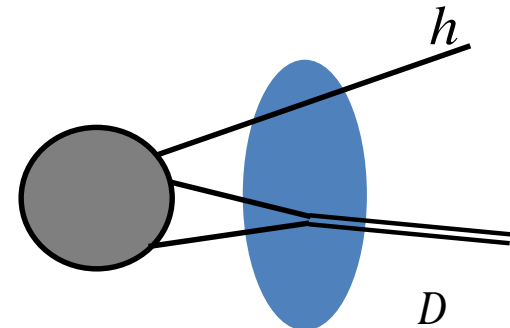


$$S_r(\mathbf{r}) = \left( \frac{1}{4\pi R^2} \right)^{3/2} \exp\left( -\frac{\mathbf{r}^2}{4R^2} \right)$$

$$S_{3r}(\mathbf{r}) = \left( \frac{1}{3\pi R^2} \right)^{3/2} \exp\left( -\frac{\mathbf{r}^2}{3R^2} \right)$$



$$\sqrt{\frac{4}{3}} \approx 1.15$$



Final state interaction  
& factorization

$$C(\mathbf{q}) = \int d^3r S_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

# Lednický-Lyuboshitz formula

$$C(\mathbf{q}) = \int d^3r S_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

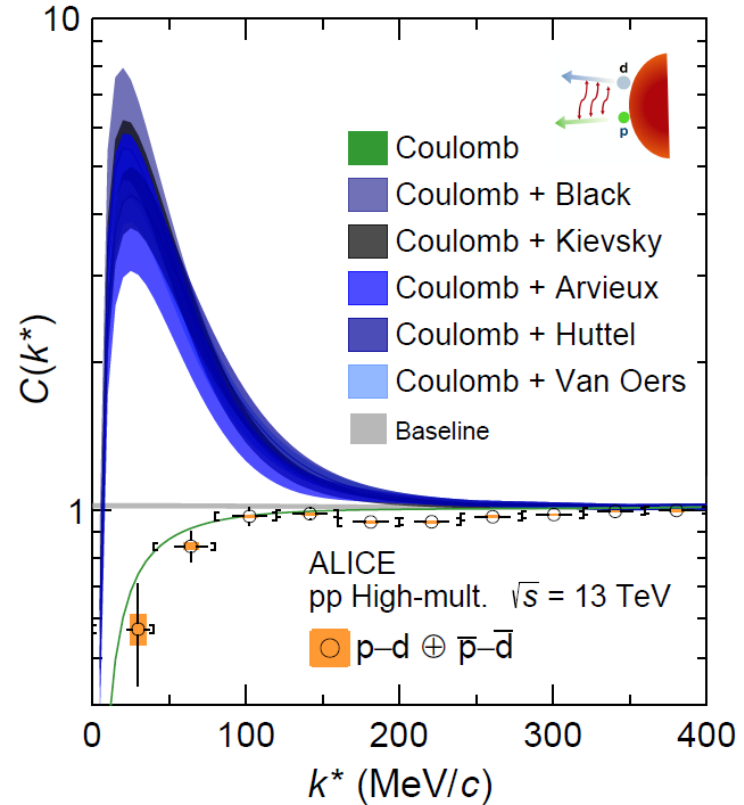
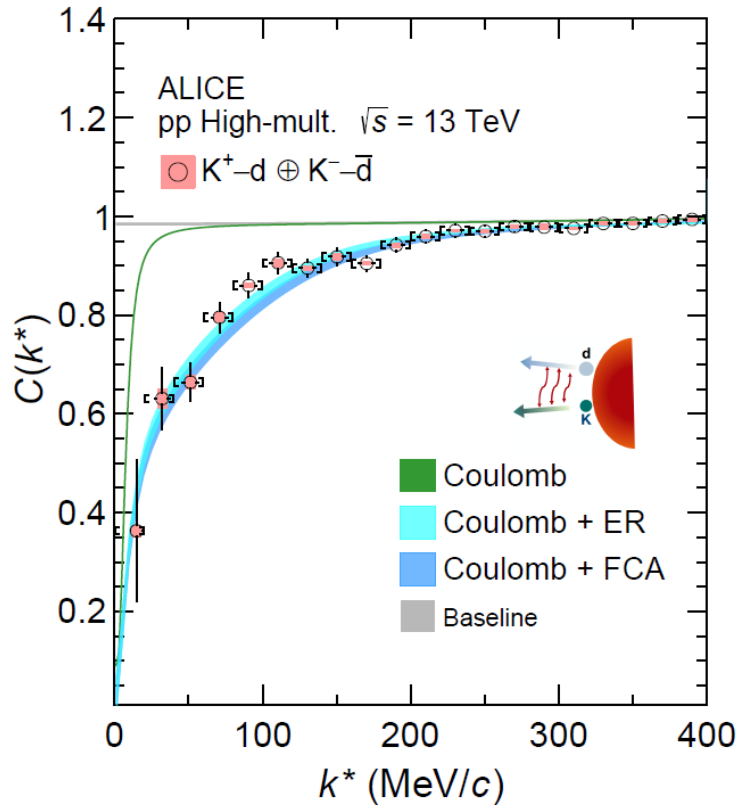
The wave function in scattering asymptotic state

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{q}\mathbf{r}} + f(\mathbf{q}) \frac{e^{iqr}}{r}$$

The s-wave amplitude  $f(\mathbf{q}) = -\frac{a}{1 - \frac{1}{2}daq^2 + iqa}$

$a$  – scattering length,  $d$  – effective range

# *K-D vs. p-D correlations*



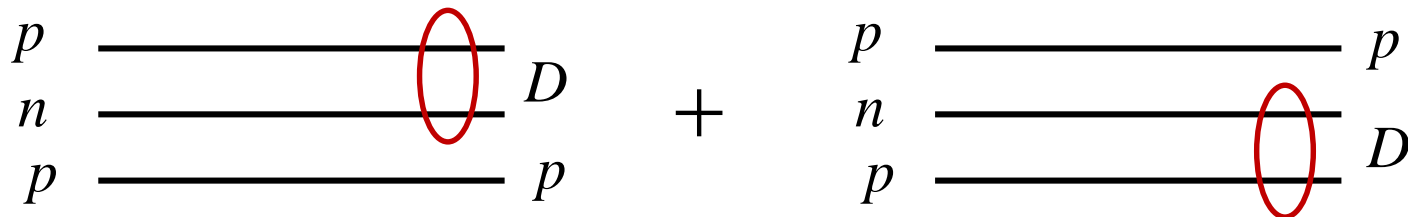
$$a_{KD} \approx 0.5 \text{ fm}$$

$$a_{pD}^{1/2} \approx 2 \text{ fm}$$

$$a_{pD}^{3/2} \approx 12 \text{ fm}$$

# $p$ - $D$ correlation function

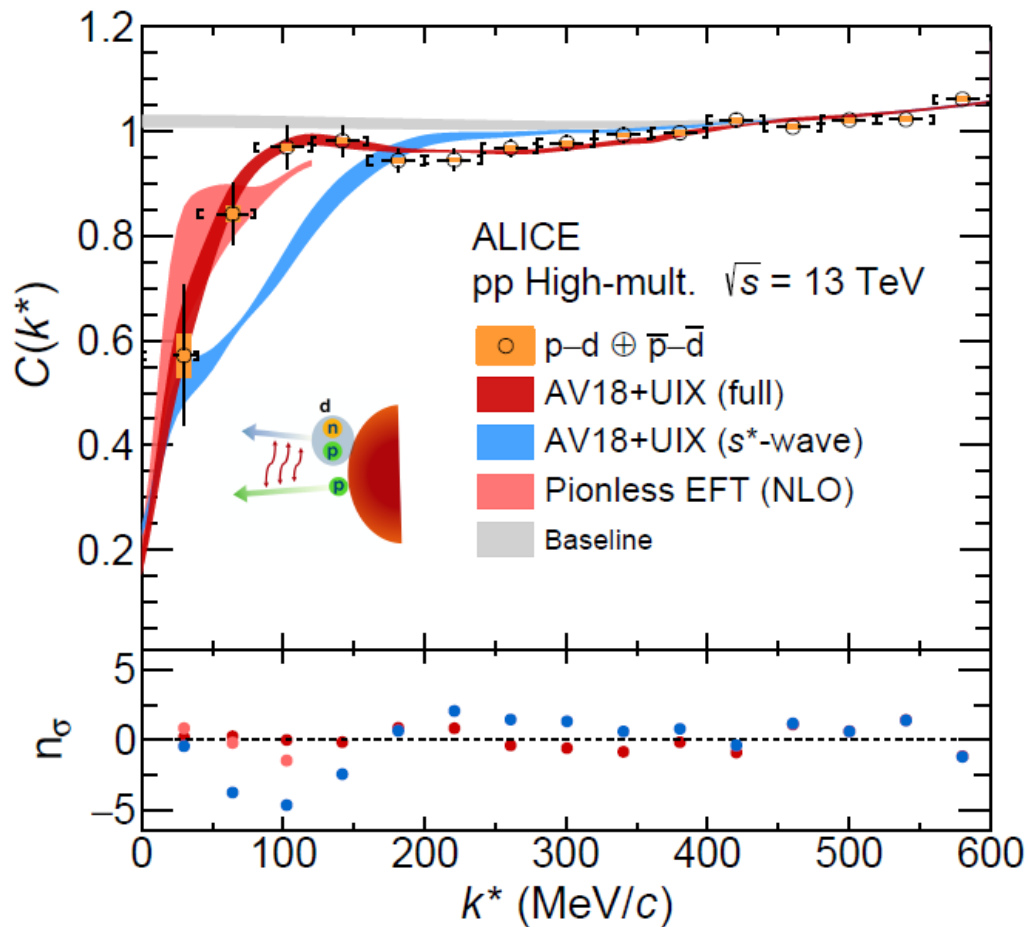
$$\psi_{pD}^{\mathbf{q}}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2})$$



Full three-body calculations

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3 r_n d^3 r_{p_1} d^3 r_{p_2} S(\mathbf{r}_n) S(\mathbf{r}_{p_1}) S(\mathbf{r}_{p_2}) \left| \psi_{pD}^{\mathbf{q}}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2}) \right|^2$$

# $p$ - $D$ correlation function



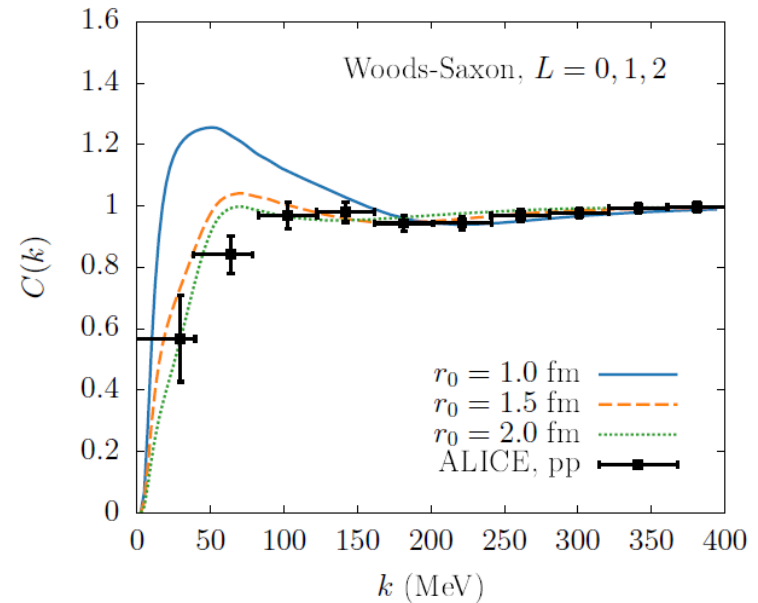
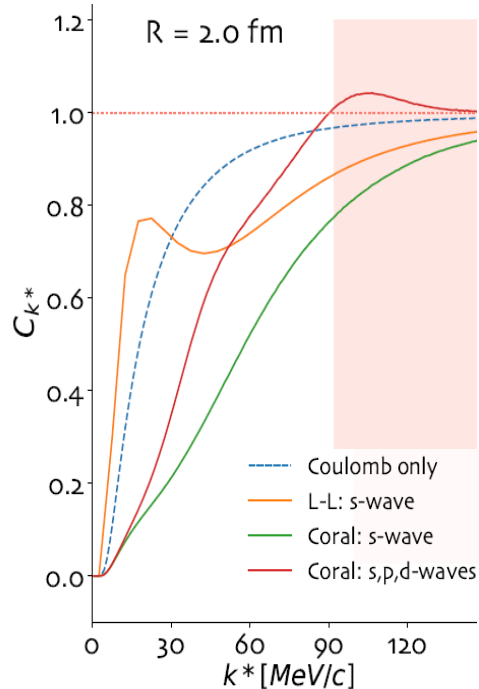
$$R_s = 1.43 \pm 0.16 \text{ fm}$$

ALICE, Phys. Rev. X **14**, 031051 (2024)

M. Viviani et al, Phys. Rev. C **108**, 064002 (2023)

# $p$ - $D$ correlation function

Two-body approaches beyond the Lednicky-Lyuboshitz formula



- The asymptotic wave function is insufficient.
- The  $p$  &  $d$  waves are important.

There is an ambiguity of deuteron source radius.

# Deuteron-deuteron correlation function

Direct production

$$C(\mathbf{q}) = \int d^3r S_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



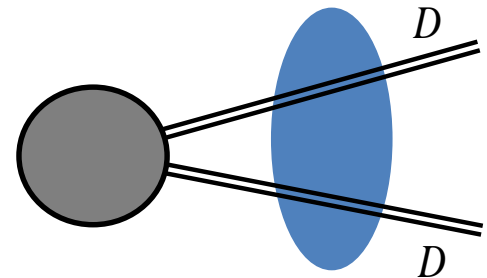
$$S_r(\mathbf{r}) = \left( \frac{1}{4\pi R^2} \right)^{3/2} \exp\left( -\frac{\mathbf{r}^2}{4R^2} \right)$$

$$S_{4r}(\mathbf{r}) = \left( \frac{1}{2\pi R^2} \right)^{3/2} \exp\left( -\frac{\mathbf{r}^2}{2R^2} \right)$$

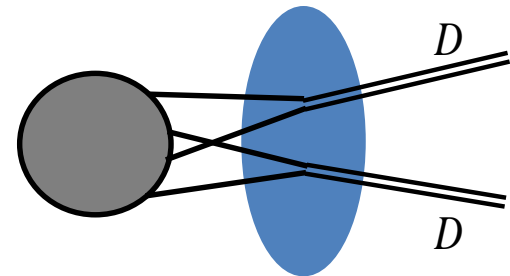
Final state interaction  
& factorization



$$C(\mathbf{q}) = \int d^3r S_{4r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

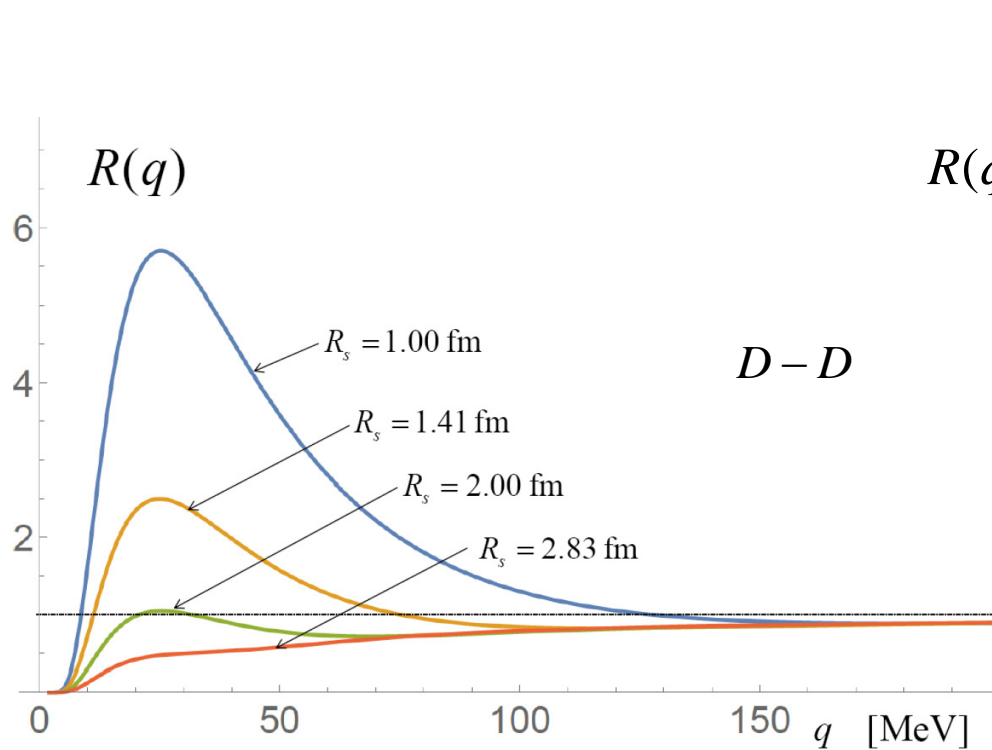


$$\sqrt{2} \approx 1.41$$





# Deuteron-deuteron correlation function



$$R(q) = \frac{1}{9} R_0(q) + \frac{3}{9} R_1(q) + \frac{5}{9} R_2(q)$$

spin 0
spin 1
spin 2

$$a_0 = (10.2 + 0.2i) \text{ fm}$$

$$a_2 = 7.5 \text{ fm}$$

$$2.83 = \sqrt{2} \cdot 2.00 = (\sqrt{2})^2 \cdot 1.41 = (\sqrt{2})^3 \cdot 1.00$$

$R_s$  from  $D-D$  correlation function vs.  $R_s$  from  $p-p$  &  $p-D$  correlation function

# Proton-<sup>3</sup>He correlation function

Direct production

$$R(\mathbf{q}) = \int d^3 r D_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



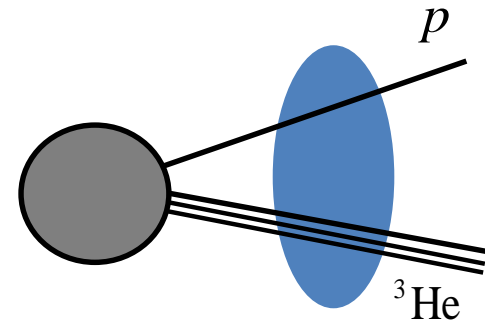
$$D_r(\mathbf{r}) = \left( \frac{1}{4\pi R^2} \right)^{3/2} \exp\left( -\frac{\mathbf{r}^2}{4R^2} \right)$$

$$D_{4r}(\mathbf{r}) = \left( \frac{3}{8\pi R^2} \right)^{3/2} \exp\left( -\frac{3\mathbf{r}^2}{8R^2} \right)$$

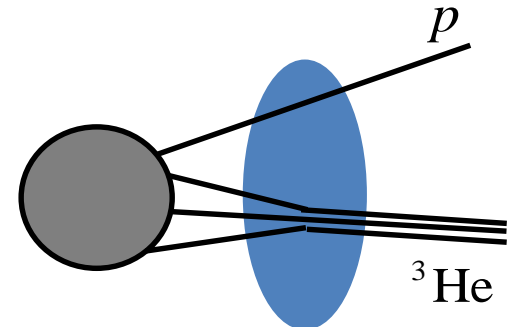


Final state interaction  
& factorization

$$R(\mathbf{q}) = \int d^3 r D_{4r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

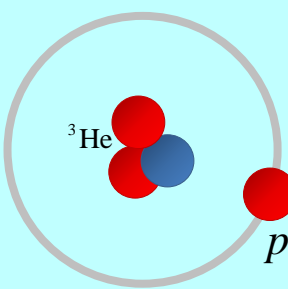


$$\sqrt{\frac{3}{2}} \approx 1.22$$

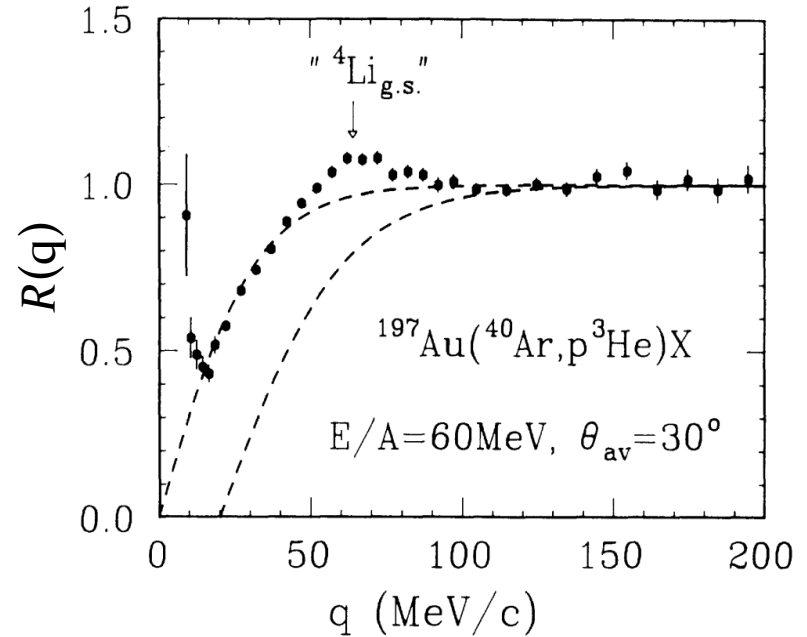


# Resonance ${}^4\text{Li}$

${}^4\text{Li}$




${}^4\text{Li} \rightarrow {}^3\text{He} + p$   
 $\Gamma = 6 \text{ MeV}$   
 $m = m_{{}^3\text{He}} + m_p + 4.1 \text{ MeV}$   
 $m = 3749.7 \text{ MeV}$   
 $s = 2$



J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)

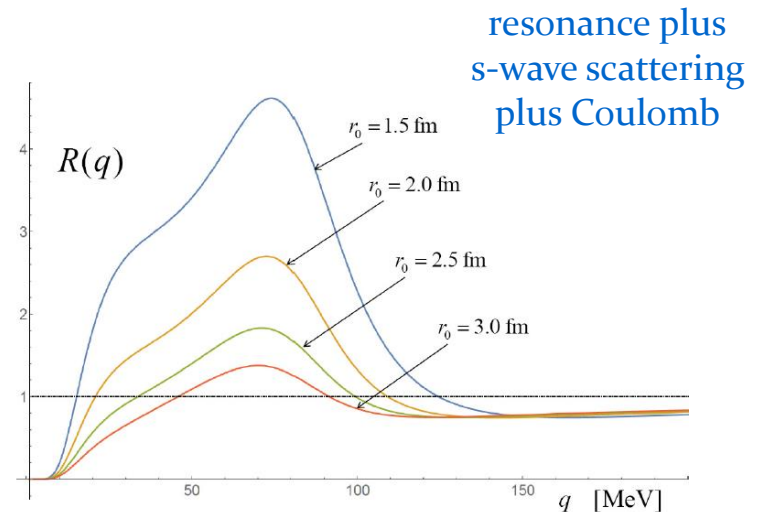
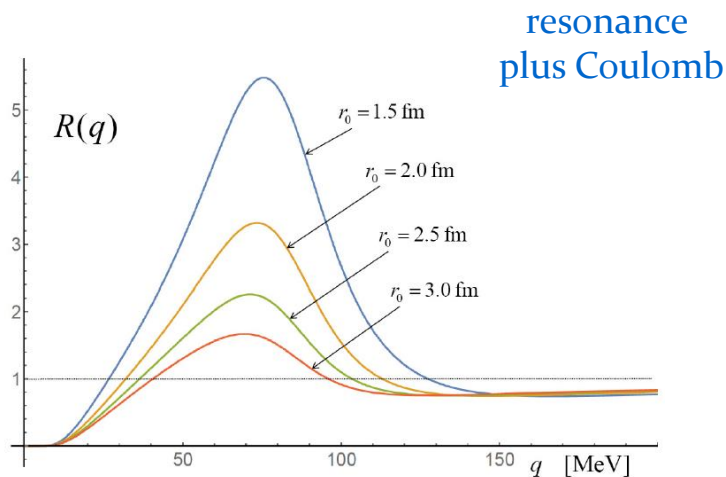
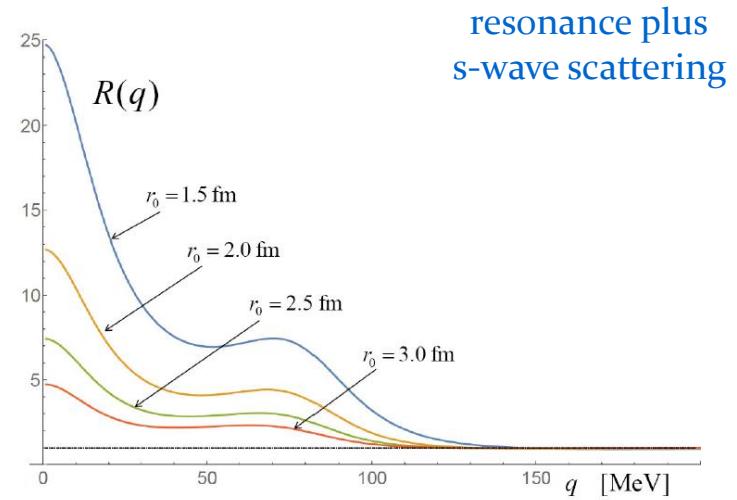
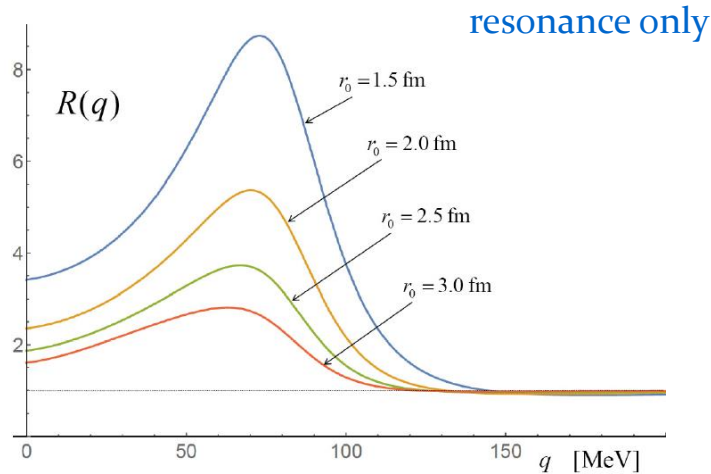
${}^4\text{He}$



$r_{\text{RMS}} = 1.68 \text{ fm}$   
 $\varepsilon_B = 28.3 \text{ MeV}$   
 $m = 3727.4 \text{ MeV}$   
 $s = 0$

M. Stefaniak for HADES Collaboration, arXiv:2402.09280

# Correlation function $p$ - $^3\text{He}$



# How to measure yield of ${}^4\text{Li}$

$$\frac{dN_{\text{Li}}}{d\mathbf{p}} = S_R \frac{dN_p}{d\mathbf{p}} \frac{dN_{{}^3\text{He}}}{d\mathbf{p}}$$

$\mathbf{p}$  - momentum per nucleon

$$S_R \equiv \int d^3q R_R(\mathbf{q})$$

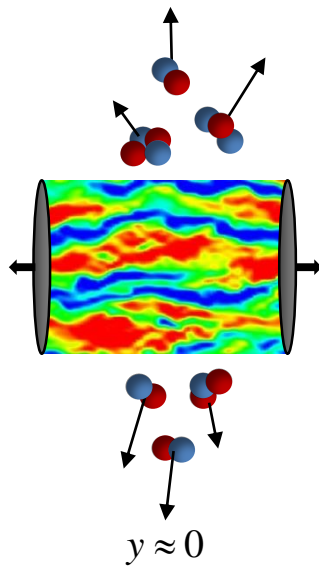
correlation function where only the  ${}^4\text{Li}$  resonance contributes

# Conclusions

- ▶ The correlation function of light nuclei depends on their production mechanism.
- ▶ Light nuclei should not be treated as elementary particles.
- ▶ The correlation functions of light nuclei can reveal an existence of various nuclear resonances.

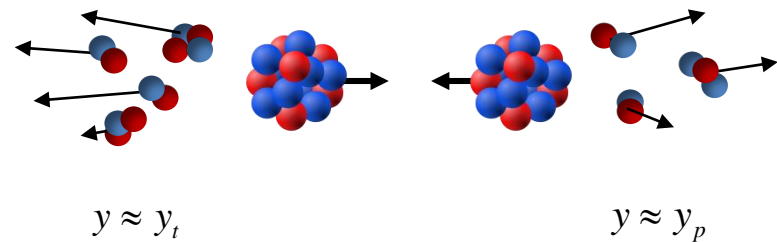
# Two very different cases of producing light nuclei

## Genuine production



hard process

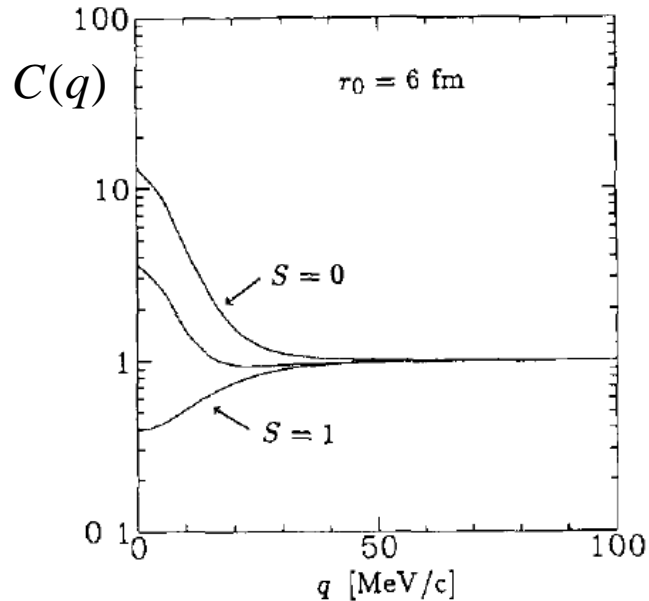
## Shattering of incoming nuclei



soft process

# n-p correlation function

Sum rule due to completeness of quantum states

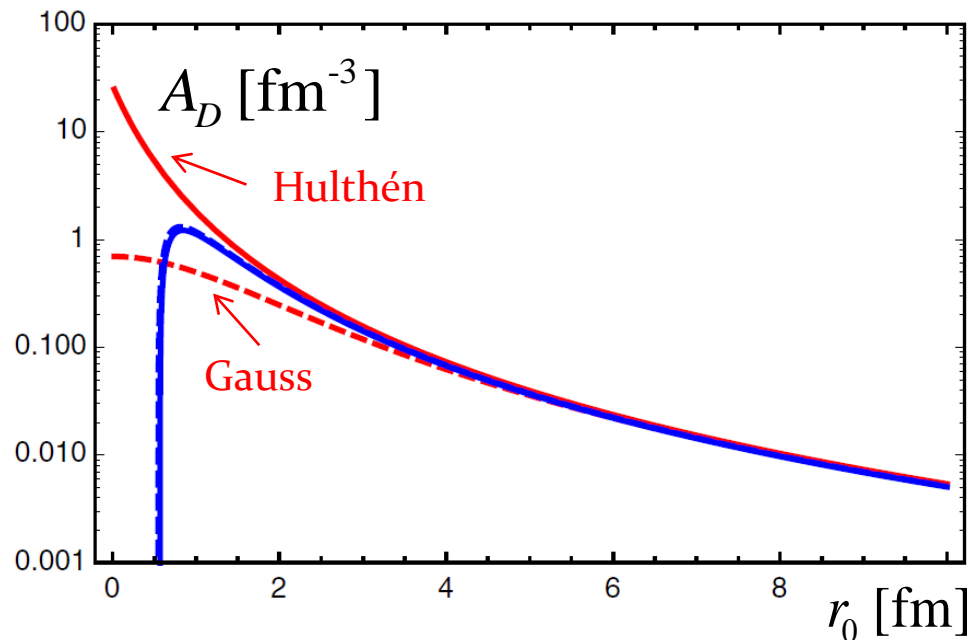


$$S(\mathbf{r}) = \left( \frac{1}{2\pi r_0^2} \right)^{3/2} \exp\left( -\frac{\mathbf{r}^2}{2r_0^2} \right)$$

Lednický-Lyuboshitz formula

St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

$$\int d^3\mathbf{q} (C_1(\mathbf{q}) - C_0(\mathbf{q})) = -A_D$$



R. Maj & St. Mrówczyński, Phys. Rev. C **101**, 014901 (2020)

R. Maj & St. Mrówczyński, Phys. Rev. C **71**, 044905 (2005)

St. Mrówczyński, Phys. Lett. B **345**, 393 (1995)