Measuring Gravitational Wave Backgrounds (GWBs) with LISA

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Gravitational Wave Orchestra in the Alps

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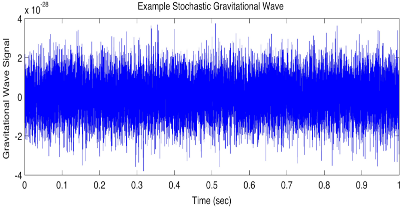
Overview

GWBs detection and characterization

GWBs are:

- Stochastic signals from the whole sky
- Signals with no phase coherency
- Of cosmological or astrophysical origin
- Invaluable source of information (HEP!)
- A target for all future detectors



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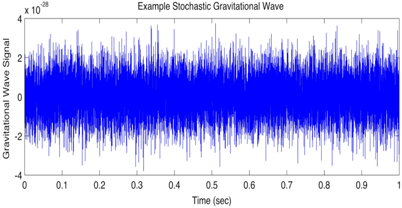

Detection prospects?

- At least two GWB components (sBHBs and CGBs) are guaranteed signals for LISA!
- News from LVK + future Earth-based interferometers (LIGO-India, ET, CE, ...)??
- Hints of GWB detection from millisecond pulsars timing experiments ...

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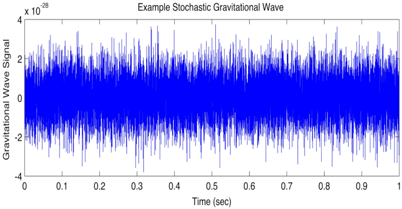

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Few characteristics to classify GWBs: $\longrightarrow$

- **Isotropy / Anisotropy**
- **Stationary / Non-stationary**
- **Polarized / Unpolarized**
- **Statistical properties**
- **Frequency shape**

* Figure from: https://www.ligo.org/science/GW-Overview/images/stochastic.jpg

Some general ingredients

Data $\tilde{d}$ (in frequency space) $\longrightarrow$ $\tilde{d} = \tilde{s} + \tilde{n}$

- For individual sources $\langle \tilde{s} \rangle \neq 0$
- For GWBs $\langle \tilde{s} \rangle = 0$
- For noise $\langle \tilde{n} \rangle = 0$

For an isotropic GWB $\longrightarrow$ $\langle h_\lambda(\vec{k})\, h^*_{\lambda'}(\vec{k}')\rangle \propto \delta_{\lambda\lambda'} P_h^\lambda(k)\delta(\vec{k} - \vec{k}')$

Assuming $\langle \tilde{s}\tilde{n} \rangle = 0$ and Gaussian signal and noise

$$\left\langle \tilde{d}^2 \right\rangle = \left\langle \tilde{s}^2 \right\rangle + \left\langle \tilde{n}^2 \right\rangle = \sum_\lambda \mathcal{R}_\lambda\, P_h^\lambda + N \equiv \mathcal{R}\left[P_h + S_n\right]$$

where we have introduced

- The (quadratic) response function of the instrument $\mathcal{R}$
- The (intensity of the) signal power spectrum P_h (in 1/Hz)
- The noise power spectrum N (in 1/Hz)
- The (square of the) Strain sensitivity S_n (in 1/Hz)

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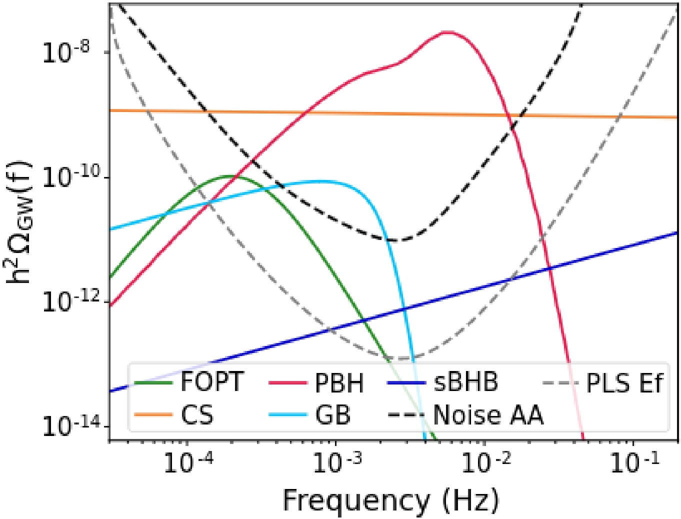
In order to compare with cosmological predictions it's customary to introduce

$$\Omega_{\rm GW} \equiv \frac{1}{3H_0^2 M_p^2}\frac{\partial \rho_{\rm GW}}{\partial \ln f} = \frac{4\pi^2}{3H_0^2} f^3 P_h \qquad \text{and} \qquad \Omega_n(f) = \frac{4\pi^2}{3H_0^2} f^3 S_n(f)\,,$$

where $H_0 \simeq h_0 \times 3.24 \times 10^{-18}\,\mathrm{Hz}$ is the Hubble parameter today.

Sources of GWBs in the LISA



* Figure from M. Colpi et al., ArXiv:2402.07571

Estimating the GWB from an astro population

First option: analytical estimation (see E.S. Phinney, ArXiv:astro-ph/0108028).

The total energy of the GWB can be computed as:

$$\frac{\rho^{(\mathrm{tot})}_{\mathrm{GWB}}}{\rho_c} = \int_0^\infty \frac{\mathrm{d}f}{f}\,\Omega_{\mathrm{GWB}}(f) = \int \mathrm{d}\xi \int \mathrm{d}V_c \int \mathrm{d}\tau_c \,\frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi\,\mathrm{d}V_c\,\mathrm{d}\tau_c}\,\frac{\rho^{(\mathrm{event})}_{\mathrm{GW}}}{\rho_c}\,,$$

where ξ are the source parameters, θ the population hyper-parameters.

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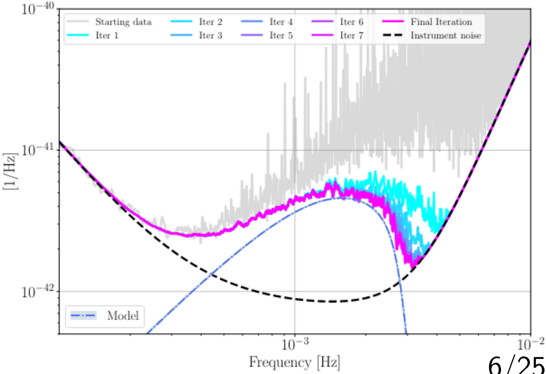
where ξ are the source parameters, θ the population hyper-parameters.

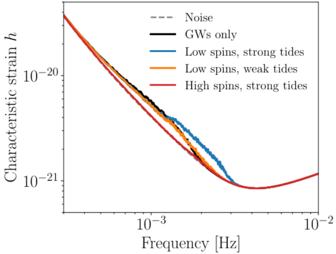
Second option: iterative method

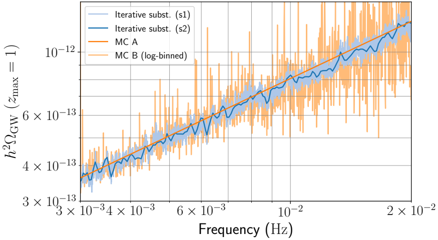
1. Get the whole data set including noise + signal from all the sources
2. Smooth it (using running mean or median) and compute the SNR of each source in the catalog
3. Remove high SNR sources (given some threshold) and go back to point until convergence is reached

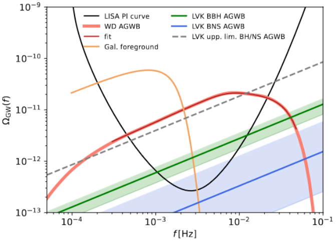
N. Karnesis et al., *Phys.Rev.D 104 (2021) 4, 043019*, ArXiv:2103.14598.

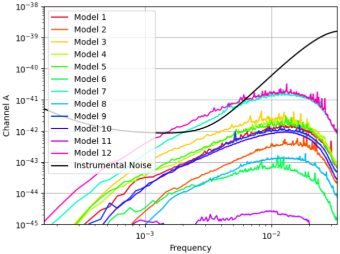


Several astrophysical populations might source GWBs









* Figures from A. Toubiana et al. 2403.16867.
S. Babak et al., *JCAP 08 (2023) 034*, ArXiv:2304.06368.
S. Staelens et al. *Astron.Astrophys. 683 (2024) A139*, ArXiv:2310.19448.
F. Pozzoli et al. *Phys.Rev.D 108 (2023) 10, 103039*, ArXiv:2302.07043.

LVK populations and general properties of the catalogs

sBHB catalogs require:

- Time-to-coalescence
- Sky localization
- Inclination / orientation
- Initial phase
- Redshift distribution
- Mass function
- Spin distribution

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Populations are provided by LVK!

Parameter	**Prior**
Time-to-coalescence (source frame)	$U[0, \tau^{(\mathrm{det})}_{c,\max}/(1+z)]$ yrs
Ecliptic Longitude	$U[0, 2\pi]$ rad
Ecliptic Latitude	$\arcsin(U[-1, 1])$ rad
Inclination	$\arccos(U[-1, 1])$ rad
Polarization	$U[0, 2\pi]$ rad
Initial Phase	$U[0, 2\pi]$ rad

Rate of events $R(z)$	**Mass distribution**	**Spin distribution**
$R_{0.2} = 28.1\,\mathrm{Gpc}^{-3}\mathrm{yrs}^{-1}$ $\kappa = 2.7$ $z_{\mathrm{peak}} = 2.04$ $r = 3.6$	$[m_{\min}, m_{\max}] \in [2.5, 100]\,M_\odot$ $\delta_{\min} = 7.8\,M_\odot$ $\alpha = 3.4$ $\lambda_{\mathrm{peak}} = 0.039$ $\mu_m = 34\,M_\odot$ $\sigma_m = 5.1\,M_\odot$ $\beta_q = 1.1$	$\mathrm{E}[a] = 0.25$ $\mathrm{Var}[a] = 0.03$ $\zeta = 0.66$ $\sigma_t = 1.5$

LVK collaboration, *Phys.Rev.X 13 (2023) 1, 011048*, ArXiV:2111.03634

Redshift distribution

LVK $\longrightarrow R(z) \propto R_0(1+z)^{\kappa}$, but observations constrain only at low z ($\lesssim 1$)!

To fix the behavior for $z \gtrsim 1$ we assume sBHBs track the Star Formation Rate:

$$R_{\rm SFR}(z) \propto R_0(1+z)^{\kappa} / \left[1 + \frac{\kappa}{r}\left(\frac{1+z}{1+z_{\rm peak}}\right)^{\kappa+r}\right]$$

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Including delay between formation and merger $\longrightarrow R(z) = \int_{t_{d,\rm min}}^{d,\rm max} R_{\rm SFR}(t(z)+t_d)p(t_d)\mathrm{d}t_d$

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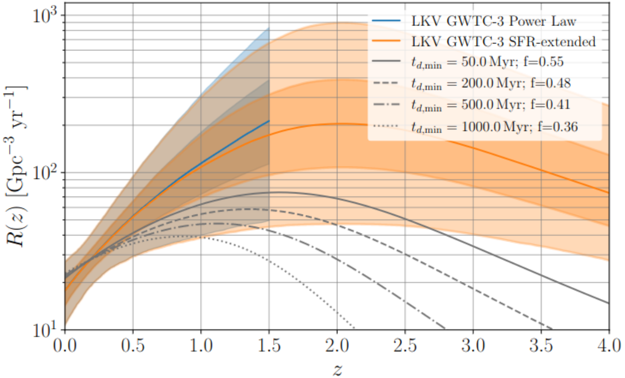

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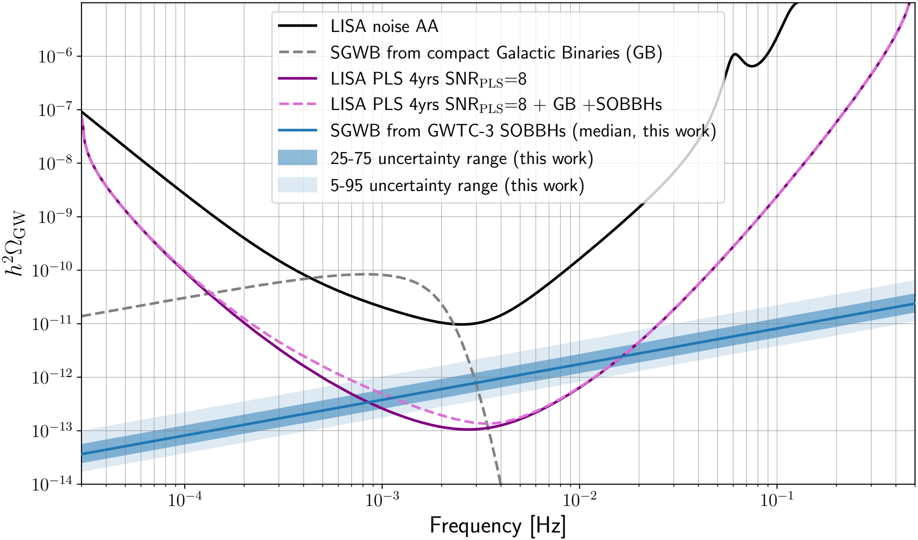
SGWB detectability and reconstruction with LISA



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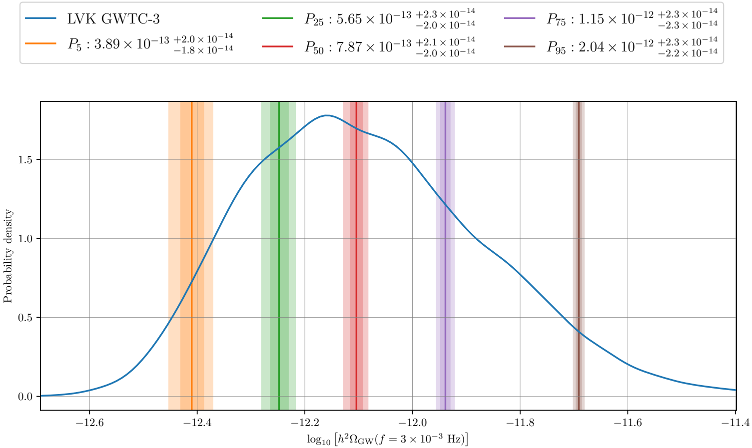
Comparison with LVK measurements

How much does the determination of the SGWB amplitude improve?



The posterior shrinks by $\sim$ one order of magnitude!

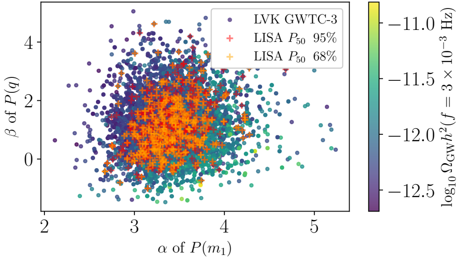
S. Babak et al., *JCAP 08 (2023) 034*, ArXiv:2304.06368.

Complementarity with LVK measurements

Improvement in the determination of the mass parameters

↓

The posterior distribution shrinks significantly



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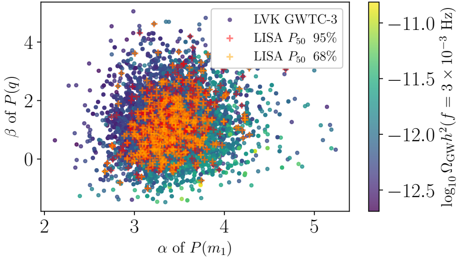
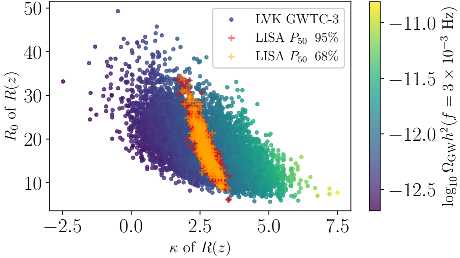


Improvement in the determination of the redshift parameters

↓

Different degeneracy and very accurate determination of κ



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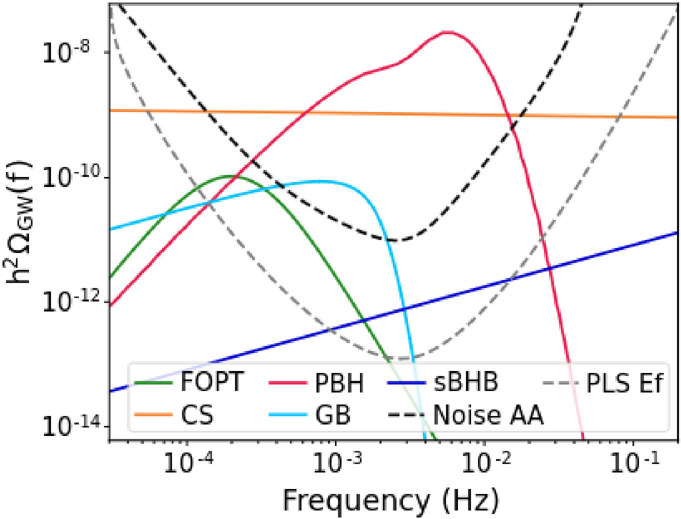


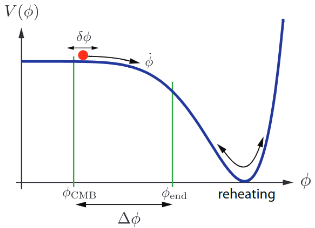
* Figure from M. Colpi et al., ArXiv:2402.07571

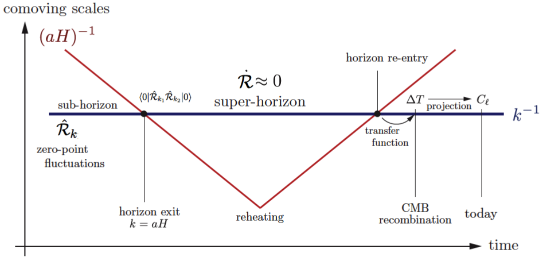
Inflation

The minimal realization of inflation: $\longrightarrow$

$$S = \int \mathrm{d}^4x \sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{\dot\phi^2}{2} - V(\phi)\right),$$

$$3H^2 = \frac{\dot\phi^2}{2} + V(\phi) \qquad -2\dot H = \dot\phi^2 \qquad (\text{where } H \equiv \tfrac{\dot a}{a})$$





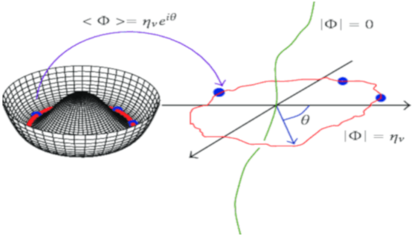
GWs from slow-roll inflation are too feeble, but things change dramatically in non-minimal scenarios: $\longrightarrow$

- Axion inflation: $\mathcal{L} \supset \frac{\alpha}{4\Lambda}\phi F\tilde F$
- Spectator fields: $\mathcal{L} \supset P(\dot\sigma, \sigma)$
- Symmetry breaking: $m_h \neq 0$
- ...

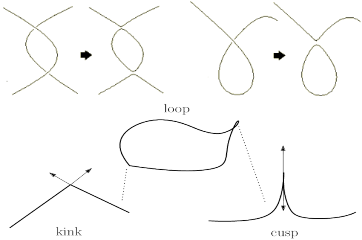
* Figures from D. Baumann, ArXiv:0907.5424
For models, see, e.g., N. Bartolo et al., *JCAP 12 (2016) 026*, ArXiv:1610.06481 or LISA Cosmology Working Group, ArXiv:2405.03740.

Cosmic Strings

CS might form in the early Universe



Evolution turn long strings into loops



GWs from CS form a (loud?) GWB (and also produce bursts)!

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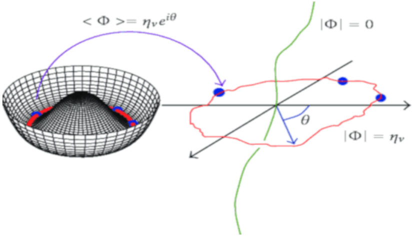

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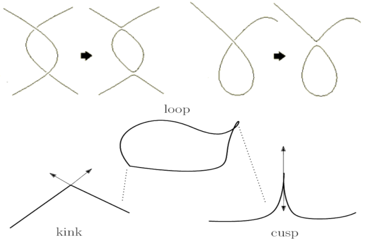

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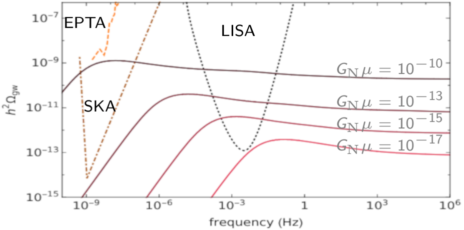
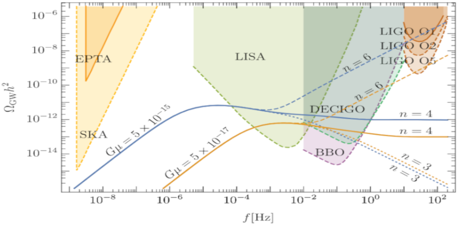
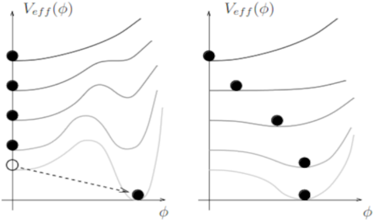




* Figures from Ringeval, *Adv.Astron. 2010 (2010) 380507*, ArXiv:1005.4842, Shellard and Vilenkin 1994, Gouttenoire, Servant and Simakachorn *JCAP 07 (2020) 032*, ArXiv:1912.02569, Auclair et al. *JCAP 04 (2020) 034*, ArXiv:1909.00819, Cui, et al. *Phys.Rev.D 97 (2018) 12, 123505*, ArXiv:1711.03104.

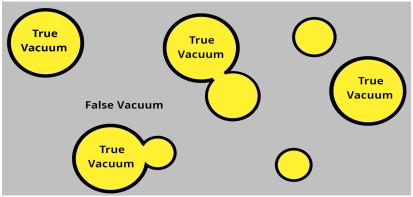
First order phase transitions

First order Second order



$\Longrightarrow$

FOPT $\to$ Bubble nucleation



Bubble collisions, sound waves in plasma, and MHD turbulence contribute to GWB!

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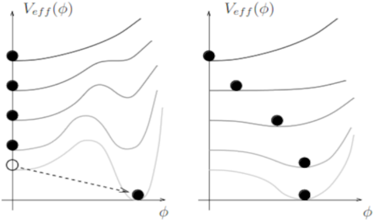

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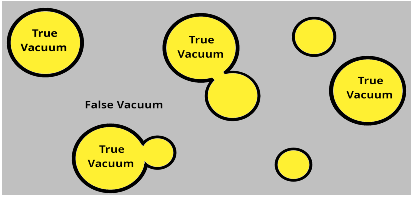
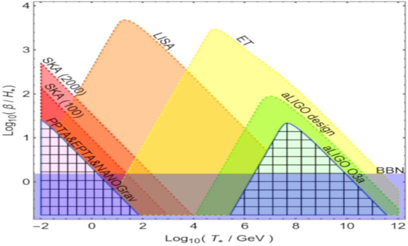
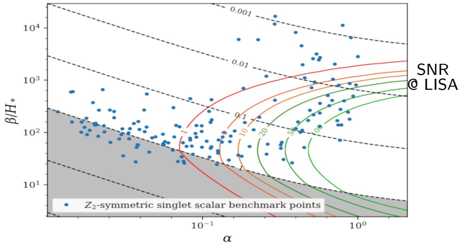


Bubble collisions, sound waves in plasma, and MHD turbulence contribute to GWB!
In SM both EW and QCD PTs should be second order $\Longrightarrow$ Detection implies BSM!





* Figures from Rubakov ArXiv:1804.11230, Caprini et al., *JCAP 03 (2020) 024*, ArXiv:1910.13125, Auclair et al. *Living Rev.Rel. 26 (2023) 1, 5*, ArXiv:2204.05434

Forecasting LISA constraints I

Choose a template

$\downarrow$

Get forecasts (e.g., using Fisher Information Matrix (FIM)) on the template parameters

$\downarrow$

Convert in constraints on model parameter

$\downarrow$

Forecast constrains on fundamental physics!

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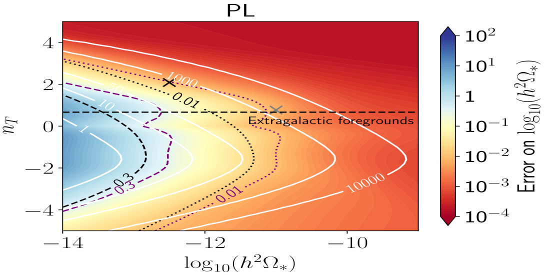
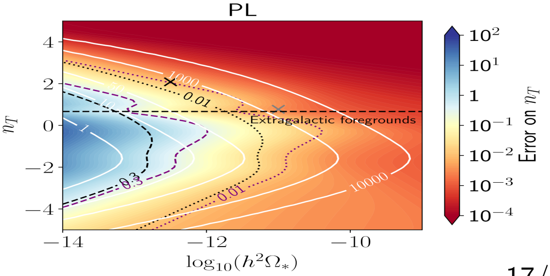
Forecast constrains on fundamental physics!

Example: a power-law

$$\Omega_{\rm GW} h^2 = 10^{\log_{10}(h^2\Omega_*)} \left(\frac{f}{f_*}\right)^{n_T}$$



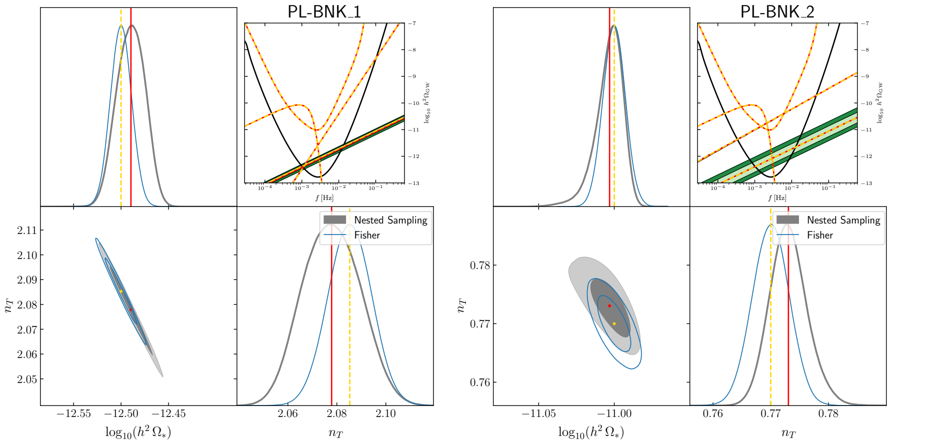


LISA Cosmology Working Group, ArXiv:2405.03740.

Forecasting LISA constraints II

Validate Fisher with some more realistic data analysis pipeline e.g, `SGWBinner`

(see C. Caprini et al. *JCAP 11 (2019) 017*, ArXiv:1906.09244.
R. Flauger et al. *JCAP 01 (2021) 059*, ArXiv:2009.11845.)



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Forecasting LISA constraints III

Again PL
(from inflation)

LISA Cosmology Working Group, ArXiv:2405.03740.

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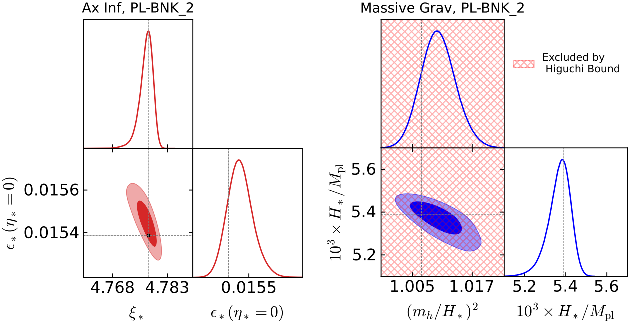

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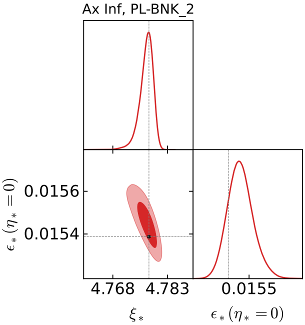

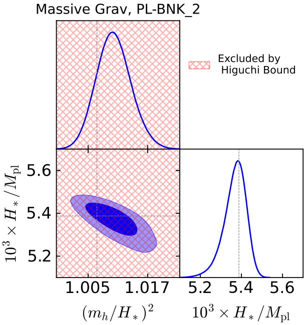

Or a BPL
(from FOPTs)

LISA Cosmology Working Group, ArXiv:2403.03723.

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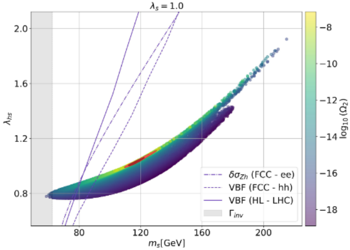

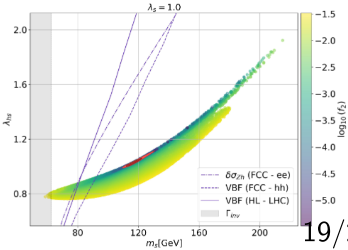

ML for GWB data analysis

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scale poorly with number of parameters and require explicit likelihoods

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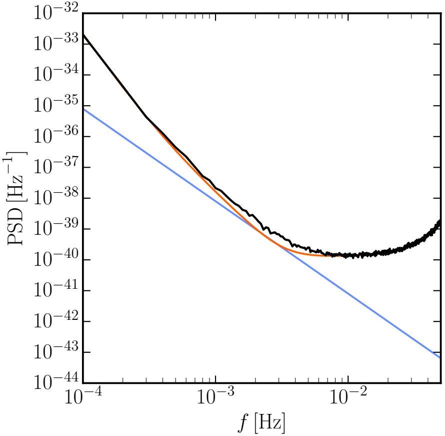
This can be cast in a minimization problem that can be solved with ML
the approach is typically referred to as Neural Ratio Estimation (NRE)
(basically build a classifier to say whether θ, d are joint or marginal...).

Recover previous results I ...

Assume we inject a power law signal:
Can we recover it with the same level of accuracy?



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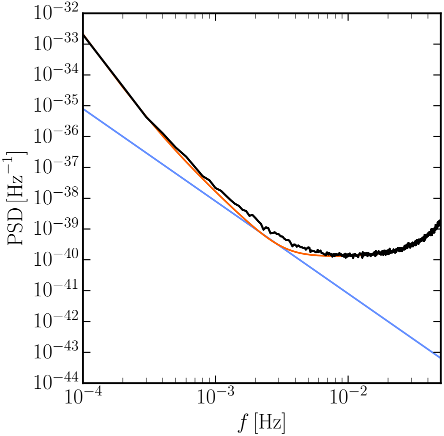
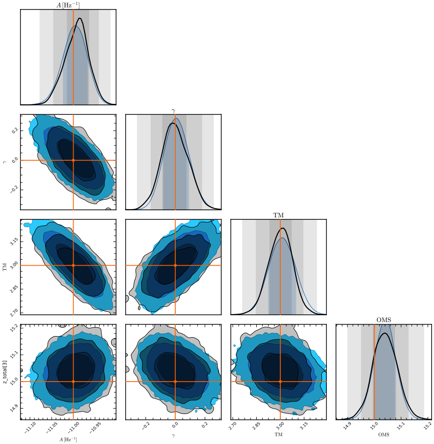




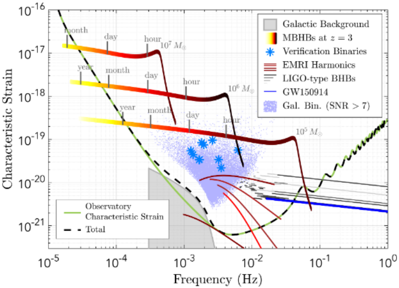
Good news!

James Alvey et al., Phys. Rev. D 109 (2024) 083008, ArXiv:2309.07954.

Code available at `https://github.com/PEREGRINE-GW/saqqara/`

... plus something completely new!

What if there's something else beyond GWB and noise?



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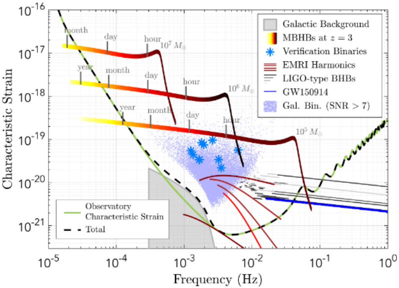

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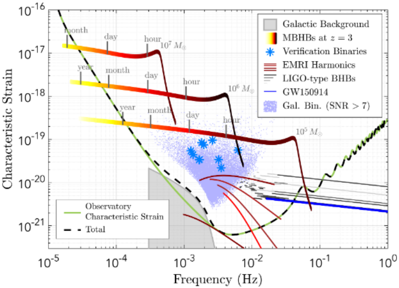
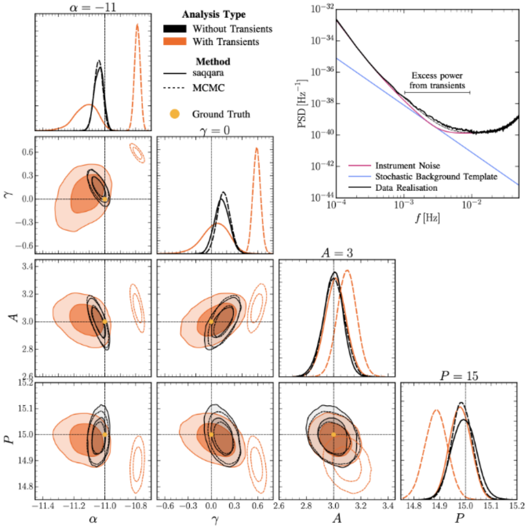


For example, assume some sources slightly below the threshold for detection are randomly injected.

Would this still work??

Yes!!



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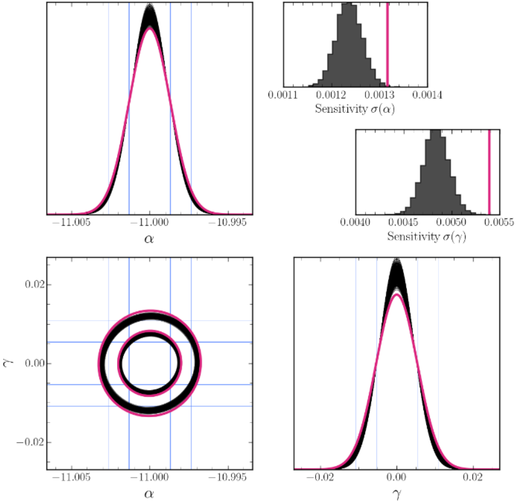
Looks like you actually do better!



James Alvey et al., ArXiv:2408.00832.
Code available at `https://github.com/PEREGRINE-GW/saqqara/`
See also `https://github.com/Mauropieroni/GW_response`

Conclusions and outlook

Some general conclusions:

- GWBs are quite interesting sources for LISA
- GWBs of astrophysical origin → info on astro populations
- GWBs of cosmological origin → new window on BSM!

New ideas and tools will be necessary:

- Identification of "smoking-gun" observables (chirality, anisotropy, time modulations, statistical properties, ...)
- Data analysis techniques to fully exploit the data
- Cross-correlations with other probes (CMB, LSS, ...?)

Last Slide

The end

Thank you for your attention