LÉVY-STABLE SOURCES FROM SPS THROUGH RHIC TO LHC

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WPCF 2024, TOULOUSE

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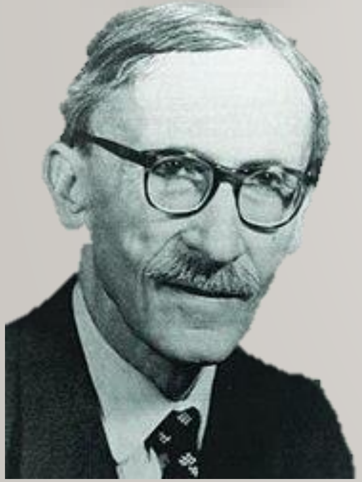


OVERVIEW OF LÉVY FEMTOSCOPY RESULTS AT WPCF

- Measurements
  - Monday 15:25: **Sándör Lökös**, Centrality dependence of Lévy-stable two-pion correlations at PHENIX
  - Tuesday 17:00: **Sneha Bhosale**, 3D measurements of pion HBT correlations at STAR
  - Tuesday 17:25: **Barnabás Pórfy**, Energy scan results with Lévy type femtoscopy at NA61/SHINE
- Simulations
  - Wednesday 16:20: **Dániel Kincses**, Lévy walk of pions in heavy-ion collisions
  - Wednesday 16:35: **Emese Árpási**, 3D source sizes and shapes of hadron emission in EPOS
  - Wednesday 17:15: **László Kovács**, Event-by-event two-kaon source function with EPOS
- Phenomenology
  - Tuesday 17:40: **Márton Nagy**, Coulomb and strong final state interactions with Lévy sources
  - *Monday 11:35: Máté Csanád, this talk ☺*

HBT OR FEMTOSCOPY IN HIGH ENERGY PHYSICS

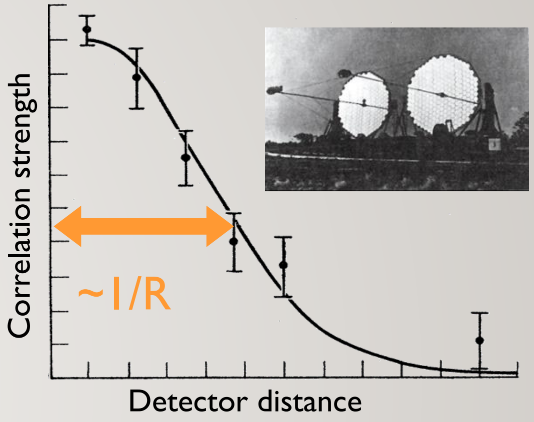
- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to source $S(r)$

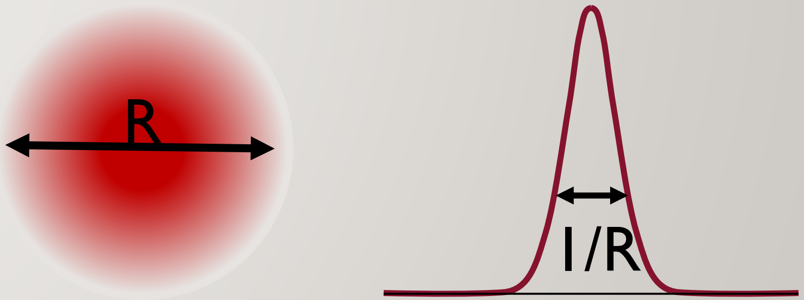
    $$C(q) \cong 1 + \left|\int S(r) e^{iqr} dr\right|^2$$

    (under some assumptions)
  - Also with distance distribution $D(r)$:

    $$C(q) \cong 1 + \int D(r) e^{iqr} dr$$

  - Neglected: pair reconstruction, final state interactions, multi-particle correlations, coherence, …
- What is the source shape? Can be explored via femtoscopy

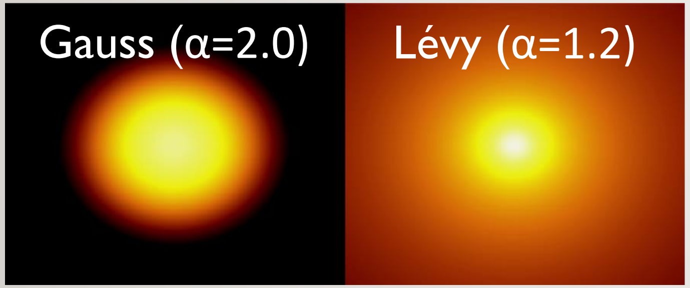


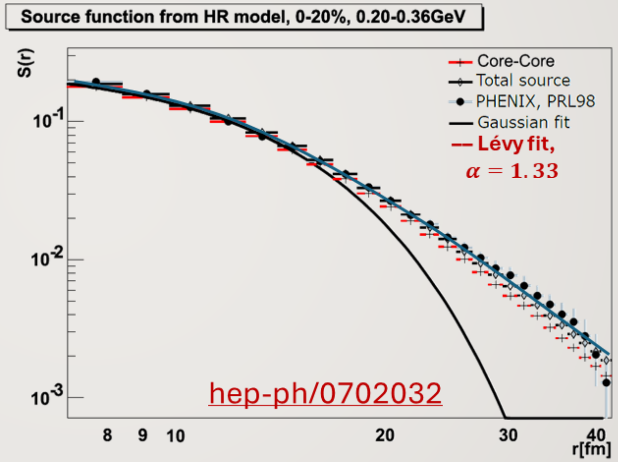


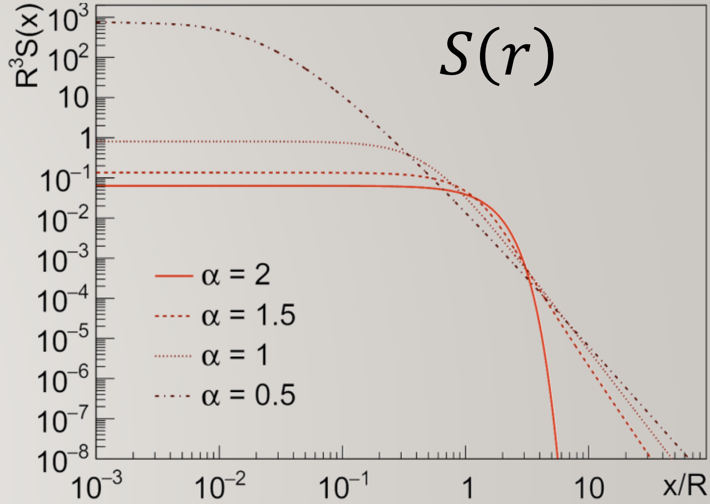
source function S(r) correlation funct. C(q)

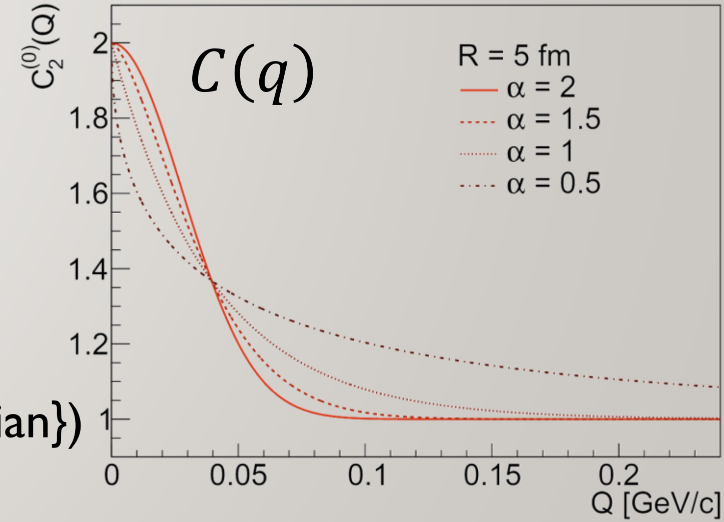
LÉVY DISTRIBUTIONS IN HEAVY ION PHYSICS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = \frac{1}{2\pi}\int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$
  - From generalized central limit theorem
  - Power-law tail $\sim r^{-1-\alpha}$
  - Special cases: α = 2 Gaussian, α = 1 Cauchy





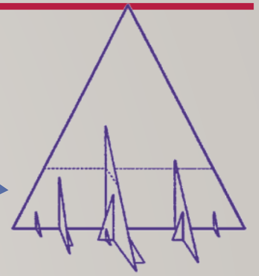


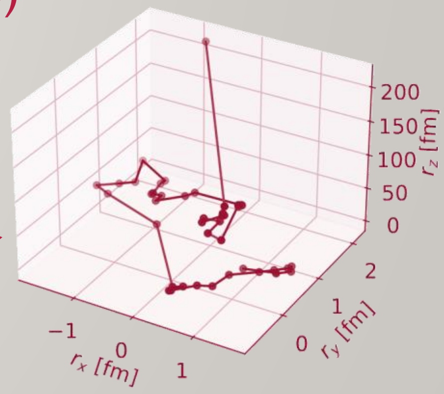


- Shape of the correlation functions with Lévy source:
  - $C_2(q) = 1 + \lambda \cdot e^{-|qR|^\alpha}$; $\alpha = 2$: Gaussian; $\alpha = 1$: exponential
    Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78
- Sidenote: Lévy-stable ≠ not Gaussian (but {Lévy-stable, $\alpha < 2$} ⊂ {not Gaussian})
- A possible reason for Lévy source: Lévy flight or Lévy walk

WHY DO LÉVY SHAPES APPEAR, WHY IS IT IMPORTANT?

- A more comprehensive list of possible reasons:
  - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36 (2005) 329-337)
    - See also talk by Yacine Mehtar-Tani at ExploreQGP workshop in Belgrade, 2023
    - Important in e^+e^- and other small systems
  - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828 (2006) no.1, 525-532)
    - Important in the 3-30 GeV region?
  - Directional or event averaging, non-sphericality (Cimerman et al., Phys.Part.Nucl. 51 (2020) 282)
    - Ruled out by event-by-event and 3D analyses
  - Lévy walk (BJP37(2007); PRB103(2021), Entropy24(2022); PLB847(2023); arXiv:2409.10373)
    - Only plausible explanation at high energies (see talk by D. Kincses on Wed)
- Importance of utilizing Lévy sources, leaving α as parameter:
  - Measuring α and R: quark-hadron transition, critical point, etc
  - Measuring λ: In-medium mass modification, coherent pion production





INTERACTIONS

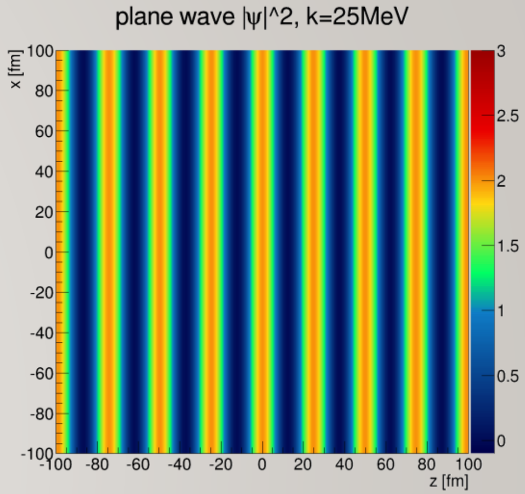
- Plane-wave result, based on $\left|\Psi^{(0)}_{2,\mathrm{q}}(r)\right|^2 = 1 + e^{iqr}$, for pair source $D(r)$

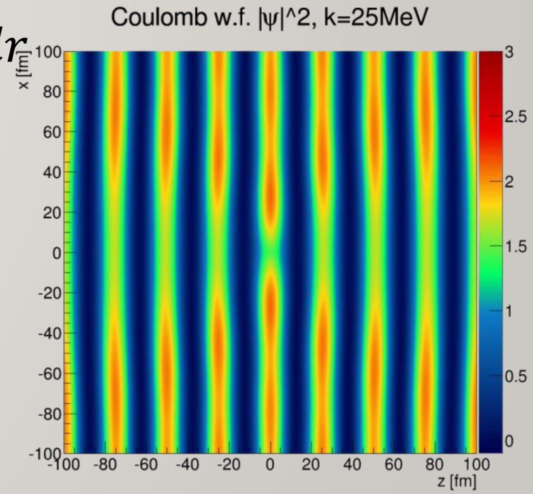
$$C_2(q,K) \cong \int D(r,K)\left|\Psi^{(0)}_{2,q}(r)\right|^2 dr = 1 + \int D(r,K)e^{iqr}\,dr$$

- If there are interactions, solve Schrödinger eq: $\Psi^{(0)}_{2,q}(r) \rightarrow \Psi^{(\mathrm{int})}_{2,q}(r_1, r_2)$
- For Coulomb, solution is known: $\left|\Psi^{(C)}_{2,\mathrm{q}}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1}\cdot$ (hypergeometric expression)
- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\mathrm{Bose-Einstein}}(q)K(q), \text{where } K(q) = \int D(r,K)\left|\Psi^{(C)}_{2,\mathrm{q}}(r)\right|^2 dr \Big/ \int D(r,K)\left|\Psi^{(0)}_{2,\mathrm{q}}(r)\right|^2 dr$$

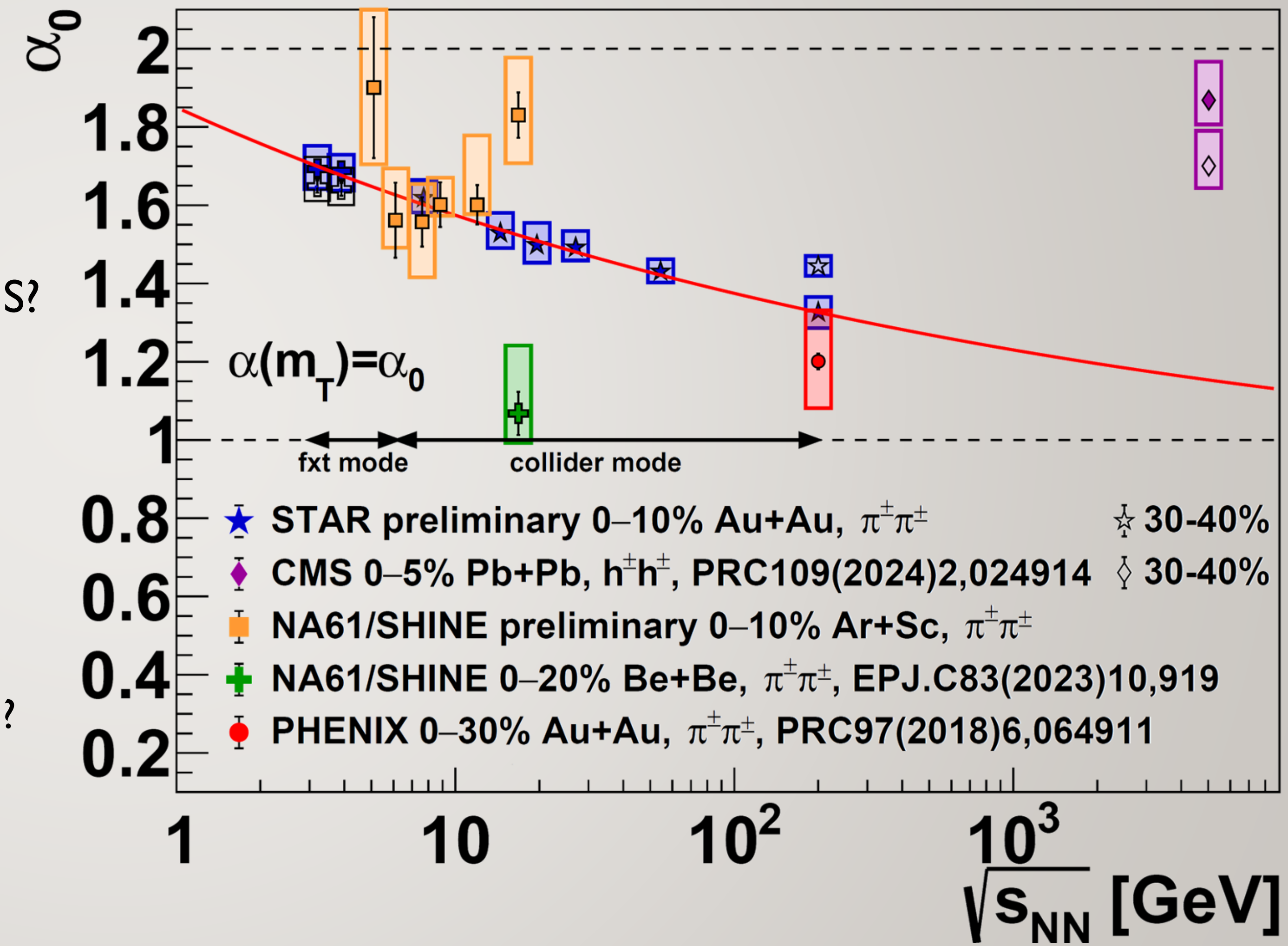
- Complication: need for integrating power-law tails
  - Precalculated in a tabular form, iterative fitting, e.g., PHENIX, PRC97(2018)064911
  - Interpolating functional form, see Csanád, Lökös, Nagy, Phys.Part.Nucl. 51(2020)238
  - Role of the strong interaction, see Kincses, Nagy, Csanád, PRC102(2020)064912
  - Recent method: EPJC83(2023)1015, code at github.com/csanadm/CoulCorrLevyIntegral
- Many new results, also for the strong interaction: see talk by M. Nagy on Tuesday





PION SOURCE SHAPE EVOLUTION, SPS→RHIC→LHC

- NA61: Be+Be at 17 GeV and Ar+Sc at 5-17 GeV
- STAR Au+Au at 3-200 GeV (also 3D now!)
- CMS: Pb+Pb at 5 TeV
- Opposite centrality trends in STAR and CMS?
- Opposite energy dependence trends?
    - Weak minimum at NA61 in Ar+Sc
    - Decrease with energy at STAR in Au+Au
    - Increase from STAR to CMS
- Origin of nonmonotonicity?
    - Difference in momentum acceptance & PID?
- Possible analysis in O+O vs Au+Au
- What do models say?



LÉVY PROCESSES IN NATURE AND IN SCATTERING

- Lévy walk and Lévy flight: known in ecology, climatology, etc
  - If stepsize distribution has no finite width: generalized central limit theorem, Lévy-stable limiting distributions
- In HIC: increasing mean free path, stepsize increase
  - Seen in expansion under Coulomb potential in solid-state physics
- Observed in UrQMD [arXiv:2409.10373], without Coulomb
  - See talk by D. Kincses on Wednesday

E. I. Kiselev, Phys. Rev. B 103, 235116 (2021)

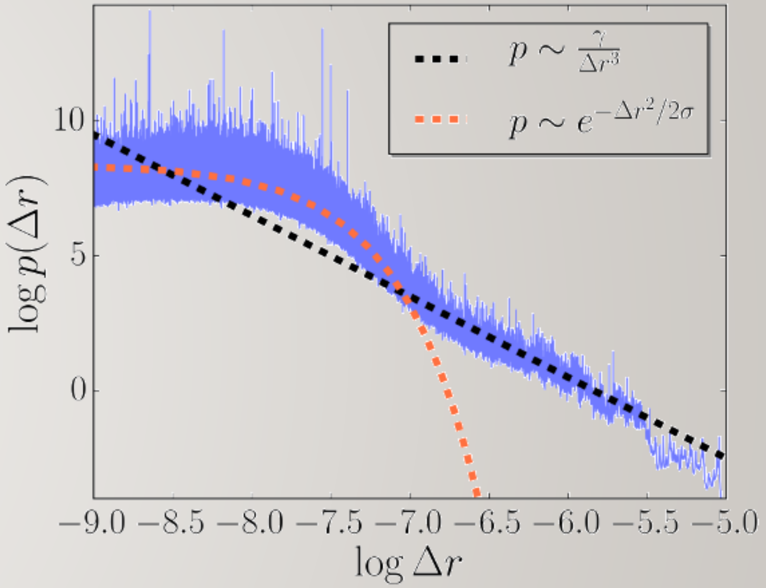
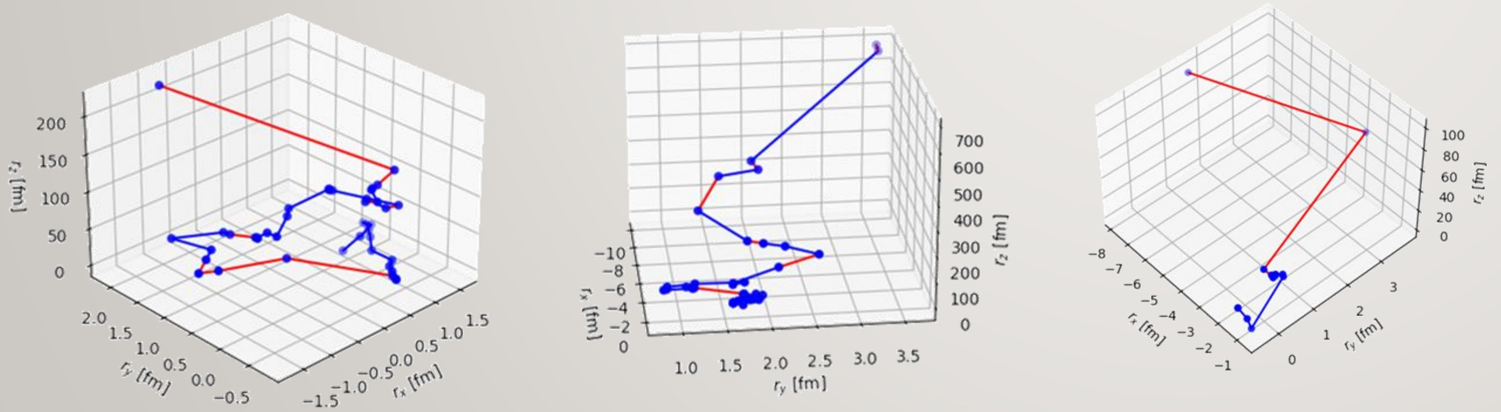
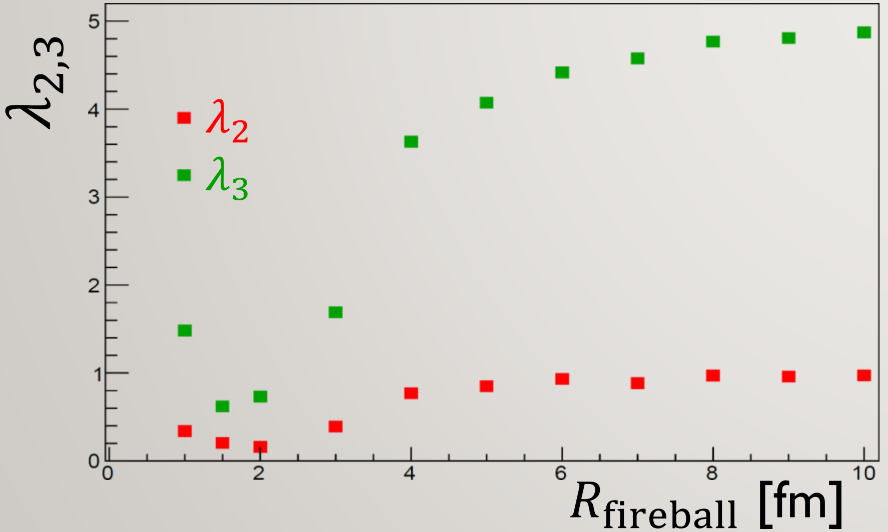


Figure 1. The Figure shows the step size distribution $p(\Delta r)$ of a random walk as performed by Coulomb interacting, diffusing particles in two dimensions. At large step sizes, the distribution clearly follows the $p \sim \Delta r^{-3}$ power-law which leads to the superdiffusive dynamics described by Eq. (1). The data was obtained by integrating the system of coupled Langevin equations of Eq. (56).



CHARGED CLOUD: ANOTHER INTERESTING EFFECT

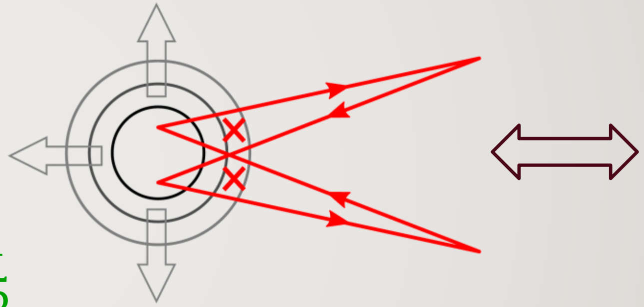
- Coulomb potential: infinite range, affecting evolution for a long time
- Solid-state physics (as mentioned on previous slide): may cause Lévy flight and power-law tails
- Another interesting effect: distortion of flight paths after kinetic freeze-out
  - Phase shift, similarly to an Aharonov-Bohm effect (arXiv:2007.07167 and arXiv:2410.15525)
- Phase shift decreases correlation strengths



$\lambda_3 = 5$

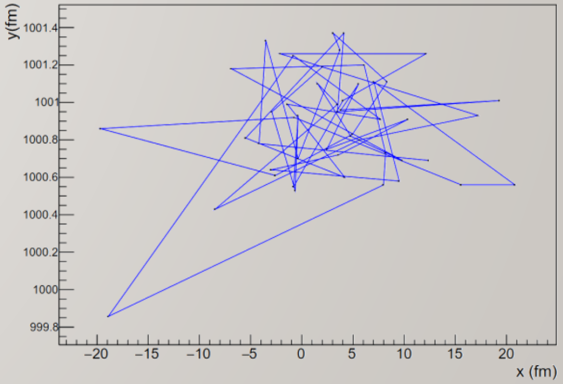
No A-B effect, pure core, fully chaotic source

$\lambda_2 = 1$



exaggerated illustration

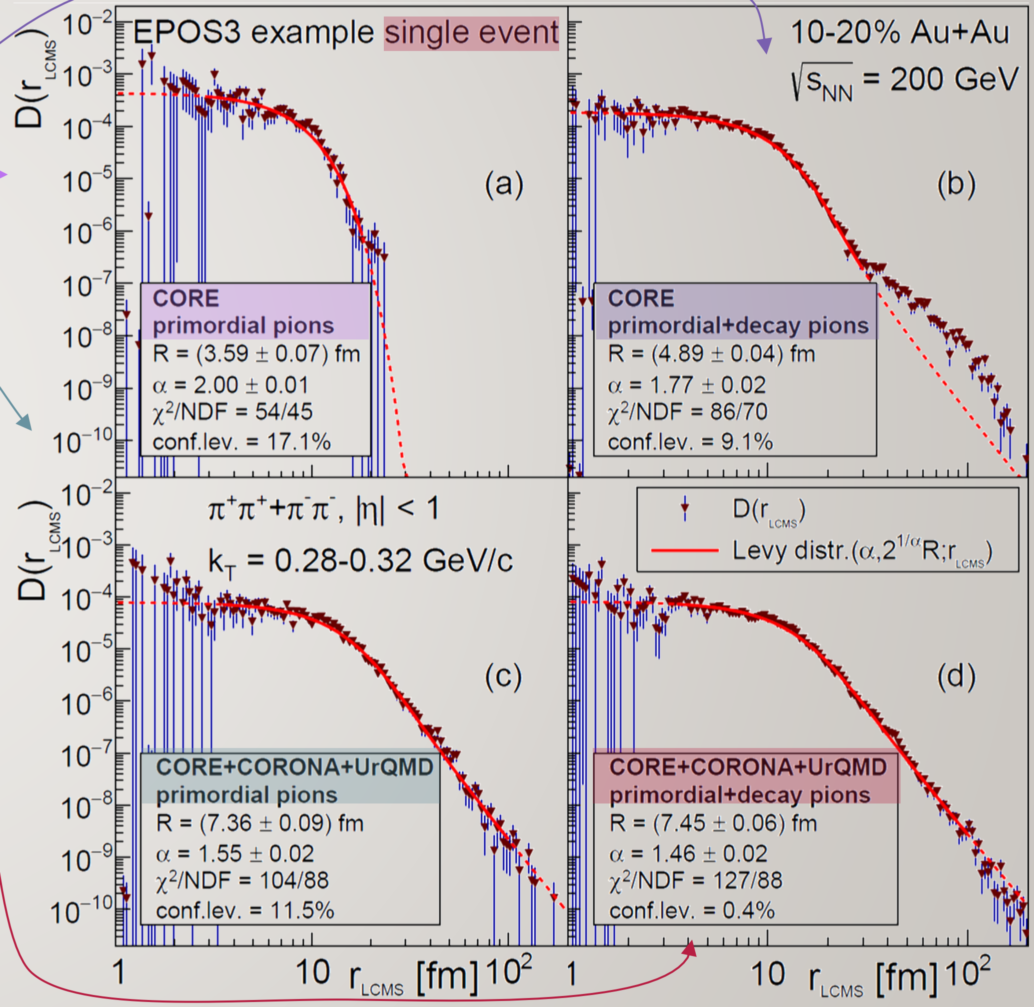
simulated transverse path



LÉVY SHAPES IN SINGLE EPOS EVENTS, 1D

- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
- Source observed in four stages:
  a) CORE, primordial pions: close to Gaussian
  b) CORE, with decay products: power-law structures
  c) CORE+CORONA+UrQMD, primordial pions: Lévy shape
  d) CORE+CORONA+UrQMD, with decay products: Lévy shape
  - Radii in the four stages (one example event)
    3.59 fm → 4.89 fm → 7.36 fm → 7.45 fm
  - Shape (α) change: 2.00 → 1.77 → 1.55 → 1.46
- Can one relate the observed HBT radii to the hydro phase homogeneity lengths?
- More investigations needed…
- Related talks by D. Kincses, E. Árpási, L. Kovács on Wed

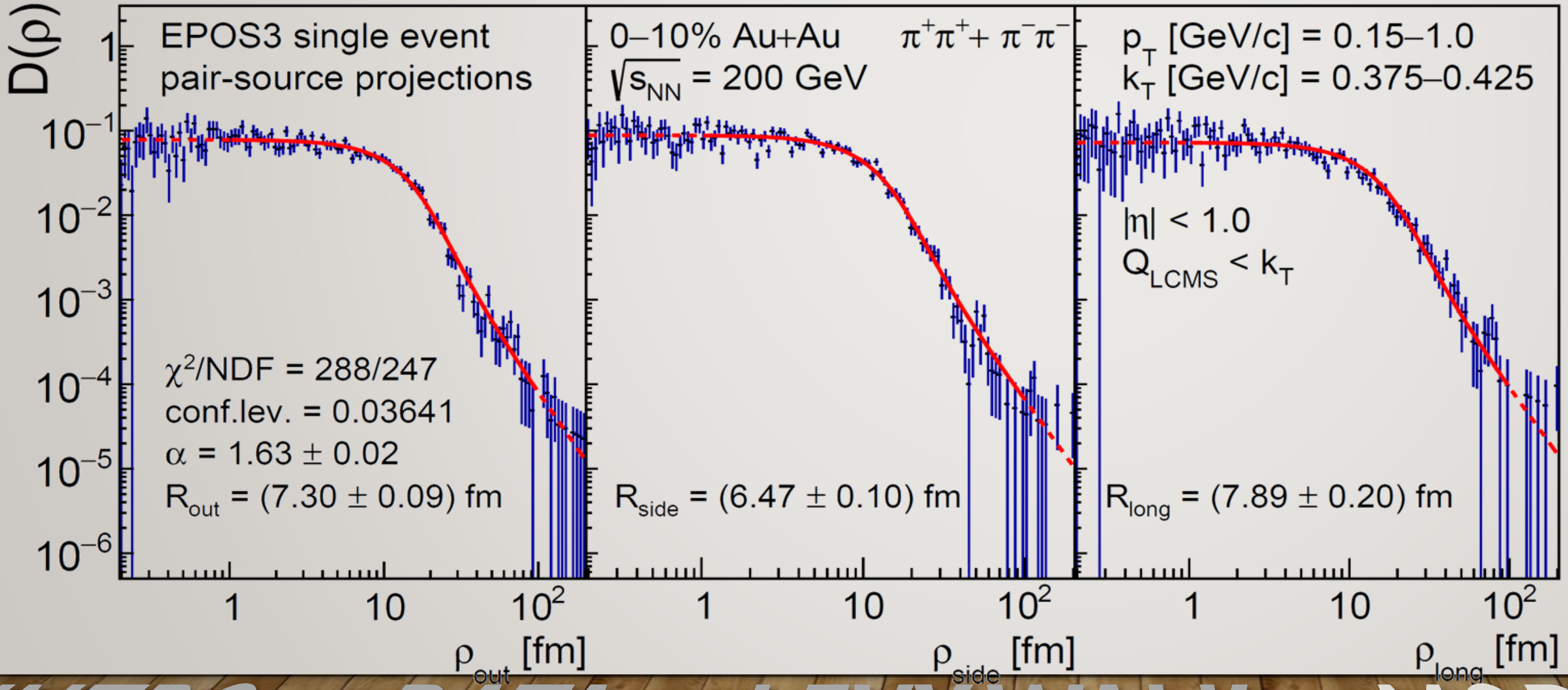
arXiv:2201.07962

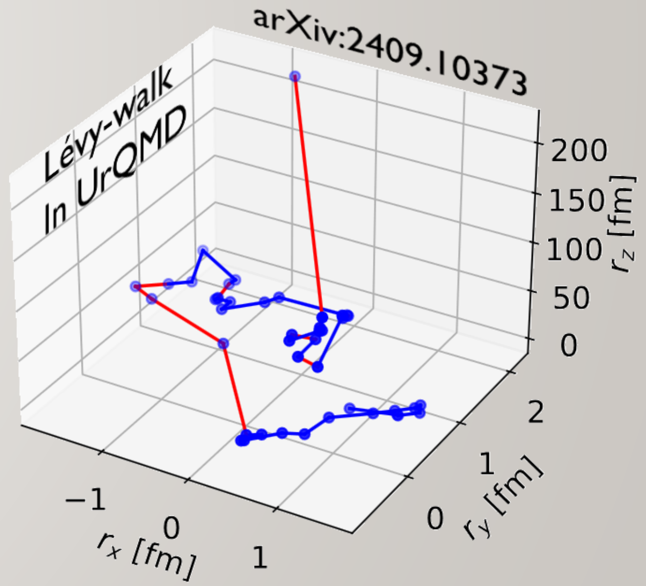


LÉVY SHAPES IN SINGLE 3D EPOS EVENTS, 3D

- What if the Lévy shapes appeared only because of directional averaging?
- Let's check 3D event shapes in EPOS! → Also Lévy, with similar α and radii (as those in 1D)
- Clear physical reason: Lévy walk (see talk by D. Kincses on Wednesday)

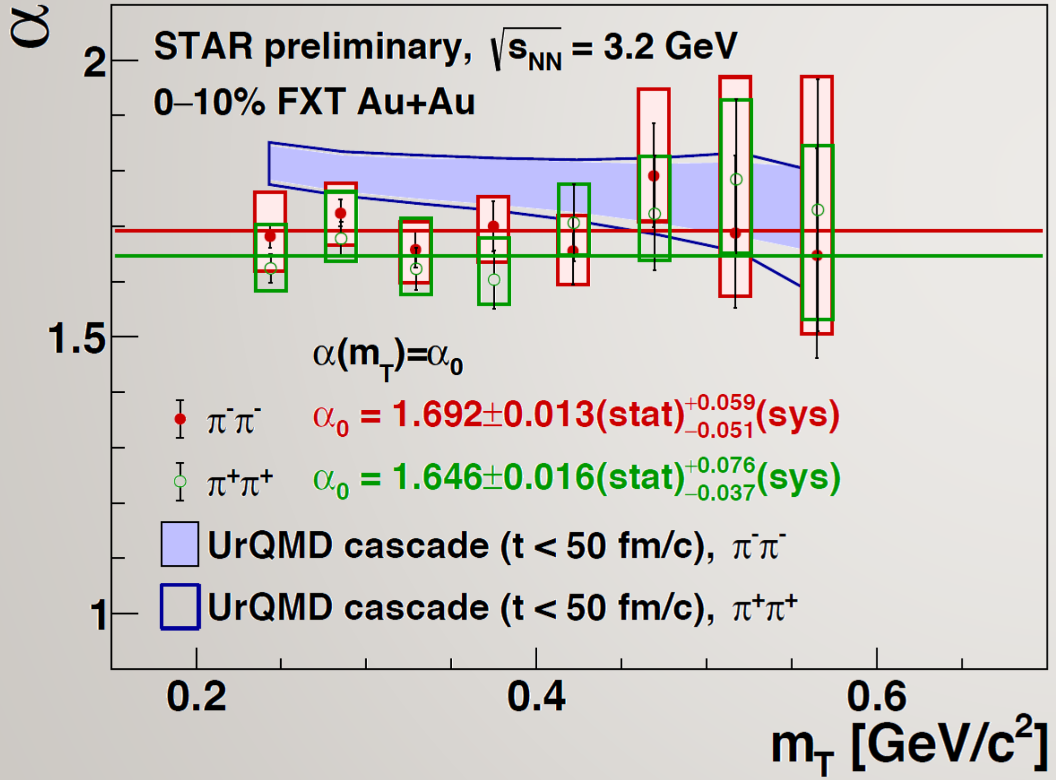
arXiv:2409.10373

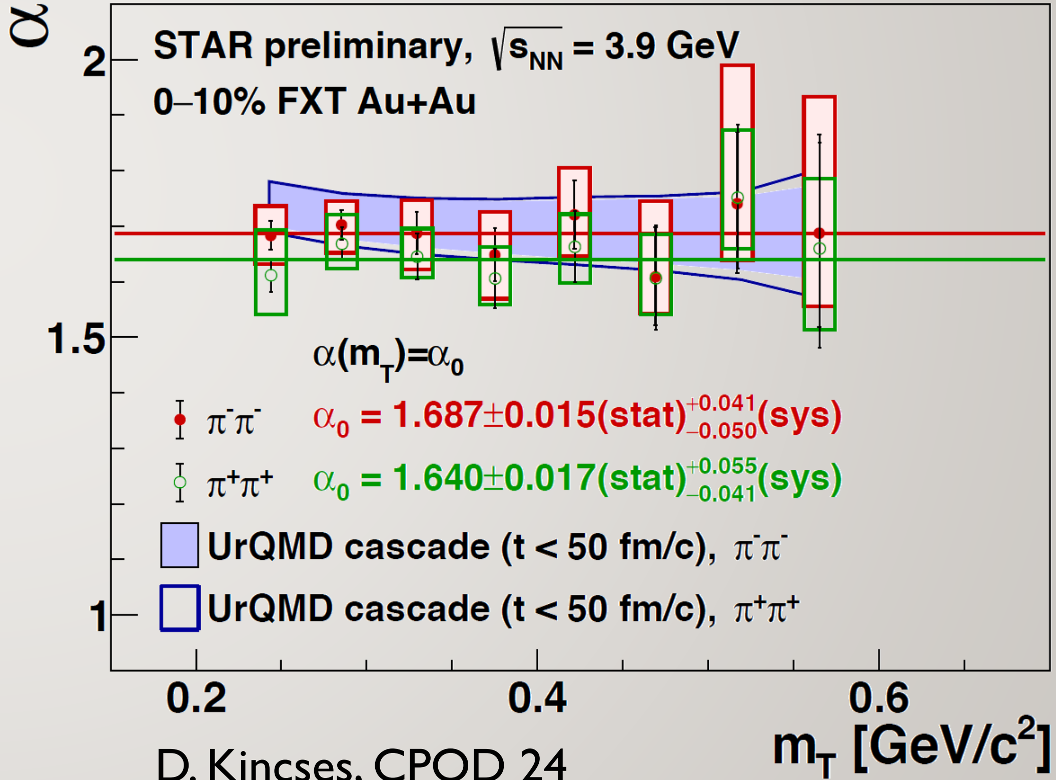




URQMD COMPARED TO STAR DATA AT 3.2 AND 3.9 GEV

- Non-Gaussian values ($\alpha < 2$); small systematic difference between $\pi^-\pi^-$ and $\pi^+\pi^+$ pairs
- 3.9 and 3.2 GeV compatible, no m_T dependence observed
- UrQMD within uncertainties – no other effect but rescattering and decays, good agreement

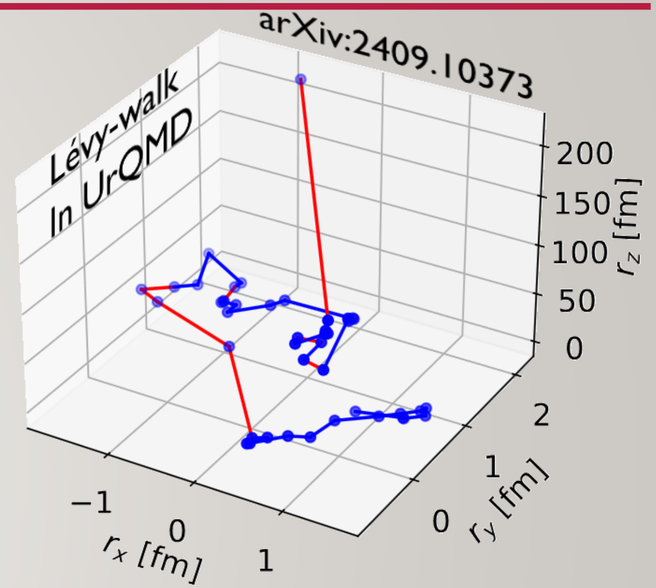


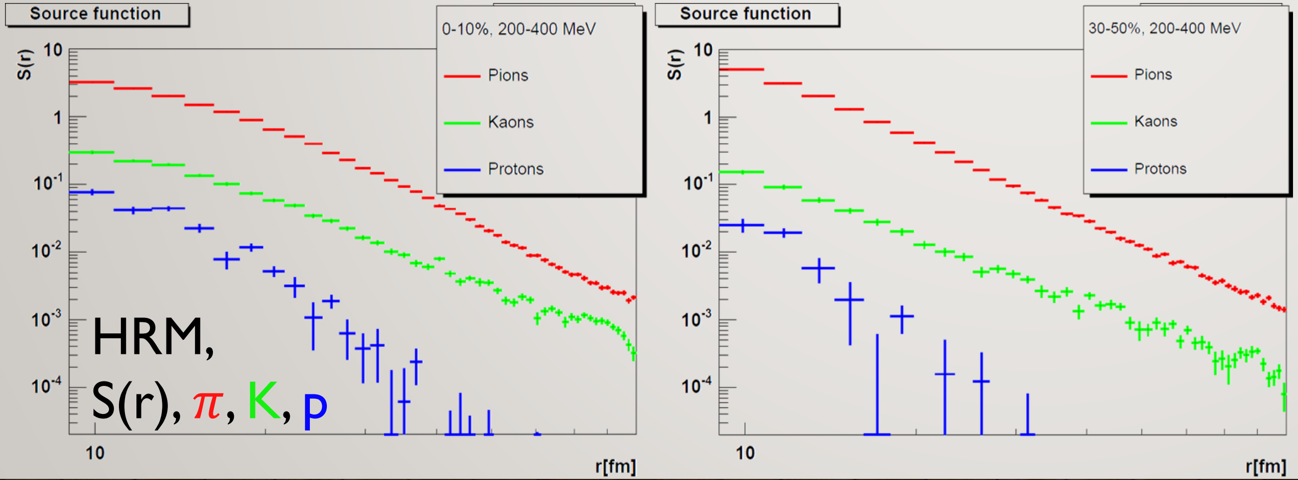


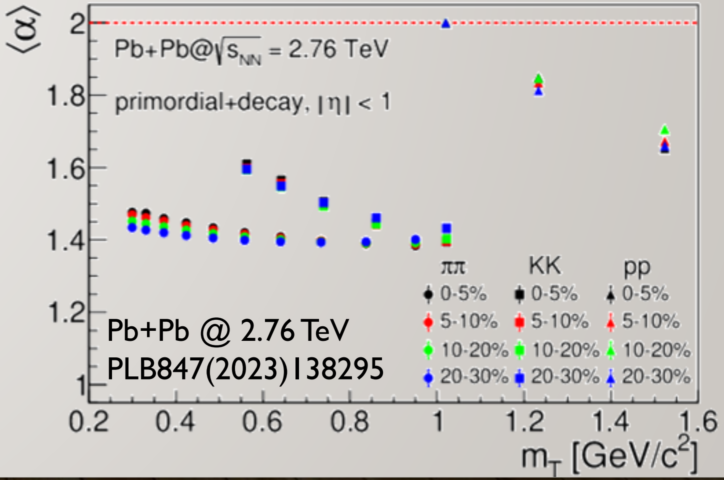
D. Kincses, CPOD 24

SO WHEN DO THE POWER-LAW TAILS FORM?

- Based on EPOS: apparently Gaussian in hydro phase
- Power-law tails due to Lévy-walk: scattering processes
  - 2-by-2, decay, coalescence, etc
- How to test? Particle type dependence!
  - Based on cross-sections: $\alpha(p) > \alpha(\pi) > \alpha(K)$
    Humanic, IJMPE15(2006)197, Csanád, Csörgő, Nagy, BJP37(2007)1002
  - Not confirmed by EPOS! Role of decays and inelastic collisions?



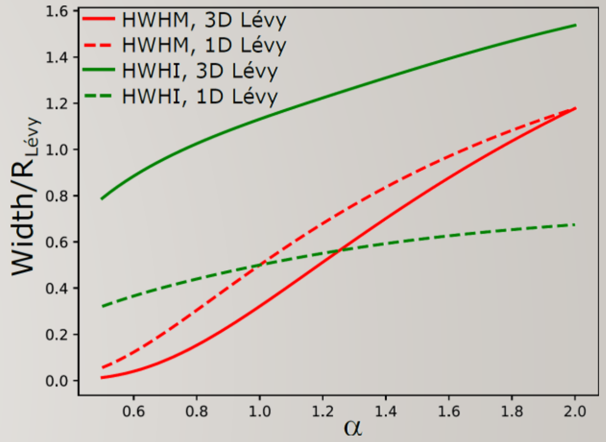


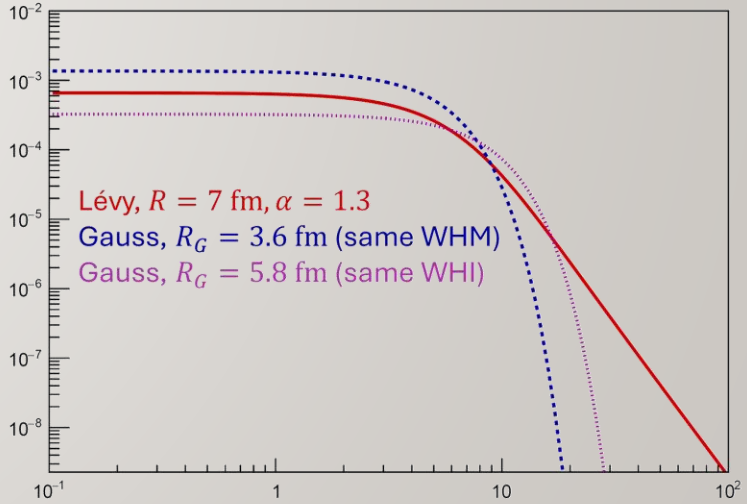


SOURCE SIZE MEASURE CHANGE WITH α

- No tail if $\alpha = 2$, power law and RMS $= \infty$ if $\alpha < 2$, in practice: depends on cutoff
- What do Gaussian HBT radii mean? Important also w.r.t. CEP search
- Alternative measures:
  - HWHM: (half) width at half maximum
  - HWHI: (half) width at half integral
  - Width (normalized by R) nontrivially depends on α
- Relations for 3D Gauss: HWHM $\approx 1.17 \cdot R_G$, HWHI $\approx 1.54 \cdot R_G$
- Relations for Lévy $\alpha = 1.3$: HWHM $\approx 0.61 \cdot R_L$, HWHI $\approx 1.27 \cdot R_L$
- **Thus (e.g.) $\alpha = 1.3$ and $R_L = 7$ fm "means":**
  - Same HWHM Gaussian: $R_G \approx 3.6$ fm
  - Same HWHI Gaussian: $R_G \approx 5.8$ fm

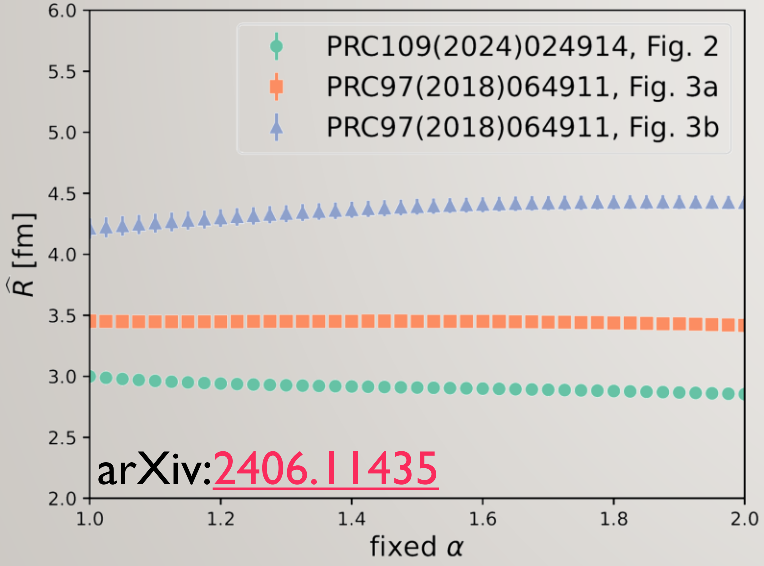
arXiv:2401.01249

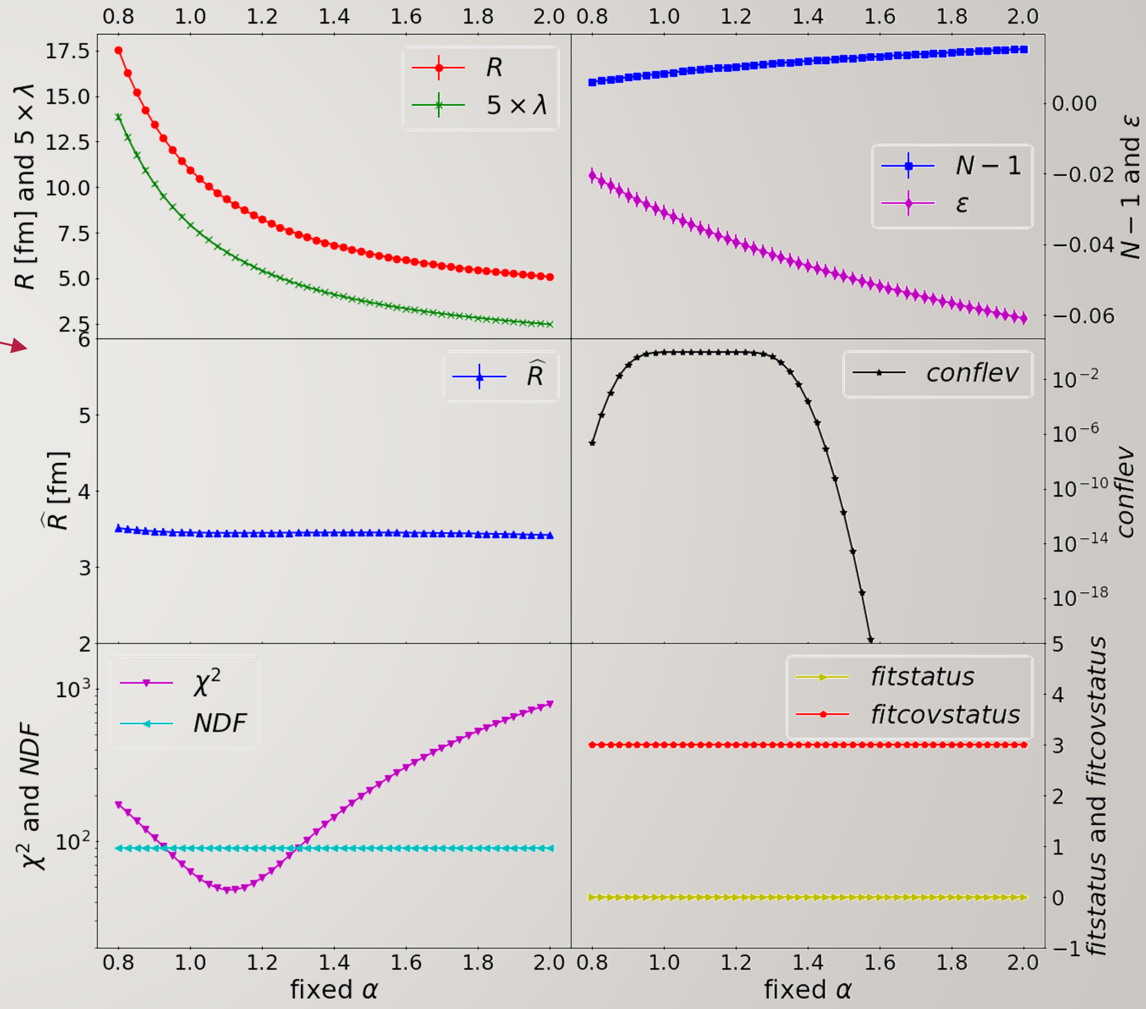




RESCALING HBT RADII FROM GAUSS TO LÉVY

- Source shape and size entangled in Gaussian radii
- Fits possible with many α values
  - Some statistically acceptable, some not
  - Fits to PHENIX HBT paper PRC 2018, Fig 3a
- $\hat{R} = R/[\lambda(1+\alpha)]$ scaling observed generally

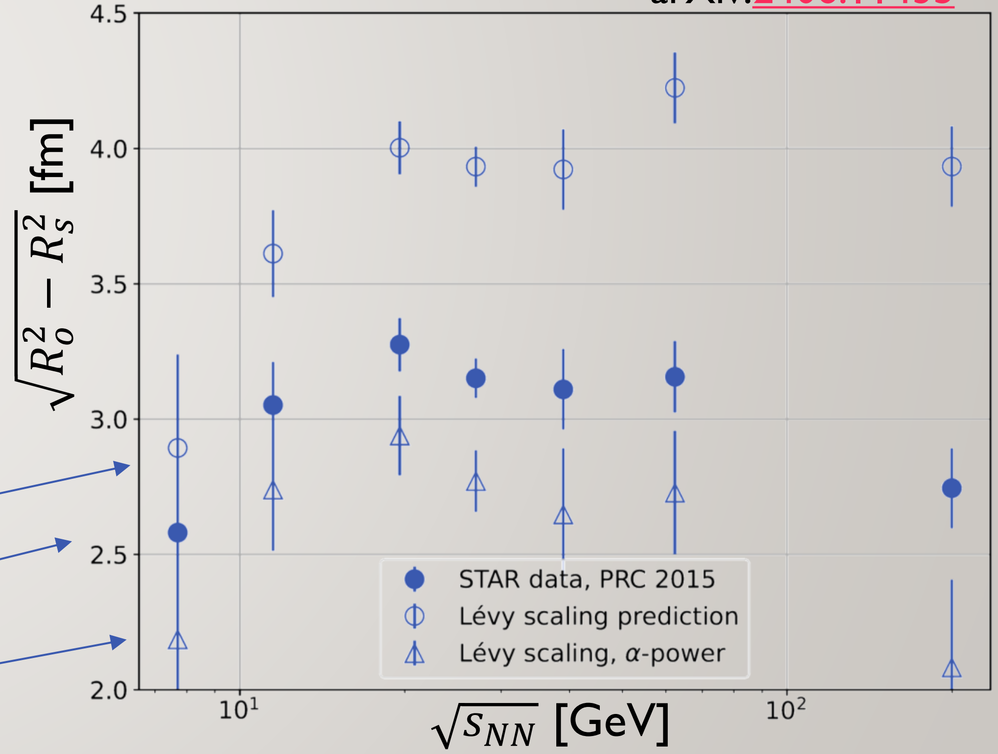




ENERGY DEPENDENCE OF LÉVY SOURCE SIZE?

- $\hat{R} = \frac{R}{\lambda(1+\alpha)}$ doesn't depend on α, can estimate $R_{\text{free }\alpha} = R_{\text{Gauss}} \frac{\lambda_{\text{free }\alpha}(1+\alpha)}{\lambda_{\text{Gauss}}(1+2)}$
  - Assuming trends of α and λ as $A \cdot \sqrt{s_{NN}}^{\,B}$, with $A_\alpha = 1.85, B_\alpha = -0.06, A_\lambda = 0.6, B_\lambda = 0.06$
- Different trends of guesstimated $R_{\text{Lévy}}$ and R_{Gauss}
- Caused by shape change with $\sqrt{s_{NN}}$
- Connection of $\sqrt{R_o^2 - R_s^2}$ to emission duration: based on Gaussian sources,
- Maybe $(R_o^\alpha - R_s^\alpha)^{1/\alpha}$ for Lévy source, Csörgő, Hegyi, Zajc, EPJC36(2004)67
- Importance of measuring $R_{o,s,l}$ with free α

arXiv:2406.11435



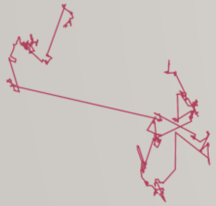
$\hat{R}$ scaling guesstimate for Lévy radii

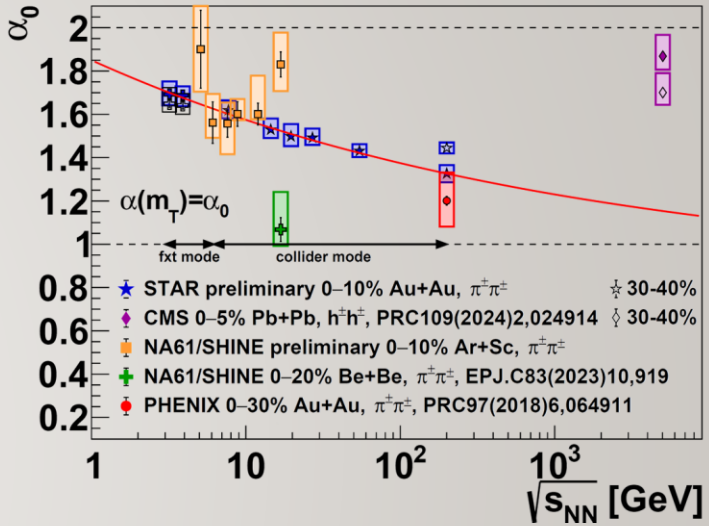
original Gaussian radii

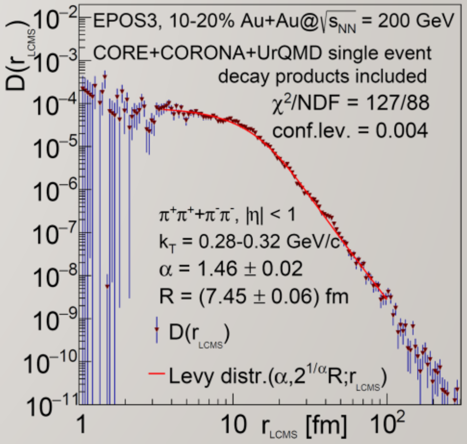
α-powered version

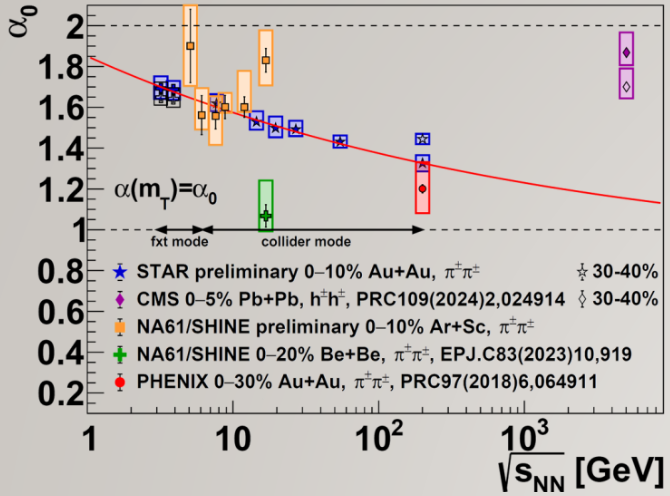
CONCLUSIONS AND OUTLOOK

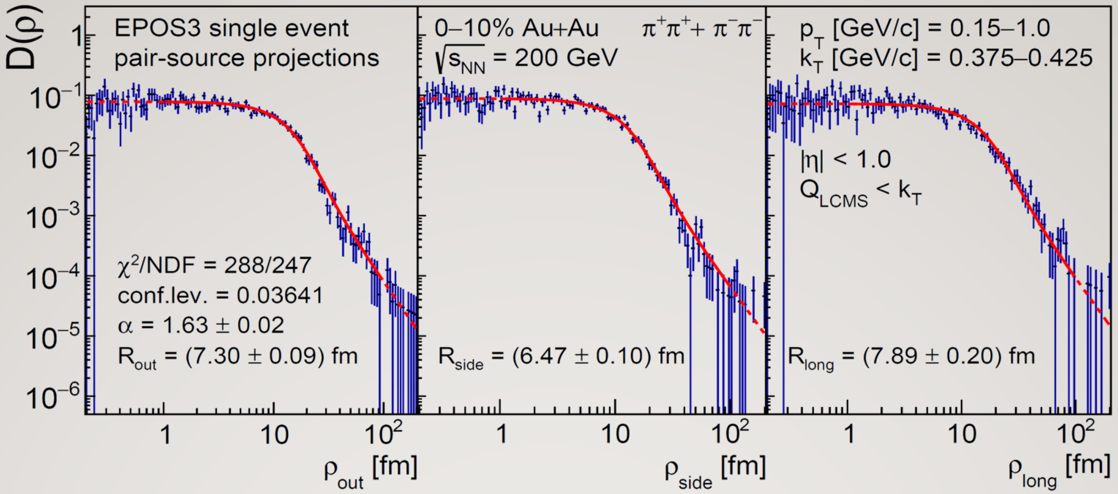
- Lévy sources from SPS to RHIC and LHC, in 3D as well
  - **Lévy α**: between 1 and 2, increases with $\sqrt{s_{NN}}$?
    - Interesting centrality and collision energy dependence
  - **Lévy R**: hydro scaling, relation to Gaussian through HWHM/HWHI
  - **Lévy λ**: signs of η' in-medium mass modification (see talk by S. Lökös)
- Possible reasons:
  - Jet fragmentation → not dominant in AA collisions
  - **Critical phenomena** → maybe at lowest RHIC energies and SPS?
  - Directional averaging → good fits and same Lévy exponent in 1D and 3D
  - Event averaging → event-by-event simulations show Lévy
  - **Resonance decays** → part of the reason, not enough alone
  - **Hadronic rescattering, Lévy walk** → explains results
- Questions to be answered:
  - When measuring α, what effects need to be considered?
  - Can there be anomalous hydrodynamics in the quark stage?
  - What is the role of finite size and finite time in critical power-laws?

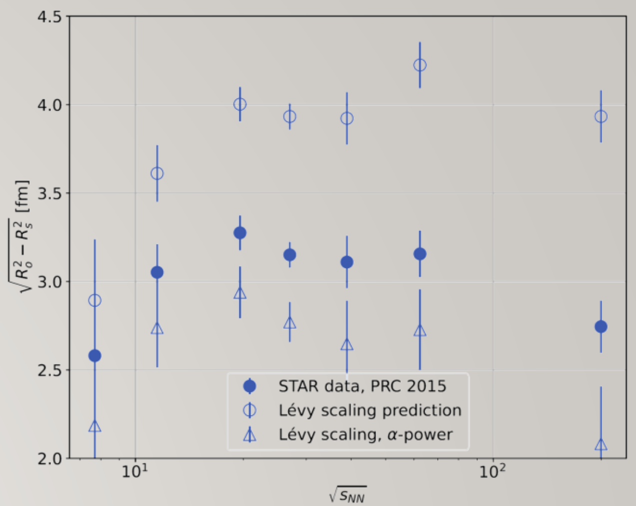












THANK YOU FOR YOUR ATTENTION

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http://zimanyischool.kfki.hu/24/