From Source Imaging to Balance Functions
Research Projects with Scott Pratt

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FRIB

Scott Edward Pratt

- BS U of Kansas 1980
- PhD U of Minnesota *Pion Pictures of Heavy Ion Collisions*
  Supervisor: Joseph Kapusta 1985
- Joined Michigan State U in 1992
- Major Impact on Development of Heavy-Ion Collisions:
  - Femtoscopy   Koonin-Pratt Eq
  - Speed of Sound
  - Viscosity
  - Balance Functions
  - Baryon Number Transport
  - ...

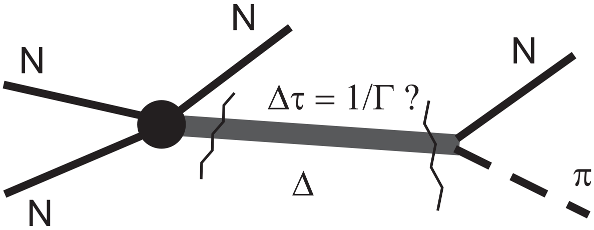
Our Joint Projects

- Our research careers largely overlapped in the same group!
- Scott Pratt’s research energy area $\gtrsim$ PD’s
- 18 joint publications
- 3 major project areas:
  - Delays in Elementary Interactions – Context of HI Simulations
  - 3D Imaging of Sources from Correlations & Other Femtoscopy
  - Clocking Hadronization with Balance Functions

Near-Threshold Resonances in Low-Energy Collisions

Transport-Theory Paradox

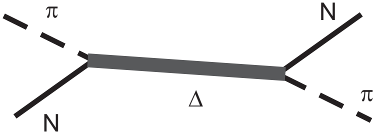
Pion Production: $N + N \longrightarrow N + \Delta$,
$\Delta \longrightarrow N + \pi$ decays at the rate Γ, but $\Gamma \propto p^3$,
where p momentum in $N\pi$ channel, i.e., decay rate vanishes near threshold!

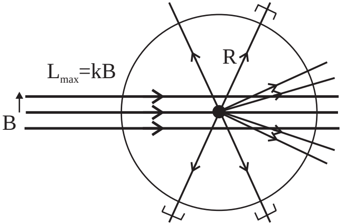


$\Delta\tau \to \infty$ Stable Δ's??

Quantal Consideration

PD & Scott Pratt PRC53(96)249





How long Δ of mass *m* lives?

Wavepacket sent into a volume of radius *R*.
How long does it stay there?

$$\tau_{\text{vol}} = \frac{1}{N} \int \mathrm{d}t\, t \oint \mathrm{d}\vec{\sigma} \cdot \vec{j}(\vec{r}, t)$$

?Change compared to free passage:

$$\Delta\tau = \tau_{\text{vol}} - \tau_{\text{vol}}^{\text{free}} = \frac{1}{N} \int \mathrm{d}t\, t \oint \mathrm{d}\vec{\sigma} \cdot \left[\vec{j} - \vec{j}^{\text{free}}\right]$$

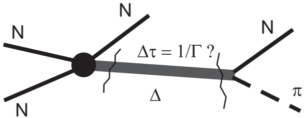
$$= \frac{\mathrm{d}\delta_J}{\mathrm{d}E} = \cdots = -\int \mathrm{d}t\, t \, \frac{\mathrm{d}}{\mathrm{d}t} \int_V \left[|\Psi|^2 - |\Psi^{\text{free}}|^2\right]$$

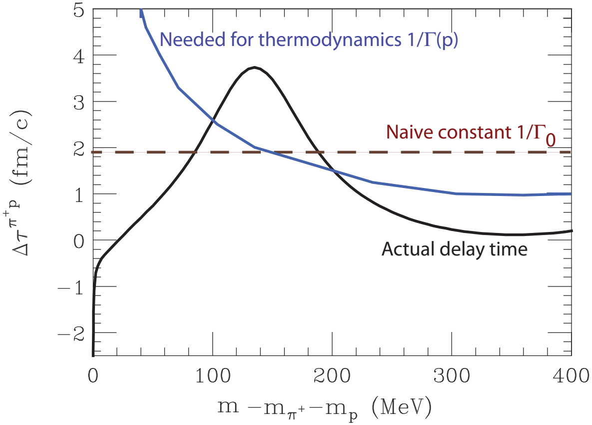
for scattering in one partial wave

Close femtoscopy connection

Delta Lifetime

PD & Scott Pratt PRC53(96)249





$$\tan\delta = \frac{\Gamma(m)/2}{m - m_\Delta^0}$$

where

$$\Gamma(m) \propto (m - m_\pi - m_N)^{3/2}$$

$$\Delta\tau = \frac{d\delta}{dE}$$

Virtual near-threshold Δs live short not long time

Near-threshold production should be isolated as an elementary process in transport!

SπRIT Coll PLB813(21)136016

Source Imaging

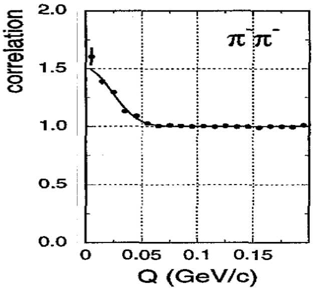
Start: Brown *et al*

Rewritten Koonin-Pratt relation - of interest deviation of C from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int \mathrm{d}\mathbf{r} \left(|\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int \mathrm{d}\mathbf{r}\, K(\mathbf{q},\mathbf{r})\, S(\mathbf{r})$$

Potential to learn on S there when scat wf $|\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair Brown&PD PLB398(97)252.

For pure interference, $\Phi^{(-)}_{\mathbf{q}}(\mathbf{r}) = \frac{1}{\sqrt{2}} \left(e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}} \right)$, kernel $K = |\Phi|^2 - 1$ results from the interference term in $|\Phi|^2$: $K(\mathbf{q},\mathbf{r}) = |\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2 - 1 = \cos(2\mathbf{q}\mathbf{r})$



Miskowiec E877

Coulomb corrected $\pi^-\pi^-$ correlation function

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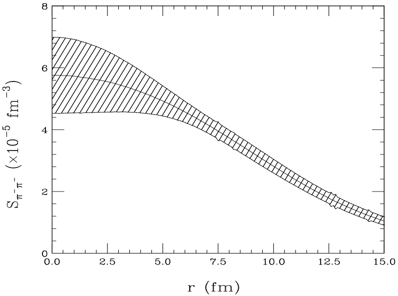


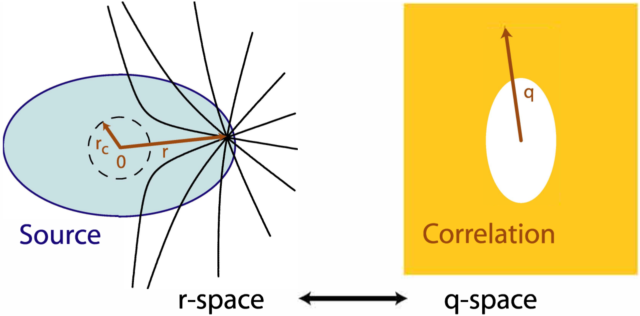
3D Source-Imaging

PD&Pratt PLB618(05)60, PRC75(07)034907; Brown, PD, Pratt *et al* PRC72(05)054902

Spin-averaged kernel K in the 3D Koonin-Pratt relation depends only on the relative angle between **q** and **r** - can be expanded in Legendre polynomials:

$$K(\mathbf{q},\mathbf{r}) = \sum_{\ell} (2\ell+1)\, K_\ell(q,r)\, P^\ell(\cos\theta)$$

Ability to learn on source deformation depends on nonvanishing K^ℓ for $\ell > 0$. E.g., repulsive Coulomb trajectories appropriate for IMF-IMF correlations, firmly map source deformation onto correlation deformation



Expansion Options for Source and Correlation

Spherical vs Cartesian

$$K(\mathbf{q},\mathbf{r}) = \sum_{\ell} (2\ell+1)\, K_\ell(q,r)\, P^\ell(\cos\theta_{\mathbf{qr}})$$

Expansion options for asymmetry handling:
spherical tesseral (Brown *et al*) or cartesian (PD&Pratt) harmonics

$$P^\ell(\cos\theta_{\mathbf{qr}}) = \frac{4\pi}{2\ell+1}\sum_m Y^*_{\ell m}(\hat{\mathbf{q}})\, Y_{\ell m}(\hat{\mathbf{r}}) = (2\ell-1)!!\sum_{\boldsymbol{\ell}} \frac{1}{\ell_x!\,\ell_y!\,\ell_z!}\, \mathcal{A}_{\boldsymbol{\ell}}(\hat{\mathbf{q}})\, \mathcal{A}_{\boldsymbol{\ell}}(\hat{\mathbf{r}})$$

These yield

$$S(\mathbf{r}) = \sqrt{4\pi}\sum_{\ell m} S^*_{\ell m}(r)\, Y_{\ell m}(\hat{\mathbf{r}}) = \sum_{\boldsymbol{\ell}} \frac{1}{\ell_x!\,\ell_y!\,\ell_z!}\, S_{\boldsymbol{\ell}}(r)\, \mathcal{A}_{\boldsymbol{\ell}}(\hat{\mathbf{r}})$$

and similar expansions for $R(\mathbf{q}) = C(\mathbf{q}) - 1$. Irrespectively which expansion is used, K_ℓ connects the corresponding deformation coefficients for R (or C) and S,

$$\boxed{\mathcal{R}_{\ell m,\boldsymbol{\ell}}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, S_{\ell m,\boldsymbol{\ell}}(r)}$$

Which Expansion?

Spherical or Cartesian Tensors

Spherical tensors, $Y_{\ell m}(\hat{n})$, and expansion coefficients in their basis are complex functions. The basis distinguishes a specific axis and transformation properties under rotations are involved.

Traceless Maxwell-Cartesian tensors, $\mathcal{A}_\ell(\hat{n})$, and expansion coefficients in their basis are real functions. The axes are treated democratically and transformation properties under rotations are straightforward.

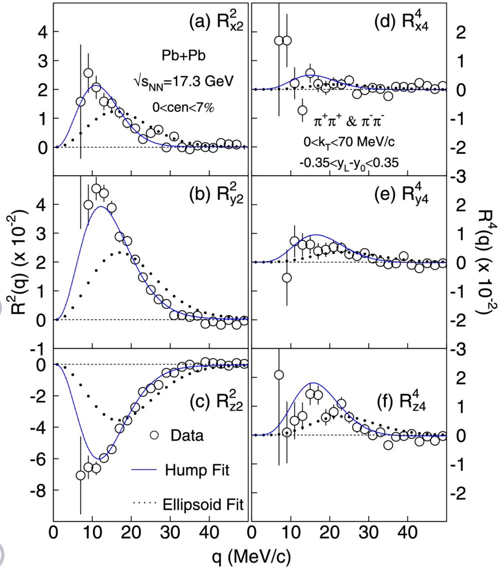
E.g., correlation function C cartesian-expanded correlation up to rank $\ell = 2$:

$$C(\boldsymbol{q}) = C^{(0)}(q) + \sum_i C_i^{(1)}(q)\,\hat{q}_i + \sum_{ij} C_{ij}^{(2)}(q)\,\hat{q}_i\,\hat{q}_j + \dots$$

Here, $C_i^{(1)}(q)$ is a vector function describing a dipole distortion and $C_{ij}^{(2)}$ is a traceless matrix, that can be diagonalized, describing a quadrupole distortion.

Identical π Correlations in Pb+Pb at $\sqrt{s} = 17.3$ GeV/u

NA49 . . . Pratt PLB 685(10)41



z - beam direction
x - $P^\perp$ of the pair direction
y - perpendicular to *x* & *z*

The data described well by the hump function, inspired by the 3D imaging,

$$S(\mathbf{r}) = \Lambda \exp\left[- \frac{f_s(r)\, r^2}{r_s^2} - (1 - f_s(r)) \left(\frac{x^2}{x_l^2} + \frac{y^2}{y_l^2} + \frac{z^2}{z_l^2} \right) \right]$$

where $f_s(r) = 1/[1 + (r/r_0)^2]$. The hump function evolves from a spherical Gaussian at short distances to an anisotropic one at long.

Balance Functions

Bass, PD, Pratt PRL 85(00)2689; JPG 27(01)635

- Steffen Bass, co-creator of URQMD, joining MSU as a postdoc
- PD ponders a talk at QM on strange-antistrange particle production. What happens to the pair members as they separate, in space and rapidity? Can one learn about system history? A good postdoc project
- Steffen explores the fate of compensating quantum numbers in URQMD
- Scott works on fluctuations in collisions - related, but no separation vble f/quantum numbers there
- Scott joins the project and formulates the balance function observable, modifying one from jet studies Drijard NPB155(79)269, to be pursued experimentally

$$B(y_2|y_1) = \frac{1}{2}\Big[P(Q, y_2|\overline{Q}, y_1) - P(Q, y_2|Q, y_1) + P(\overline{Q}, y_2|Q, y_1) - P(\overline{Q}, y_2|\overline{Q}, y_1)\Big]$$

Here, $y_{1,2}$ are different values of a kinematic vble, such as rapidity, Q & $\overline{Q}$ are compensating quantum Nos and P are conditional probabilities

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Thanks, Scott, for the physics opportunities you provided to all of us !