Stochastic gravitational wave background constraints from Gaia Data Release 3

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Gravitational Wave Orchestra in the Alps. Annecy, 19th September 2024

Based on S. Jaraba, J. García-Bellido, S. Kuroyanagi, S. Ferraiuolo, M. Braglia, MNRAS 524 (2023) 3, 3609-3622

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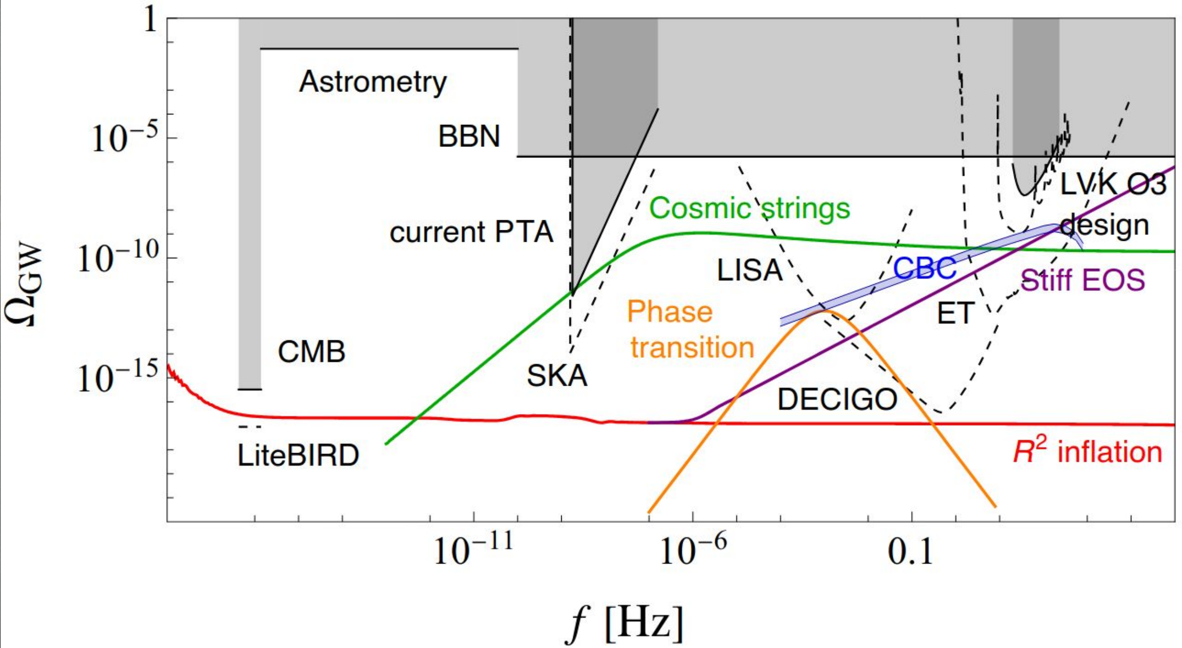
The stochastic gravitational wave background

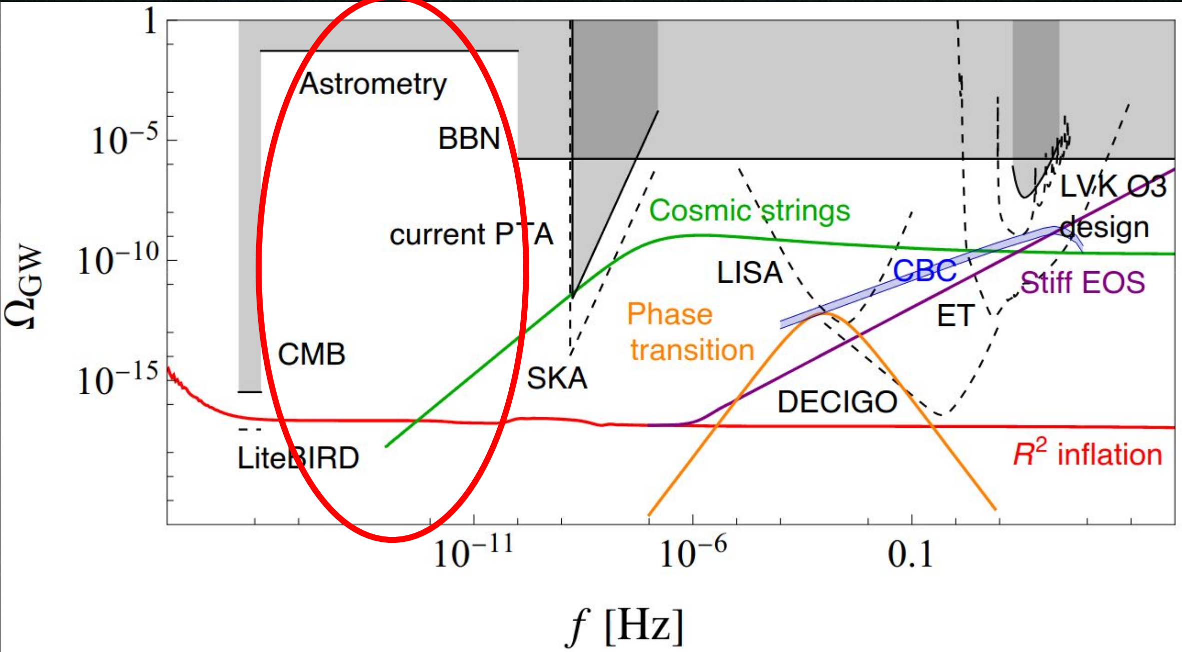
The SGWB should be constrained in all frequency spectrum.



A. Romero, S. Kuroyanagi, arXiv:2407.00205

The stochastic gravitational wave background

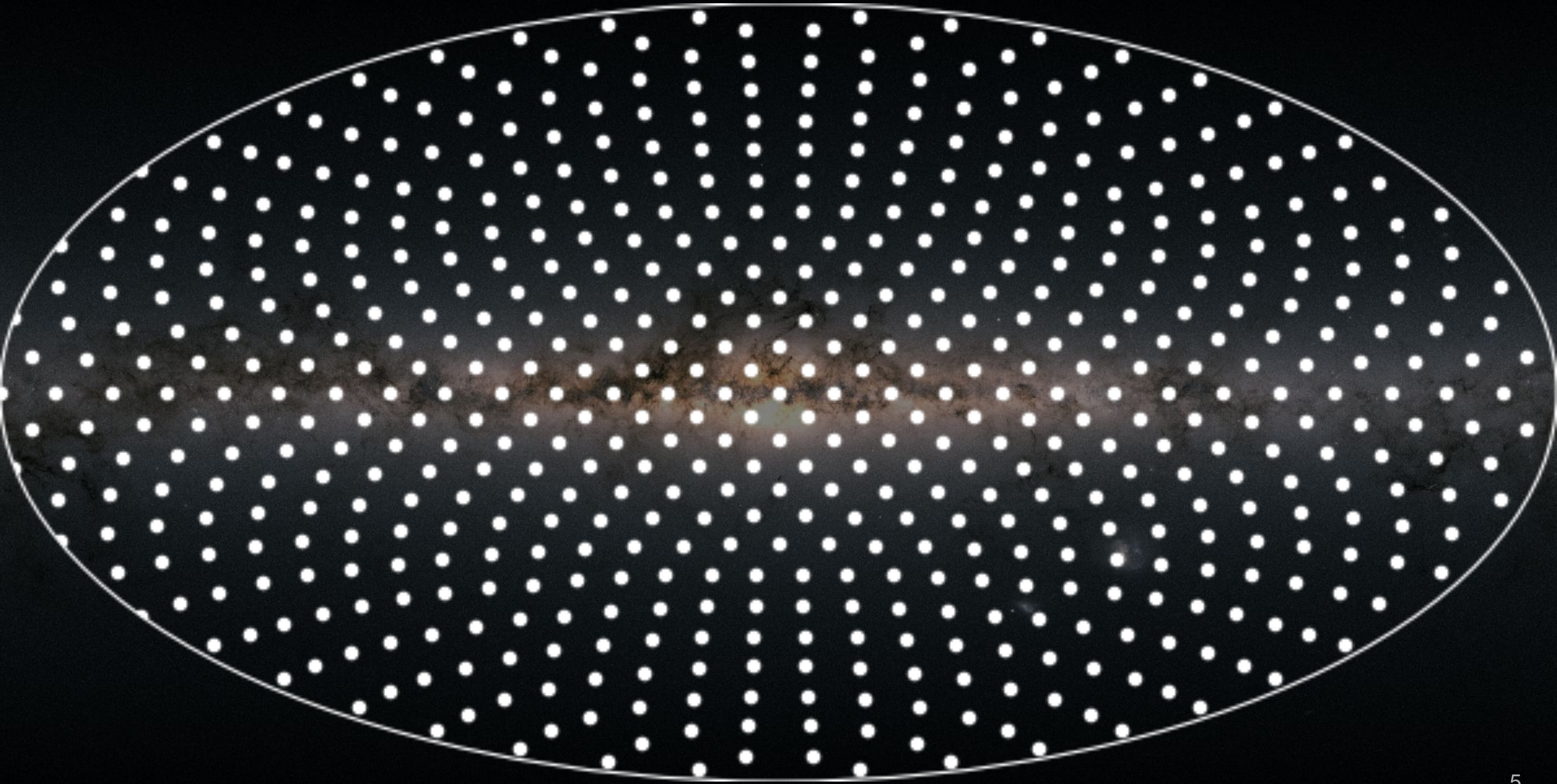
Astrometry can constrain between PTAs and CMB!

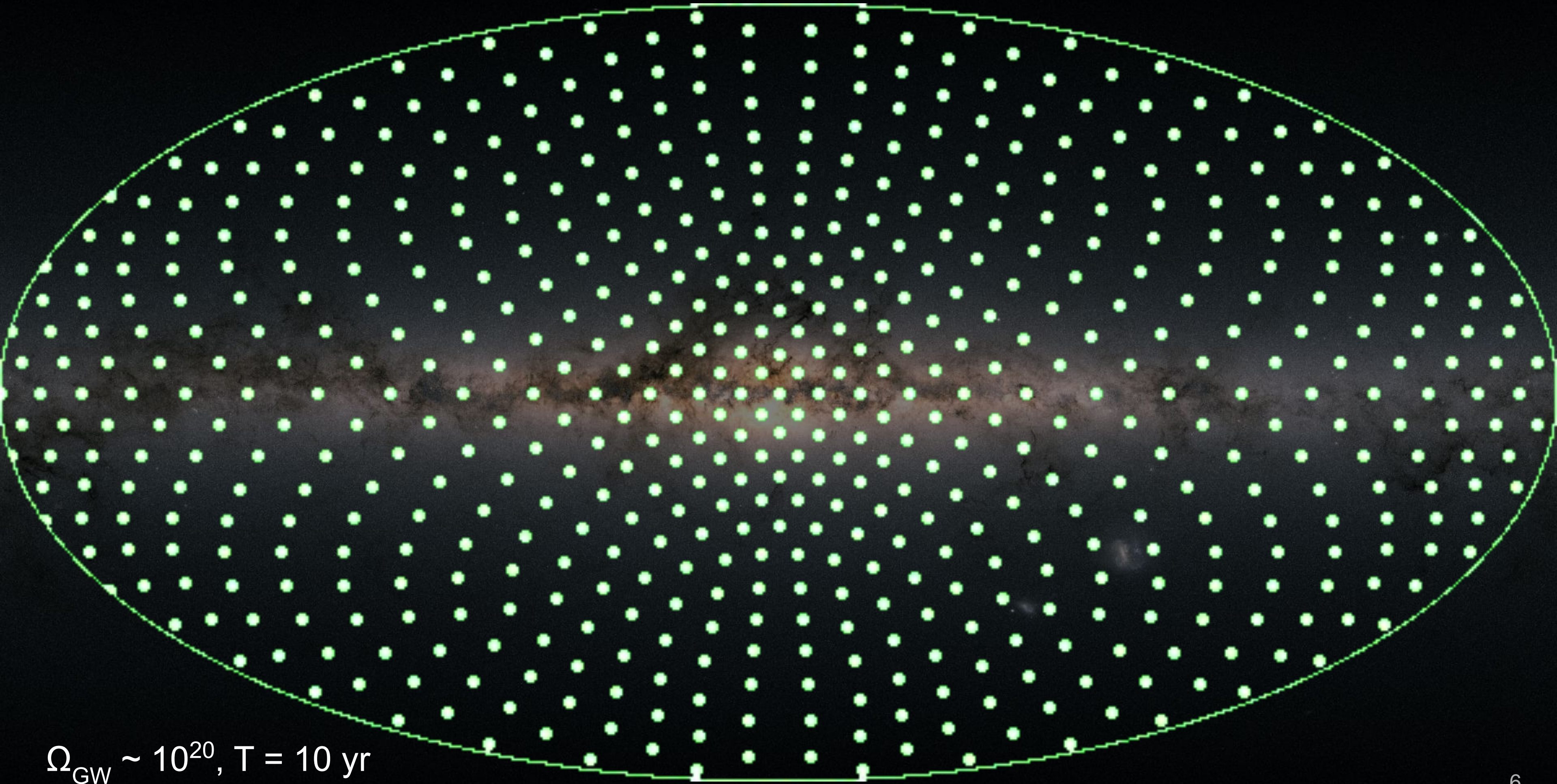


A. Romero, S. Kuroyanagi, arXiv:2407.00205

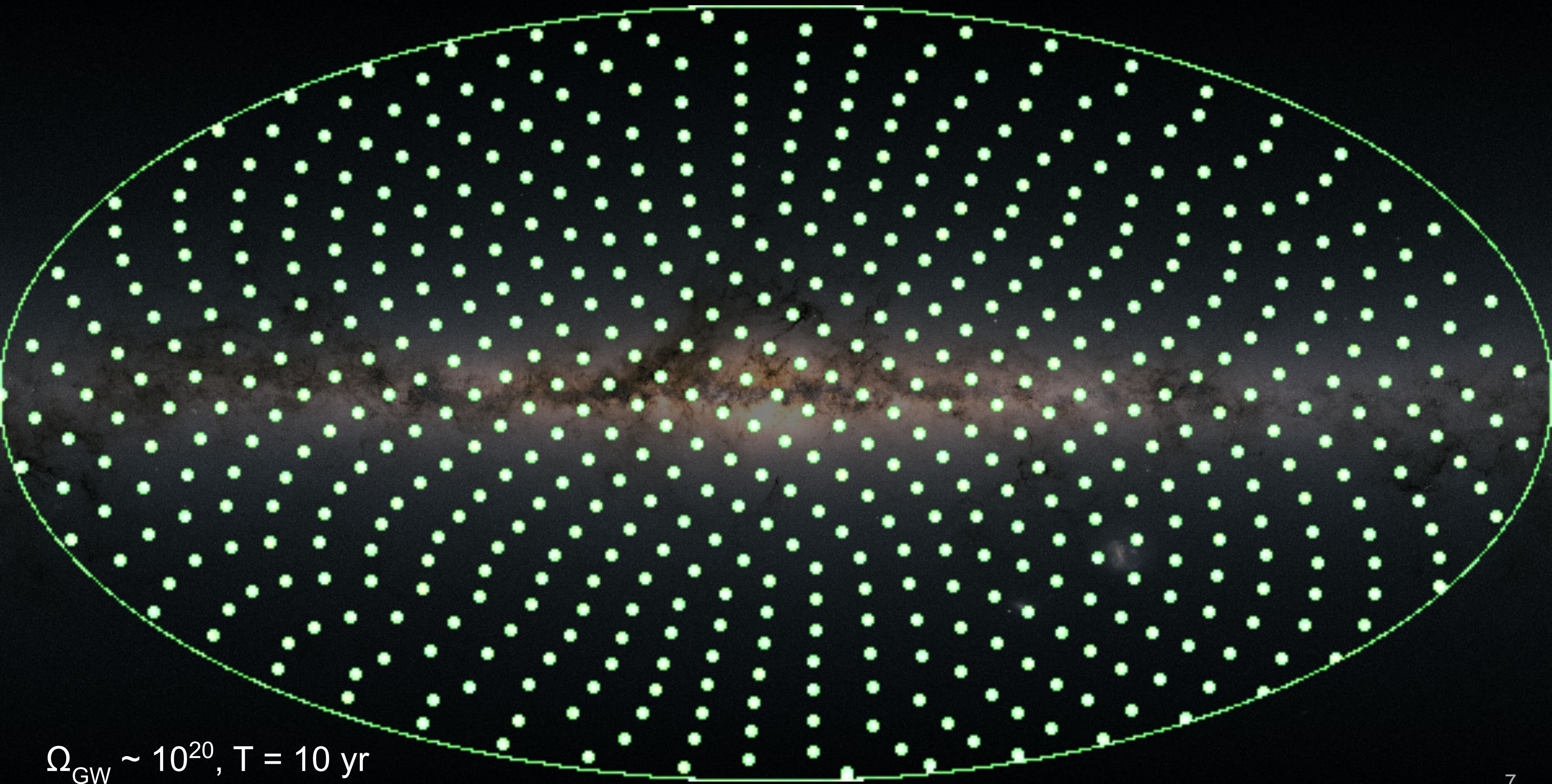
Astrometry

- Astrometry: precise measurements of the positions and movements of stars and other celestial bodies.
- For us → dataset of bodies with their measured positions and proper motions over a time period T.
- *How much bodies in a given position have moved over the period T*.





$\Omega_{GW} \sim 10^{20}$, T = 10 yr

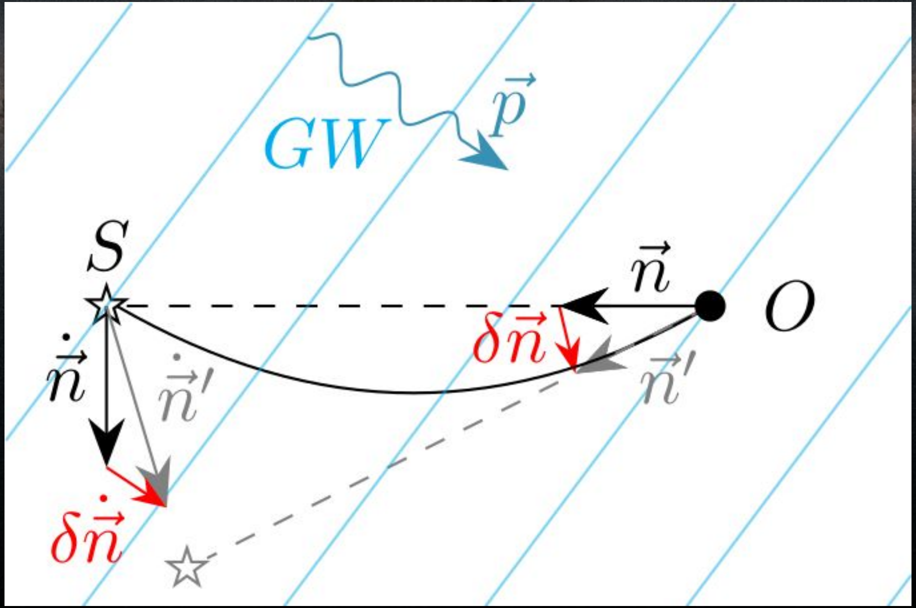

$\Omega_{GW} \sim 10^{20}$, T = 10 yr

Gravitational waves from astrometry

- Studied in the 1980s and 90s (E. V. Linder 1986, Braginsky et al. 1990, Pyne et al. 1995, Gwinn et al. 1996, etc.).
- New wave of publications after L. Book and E. E. Flanagan review, arXiv:1009.4192. and the launch of the Gaia mission.
  - Formalism-focused: F. Mignard and S. Klioner arXiv:1207.0025.
  - Theoretical/mock data: C. J. Moore et al. arXiv:1707.06239, D. Mihaylov et al. arXiv:1804.00660, D. Mihaylov et al. arXiv:1911.10356.
  - Forecasts for future missions: J. García-Bellido et al. arXiv:2104.04778.
  - Data analysis works: J. Darling et al. arXiv:1804.06986, S. Aoyama et al. arXiv:2105.04039 (!), **S. Jaraba** et al. arXiv:2304.06350.

Gravitational waves from astrometry

- We observe light from distant stars.
- The passage of a GW can alter the observed position and proper motion.



Angular deflection from a gravitational wave

Idea of formalism: first, effect of generic GW in perturbed Minkowski background

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + (\delta_{ij} + h_{ij}) dx^i dx^j.$$

- Observer at origin, star at direction **n**.
- Consider trajectory of light ray. Unperturbed: $x_0^\alpha(\lambda) = \omega_0(\lambda, -\lambda\mathbf{n}) + (t_0, 0, 0, 0)$
- ω_0 unperturbed frequency, t_0 time of photon observation at origin.
- Unperturbed photon momentum: $k_0^\alpha = \omega_0(1, -\mathbf{n})$
- Both source and observer are stationary:

$$x^i_{obs}(t) = 0$$
$$x^i_s(t) = x^i_s = \text{constant}.$$

$$\lambda_s = -\frac{|\mathbf{x}_s|}{\omega_0}.$$

Angular deflection from a gravitational wave

- Perturbed trajectory and momentum: $x^\alpha(\lambda) = x_0^\alpha(\lambda) + x_1^\alpha(\lambda)$ $k^\alpha(\lambda) = k_0^\alpha(\lambda) + k_1^\alpha(\lambda)$
- Apply geodesic equations:

$$\frac{d^2 x_1^0}{d\lambda^2} = -\frac{\omega_0^2}{2} n^i n^j h_{ij,0}$$

$$\frac{d^2 x_1^k}{d\lambda^2} = -\frac{\omega_0^2}{2}\left[-2n^i h_{ki,0} + n^i n^j \left(h_{ki,j} + h_{kj,i} - h_{ij,k}\right)\right]$$

- Solve:

$$k_1^0(\lambda) = -\frac{\omega_0^2}{2} n^i n^j \mathcal{I}_{ij}(\lambda) + I_0,$$
$$x_1^0(\lambda) = -\frac{\omega_0^2}{2} n^i n^j \mathcal{K}_{ij}(\lambda) + I_0\lambda + K_0,$$

$$k_1^j(\lambda) = -\frac{\omega_0^2}{2} n^i R_{ij} + J_0^j,$$
$$x_1^j(\lambda) = -\frac{\omega_0^2}{2} n^i S_{ij} + J_0^j\lambda + L_0^j,$$

$$\mathcal{I}_{ij}(\lambda) = \int_0^\lambda d\lambda' h_{ij,0}(\lambda'),$$
$$\mathcal{K}_{ij}(\lambda) = \int_0^\lambda d\lambda' \int_0^{\lambda'} d\lambda'' h_{ij,0}(\lambda''),$$

$$\mathcal{J}_{ijk}(\lambda) = \int_0^\lambda d\lambda' h_{ij,k}(\lambda'),$$
$$\mathcal{L}_{ijk}(\lambda) = \int_0^\lambda d\lambda' \int_0^{\lambda'} d\lambda'' h_{ij,k}(\lambda''),$$

$$R_{ij}(\lambda) \equiv \left[-2\mathcal{I}_{ij}(\lambda) + n^k \left(\mathcal{J}_{ijk}(\lambda) + \mathcal{J}_{jki}(\lambda) - \mathcal{J}_{ikj}(\lambda)\right)\right]$$

$$S_{ij}(\lambda) \equiv \left[-2\mathcal{K}_{ij}(\lambda) + n^k \left(\mathcal{L}_{ijk}(\lambda) + \mathcal{L}_{jki}(\lambda) - \mathcal{L}_{ikj}(\lambda)\right)\right]$$

Angular deflection from a gravitational wave

- Determination of integration constants I_0, J_0^j , K_0, and L_0^j with boundary conditions:
  - Photon path must pass through detection event: $x^\mu(0) = x_0^\mu(0) + x_1^\mu(0) = (t_0, 0, 0, 0)$
  - Photon geodesic is null.
  - Photon must be emitted with frequency ω_0.
  - Perturbed photon path must start at the source: $x^j(\tilde{\lambda_s}) = x_s^j = x_0^j(\tilde{\lambda_s}) + x_1^j(\tilde{\lambda_s})$
- More steps:
  - Compute perturbed observed frequency: $z \equiv \frac{\omega_0 - \omega_{obs}}{\omega_0} = -\frac{1}{2(1+\gamma)} n^i n^j \left[h_{ij}(\lambda_s) - h_{ij}(0)\right]$
  - Compute changes in local reference frame of observer due to the GW.
- Final result:

$$\delta n^{\hat{i}} = \frac{1}{2}\left\{ n^j h_{ij}(0) - n^i n^j n^k h_{jk}(0) - \frac{\omega_0}{\lambda_s}\left(\delta^{ik} - n^i n^k\right) n^j \right.$$
$$\left. \times \left[-2\int_0^{\lambda_s} d\lambda' \int_0^{\lambda'} d\lambda'' h_{jk,0}(\lambda'') + n^l \int_0^{\lambda_s} d\lambda' \int_0^{\lambda'} d\lambda'' \left(h_{jk,l}(\lambda'') + h_{kl,j}(\lambda'') - h_{jl,k}(\lambda'')\right)\right] \right\}$$

Angular deflection from a gravitational wave

- For plane wave propagating in direction **p**, $h_{ij}(\lambda) = \mathrm{Re}[\mathcal{H}_{ij} e^{-i\Omega\{t_0+\omega_0(1+\vec{p}\cdot\vec{n})\lambda\}}]$

$$\delta n^i = \mathrm{Re}\left[\left(\left(\left\{1 + \frac{i(2+\vec{p}\cdot\vec{n})}{\omega_0\lambda_S\Omega(1+\vec{p}\cdot\vec{n})}\left[1 - e^{-i\Omega\omega_0(1+\vec{p}\cdot\vec{n})\lambda_S}\right]\right\} n^i + \left\{1 + \frac{i}{\omega_0\lambda_S\Omega(1+\vec{p}\cdot\vec{n})}\left[1 - e^{-i\Omega\omega_0(1+\vec{p}\cdot\vec{n})\lambda_S}\right]\right\} p^i\right) \frac{n^j n^k \mathcal{H}_{jk} e^{-i\Omega t_0}}{2(1+\vec{p}\cdot\vec{n})} - \left\{\frac{1}{2} + \frac{i}{\omega_0\lambda_S\Omega(1+\vec{p}\cdot\vec{n})}\left[1 - e^{-i\Omega\omega_0(1+\vec{p}\cdot\vec{n})\lambda_S}\right]\right\} n^j \mathcal{H}^i_j e^{-i\Omega t_0}\right].$$

- **Distant source limit**: distance to the source $\omega_0|\lambda_s|$ much larger than GW wavelength c/Ω.

$$\delta n^{\hat{i}}(\tau, \mathbf{n}) = \frac{n^i + p^i}{2(1+\mathbf{p}\cdot\mathbf{n})} h_{jk}(0) n_j n_k - \frac{1}{2} h_{ij}(0) n_j$$

- Same expression for FLRW spacetime. Some extra steps needed.

Angular deflection spectrum from a SGWB

- First, we decompose the GW:

$$h_{ij}(\mathbf{x},t) = \sum_{A=+,\times} \int_0^\infty df \int d^2\Omega_{\mathbf{p}}\, h_{A\mathbf{p}}(f)\, e^{2\pi i f(\mathbf{p}\cdot\mathbf{x}-t)}\, e_{ij}^{A,\mathbf{p}} + c.c.$$

- Our deflection is then expressed as

$$\delta n^i(\mathbf{n},t) = \sum_{A=+,\times} \int_0^\infty df \int d^2\Omega_{\mathbf{p}}\, h_{A\mathbf{p}}(f)\, e^{-2\pi i f t}\, \mathcal{R}_{ikl}(\mathbf{n},\mathbf{p})\, e_{kl}^{A,\mathbf{p}} + c.c.$$

$$\mathcal{R}_{ikl}(\mathbf{n},\mathbf{p}) = \frac{1}{2}\left[\frac{(n_i+p_i)\,n_k n_l}{1+\mathbf{p}\cdot\mathbf{n}} - n_k\delta_{il}\right]$$

- Gaussian random process assumption:

$$\langle h_{A\mathbf{p}}(f)\, h_{B\mathbf{p}'}(f')\rangle = 0$$

$$\langle h_{A\mathbf{p}}(f)\, h_{B\mathbf{p}'}(f')^*\rangle = \frac{3H_0^2\Omega_{\rm gw}(f)}{32\pi^3 f^3}\,\delta(f-f')\,\delta_{AB}\,\delta^2(\mathbf{p},\mathbf{p}')$$

- Taking the spectrum and applying Ω_{GW} definition,

$$\langle \delta n^i(\mathbf{n},t)\, \delta n^j(\mathbf{n}',t')\rangle = \int_0^\infty df\, \frac{3H_0^2}{32\pi^3} f^{-3}\Omega_{\rm gw}(f) e^{-2\pi i f(t-t')} H_{ij}(\mathbf{n},\mathbf{n}') + c.c.$$

$$H_{ij}(\mathbf{n},\mathbf{n}') = \sum_{A=+,\times} \int d^2\Omega_{\mathbf{p}} \mathcal{R}_{ikl}(\mathbf{n},\mathbf{p})\, e_{kl}^{A,\mathbf{p}}\, \mathcal{R}_{jrs}(\mathbf{n}',\mathbf{p})\, \left(e_{rs}^{A,\mathbf{p}}\right)^*$$

Angular deflection spectrum from a SGWB

- For the particular case **n** = **n'**, after heavy simplification,

$$\langle \delta\mathbf{n}(\mathbf{n},t)^2 \rangle = \theta^2_{\rm rms} = \frac{1}{4\pi^2}\int d\ln f \left(\frac{H_0}{f}\right)^2 \Omega_{\rm gw}(f)$$

- Differentiating,

$$\langle \delta\dot{\mathbf{n}}(\mathbf{n},t)^2 \rangle = \int d\ln f \, H_0^2 \Omega_{\rm gw}(f)$$

Angular deflection spectrum from a SGWB

$$\langle \delta \dot{\mathbf{n}}(\mathbf{n}, t)^2 \rangle = \int d\ln f \, H_0^2 \Omega_{\rm gw}(f)$$

- Spectrum of averaged proper motions over a time period T → constrained quantity is

$$\int_{f \lesssim T^{-1}} d\ln f \, \Omega_{\rm gw}(f)$$

- Above T^{-1}, the proper motions average out to around zero → $f_{max} \approx T^{-1}$.
- Distant source limit ⟶ $f_{min} \approx c/D$, D distance to the nearest source.
- In our case (Gaia DR3, T = 2.84 years), 4×10^{-18} Hz $\lesssim f \lesssim 1 \times 10^{-8}$ Hz.
- Assuming a dominant $\Omega_{GW}(f)$ over an order of magnitude,

$$\Omega_{\rm GW}(f) \sim \langle \mu(f)^2 \rangle / H_0^2$$

Multipole decomposition

$$\vec{S}_{lm}(\alpha,\delta) = \frac{1}{l(l+1)} \nabla Y_{lm}(\alpha,\delta)$$

$$\vec{T}_{lm}(\alpha,\delta) = -\frac{1}{l(l+1)} \hat{n} \times \nabla Y_{lm}(\alpha,\delta)$$

- A vector field needs two basis: spheroidal/electric and toroidal/magnetic.
- We run MCMCs to fit proper motion data to a generic vector field up to l = 2.

$$\vec{V}(\alpha,\delta) = \sum_{\substack{r=s,t\\R=S,T}} \sum_{l=1}^{\infty} \left[r_{l0}\vec{R}_{l0} + 2\sum_{m=1}^{l} \left(r_{lm}^{\mathrm{Re}}\vec{R}_{lm}^{\mathrm{Re}} - r_{lm}^{\mathrm{Im}}\vec{R}_{lm}^{\mathrm{Im}} \right) \right]$$

- Power per multipole and mode

$$P_l^r = \sum_{m=-l}^{l} |r_{lm}|^2 = r_{l0}^2 + 2\sum_{m=1}^{l} (r_{lm}^{\mathrm{Re}})^2 + (r_{lm}^{\mathrm{Im}})^2$$

- When dominated by noise, only the quadrupole is relevant to set constraints.

$$\Omega_{\mathrm{GW}} = \frac{6}{5}\frac{1}{4\pi}\frac{P_2}{H_0^2} = 0.000438 \frac{P_2}{(1\ \mu\mathrm{as/yr})^2} h_{70}^{-2}$$

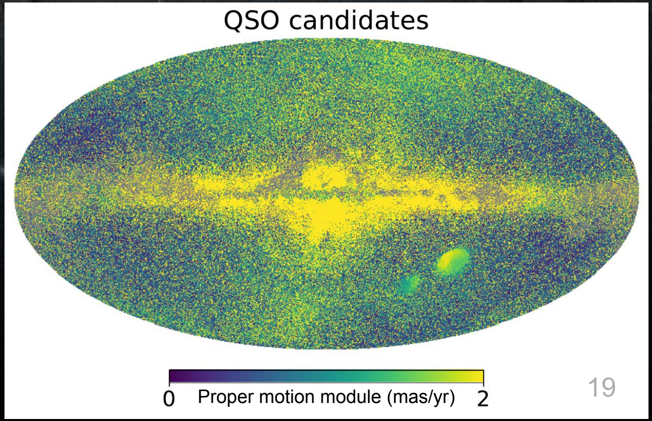
$$P_l = P_l^s + P_l^t$$

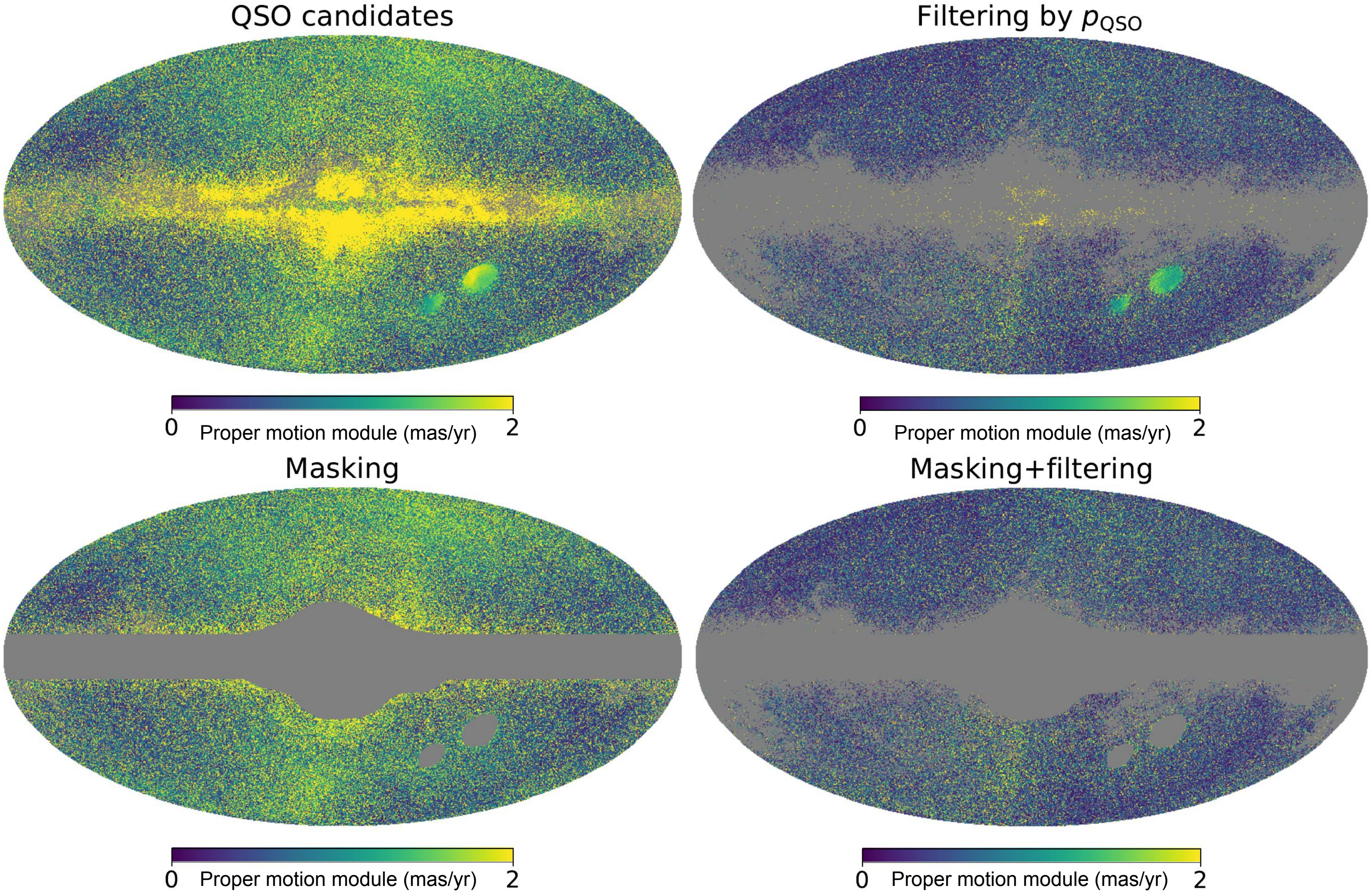
$$H_0 = 70h_{70}\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

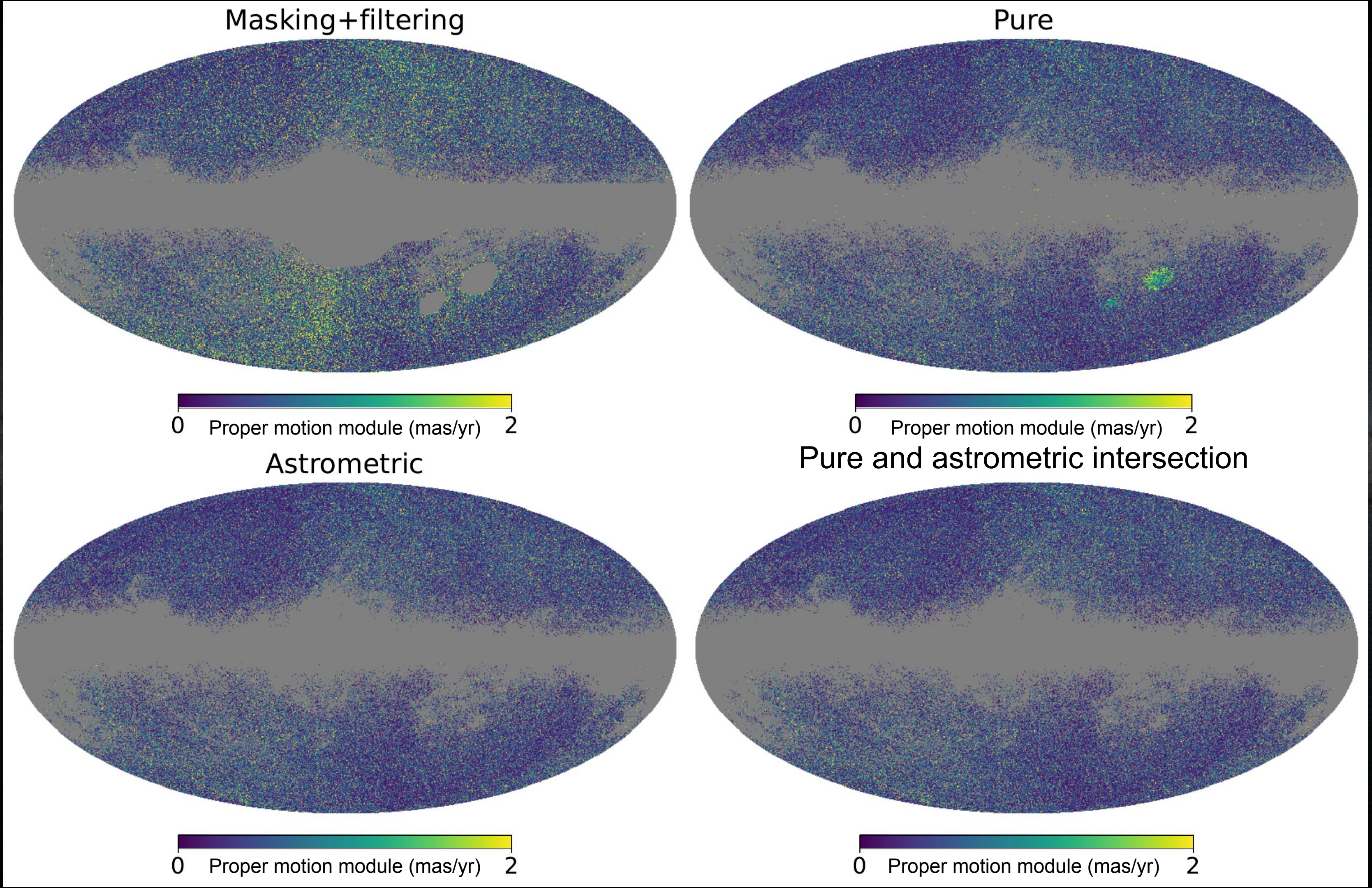
The datasets

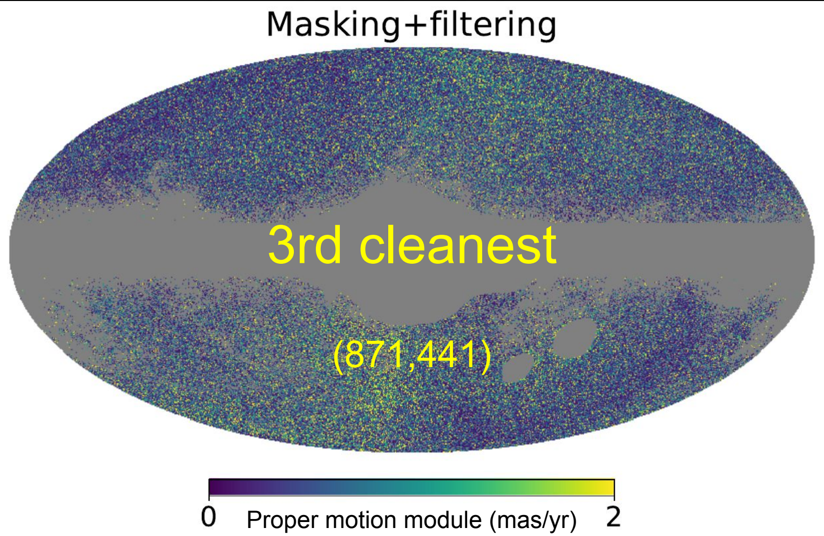
Overview of Gaia mission

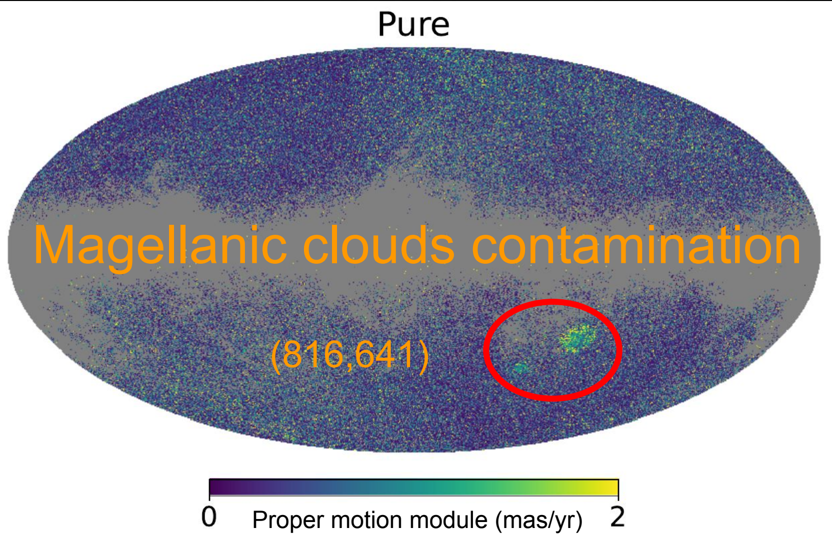
- Launched by ESA in December 2013, expected to operate until 2025.
- Data Release 3 (June 2022): 1.81 billion objects, 34 months of operation.
- Low intrinsic proper motions needed → focus on Quasi Stellar Objects (QSO).
- No “official” Gaia QSO catalog, but “QSO candidate” list provided.
- By cleaning this sample, we can get purer datasets.
- Dataset: list of objects with positions and proper motions.
- No time series: single, averaged value for each source.

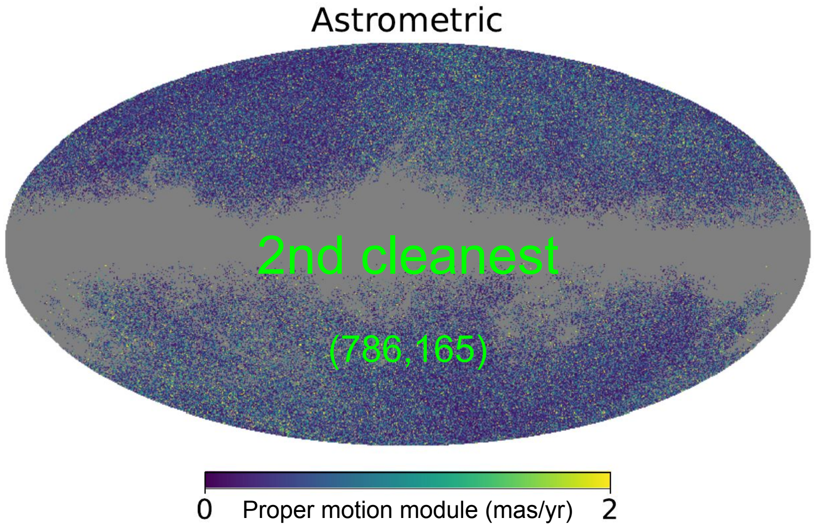


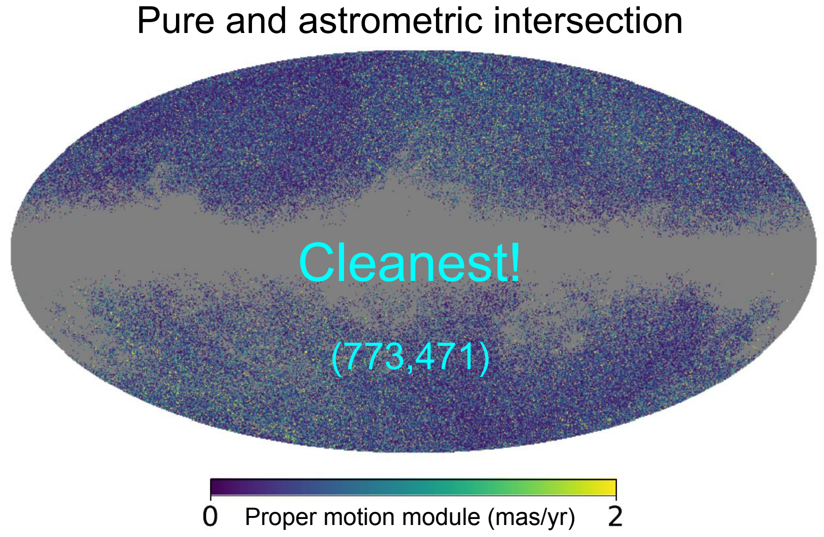

QSO candidates
0
Proper motion module (mas/yr)
2
Filtering by p_{QSO}
0
Proper motion module (mas/yr)
2
Masking
0
Proper motion module (mas/yr)
2
Masking+filtering
0
Proper motion module (mas/yr)
2


Masking+filtering
0 Proper motion module (mas/yr) 2
Pure
0 Proper motion module (mas/yr) 2
Astrometric
0 Proper motion module (mas/yr) 2
Pure and astrometric intersection
0 Proper motion module (mas/yr) 2


Masking+filtering
3rd cleanest
(871,441)
0 Proper motion module (mas/yr) 2


Pure
Magellanic clouds contamination
(816,641)
0 Proper motion module (mas/yr) 2


Astrometric
2nd cleanest
(786,165)
0 Proper motion module (mas/yr) 2


Pure and astrometric intersection
Cleanest!
(773,471)
0 Proper motion module (mas/yr) 2

Previous work (Darling, Truebenbach, Paine, arXiv:1804.06986)

- Darling et al. followed a very similar procedure to ours to get $\Omega_{GW} \lesssim 0.0064$.
- Two datasets: quasars from VLBA, combination of VLBA + Gaia DR1.
- Only 711 and 508 sources (1000 times less than our datasets).
- However, much better resolution! A factor 30-40 better than ours.
  - The combination of both makes the expected results comparable.
- Main differences with our analysis:
  - They fit the dipole and quadrupole separately → less conservative result.
  - Their code underestimates the errors in a factor ~2.
- We thus decided to reanalyse their work for a better comparison.

Results

- No evidence for detection → we provide 95% upper bound $\Omega_{GW} \lesssim 0.087$.
- Control datasets behave as expected.
  - The astrometric is similar but slightly more contaminated.
  - The masked behaves a bit worse, but still within 30%.
  - The pure one does much worse due to contamination. Still, within the order of magnitude.
- For VLBA and VLBA+Gaia datasets, more conservative results than Darling et al.
- VLBA more constraining than Gaia DR3 for now.
- Expected due to much better resolution caused by larger obs. period (22.2 years vs 2.84).

Dataset	$\sqrt{P_2}$ (μas/yr)	$h_{70}^2\Omega_{\rm GW}$	$h_{70}^2\Omega_{\rm GW}^{\rm up}$ (95%)
Masked	12.51(1.81)	0.069(0.021)	0.114
Pure	23.15(2.01)	0.235(0.040)	0.295
Astrometric	10.13(1.73)	0.045(0.017)	0.089
Intersection	9.53(1.73)	0.040(0.017)	0.087
VLBA	2.73(1.23)	0.0033(0.0056)	0.024
VLBA+Gaia DR1	5.30(1.36)	0.0123(0.0077)	0.034

Conclusions and future prospects

- Gaia DR3 (2.84 yr) → $\Omega_{GW} \lesssim 0.087$ for 4×10^{-18} Hz $\lesssim f \lesssim 1 \times 10^{-8}$ Hz.
- VLBA update → $\Omega_{GW} \lesssim 0.024$ for 6×10^{-18} Hz $\lesssim f \lesssim 1 \times 10^{-9}$ Hz.
- Gaia improves proper motion resolution like $T^{3/2}$: 2.7x and 6.6x improvement factors for DR4 (5.5 yr) and DR5 (10 yr) → 7.2x and 44x improvement for Ω_{GW}.
- Extrapolating our constraints, $\Omega_{GW} \lesssim 0.012$ (DR4, "not before mid 2026") and $\Omega_{GW} \lesssim 0.0020$ (DR5, "not before the end of 2030").
  - Conservative prediction: number of sources will likely increase.
  - For DR5, we will also have the full time series, which will help further cleaning the data.
- Proposed mission Theia with 60x better angular resolution & 100x more sources → $O(10^{-10})$!
- More modest constraints than usual methods, but different constrained frequencies.
  - Supermassive black hole binaries, cosmic strings, better characterize signal from PTAs, etc.

Thank you for your attention!

Backup: angular deflection spectrum from a SGWB

- Assuming pure Gaussian fluctuations and equal distribution of sources in the sky, it is usually assumed we can get root mean square proper motions of order

$$\Delta\theta/(T\sqrt{N})$$

so we could set constraints

$$\Omega_{\rm gw}(f) \lesssim \frac{\Delta\theta^2}{NT^2H_0^2}$$

  - $\Delta\theta$ angular resolution, T observing period, N number of sources.

- These estimations tend to be optimistic. However, good to have them in mind.
- Ω_{GW} scales like N^{-1}, while the resolution enters squared.