Studying the proton source in Pb–Pb collisions at $\sqrt{s_{\mathrm{NN}}}$ = 5.36 TeV

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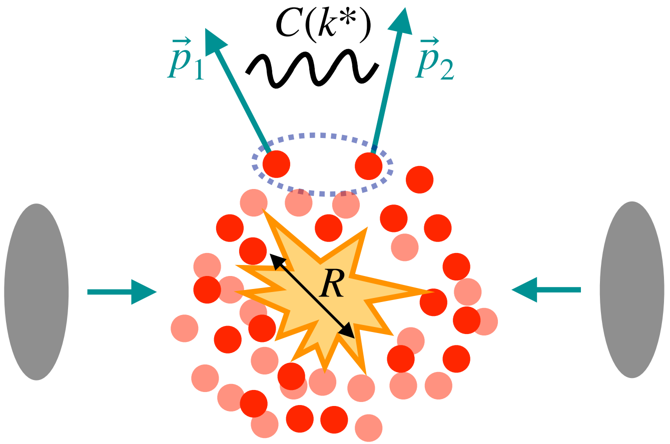
on behalf of the ALICE Collaboration

WPCF 2024
Toulouse, 4/11/2024

This research has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 950692).

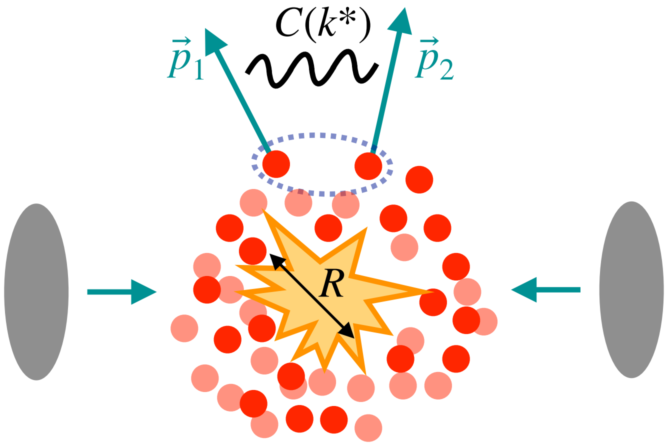
Femtoscopy



$2k^* = |\vec{p}_1 - \vec{p}_2|$ —> **rel. momentum of a pair**

Correlation femtoscopy is used for studying space–time properties of an emission source via particle correlations based on quantum statistics (QS), strong and Coulomb interactions.

Femtoscopy



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Correlation femtoscopy is used for studying space–time properties of an emission source via particle correlations based on quantum statistics (QS), strong and Coulomb interactions.

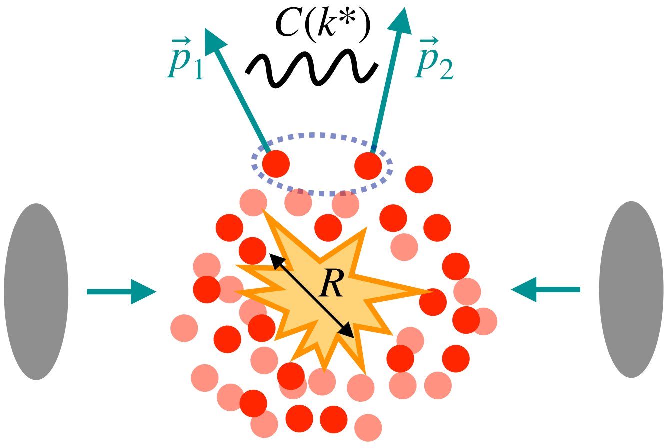
Femtoscopic correlation function (CF) experimentally obtained as a ratio:

$$\mathbf{C}(k^*) = \mathbf{N} \cdot \frac{\mathbf{S}(k^*)}{\mathbf{B}(k^*)}$$

$\mathbf{S}(k^*)$ — rel. momentum distribution of pairs measured in the same event;

$\mathbf{B}(k^*)$ — rel. momentum distribution of pairs measured in different events;

Femtoscopy



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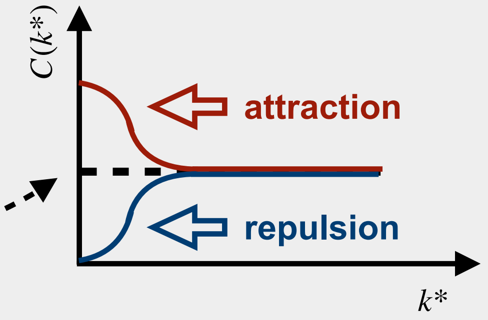
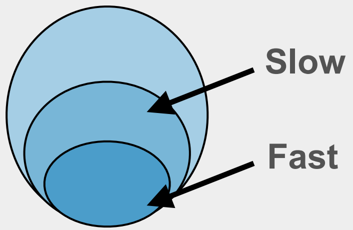
Correlation femtoscopy is used for studying space–time properties of an emission source via particle correlations based on quantum statistics (QS), strong and Coulomb interactions.

Motivation:

- Measure the spatial & temporal characteristics of the particle-emitting regions;
- Study strong interaction;
- Study collective dynamics (e.g. radial flow);
- Check and constrain theoretical models;



Expanding source -> x:p correlation



Parametrisation of the correlation function

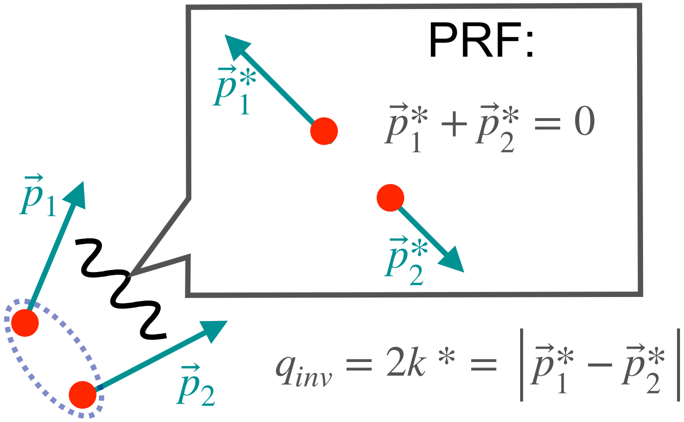
1D parametrisation in Pair Rest Frame (PRF*):

$$C_{exp}(R_{inv}, k^*) = N\left[1 - \lambda\left(C_{th}(R_{inv}, k^*) - 1\right)\right]$$

λ — correlation strength

N — normalisation

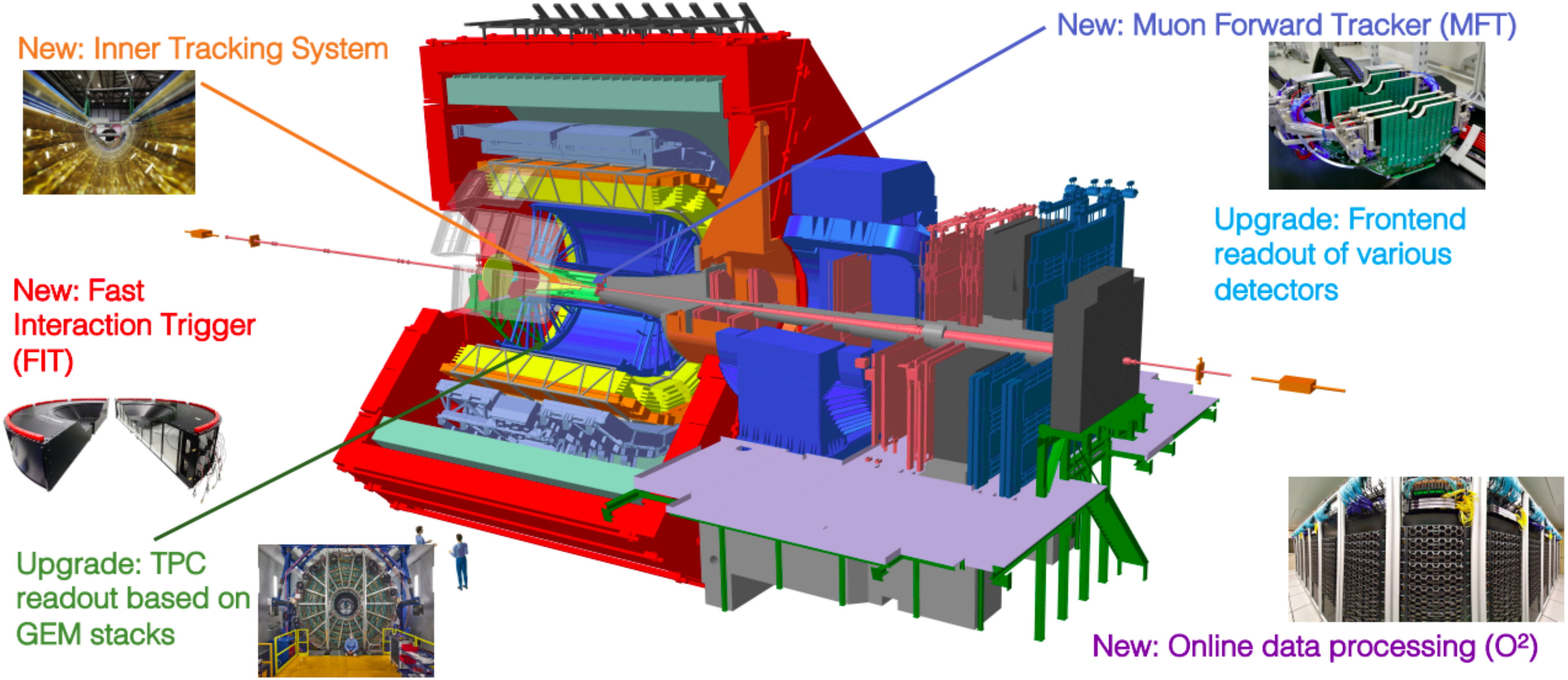
R_{inv} — 1D radius — corresponds to geometrical size of the system



For the theoretical proton correlation function we used a custom analytical model that is based on the Lednicky's one but with an additional square-well potential to take into account the strong interaction at the small distances.

(more detail in the backup)

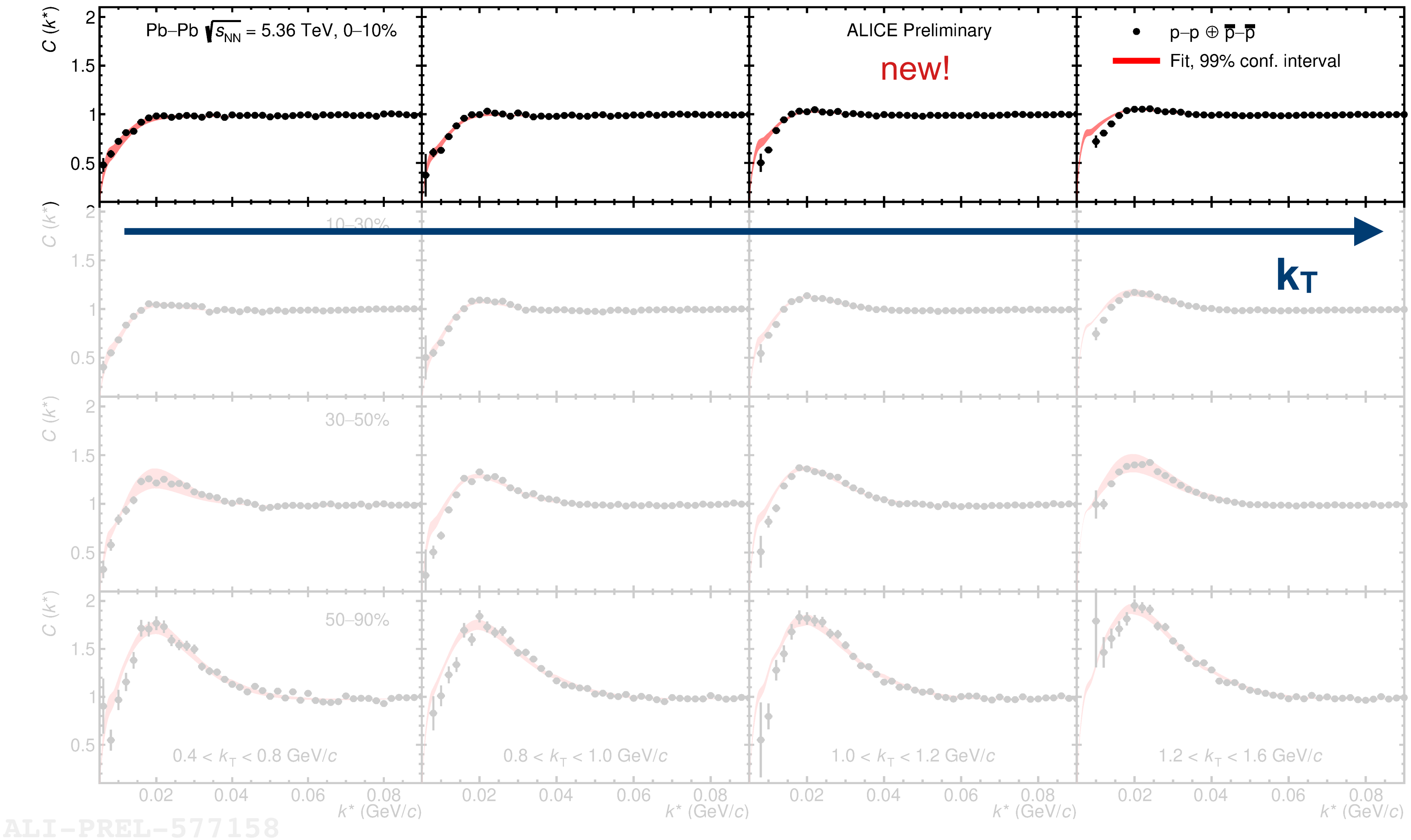
Results from the first Pb–Pb data of Run 3 with a “new” ALICE detector



Proton source

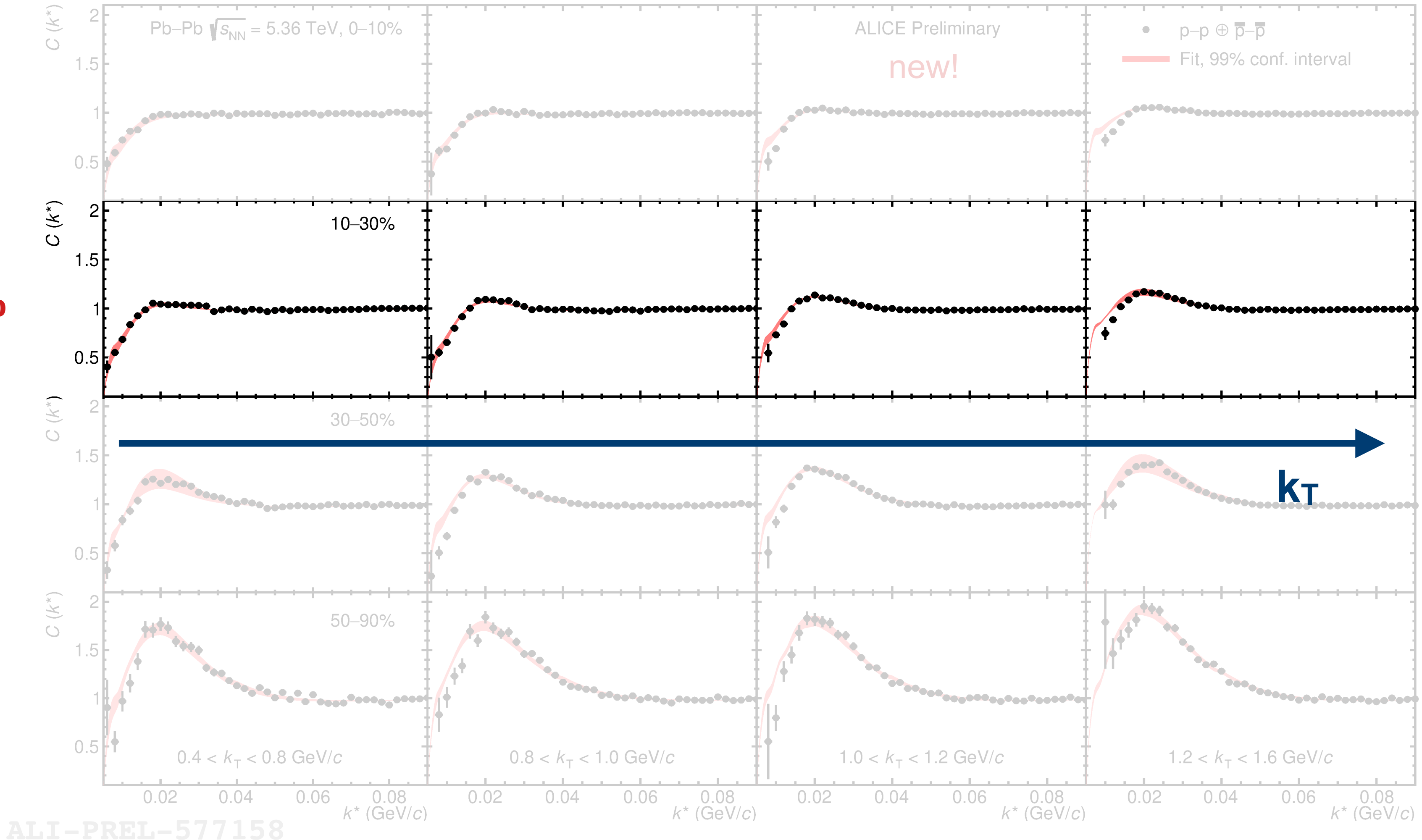
Proton CFs in Pb–Pb at 5.36 TeV

0-10%



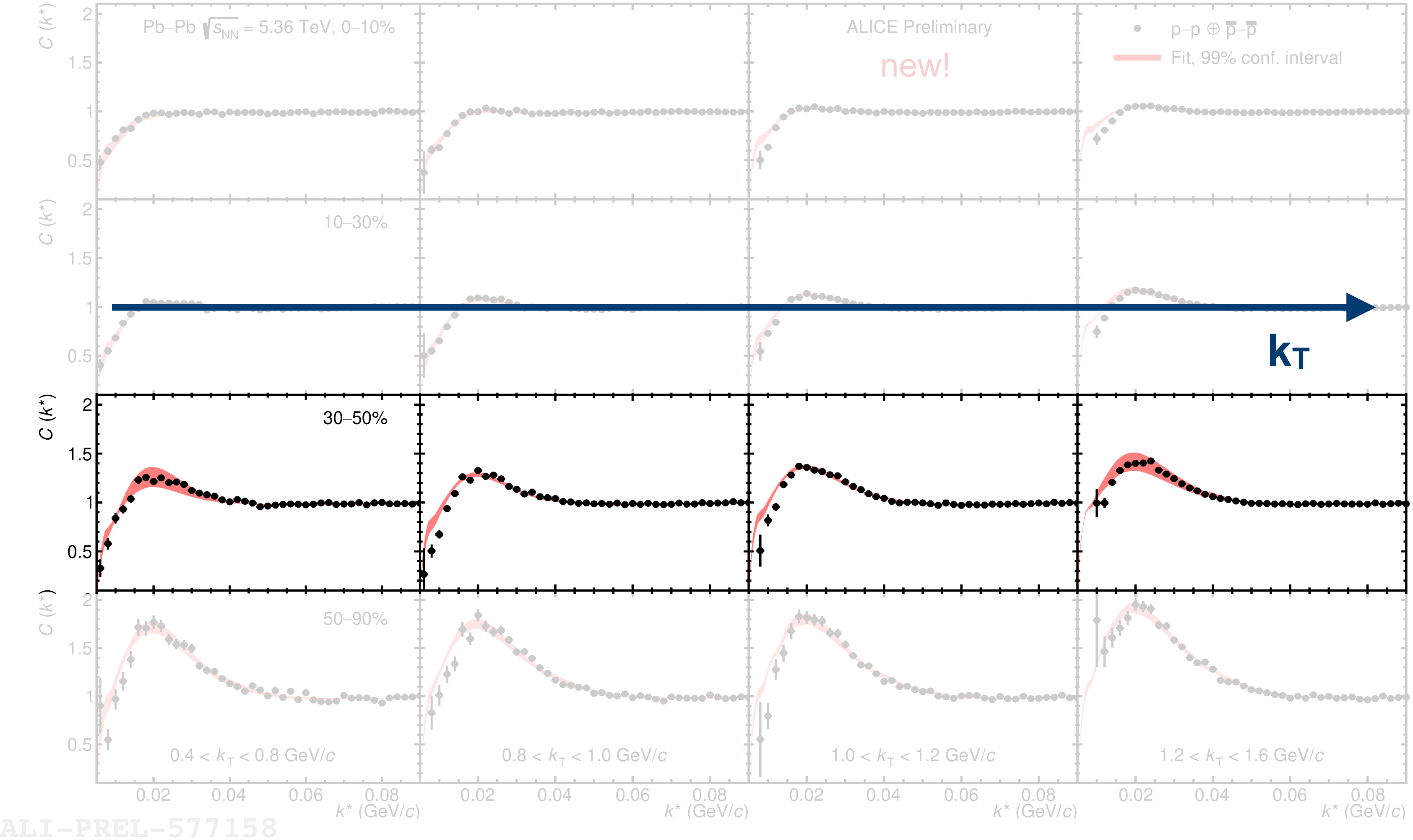
Proton CFs in Pb–Pb at 5.36 TeV

10-30%



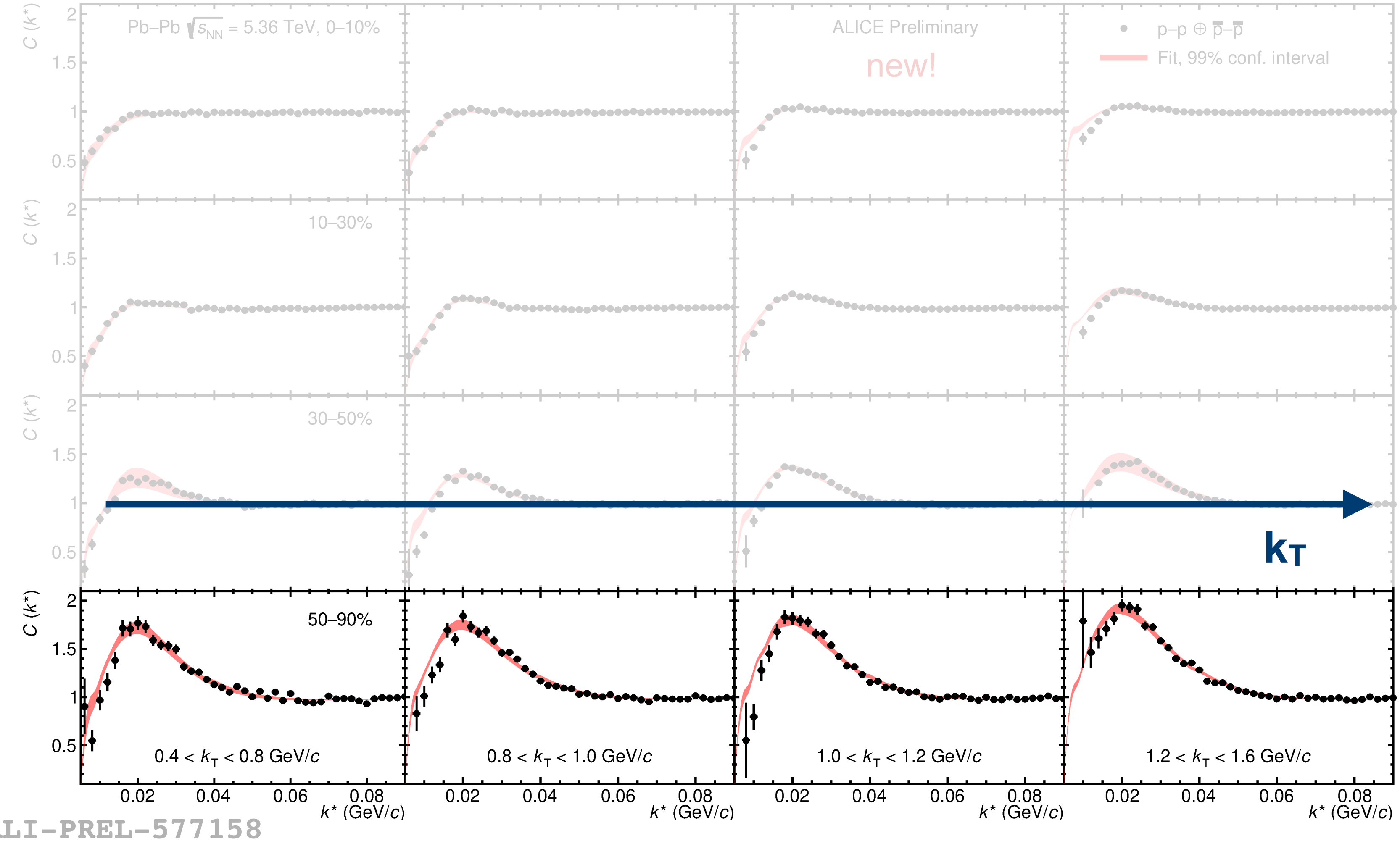
Proton CFs in Pb–Pb at 5.36 TeV

30-50%

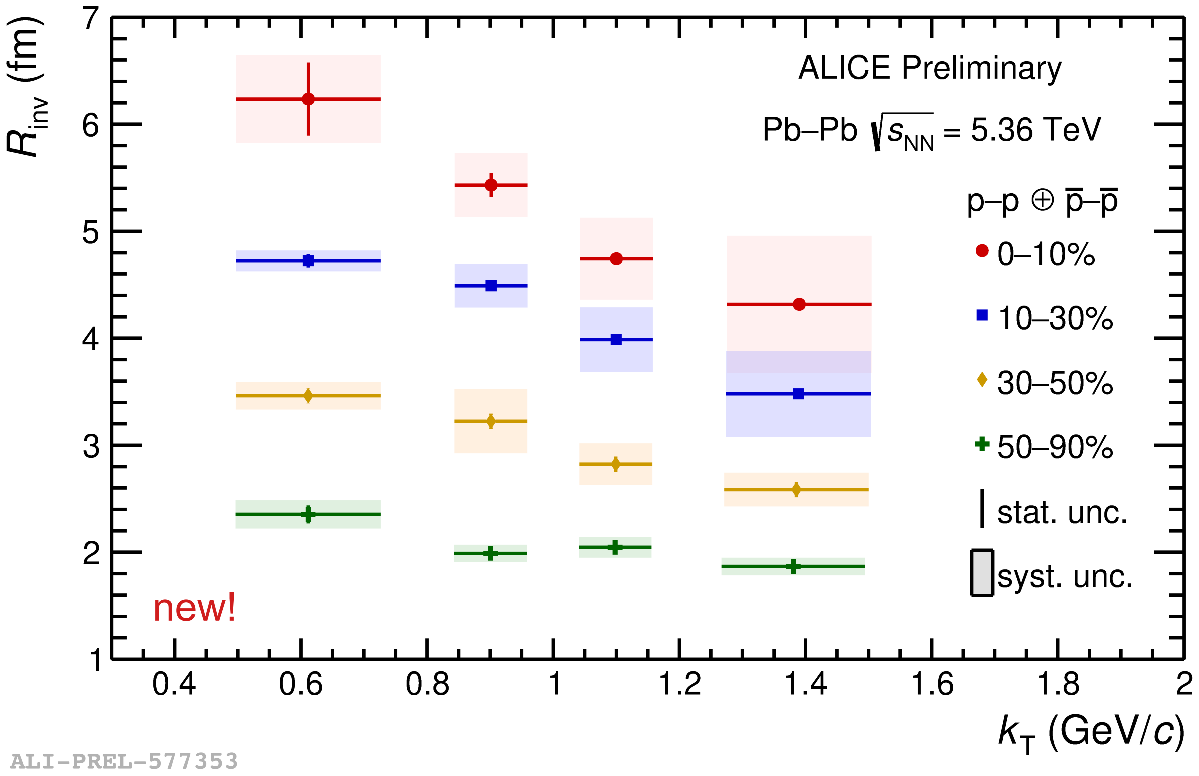


Proton CFs in Pb–Pb at 5.36 TeV

50-90%

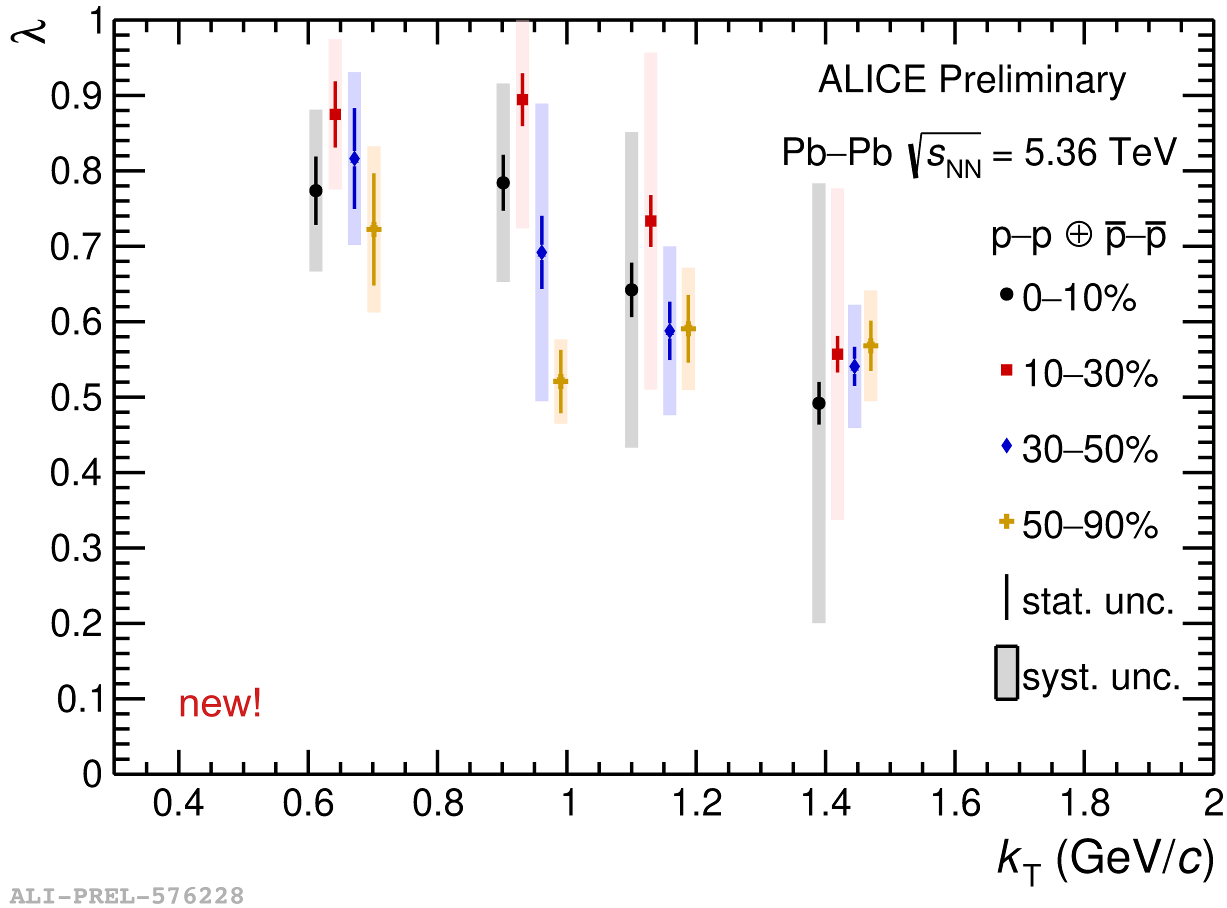


Proton 1D radii in Pb–Pb at 5.36 TeV



- Proton radii demonstrate the dynamics that is typical for heavy-ion collisions.
- R_{inv} decreases with increasing k_T → collective (radial) flow (weaker for more peripheral events)

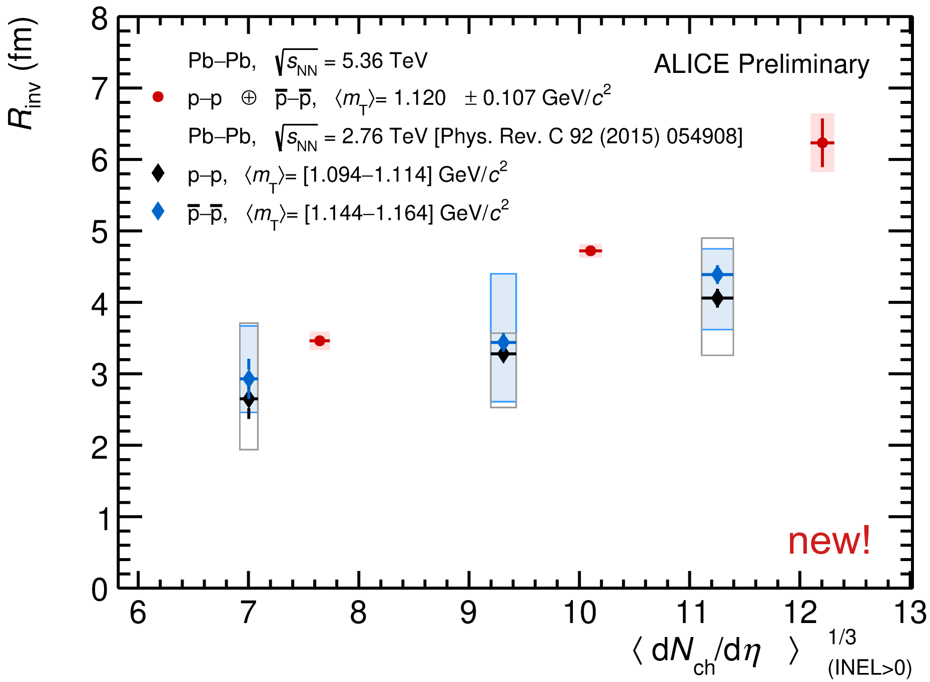
Proton 1D lambdas in Pb–Pb at 5.36 TeV



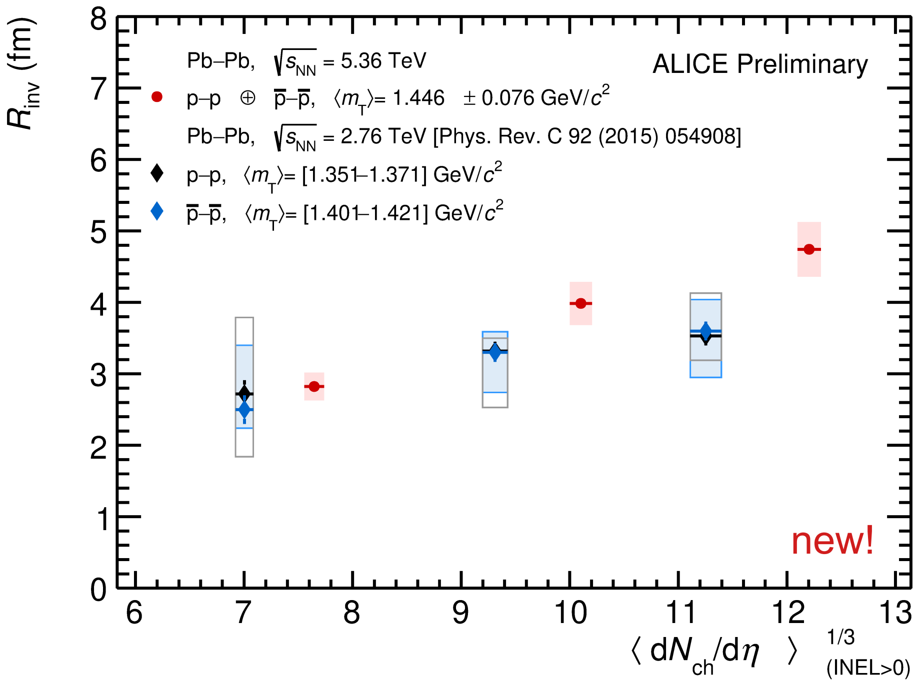
- The extracted λ parameters are consistent throughout all the centrality bins
- The decreasing trend with increasing the k_T is caused by decrease in purity

*k_T binning and errors along X axis for λ parameters are the same as for the radii, the points have been shifted for clarity.

Proton 1D radii: comparison with Run 1



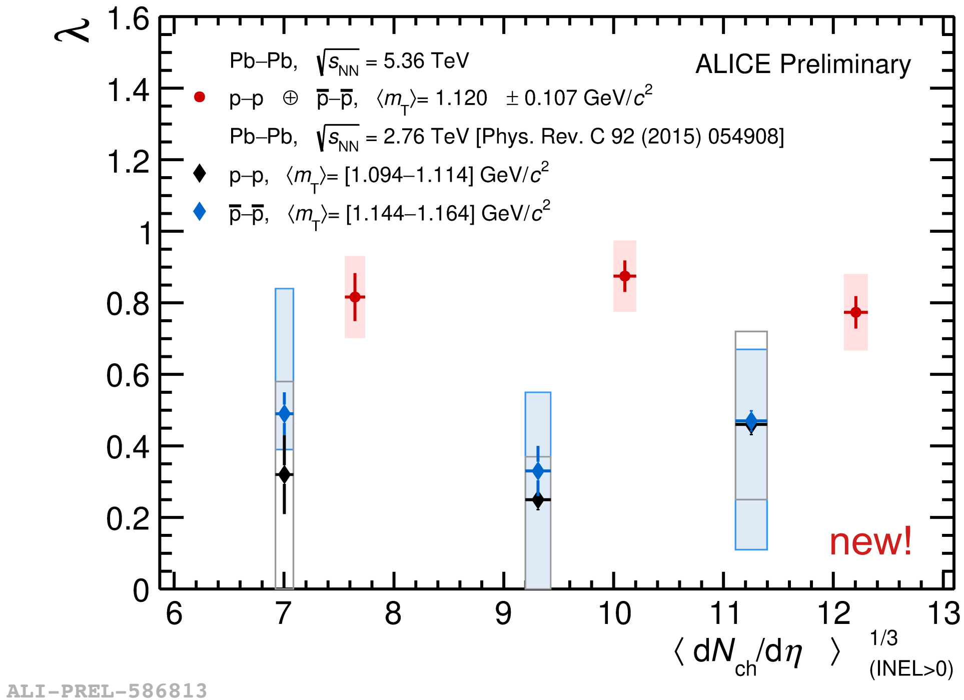
ALI-PREL-586808

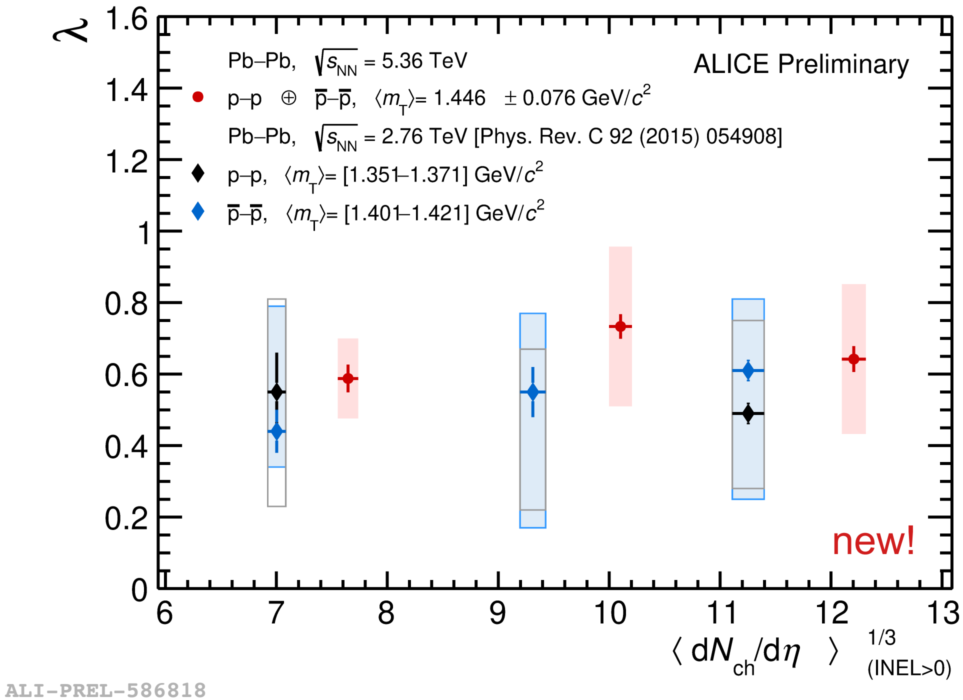


ALI-PREL-586803

- The new Run 3 results are consistent with Run 1 data (at close <m_T>)
- The precision has improved w.r.t. Run 1
- More peripheral events are accessed w.r.t. Run 1 results
  (50-90% from Run 3 not shown here)

Proton 1D Lambdas: comparison with Run 1





- The λ parameters obtained with Run 3 results are higher than the one of Run 1 data (at close <m_T>)
- Probably comes from the different in theoretical models — usual Lednicky's approach is known to overestimate the CF (more in the backup).

Summary

- First femtoscopic measurement with ALICE's Run 3 Pb–Pb data is performed;
- Proton radii demonstrate the dynamics typical for heavy-ion collisions → collectivity;
- New Run 3 results are in a good agreement with Run 1 ones;
- "New" theoretical approach is tested;
- Significant improvements are expected (more statistics, better reconstruction, etc.)

Backup slides

Lednický-Lyuboshitz model

Lednický-Lyuboshitz model simply introduces the short-range (strong) potential asymptotically by adding an additional phase shift to the WF:

$$\psi_{c+s} \underset{r>d}{=} \frac{1}{2\rho} \sum_{l=0}^{\infty} (2l+1)\, i^{l+1} \left(u_l^-(\eta,\rho) - e^{2i\sigma_l} e^{2i\delta_l} u_l^+(\eta,\rho) \right) P_l(\cos\theta)$$

δ_l — phase shift corresponding to the short-range potential

We can rewrite:

$$\psi_{c+s} \underset{r>d}{=} \frac{1}{r} \sum_{l=0}^{\infty} (2l+1)\, i^l e^{i\sigma_l} \left(\frac{F_l(\eta,\rho)}{k} + f_l(k) \left(G_l(\eta,\rho) + i\, F_l(\eta,\rho) \right) \right) P_l(\cos\theta)$$

If we consider a short-range potential only in s-wave:

$$\psi_{c+s}^{l=0} \underset{r>d}{=} \sqrt{A_c(\eta)}\, e^{i\sigma_0} e^{i\vec{k}\vec{r}}\, {}_1F_1\left(-i\eta,\, 1,\, i(kr - \vec{k}\vec{r}) \right) + e^{i\sigma_0} f_0(k) \frac{G_0(\eta,\rho) + i\, F_0(\eta,\rho)}{r}$$

But that is an asymptotical solution (also singular at 0) to the Schrodinger's equation and can be used only outside the strong potential range.

(Ledndicky also noted that (for example): https://arxiv.org/abs/nucl-th/0501065v3)

Starting with the Coulomb

The WF that satisfies the Schrodinger's equation for the Coulomb potential is well known (here we already anticipate the partial wave expansion):

$$\psi^{reg} = \sum_{l=0}^{\infty} (2l+1) i^l e^{i\sigma_l} \frac{F_l(\eta, \rho)}{\rho} P_l(cos\ \theta)$$

we need the solution to be regular at 0 so we close the regular Coulomb WF

or

$$\psi^{reg} = \frac{1}{2\rho} \sum_{l=0}^{\infty} (2l+1)\ i^{l+1} \left(u_l^-(\eta,\rho) - e^{2i\sigma_l} u_l^+(\eta,\rho) \right) P_l(cos\ \theta)$$

Adding here an additional phase shift one can obtain the analytical solution for a short-range potential — e.g. Lednicky's model

$$\rho_l = k_l \cdot r$$

$$\eta_l = \frac{1}{k_l a_B}$$

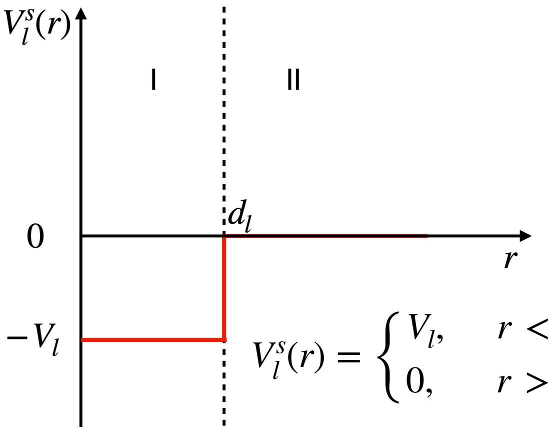
Albert Messiah, Quantum Mechanics, v1

Adding box potential

Let's introduce a short-range potential as a square-well for each value of l.

Radial Schrodinger's equation in (I) sector:

$$\frac{\mathrm{d}^2 R_l^{(I)}}{\mathrm{d}r^2} + \left[k^2 - \frac{l(l+1)}{r^2} - \frac{2}{a_B r} - \frac{2\mu}{\hbar^2} V_l\right] R_l^{(I)} = 0$$



$$V_l^s(r) = \begin{cases} V_l, & r < d_l \quad (I) \\ 0, & r > d_l \quad (II) \end{cases}$$

Substituting the sq.-well potential:

$$\frac{\mathrm{d}^2 R_l^{(I)}}{\mathrm{d}r^2} + \left[\tilde{k}^2 - \frac{l(l+1)}{r^2} - \frac{2}{a_B r}\right] R_l^{(I)} = 0 \qquad \tilde{k}_l = \sqrt{k^2 - \frac{2\mu}{\hbar^2} V_l}$$

But we already know the solution (here we put directly the total):

$$\psi^{reg} = \sum_{l=0}^{\infty} (2l+1) i^l e^{i\tilde{\sigma}_l} \frac{F_l(\tilde{\eta}, \tilde{\rho})}{\tilde{\rho}} P_l(\cos\theta)$$

Final solution and CF

After matching the two WFs (within the box potential and the asymptotical one) one can get the final WF:

$$\psi_{c+s}(k, r) = \frac{1}{r} \sum_{l=0}^{\infty} (2l+1)\, i^l e^{i\sigma_l}\, u_l(k, r)\, P_l(cos\ \theta)$$

$$u_l(k, r) = \begin{cases} \frac{F_l(\tilde{\eta}_l, \tilde{k}_l r)}{F_l(\tilde{\eta}_l, \tilde{k}_l d)} \left(\frac{F_l(\eta, kd)}{k} + f_l(k)\left(G_l(\eta, kd) + i\ F_l(\eta, kd)\right)\right), & r < d \\ \left(\frac{F_l(\eta, \rho)}{k} + f_l(k)\left(G_l(\eta, \rho) + i\ F_l(\eta, \rho)\right)\right), & r \geq d \end{cases}$$

General expression for the CF: $C(k, R_{inv}) = \int d^3r \cdot S(r, R_{inv}) \cdot |\psi(\vec{k}, \vec{r})|^2$

$$S(r, R_{inv}) = \frac{1}{8\pi^{\frac{3}{2}} R_{inv}^3} exp(-\frac{r^2}{4R_{inv}^2})$$

— assuming Gaussian source

For a pair of protons with L=[0, 1]. Corresponding states:

$$C_{pp}(k*, R_{inv}) = \frac{1}{2} \sum_{S=0}^{1} \frac{2S+1}{(2s_p+1)^2} \sum_{L, J} \omega_{LJ} \int d^3r\ S(r, R_{inv})\, |\psi^S_{-\vec{k}}(\vec{r}) + (-1)^S \psi^S_{\vec{k}}(\vec{r})|^2$$

$$\omega_{LJ} = \frac{2J+1}{(2L+1)(2S+1)}$$

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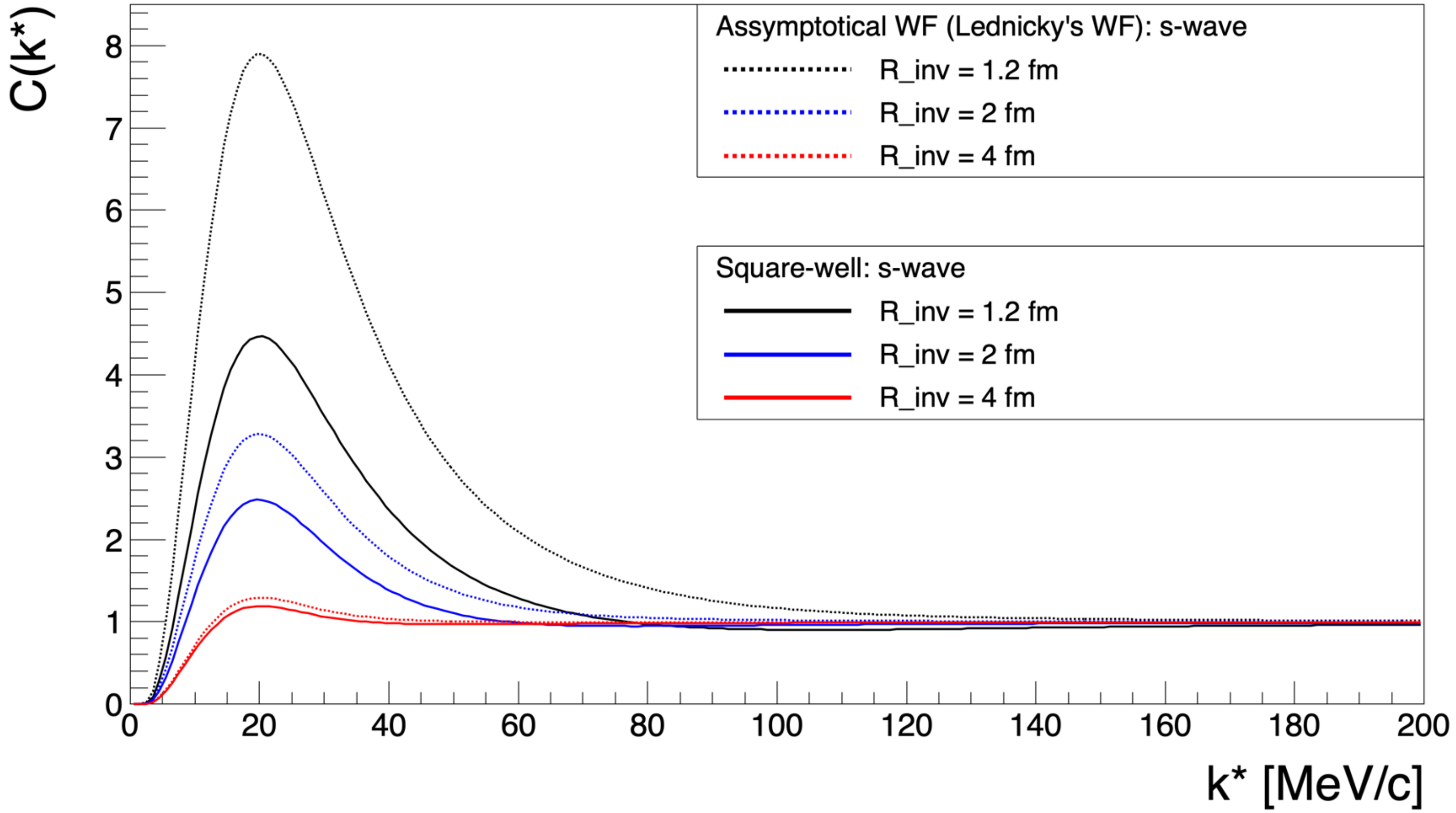
$$\omega_{LJ} = \frac{2J+1}{(2L+1)(2S+1)}$$

Result: with or without square-well potential



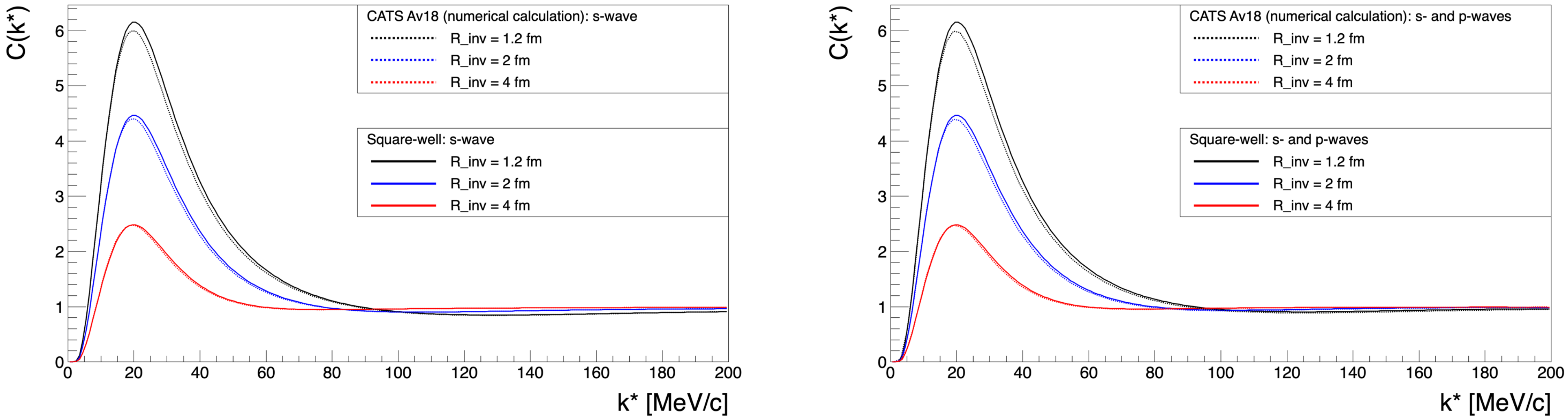
Can we use a square-well?

Square-well is indeed a very rough approximation. But is it so bad?

It was shown in phase shifts analyses that for small energies we are not sensitive to the shape of the strong potential (generally for a short-range one). Which means that we cannot extract any particular information about its shape from experimental data (at least from phase shifts).
(for example: H. A. Bethe, Phys. Rev. 76, 38 – Published 1 July 1949)

But we see our femtoscopic peak in small energy (momentum) region!

Result: comparison with the CATS



The agreement is quite good, though there is a discrepancy (especially for small sizes) that has to be investigated. Nevertheless, in term of application to the experimental data that are no perfectly precise both approaches might be within the uncertainties.

Thanks to Dimitar Mihaylov for the assistance with the CATS.

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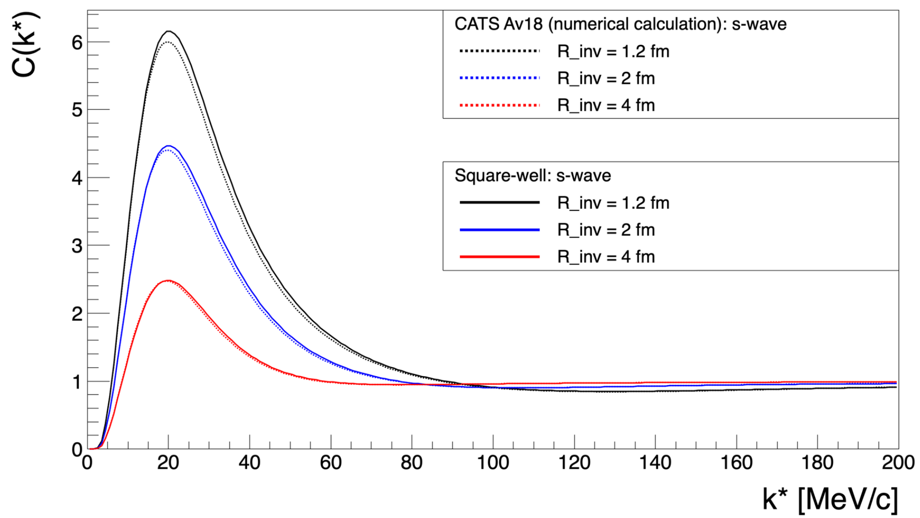

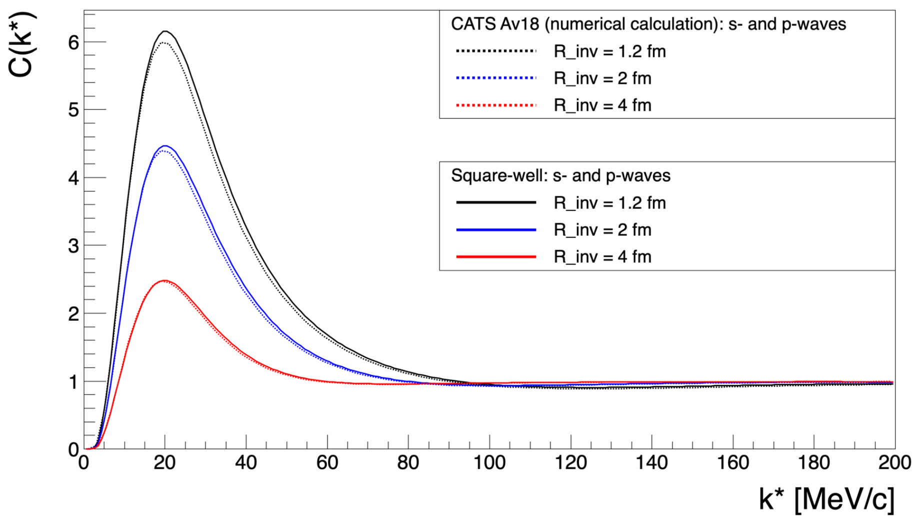

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Defining the potential parameters

OK, we have an analytical WF for a square-well potential, but what are its parameters (depth and width)?

One can obtain them by fitting momenta(energy)-dependent phase shifts with our matching condition:

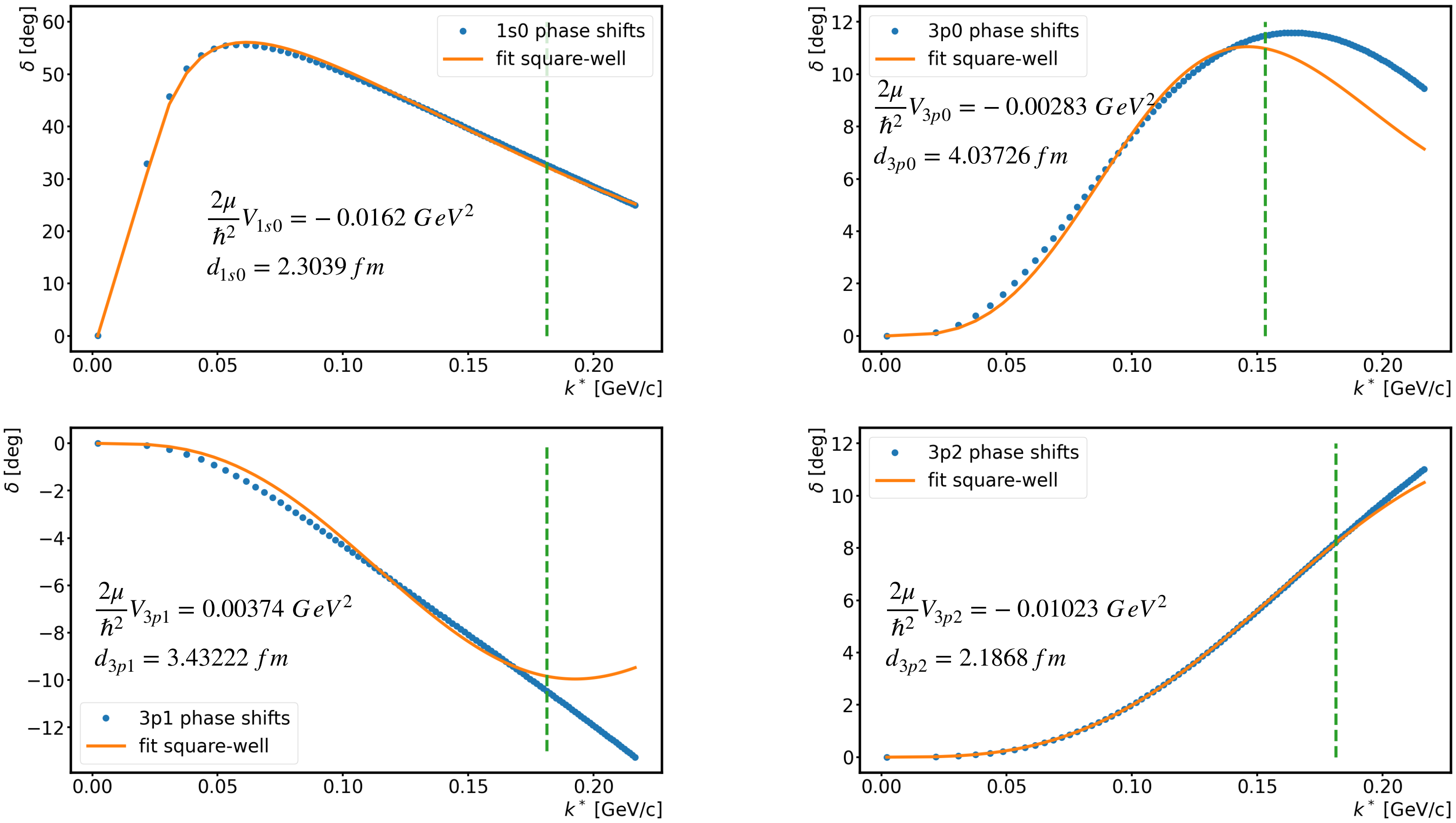
$$ctg\,\delta_l = \frac{G_l(\eta, kd)}{F_l(\eta, kd)}\,\frac{\tilde{k}\,f_l(\tilde{\eta}, \tilde{k}d) - k\,g_l(\eta, kd)}{k\,f_l(\eta, kd) - \tilde{k}\,f_l(\tilde{\eta}, \tilde{k}d)}$$

$$f_l(\eta,\rho) = \frac{\mathrm{d}}{\mathrm{d}r}\Big(ln\,(F_l(\eta,\rho))\Big)\Big|_{r=d}$$

$$g_l(\eta,\rho) = \frac{\mathrm{d}}{\mathrm{d}r}\Big(ln\,(G_l(\eta,\rho))\Big)\Big|_{r=d}$$

Defining the potential parameters

Phase shifts data are taken from: https://nn-online.org/



Green line — chosen range of the fit

Effective range parameters (Bethe's formula)

By fitting the momenta-dependent phase shifts:

$$f(k) = \frac{1}{k\ ctg(\delta_l) - ik}$$

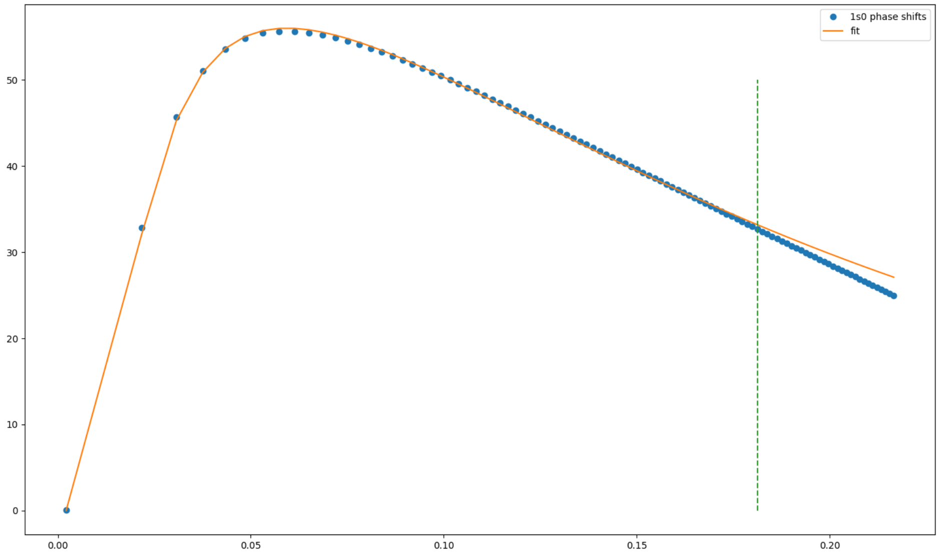
$$k\ ctg(\delta_l) \sim -\frac{1}{f_0} + \frac{1}{2}d_0k^2 + (-Pd_0^3k^4) + O(k^6)$$

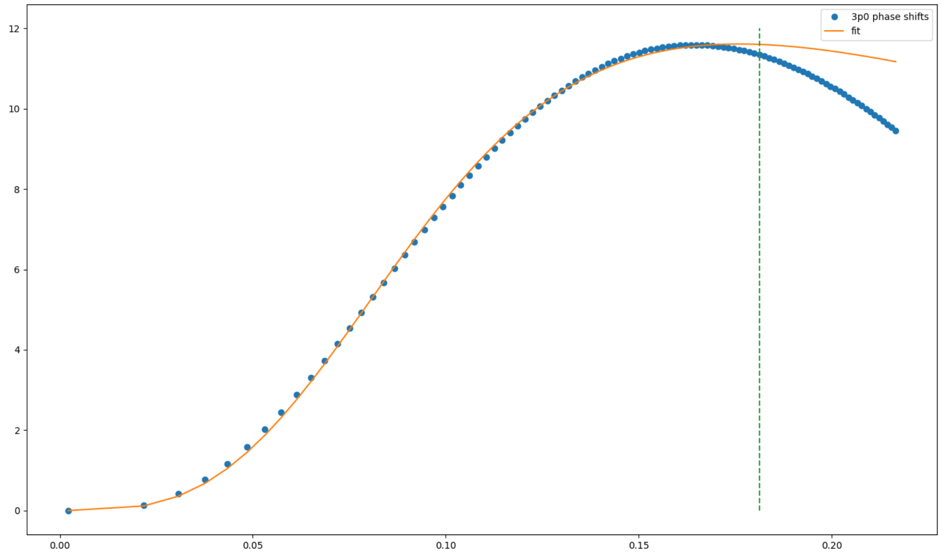
H. A. Bethe, Phys. Rev. 76, 38 – Published 1 July 1949

Extract effective range parameters.

I used coefficient P != 0 only for 1s0 and 3p0 states.

Phase shifts fit: eff. range parameters





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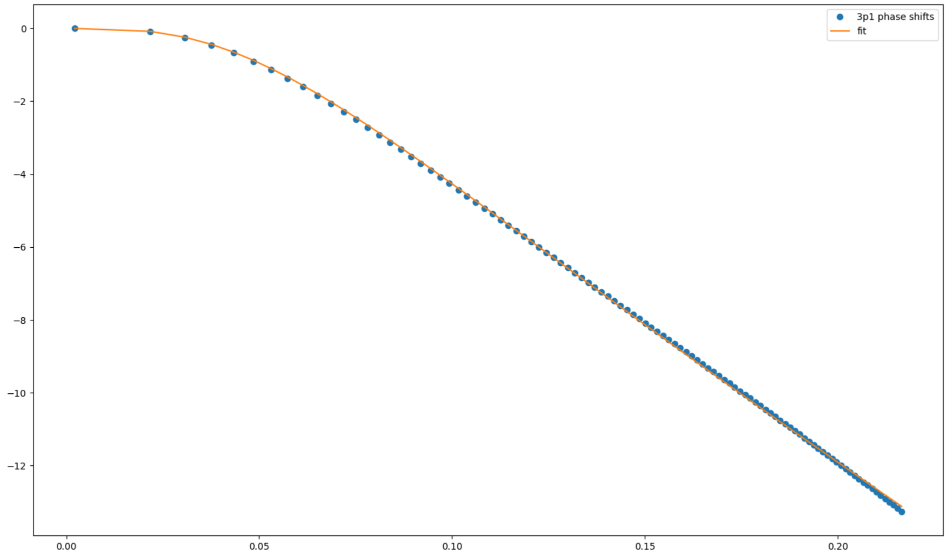

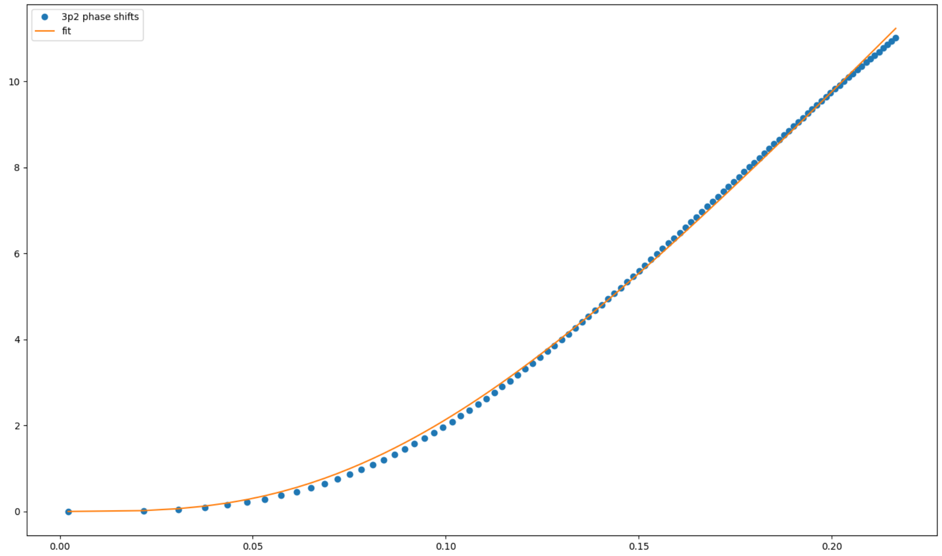

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