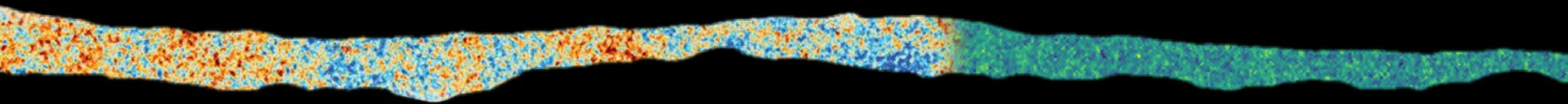
Gravitational Waves to unveil primordial non-Gaussianity



Based on arXiv: 2302.08429
arXiv: 2403.06962

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Annecy, 18/09/24

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OVERVIEW OF THE TALK

1. INTRODUCTION

- Recap on GWs
- Importance of primordial non-Gaussianity

RECAP ON GWs

Focus on the **S**tochastic **G**ravitational **W**ave **B**ackground (SGWB):

Two contributions
- Cosmological
- Astrophysical

We characterize them with

(DIMENSIONLESS)
GW ENERGY
DENSITY

$$\Omega_{\rm GW} \equiv \frac{f_{\rm o}}{\rho_c} \frac{d\rho_{\rm GW}}{df_{\rm o} d\Omega_{\rm o}}$$

Observed frequency ($f_{\rm o}$)

Critical Density (ρ_c)

Regimbau, [1101.2762]
Caprini et al., [1910.13125]
Bartolo et al., [1912.09433]

PRIMORDIAL NON-GAUSSIANITY

Deviations of primordial perturbations from Gaussian behaviour.

(e.g., non vanishing high order connected correlators)

Lowest order Correlator: **Bispectrum**

→ Amplitude $f_{\rm NL}$

→ Shape $F(k_1, k_2, k_3)$

Standard Scenario: Single Field, Slow Roll

Low Level of NG
$f_{NL} \sim 10^{-2} - 10^{-3}$

Latest constraints by Planck

$$f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$$
$$f_{\rm NL}^{\rm equil} = -26 \pm 47$$
$$f_{\rm NL}^{\rm ortho} = -38 \pm 24$$

Planck Collaboration [1905.05697]

There is still space for models predicting a non-vanishing, sufficiently high primordial non-Gaussianity.

Focus on **local** primordial non-Gaussianity.

We can write the NG field as

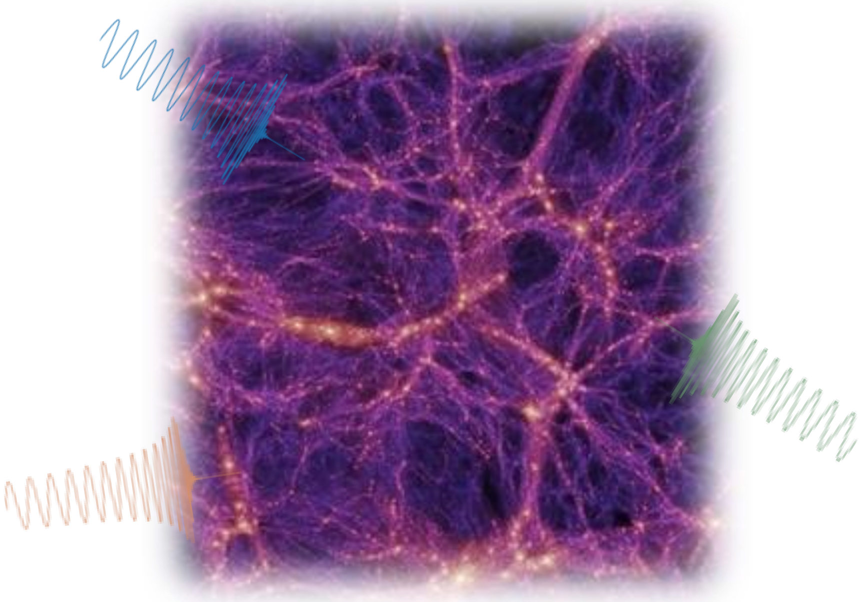
Gaussian Field

$$\Phi(\vec{x}) = \Phi_L(\vec{x}) + f_{\rm NL}\left(\Phi_L^2(\vec{x}) - \langle\Phi_L^2(\vec{x})\rangle\right)$$

HOW DO GWs COULD BE AFFECTED BY THE PRESENCE OF NG?

2. AGWB, CMB AND NON-GAUSSIANITY

- Anisotropies
- Bias and primordial nG
- Cross-correlation prescription
- Results

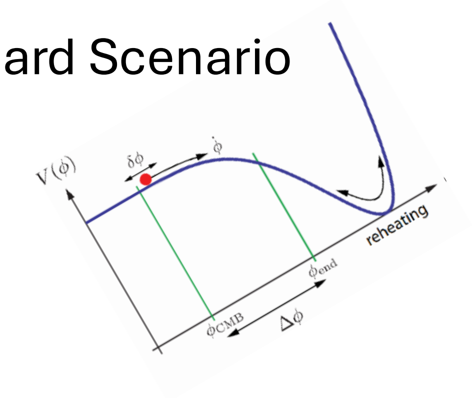


AIM

Extract cosmological information combined with other cosmological probes

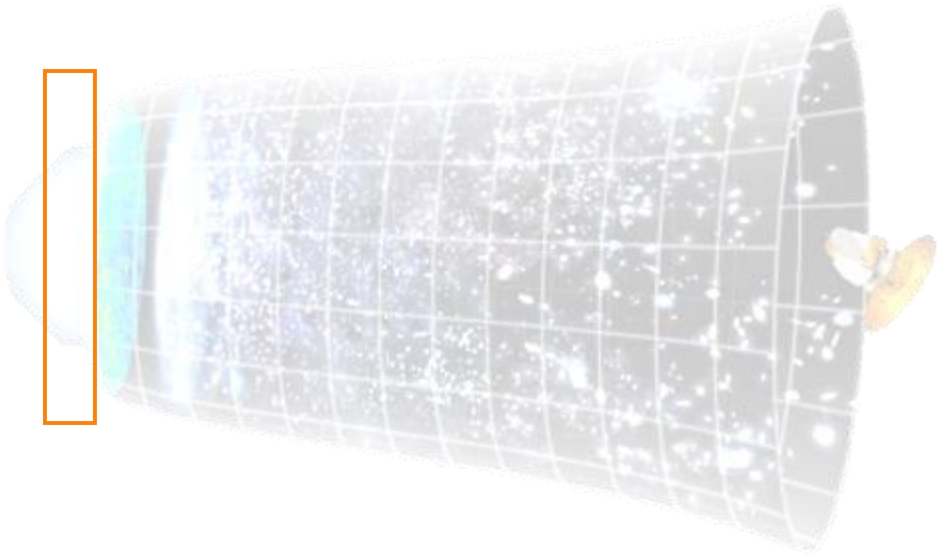
Primordial
non-Gaussianity

Standard Scenario



Different models of inflation predict
different types of non Gaussianity,
So a better constraint would help us
to discard many of them!

Cosmic Microwave
Background



Anisotropies - AGWB

AGWB: Superposition of a large number of unresolved sources since the beginning of stellar activity.

Two contributions:

$$\Omega_{\rm GW} \equiv \frac{f_{\rm o}}{\rho_c}\frac{d\rho_{\rm GW}}{df_{\rm o}d\Omega_{\rm o}} = \frac{\bar{\Omega}_{\rm GW}(f)}{4\pi} + \Delta\Omega_{\rm GW}(f,\hat{\mathbf{n}})$$

Isotropic ← $\frac{\bar{\Omega}_{\rm GW}(f)}{4\pi}$

Anisotropic ← $\Delta\Omega_{\rm GW}(f,\hat{\mathbf{n}})$

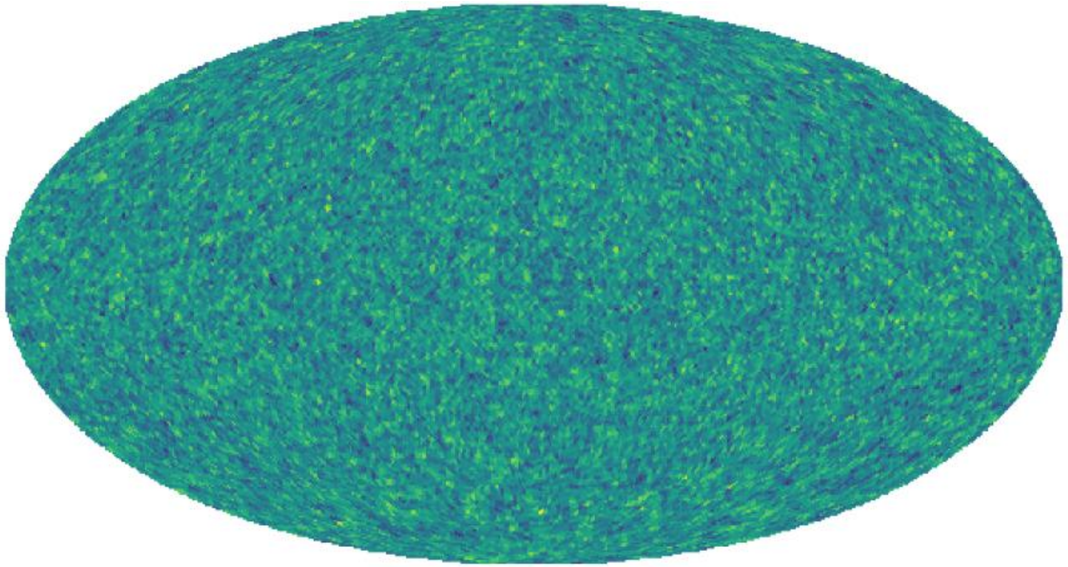


Image Credit: Ricciardone et al., [2106.02591]

Contaldi, [1609.08168]
Cusin et al., [1704.06184]
Jenkins et al., [1806.01718]
Cusin et al.,[1904.07757]
Bertacca et al., [1909.11627]

In Poisson Gauge:

$$\Delta\Omega^{[i]}_{\mathrm{GWB}}(f_o,\hat{\mathbf{n}}) = \frac{f_o}{\rho_{0c}c^2}\int \frac{dz}{H(z)}\int d\boldsymbol{\theta}\, p^{[i]}(\boldsymbol{\theta}|z)\,[1-\varepsilon_{\mathrm{res}}(z,\boldsymbol{\theta})]\,\frac{\bar{R}^{[i]}}{1+z}\,\overline{\frac{dE^{[i]}_{\mathrm{GW},e}}{df_e d\Omega_e}}$$

$$\times\Bigg\{\delta^{[i]}_{\mathrm{GW}}$$

$$+\left(b^{[i]}_{\mathrm{evo}}-2-\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\hat{\mathbf{n}}\cdot\mathbf{V}-\frac{1}{\mathcal{H}}\partial_{\|}(\hat{\mathbf{n}}\cdot\mathbf{V})-(b^{[i]}_{\mathrm{evo}}-3)\mathcal{H}V$$

$$+\left(3-b^{[i]}_{\mathrm{evo}}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\Psi+\frac{1}{\mathcal{H}}\Phi'+\left(2-b^{[i]}_{\mathrm{evo}}+\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\int_0^{\chi(z)}d\chi\,(\Psi'+\Phi')$$

$$+\left(b^{[i]}_{\mathrm{evo}}-2-\frac{\mathcal{H}'}{\mathcal{H}^2}\right)\left(\Psi_o-\mathcal{H}_0\int_0^{\tau_0}d\tau\,\frac{\Psi(\tau)}{1+z(\tau)}\Bigg|_o-(\hat{\mathbf{n}}\cdot\mathbf{V})_o\right)\Bigg\},$$

Density

Velocity

Gravity

Observer

GW Sources overdensity

$b^{[i]}\delta_m$

Bias

Dark Matter overdensity

Bertacca et al., [1909.11627]
Bellomo et al., [2110.15059]

$$\Delta\Omega_{\rm GW}(\mathbf{n}) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{+\ell} a_{\ell m} Y_{\ell m}(\mathbf{n}) \quad \Longrightarrow \quad a_{\ell m} = \int d\mathbf{n} Y^*_{\ell m}(\mathbf{n})\Delta\Omega_{\rm GW}(\mathbf{n})$$

Spherical harmonics

Physics inside!

In Fourier Space and considering the sum over different contributions

$$a_{\ell m} = \sum_{[i]\beta} \int \frac{d^3k}{(2\pi)^3} Y^*_{\ell' m}(\hat{\mathbf{k}}) \mathcal{S}_\ell^{[i]\beta}(k) \delta_m(\mathbf{k}, \eta_0)$$

Source Function

$[i]$ = source

β = anisotropy term

with

$$\mathcal{S}_\ell^{[i]\delta_m^{(\rm SC)}}(k) = (4\pi) i^\ell \int d\bar{\chi} \mathcal{W}^{[i]}(\bar{\chi}) b_{\rm GW}^{[i]} \frac{D(\eta)}{D(\eta_0)} j_\ell(k\bar{\chi}) \quad \ldots$$

$D(\eta)$ = growth function

ANISOTROPIES - CMB

$$T(\mathbf{x}, \hat{\mathbf{n}}, \eta) = T(\eta)\left[1 + \Theta(\mathbf{x}, \hat{\mathbf{p}}, \eta)\right]$$

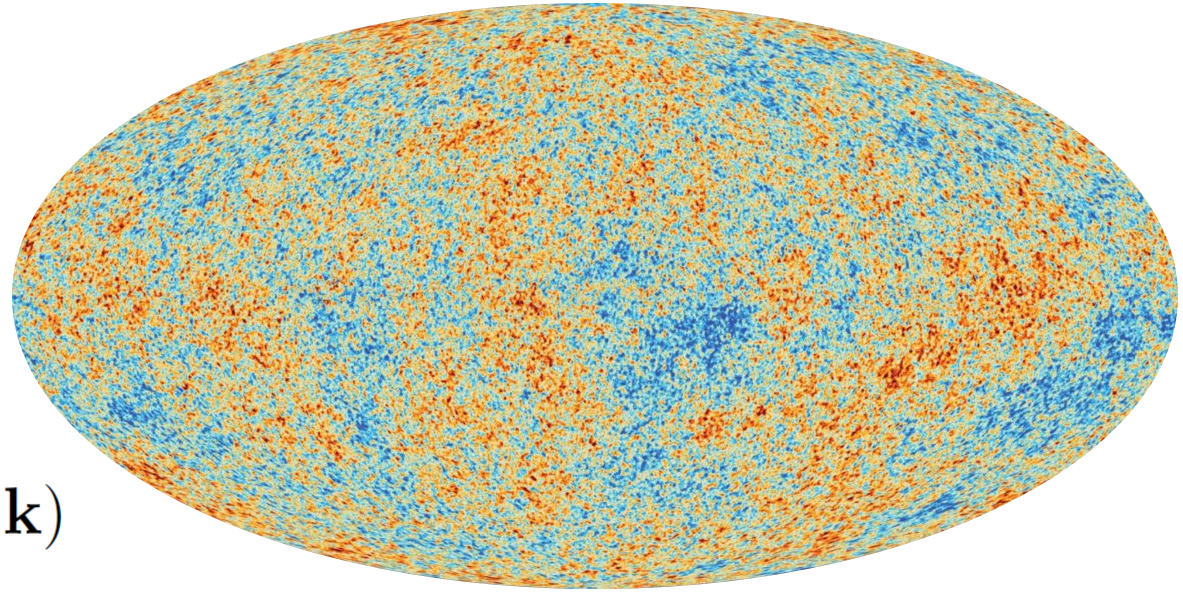


Image Credit: ESA

In this case

$$a_{\ell m}^{\rm CMB}(\mathbf{x}, \eta_0) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} Y_{\ell m}^*(\hat{k}) \mathcal{T}_\ell(\hat{p}, \eta_0) \zeta(\mathbf{k})$$

And the transfer functions

$$\mathcal{T}_\ell^{\rm ISW}(k, \eta_0) = (4\pi) i^\ell \int_0^{\eta_0} d\eta \left[T'_\Psi + T'_\Phi\right] e^{-\tau} j_\ell(k(\eta_0 - \eta))$$

← Integrated Sachs-Wolfe

$$\mathcal{T}_\ell^{\rm SW}(k, \eta_0) = (4\pi) i^\ell \int_0^{\eta_0} d\eta \delta(\eta - \eta_*) T_\Phi j_\ell(k(\eta_0 - \eta))$$

← Sachs-Wolfe

$$\mathcal{T}_\ell^{\rm DOP}(k, \eta_0) = (4\pi) i^\ell \int_0^{\eta_0} d\eta \delta(\eta - \eta^*) T_v \frac{d}{d\eta}\left[j_\ell[k(\eta_0 - \eta)]\right]$$

← Doppler

BIAS AND NON-GAUSSIANITY

GOAL: Understand clustering properties of **Dark Matter**...

...by means of «luminous» sources

TRACER [i]

- $\text{Tracer}_1 = b_1 \delta_m$
- $\text{Tracer}_2 = b_2 \delta_m$

$b_1 \neq b_2$!

Tracers in this work: GW Binaries

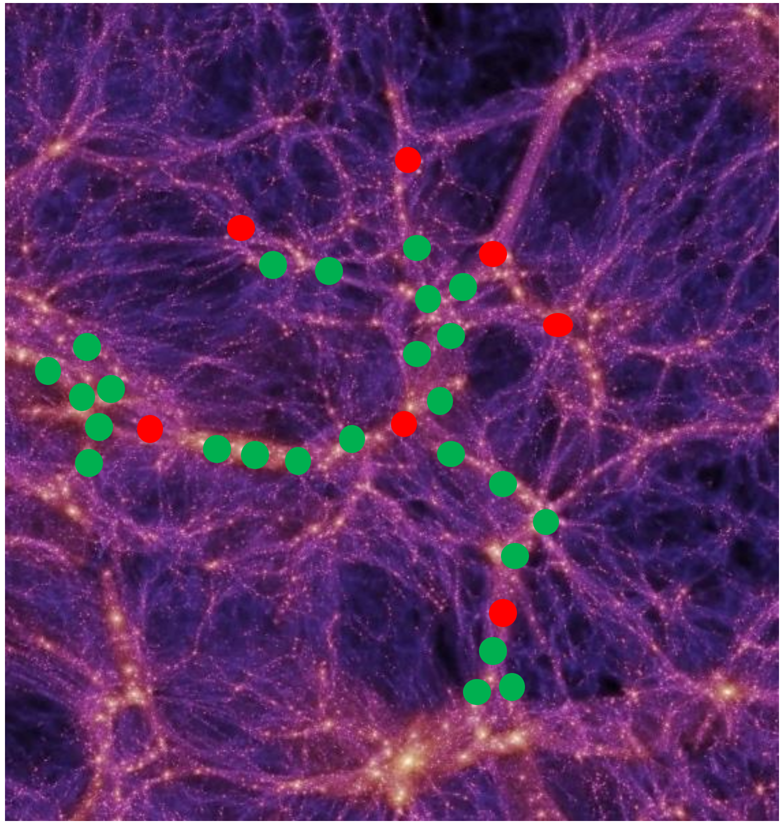


Image Credit: Millenium Simulation

Desjaques et al, [1611.09797]

How does primordial non-Gaussianity affect the evolution of structures in the Universe?

Additional scale dependent contribution

$$\Delta b(k) = 3 f_{\mathrm{NL}} (b - p) \delta_c \frac{\Omega_m}{T(k) D(z) k^2} \left(\frac{H_0}{c} \right)^2$$

Matter Transfer Function

1. Models of Inflation
2. Large Scales

Dalal et al., [0710.4560]
Matarrese, Verde [0801.4826]
Slosar et al., [0805.3580]

CROSS-CORRELATION PRESCRIPTION

Evaluation of the Cross-Correlation between the AGWB and the CMB anisotropies considering the presence of an additional contribution to the bias due to the presence of NG

$$\langle a^{\alpha}_{\ell m} a^{\beta *}_{\ell' m'} \rangle = \delta_{mm'}\delta_{\ell\ell'} C^{\alpha\beta}_{\ell}$$

$a^{\mathrm{CMB}}_{\ell m}$ → $a^{\alpha}_{\ell m}$

$a^{*\mathrm{AGWB}}_{\ell' m'}$ → $a^{\beta *}_{\ell' m'}$

$$C^{\mathrm{GW}\times\mathrm{CMB}}_{\ell} = \sum_{[i]\alpha} \int \frac{dk_1}{(2\pi)^3} k_1^2 \mathcal{S}^{[i]\alpha}_{\ell}(\hat{k}_1) \mathcal{T}^{*}_{\ell}(k_1) P_{\zeta}(k_1)$$

$$C^{\mathrm{GW}\times\mathrm{CMB}}_{\ell} = C^{\mathrm{ISW}\times\delta^{(SC)}_{m}}_{\ell} + C^{\mathrm{ISW}\times(\hat{\mathbf{n}}\cdot\mathbf{v})}_{\ell} + \;.....$$

Dominant Contribution

RESULTS (1)

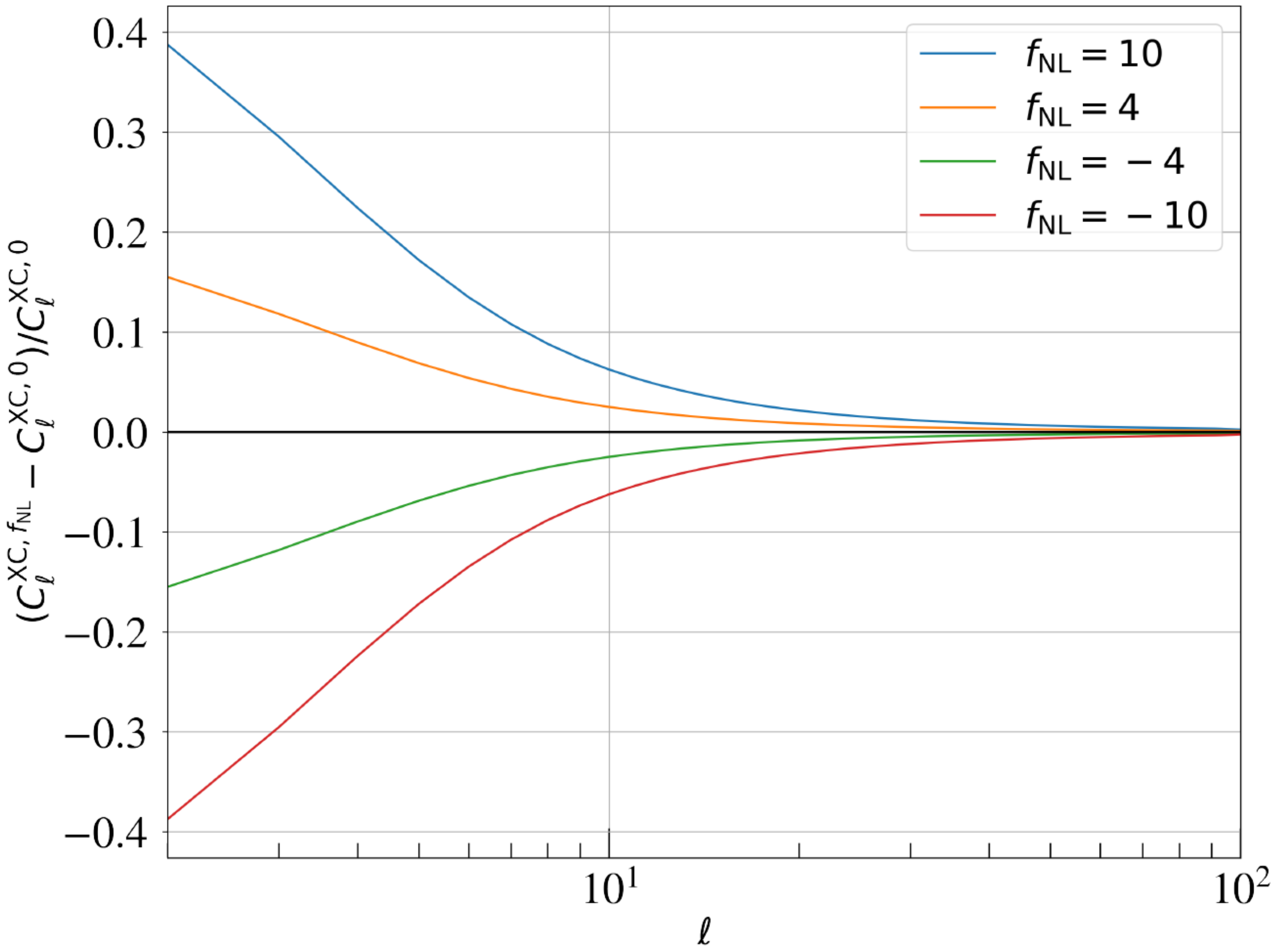
b = 1.5

I modified the CLASS Code

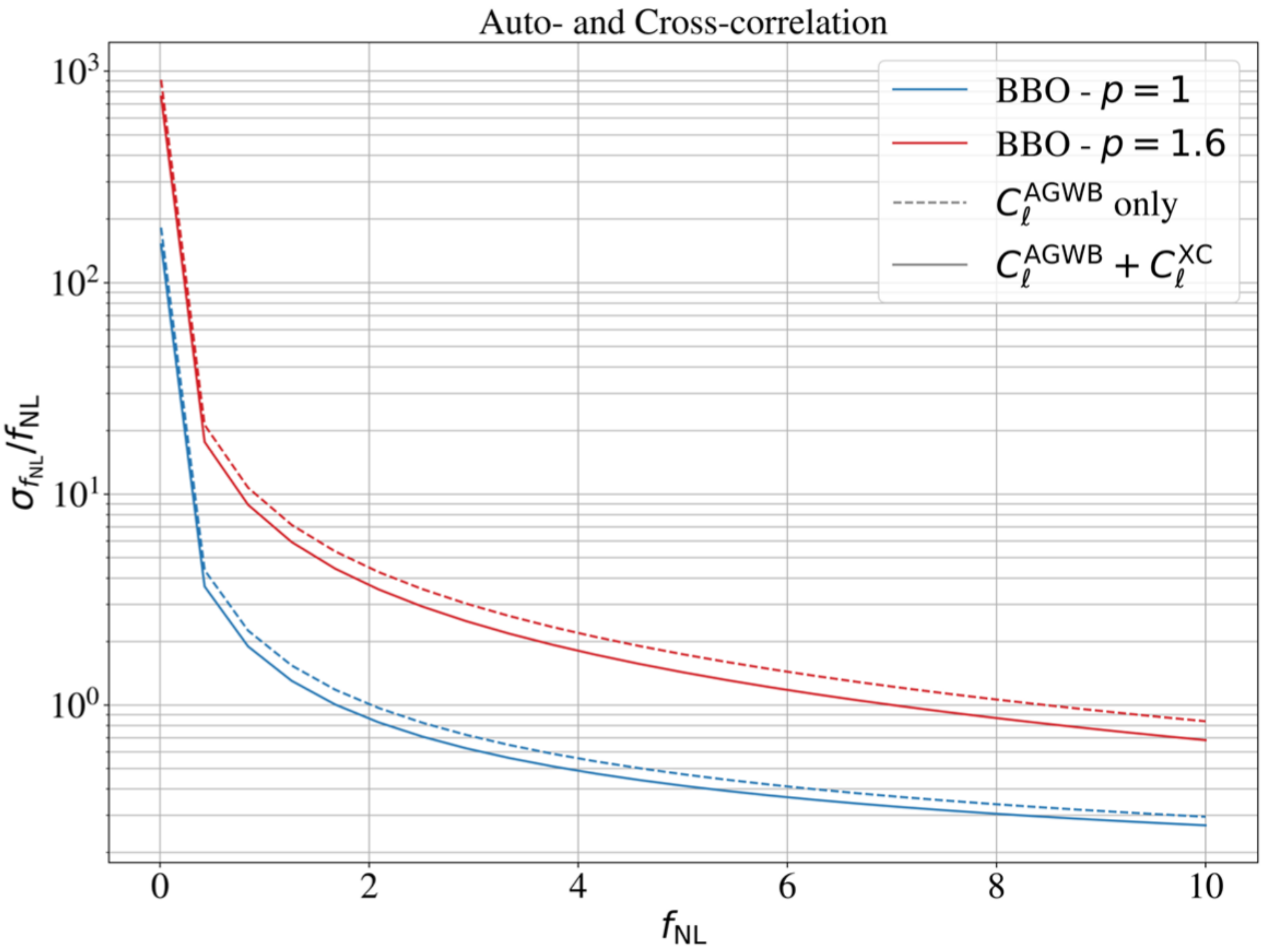
- AGWB anisotropies
- Bias correction

  +

  Astrophysical Module



RESULTS (2)



Auto- and Cross-correlation
BBO - $p = 1$
BBO - $p = 1.6$
$C_\ell^{\rm AGWB}$ only
$C_\ell^{\rm AGWB} + C_\ell^{\rm XC}$
$\sigma_{f_{\rm NL}}/f_{\rm NL}$
10^3
10^2
10^1
10^0
0
2
4
6
8
10
$f_{\rm NL}$

3. IMPRINTS OF NG ON THE SIGWB

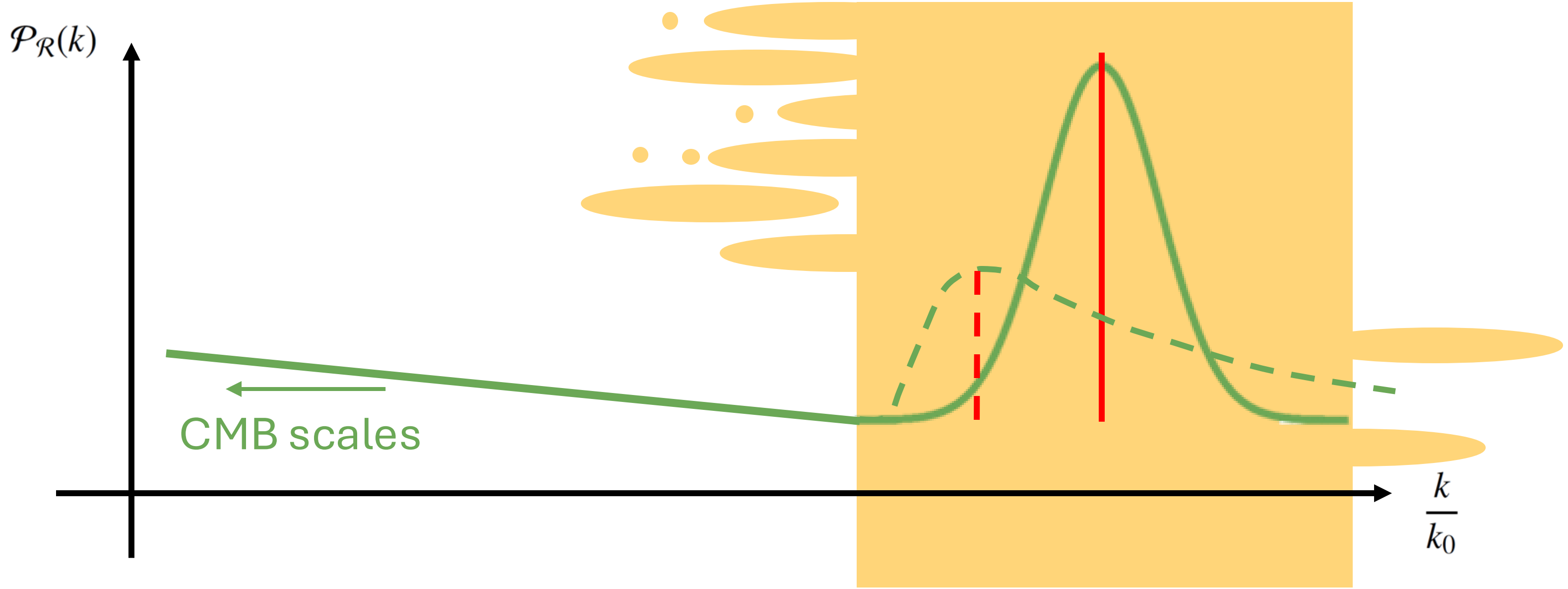
- Theoretical treatment
- SIGWB Spectrum
- Imprints of nG
- Results

SCALAR-INDUCED GWS

- It is possible to generate GWs by means of mode coupling of scalar modes at **second order** in perturbation theory.

- A large range of scales has been constrained by the CMB, **BUT** a huge window at the smaller scales remains *unconstrained*...

- SIGWs provide a *complementary* way to probe the power spectrum at the scales of interest.



[Matarrese et al; ‘93,’94]
[Ananda et al; ‘06]
[Baumann et al; ‘07] ...

SOURCING SIGW

Start from the conformal metric, written up to **second** order in perturbations

$$ds^2 = a^2(\eta)\Big\{-\left(1+2\Phi^{(1)}+\Phi^{(2)}\right)d\eta^2 + 2\omega_i^{(2)}d\eta dx^i + \left[(1-2\Psi^{(1)}-\Psi^{(2)})\delta_{ij}+\frac{1}{2}h_{ij}\right]dx^i dx^j\Big\}$$

To obtain the equation for GWs, accounting for the presence of scalar perturbation.

Einstein equations

Projecting on the *Transverse-Traceless* Gauge to extract the gauge invariant modes

Projection tensor

$$\hat{\mathcal{T}}_{ij}^{lm}G_{\mu\nu}^{(2)} = 8\pi G\hat{\mathcal{T}}_{ij}^{lm}T_{\mu\nu}^{(2)}$$

SIGWB SPECTRUM

$$\langle h_{\lambda_1}(\mathbf{k}_1,\eta_1)h_{\lambda_2}(\mathbf{k}_2,\eta_2)\rangle = 16\int \frac{d^3q_1}{(2\pi)^{3/2}}\int \frac{d^3q_2}{(2\pi)^{3/2}} Q_{\lambda_1}(\mathbf{k}_1,\mathbf{q}_1) I(|\mathbf{k}_1-\mathbf{q}_1|,q_1,\eta_1)$$
$$\times Q_{\lambda_2}(\mathbf{k}_2,\mathbf{q}_2) I(|\mathbf{k}_2-\mathbf{q}_2|,q_2,\eta_2)$$
$$\times \langle \mathcal{R}(\mathbf{q}_1)\mathcal{R}(\mathbf{k}_1-\mathbf{q}_1)\mathcal{R}(\mathbf{q}_2)\mathcal{R}(\mathbf{k}_2-\mathbf{q}_2)\rangle$$

Mapping to observable quantities

$$\Omega_{\rm GW}(k,\eta) \equiv \frac{\rho_{\rm GW}(k,\eta)}{\rho_{tot}(\eta)} = \frac{1}{48}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \sum_{\lambda=+,\times} \overline{\Delta^2_{h,\lambda}(k,\eta)}$$

At the end

$$\Omega_{\rm GW,0}(k)h^2 = \Omega_{rad,0}h^2\left(\frac{g_{*,0}}{g_{*,e}}\right)^{1/3}\Omega_{\rm GW,e}(k)$$

e.g. [Watanabe, Komatsu; ‘06]

Imprints of non-Gaussianity

$$\Omega_{\mathrm{GW},0}(k)h^2 \propto \int d^3q_1\, d^3q_2 \ \langle \mathcal{R}(\mathbf{q}_1)\mathcal{R}(\mathbf{k}_1-\mathbf{q}_1)\mathcal{R}(\mathbf{q}_2)\mathcal{R}(\mathbf{k}_2-\mathbf{q}_2)\rangle$$

Focus on local non-Gaussianity

$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) + f_{\mathrm{NL}}(\mathcal{R}_g^2(\mathbf{x}) - \langle \mathcal{R}_g^2\rangle) + g_{\mathrm{NL}}\mathcal{R}_g^3(\mathbf{x}) + h_{\mathrm{NL}}(\mathcal{R}_g^4(\mathbf{x}) - 3\langle \mathcal{R}_g^2\rangle^2) + i_{\mathrm{NL}}\mathcal{R}_g^5(\mathbf{x})$$

Why considering up to the 5$^{\text{th}}$ order in the expansion?

- Properly account for all the possible contributions at the next-to and next-to-next-to leading order in the perturbations
- Perform the calculations without any assumption on the hierarchy among the nG parameters.

Imprints of non-Gaussianity

$$\mathcal{R}^{\rm NG}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) + f_{\rm NL}(\mathcal{R}_g^2(\mathbf{x}) - \langle \mathcal{R}_g^2 \rangle) + g_{\rm NL}\mathcal{R}_g^3(\mathbf{x}) + h_{\rm NL}(\mathcal{R}_g^4(\mathbf{x}) - 3\langle \mathcal{R}_g^2 \rangle^2) + i_{\rm NL}\mathcal{R}_g^5(\mathbf{x})$$

In Fourier space

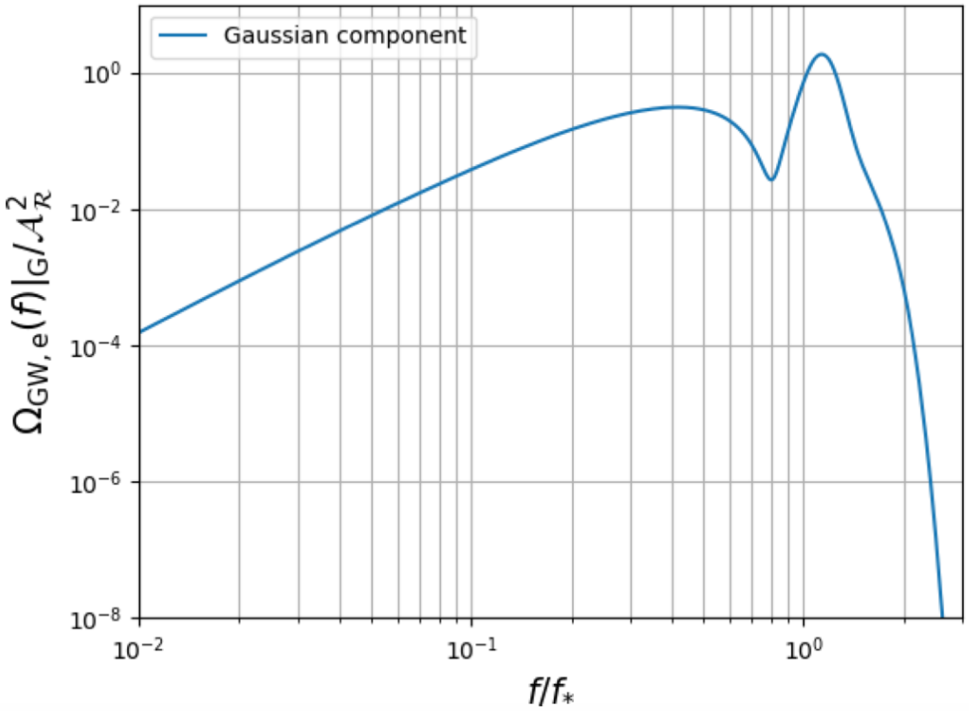
$$\langle \mathcal{R}^{\rm NG}_{\mathbf{k}_1}\mathcal{R}^{\rm NG}_{\mathbf{k}_2}\mathcal{R}^{\rm NG}_{\mathbf{k}_3}\mathcal{R}^{\rm NG}_{\mathbf{k}_4} \rangle$$

Leading Order 4-point $= \langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\mathcal{R}_{\mathbf{k}_4} \rangle$

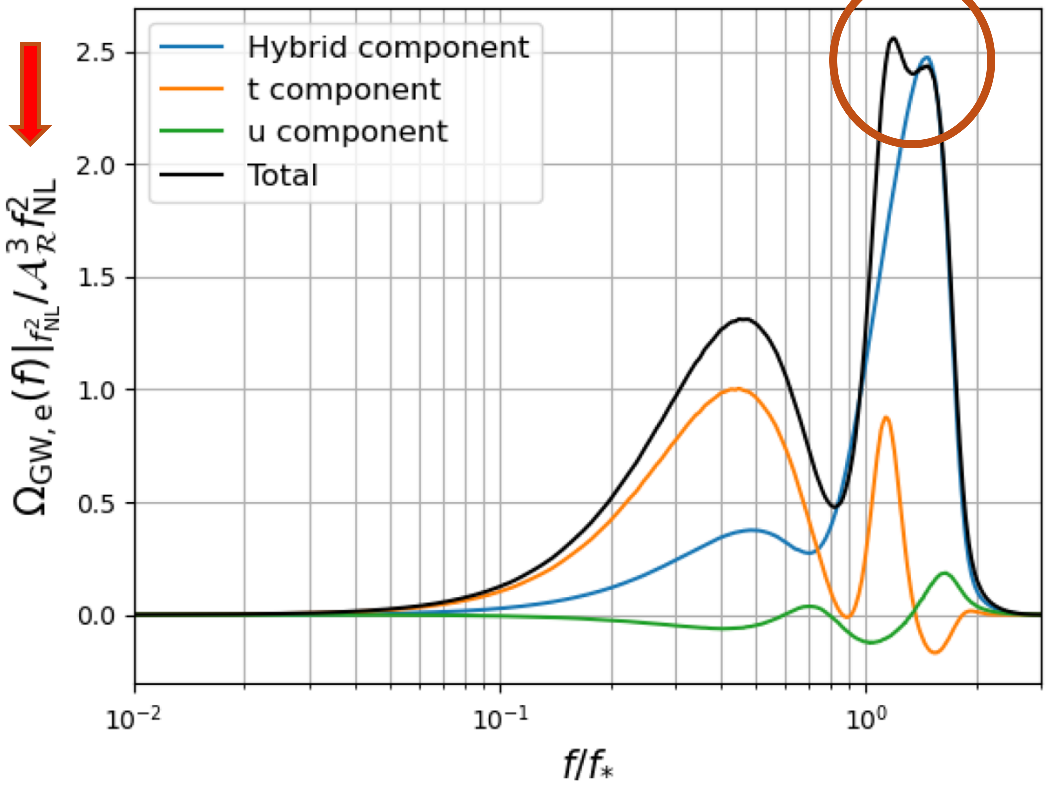
NT LO 6-point $+ \langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\mathcal{R}_{\mathbf{k}_4}\mathcal{R}_{\mathbf{k}_5}\mathcal{R}_{\mathbf{k}_6} \rangle$

NTNT LO 8-point $+ \langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\mathcal{R}_{\mathbf{k}_4}\mathcal{R}_{\mathbf{k}_5}\mathcal{R}_{\mathbf{k}_6}\mathcal{R}_{\mathbf{k}_7}\mathcal{R}_{\mathbf{k}_8} \rangle$

Leading Order



NT LO

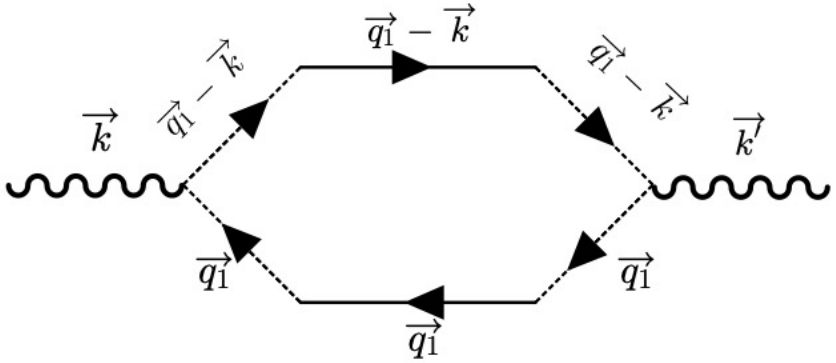


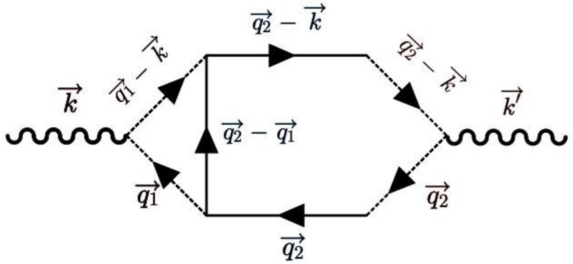
LogNormal Spectrum:

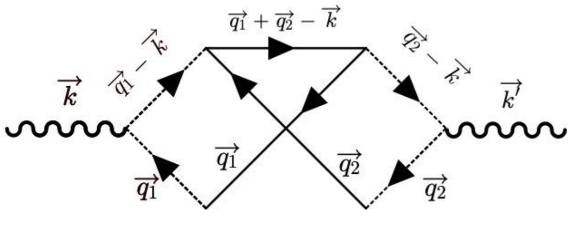
$$\Delta_g^2(k) = \frac{\mathcal{A}_\mathcal{R}}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\ln^2(k/k_*)}{2\sigma^2}\right)$$

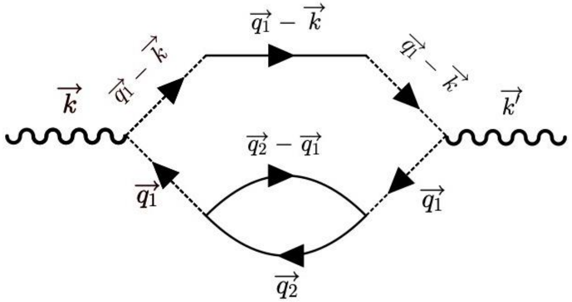
FEYNMAN DIAGRAMS

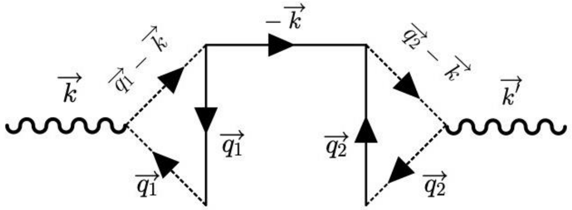
$$\Omega_{\mathrm{GW},0}(k)h^2 \propto \int d^3q_1\, d^3q_2\ \langle \mathcal{R}(\mathbf{q}_1)\mathcal{R}(\mathbf{k}_1-\mathbf{q}_1)\mathcal{R}(\mathbf{q}_2)\mathcal{R}(\mathbf{k}_2-\mathbf{q}_2)\rangle$$



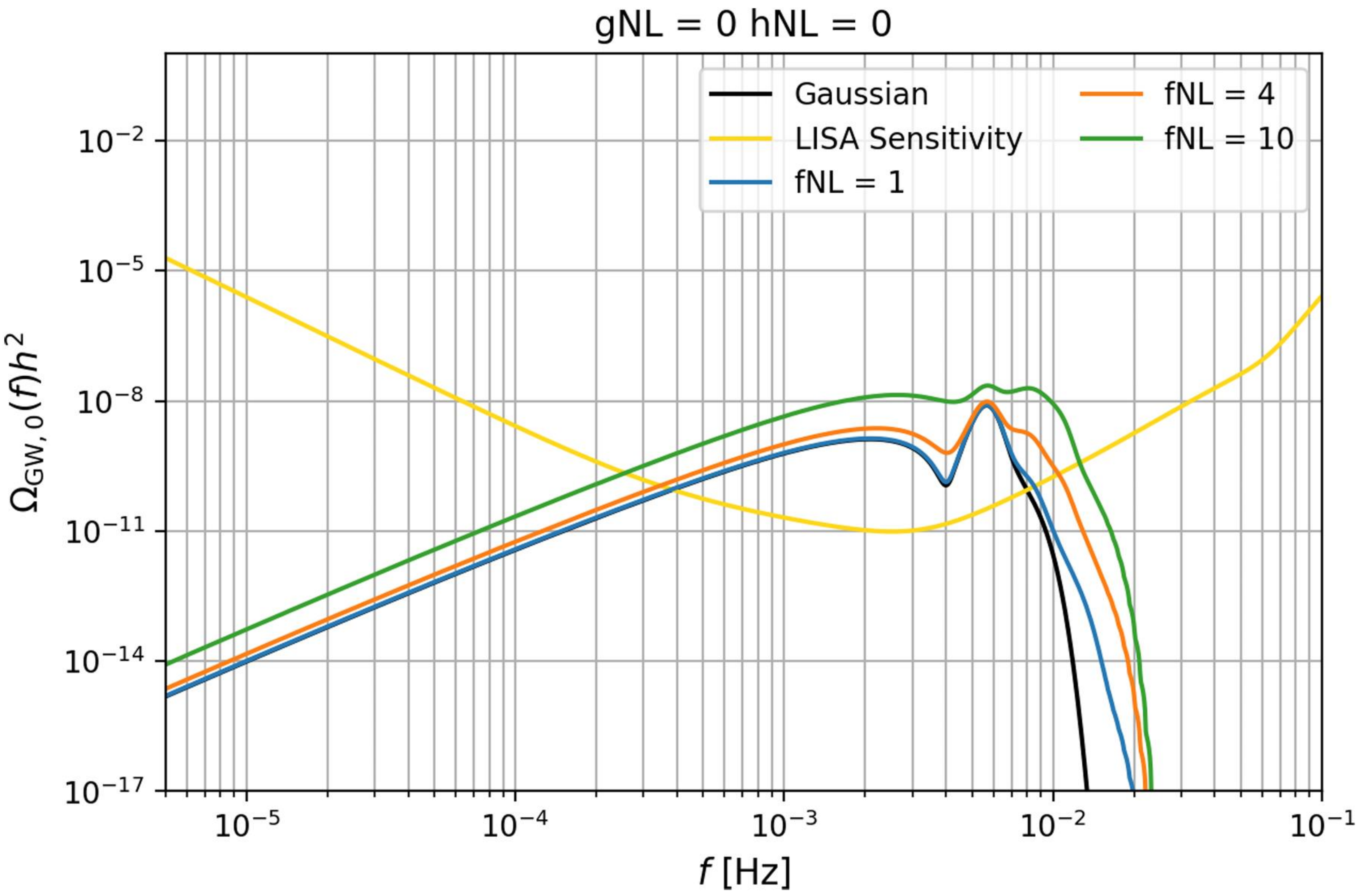






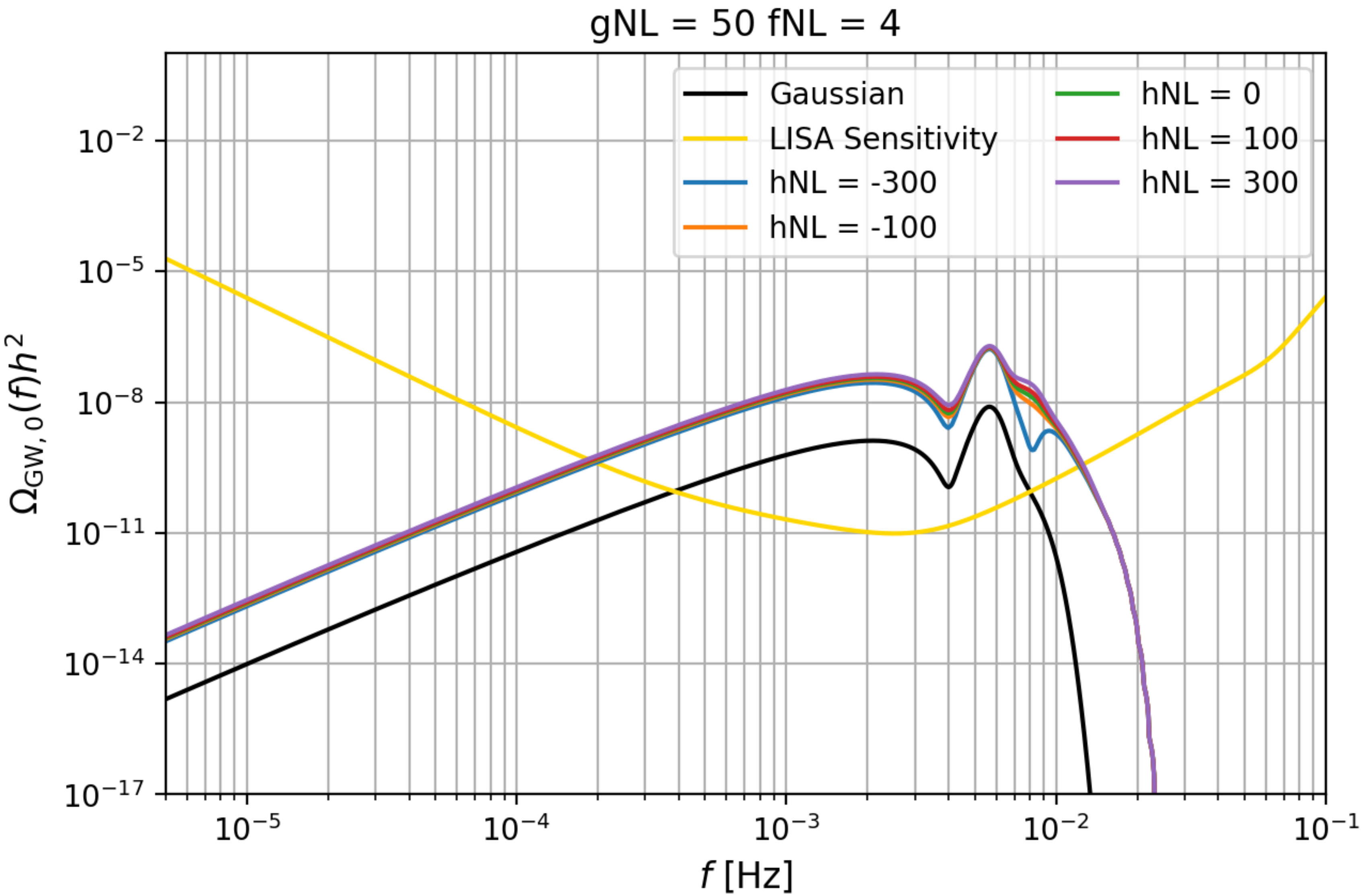


RESULTS



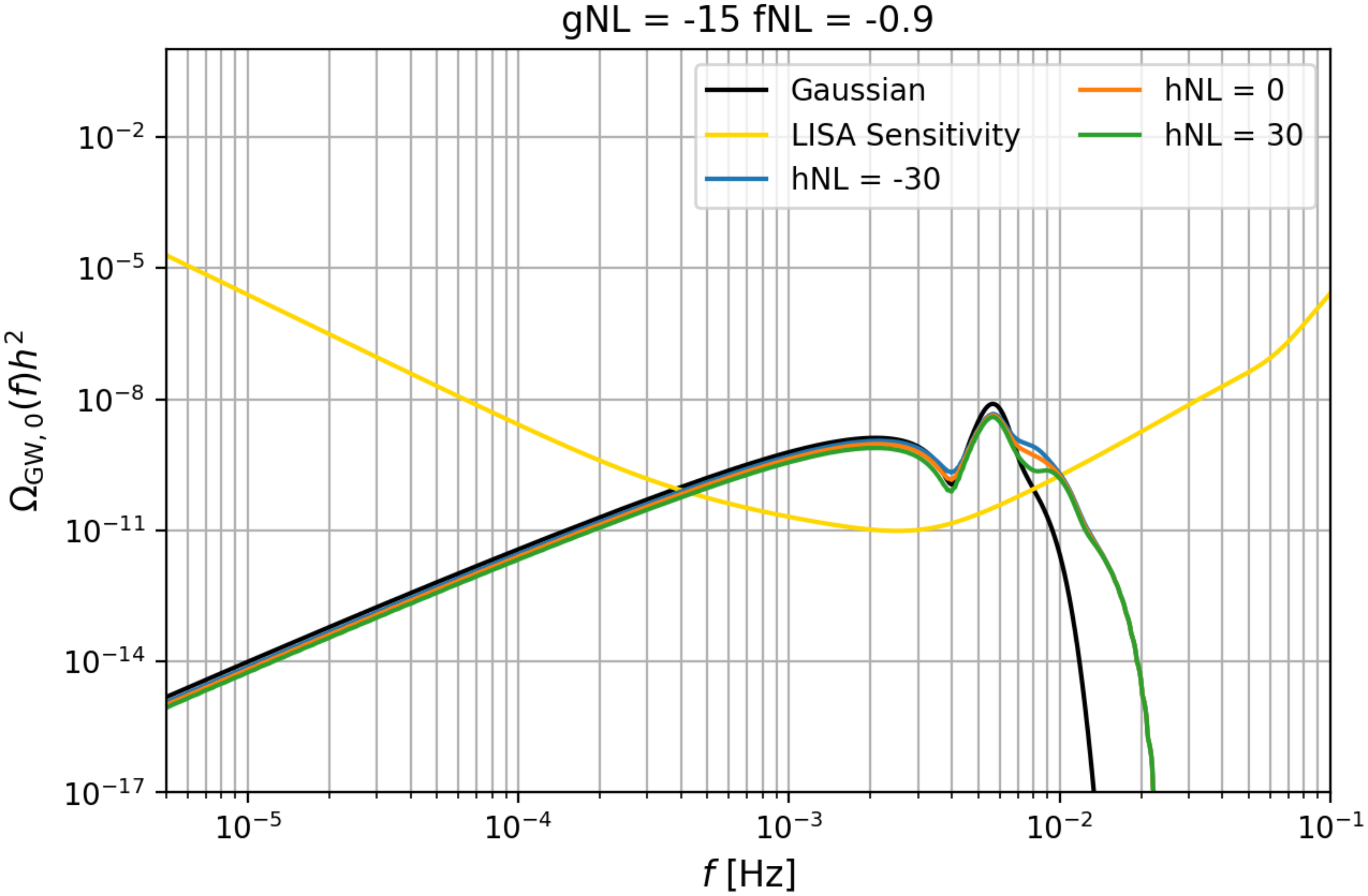
gNL = 0 hNL = 0
Gaussian
LISA Sensitivity
fNL = 1
fNL = 4
fNL = 10
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
$\Omega_{GW,0}(f)h^2$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
f [Hz]

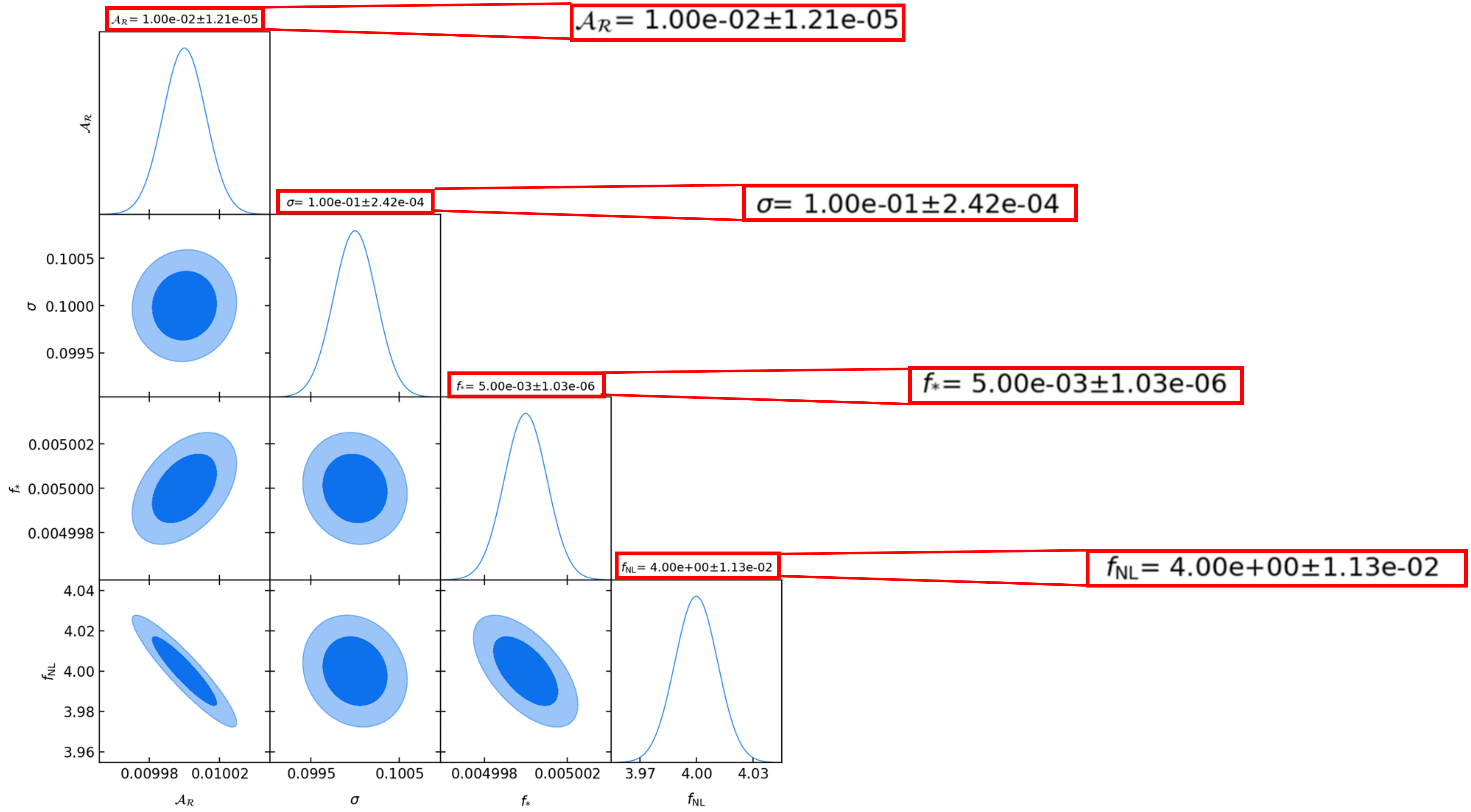
RESULTS

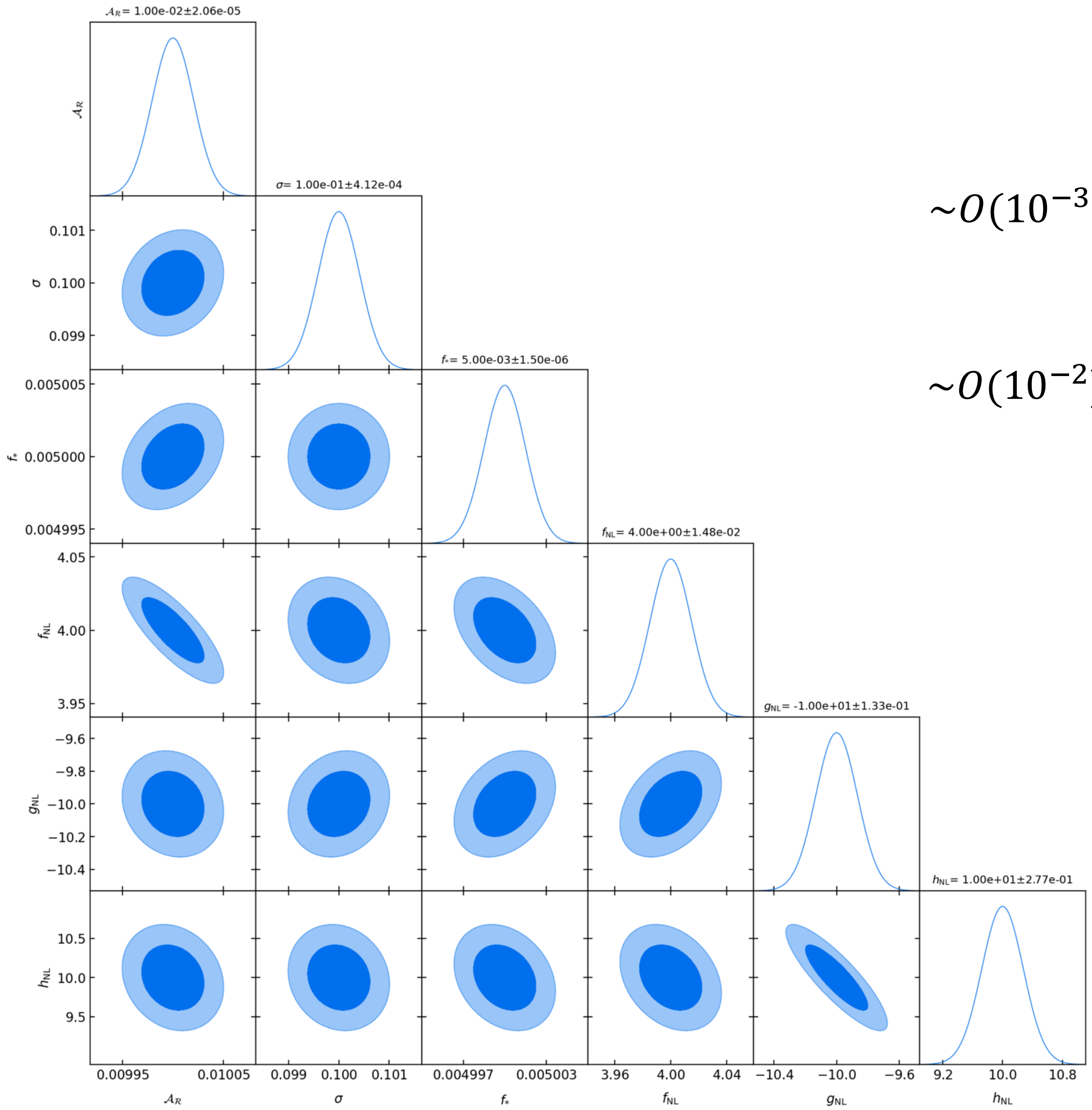


gNL = 50 fNL = 4
Gaussian
LISA Sensitivity
hNL = -300
hNL = -100
hNL = 0
hNL = 100
hNL = 300
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
$\Omega_{GW,0}(f)h^2$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
f [Hz]

Results


gNL = -15 fNL = -0.9
Gaussian
LISA Sensitivity
hNL = -30
hNL = 0
hNL = 30
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
$\Omega_{GW,0}(f)h^2$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
f [Hz]


$\mathcal{A}_\mathcal{R}$= 1.00e-02±1.21e-05
σ= 1.00e-01±2.42e-04
f_*= 5.00e-03±1.03e-06
f_{NL}= 4.00e+00±1.13e-02
0.1005
0.1000
0.0995
0.005002
0.005000
0.004998
4.04
4.02
4.00
3.98
3.96
0.00998
0.01002
0.0995
0.1005
0.004998
0.005002
3.97
4.00
4.03
$\mathcal{A}_\mathcal{R}$
σ
f_*
f_{NL}



$\sim O(10^{-3})$

$\mathcal{A}_\mathcal{R}$ = 1.00e-02±2.06e-05

σ = 1.00e-01±4.12e-04

f_* = 5.00e-03±1.50e-06

$\sim O(10^{-2})$

f_{NL} = 4.00e+00±1.48e-02

g_{NL} = -1.00e+01±1.33e-01

h_{NL} = 1.00e+01±2.77e-01

CONCLUSIONS

- Constraining primordial non-Gaussianity is fundamental to unveil the dynamics of the early-Universe
- The resulting imprints on the GW anisotropies of the astrophysical background could be important on the largest-scales and in principle detectable by future interferometers.
- On the other hand, the presence of nG could leave specific imprints on the SIGW signal that could be detectable with LISA.
- More in detail LISA could be able to measure both the shape parameters and the nG parameters, with higher order terms able to break degeneracies

Thank you!