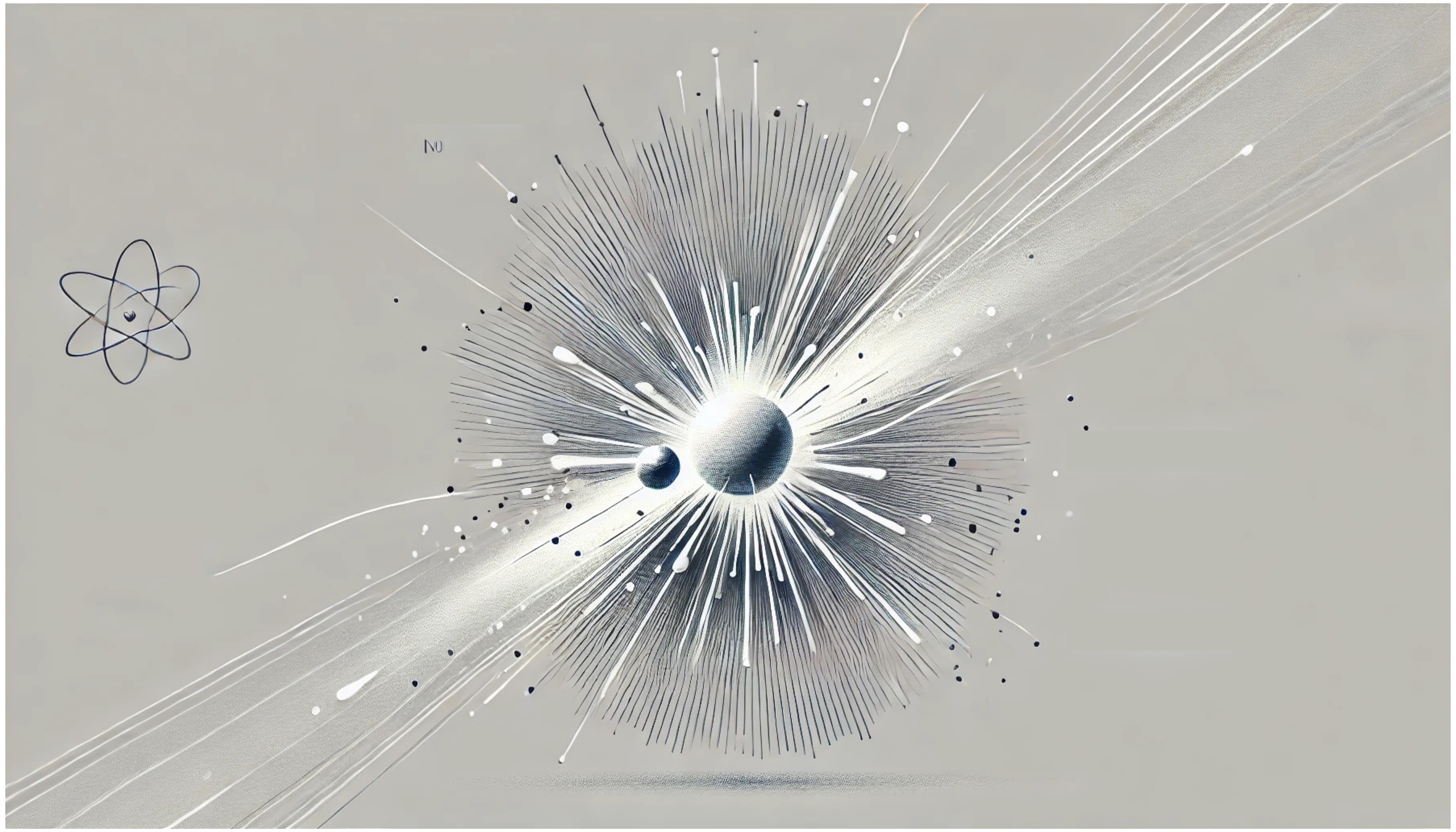
Theoretical description of proton-deuteron interactions

Based on: arXiv:2410.13983

W. Rzęsa, **M. Stefaniak**, S. Pratt



THE OHIO STATE UNIVERSITY

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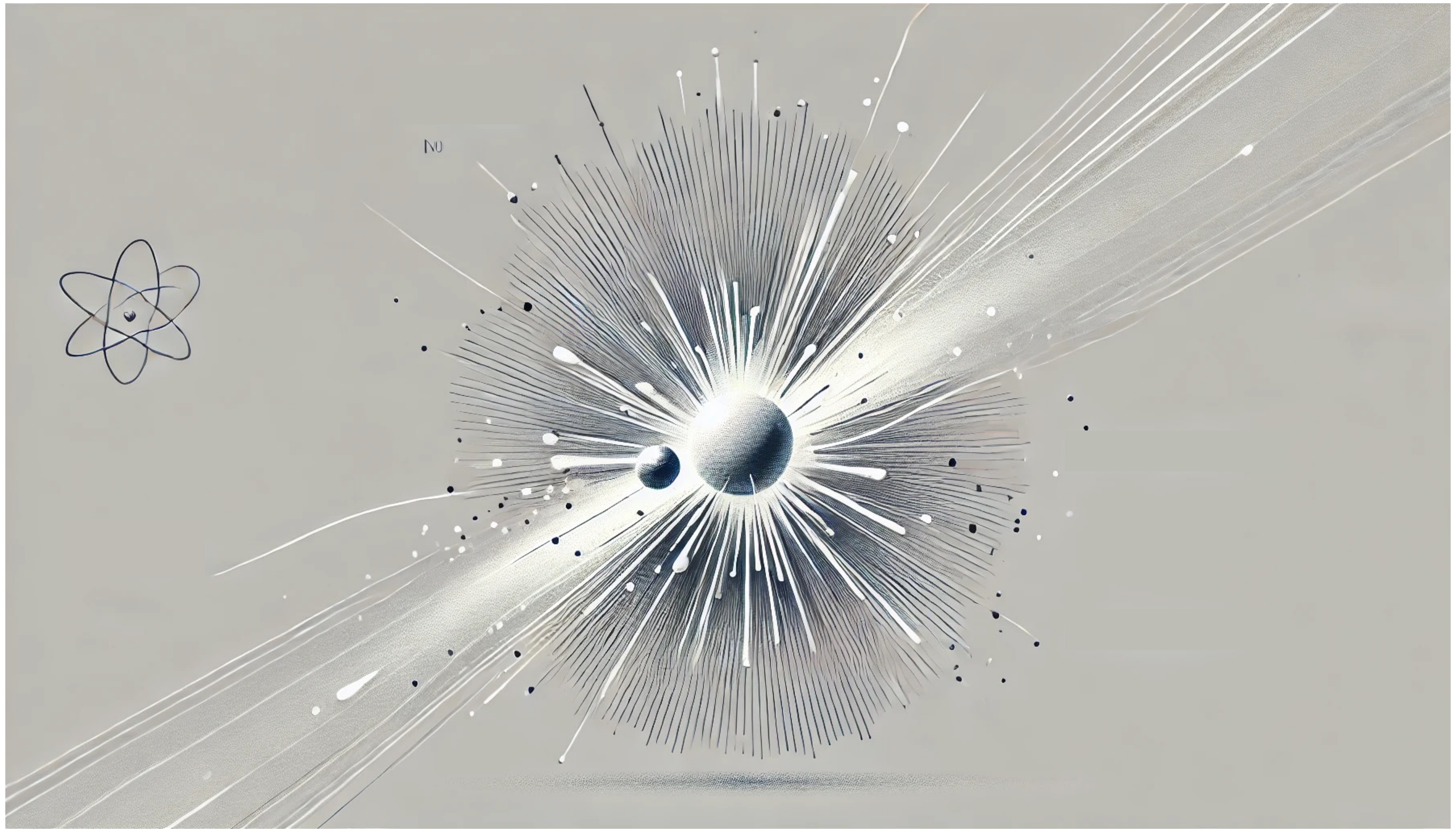
How this work was created



The Ohio State University

Everything started 2 years ago at WPCF at MSU

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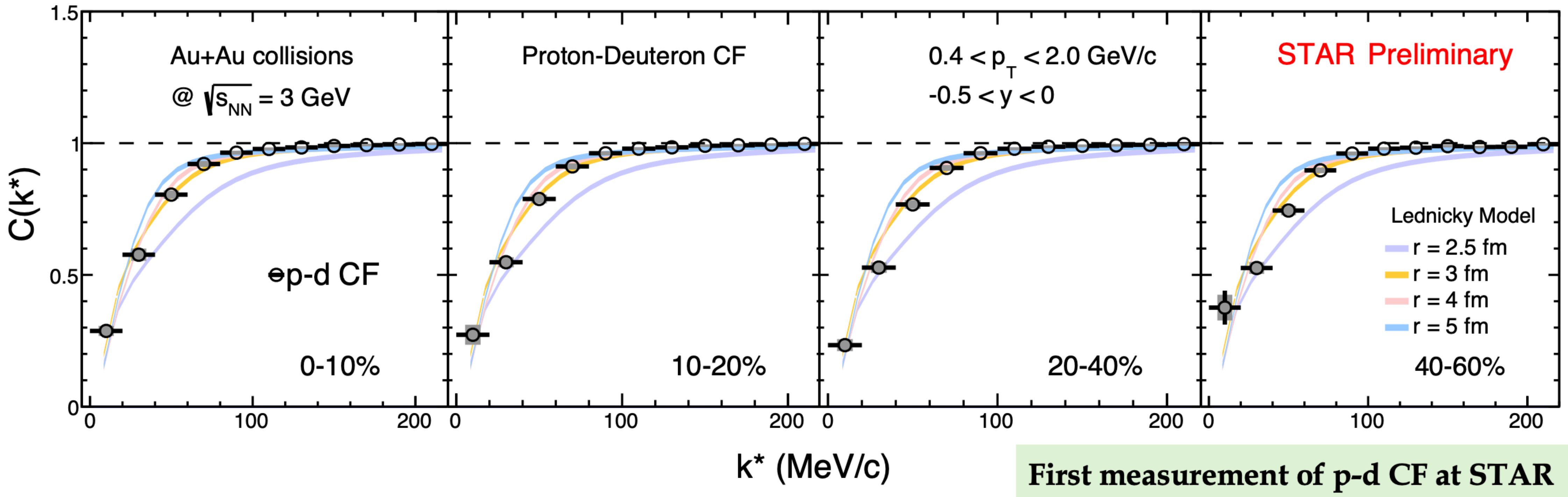
And followed in GSI on a first meeting HBT camp

Motivation

- Studies of **strong interactions**
- Proton - deuteron: interesting system, possible studies of light nuclei creation
- Three body system? I will talk only about 2-body "approach"
- Both me and Wiola (and others) needed the robust **theoretical description** of our experimental results on p-d femtoscopic correlations

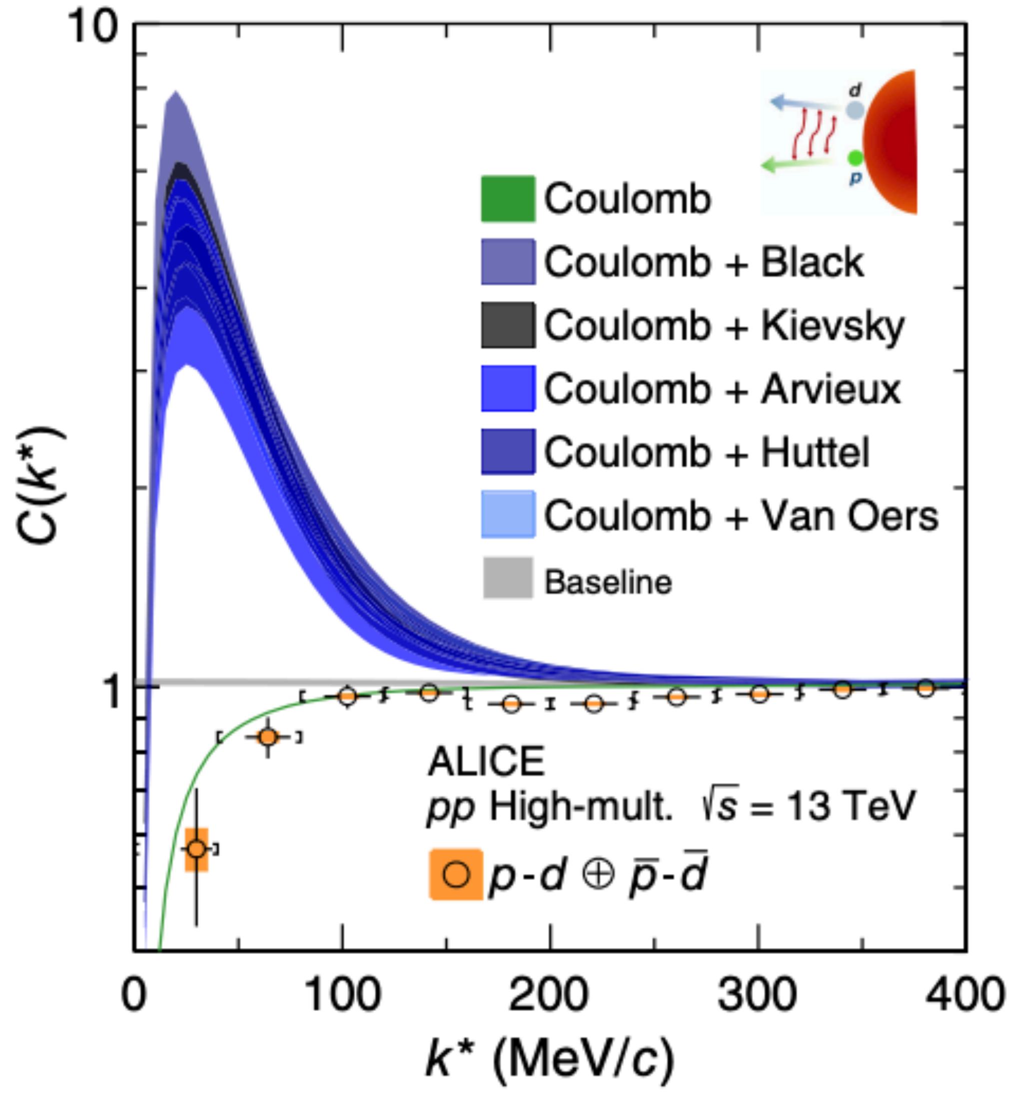
STAR



First measurement of p-d CF at STAR

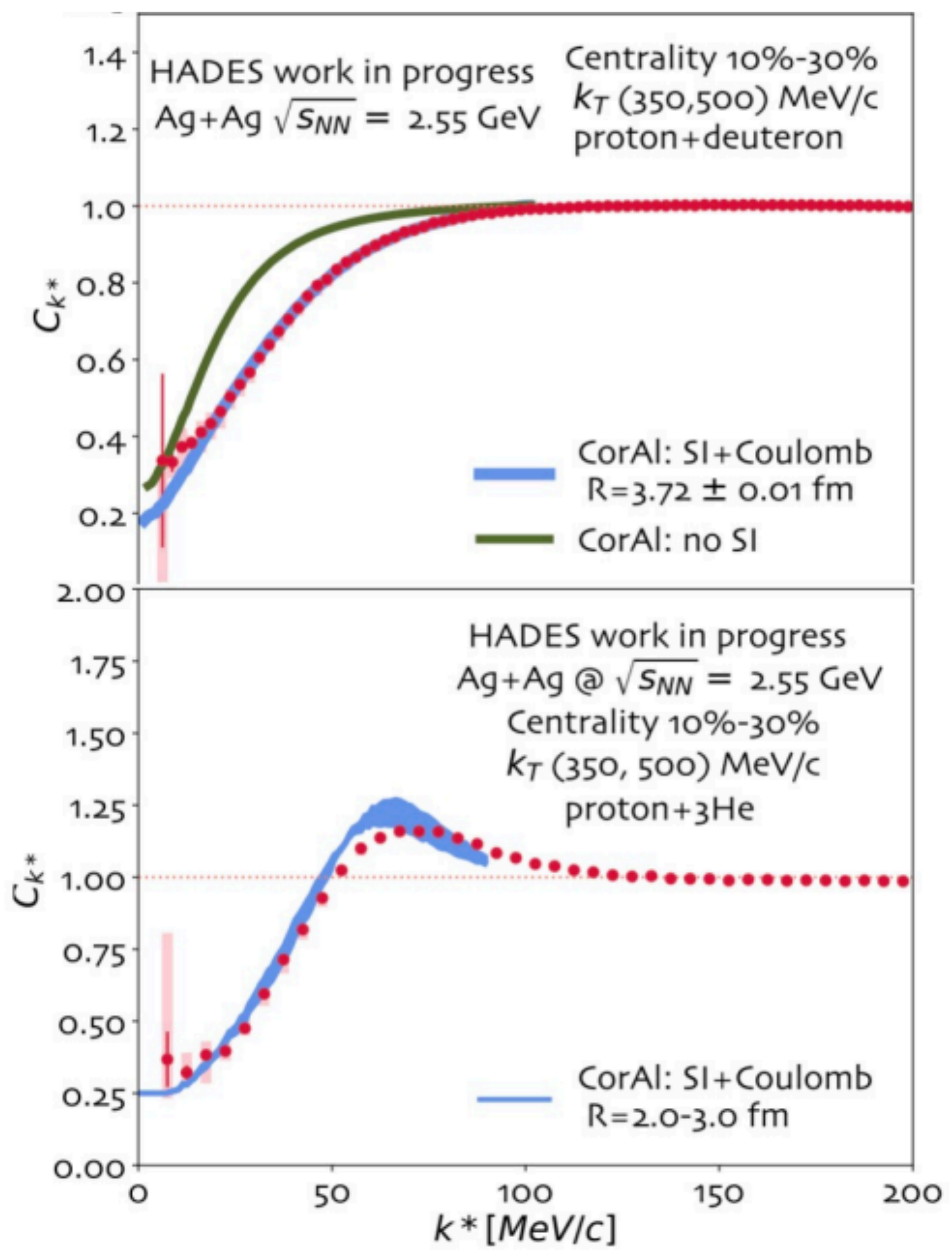
Acta Phys. Polon. Supp., vol. 16, no. 1, pp. 1–A91, 2023

ALICE



ALICE: Phys. Rev. X 14, 031051

HADES



EPJ Web Conf., vol. 296, p. 02001, 2024.

Correlation function

S.E. Koonin: Phys.Lett.B 70 (1977) 43-47, S.Pratt: *Phys.Rev.D* 33 (1986) 1314-1327

$$C(\mathbf{k}^*) = \int S(\mathbf{r}^*)\,|\,\Psi(\mathbf{r}^*, \mathbf{k}^*)\,|\,\mathrm{d}^3 r^*$$

* - denotes variables in the pair-rest frame

$k^* = |\,\mathbf{p}_1^* - \mathbf{p}_2^*\,|/2$

$\mathbf{p}_{1,2}^*$ - the particle momenta

r^* - relative distance between the two particles

- $S(\mathbf{r}^*)$ - probability that two particles emitted with the same velocity would be separated by a relative distance r^*
  - The formalism is contingent on the assumption that other sources of correlation, such as energy or momentum conservation are ignored

- We assume the Gaussian profile of source!

$$S(\mathbf{r}^*) \sim \exp\left(-\frac{r^{*2}}{4R^2}\right)$$

R- femtoscopic source size

Correlation function

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- $S(\mathbf{r}^*)$ - probability that two particles emitted with the same velocity would be separated by a relative distance r^*
- $\Psi(\mathbf{k}^*, \mathbf{r}^*)$ - wave function, describing the interactions between the two particles:
  - A. derived by solving the Schrodinger equation for a given potential
  - B. obtained through parameterization of the interaction

Here, the challenging part starts...

Proton-deuteron system is relatively **large** (deuteron radius is already ~2.1 fm)

Lednicky-Lyuboshits

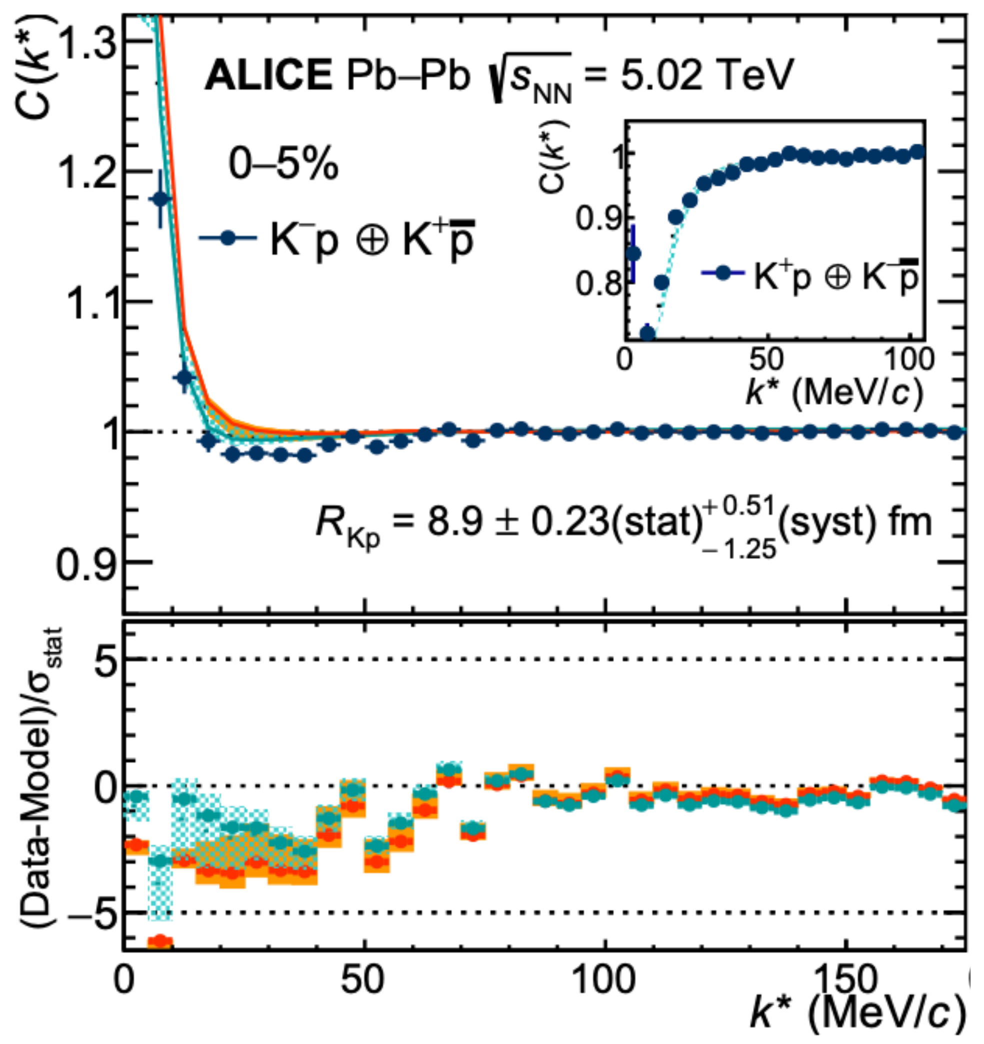
R. Lednicky and V. L. Lyuboshits: Yad. Fiz., vol. 35, pp. 1316–1330, 1981
R. Lednicky: Acta Phys. Polon. B, vol. 40, pp. 1145–1154, 2009

- Enables the parametrization of $\Psi(\mathbf{k}^*, \mathbf{r}^*)$ for both Coulomb and SI
- For multiple spin state systems applies sum of Clebsch-Gordan coefficients:

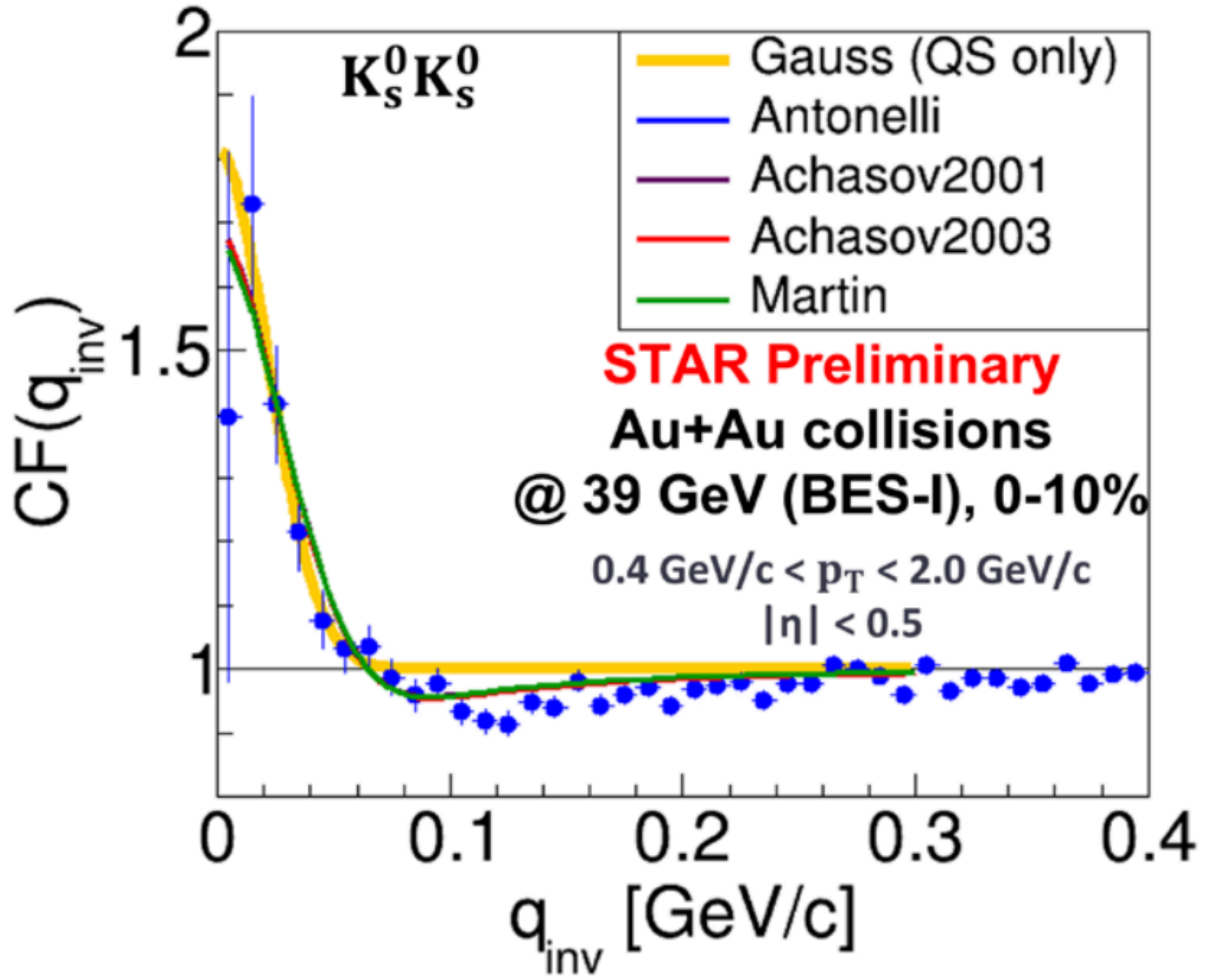
$$C_{pd}^{total} = \frac{1}{3} C_{pd}^{^2S_{1/2}} + \frac{2}{3} C_{pd}^{^4S_{3/2}} \qquad \textit{(for proton-deuteron)}$$

- Includes s-wave
- Based on zero effective-range approximation ($r_0 = 0$)
- Highly successful in multiple $C(\mathbf{k}^*)$ descriptions: kaon-kaon, Lambda-Lambda...

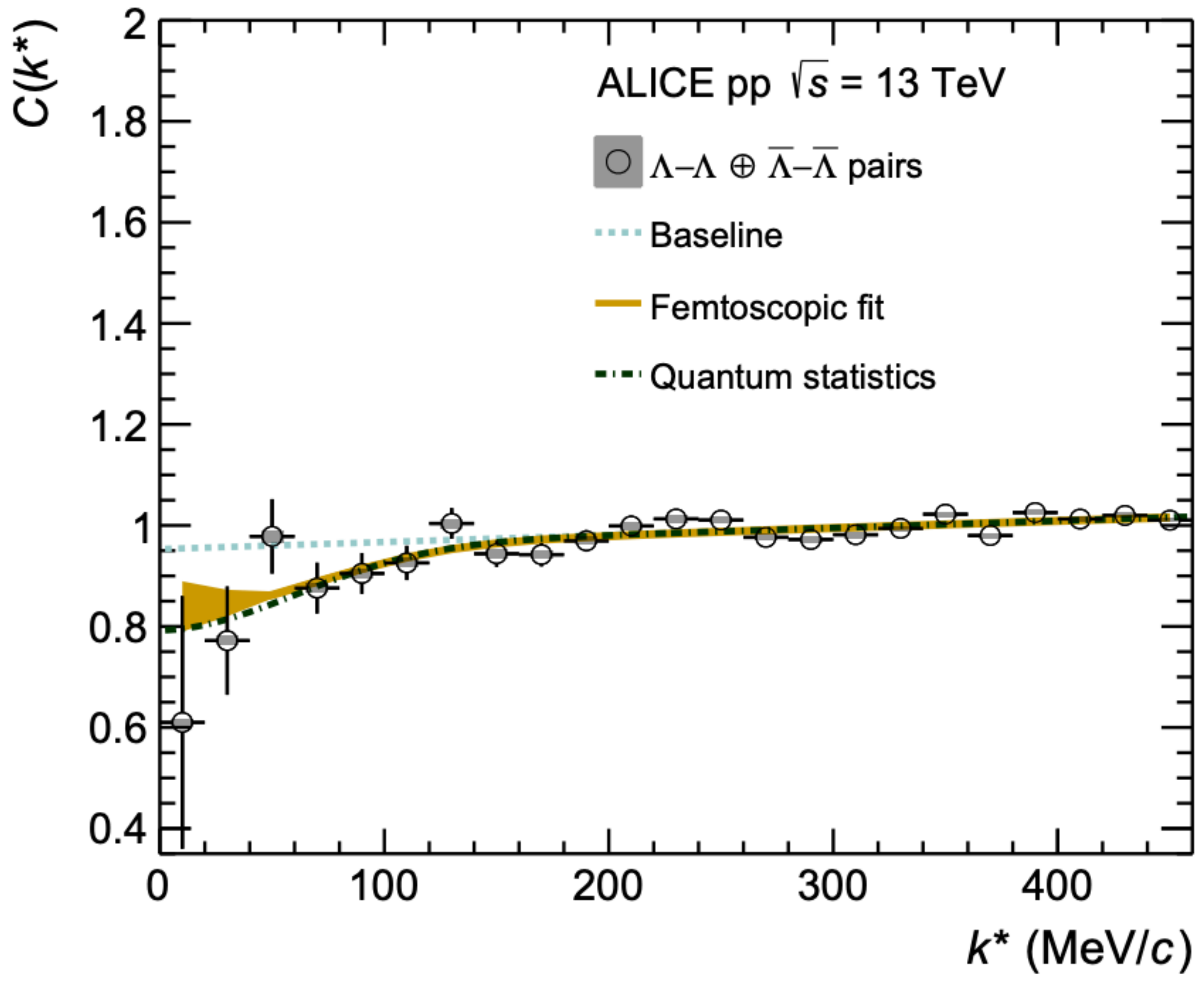
ALICE: Phys.Lett.B 822 (2021) 136708



D. Pawłowska-Szymańska (STAR): EPJ Web Conf., 276 (2023) 01016



ALICE: Phys. Lett. B 797 (2019) 134822



Lednicky-Lyuboshits

- Wave function disturbed by interactions (Coulomb neglected):

$$\phi_{k*}(r) = e^{i\delta}\sin(k^*r + \delta)$$

r - range of interaction

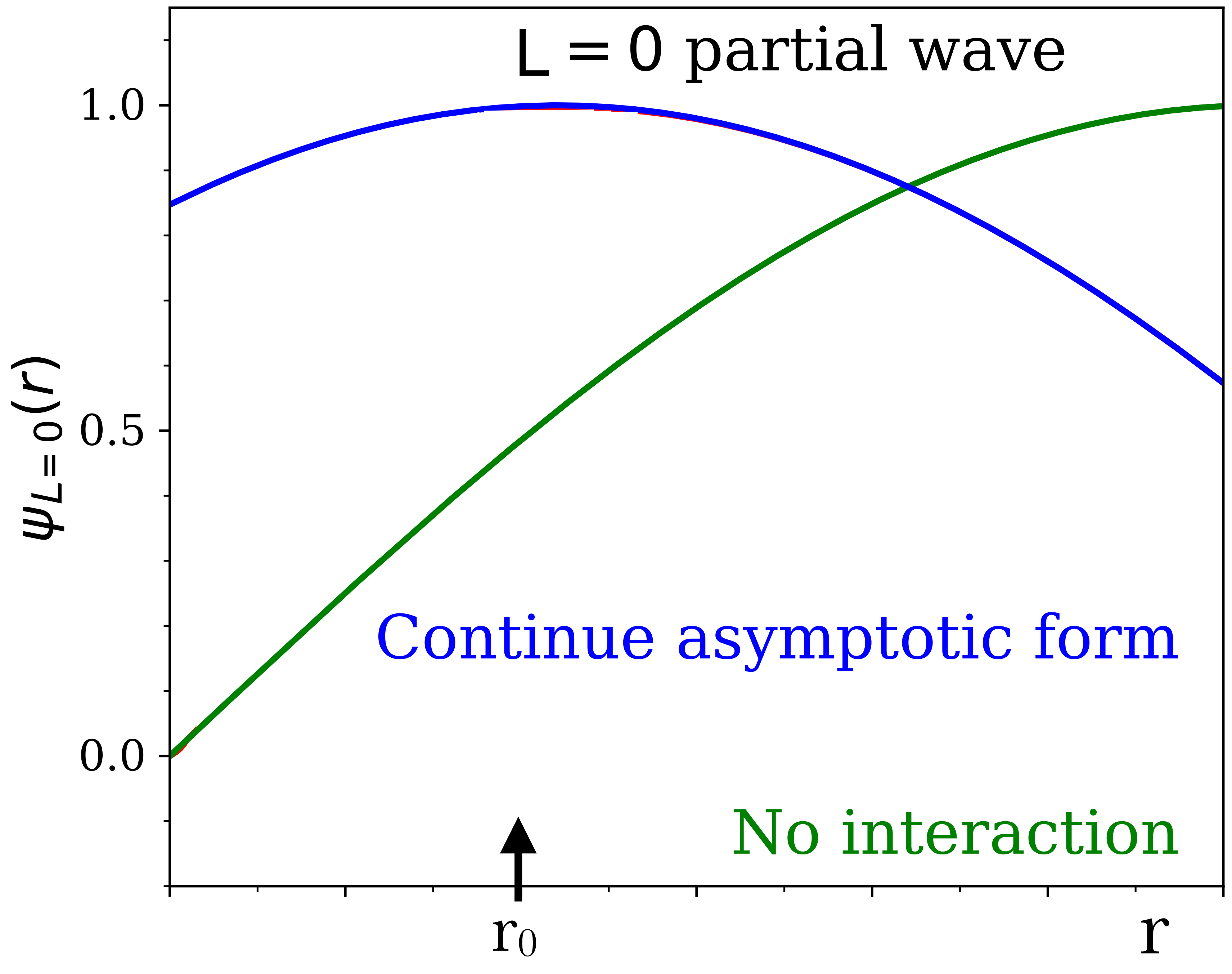
δ - parameter related to phase shift

Exact only when r larger than the range of nuclear reaction r_0!

- Proposed asymptotic form does not satisfy the boundary condition

$$\phi(k^*, r) \to 0 \text{ as } r \to 0$$

where true function vanishes completely



r_0 - zero effective-range of the interaction

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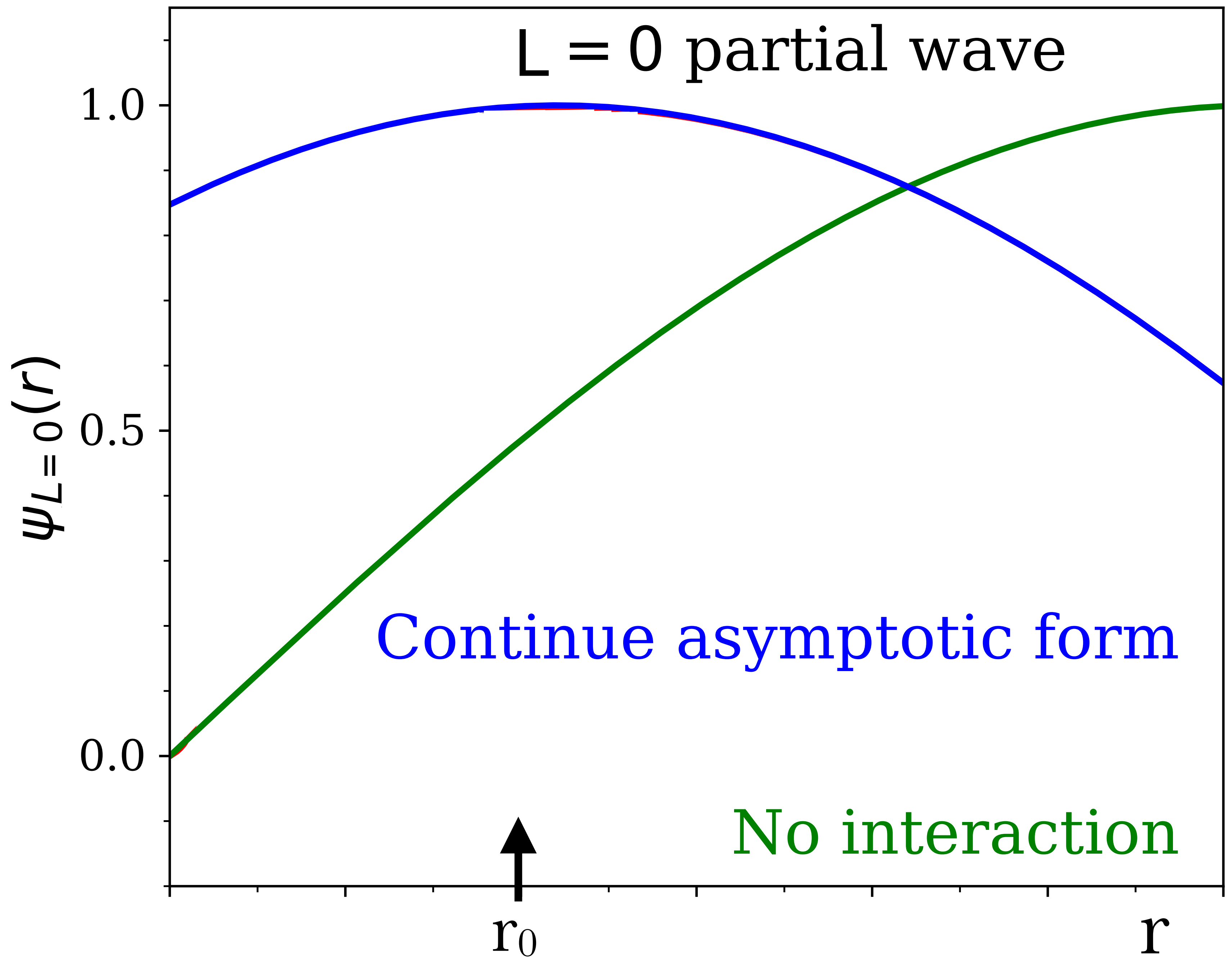
As a result of the size of the interacting p-d: for many investigated systems $r<r_0$

What are other options?



r_0 - zero effective-range of the interaction

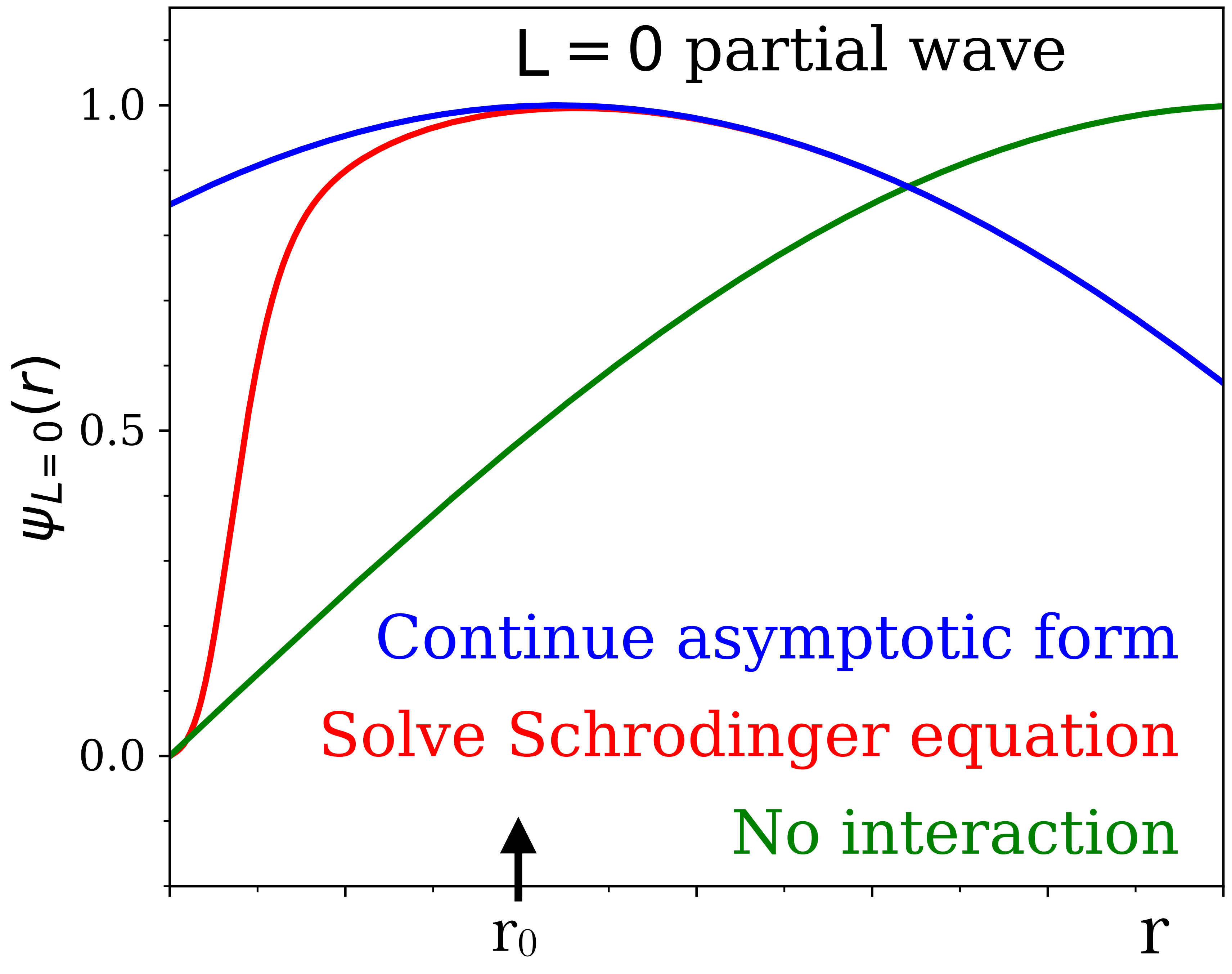
Solving Schrödinger equation numerically

- Using potentials derived from fits to experimentally measured phase shifts
- This method allows for a calculation of wave function down to $r = 0$
- Possible to capture the full dynamics of interaction including shorter distances

Applied in CorAL package ! https://github.com/scottedwardpratt/coral

- Potentials chosen to be combinations of simple square wells
- Altered to best fit the experimental phase shifts
- Potentials found for total intrinsic spins of deuteron: $S = 0{,}1$
- And for orbital momenta: $L = 0{,}1{,}2$

More details about treatment of orbital angular momenta J in arXiv:2410.13983



r_0 - zero effective-range of the interaction

Solving Schrödinger equation numerically: CorAL

What about potentials determination? More than one can:

- Produce the same phase shift
- Have the same derivative of the phase shifts:

$$\int dr \left(|\psi(r)|^2 - |\psi_0(r)|^2 \right) = \frac{1}{2}\frac{d\delta}{dk^*}$$

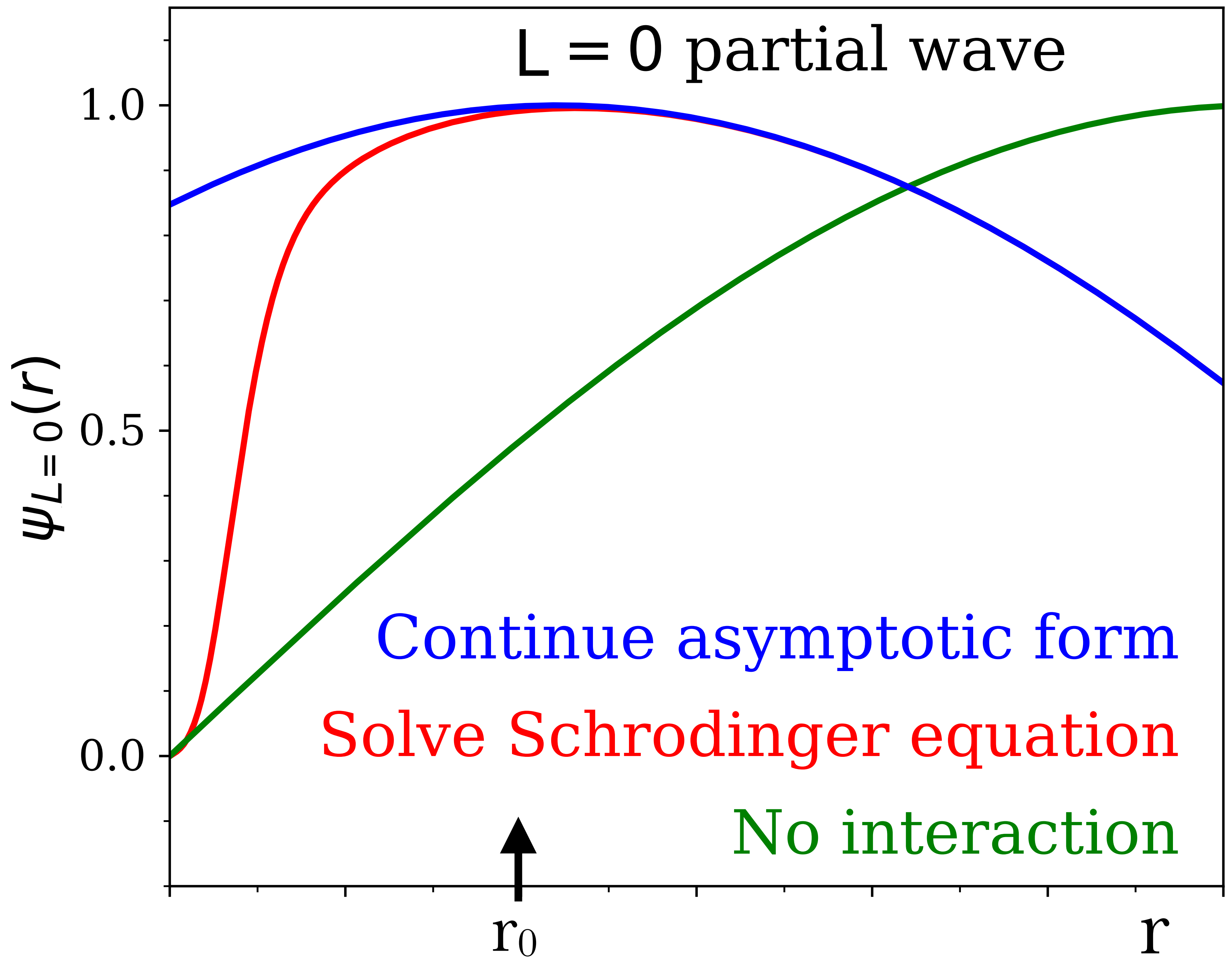
ψ_0 - wave function in the absence of SI
δ - parameter related to phase shift

Integral of $|\psi(r)|^2$ is the same in range of r_0

Not trivial to decide between two potentials

Important note: in scattering theory phase shifts that differ by π are indistinguishable

Look at the experimental data!



r_0 - zero effective-range of the interaction

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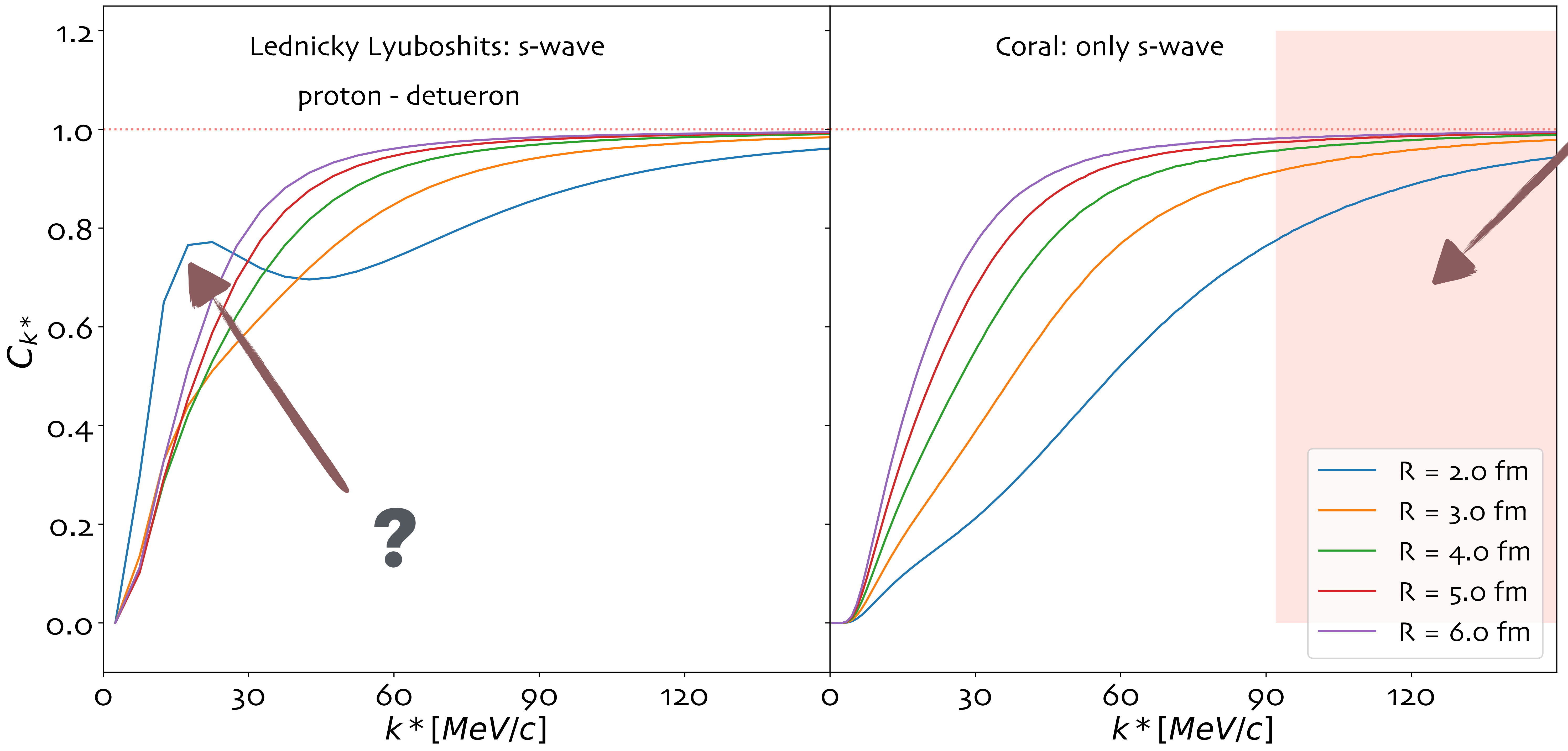
Comparison of two approaches

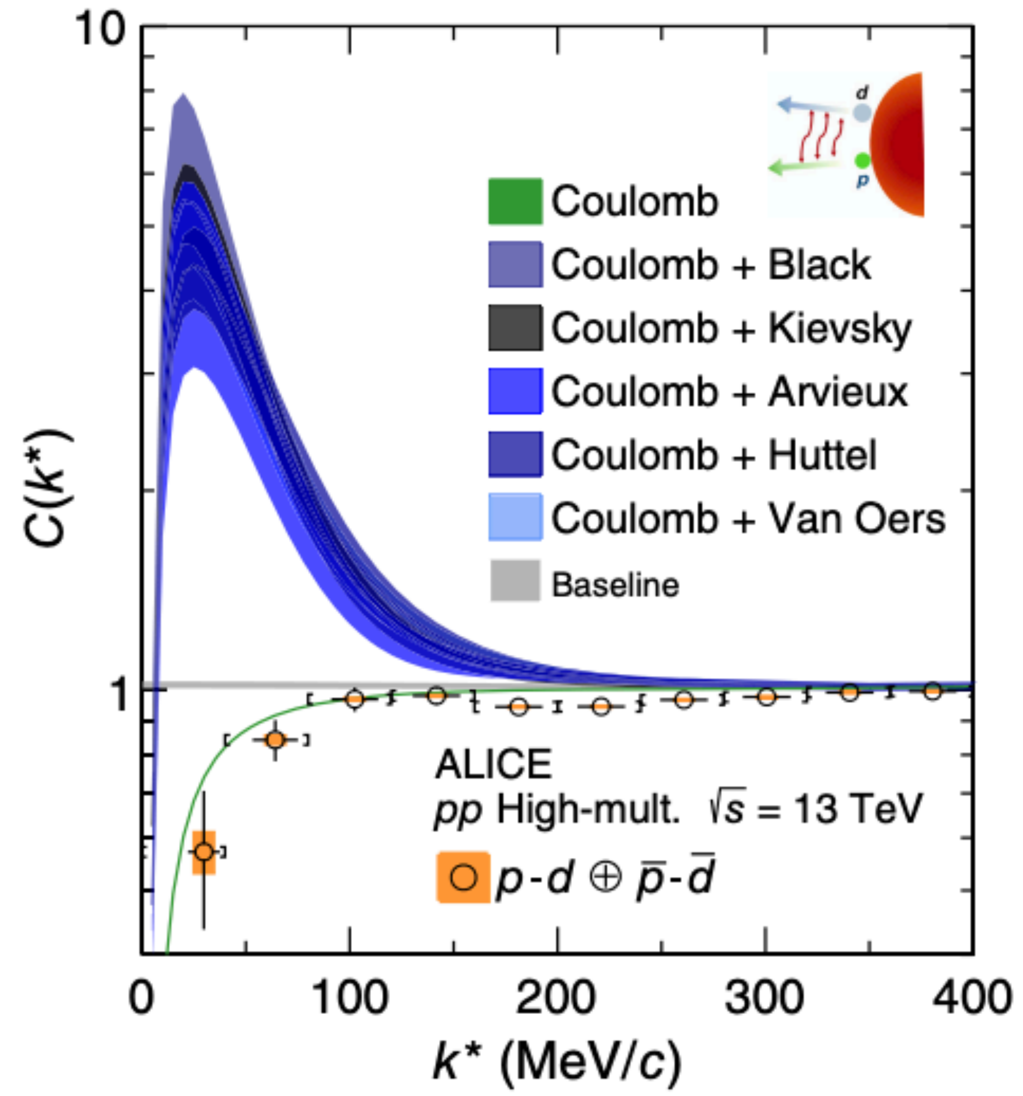


phase shifts measured up to $k^* < 92$ MeV/c

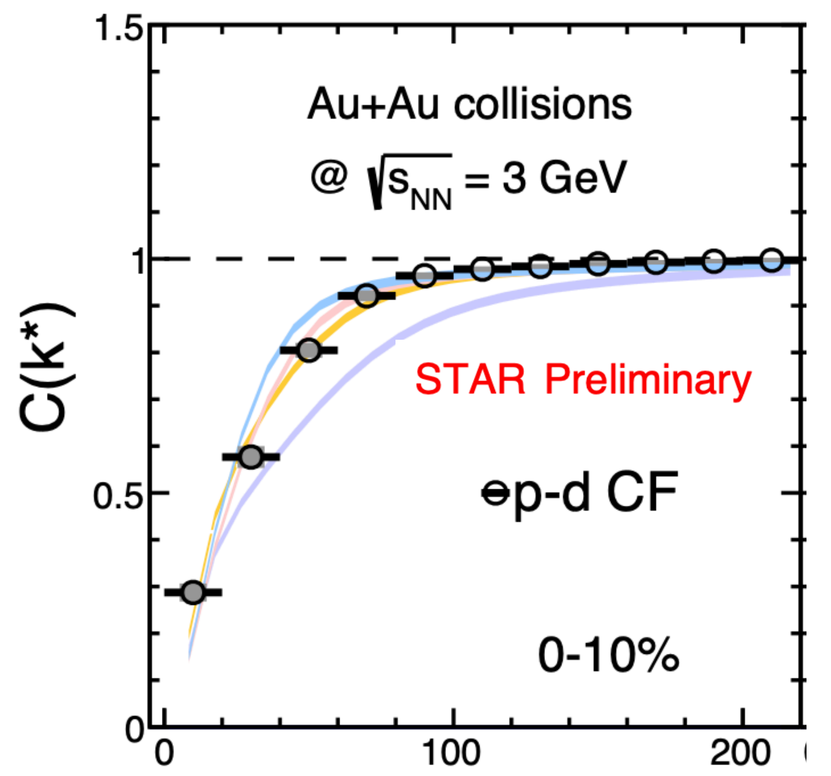
W.Tornow, A. Kievsky, H. Witała: Few-Body Systems, vol. 32, no. 9, 2002

T. Black at al: Physics Letters B, vol. 471, no. 2, pp. 103–107, 1999

higher k^* - extrapolation



ALICE: Phys. Rev. X 14, 031051



Acta Phys. Polon. Supp., vol. 16, no. 1, pp. 1–A91,

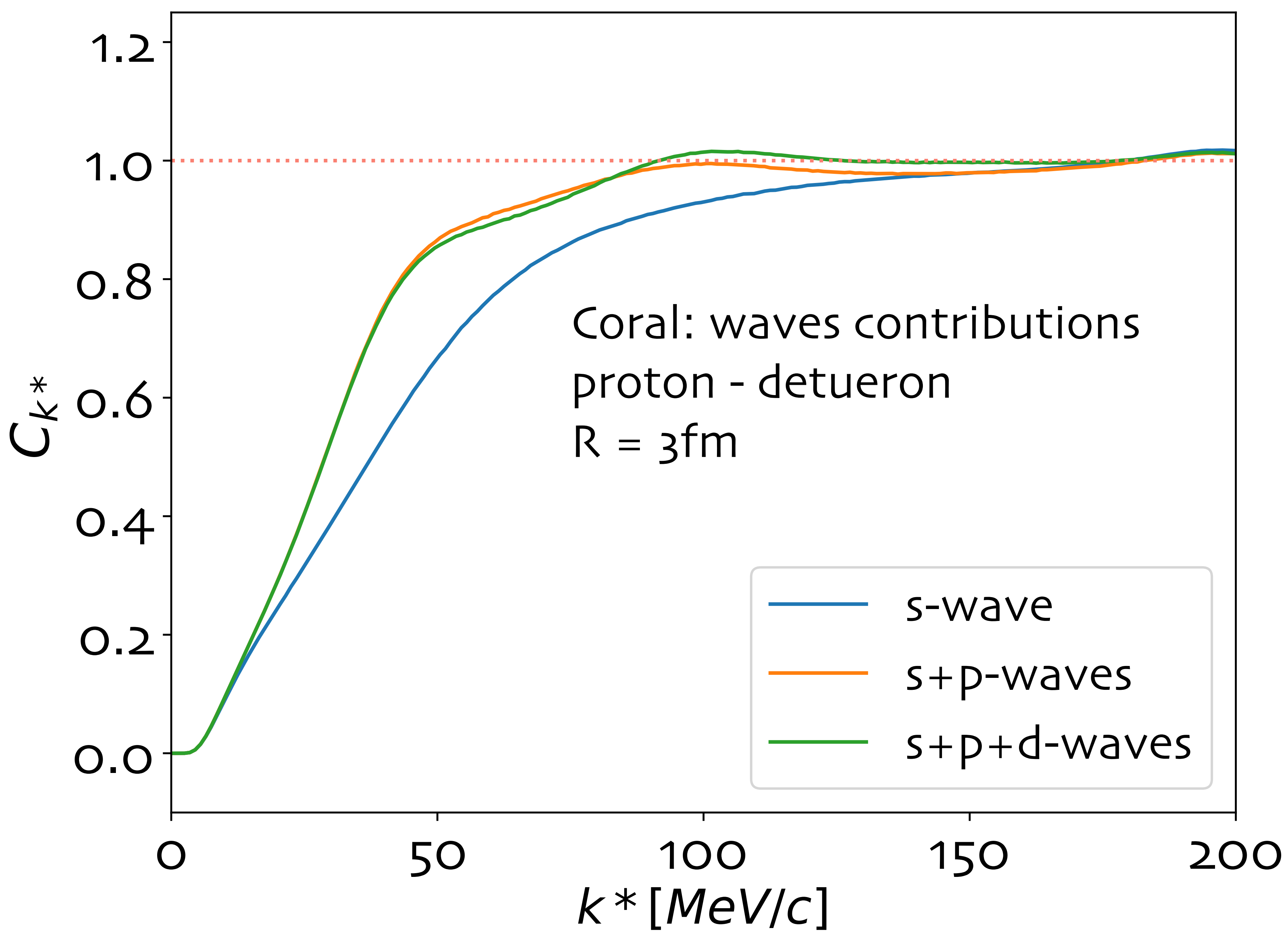
- S-wave only!
- Not clear source of peak structure at k^* around 20 MeV/c

Not visible in any available experimental data!

Higher orbital momentum waves

- p-wave contribution significant for capturing the dynamics od p-d interactions
- Reduces the strength of in traction between p and d, less repulsive
- Possible formation of ³He bound state in $S = 1/2$ spin state

M. Viviani, S. K¨onig, A. Kievsky, L. E. Marcucci, B. Singh, and O. V. Doce: Phys. Rev. C, vol. 108, p. 064002, Dec 2023



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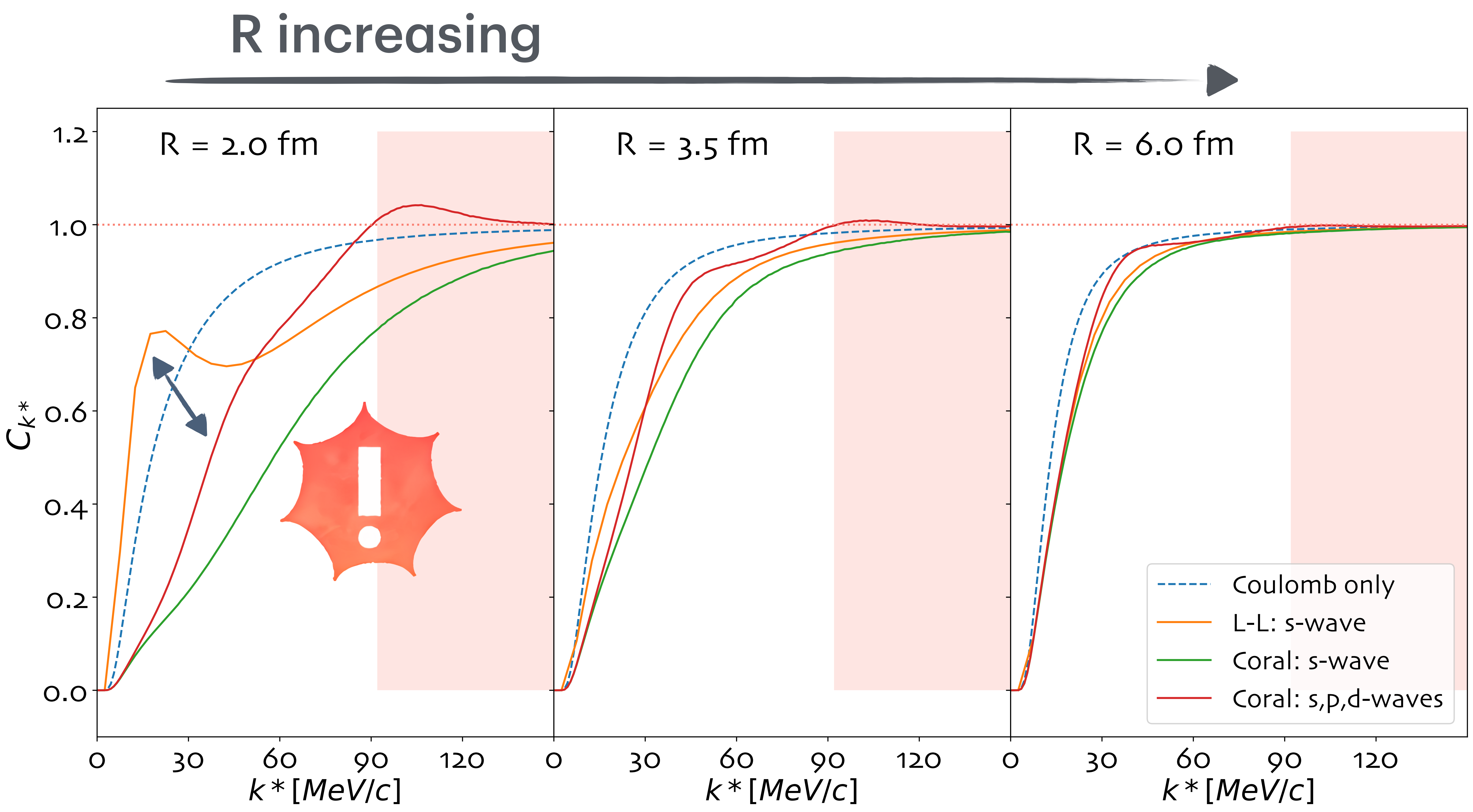

- Coulomb only - reference
- Clear discrepancies between two models
- Difference relevant for smaller sources
- R = 2 fm: peak structure in LL
- “Wiggles” in CorAL incorporate s-, p-, d- waves arise from complexity of the potential.

Also seen in:

M. Viviani, S. K¨onig, A. Kievsky, L. E. Marcucci, B. Singh, and O. V. Doce: Phys. Rev. C, vol. 108, p. 064002, Dec 2023
ALICE:, Phys. Rev. X, 2023

Key difference between the two-body approaches: asymptotic approximation

Conclusions

- Significant impact of asymptotic approximation on the description of the *p-d* interactions
- Relevant influence of higher -order waves (especially p-wave!)

Message: Employ a full solution of the Schrödinger equation in any studies of correlations including deuteron!

W. Rzęsa, M. Stefaniak, S. Pratt: arXiv:2410.13983

V. ACKNOWLEDGMENTS

We would like to express our gratitude to the entire "HBT camp" community for their insightful discussions and the inspiration that led to this paper.

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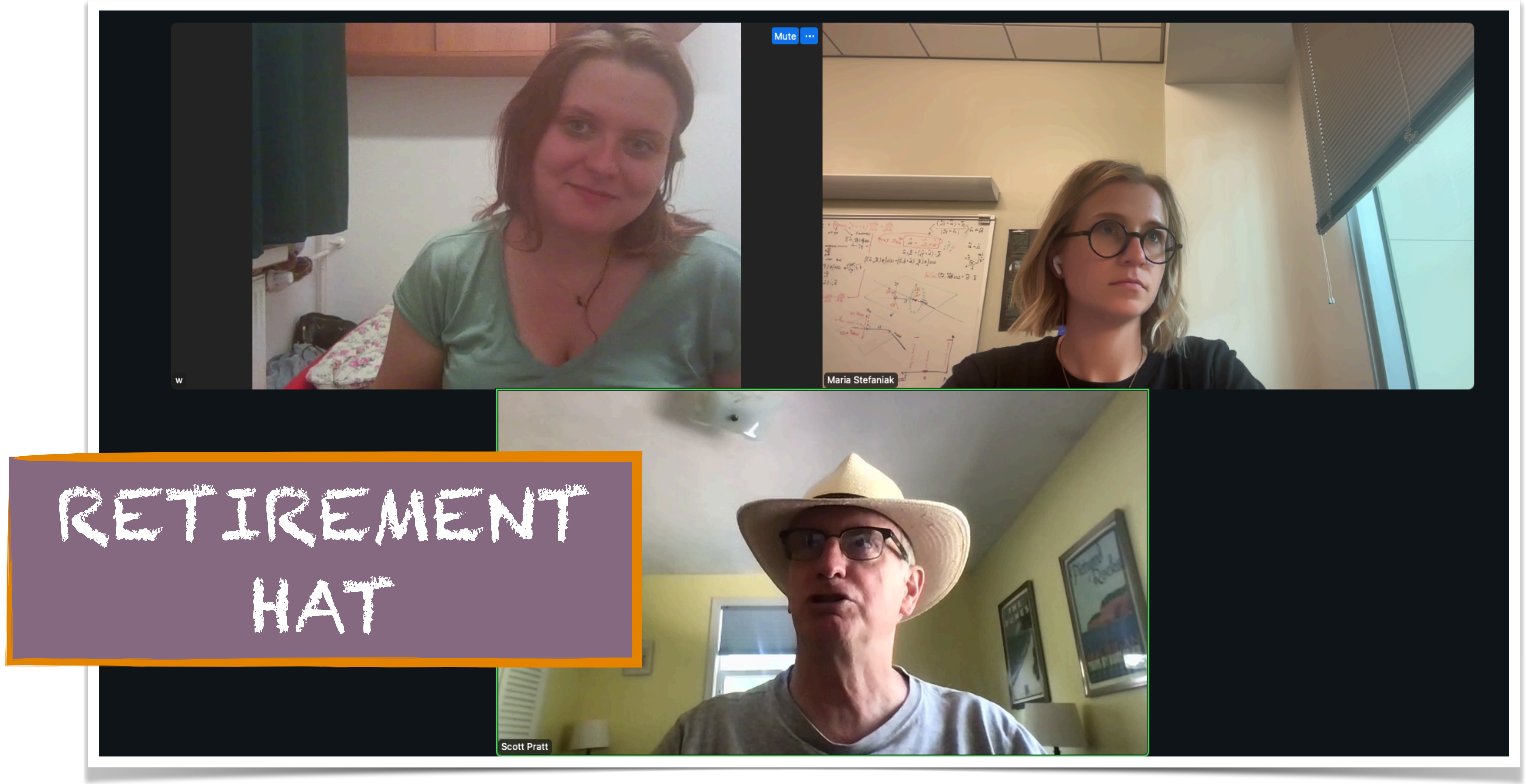

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Happy retirement Scott!
Thank you for the time you dedicated to this work and HBT camp :)