Searches for gluon saturation at the future Electron-Ion Collider

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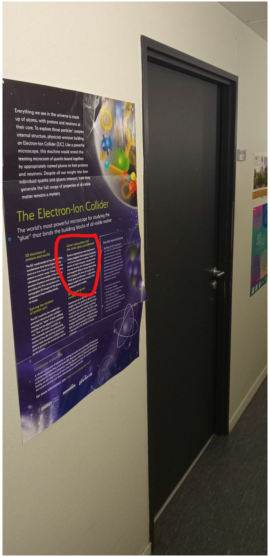
Motivation

From Subatech's administrative corridor.



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QCD in a nutshell

- A QFT for the strong interaction, with **quarks** and **gluons** as elementary fields.
- Quarks are fermions and gluons are the mediating gauge bosons.
- These "partons" carry an internal degree of freedom, called the color.
- QCD is asymptotically free: $\alpha_s(Q) \to 0$ as $Q \to \infty$.
- Emergent scale below which QCD becomes strongly coupled, $\alpha_s(Q) \sim 1 \Leftrightarrow Q \sim \Lambda_{\rm QCD} \sim 200 \div 300$ MeV.

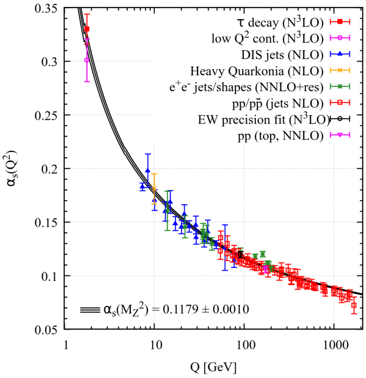
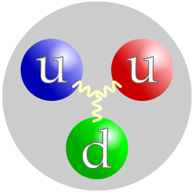


Fig. from PDG 2023

The proton: a very complicated physical system

- The proton is made of 3 valence quarks: two "up" and one "down".
- These quarks are binded together by gluons with momenta $\mathcal{O}(\Lambda_{\rm QCD})$.
- Binding is strong, so weak coupling techniques do not work. Unless...



Snapshot of a proton's movie available here.

Deep inelastic scattering: a proton's microscope

- Unless we can observe proton substructure over very short distances thanks to asymptotic freedom!
- The probe: a **virtual photon**, which couples to the internal quarks.
- This probe is characterized by two scales: Q^2 and $x_{\rm Bj}$:

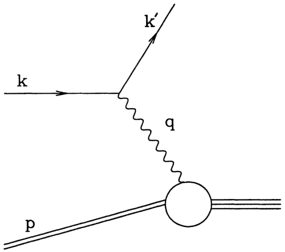
$$Q^2 = -q^2 \quad , \; x_{\rm Bj} = \frac{Q^2}{2P \cdot q} \sim \frac{Q^2}{s}$$

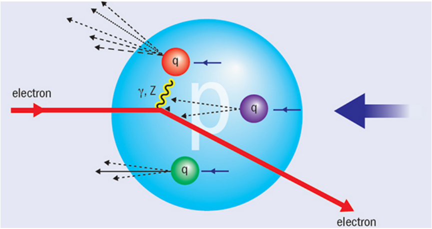
- $Q^2 \gg \Lambda^2_{\rm QCD}$ determine the transverse resolution of the imager.

$$\lambda_\perp^2 \sim \frac{1}{Q^2}$$

Order of magnitude: $\lambda_\perp \sim 10^{-17}$m $\ll 1/m_p \sim 10^{-15}$m for $Q = 100$ GeV

- $x_{\rm Bj} P^+ / Q^2$ interaction time of the microscope.



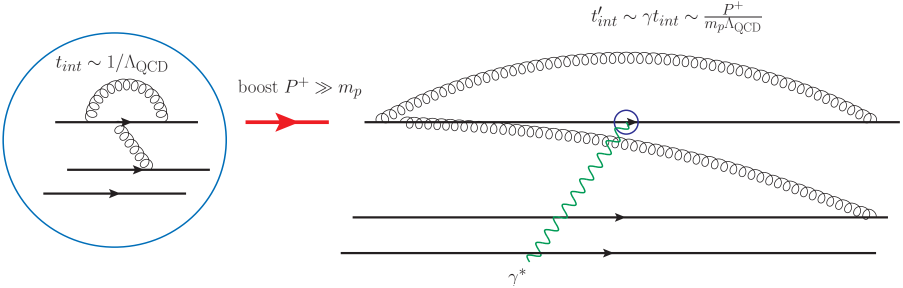


Proton in a boosted frame

- Proton in a boosted frame: time dilatation of hadronic fluctuations.
- They appear "frozen" during the measurement time of our microscope.

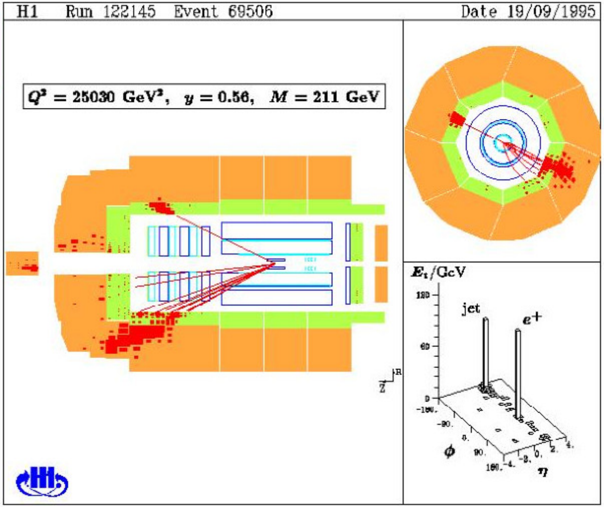
$$\frac{x_{\rm Bj} P^+}{Q^2} \ll \frac{P^+}{m_p} \times \frac{1}{\Lambda_{\rm QCD}}$$

- Struck parton behaves as free particles, $x_{\rm Bj}$ measures its longitudinal momentum fraction.



DIS in real life

- Ex: H1 experiment at HERA (Germany, 1992-2007).
- A parton is freed by the collision and creates a hadronic "jet" in the detector.
- Very successful proton imaging!



The small-x or Regge limit of DIS

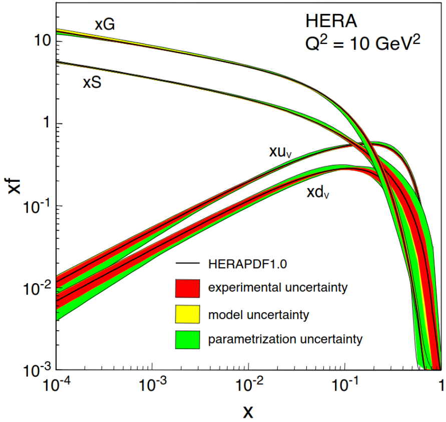
- In this talk: I will consider a particular regime of the microscope

$$Q^2 \text{ fixed}, \ x_{\text{Bj}} \to 0$$

- Since $x_{\text{Bj}} \sim Q^2/s$, this is also the high energy $s \to \infty$ limit.

- In this regime, the proton/nucleus is essentially made of gluons.

- $xg(x, Q^2)$ is the integrated gluon distribution.
  Number of gluons with transverse momentum $k_\perp \sim 1/\lambda_\perp < Q^2$ and a given x.



Multiple gluon emissions

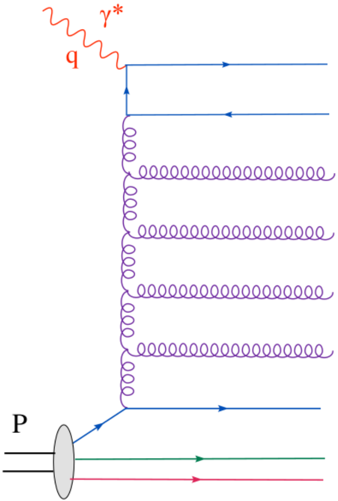
- Probability of radiating gluons: enhanced at small x and $k_\perp$:

$$dP_{\mathcal{B}} = \frac{\alpha_s C_R}{\pi^2} \frac{dx}{x} \frac{d^2 k_\perp}{k_\perp^2}$$

- At small x_{Bj}, large phase space $x_{\mathrm{Bj}} \ll x \ll 1$ for radiating gluons $\Rightarrow$ growth of the gluon distribution:

$$x \frac{dg(x, k_\perp^2)}{dk_\perp^2} \sim \frac{\alpha_s N_c}{\pi} \frac{1}{k_\perp^2} e^{c\alpha_s \ln(1/x)}$$

- Physically, gluons with shorter and shorter lifetime, down to the interaction time of the microscope.

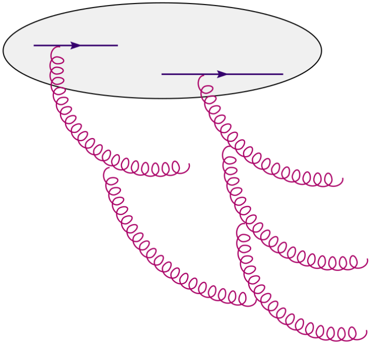


Non-linear gluon recombinations

- This growth cannot go forever: it would violate the Froissard bound on cross-section behaviour as $\sqrt{s} \to \infty$.

$$\sigma_{\rm tot}(s) \leq \sigma_0 \ln^2(s) \;\; \text{as } s \to \infty$$

Froissard, Phys. Rev. 123, 1961



- Gluons can self-interact in QCD $\Rightarrow$ non-linear recombination.
- An old model for this recombination process:

$$\frac{dxg(x,k_\perp^2)}{d\ln(1/x)} = c\alpha_s xg(x,k_\perp^2) - \frac{\alpha_s^2}{k_\perp^2 R^2}\left[xg(x,k_\perp^2)\right]^2$$

Gribov, Levin, Ryskin, Phys. Rept 100, 1983

Important message

Saturation is an emergent mechanism which makes QCD a self-consistent quantum field theory!

The saturation scale and DIS phase space

- From the GLR equation, non-linear effects matter when

$$\frac{\alpha_s}{k_\perp^2 R^2} xg(x, k_\perp^2) = \mathcal{O}(1) \Rightarrow k_\perp^2 \leq Q_s(x)$$

with

$$Q_s^2(x) \sim \alpha_s \frac{xg(x, Q_s^2(x))}{R^2}$$

- x and A dependence of the saturation scale:

$$Q_s^2(x) \sim Q_0^2 A^{1/3} \left(\frac{x_0}{x}\right)^{\lambda} , \lambda \sim 0.2$$

- At small x, $Q_s(x) \gg \Lambda_{\rm QCD} \Rightarrow$ **weak coupling** techniques in strong field regime.

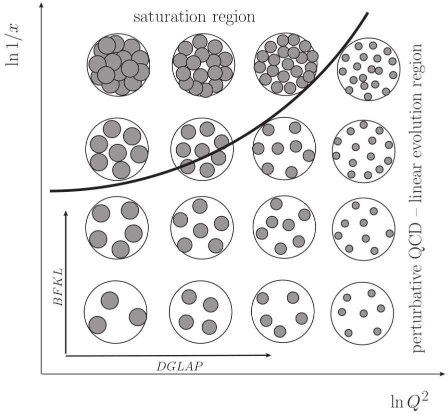
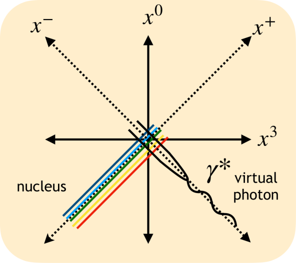
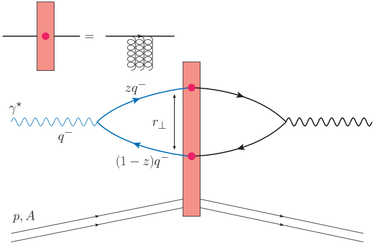


Fig. from Kovchegov & Levin's book

Dipole picture of the DIS microscope

- The photon splits into a quark-antiquark pair,
- which subsequently probes the dense gluon matter of the nucleus.
- Motivated by time-scale argument: the coherence length of the virtual photon is $\sim 1/(x_{\rm Bj} P^+) \gg 1/P^+$ longitudinal size of the proton/nucleus.
- Saturation encoded in multiple scattering of the $q\bar{q}$ dipole off small-x gluons.

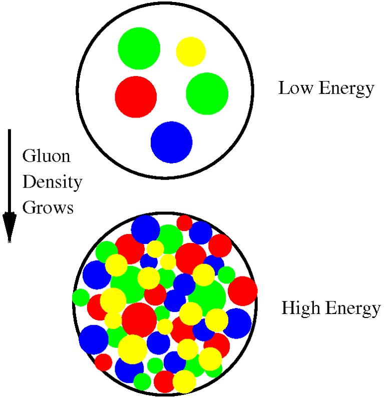




A Color Glass Condensate

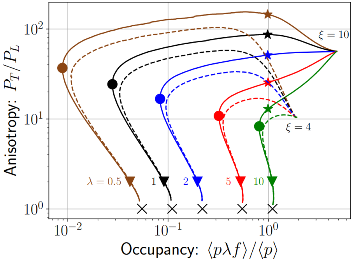
- High occupancy number of small-x gluons $\Rightarrow$ gluons can be described by a **classical field**.
- Classifical field generated by the large-x color charges (valence quarks) which are frozen on the time scales of the small x gluons.
- Similar to the physics of a glass ! $\Rightarrow$ **Color Glass Condensate**


a CGC?



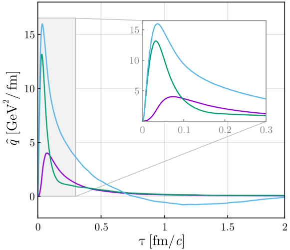
CGC in early stages of heavy-ion collisions

- CGC is the first stage of the bottom-up thermalization scenario in heavy-ion collisions.
Baier, Mueller, Schiff, Son, PLB 502, 2001



Boguslavski, Kurkela, Lappi, Lindenbauer, Peuron, arXiv:2303.12595

- Recent CGC-based calculations predicts a large diffusion coefficient $\hat{q}$ before hydrodynamic.
Avramescu, Baran, Greco, Ipp, Müller, Ruggieri, PRD 107, 2023



DIS structure functions

- Total DIS cross-section at small-x:

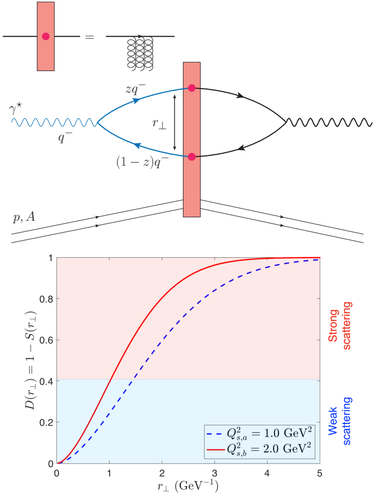
$$\sigma^{\gamma^\star A}(x, Q^2) \propto 2 \int d^2 r_\perp \int_0^1 dz |\psi^{\gamma^\star \to q\bar{q}}(r_\perp, Q^2, z)|^2 (1 - S_x(r_\perp))$$

- $S_x(r_\perp)$ encodes the scattering of a $q\bar{q}$ dipole off the dense gluon field.

- Related to the gluon distribution

$$1 - S_x(r_\perp) \sim xg(x, 1/r_\perp^2) r_\perp^2$$

- $Q_s^2 \propto A^{1/3}$ transition scale between weak and strong scattering regime.



Morreale, Salazar review 2108.08254

Geometric scaling at HERA

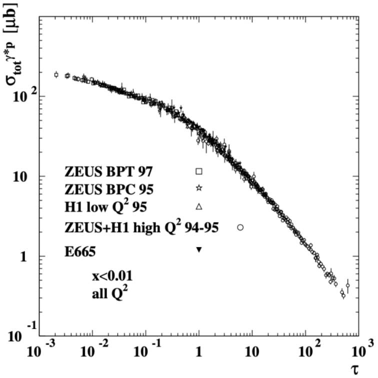
- Predicts geometric scaling of the DIS cross-section:

$$\sigma^{\gamma^* p}(x_{\rm Bj}, Q^2) = f\left(\tau = \frac{Q^2}{Q_s^2(x_{\rm Bj})}\right)$$

Stasto, Golec-Biernat, Kwiecinski, PRL 86, 2001

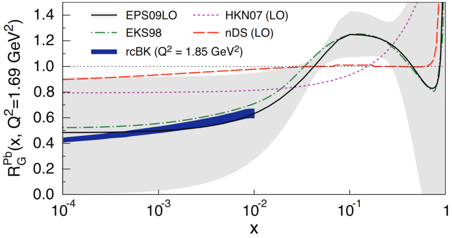
- Universal property of the saturated gluonic matter at small-x.

- But: not the end of the story, because linear DGLAP evolution also leads to geometric scaling...
Kwiecinski, Stasto, PRD 66 2002, Caola, Forte, PRL 101 2008



What does "measuring saturation" mean?

- Measure the universal properties of this Color Glass Condensate.
- In particular, the x and A dependence of the saturation scale.
- Calculation without saturation should be able to reproduce DIS data, but with many fitted nPDFs.

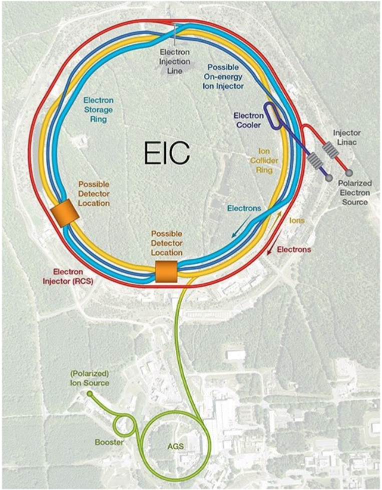


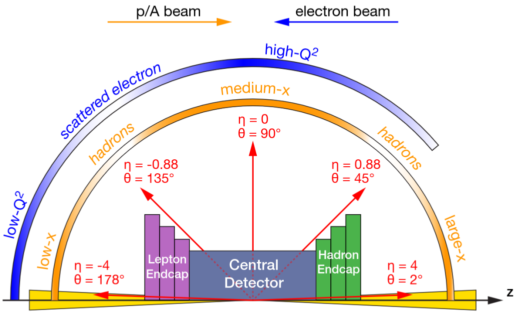
From "EIC, the next QCD frontier" report

- CGC predicts simple scaling behaviour of gluon distributions with A from QCD first principles.

The future Electron-Ion Collider 1/2

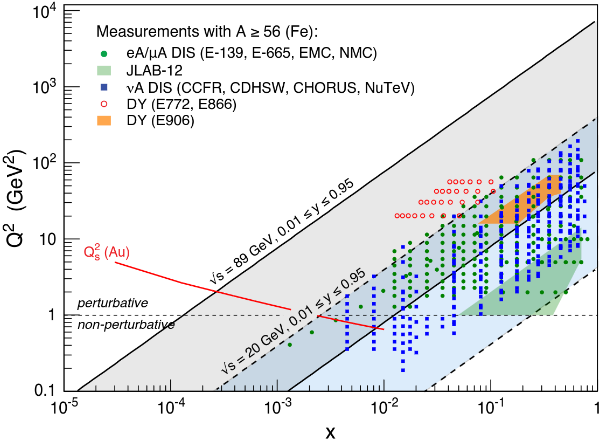
- Two slides about the EIC made by a theorist… (sorry)





- 2 interactions regions ⇒ 2 detectors.
- Current proposals: ECCE, ATHENA, CORE, …

The future Electron-Ion Collider 2/2



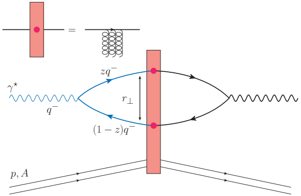
From EIC yellow report

- New opportunities with DIS on heavy nuclei!
- Window for saturation searches at small x.
- Precision QCD calculations in the CGC are needed to discriminate models.

Why going beyond inclusive observables?

- Total DIS cross-section at small-x:

$$\sigma^{\gamma^\star A}(x,Q^2) \propto 2\int d^2r_\perp \int_0^1 dz |\psi^{\gamma^\star\to q\bar{q}}(r_\perp,Q^2,z)|^2 (1-S_x(r_\perp))$$



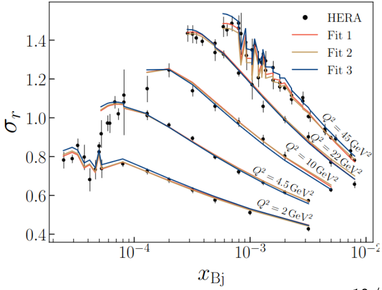
- Known at NLO and successful to describe HERA data.

NLO calculation and fit from Beuf, Hänninen, Lappi, Mäntysaari 2007.01645. See also Ducloué, Iancu, Soyez, Triantafyllopoulos, 1912.09196

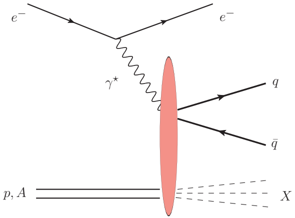
- Total DIS cross-section sensitive to saturation when $Q^2 \sim 1/r_\perp^2 \lesssim Q_s^2 \sim 1$ GeV^2.

- **But** NP contamination for semi-hard Q^2.

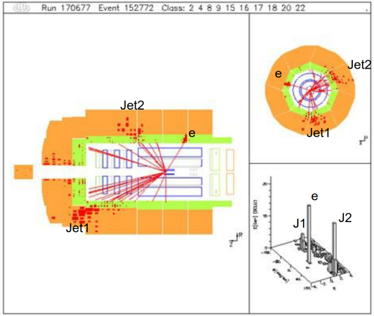
Mäntysaari, Zurita, 1804.05311, Mäntysaari, Schenke, 1806.06783



A "golden channel": inclusive back-to-back dijet production in DIS



- Production of two collimated sprays of hadrons, nearly back-to-back in the transverse plane.
- Measure azimuthal correlation between the two jets.

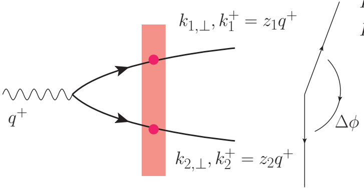


Interest of semi-inclusive processes: back-to-back dijets

- More than one transverse scale! For back-to-back dijet: $P_\perp \gg K_\perp$
- $P_\perp$ hard, $K_\perp \sim Q_s$ semi-hard.
- Imprint of saturation on final state correlations.

  Dominguez, Marquet, Xiao, Yuan, 1101.0715, Dumitru, Lappi, Skokov, 1508.04438, Dumitru, Skokov, Ullrich, 1809.02615
- **But:** soft gluon radiation effects spoil this nice picture.

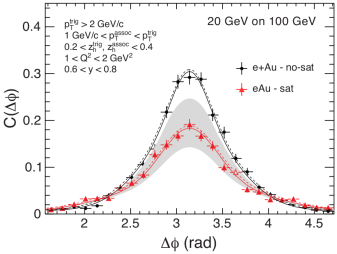
  Mueller, Xiao, Yuan, 1308.2993



$$P_\perp = z_2 k_{1,\perp} - z_1 k_{2,\perp}$$
$$K_\perp = k_{1,\perp} + k_{2,\perp}$$

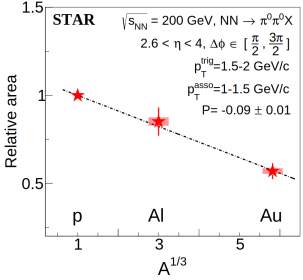
Ideal probe

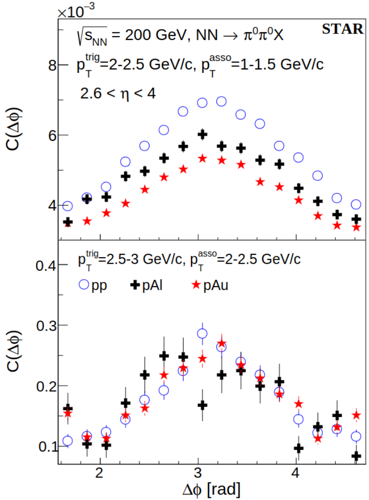
Semi-inclusive observables with a hard scale, sensitive to Q_s through final state correlations.



Zheng, Aschenauer, Lee, Xiao, 1403.2413

Similar measurement at RHIC (STAR collab.) in *pA* collisions



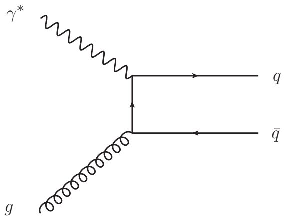


STAR, PRL 129, 2022

Azimuthal correlation in forward di-π^0 production

- Clear system size dependence of the suppression of back-to-back azimuthal correlations.
- Hint of $A^{1/3}$ scaling of Q_s...

Leading order: connection with TMD physics



- LO in photon-gluon fusion channel: TMD factorization

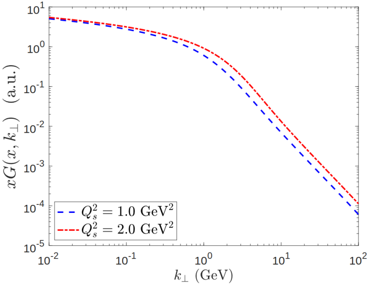
  Dominguez, Marquet, Xiao, Yuan, 1101.0715

$$\left.\frac{d\sigma^{\gamma^\star\to q\bar{q}+X}}{d^2P_\perp d^2K_\perp}\right|_{\rm LO} \propto \mathcal{H}(P_\perp)xG(x,K_\perp)+\mathcal{O}\left(\frac{K_\perp}{P_\perp}\right)+\mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

  See also del Castillo, Echevarria, Makris, Scimemi, 2008.07531

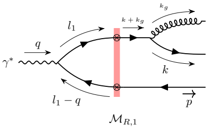
- $xG(x,K_\perp)$: Weizsäcker- William gluon TMD counts the number of gluons in the proton/nucleus wave-function.

- Behave like $Q_s^2/k_\perp^2$ at large $k_\perp$ and $\ln(Q_s^2/k_\perp^2)$ for $k_\perp < Q_s$.



Effects of soft gluon radiations

- Beyond leading order in pQCD, soft gluons are enhanced by double logarithms $\alpha_s \ln^2(P_\perp^2/K_\perp^2)$.

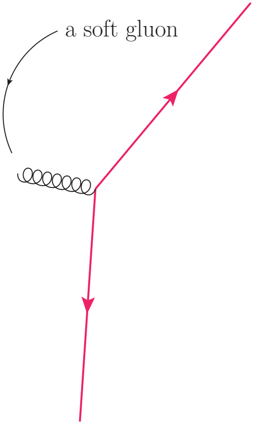


- Sudakov resummation required ⇒ suppression of the x-section

$$\mathcal{S}(P_\perp, K_\perp) = \exp\left(-\int_{K_\perp^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2}\frac{\alpha_s N_c}{\pi}\left[\ln\left(\frac{P_\perp^2}{\mu^2}\right) + s_L\right]\right)$$

- Well separated in phase space from small-x gluons.
  ⇒ simultaneous small-x and Sudakov resummation.

Mueller, Xiao, Yuan, PRD 88 (2013), PC, Salazar, Schenke, Venugopalan, JHEP 11 (2022) 169

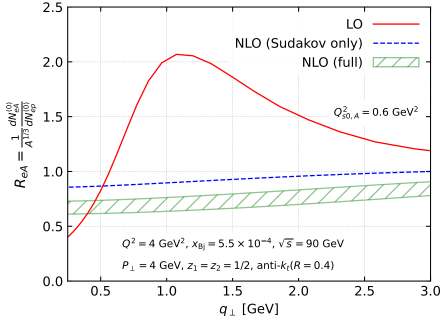


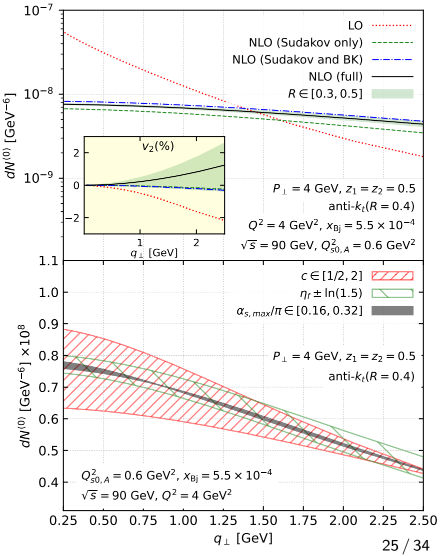
Factorization at NLO and predictions for the EIC

- Factorization of the back-to-back dijet cross-section in terms of the WW gluon TMD persists at NLO.

PC, Salazar, Schenke, Stebel, Venugopalan, PRL 132, 2024

$$\frac{\mathrm{d}\sigma^{\gamma^\star \to q\bar{q}+X}}{\mathrm{d}^2P_\perp \mathrm{d}^2K_\perp} = [\mathcal{H}_{\rm LO}(P_\perp) + \alpha_s \mathcal{H}_{\rm NLO}(P_\perp)]\, \mathcal{S}(P_\perp, K_\perp) xG(x, K_\perp)$$





Conclusion and future prospects

- QCD at high energies is a rich and wide topic with many connections to other fields.
  Statistical physics, black hole physics, ...

- Some recent progresses towards the unambiguous determination of saturation dynamics with systematic NLO calculations of key observables at the EIC.

- Crucial role of saturation in the EPOS4 event generator. K. Werner, PRC 108, 2023

- Connection between DGLAP and small-x frameworks is a topical issue nowadays: on-going efforts to understand this link using transverse momentum dependent distributions.

  PC, Iancu, Mueller, Yuan, in prep.

- Calculation of processes sensitive to Q_S in DIS, ultraperipheral AA collisions and in forward pA collisions at the LHC.

  PC, Ferrand, Salazar, 2401.01934, PC, Salazar, Yuan, in prep.

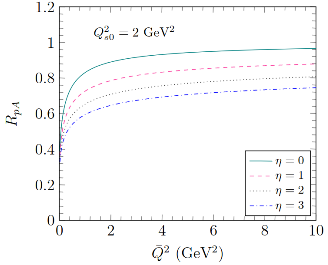
SIDIS at very forward rapidity

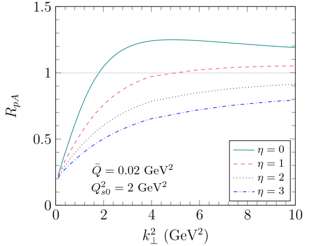
- We only measure a jet in hard $Q^2 \gg Q_s^2$ event, but very forward

$$(1-z)Q^2 \sim Q_s^2$$

- Dipole undergoes strong scattering $\Rightarrow$ sensitive to saturation.
- Complete NLO calculation in PC, Ferrand, Salazar, 2401.01934
- But: Sudakov-like suppression as $z \to 1$.

$$S(z) = \exp\left[-\frac{\alpha_s C_F}{2\pi}\ln^2(1-z) + \ldots\right]$$





LO study in Iancu, Mueller, Triantafyllopolous, Wei, JHEP 196, 2021

Non-linear evolution of the dipole amplitude

- GLR equation is a nice toy model, but the actual small-x evolution equations of gluon TMDs are more complicated.

- For the dipole S-matrix, it satisfies the BK-equation in the large N_c limit:

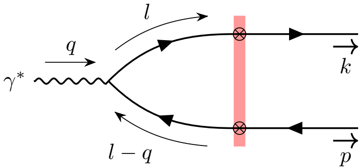
$$\frac{\partial S_Y(\boldsymbol{r}_\perp)}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2} \int \mathrm{d}^2 \boldsymbol{r}'_\perp \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}'^2_\perp (\boldsymbol{r}'_\perp + \boldsymbol{r}_\perp)^2} \left[S_Y(r'_\perp) S_Y(|\boldsymbol{r}_\perp + \boldsymbol{r}'_\perp|) - S_Y(r_\perp) \right]$$

with $Y = \ln(1/x)$.

- For other operators or beyond the large N_c limit, JIMWLK equation...

Dipole picture, CGC EFT, covariant perturbation theory

- We work in the dipole picture of DIS, large q^-.



- Covariant perturbation theory.

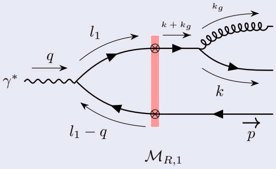
- CGC effective vertex:
$= (2\pi)\delta(q^- - p^-)\gamma^- \int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp)\boldsymbol{x}_\perp} V_{ij}(\boldsymbol{x}_\perp)$

$\Rightarrow$ multiple gluon interactions with the target resummed via Wilson lines $V(\boldsymbol{x}_\perp)$

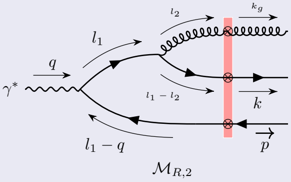
$$V(\boldsymbol{x}_\perp) = \mathcal{P}\exp\left(ig\int_{-\infty}^{\infty} dz^- A_{\mathrm{cl}}^{+,a}(z^-, \boldsymbol{x}_\perp)t^a\right)$$

NLO computation: real amplitudes

Real diagrams



$+(q \longleftrightarrow \bar{q})$

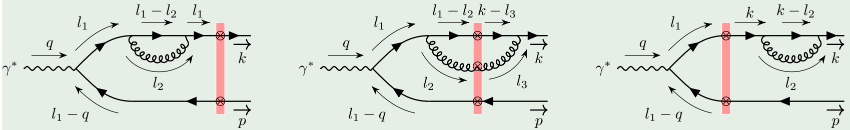


$+(q \longleftrightarrow \bar{q})$

- See also Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans, 1701.07143 using spinor helicities techniques.

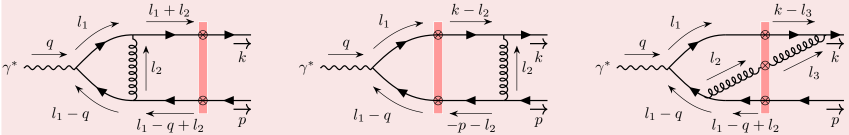
NLO computation: virtual amplitudes

Self-energies



$+(q \longleftrightarrow \bar{q})$ $\quad$ $+(q \longleftrightarrow \bar{q})$ $\quad$ $+(q \longleftrightarrow \bar{q})$

Vertex corrections

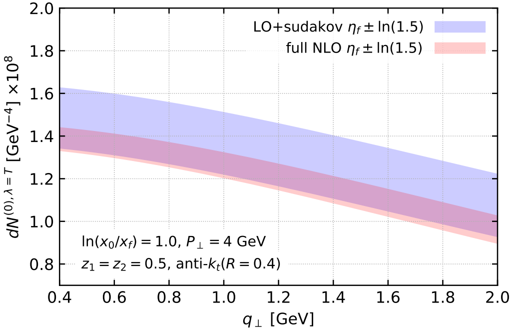


$+(q \leftrightarrow \bar{q})$

See also:

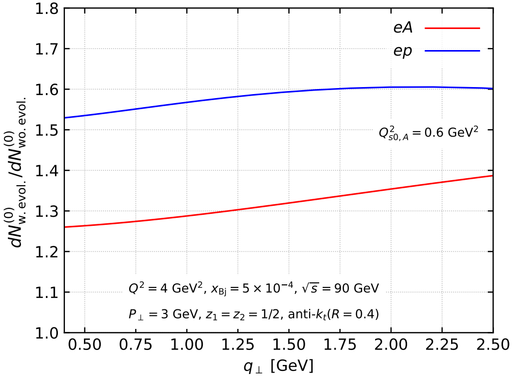
- Beuf, 1606.00777 (LCPT)
- Hänninen, Lappi, and Paatelainen 1711.08207 (LCPT)
- Boussarie, Grabovsky, Szymanowski, Wallon. 1606.00419 - 1905.07371 (exclusive dijet)
- Taels, Altinoluk, Beuf, Marquet, 2204.11650 (LCPT, $Q^2 = 0$)

Rapidity factorization scale dependence at EIC kinematics



- x_f variation around a central value to gauge the sensitivity to missing N^{2}LO corrections.
- Scale variations shrink from LO to NLO.
- One expects thinner NLO bands when $\alpha_s \ln(x_0/x_f) = \mathcal{O}(1)$.

Non-linear saturation effects in back-to-back dijet



- $q_\perp$ dependence of the x-section ratio with/without high energy resummation.
- In *ep*: mild $q_\perp$ dependence.
- In *eA*: slower evolution especially at small $q_\perp$.

How small is x at the EIC?

- Depends on the process.
- For back-to-back dijet production, $P_\perp$ down to $4 \div 5$ GeV should be ok, meaning $x_g \sim 10^{-3}$.
- For dihadrons, lower $P_\perp \Rightarrow$ lower x_g!

Expected yield of charged particles with $p_\perp > 1$ GeV in ep at 20×100 GeV

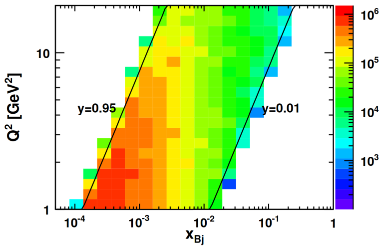


Fig. from Zheng, Aschenauer, Lee, Xiao, 1403.2413