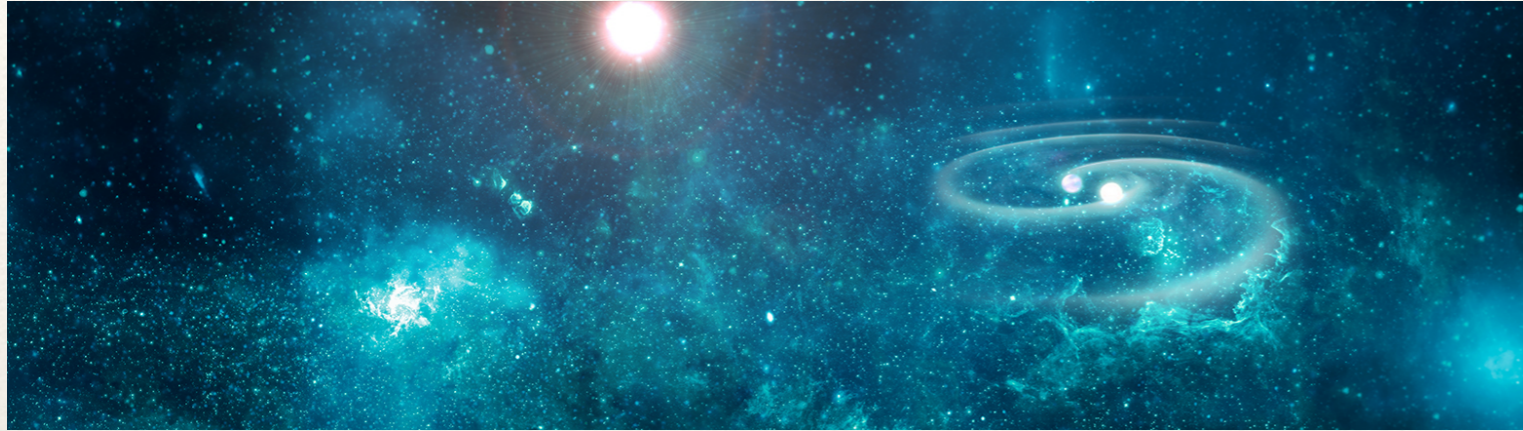


Stanislav (Stas) Babak.

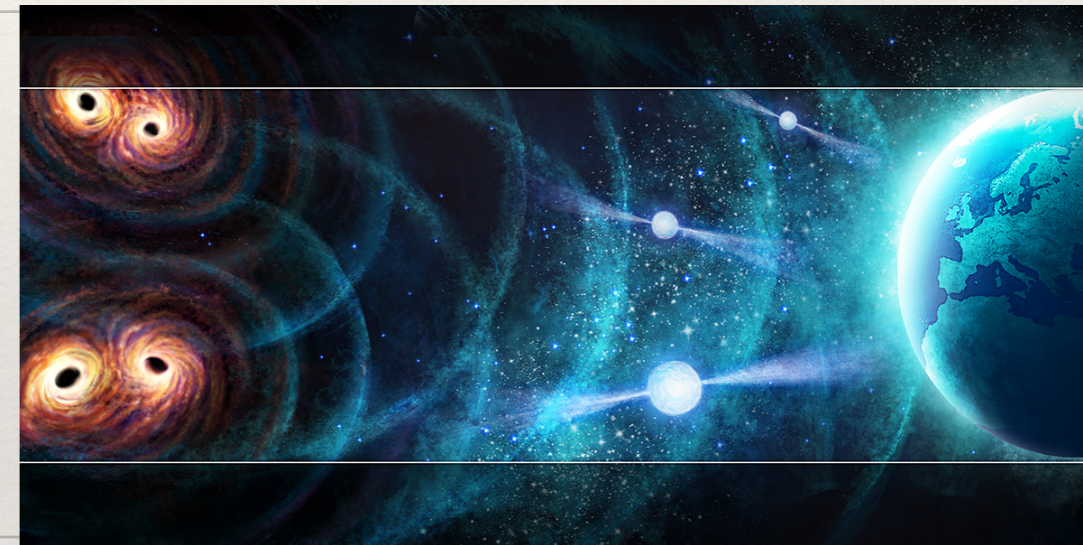
AstroParticule et Cosmologie, CNRS (Paris)



Université
Paris Cité



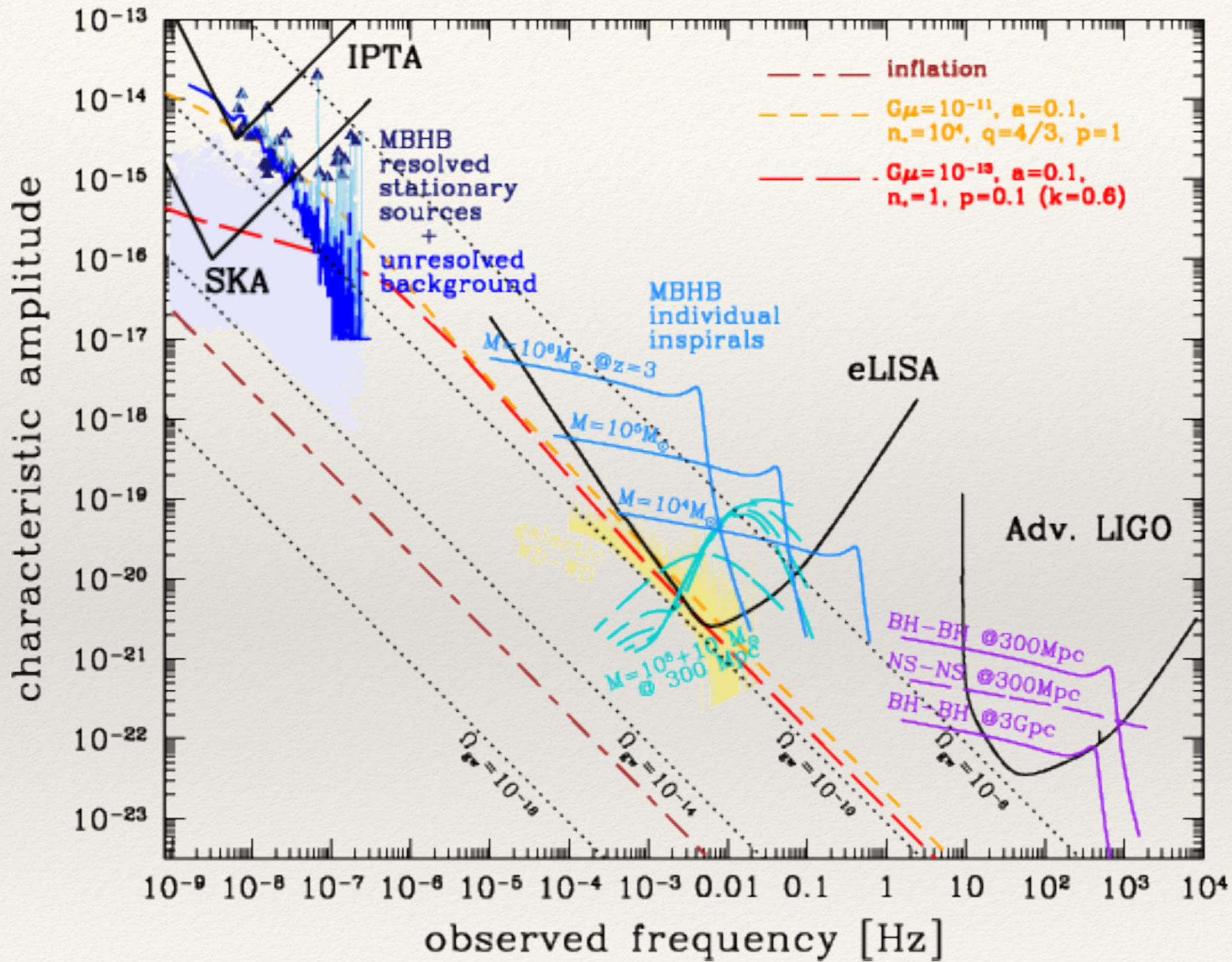
Pulsar Timing Array



September 2024



GW landscape

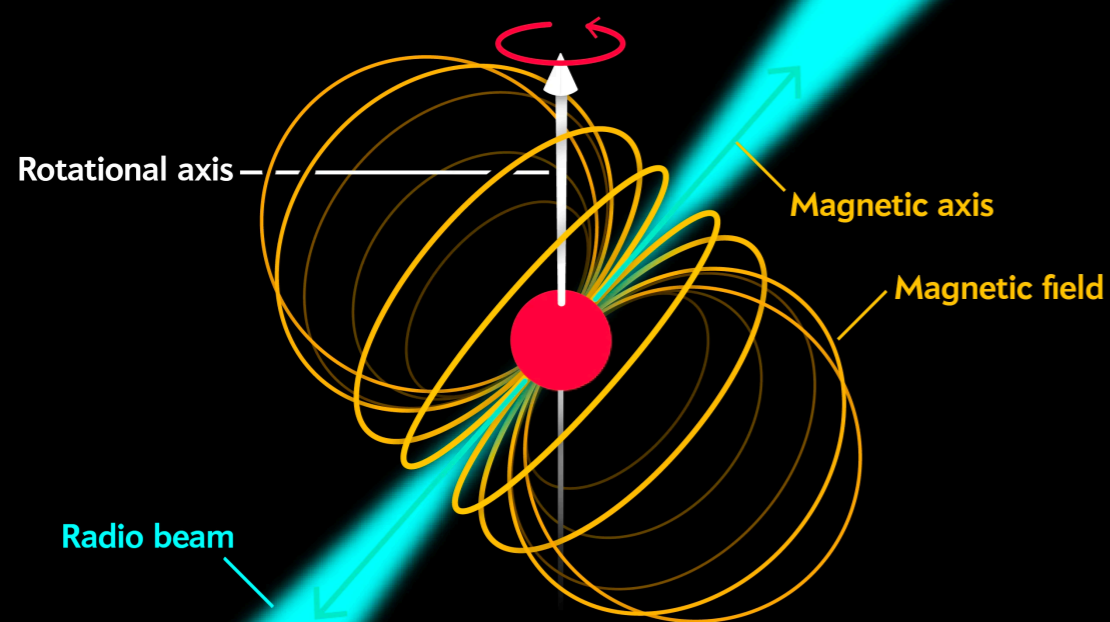
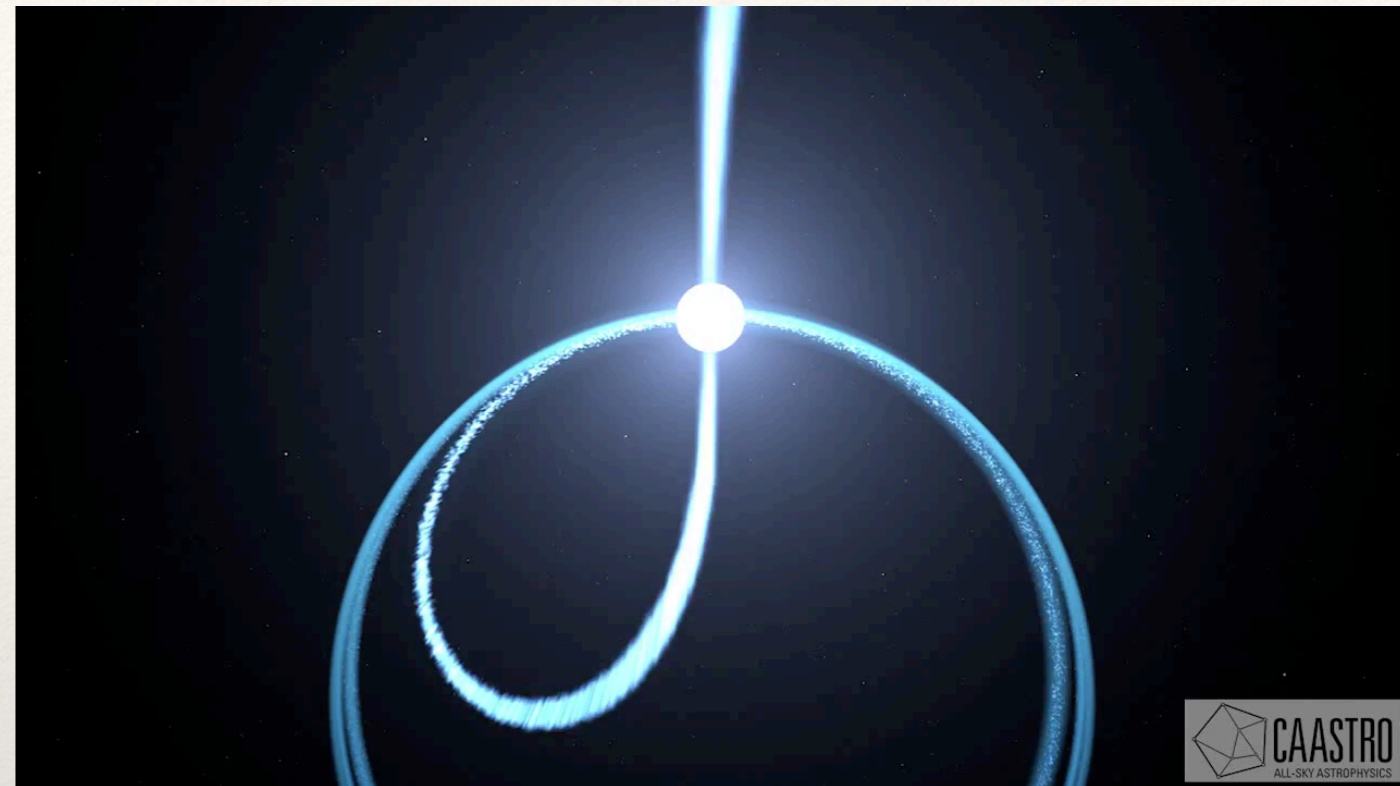


Millisecond pulsars

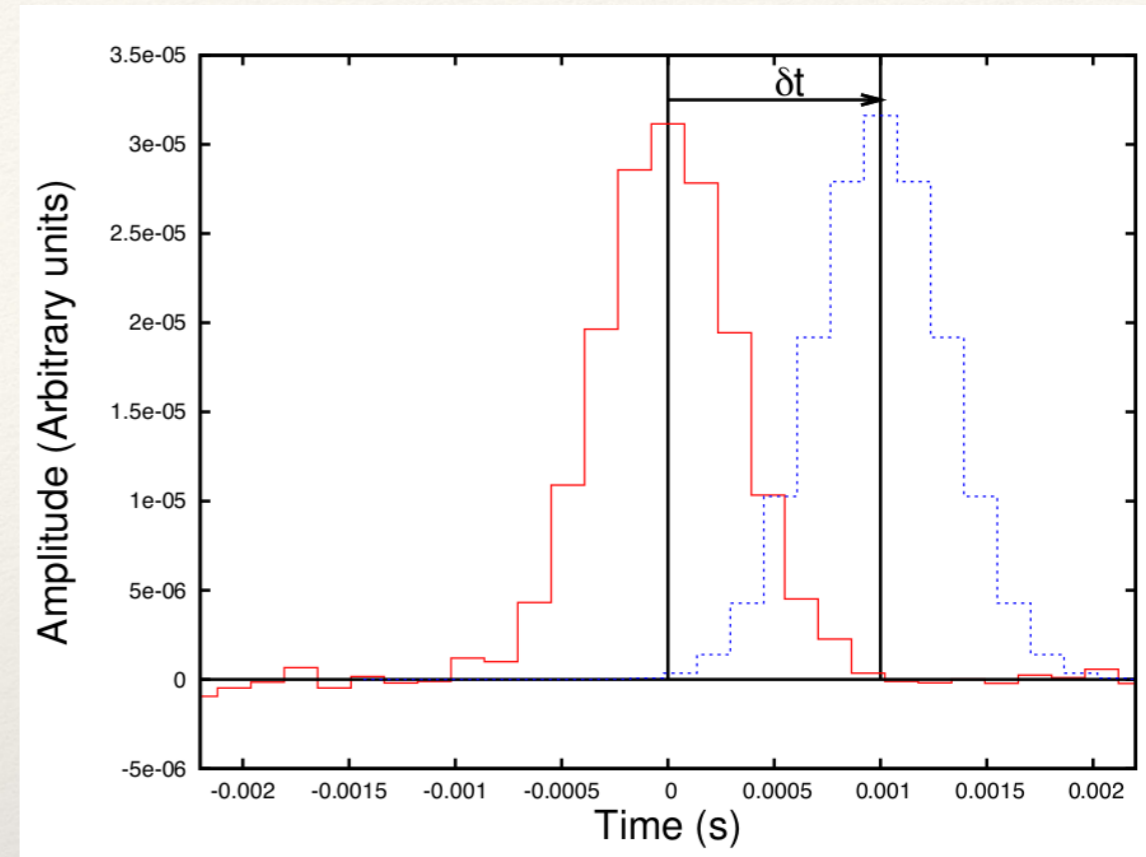
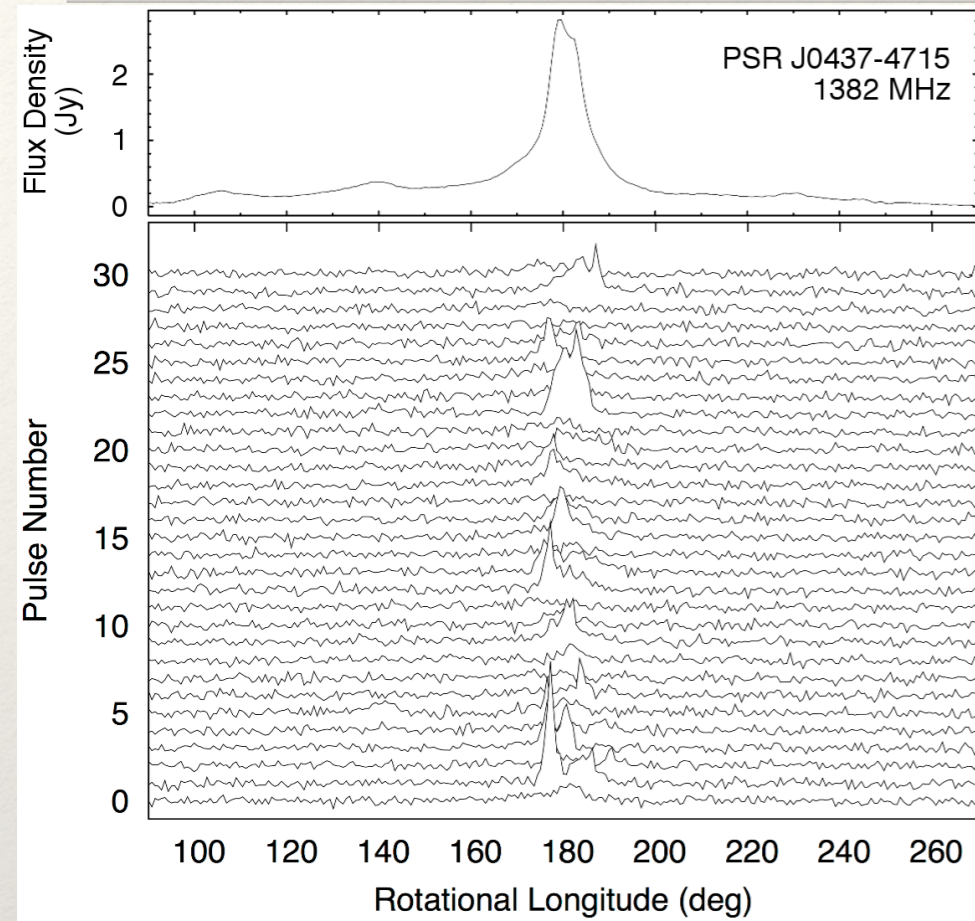


- Millisecond pulsars: period of rotation \sim millisecc
- Often in binaries
- Very old NSs, very stable rotation
- The most accurate clock on the long time scale (decades)

Jen Christiansen



Pulsar timing



[Figs: credits
S. Burke-Spolar & L. Lentati]

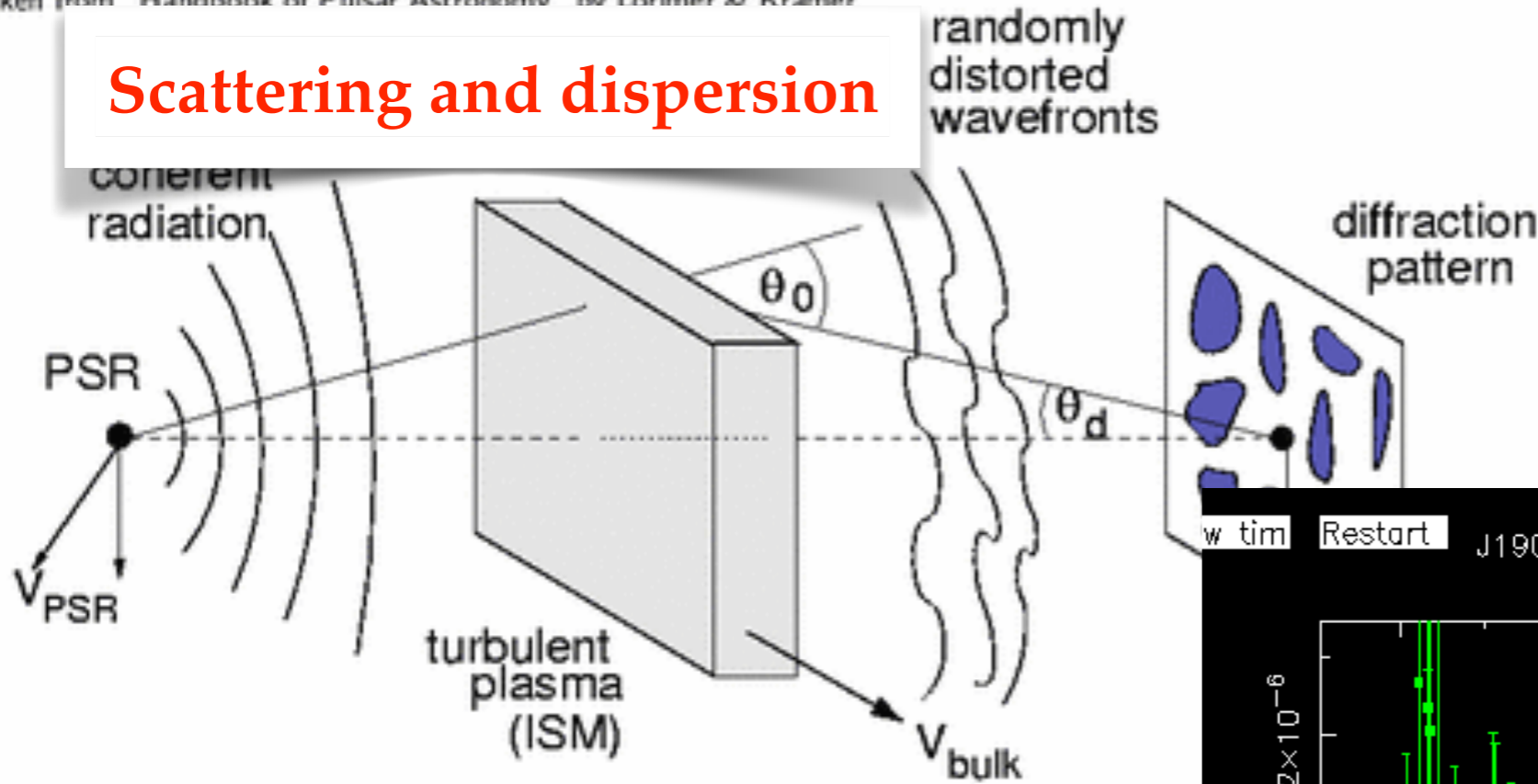
- Each observed radio pulse profile has a lot of micro-structure. If we average over ~hour the (average) profile is very stable
- We can use the average pulse profile to estimate the time-of-arrival (TOA) of the pulses.
- The idea is to measure the TOA, and compare to the expected TOA. We know the spin of the pulsars, so we can predict the TOA. The difference between measure and expected TOA: *residuals*



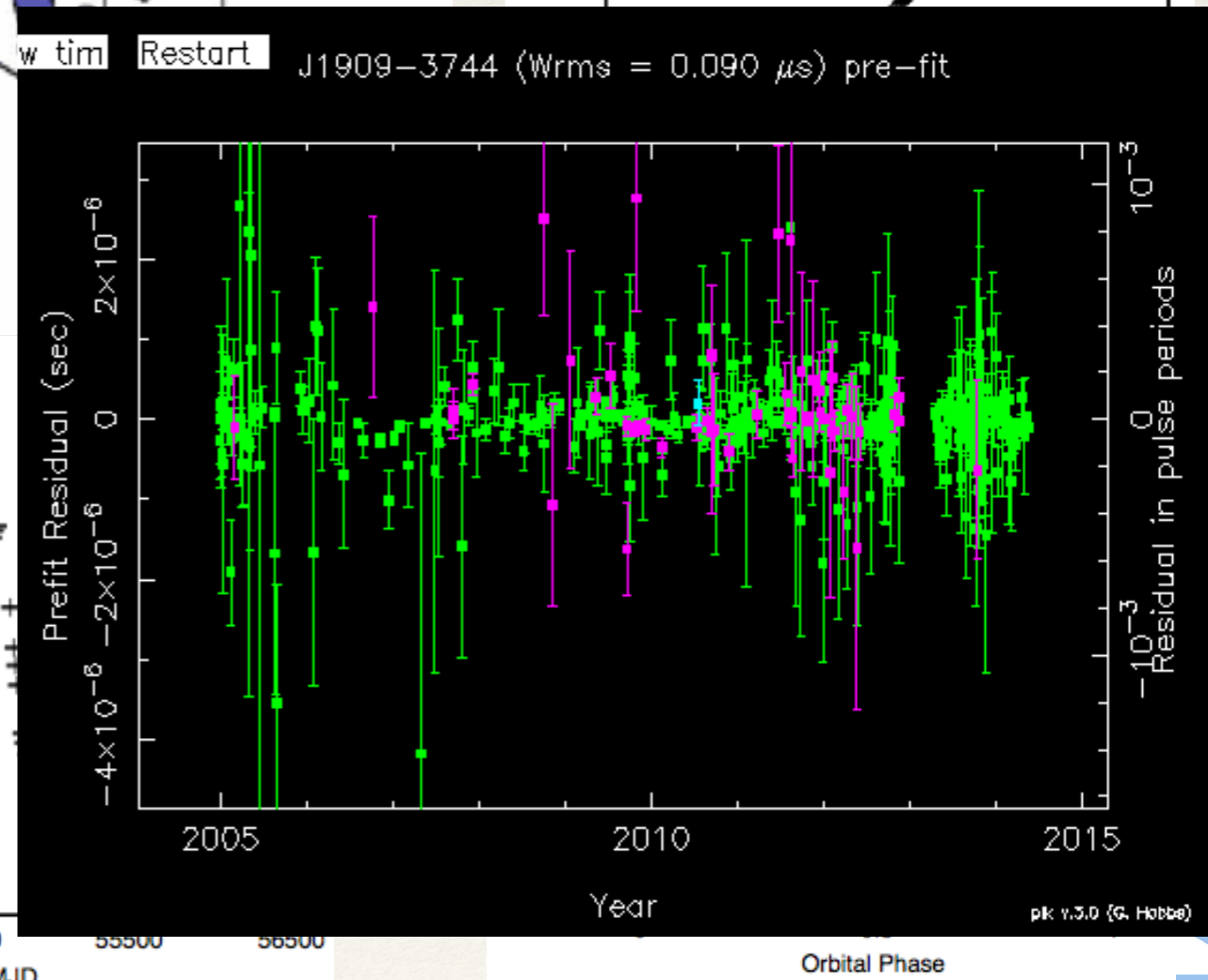
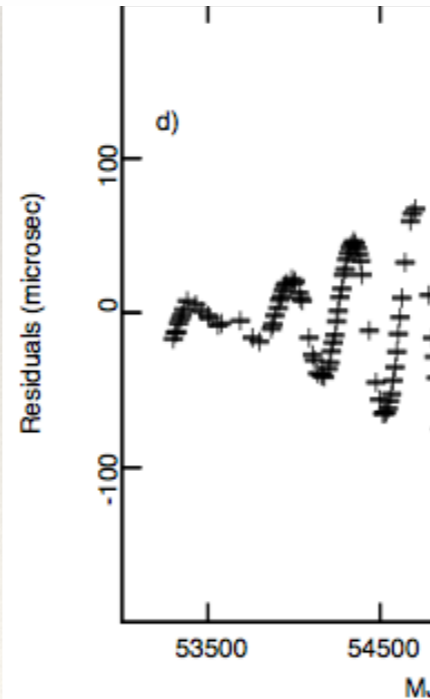
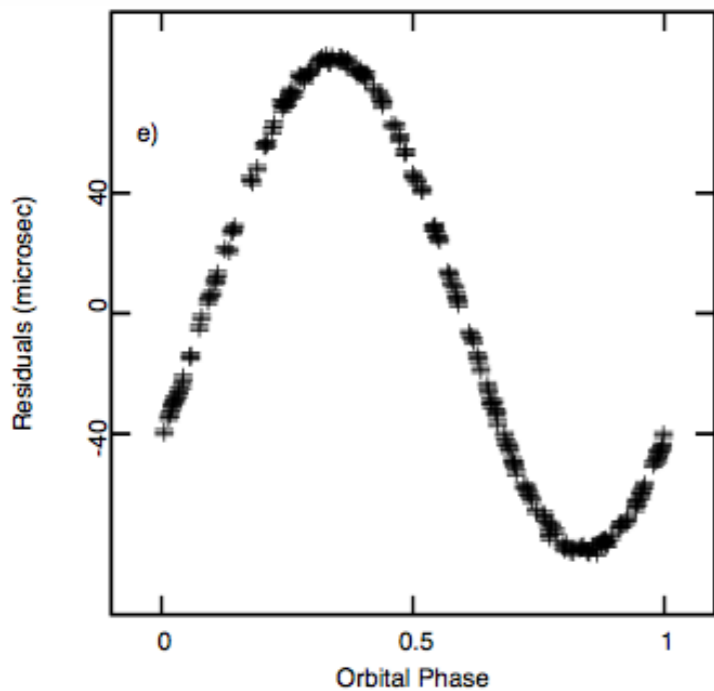
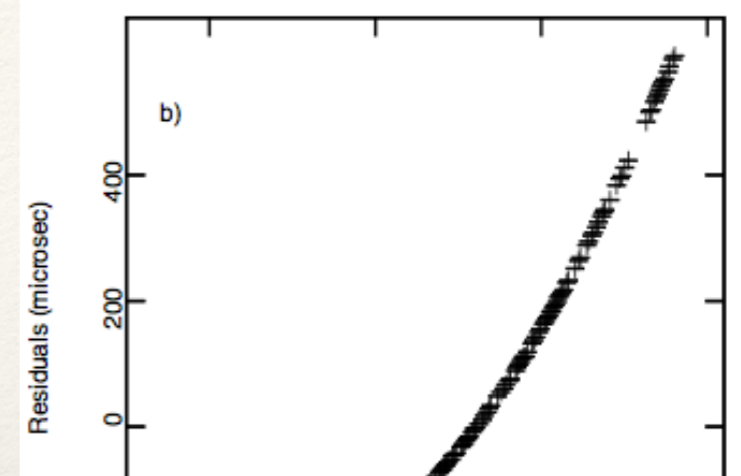
Predicting arrival time

Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Scattering and dispersion



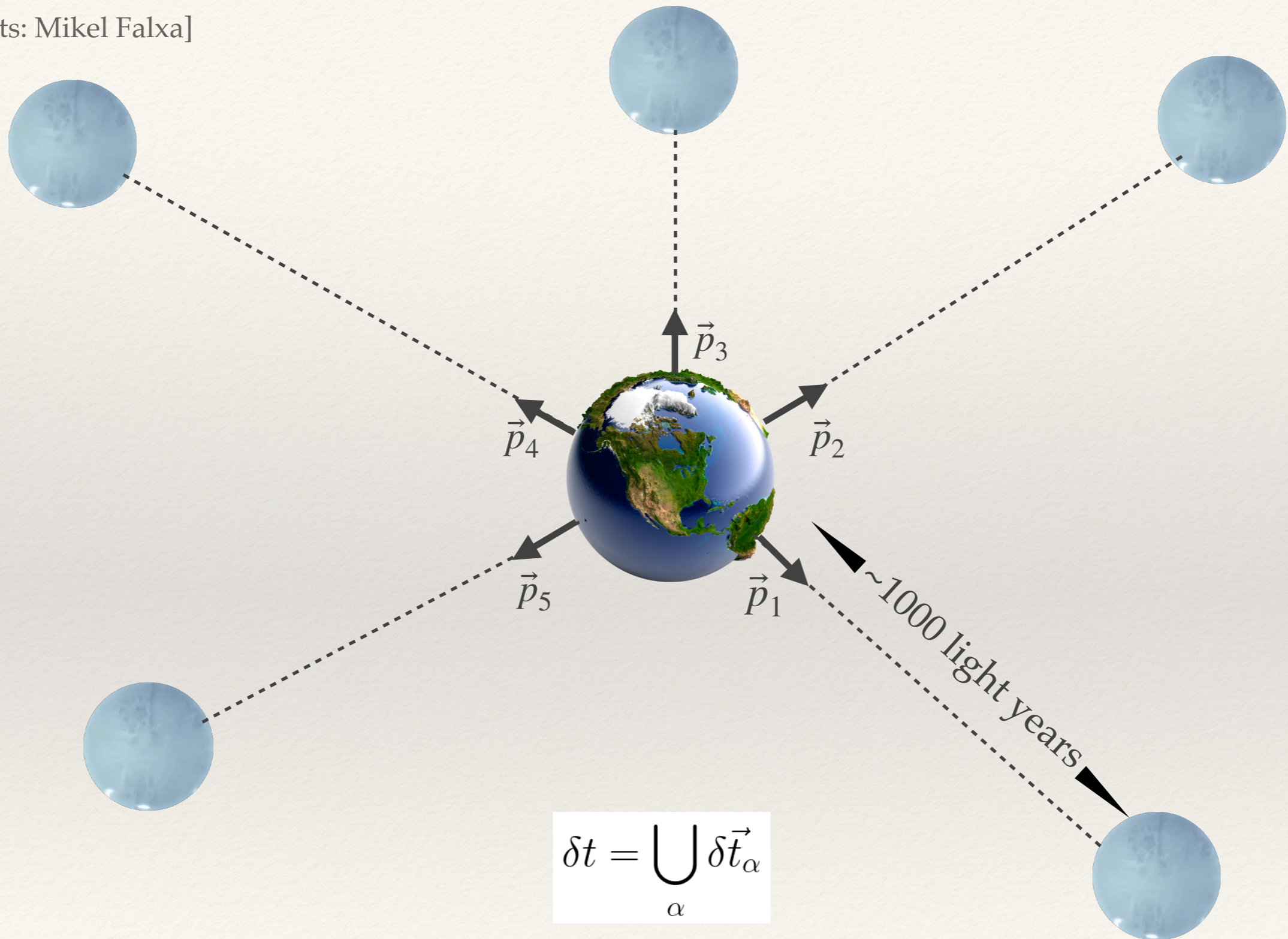
rate of change of spin



Pulsar Timing Array



[credits: Mikel Falxa]



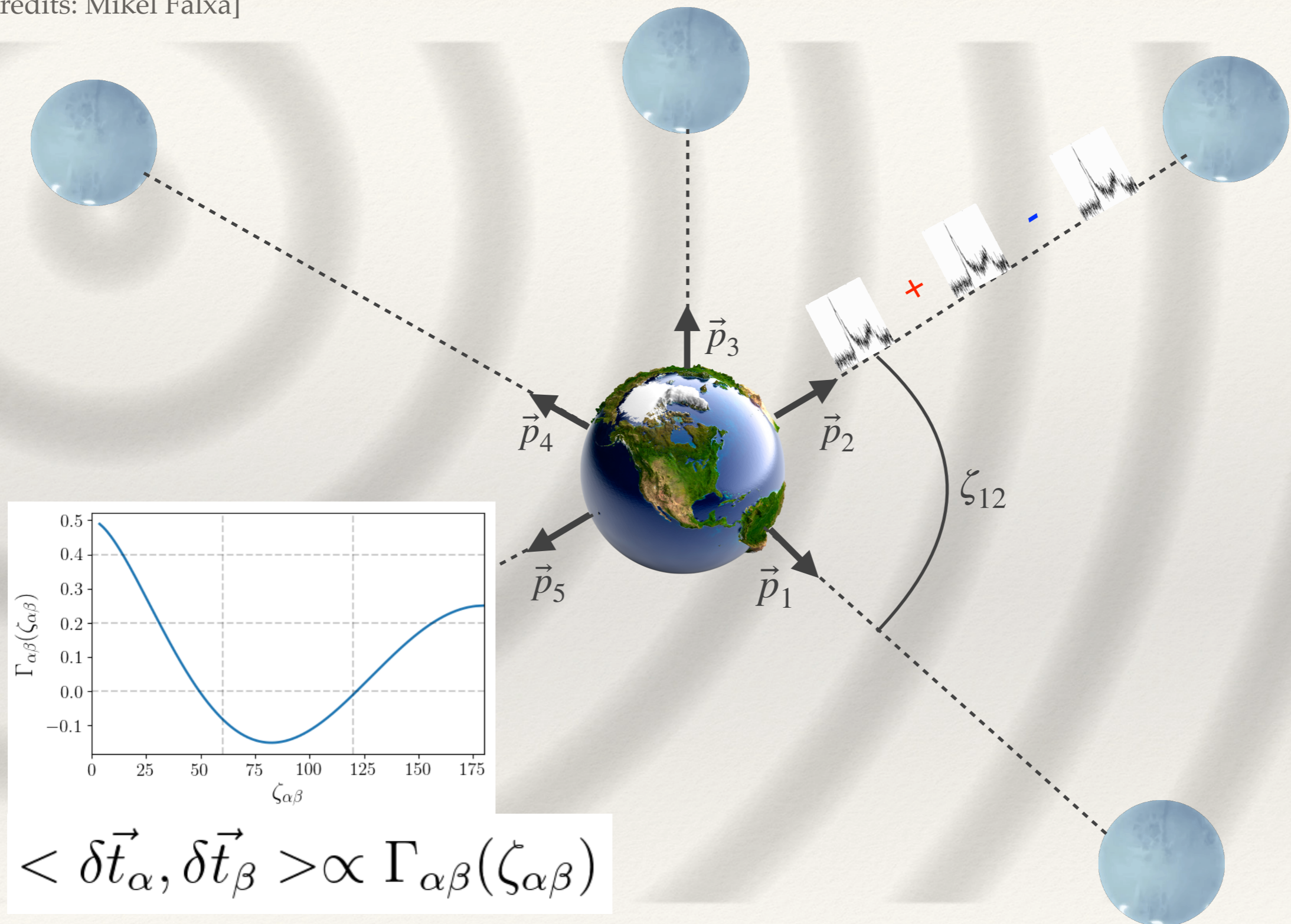
$$\delta t = \bigcup_{\alpha} \delta \vec{t}_{\alpha}$$



Pulsar Timing Array



[credits: Mikel Falxa]

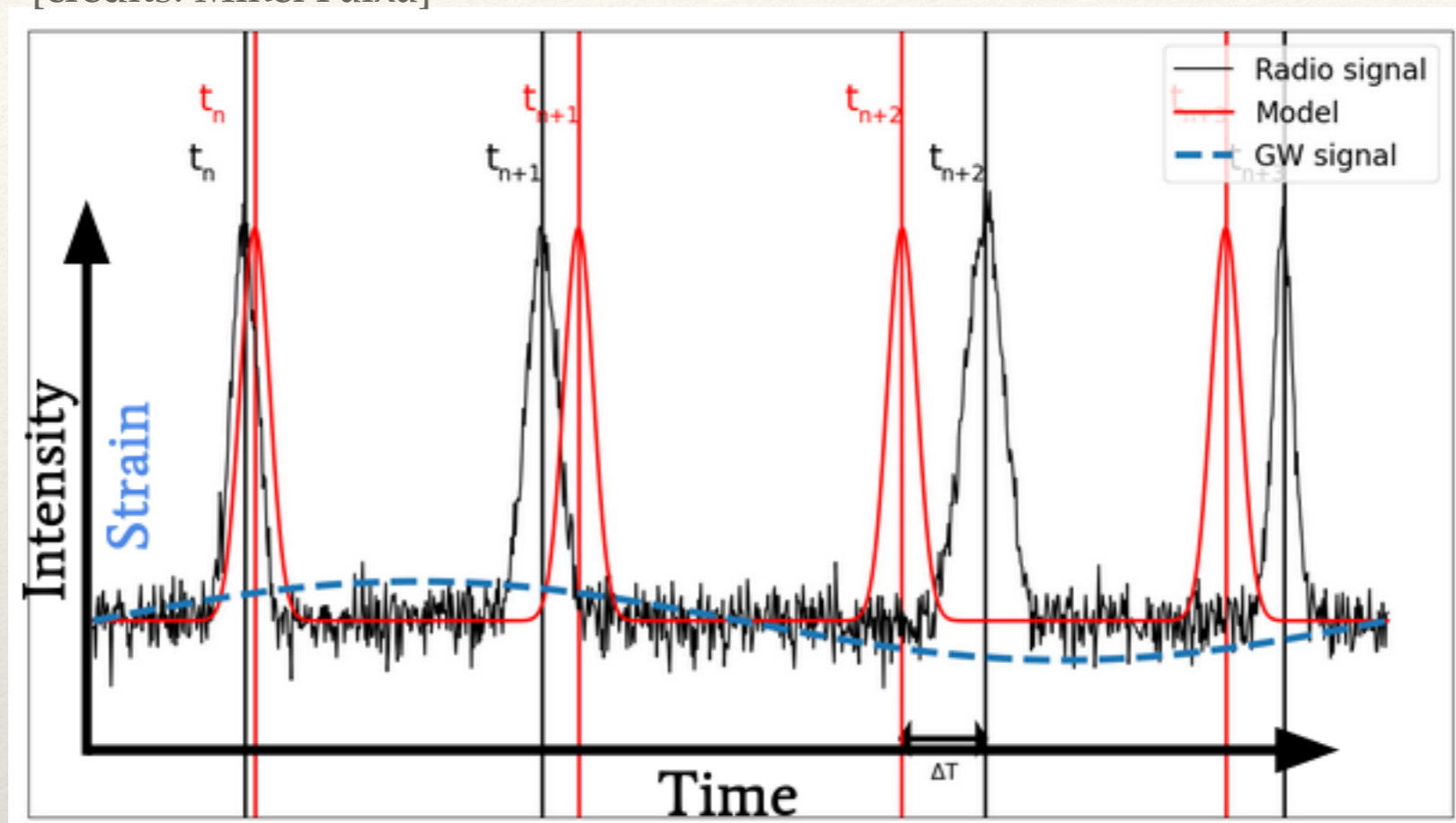


$$\langle \delta \vec{t}_\alpha, \delta \vec{t}_\beta \rangle \propto \Gamma_{\alpha\beta}(\zeta_{\alpha\beta})$$



Timing Residuals

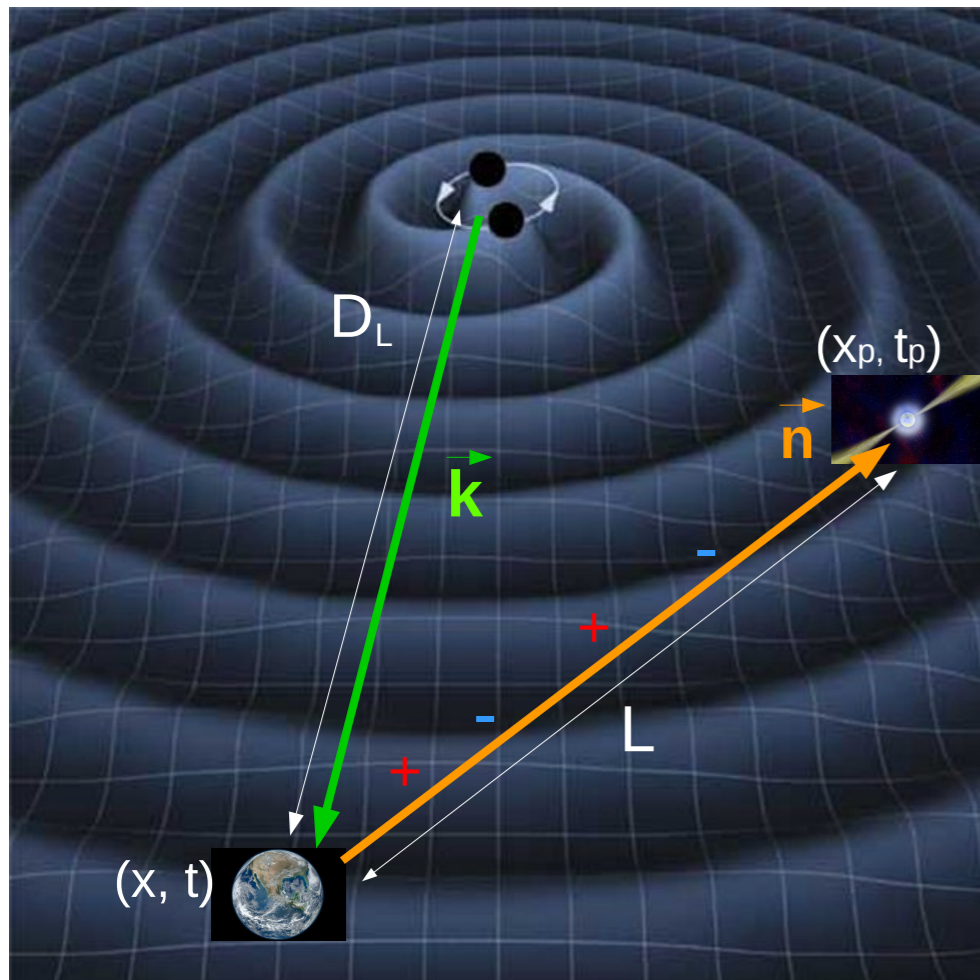
[credits: Mikel Falxa]



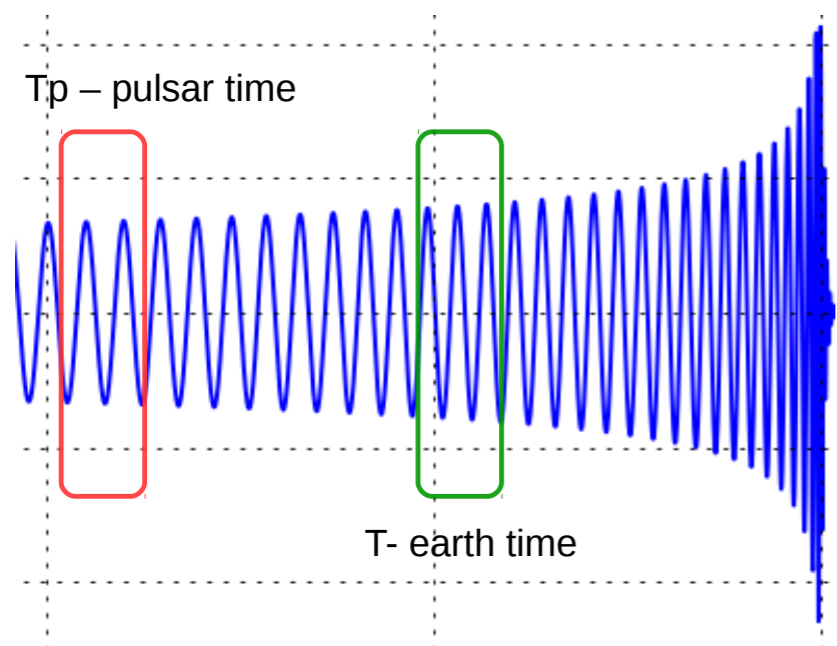
$$dt = t_{toa}^p - t_{toa}^o = dt_{errors} + \delta\tau_{GW} + noise$$

Errors in fitting the model \nearrow \nwarrow due to GWs

Response to GW signal



- PTA can be seen as a multi-arm detector where e/m signal travels only in one direction (from a pulsar to the Earth). Pulsar plays role of an accurate clock, and we measure change in phase (frequency) of arriving pulses (similar to the frequency (phase) of the laser light)
- Important quantity which characterizes the response of any GW observatory is $\epsilon = (2\pi f_* L/c)$
↙ size of GW detector



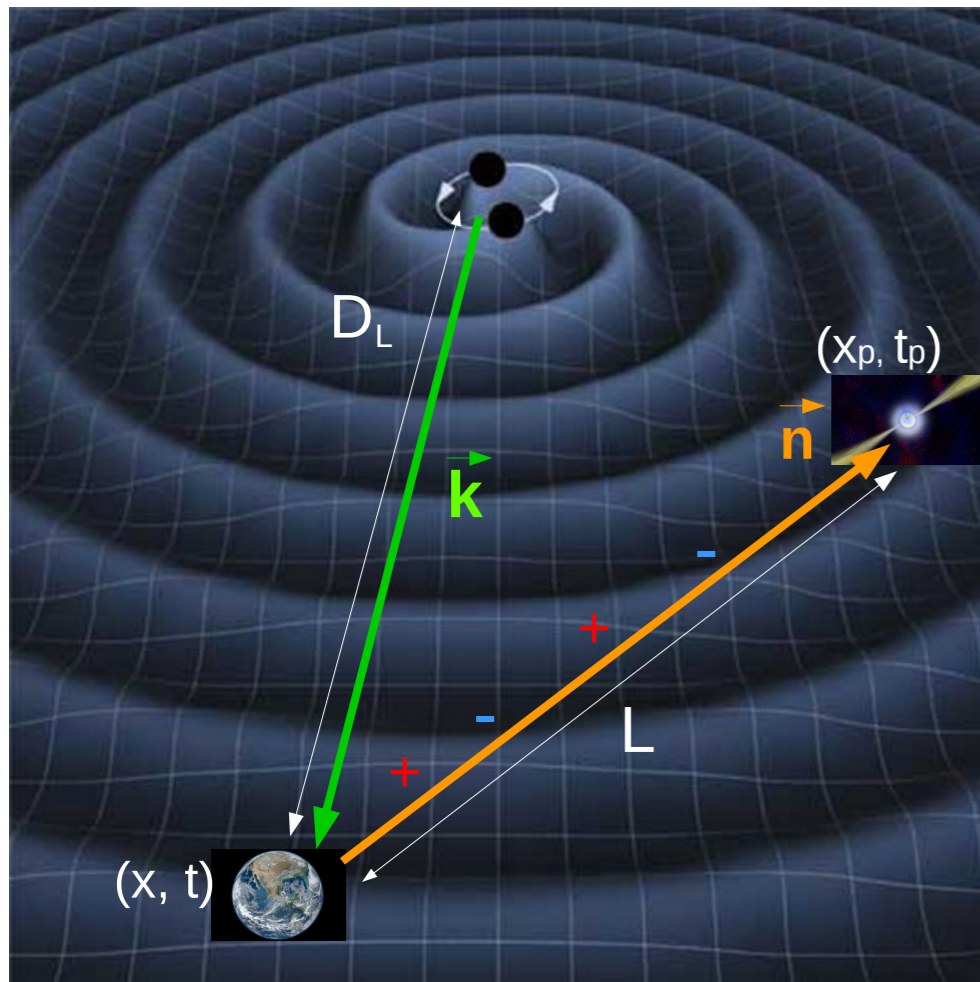
$$\epsilon \ll 1 \rightarrow R \propto h_{ij} n^i n^j \quad \text{long wavelength approximation: LIGO/Virgo}$$

$$\epsilon = 1 \rightarrow \text{LIGO: } f^* \sim 12 \text{ kHz, LISA: } f^* \sim 0.05 \text{ Hz, PTA: } f^* \sim 0.002 \text{ nHz}$$

$$\text{PTA: } \epsilon \gg 1$$



Response to GW signal



$$dt = t_{toa}^p - t_{toa}^o = dt_{errors} + \delta\tau_{GW} + noise$$

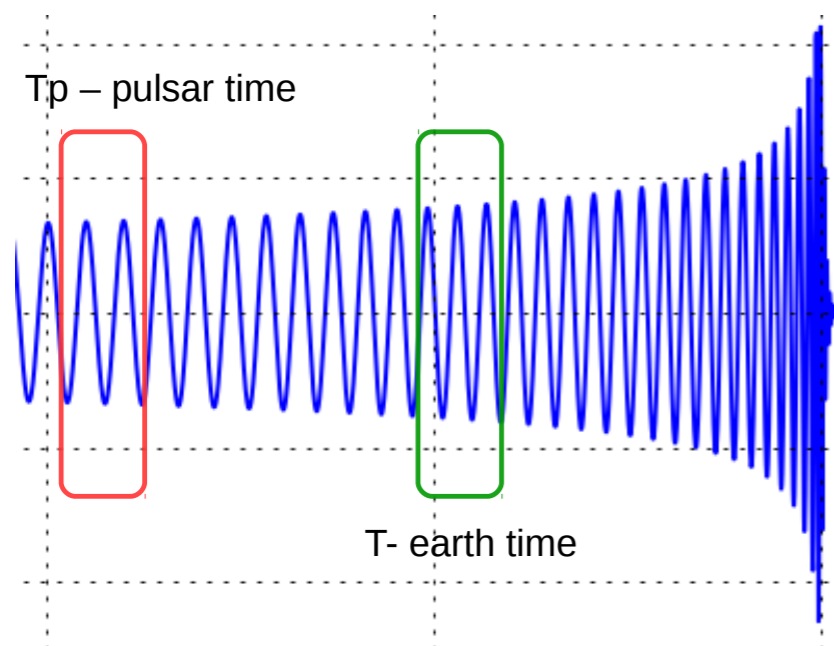
$$\delta\tau_{GW} = r(t) = \int_0^t \frac{\delta\nu}{\nu_0}(t') dt'; \quad \frac{\delta\nu}{\nu_0} = \frac{1}{2} \frac{\hat{n}^i \hat{n}^j \Delta h_{ij}}{1 + \hat{n} \cdot \hat{k}}$$

Familiar from LISA

$$\Delta h_{ij} = h_{ij}(t_p = t - L(1 + \hat{n} \cdot \hat{k})) - h_{ij}(t)$$

t_p — pulsar time, ~ time of emission of the radio pulse:

- depends on the relative position of a pulsar and GW source
- depends on the distance to the pulsar L
- $L \sim$ few kpc $\sim 10^4$ years — “pulsar” term $h(t_p)$ contains info about the system 10^5 years in the past as compared to the “earth” term
- pulsar term depends on the pulsar.



Detection statistic and search algorithm



- We assume that noise is Gaussian: the likelihood function (likelihood of the signal with given parameters) is

$$P(\vec{\delta t}, \vec{\theta}) = \frac{1}{\sqrt{(2\pi)^n \det(C)}} \exp\left(-\frac{1}{2}(\vec{\delta t} - \vec{s})^T C^{-1}(\vec{\delta t} - \vec{s})\right),$$

- $\vec{\delta t}$ - concatenated residuals from all pulsars in the array: total size n
- \vec{s} - is a model of deterministic signals (for example GW signals from individually resolvable SMBHBs)
- C is the noise variance-covariance matrix (size $n \times n$)

$$C_{\alpha i, \beta j} = C^{\text{wn}} \delta_{\alpha\beta} \delta_{ij} + C_{ij}^{\text{rn}} \delta_{\alpha\beta} + C_{ij}^{\text{dm}} \delta_{\alpha\beta} + C_{\alpha i, \beta j}^{\text{GW}} + \dots$$

white measurement noise	red noise spin noise	dispersion variation noise	stochastic GW signal
-------------------------------	----------------------------	----------------------------------	-------------------------



Noise modelling in PTA



- White noise — not very interesting. Two parameters per backend per pulsar: unaccounted noise.
- Red noise: very generic noise description in freq. domain

$$S(f) = A_{rn}^2 f^{-\gamma}$$

common, uncorrelated
red noise

$$S_{\alpha}(f) = A_{rn,\alpha}^2 f^{-\gamma_{\alpha}}$$

red noise in each pulsar

- DM (dispersion measurement variation) noise: depends on the radio-frequency of observation

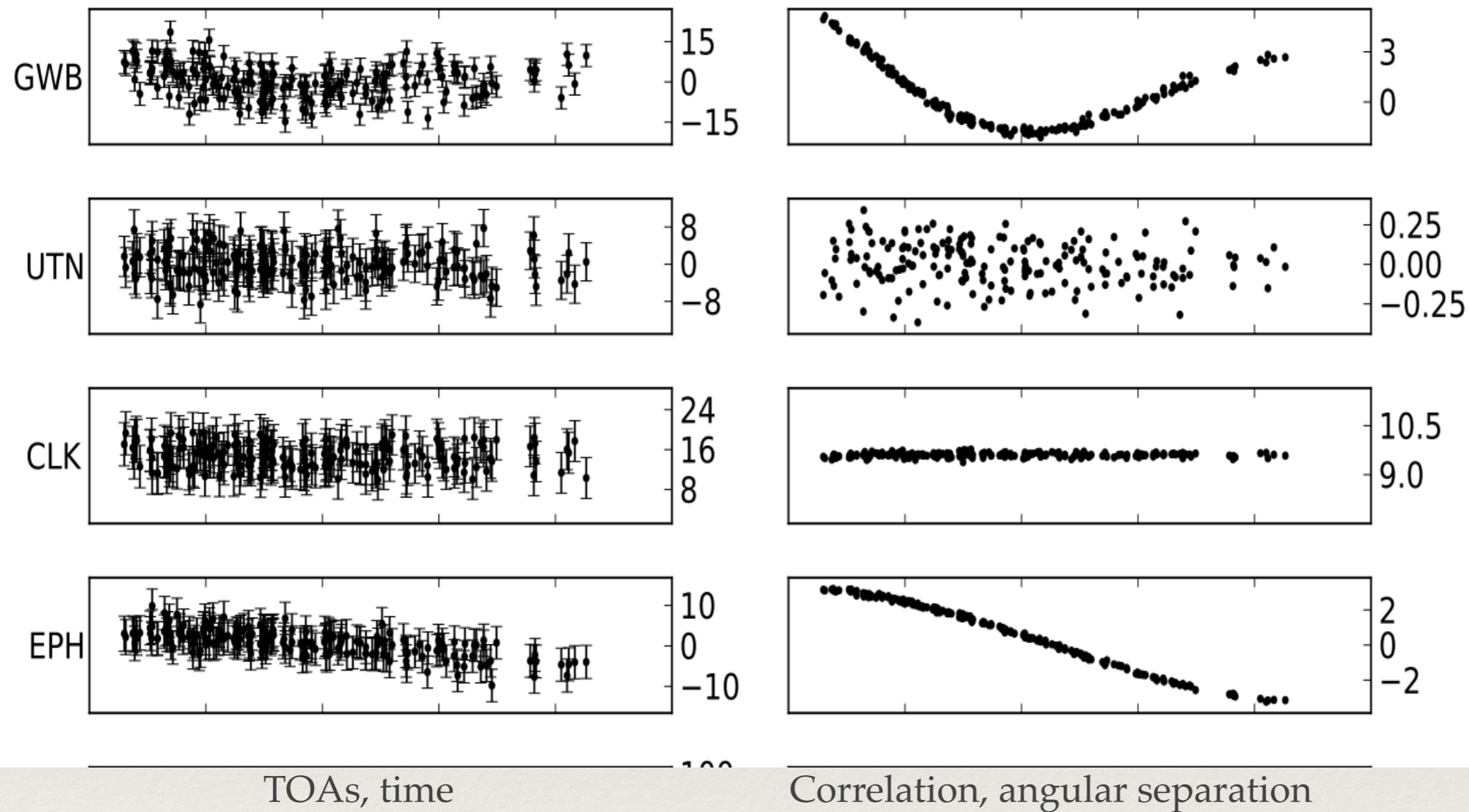
$$S_{DM}(f) \propto \frac{A_{dm}^2}{\nu^2} f^{-\gamma_{dm}}$$

- Correlated red noise processes

$$S_{\alpha\beta} = \Gamma_{\alpha\beta} A_{cor}^2 f^{-\gamma_{cor}} \quad \text{— includes also cross spectrum between each pair of pulsars: } \Gamma_{\alpha\beta} \text{ - spacial correlation coefficients}$$



Correlated noise



GWB (Quadrupole)
Hellings-Downs

Uncorrelated RN

Clock error (monopole)

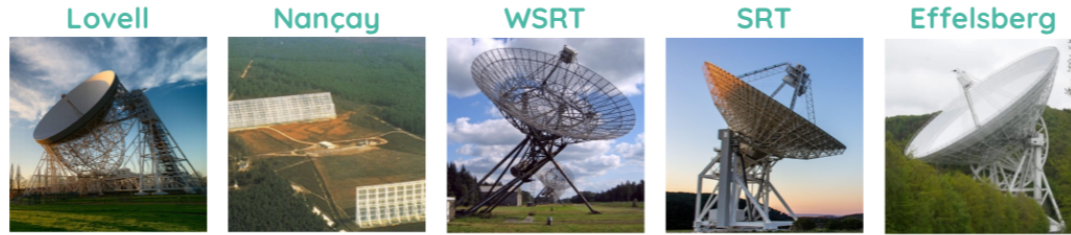
Ephemeris (dipole)

stochastic GW from population of SMBHBs:

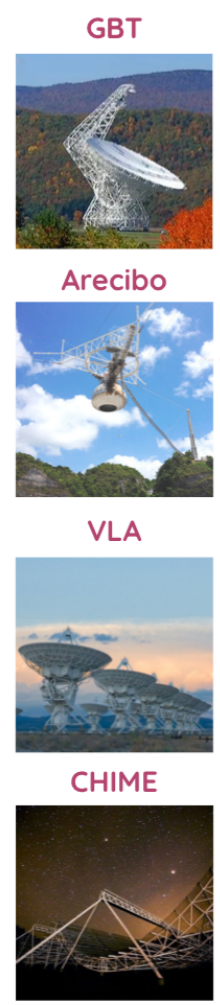
$$S_{\alpha\beta}^{SMBHB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-13/3}$$



IPTA



EPTA



NANOGrav

IPTA

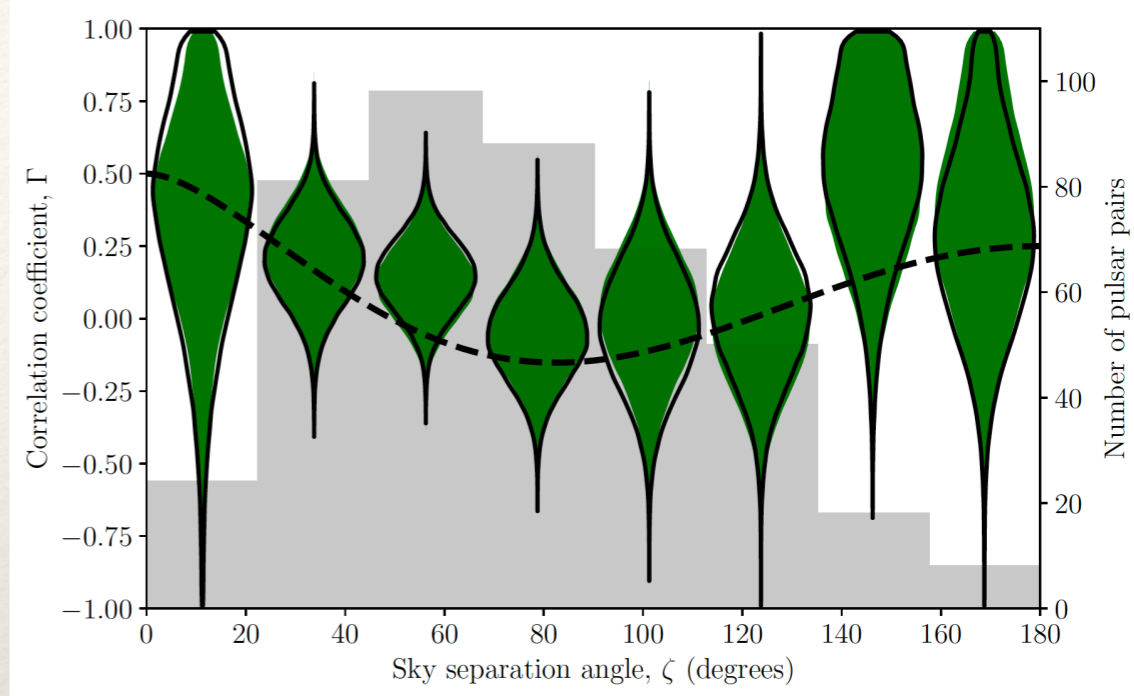
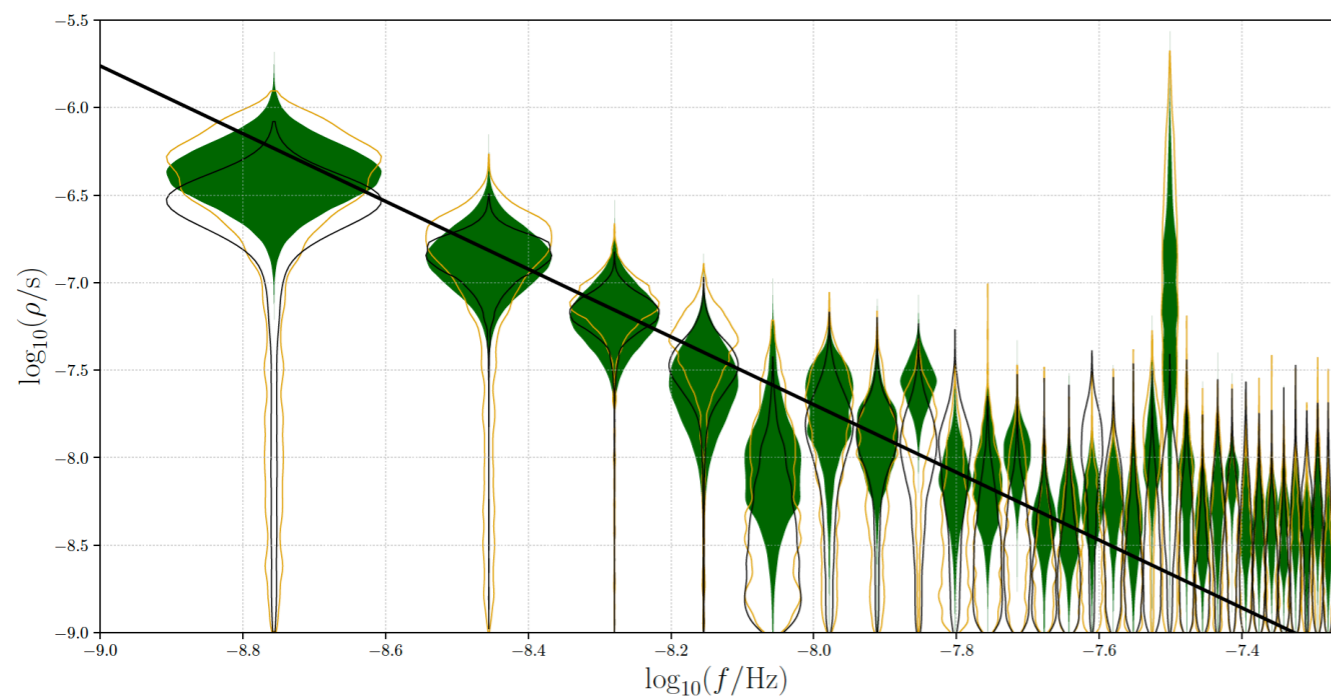


PPTA results

[PPTA 2306.16215]

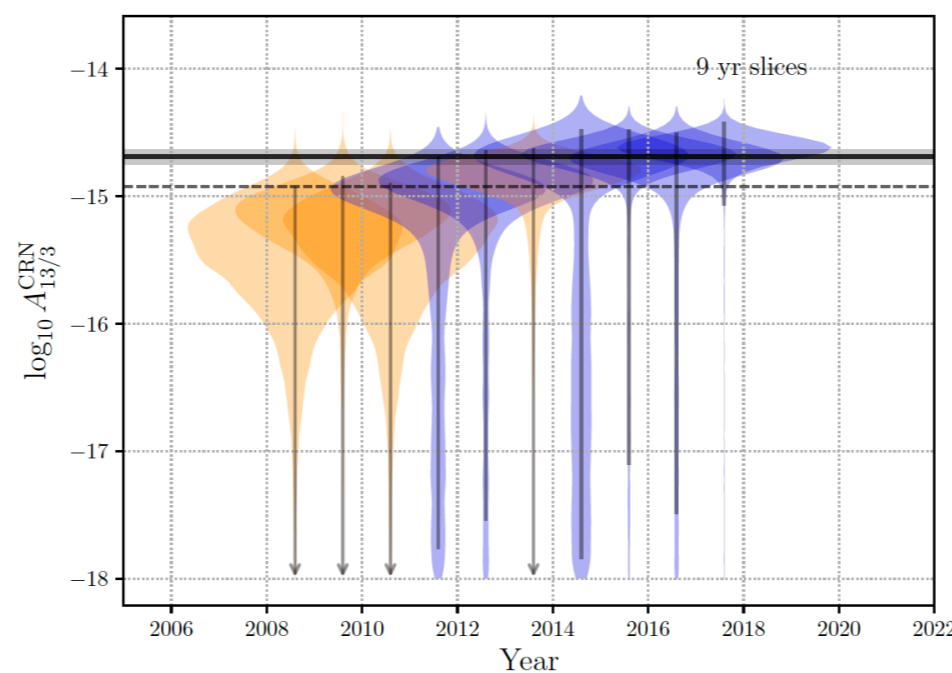
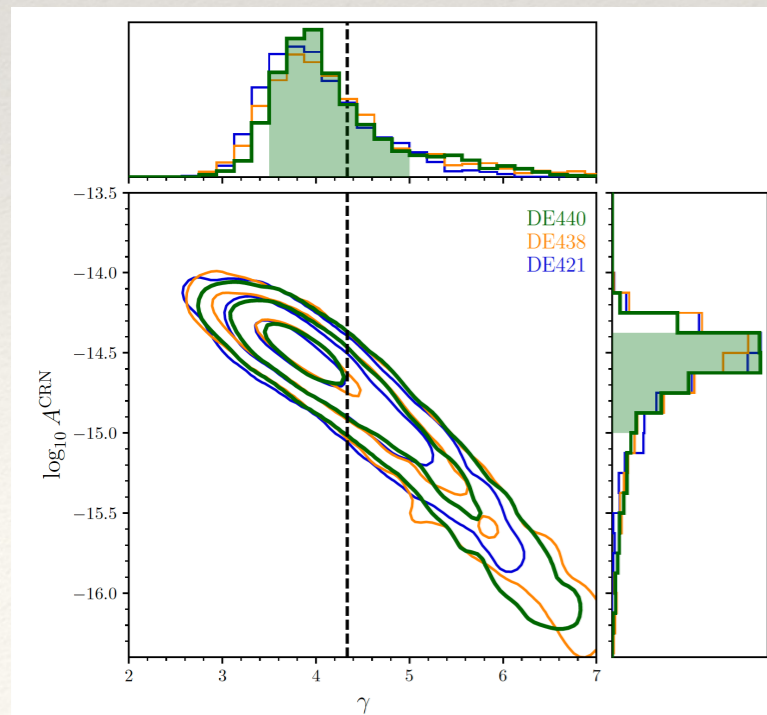


PPTA data: 18 years, 30 pulsars. 3 years of new ultra-widebandwidth radio observations



Estimating power at Fourier freq. (assuming independence).

Black: CURN, Gold: H-D



$$S_{\alpha\beta}^{SGWB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-\gamma}$$

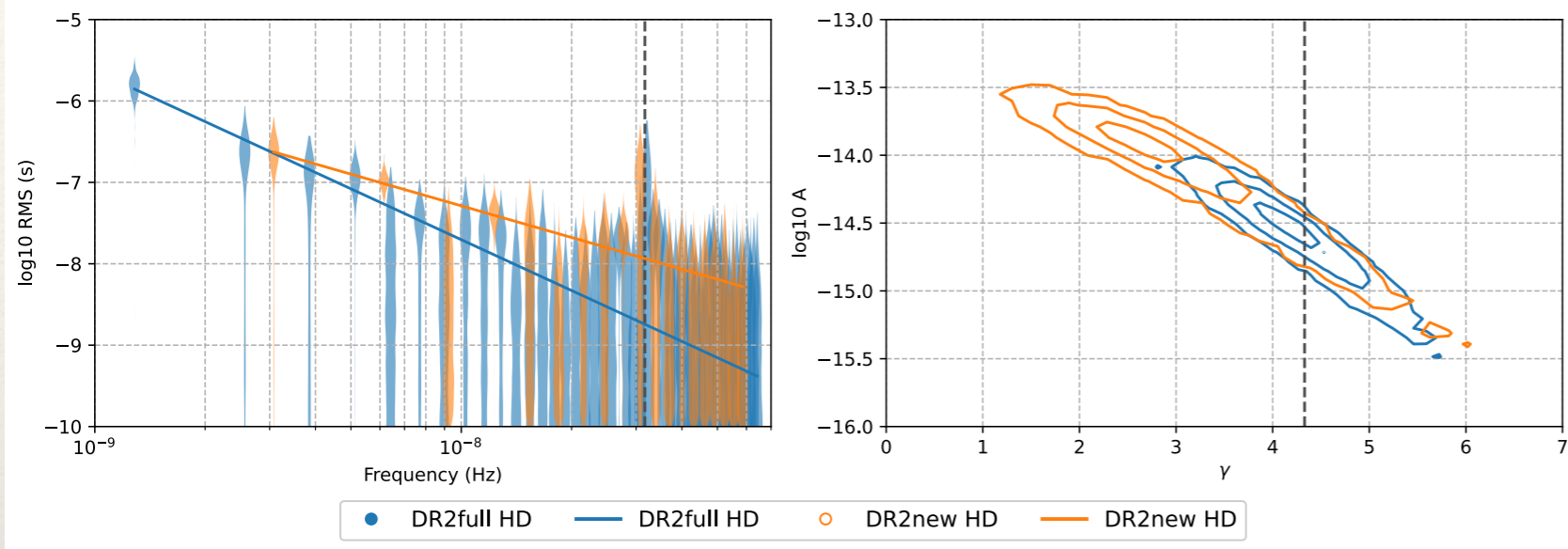


EPTA + InPTA



[EPTA+InPTA2306.16214]

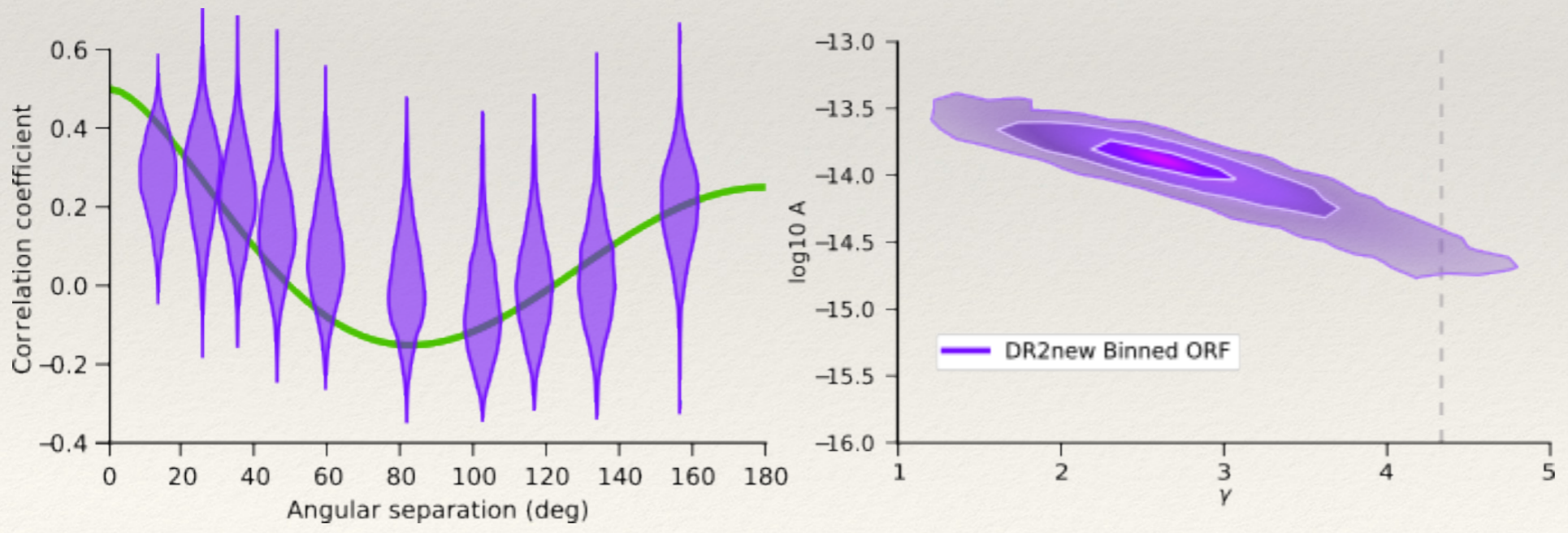
25 plsr, DR2full: up to 25yrs, DR2new: latest 14 yrs, DR2new+ Includes InPTA data (3.5 yrs)



$$S_{\alpha\beta}^{SGWB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-\gamma}$$

Statistic	DR2full	DR2new	DRnew+
\mathcal{B}_{CURN}^{HD}	4	60 33(+Ephem)	65 43(+Ephem)

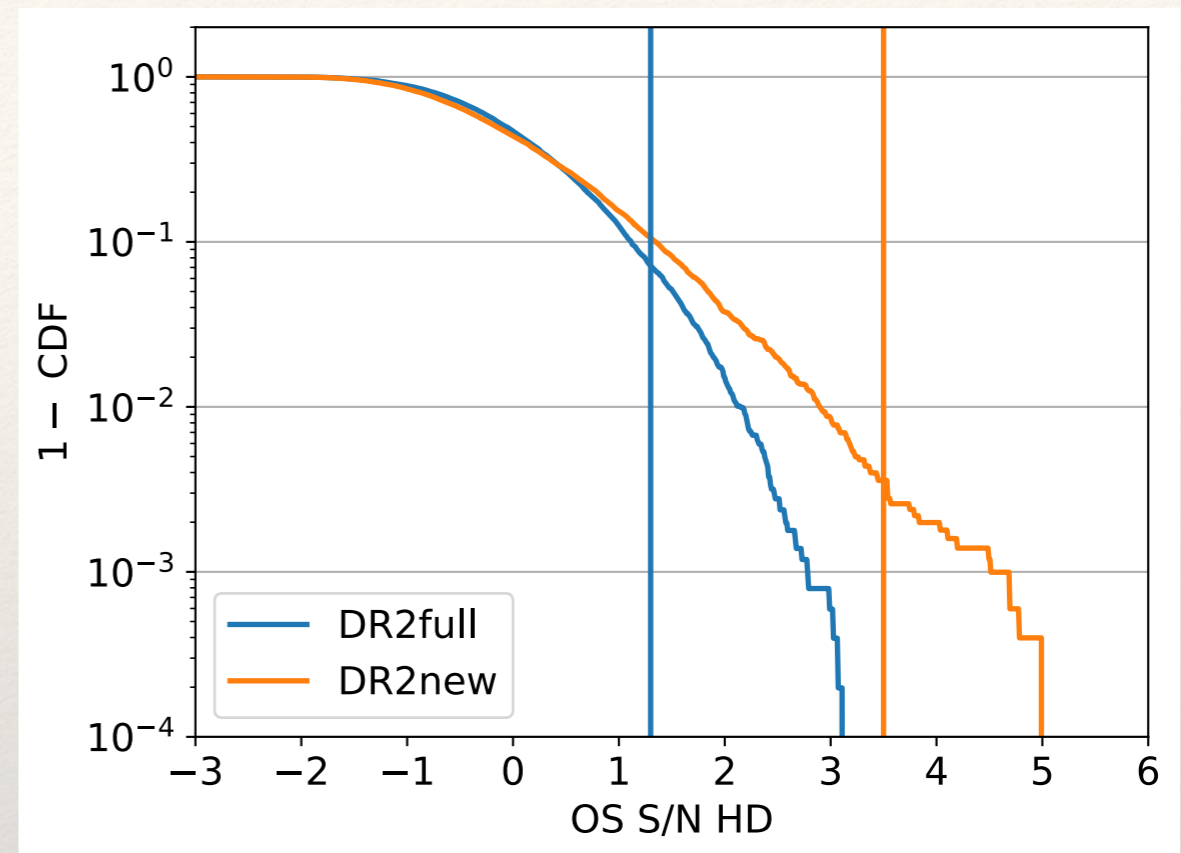
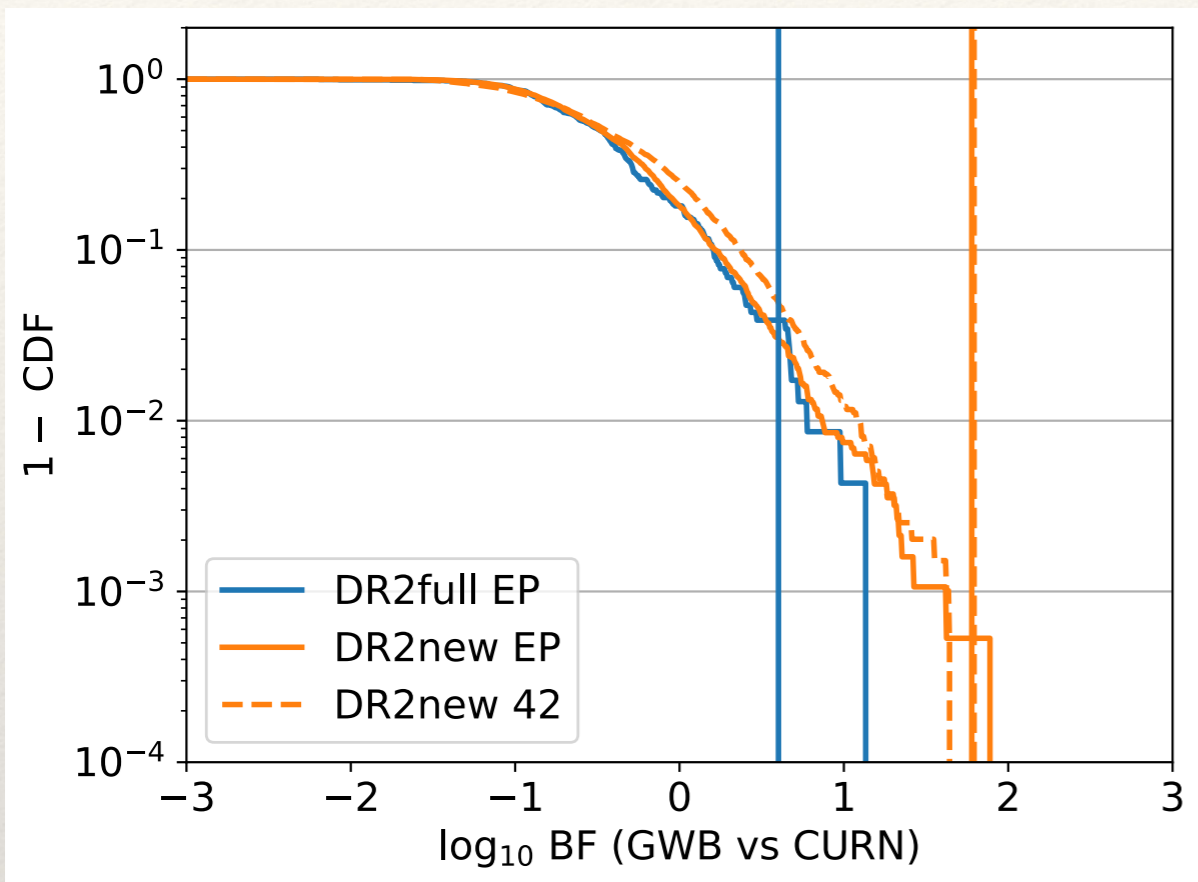
DR2new results: spatial correlations and amplitude-slope of power-law model



EPTA + InPTA



Significance: how likely to observe what we observe in absence (null hypothesis) of GW signal



We want [Cornish & Sampson 2015, Taylor+ 2016]

- Preserve properties of the noise (use observations)
- Data free of GW signal: not possible, instead we try to mimick measurements insensitive to GWs
 - Sky shuffling: change position of pulsars: observed correlation is not consistent with GW
 - Phase shift: introduce a random shift in phase at each frequency bin: destroy correlations

The question we are asking:

- how likely to get observed H-D pattern by randomly choosing pulsars on the sky
- How likely that the phases at low frequencies in all pulsars align to form observed H-D

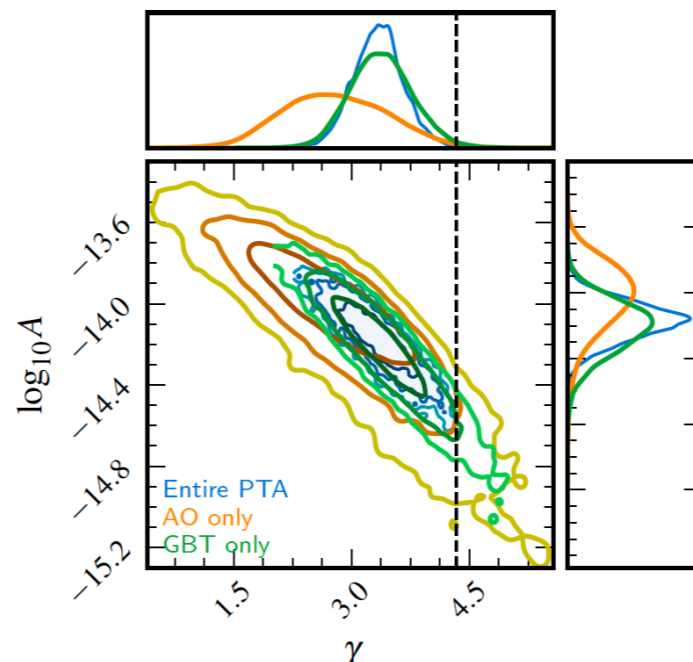
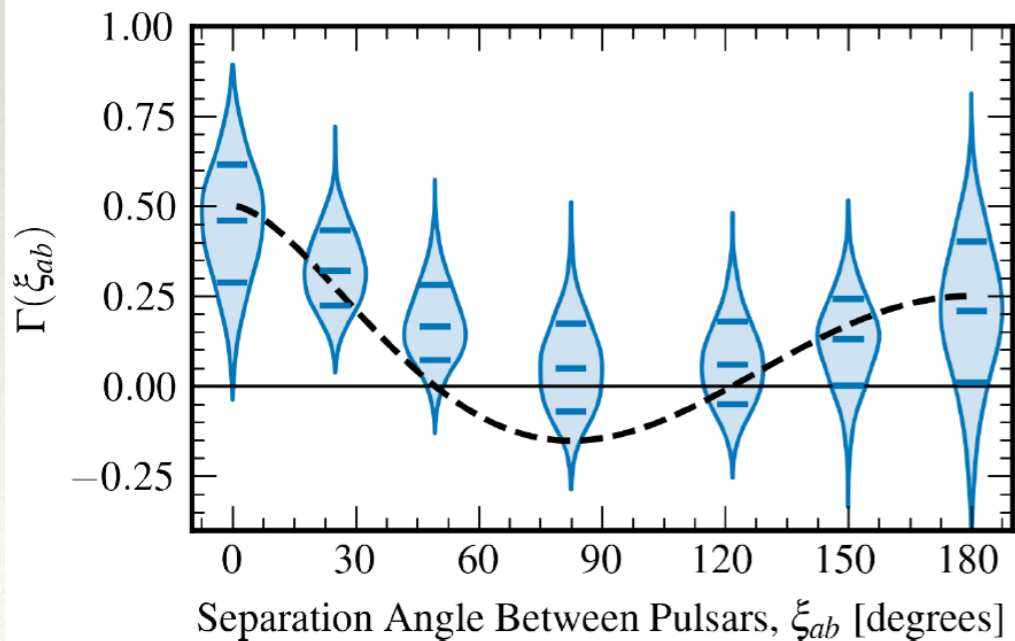
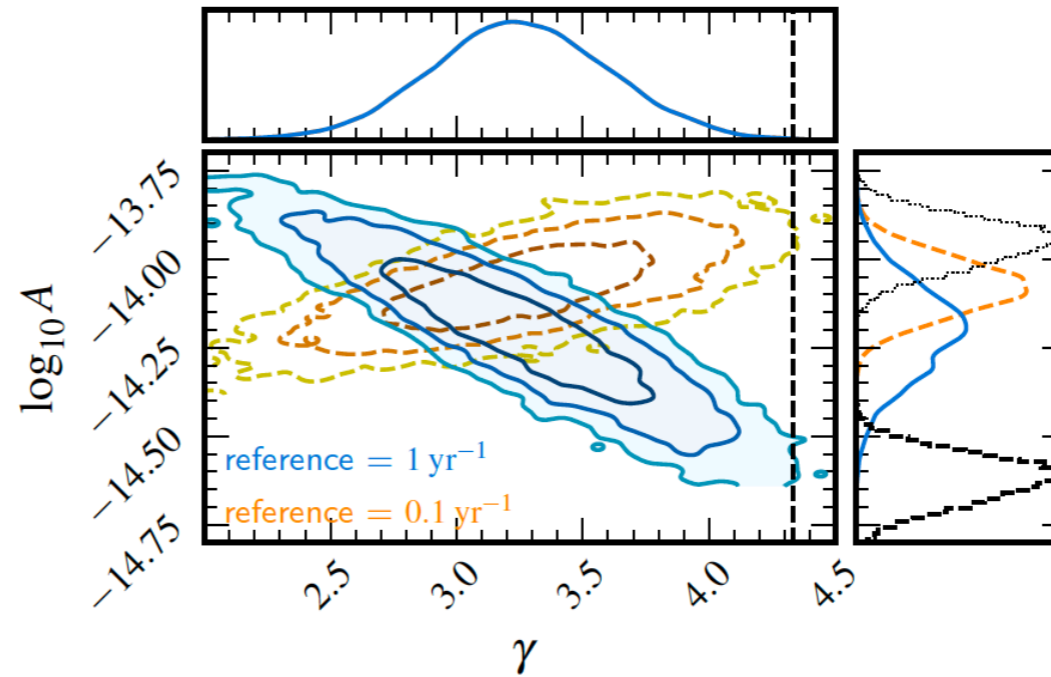
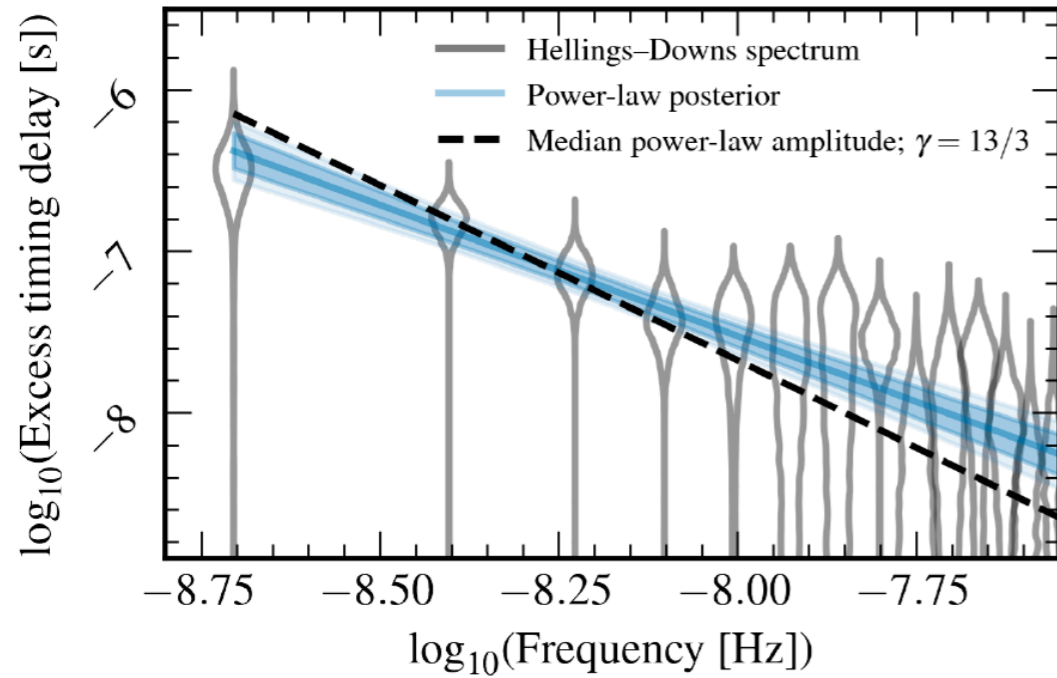


NanoGrav results

[NG 2306.16213]



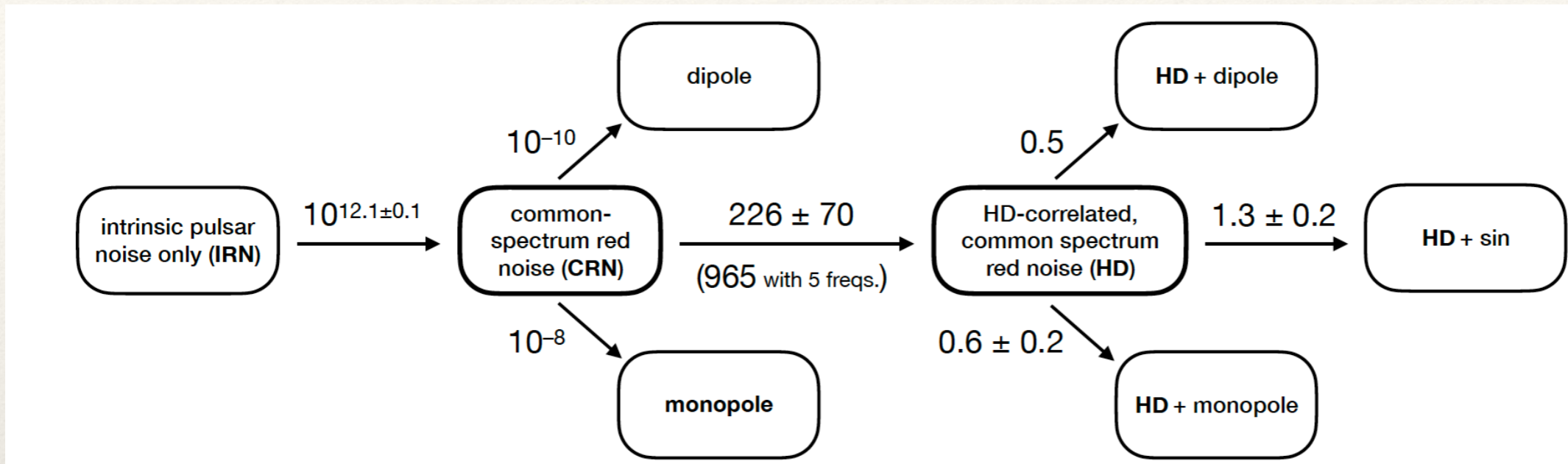
NG data: 15 years of data, 67 pulsars. Arecibo + Green Bank



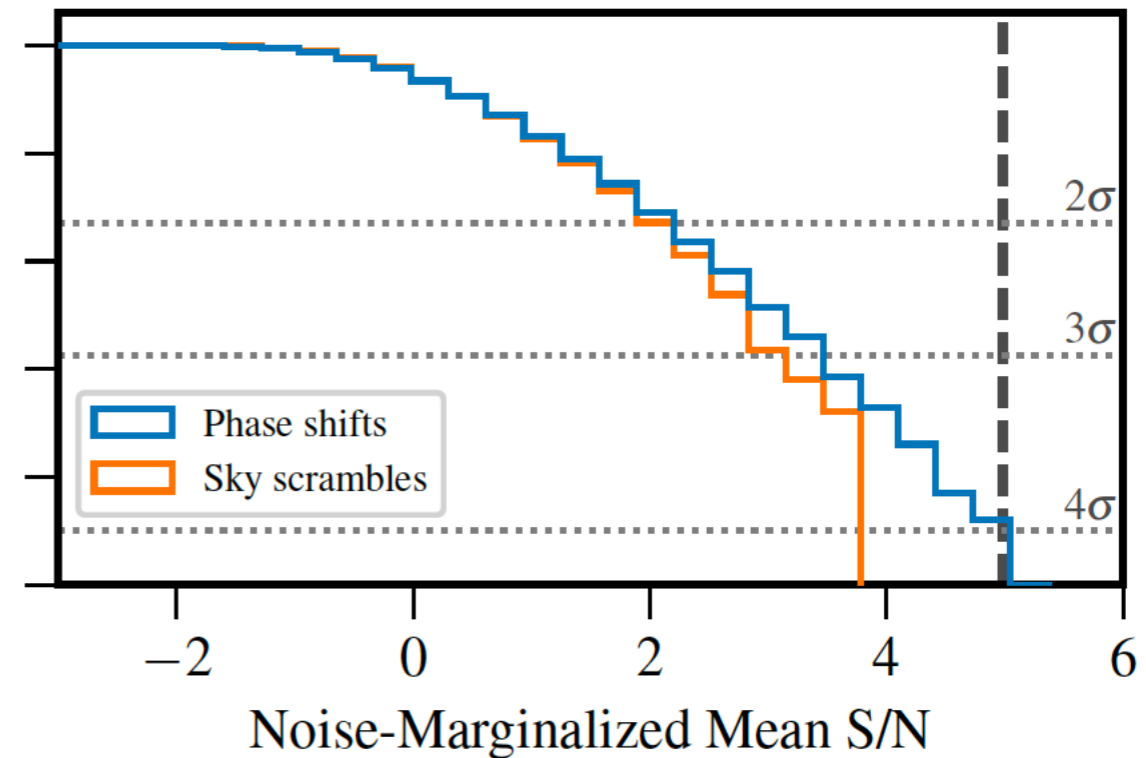
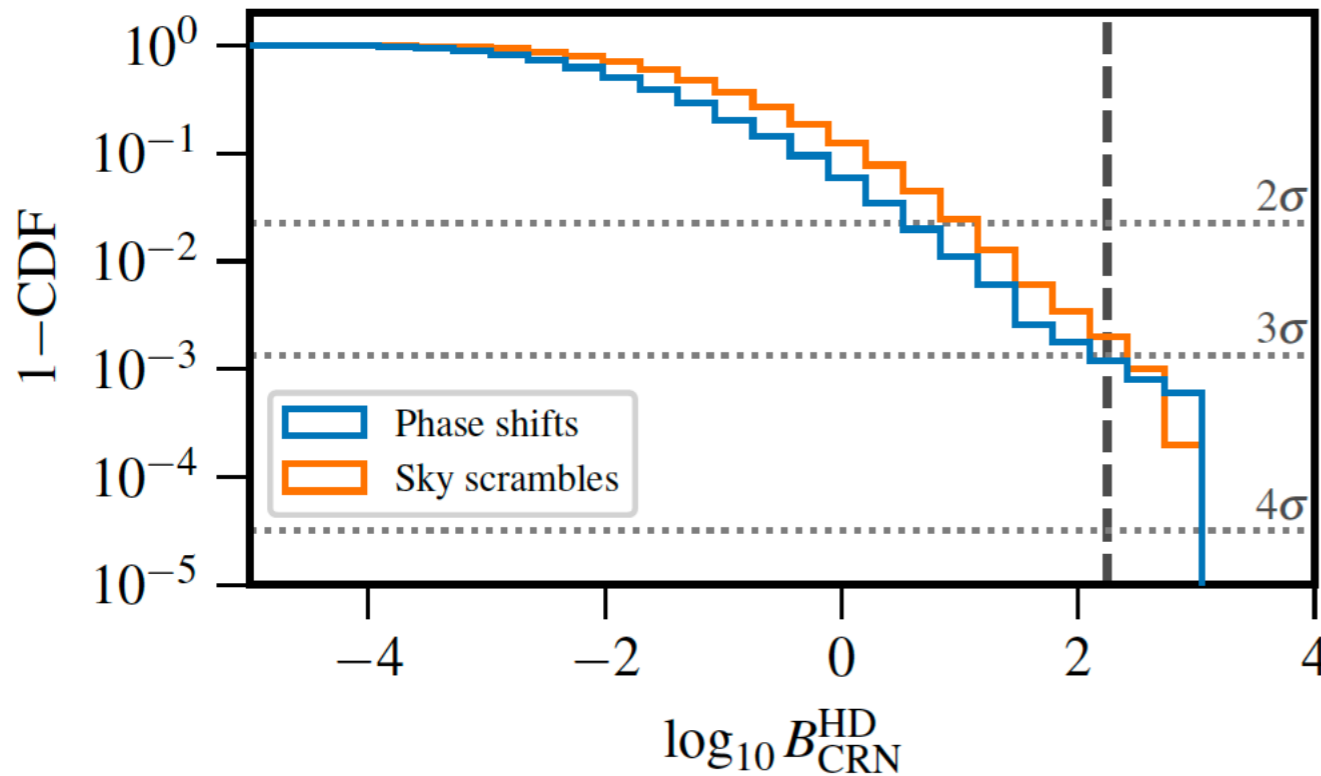
$$S_{\alpha\beta}^{SGWB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-\gamma}$$



NanoGrav results



Significance



Interpretation



LET US ASSUME THAT WHAT WE OBSERVE IS STOCHASTIC GW BACKGROUND
(SGWB)

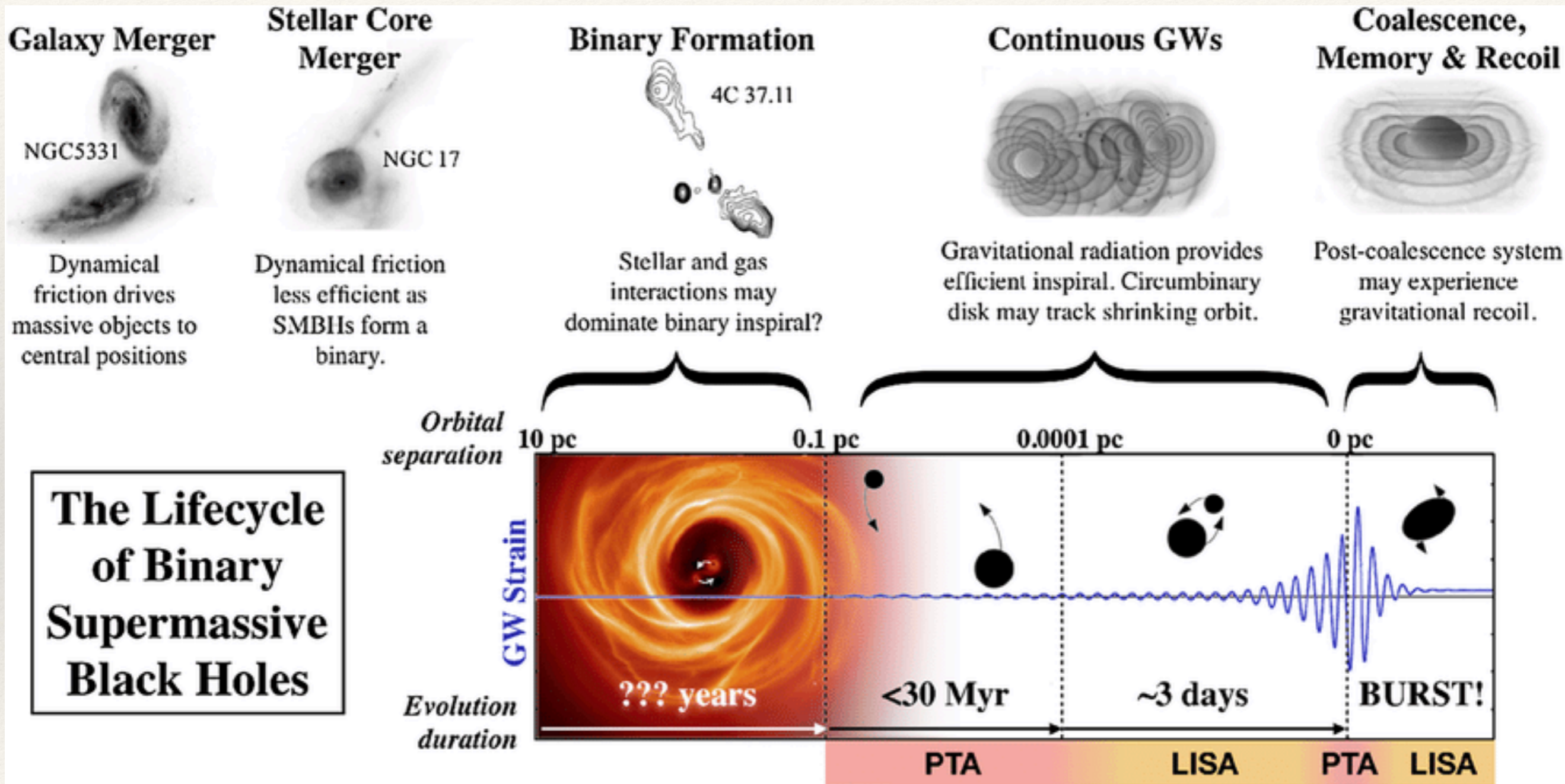
What could produce SGWB with the power-law-like spectrum?
Apparently almost anything that falls in nHz band... and even more
I'll give only few examples

DISCLAIMER

preference in interpretation of observed signal and its significance: my personal view



Massive black hole binaries



[S. Burke-Splolaor A&A review (2019)]

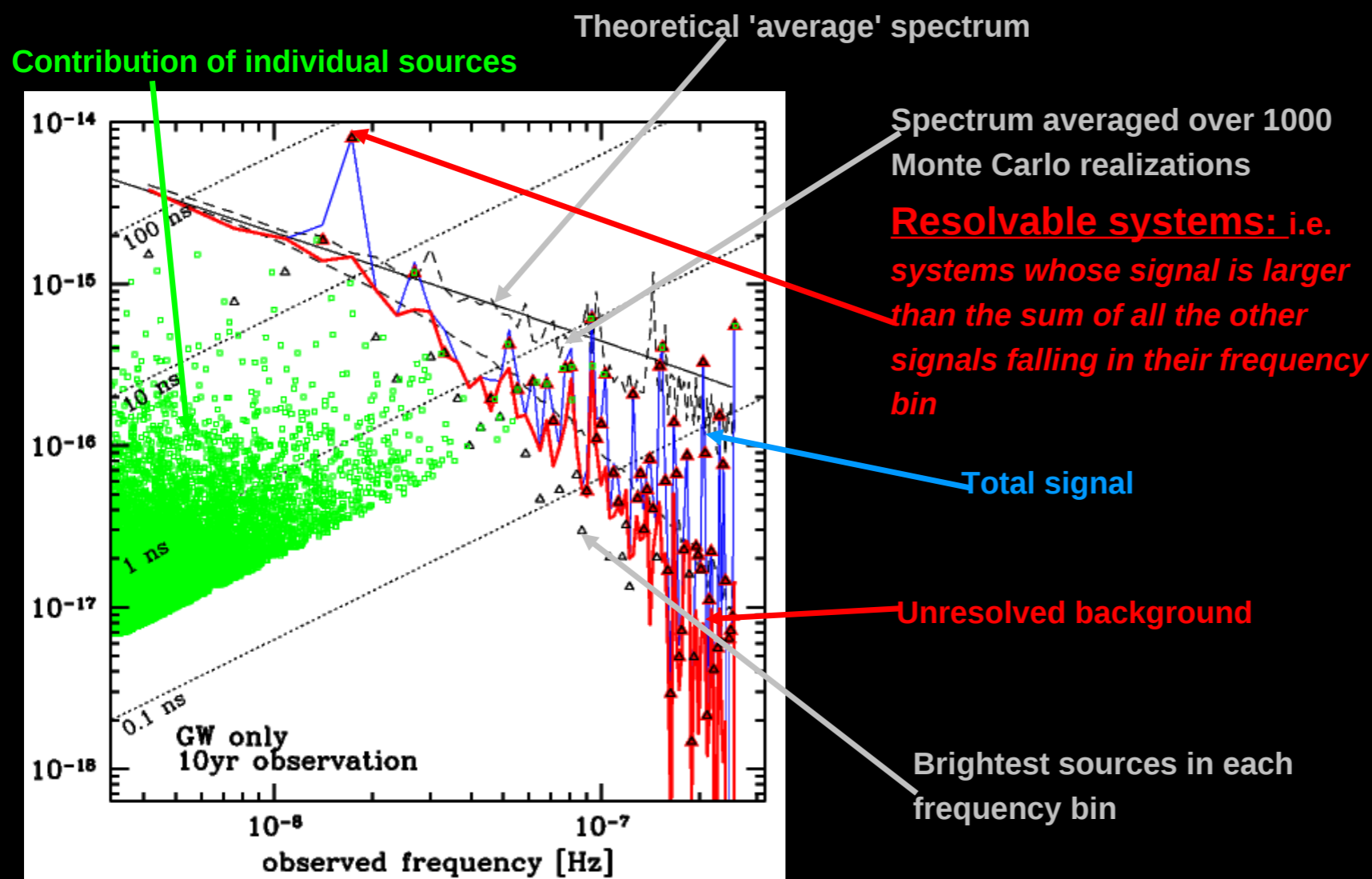


Supermassive black hole binaries



- Main sources are supermassive black hole binaries (mass 10^7 — 10^{10} solar) on very broad orbit (period \sim year(s))
- The orbital evolution due to GW emission is very slow: $\frac{dE}{dt} \propto \eta(M/r)^5$
signal is (almost) monochromatic over period of observations

Signal from a MBHB population



GW signal from the population of SMBH binaries: forms a stochastic signal at low freqs. (similar to Galactic binaries in LISA)

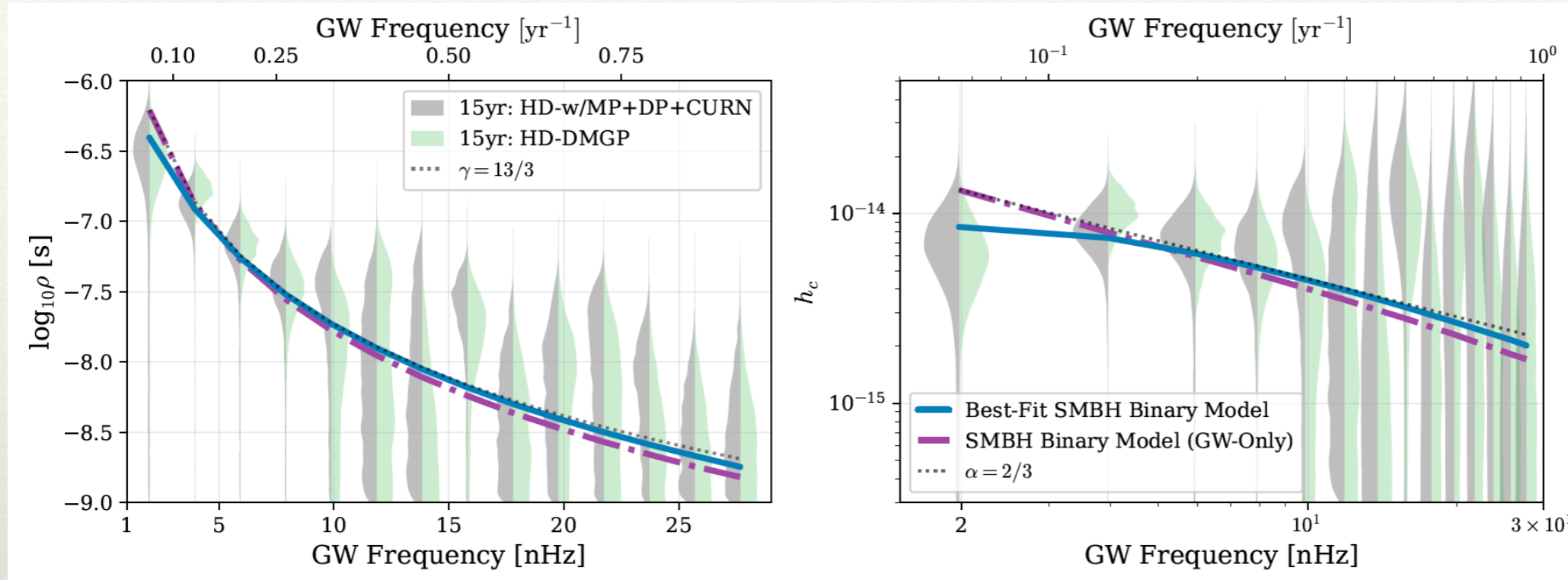
[Credits: A. Sesana]



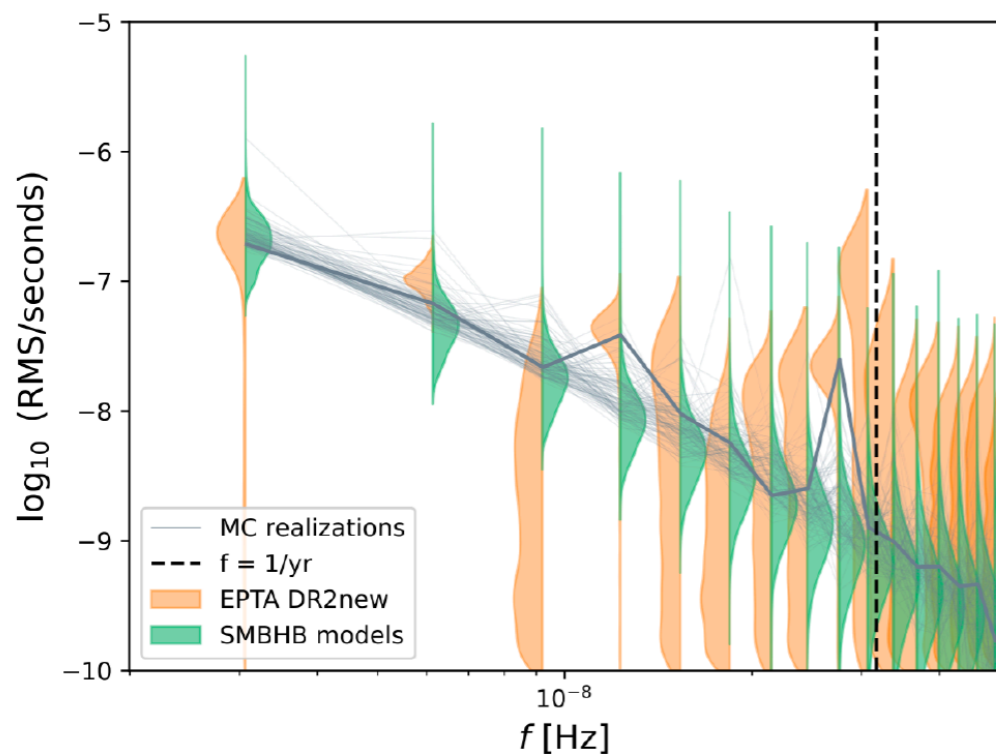
SGWB from population of SMBHBs



[NG: 2306.16220]



[EPTA 2306.16227]



The observed PTA signal could be stochastic GW signal from the population of SMBHBs in the local UNiverse

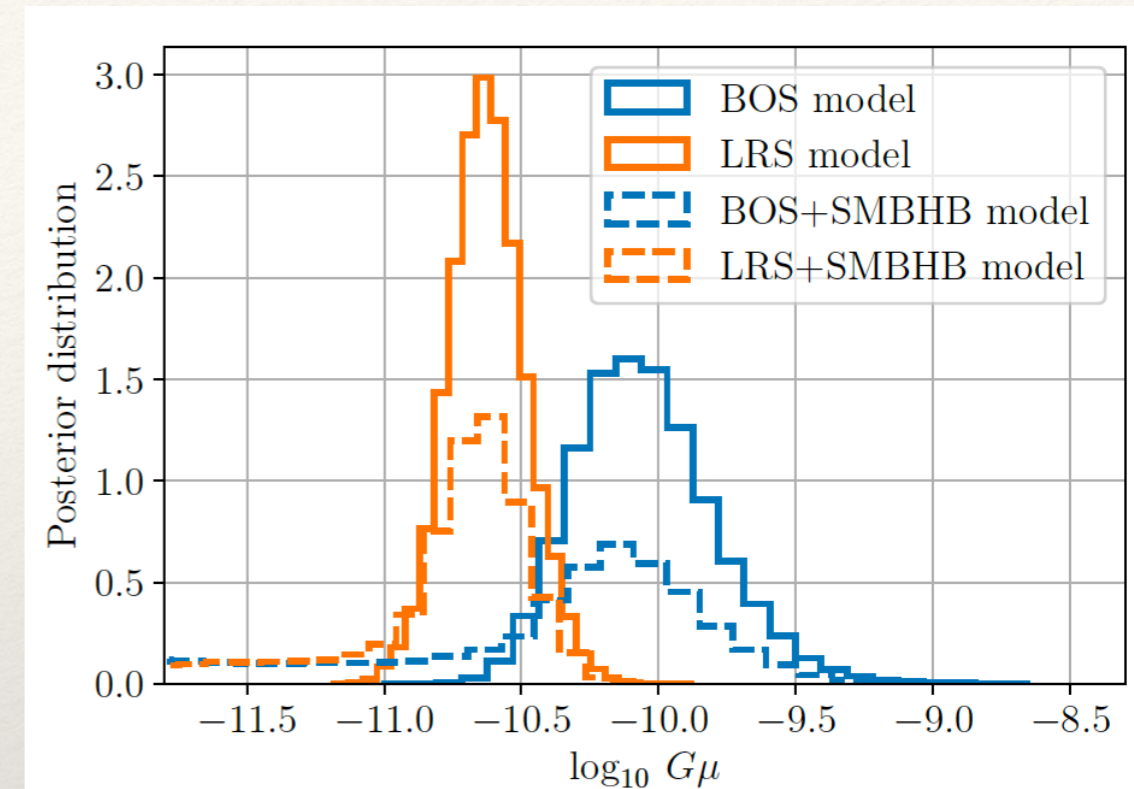
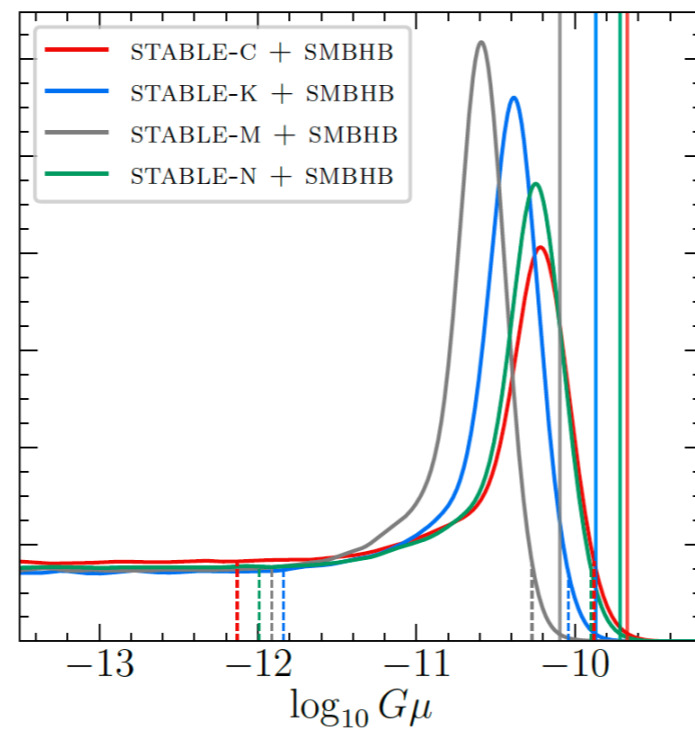
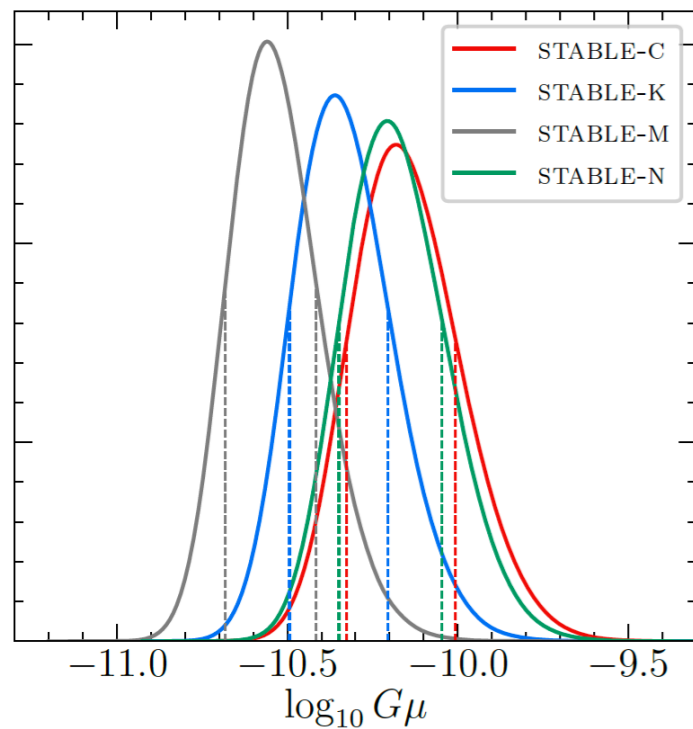


SGWB: Network of cosmic strings



[NG, 2306.16219]

[EPTA+InPTA, 2306.16227]



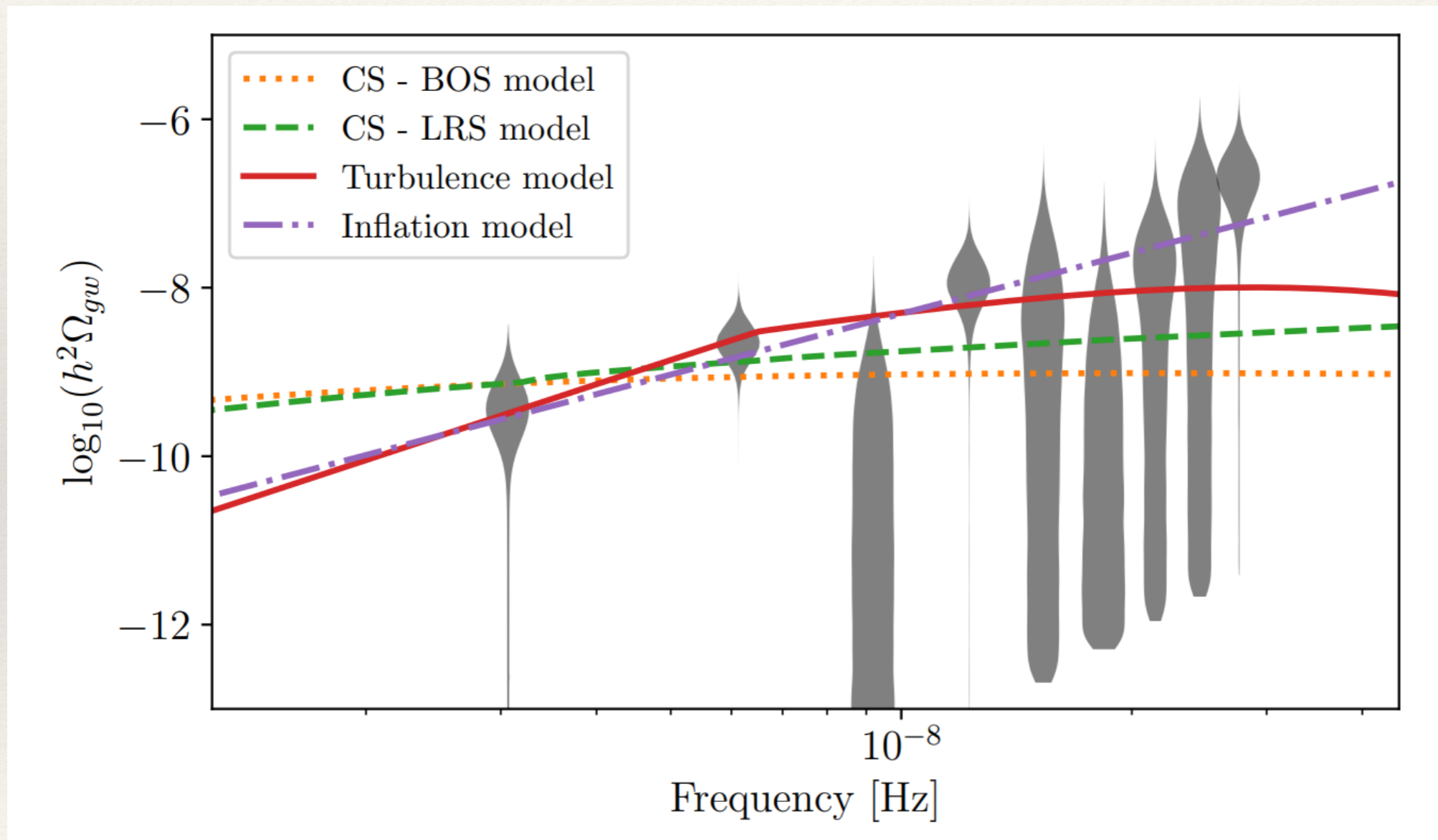
- We can constrain the tension of cosmic strings (model dependent) assuming the observed signal is entirely produced by the network of cosmic strings
- We can set an upper limit in two-component model of SGWB: CSs + SMBHBs



Interpretation of PTA signal



The signal is weak and poorly constrained:
almost “anything” can more-or-less explain it



[Credits: Hippolyte Quelquejey]



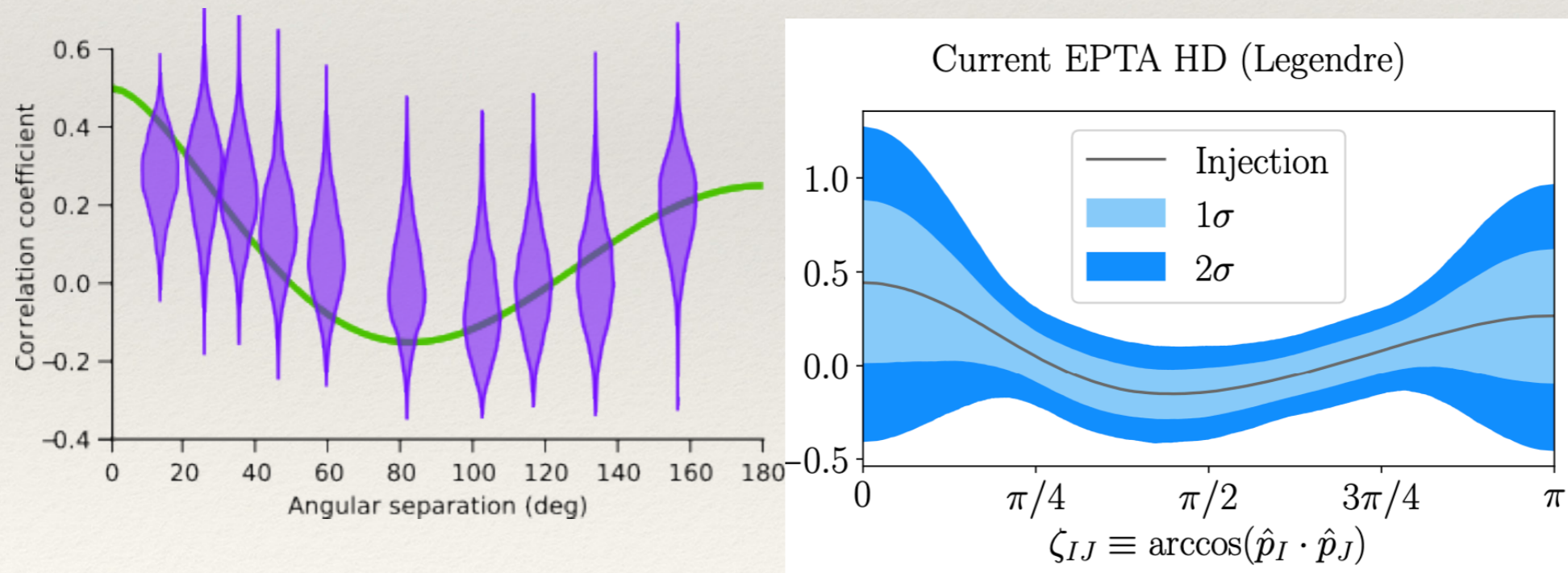
Do it yourself



[ArXiv:2404.02864]

<https://github.com/MauroPieroni/fastPTA/>

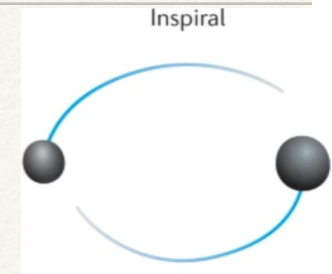
- You create your own PTA
 - based on existing and MaxLik estimation of noise parameters
 - make your own future PTA (based on SKA)
- Check how well we can detect and measure parameters of your favourite SGWB model



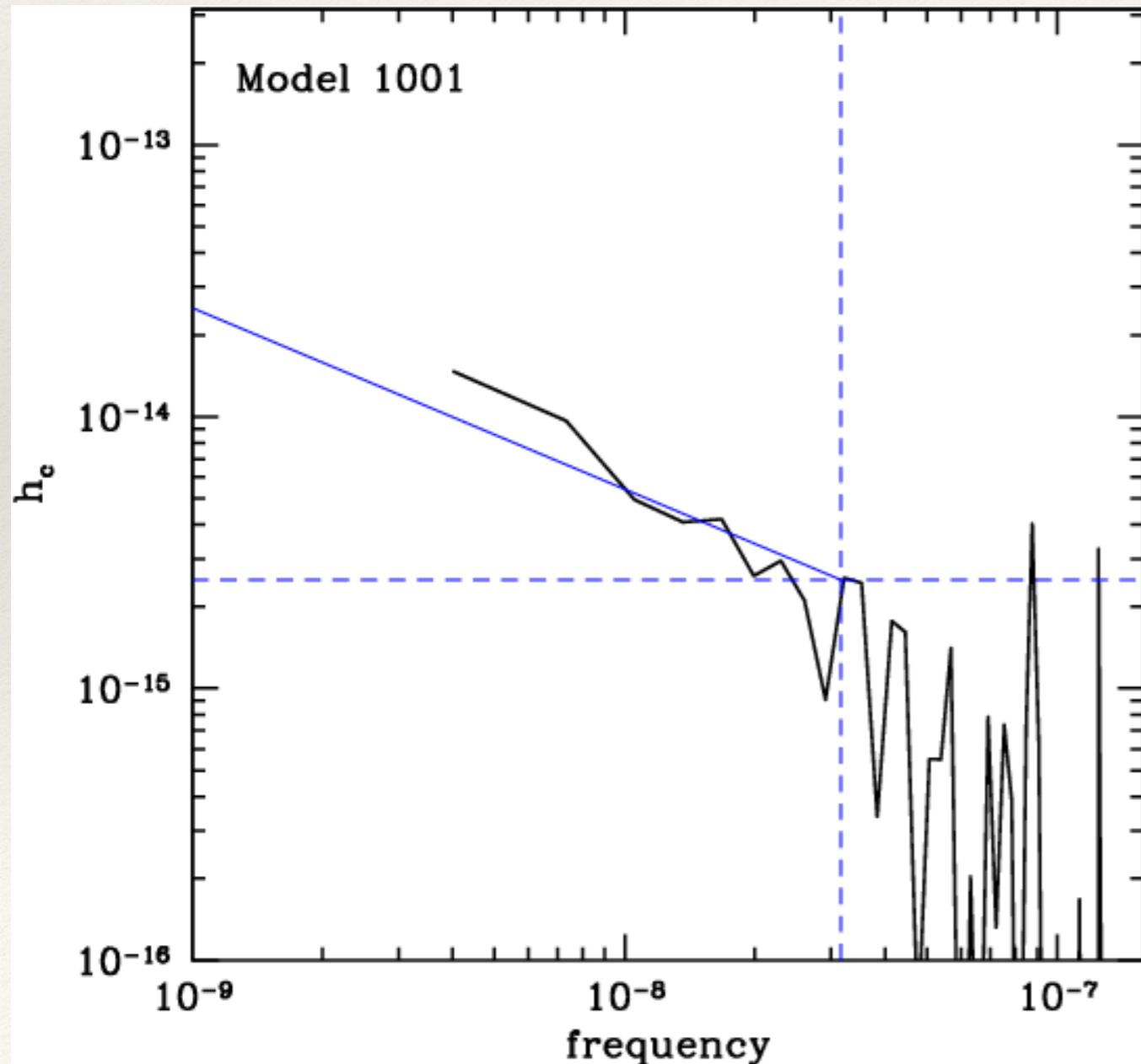
Search for individual MBHBs: continuous GW signal



Searching for GW signal from individual SMBHB binary:

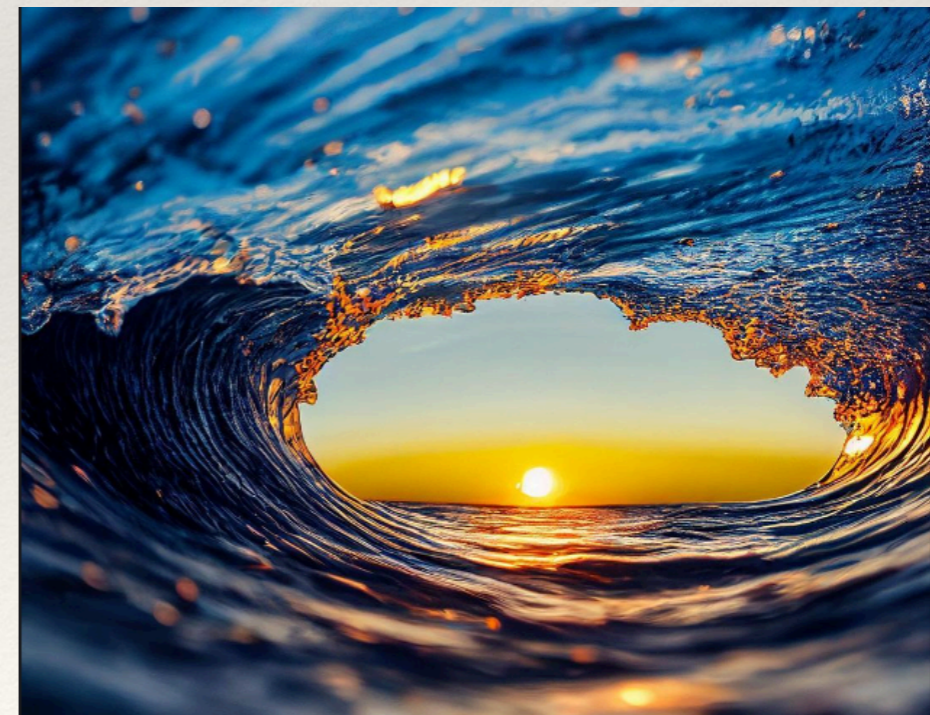


- Assume circular orbit
- Bayesian approach
- Strategy: all-sky search with simplistic model -> follow up candidates relaxing simplified assumptions on the reduced prior range



[NG: 2306.16222]

[EPTA+InPTA: 2306.16226]



CGW signal

Consider non-spinning SMBH binary in circular orbit

- pulsar and earth terms: each is monochromatic signal
- frequency. of pulsar term might or might not coincide with the earth term:
 $t_p = t - L(1 + \hat{n} \cdot \hat{k})$
- amplitude of the pulsar term is larger: $\sim \omega^{-1/3}$

$$s_\alpha = F_\alpha^+(\hat{k}, \hat{n}_\alpha) \left[\frac{h_+(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_+(t, \omega)}{2\pi f} \right] + \alpha - \text{pulsar index}$$

$$F_\alpha^\times(\hat{k}, \hat{n}_\alpha) \left[\frac{h_\times(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_\times(t, \omega)}{2\pi f} \right]$$

relative position pulsar and GW source

Pulsar term

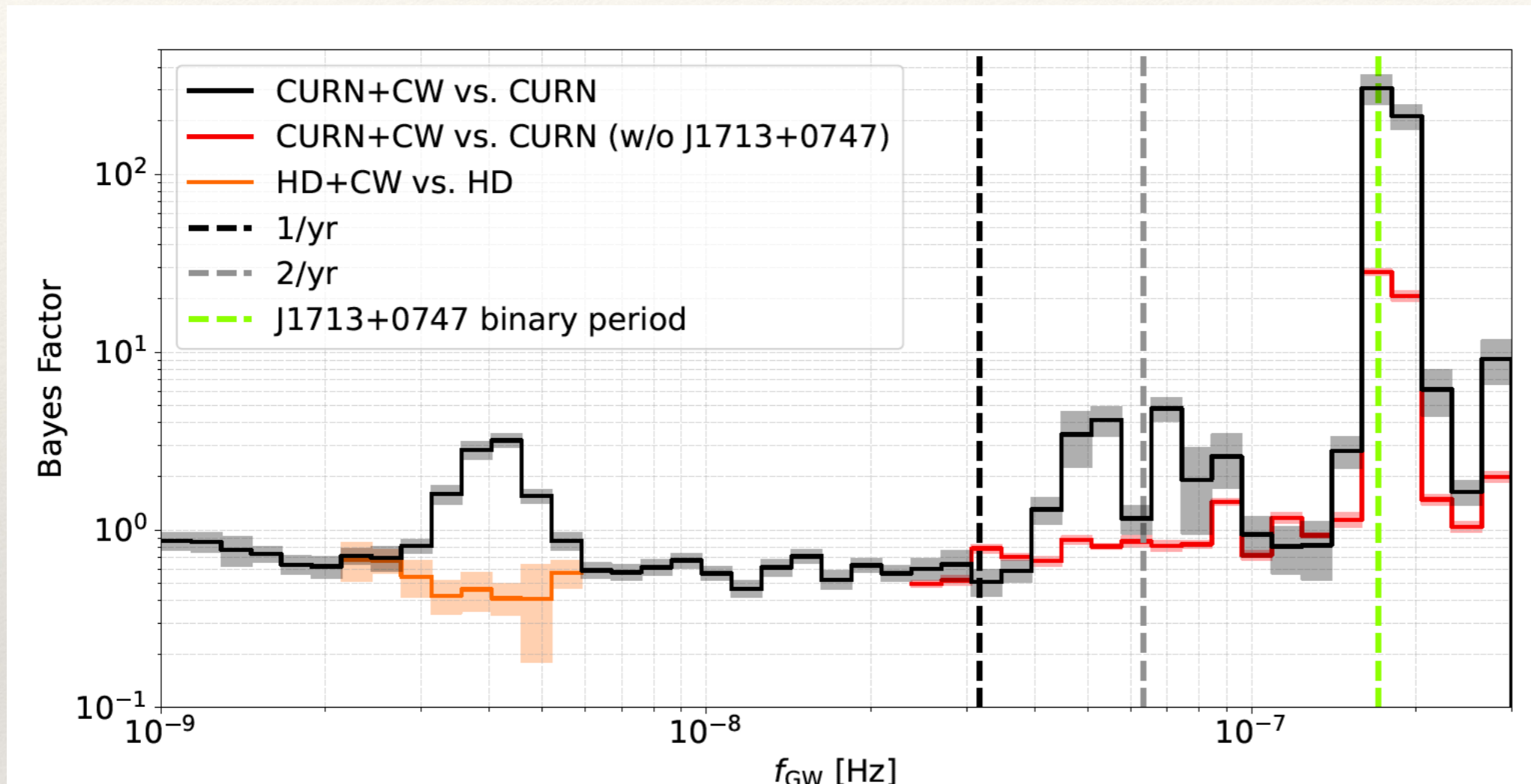
Earth term coherent across pulsars

$$\omega_\alpha = \omega(t - L_\alpha(1 + \hat{n}_\alpha \cdot \hat{k}))$$

CGW signal in NanoGrav



[NG: 2306.16222]



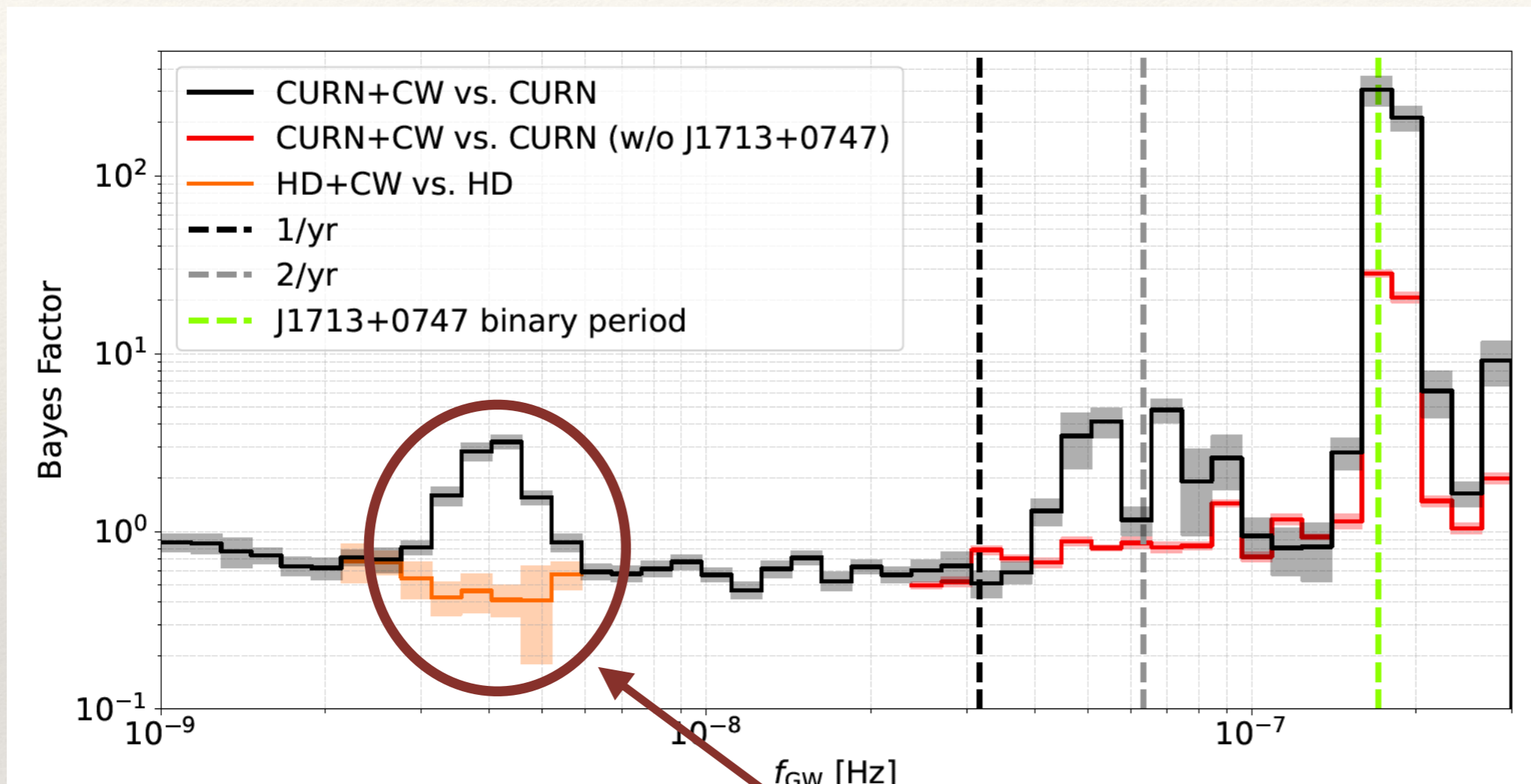
Bayesian all-sky search for a SMBHB in circular orbit: Bayes factor for presence of CGW



CGW signal



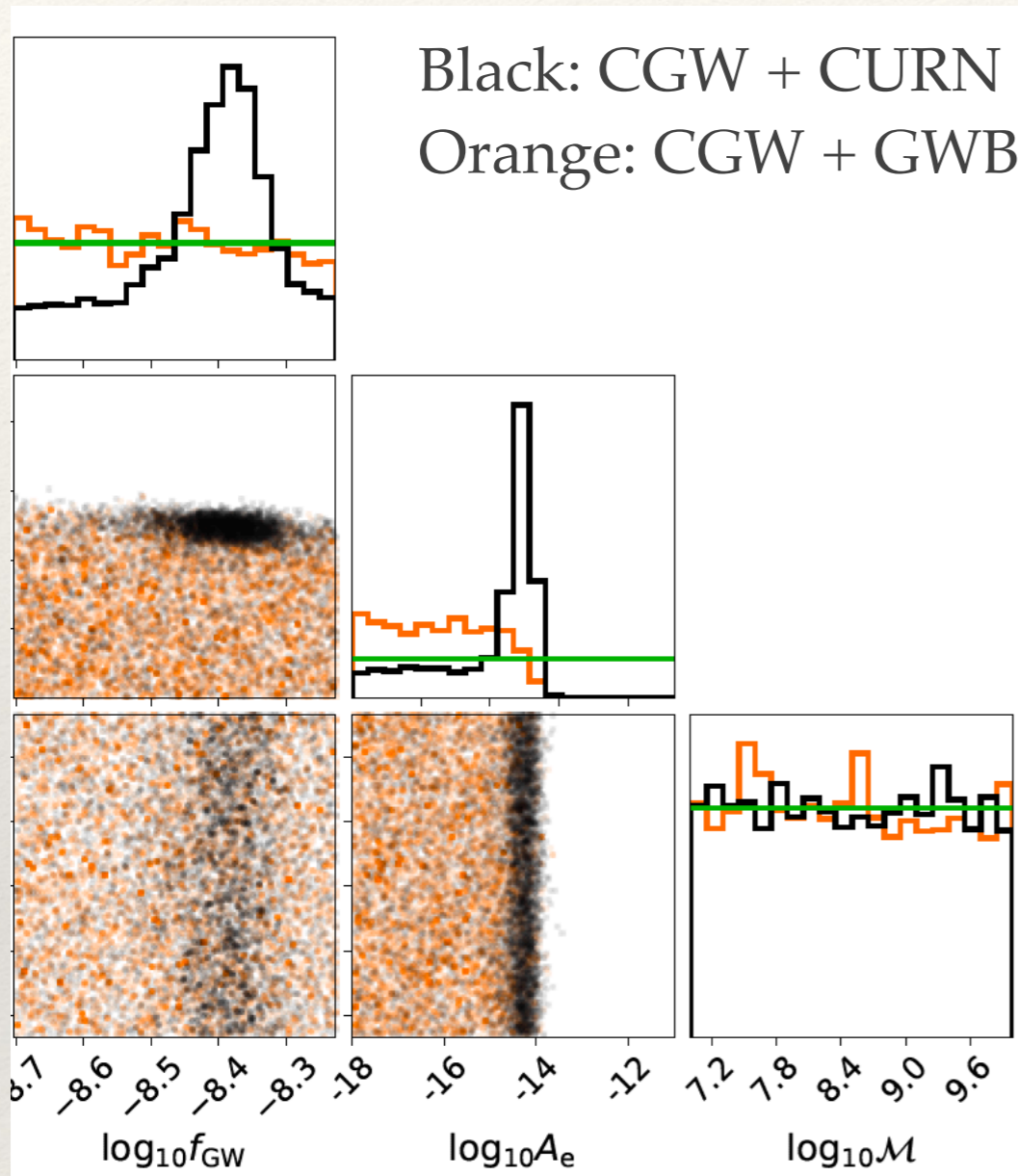
[NG: 2306.16222]



- Bayesian all-sky search for a SMBHB in circular orbit: Bayes factor for presence of CGW
- I concentrate on the low-frequency candidate

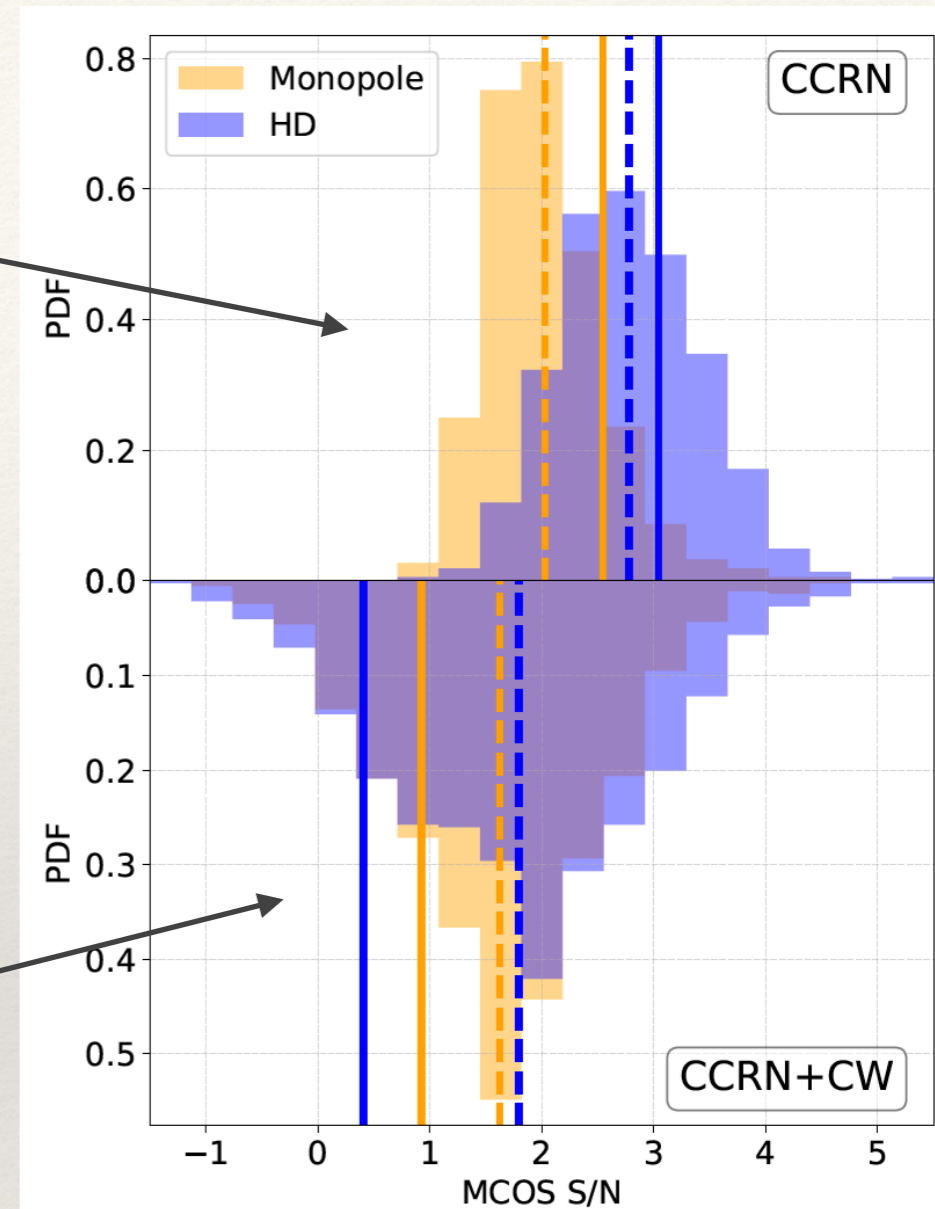


CGW in NG data

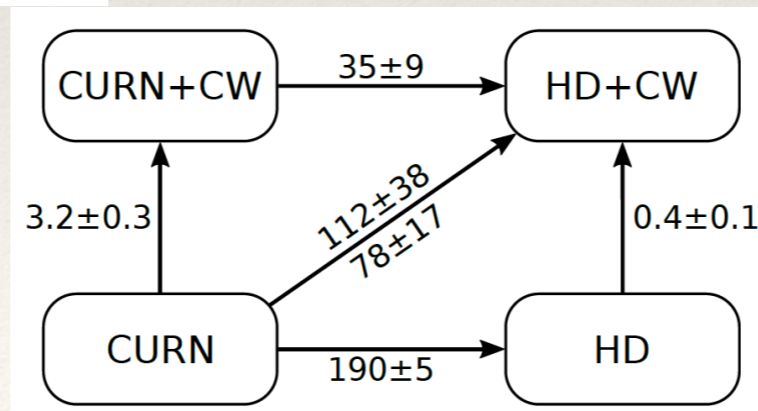


SNR of GWB and monopole in NG data

SNR of GWB and monopole in NG data after removing CGW



$f \in (3.2, 5.0)$ nHz

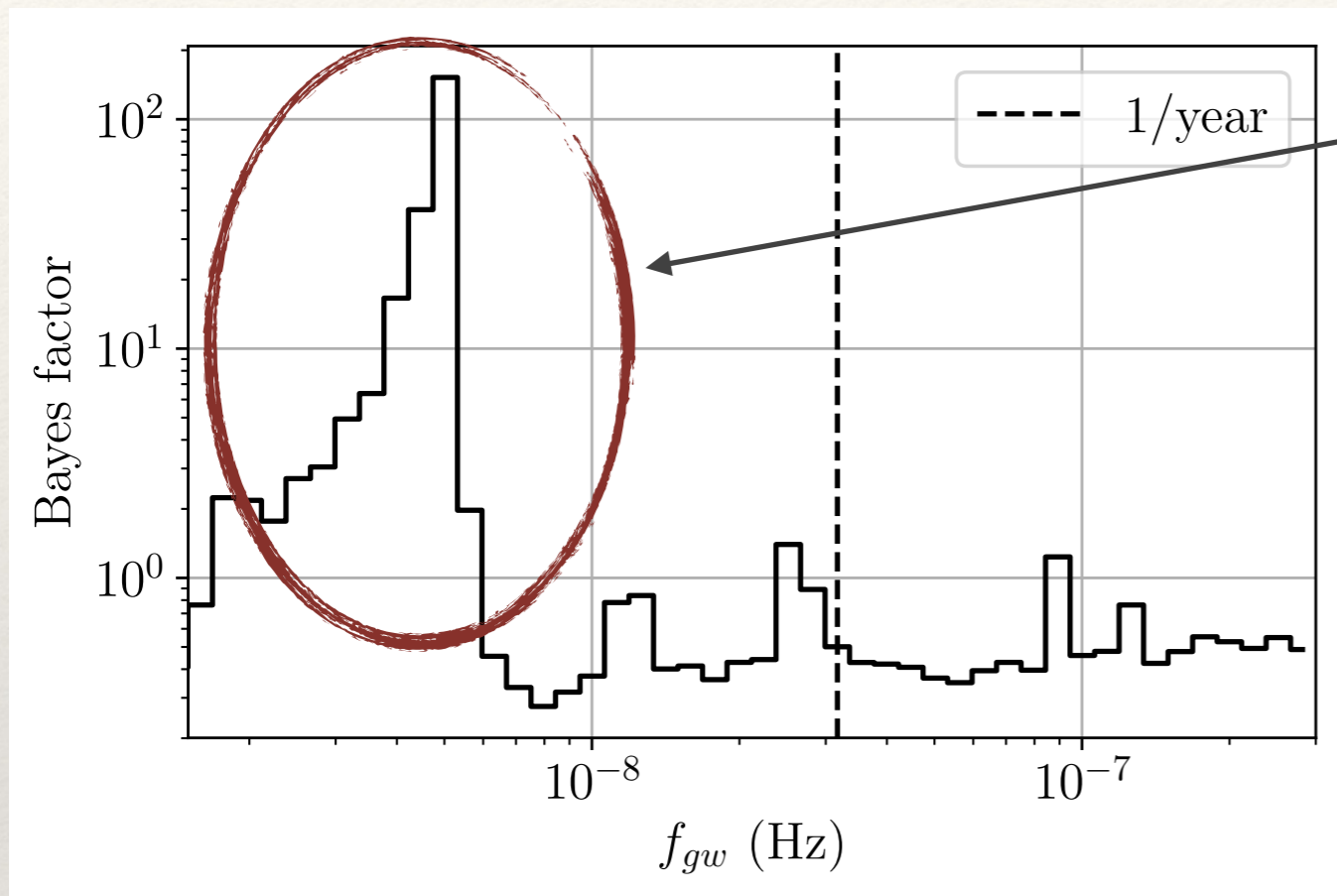


CGW signal in EPTA



BF: PSRN+CURN+CGW / PSRN+CURN

[EPTA+InPTA: 2306.16226]

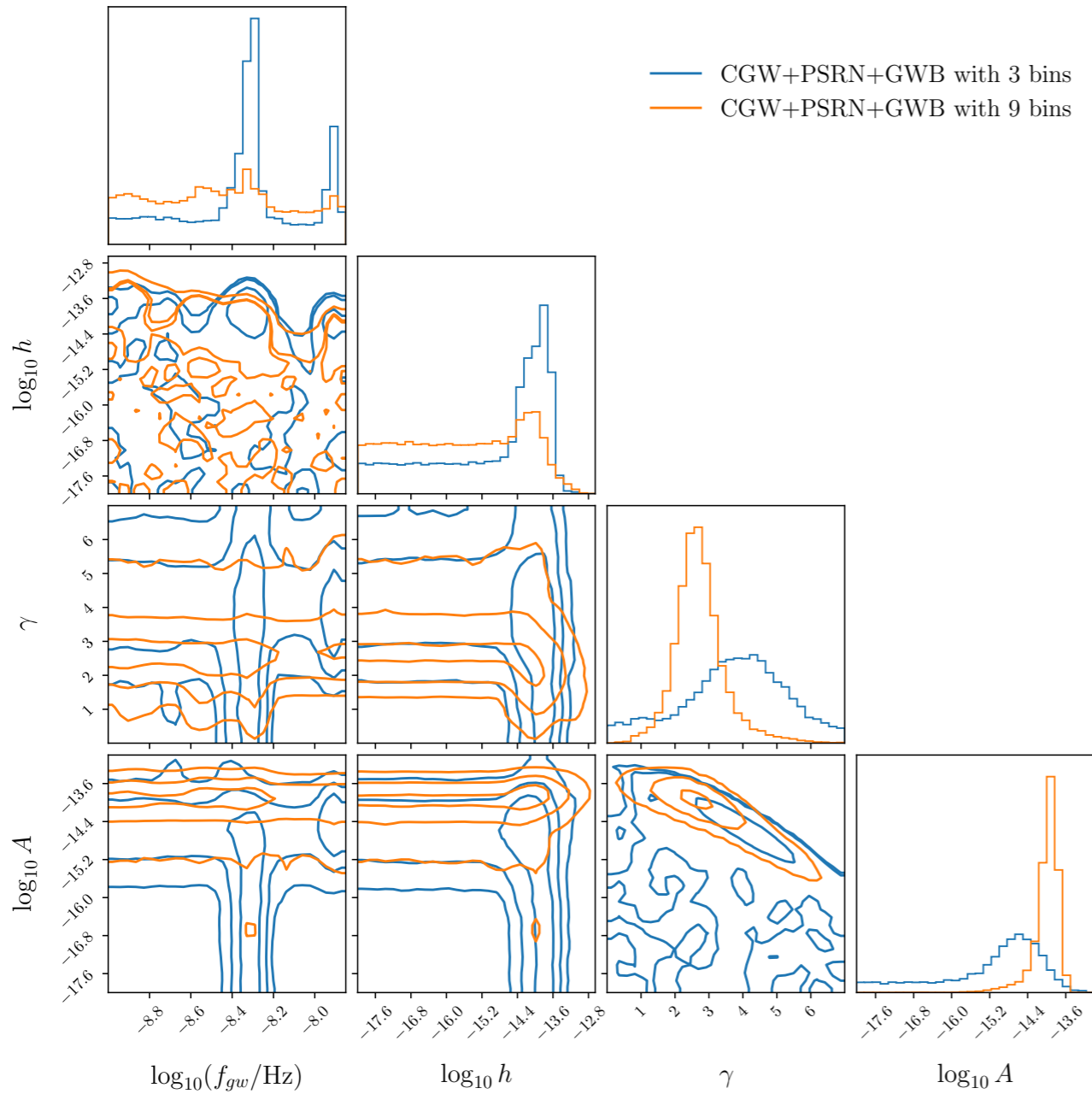


CGW candidate at low frequency

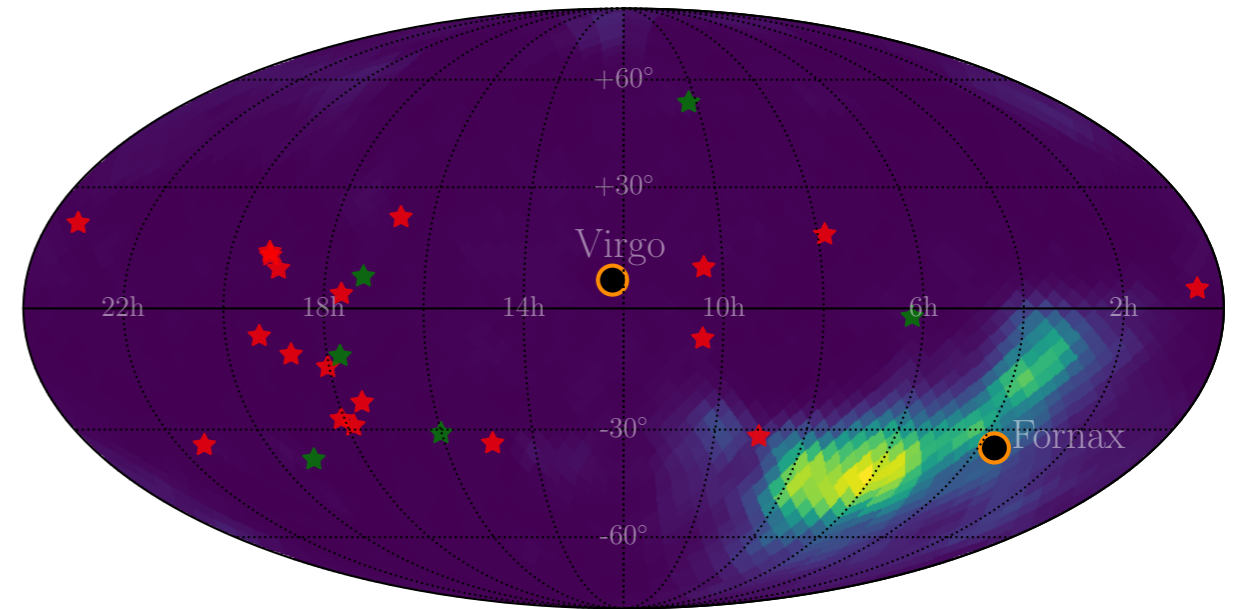
Bayesian search for CGW using Earth term only



CGW signal in EPTA



$f \in (3.2, 6.0)$ nHz

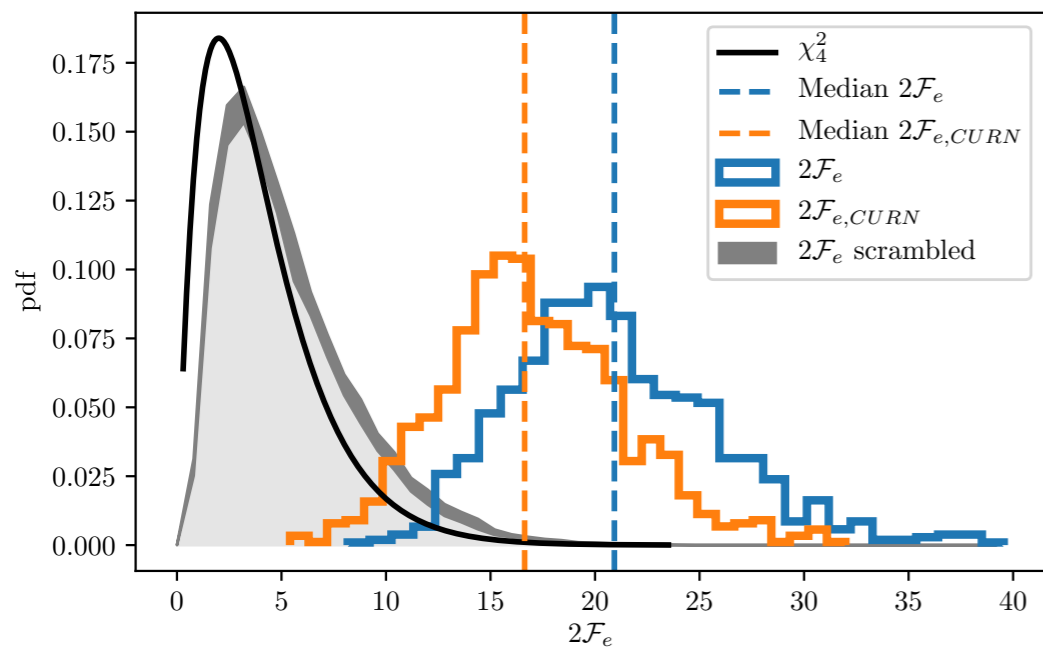


CGW: circular, Earth and Pulsar terms

Model comparison	Bayes factor
CGW+PSRN vs PSRN	4000
CGW+PSRN+CURN vs PSRN+CURN, 3 bins	12
CGW+PSRN+CURN vs PSRN+CURN, 9 bins	4
CGW+PSRN+GWB vs PSRN+GWB, 3 bins	1
CGW+PSRN+GWB vs PSRN+GWB, 9 bins	0.7



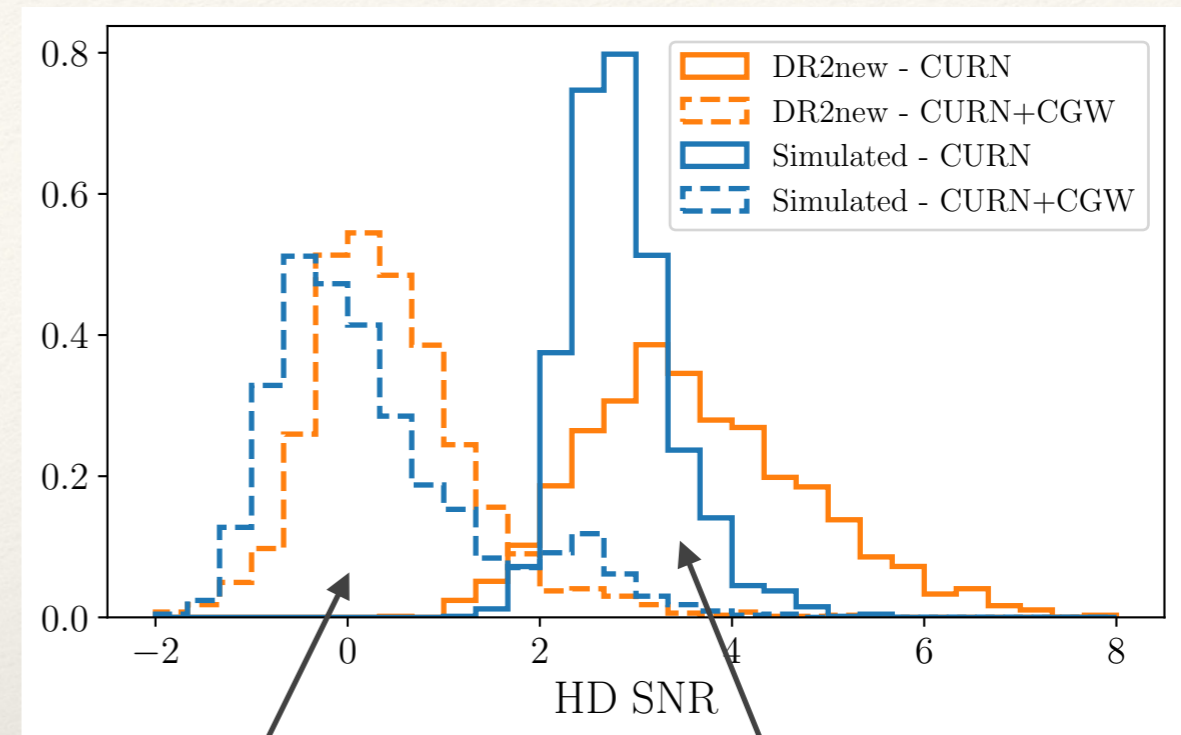
CGW signal in EPTA



Frequentist analysis (but taking into account large uncertainties in the noise)

Significance

	$p(\mathcal{F}_e)$	$p(\mathcal{F}_{e,CURN})$
χ_4^2	5×10^{-4}	1×10^{-3}
Random sky	$(7 \pm 4) \times 10^{-4}$	$(6 \pm 1) \times 10^{-3}$



GWB SNR after removing CGW

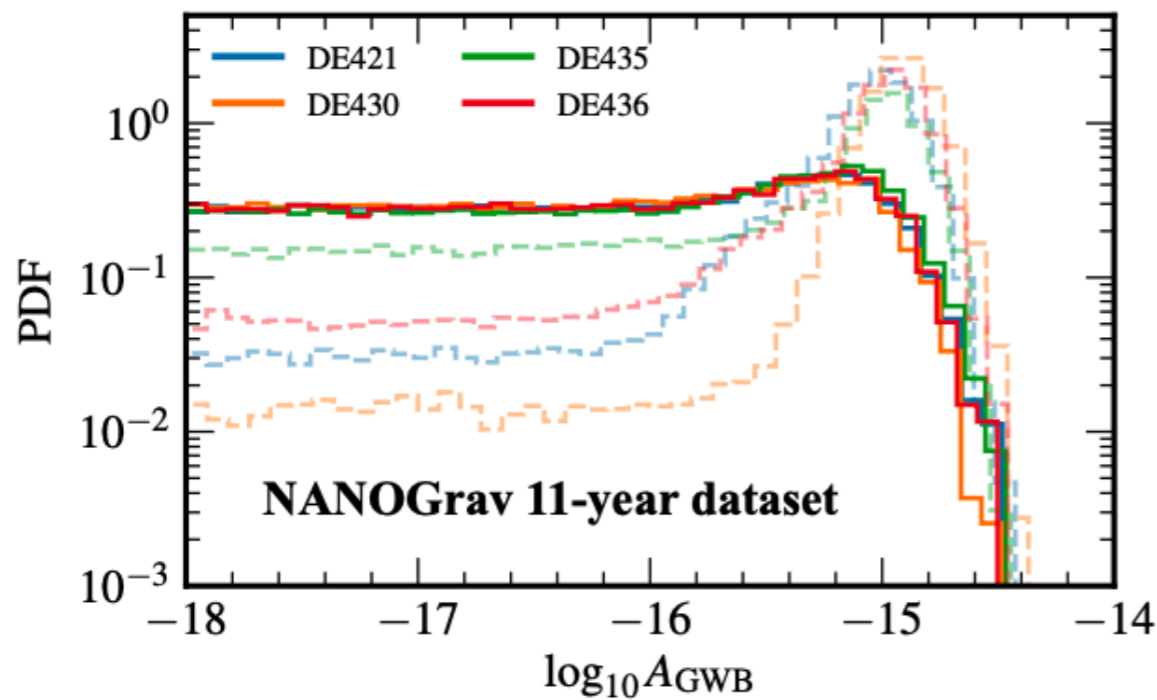
GWB SNR



Is it really GW signal?



- Error in ephemerides: JPL ephemerides D440, good measurement of Jupiter



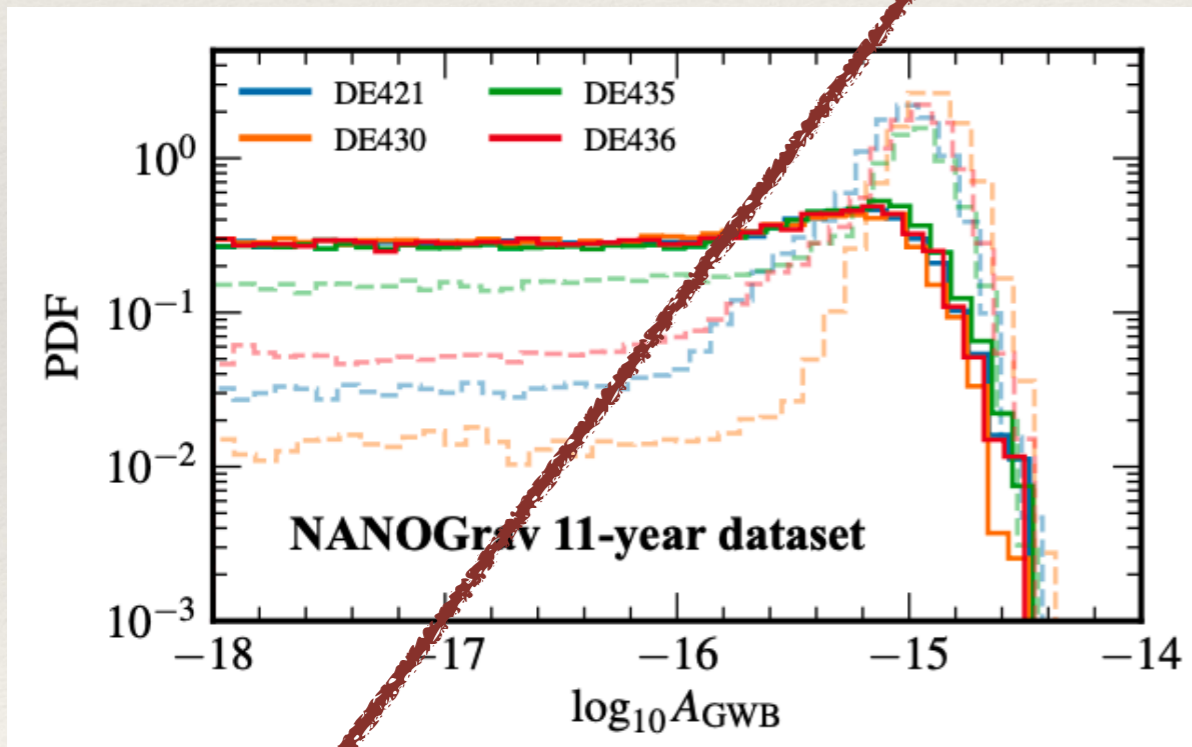
[Arzoumanian+ 2018]



Is it really GW signal?



- Error in ephemerides: JPL ephemerides D440, good measurement of Jupiter

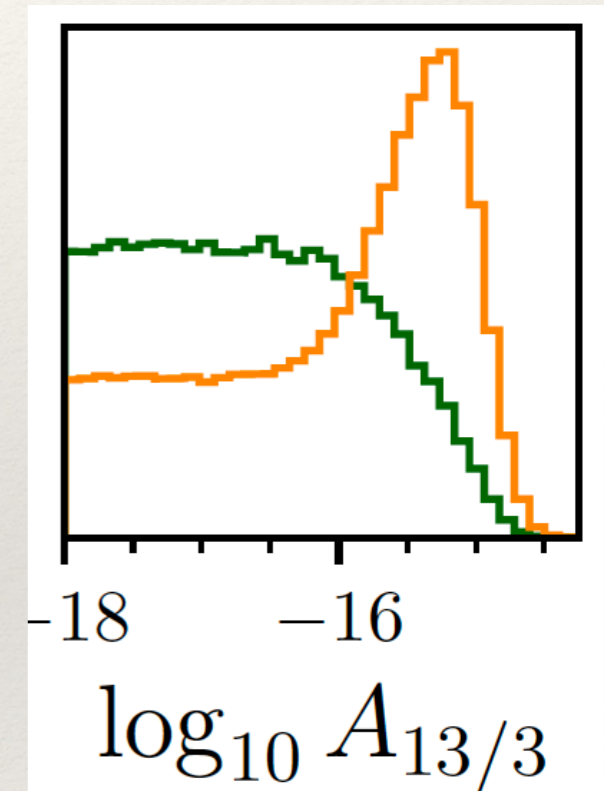
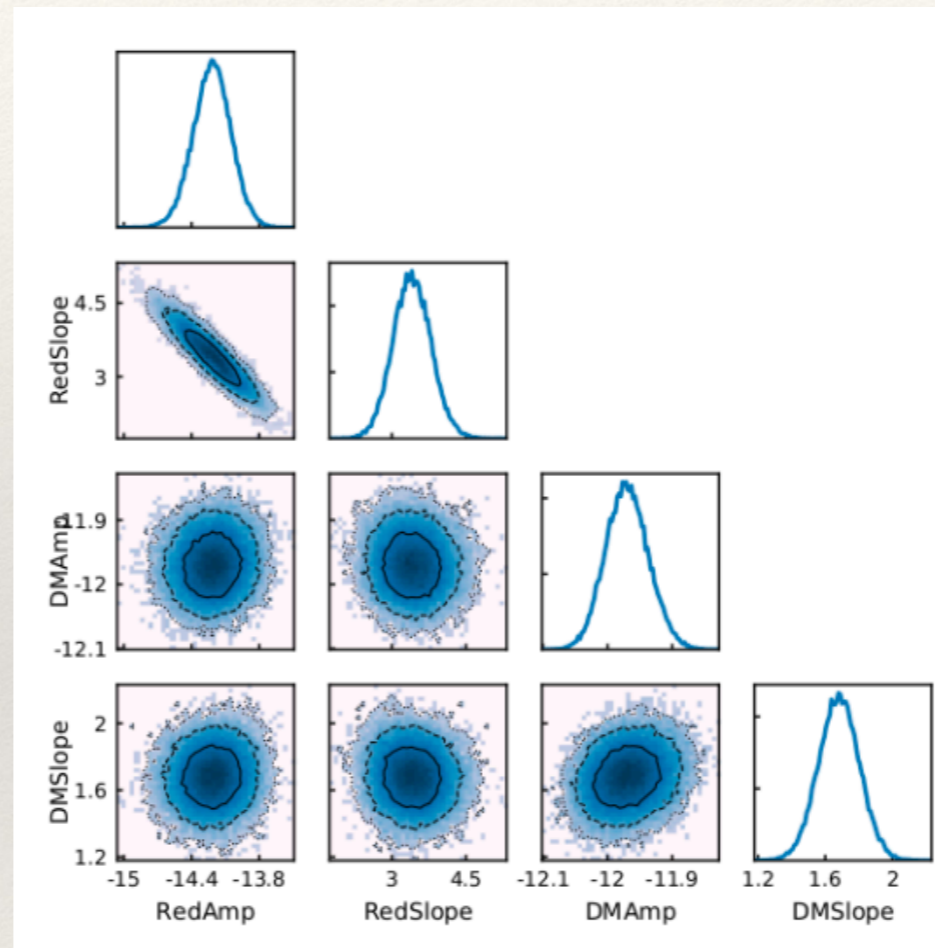
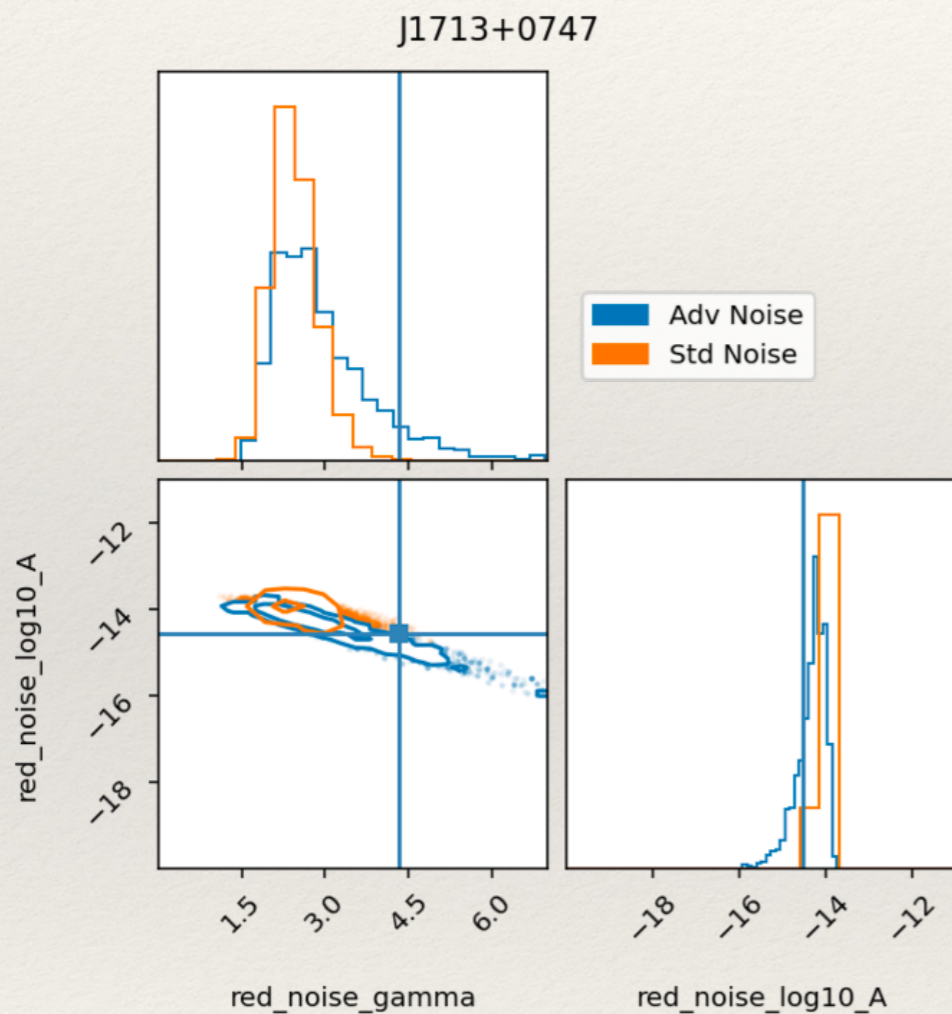


[Arzoumanian+ 2018]



Is it really GW signal?

- Error in ephemerides: JPL ephemerides D440, good measurement of Jupyter
- Modelling noise of each pulsar is very important: J1713+0747



Is it really GW signal?



- Error in ephemerides: JPL ephemerides D440, good measurement of Jupyter
- Modelling noise of each pulsar is very important: J1713+0747

Pulsar	Sel. model
J0613-0200	<i>RN10 DMv30 DMv-SN_NUP_1.4</i>
J1012+5307	<i>RN150 DMv30 DMv-SN_NUP_1.4 SN_NUP_2.5</i>
J1600-3053	<i>DMv30 Sv150 SN_LEAP_1.4</i>
J1713+0747	<i>RN15 DMv150 2 Exp. dips DMv-SN_NUP_1.4 SN_JBO_1.5 SN_LEAP_1.4 SN_BON_2.0 BN_Band.3</i>
J1744-1134	<i>RN10 DMv100 DMv-SN_NUP_1.4 BN_Band.2</i>
J1909-3744	<i>RN10 DMv100 Sv150</i>

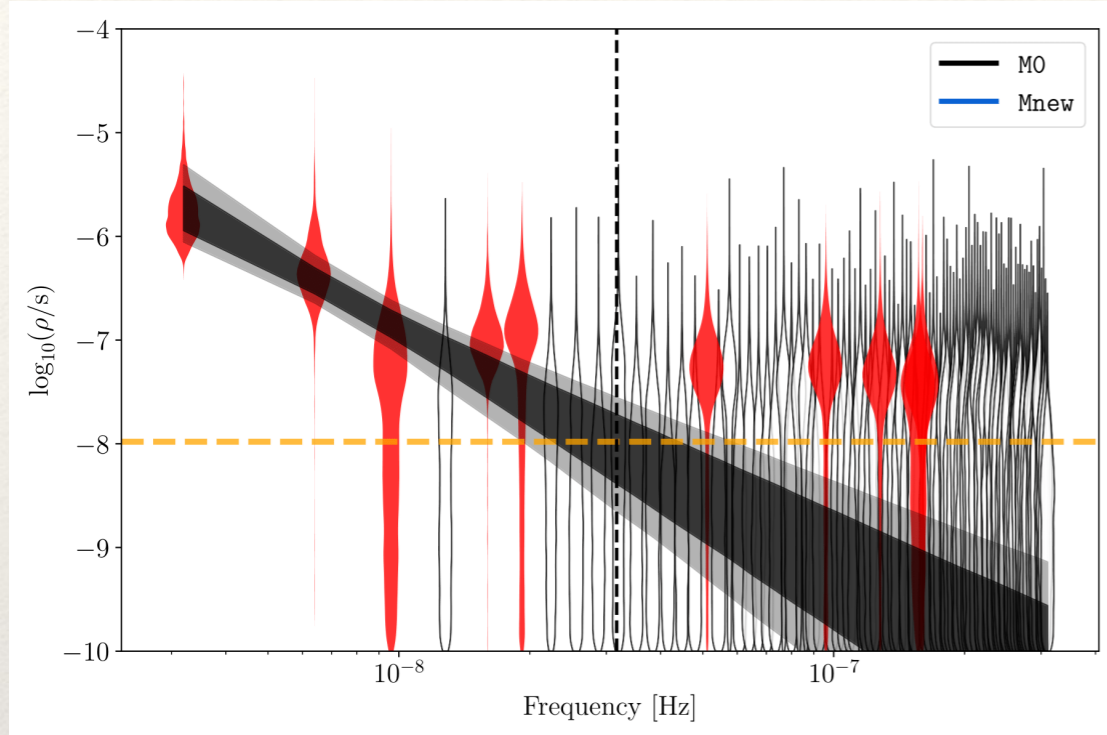
EPTA 6 best pulsars, custom noise models
[Chalumeau+ 2021]



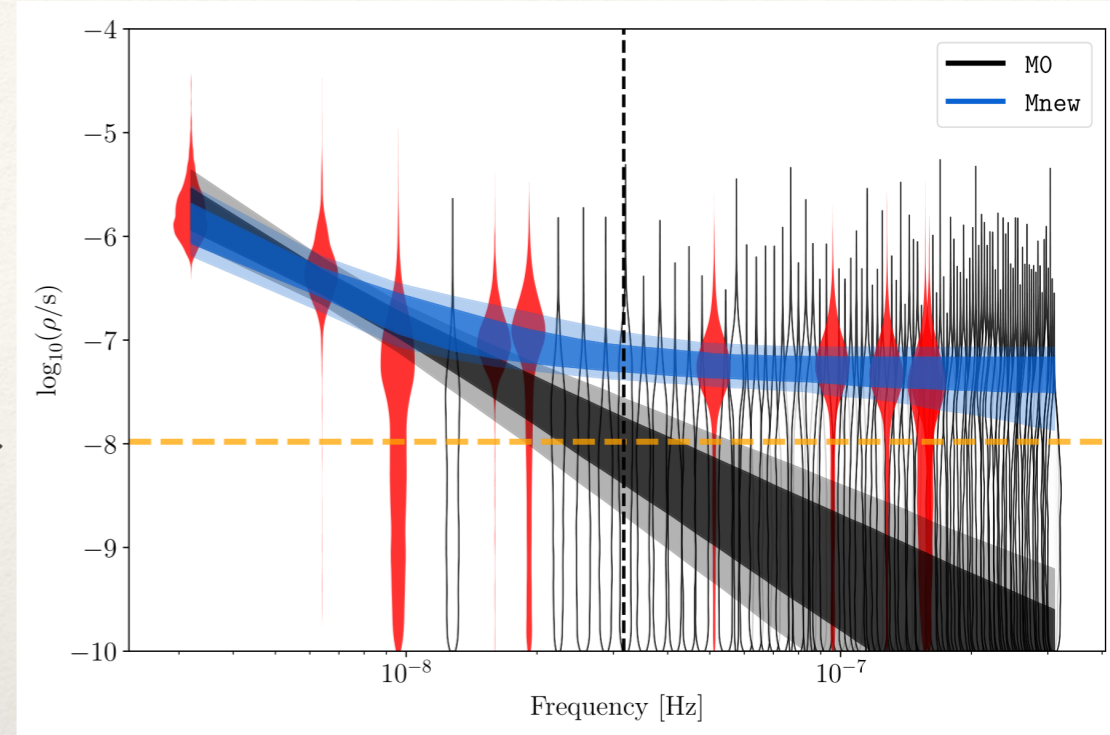
Noise model?



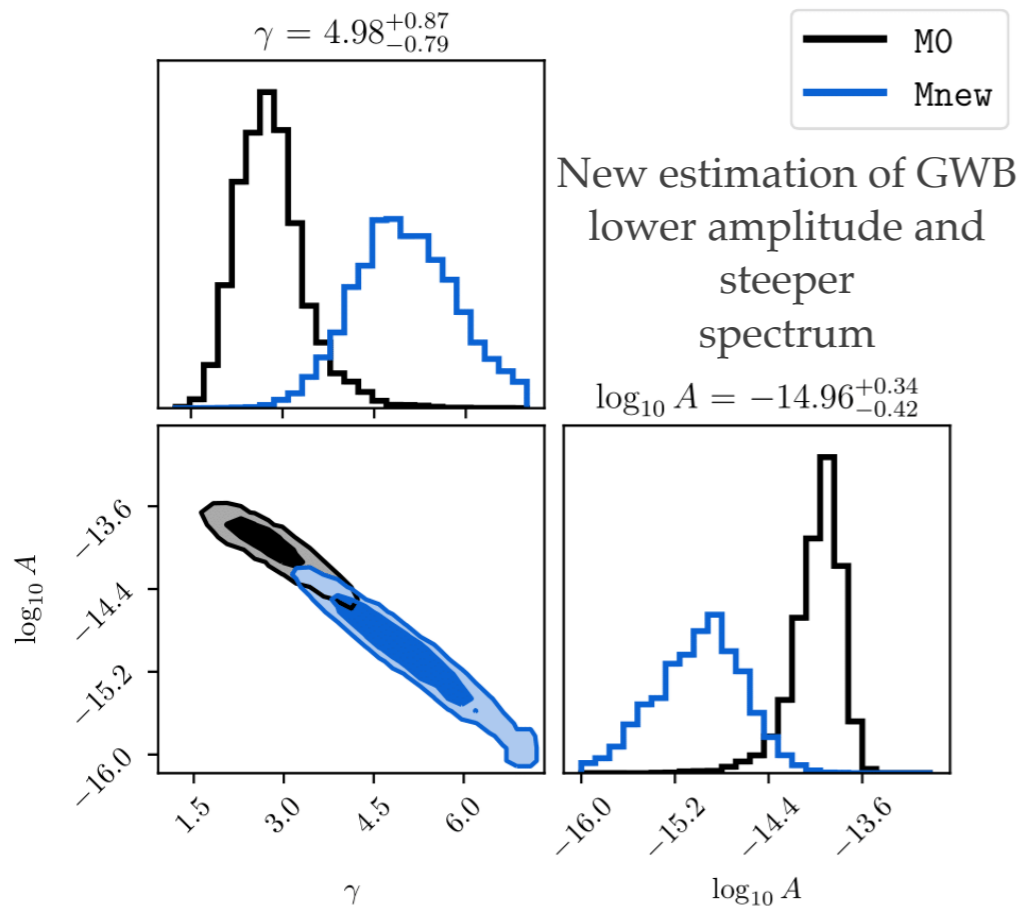
DMv noise PSR J1600-3053



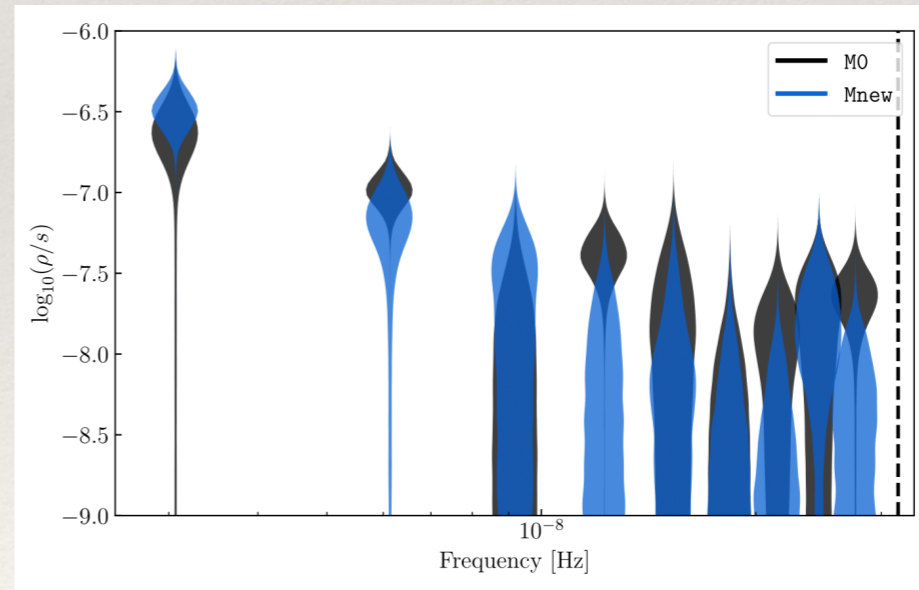
Doesn't look like power-law is a good model



$$\gamma = 4.98^{+0.87}_{-0.79}$$



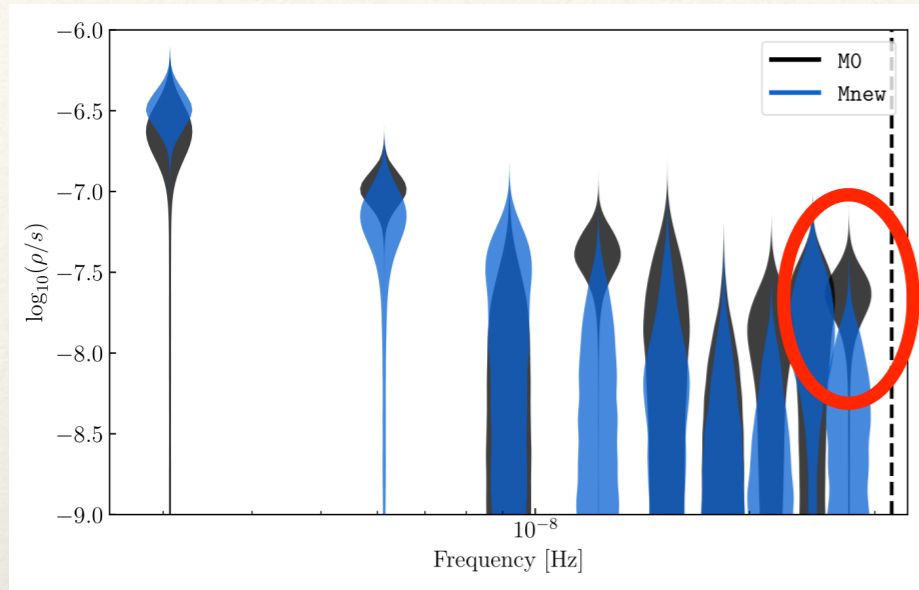
New free spectrum estimation



Models compared	M0	Mnew
CURNvsHD (9)	63 ⁺⁴ ₋₄ 68 ⁺⁴ ₋₄	59 ⁺³ ₋₃ 55 ⁺² ₋₂
CURNvsHD (8)	31 ⁺¹ ₋₁ 30 ⁺¹ ₋₁	63 ⁺³ ₋₃ 53 ⁺² ₋₂



Noise model?

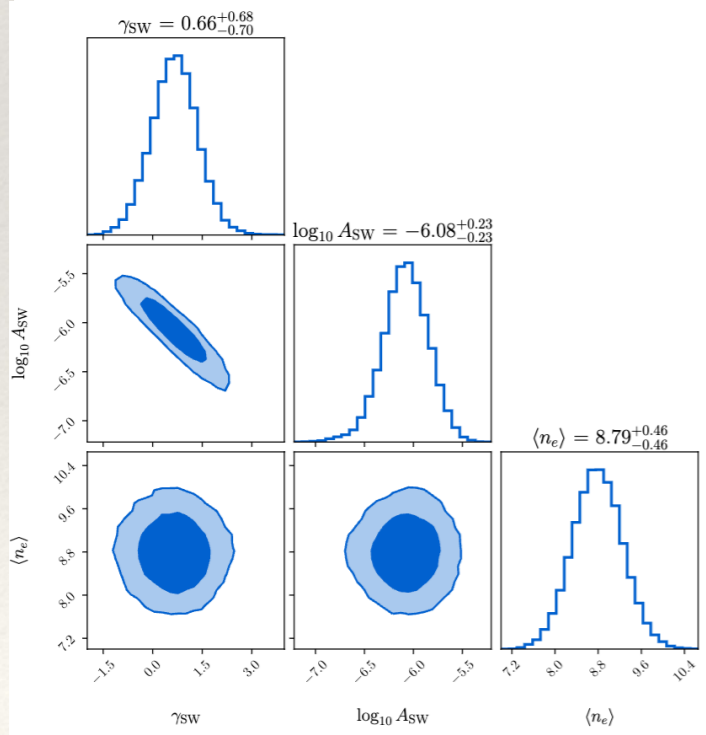


9-th bin close to 1/year frequency, sign of dipolar correlation
 Might be chromatic: DM \rightarrow solar wind?

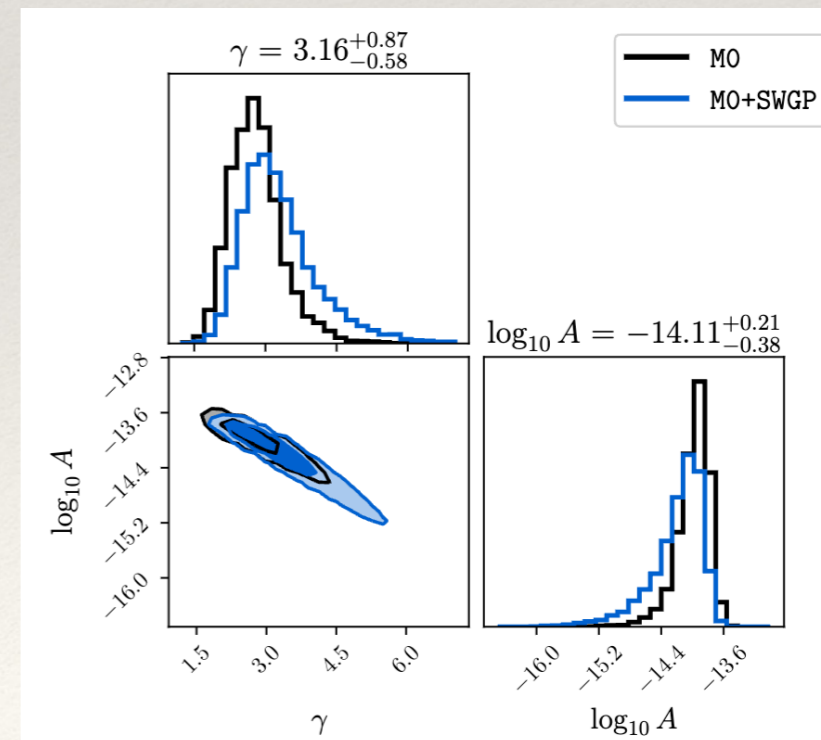
Some pulsar were previously identified to show annual DM variability

Show common solar wind (correlated DM variations with annual variability)

J0030+0451, J0751+1807, J1022+1001, J1730-2304



also affects HD (GW) spectrum

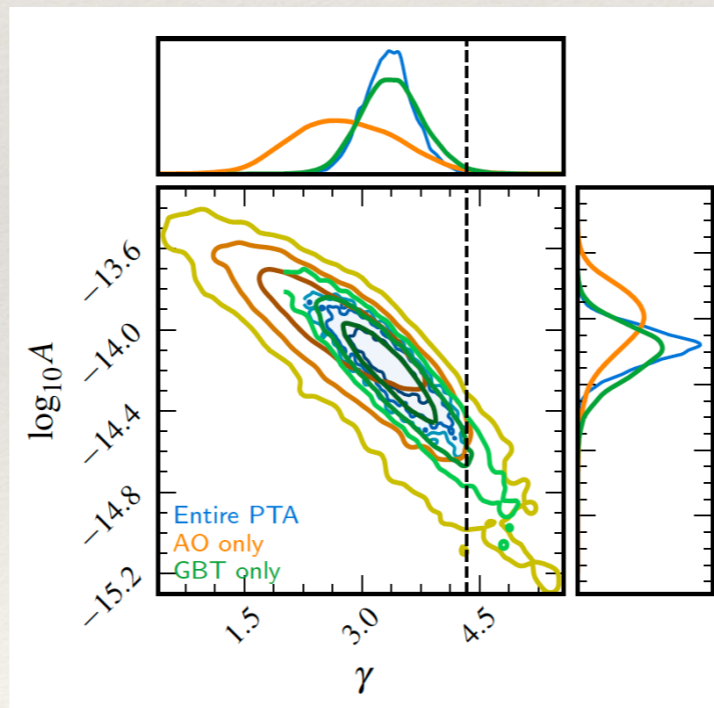
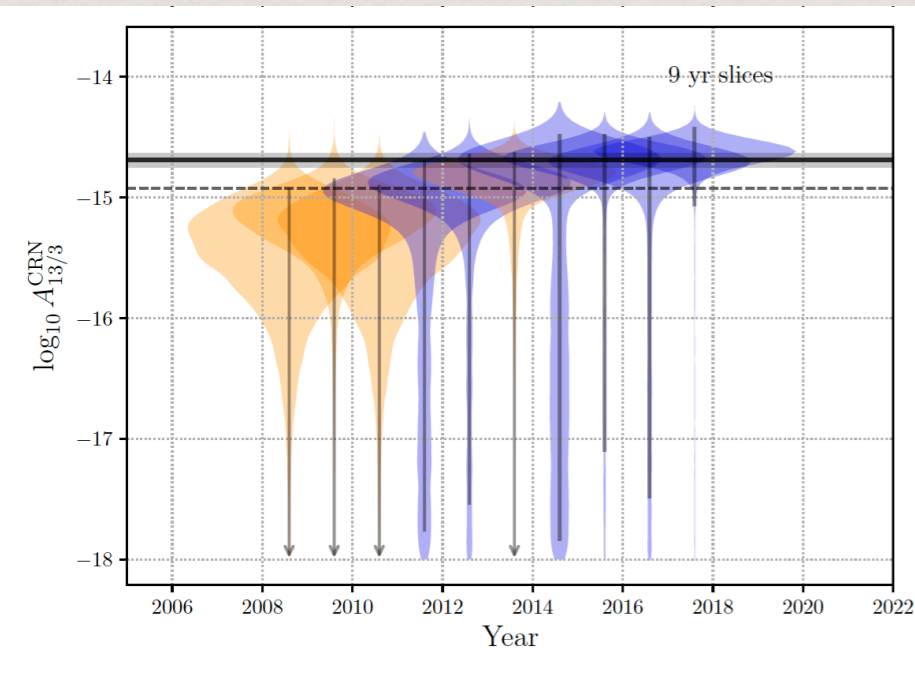


Is it really GW signal?



- Error in ephemerides: JPL ephemerides D440, good measurement of Jupyter
- Modelling noise of each pulsar is very important: J1713+0747
- Quite different BF from each PTA: 1-2 (PPTA), 60-70 (EPTA), 230-950 (NG)
- EPTA “sees” the signal only in last 14 years, PPTA sees signs of non-stationarity
 - Is it non stationarity in the GWB?
 - or in the PSR noise model?
 - or evolution of how we deal with radioobservations?

Non-stationarity modelling: [Falxa+, PRD 2024]



Statistic	DR2full	DR2new	DRnew+
\mathcal{B}_{CURN}^{HD}	4	60 33(+Ephem)	65 43(+Ephem)



New IPTA dataset (DR3)



Credits: Kuo Liu

IPTA DR3 dimensions



- In total **121** pulsars in full DR3;
 - The biggest / most sensitive PTA dataset ever made !!

Dataset	Number of pulsars	Time span	Frequency range
EPTA DR2	25	24.5 yr	283 – 5107 MHz
NANOGrav 15-yr	68	15.9 yr	302 – 3988 MHz
PPTA DR3	24	18.1 yr	704 – 4032 MHz
InPTA DR1	15	3.5 yr	300 – 1460 MHz
MeerKAT DR2	83	4.5 yr	856 – 1712 MHz
CHIME DR1	11	2.5 yr	400 – 800 MHz
LOFAR+NenoFar	17	9.6 yr	35 – 190 MHz
IPTA DR3	121	~25 yr	~30 – 5000 MHz

3



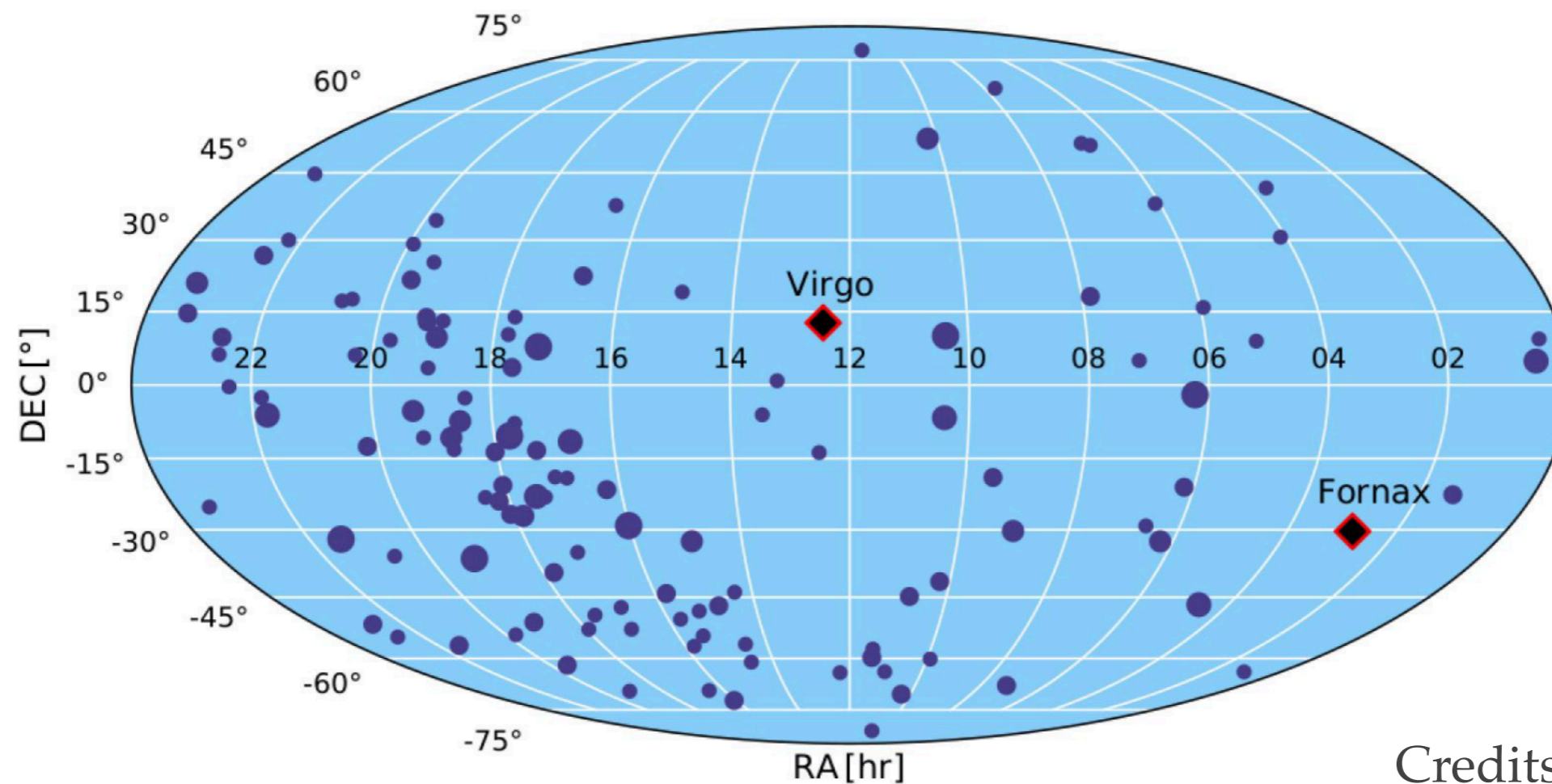
New IPTA dataset (DR3)



IPTA DR3 dimensions



- In total **121** pulsars in full DR3;
 - The biggest / most sensitive PTA dataset ever made !!



Credits Kuo Liu

4



What's next?



IPTA data combination:

- We combine the data from IPTA: EPTA, NG, InPTA, PPTA
- We use additional data (MeerKAT, Chime)
- Better coverage (dense) in time (smaller cadence)
- Better coverage in radio freq: DM and scattering variations
- Not dominated by a single radiotelescope: should see/handle systematics

Kind of summary...

- We are pretty sure that the observed signal is GW
- We are not sure about its nature
- We got so excited that made a big press release
- In relativity we need to look at IPTA data, we need longer high quality data. It is "GW detection in slow motion"



Back up



Consistency?



IPTA ArXiv:2309.00693

