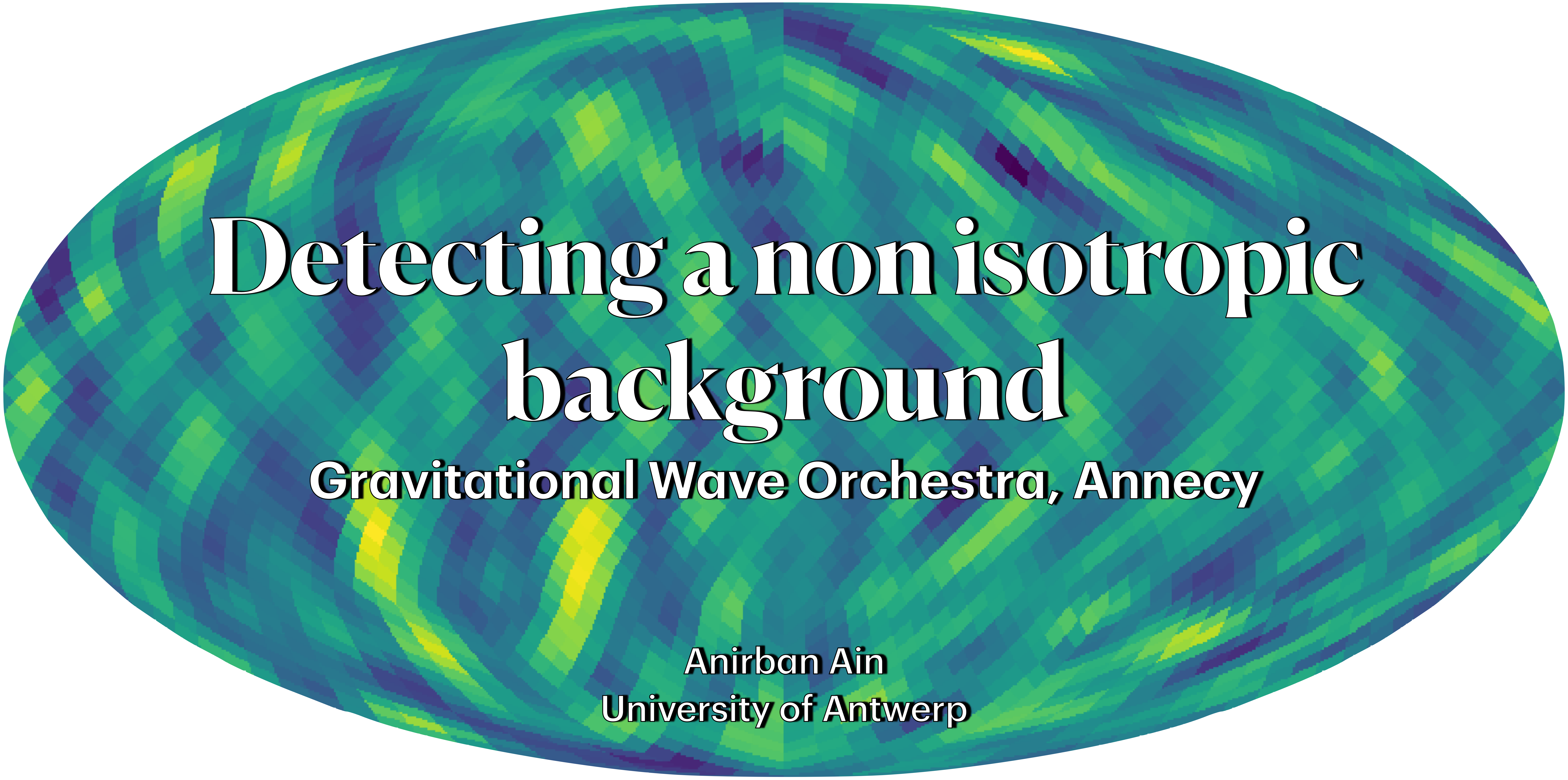


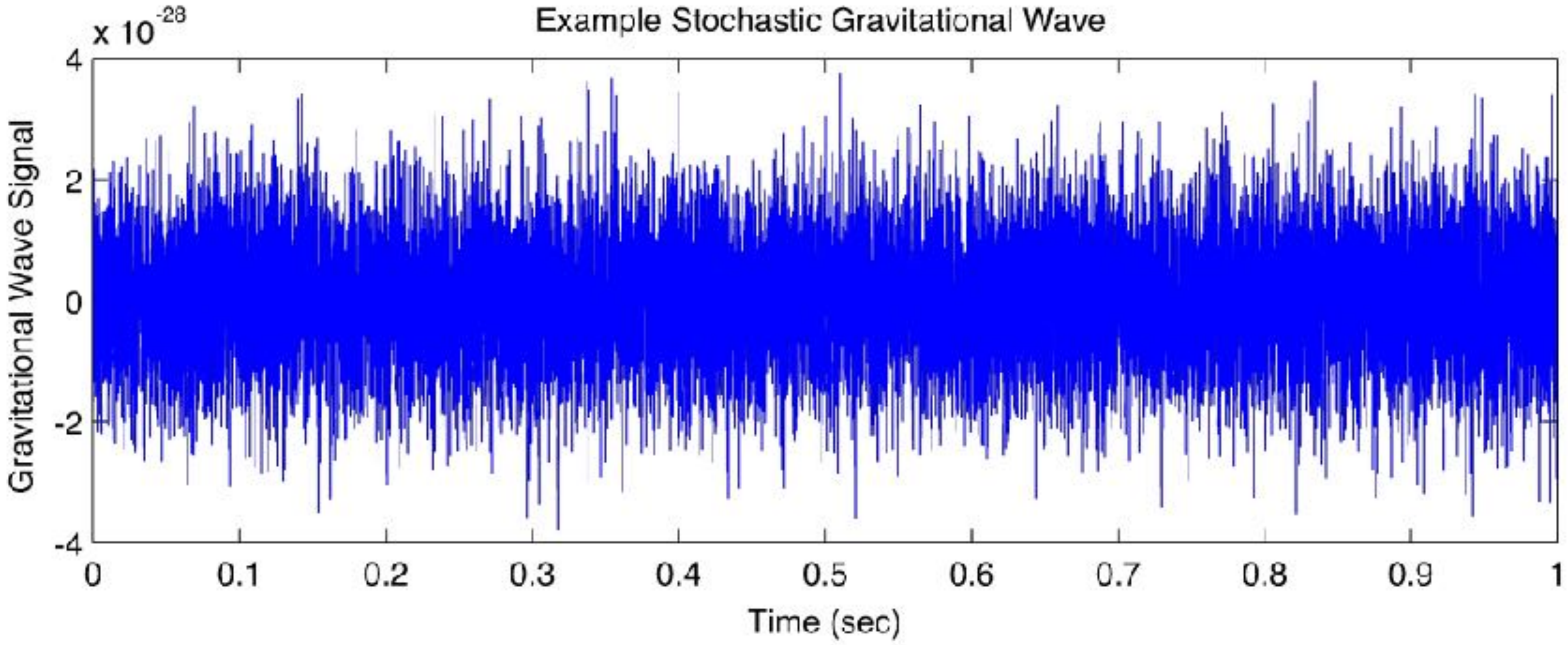
Detecting a non isotropic background

Gravitational Wave Orchestra, Annecy

Anirban Ain
University of Antwerp

Stochastic Gravitational Wave Background

SGWB

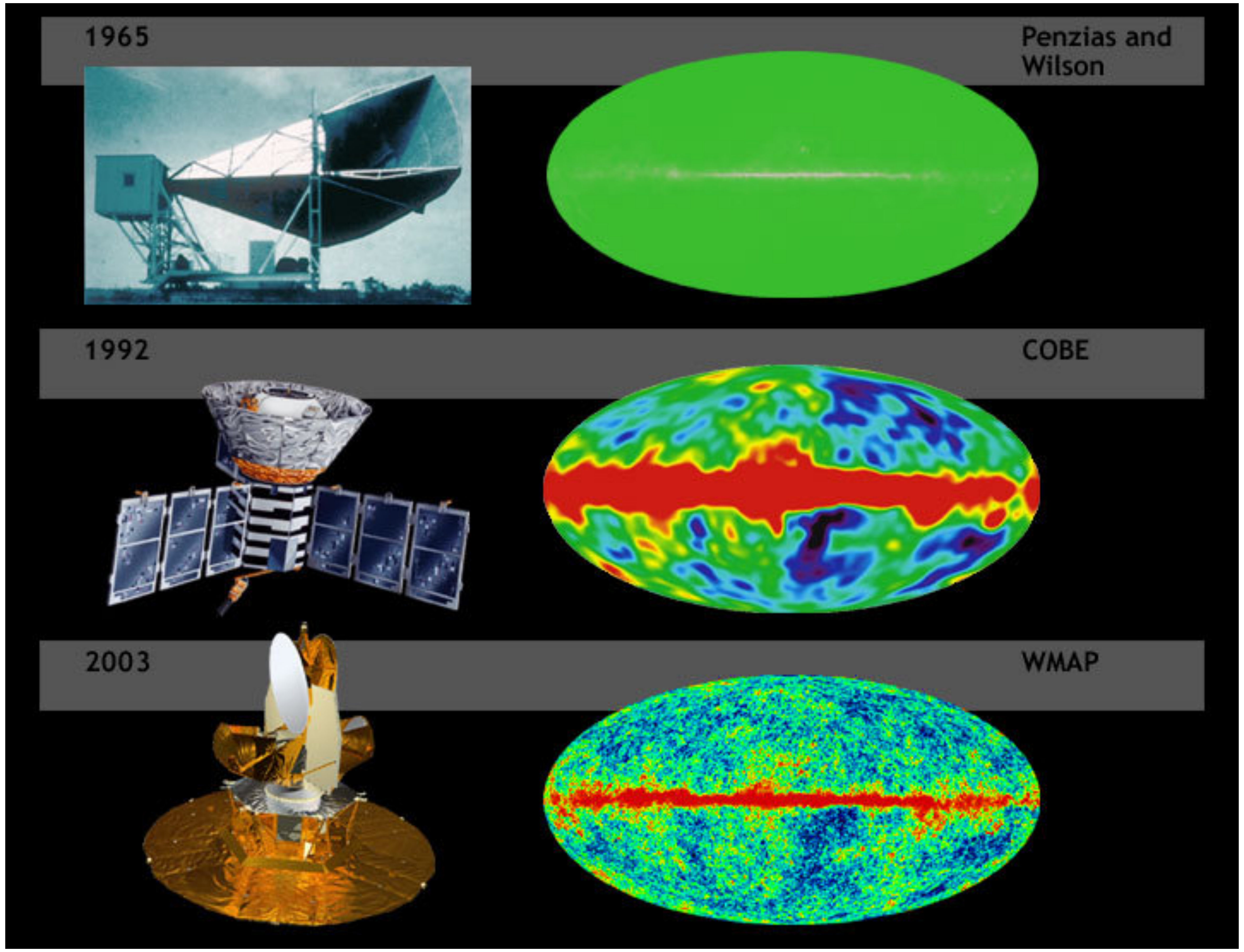


Characterised by its power spectrum $\Omega_{gw}(f) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{dlnf}, \quad \rho_c = 3H_0^2c^2/8\pi G$

- **Isotropic**
- **Unpolarized**
- **Stationary**
- **Gaussian**

Image: http://www.ligo.org/science/GW-Overview/images/stochastic.jpg

The 'isotropic' cosmic microwave background

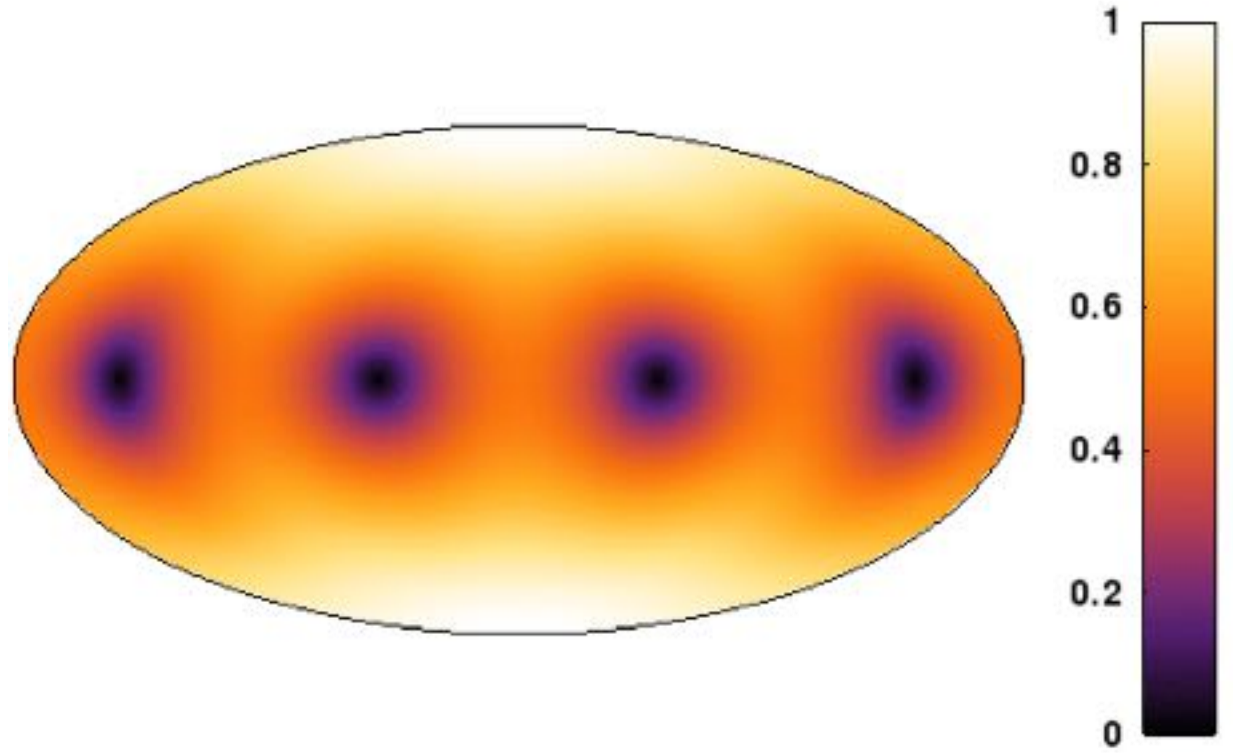


Just like CMB we expect to see some anisotropies.

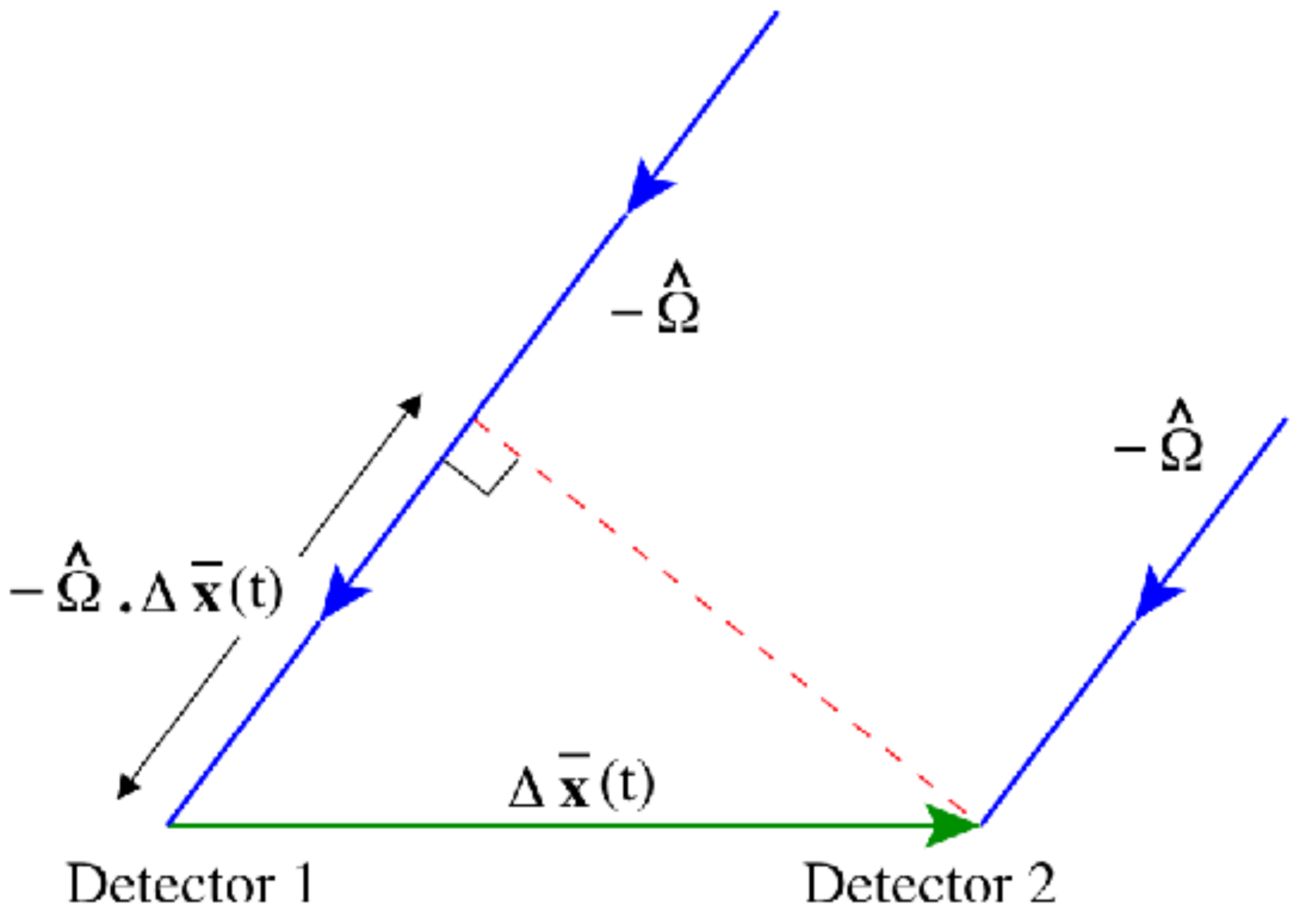
Image: https://map.gsfc.nasa.gov/m_ig/030644/030644.html

Aperture Synthesis

Using time of arrival to determine source direction



- With data from 2 detectors using interferometry (similar to radio astronomy) it is possible to map the entire sky using Aperture synthesis techniques.



- Cross-correlate detector outputs
- Make maps using time-dependent phase delay

Image: http://iopscience.iop.org/0264-9381/32/1/015014/downloadHRFigure/figure/cqg503659f1

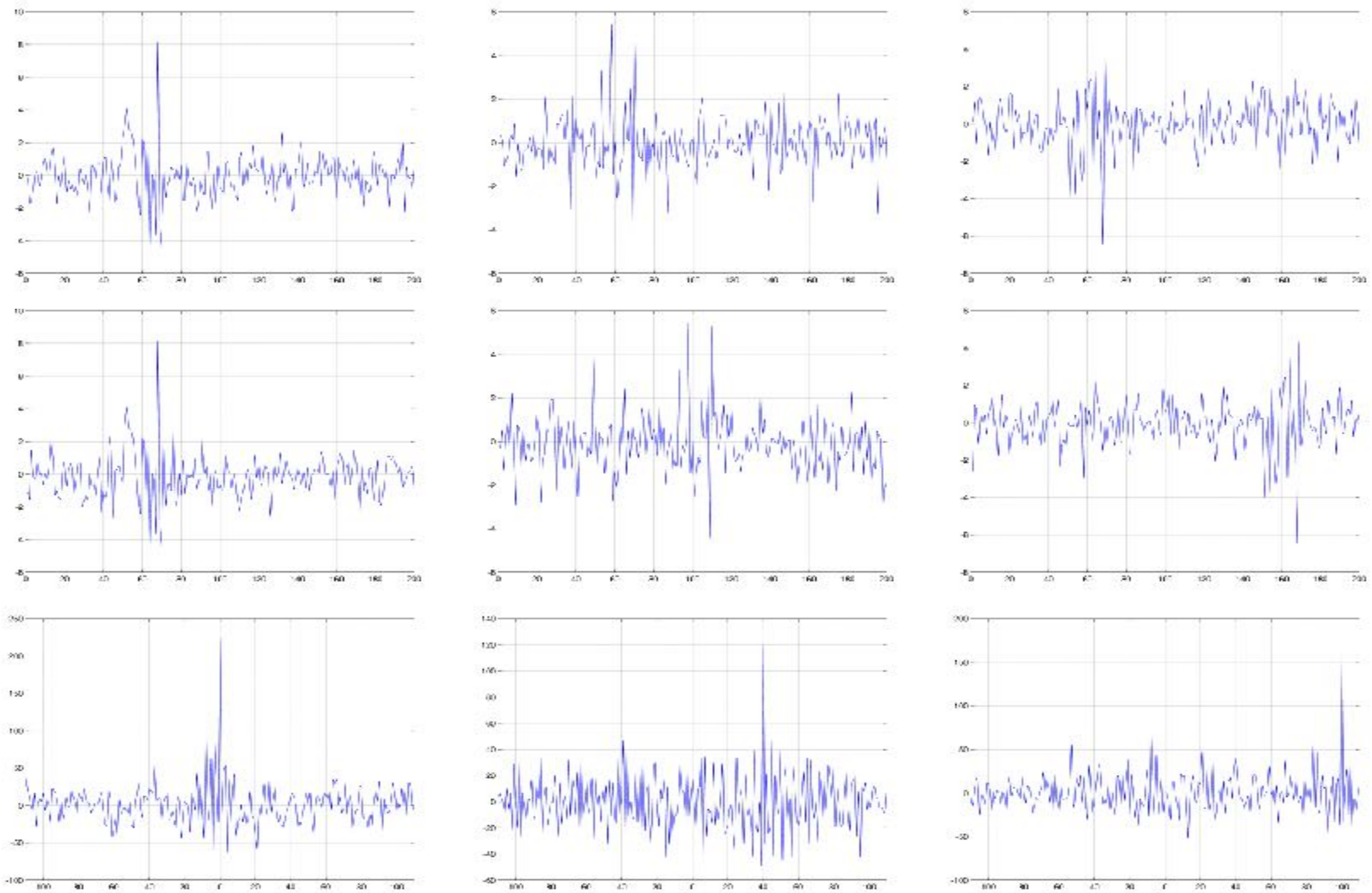
Image: http://journals.aps.org/prd/article/10.1103/PhysRevD.77.042002/figures/1/medium

Cross-Correlation to Detect Pattern

Detector 1

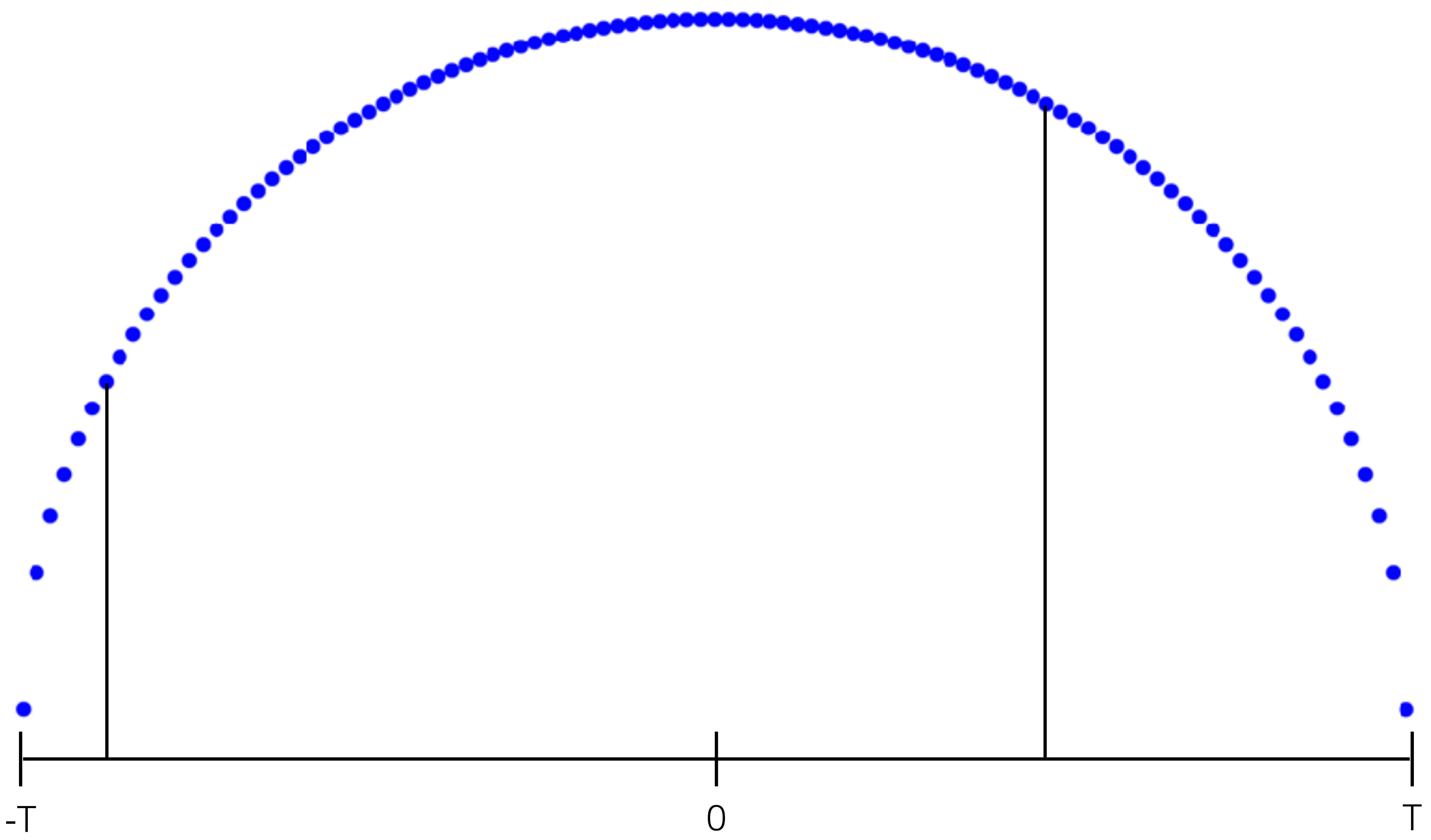
Detector 2

Cross-Correlation
of D1 & D2

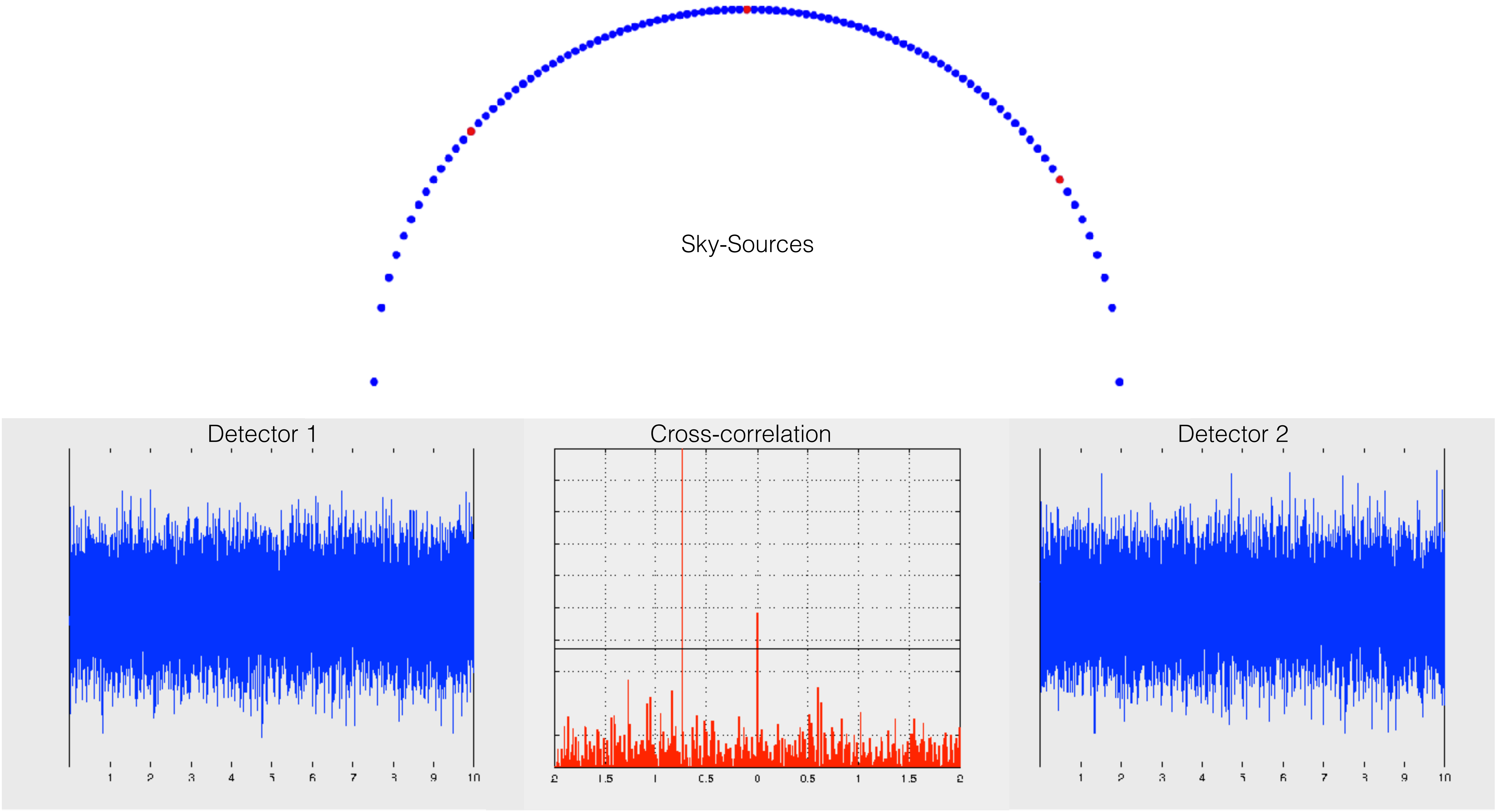


no Delay small Delay large Delay

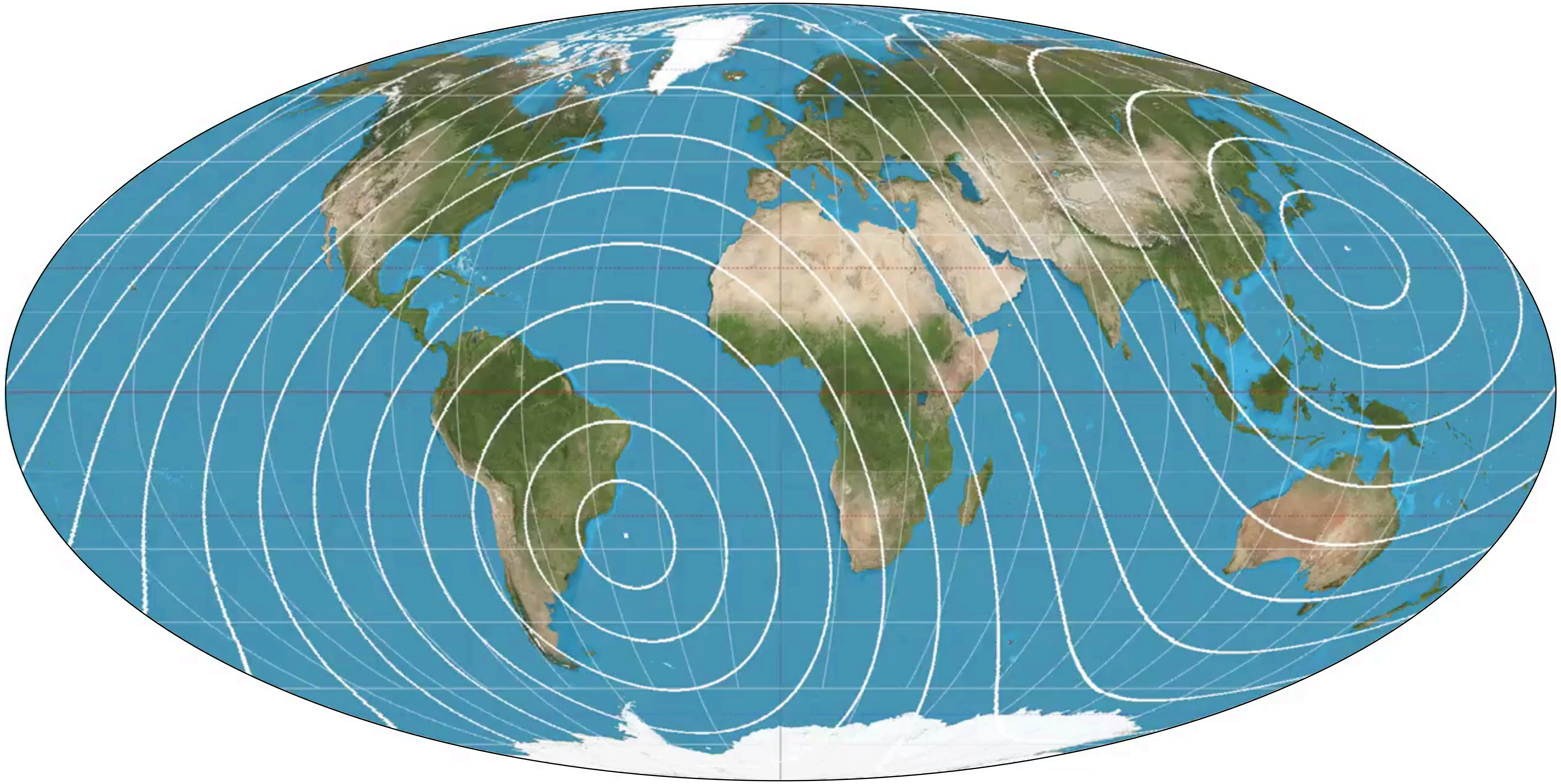
Sky-Locations and their Projection



Cross-Correlation as 1D Map

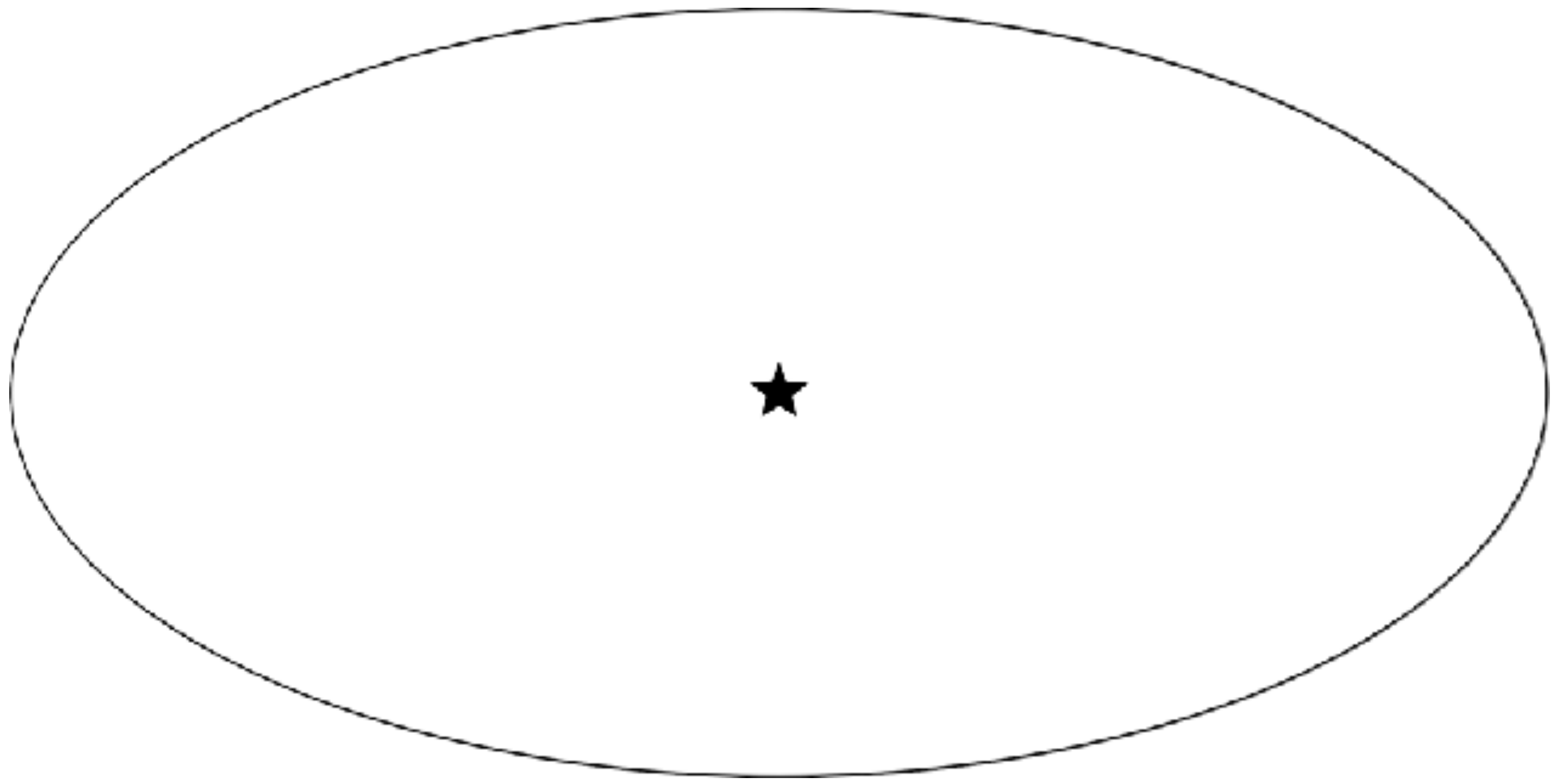


Directions with equal Time delay

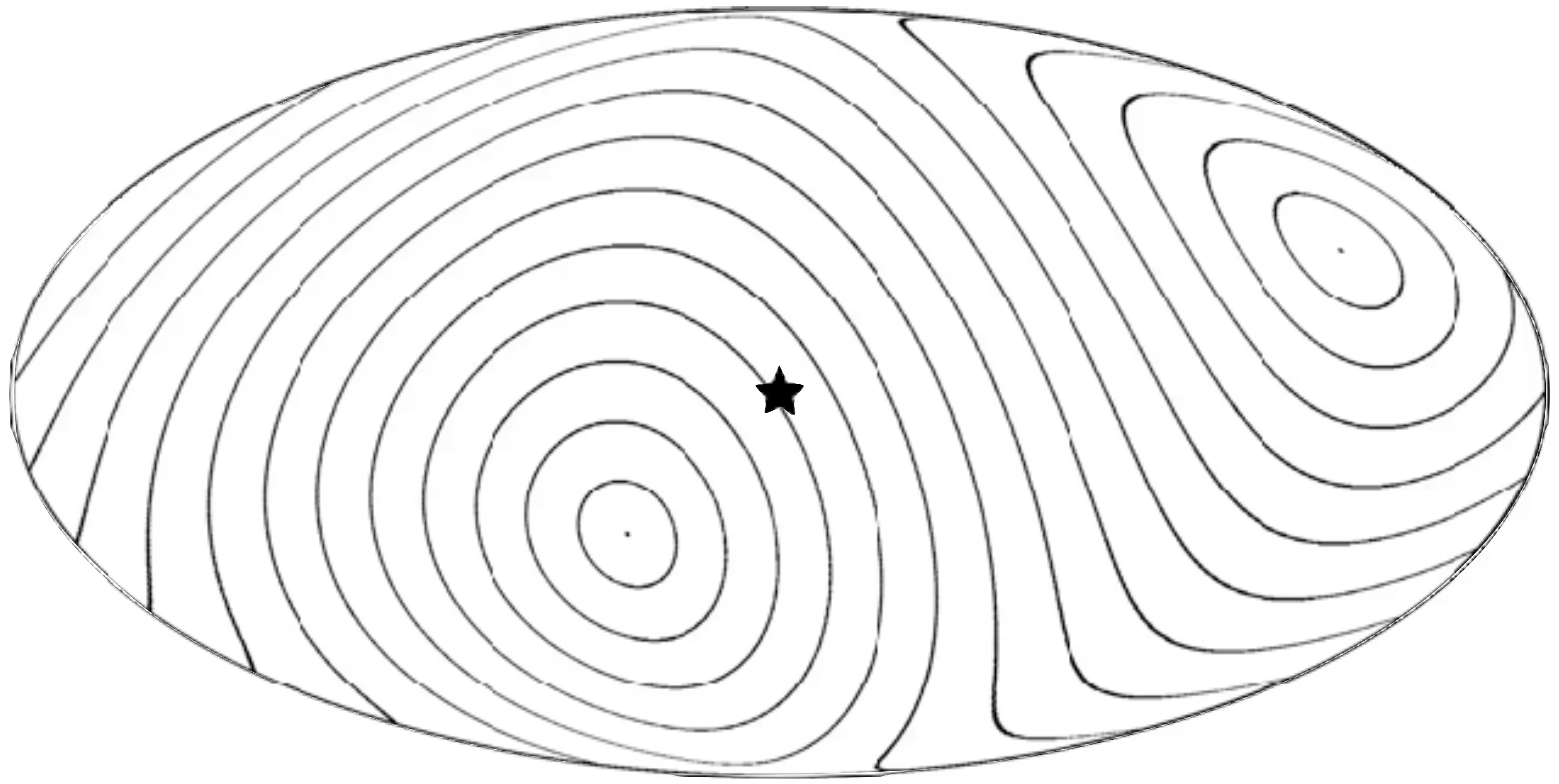


Background Image: https://upload.wikimedia.org/wikipedia/commons/9/9e/Mollweide_projection_SW.jpg

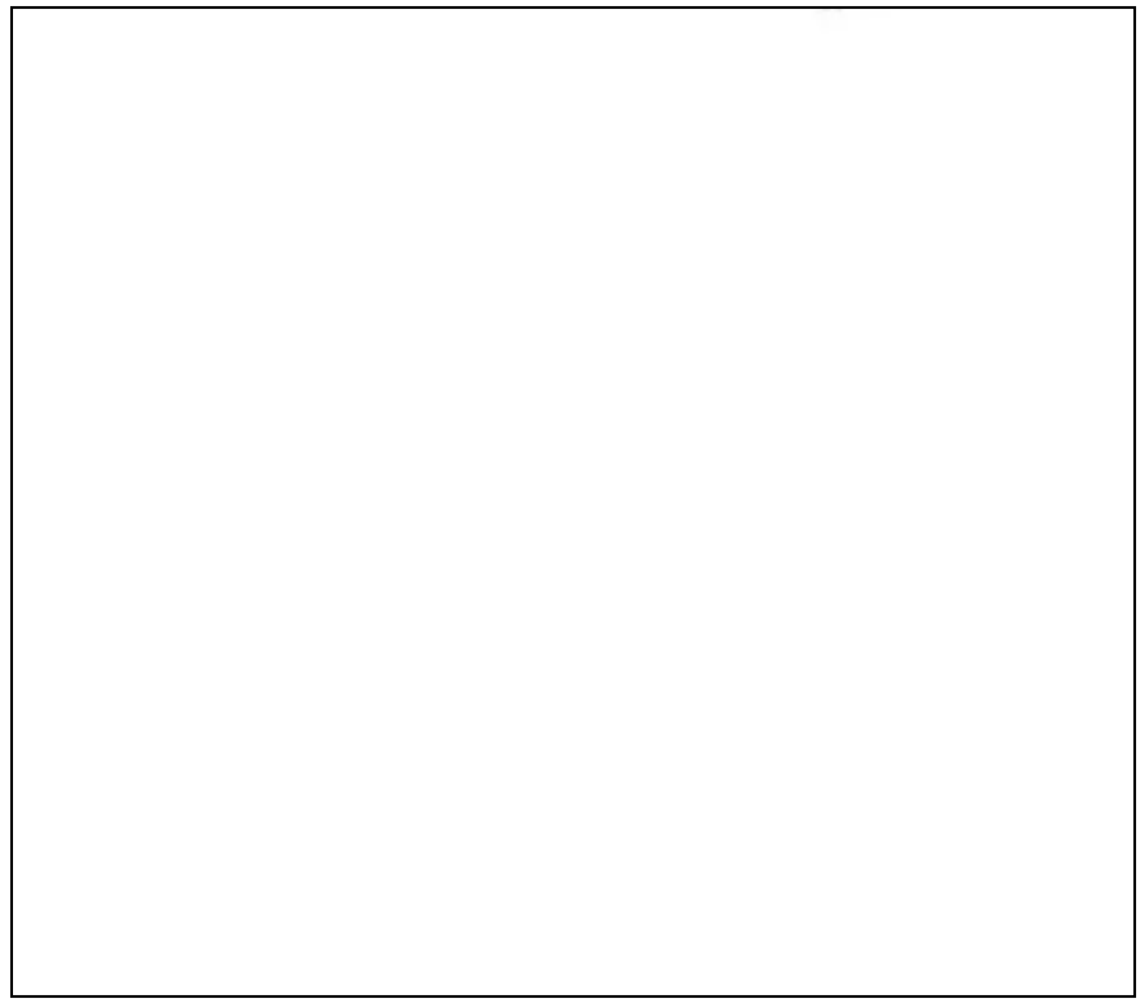
Radiometer Algorithm (Point Source)



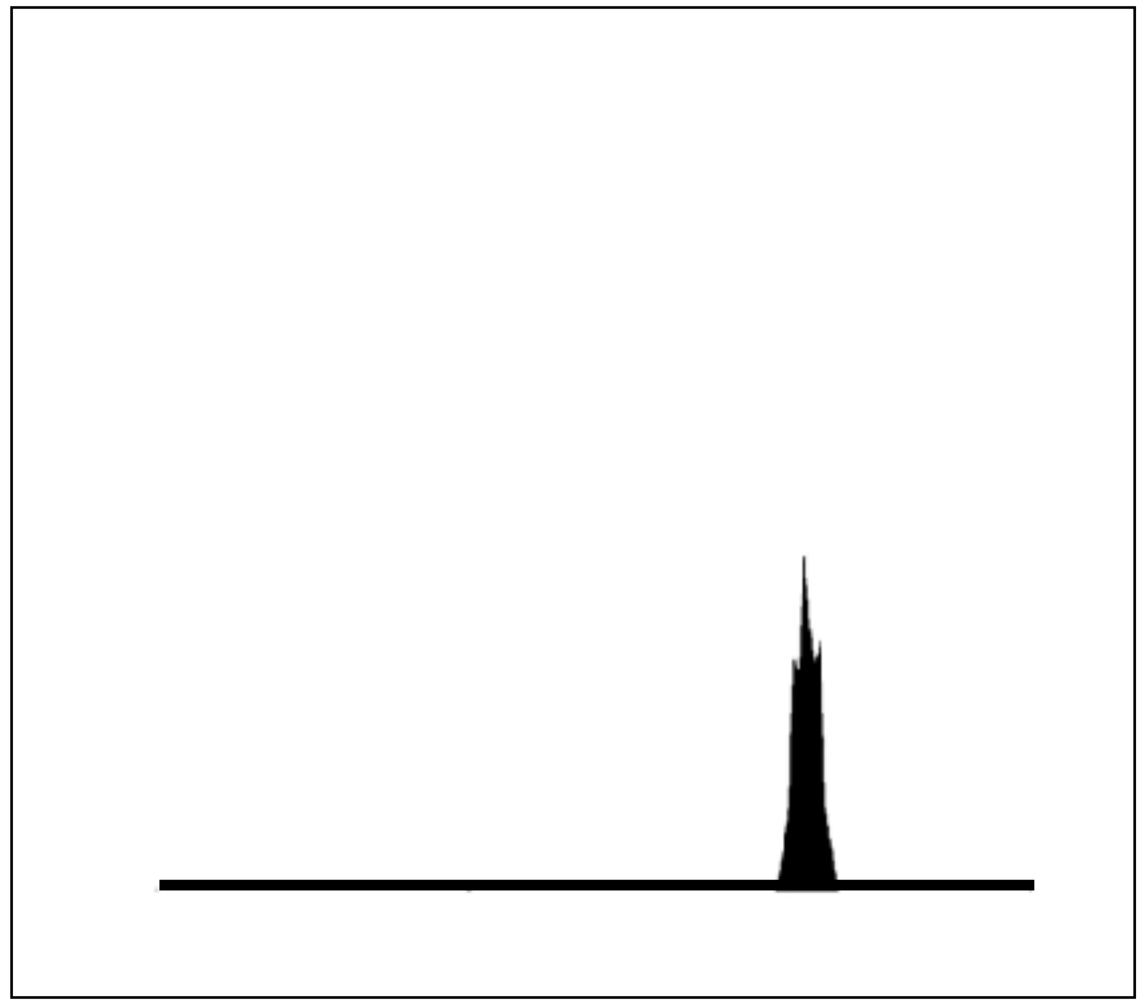
Actual Sky



Lines of Equal Time Delay

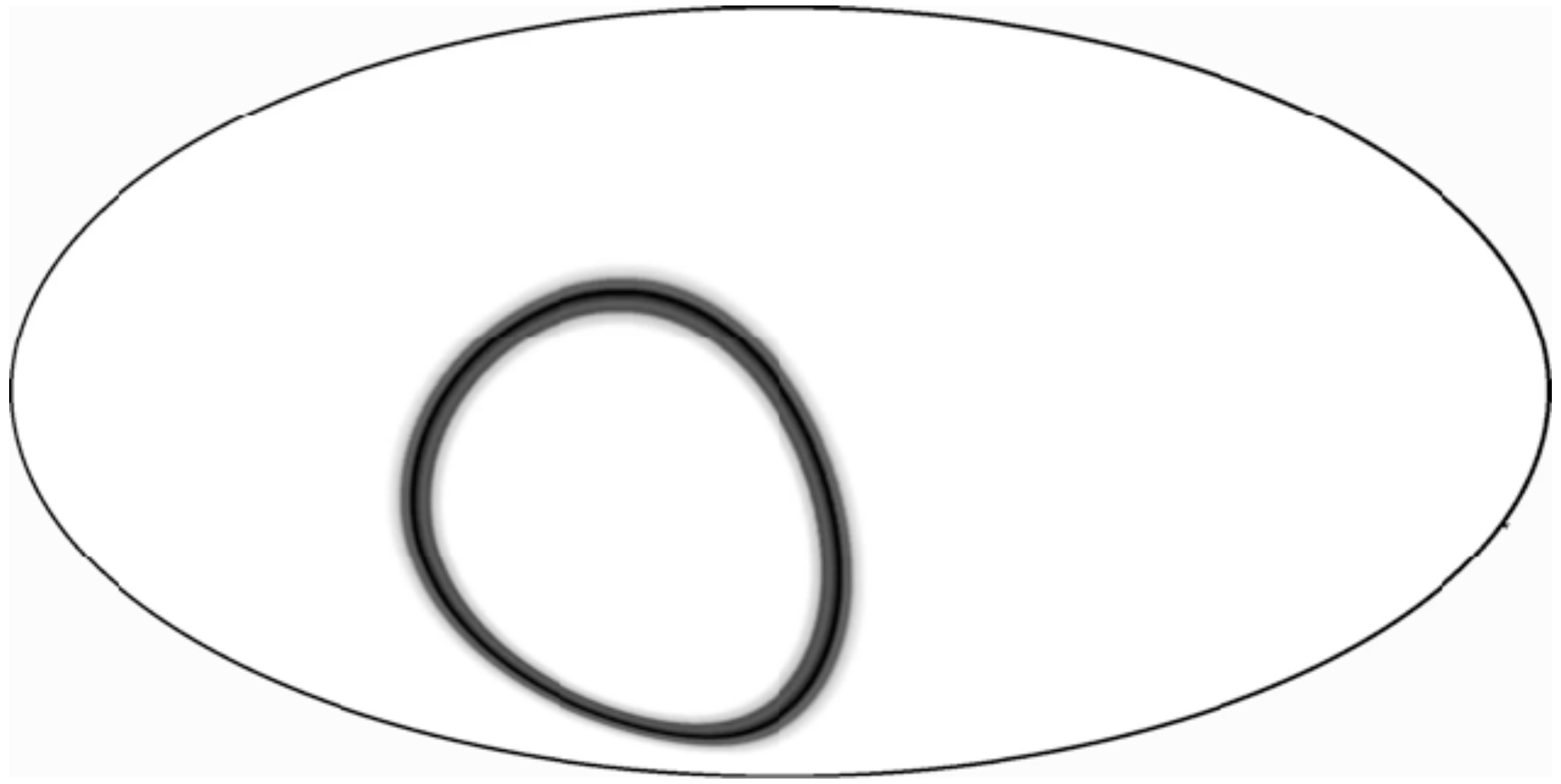


CC results of one Day

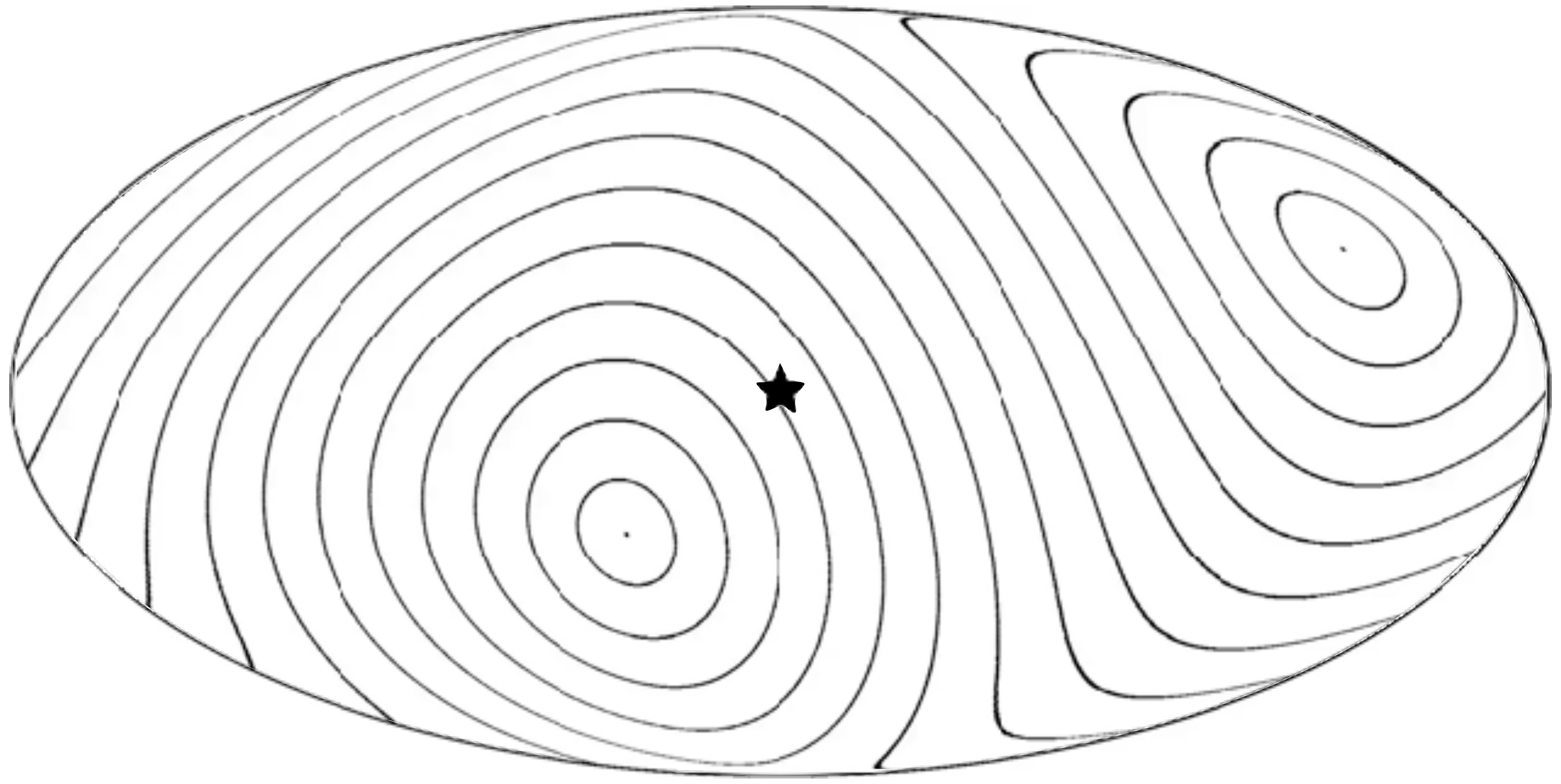


Cross-Correlation of Data Streams

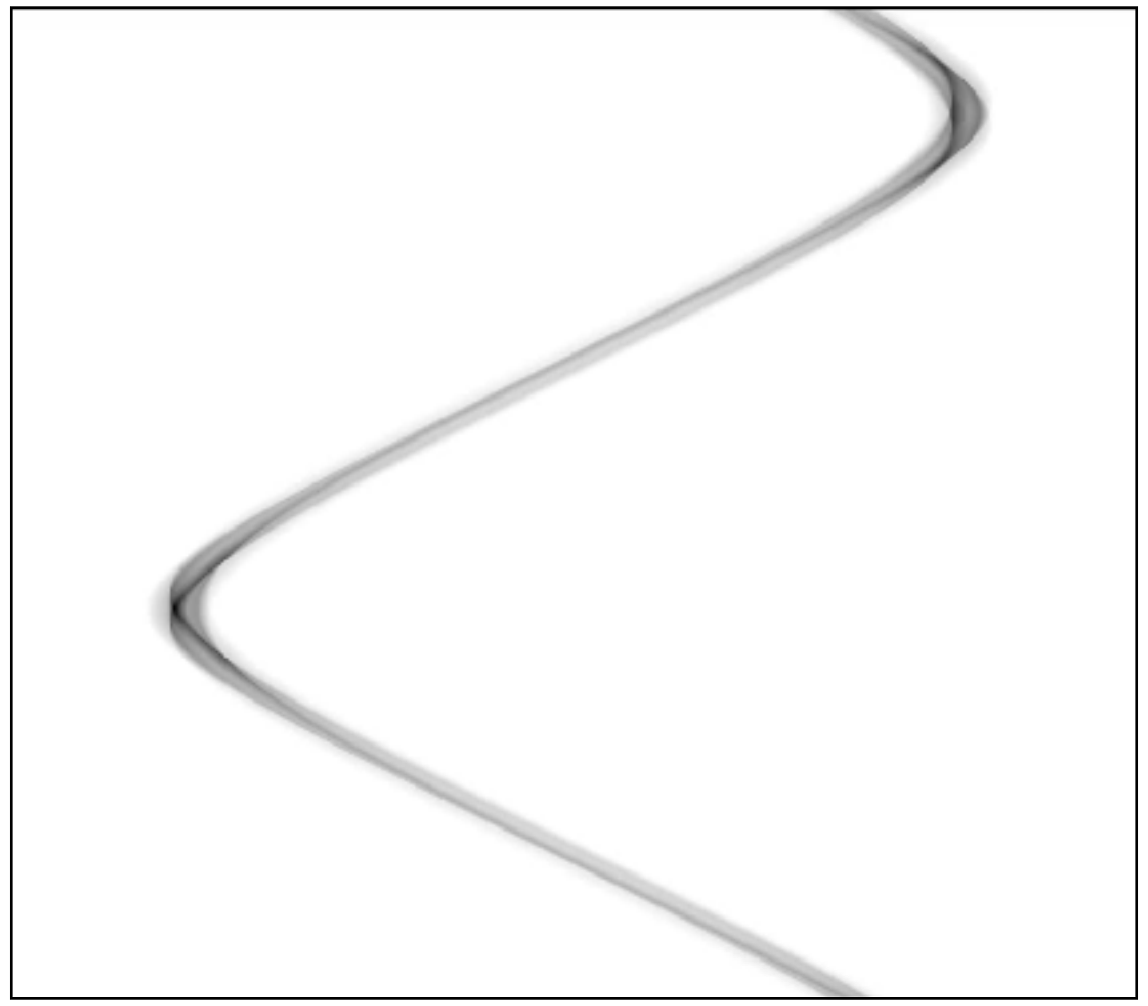
Radiometer Algorithm (Point Source)



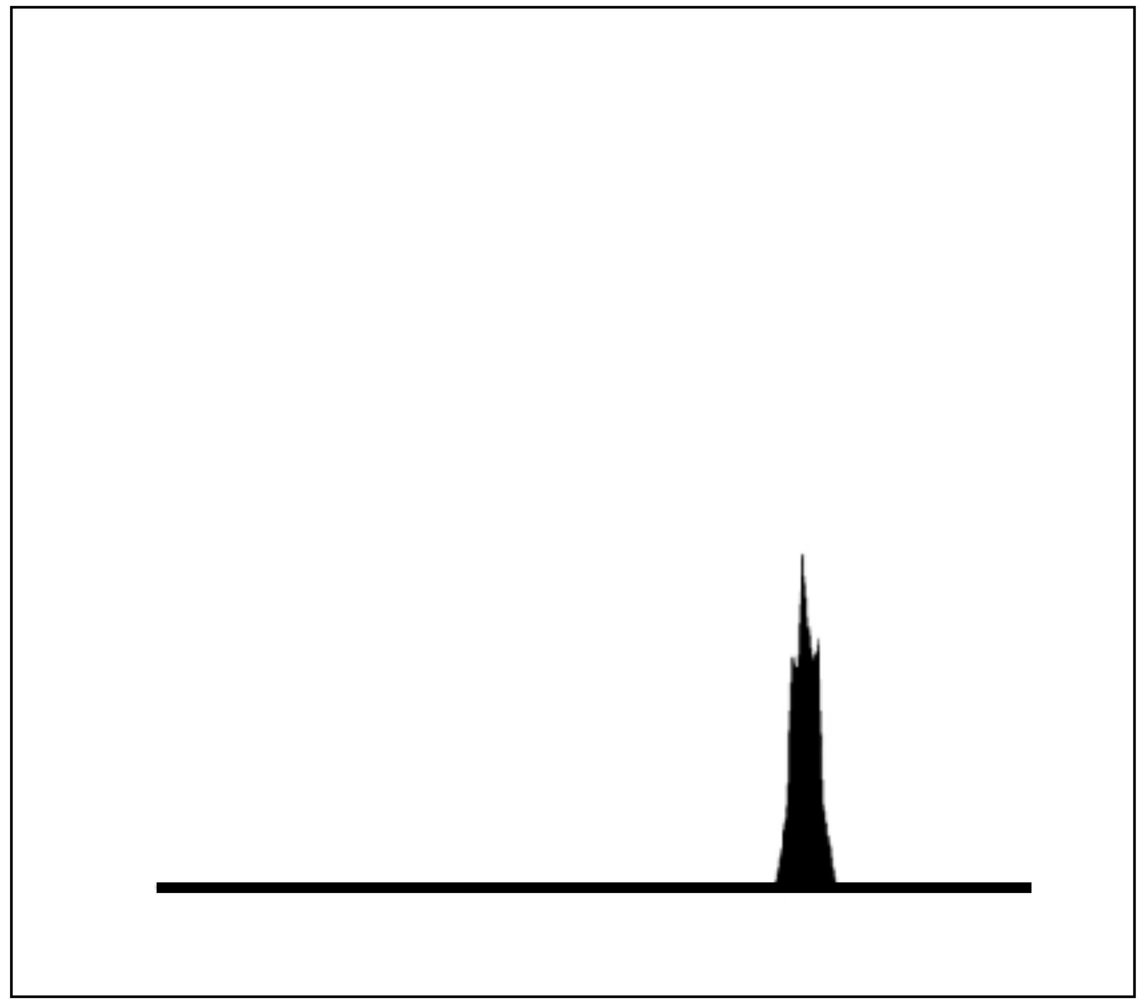
Map from CC results



Lines of Equal Time Delay

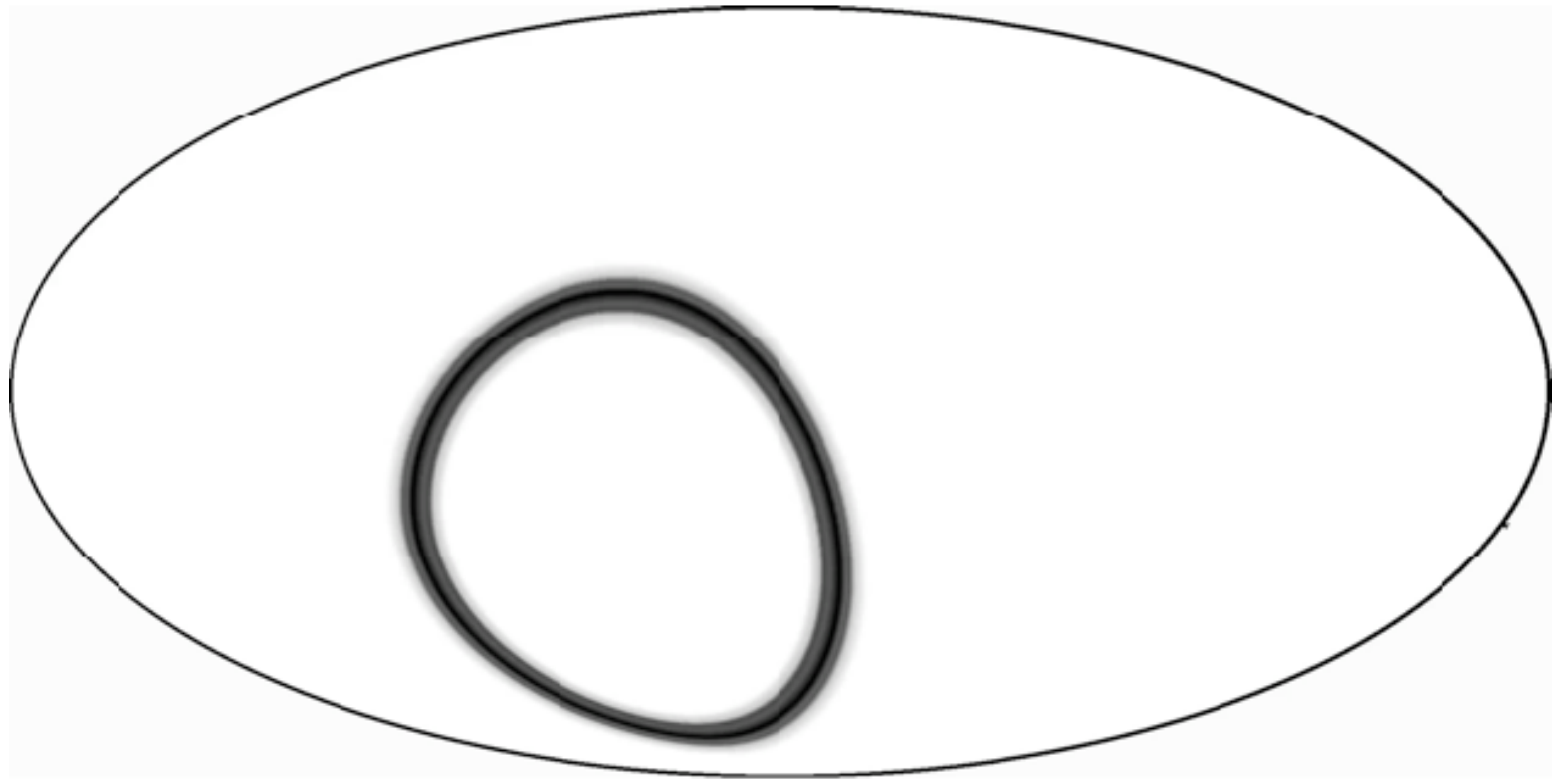


CC results of one Day

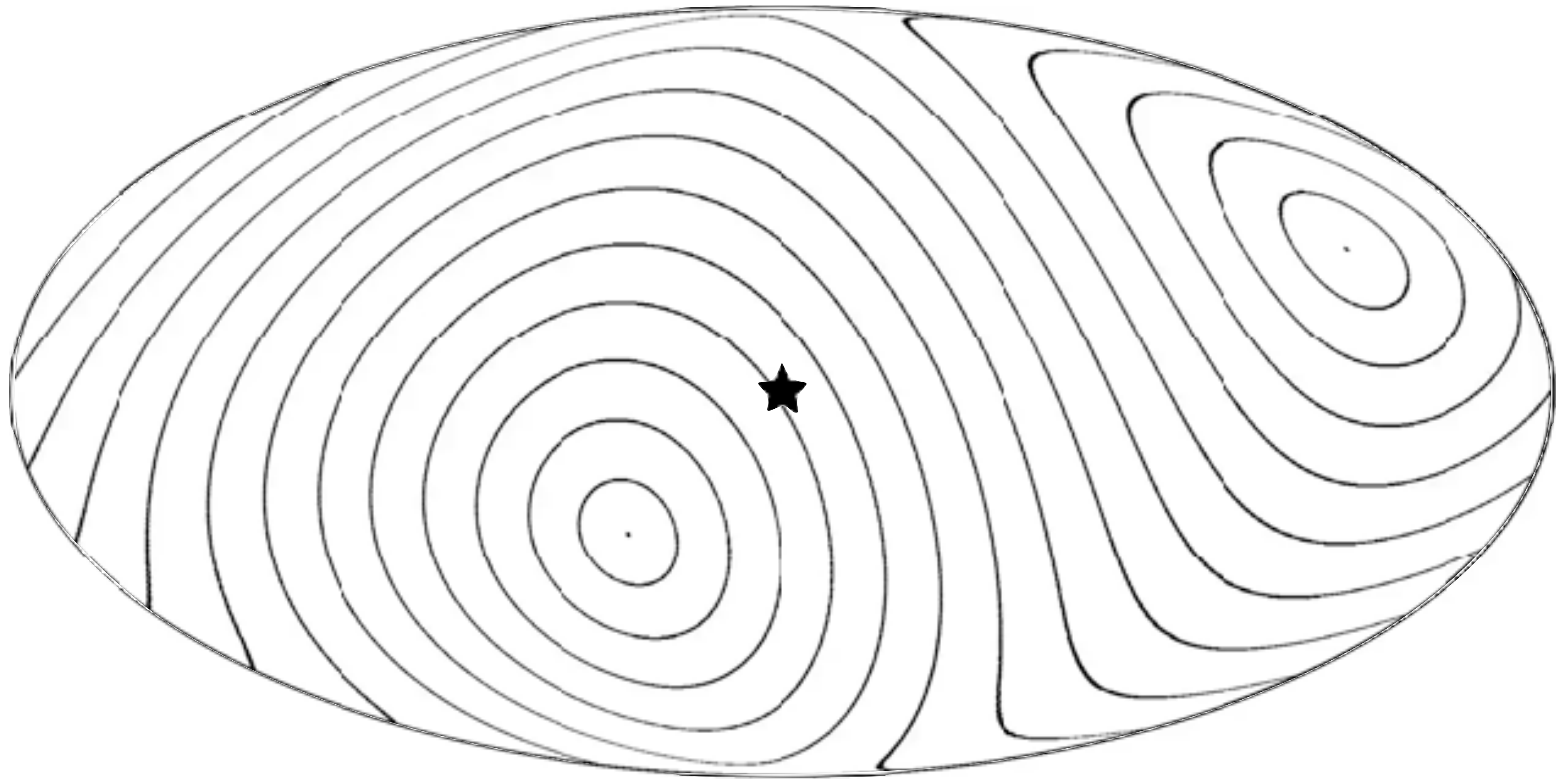


Cross-Correlation of Data Streams

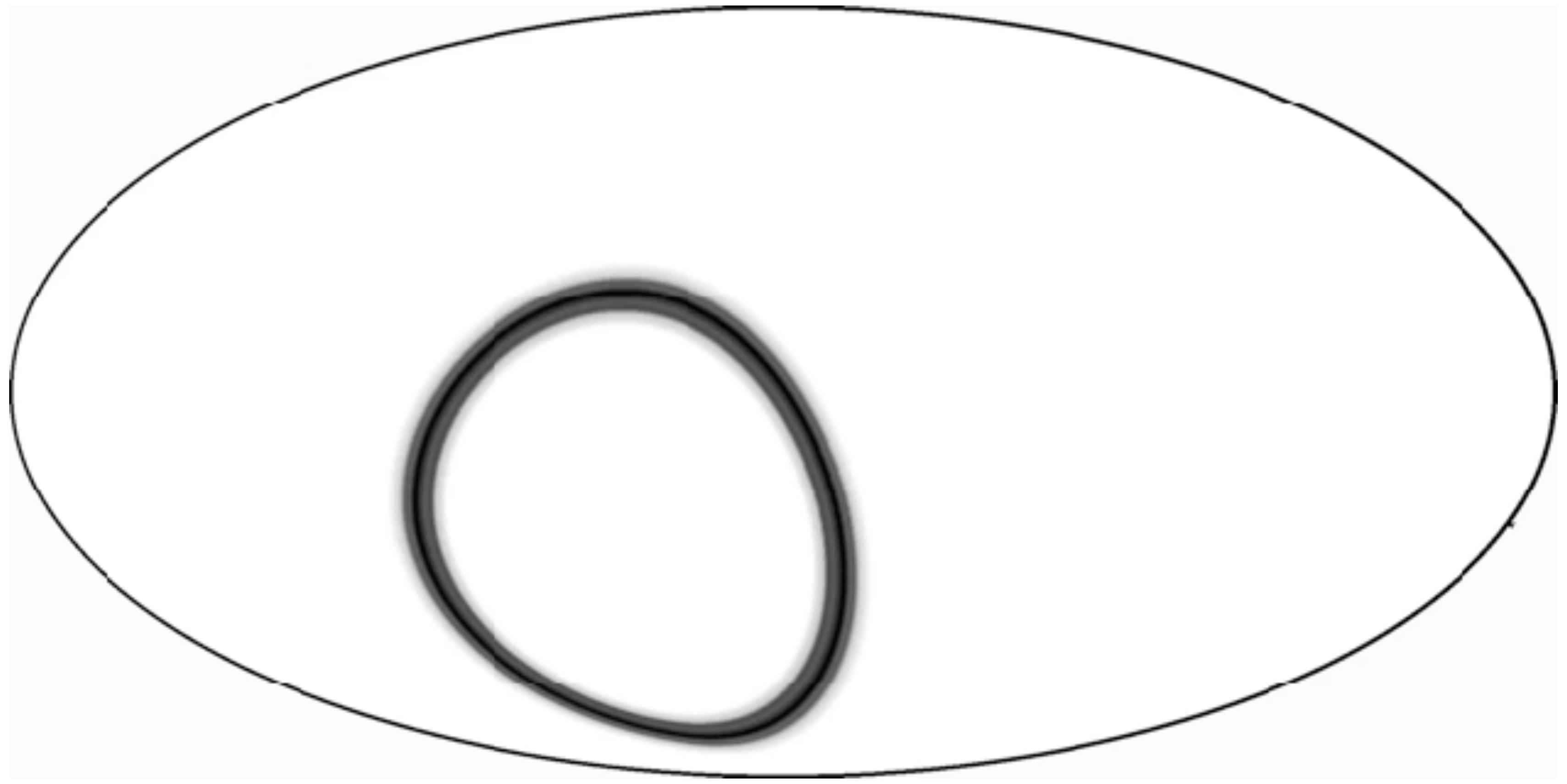
Radiometer Algorithm (Point Source)



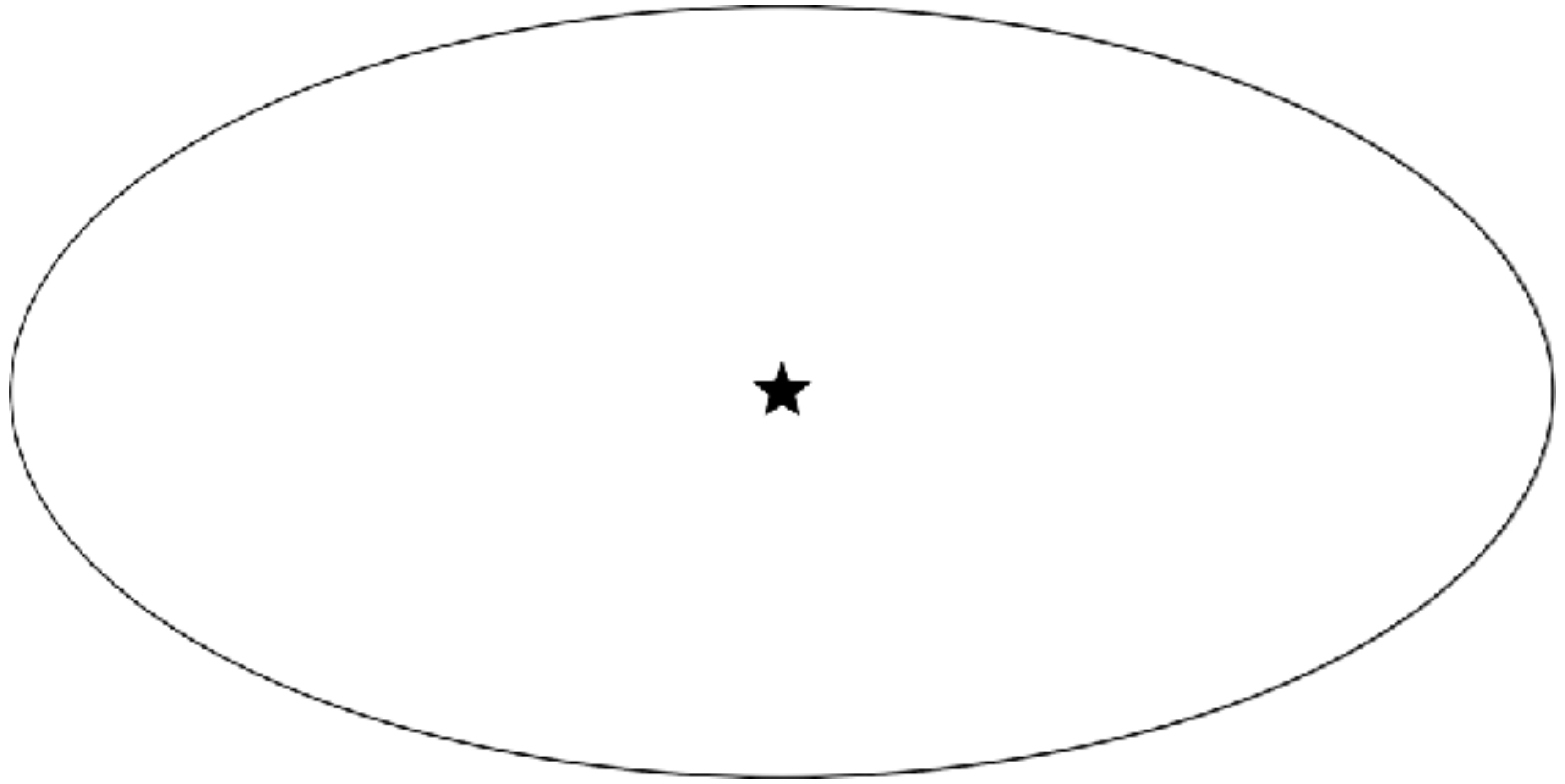
Map from CC results



Lines of Equal Time Delay

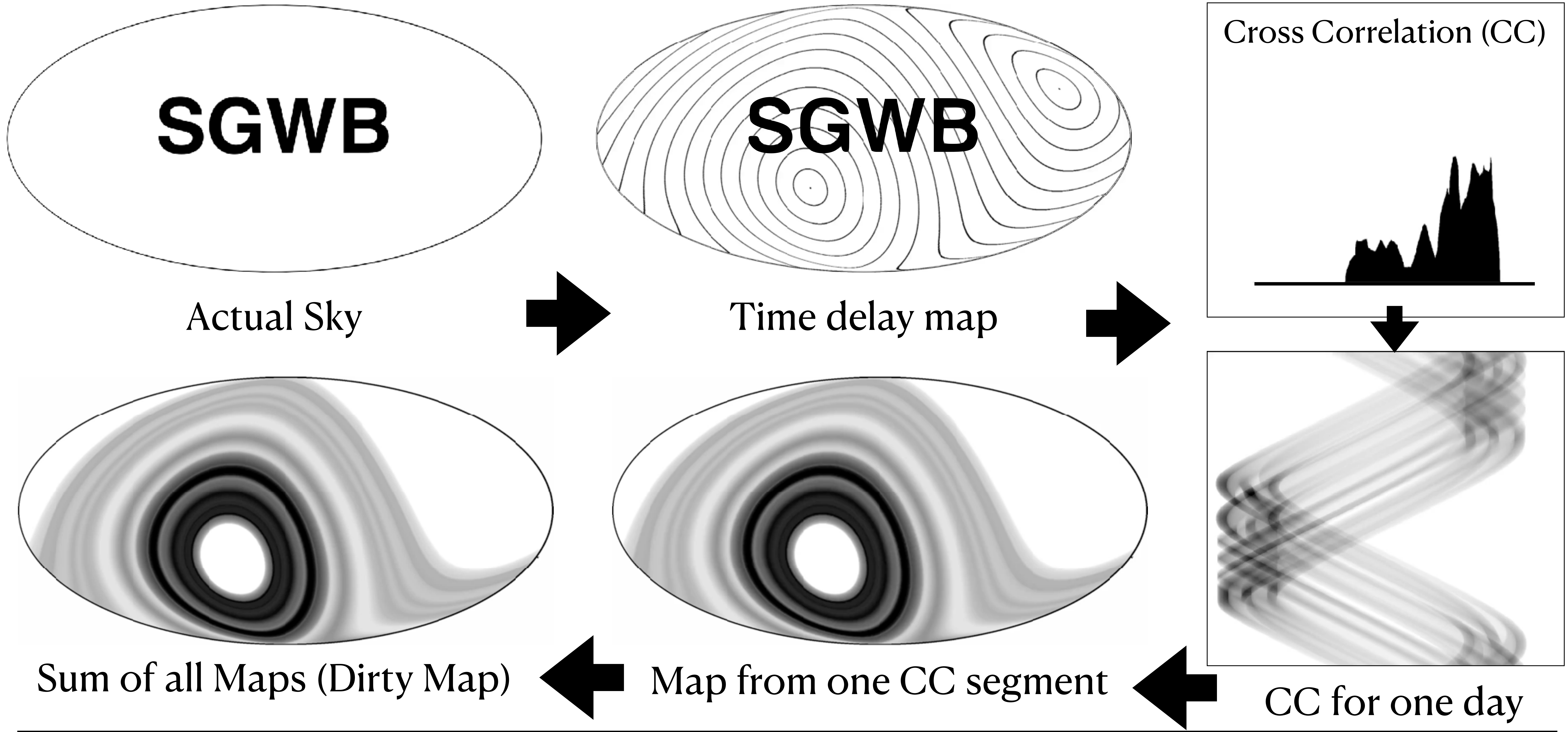


Sum of all CC Map results



Actual Sky

GW Radiometer



CSD and PSD

Time Series data segments, Hann window, Fourier transform

$$\tilde{s}(t; f) \;:=\; \int_{t-\tau/2}^{t+\tau/2} \mathrm{d}t' \, s(t') \, e^{-i2\pi f t'}$$

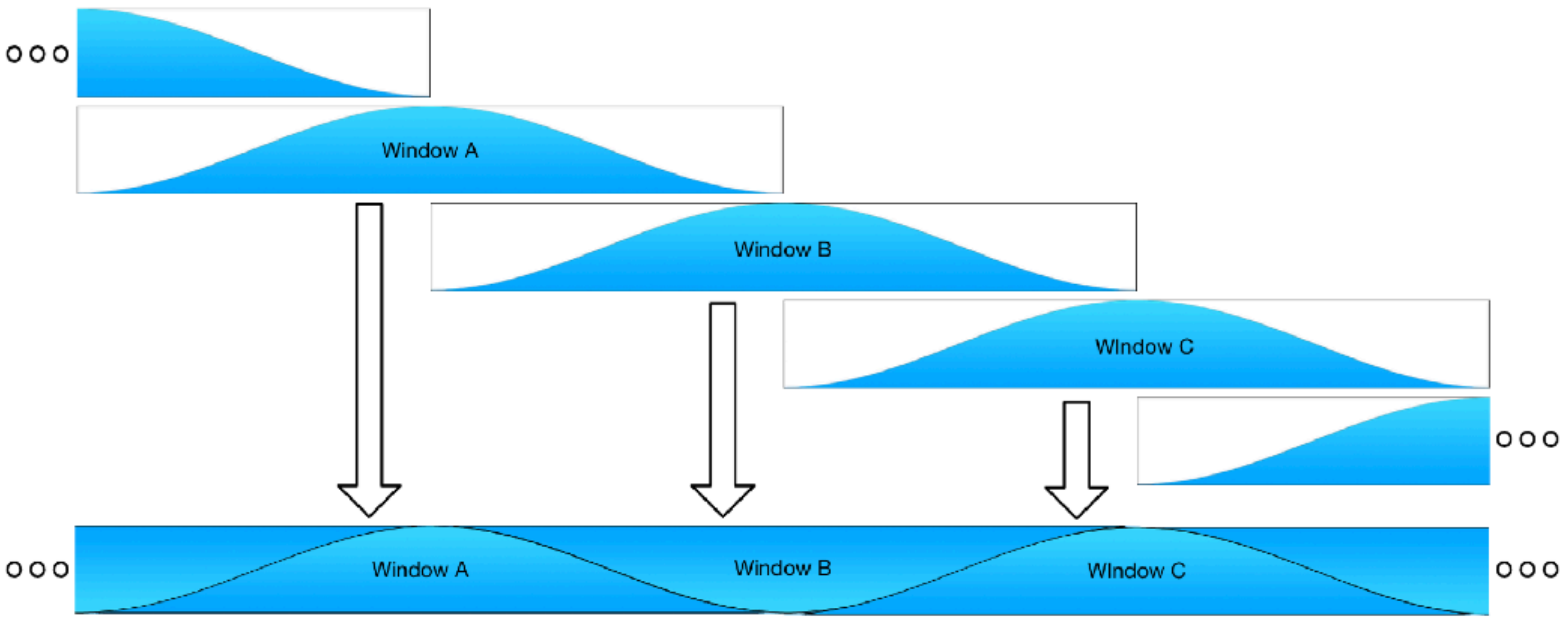
$$\mathbf{C}^I = \widetilde{s}^*_{\mathcal{I}_1}(t; f) \, \widetilde{s}_{\mathcal{I}_2}(t; f)$$

cross spectral density (CSD)

$$\sigma^2_{Ift} = \frac{\tau^2}{4} P_{\mathcal{I}_1}(t; f) \, P_{\mathcal{I}_2}(t; f)$$

Power spectral density (PSD)

192-second, 50% overlapping, Hann-windowed segments



https://iopscience.iop.org/article/10.1088/1361-6382/ab01c5

Map from the Data

Maximum Likelihood estimate of sky intensity from observed data

Overlap Reduction Function

$$\gamma^{I}_{ft,\alpha} := \sum_{A} \int_{S^2} d\hat{\mathbf{\Omega}} F^{A}_{\mathcal{I}_1}(\hat{\mathbf{\Omega}}, t) F^{A}_{\mathcal{I}_2}(\hat{\mathbf{\Omega}}, t) e^{2\pi i f \frac{\hat{\mathbf{\Omega}} \cdot \mathbf{\Delta x}_I(t)}{c}} e_{\alpha}(\hat{\mathbf{\Omega}})$$

Anisotropy is achieved by preserving this index

$$\mathbf{K}^{I} = \tau\, H(f)\, \gamma^{I}_{ft,\alpha}$$

Kernel

$$X_{\alpha} = \sum_{Ift} K^{I*}_{ft,\alpha} \sigma^{-2}_{Ift} C^{I}_{ft}$$

Dirty Map

$$\Gamma_{\alpha\alpha'} = \sum_{Ift} K^{I*}_{ft,\alpha}\, \sigma^{-2}_{Ift}\, K^{I}_{ft,\alpha'}$$

Fisher Matrix

$$\hat{\mathcal{P}}_{\alpha} = \mathbf{\Gamma}^{-1} \cdot \mathbf{X}$$

Clean Map

Pipeline and Analyses

- Pipelines
  - Folding
  - PyStoch

- Analyses
  - Broadband radiometer (BBR)
  - Spherical harmonics decomposition(SHD)
  - Narrowband radiometer (NBR)
  - All-sky all-frequency search (ASAF)

- Spherical harmonics PE
- GW-EM correlation
- Model development
- Jax Implementation

Data Folding

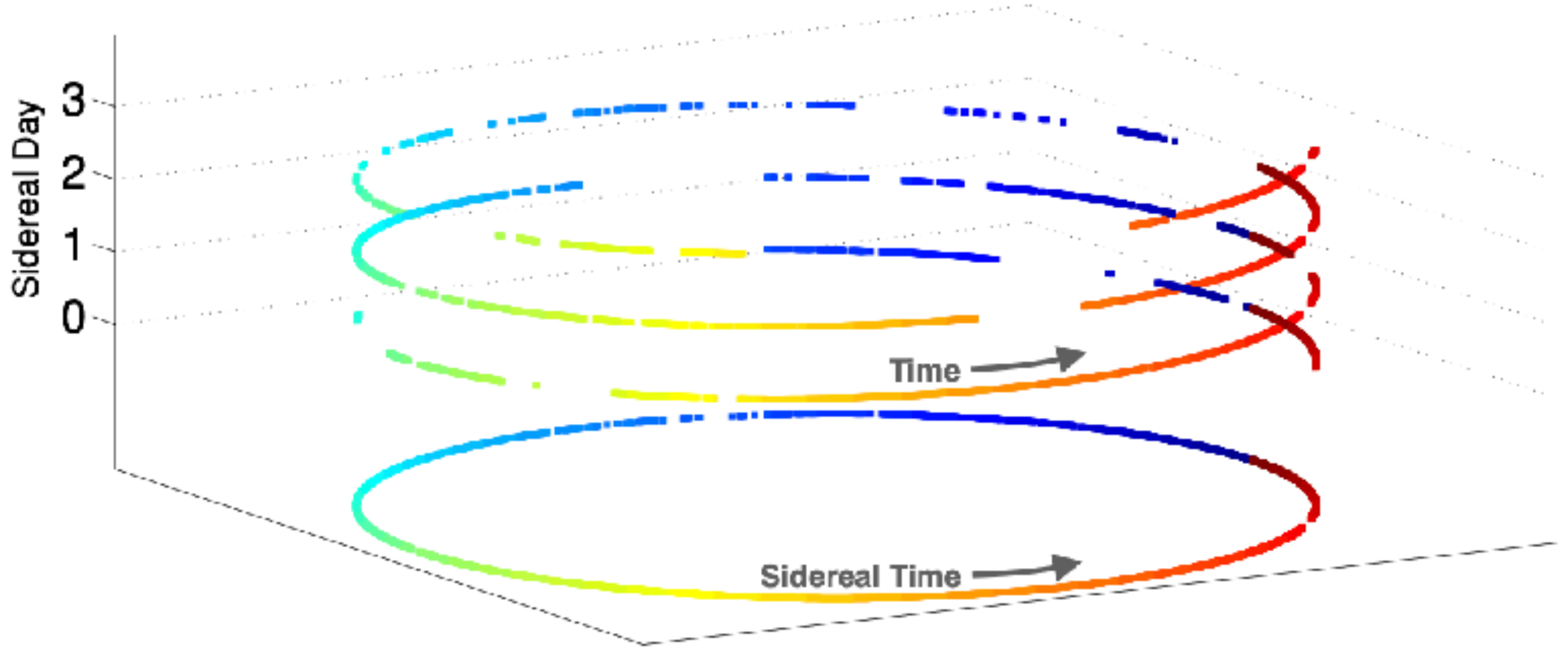
Compressing long duration data into one day

The main idea behind folding the data is that the mapping kernel has a period of 1 sidereal day (i.e. 23 hr 56 min 4 sec). So radiometric data can be summed before the kernel operation.

$$t = i_{day} + t_{sid}$$

$$\sum_t \to \sum_{i_{day}} \sum_{t_{sid}}$$

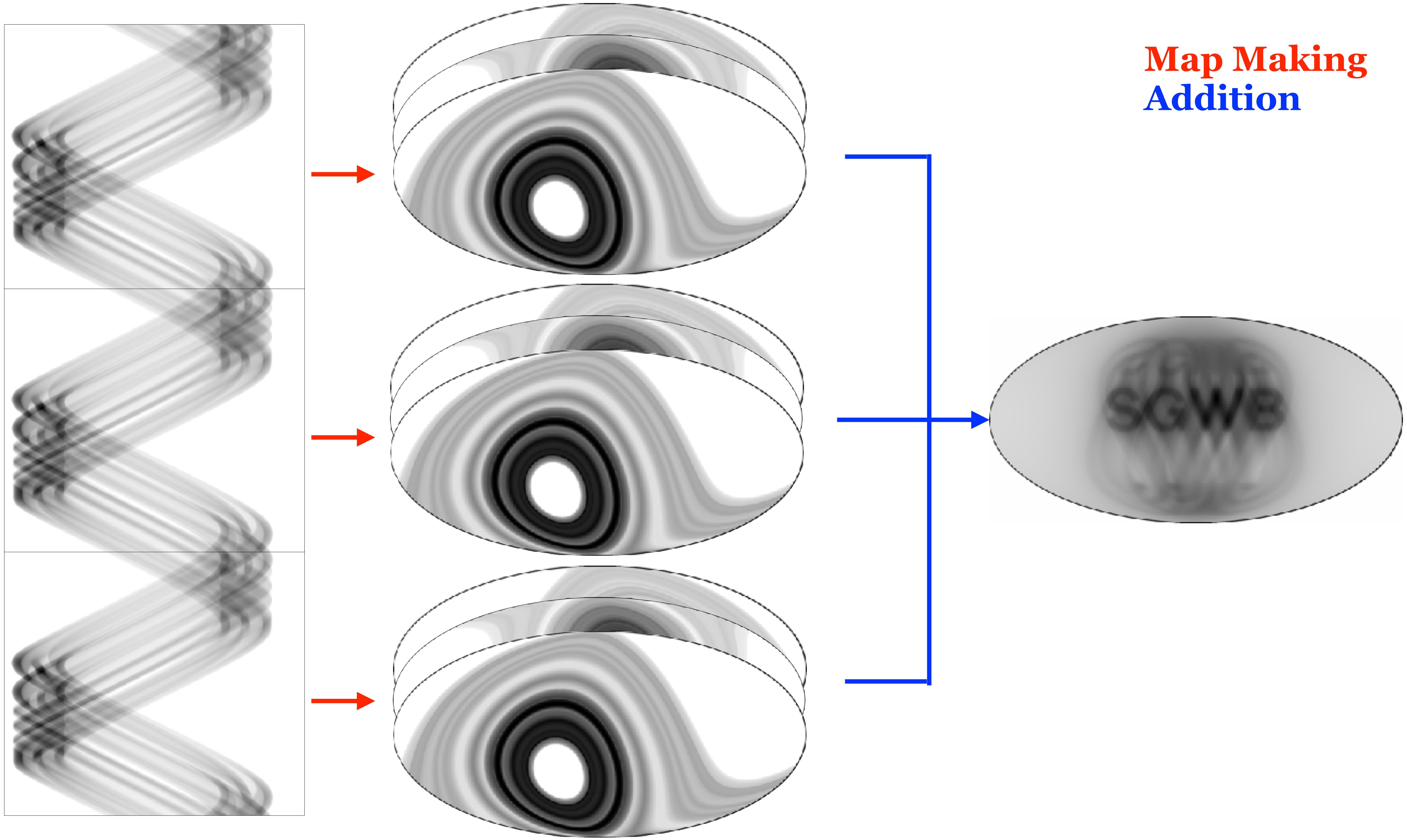
$$X_\alpha = \sum_{Ift} K^{I*}_{\alpha,ft}\, \sigma^{-2}_{Ift}\, C^I_{ft}$$
$$= \sum_{Ift_{sid}} K^{I*}_{\alpha,ft_{sid}} \sum_{i_{day}} \sigma^{-2}_{If(i_{day}+t_{sid})}\, C^I_{f(i_{day}+t_{sid})}$$



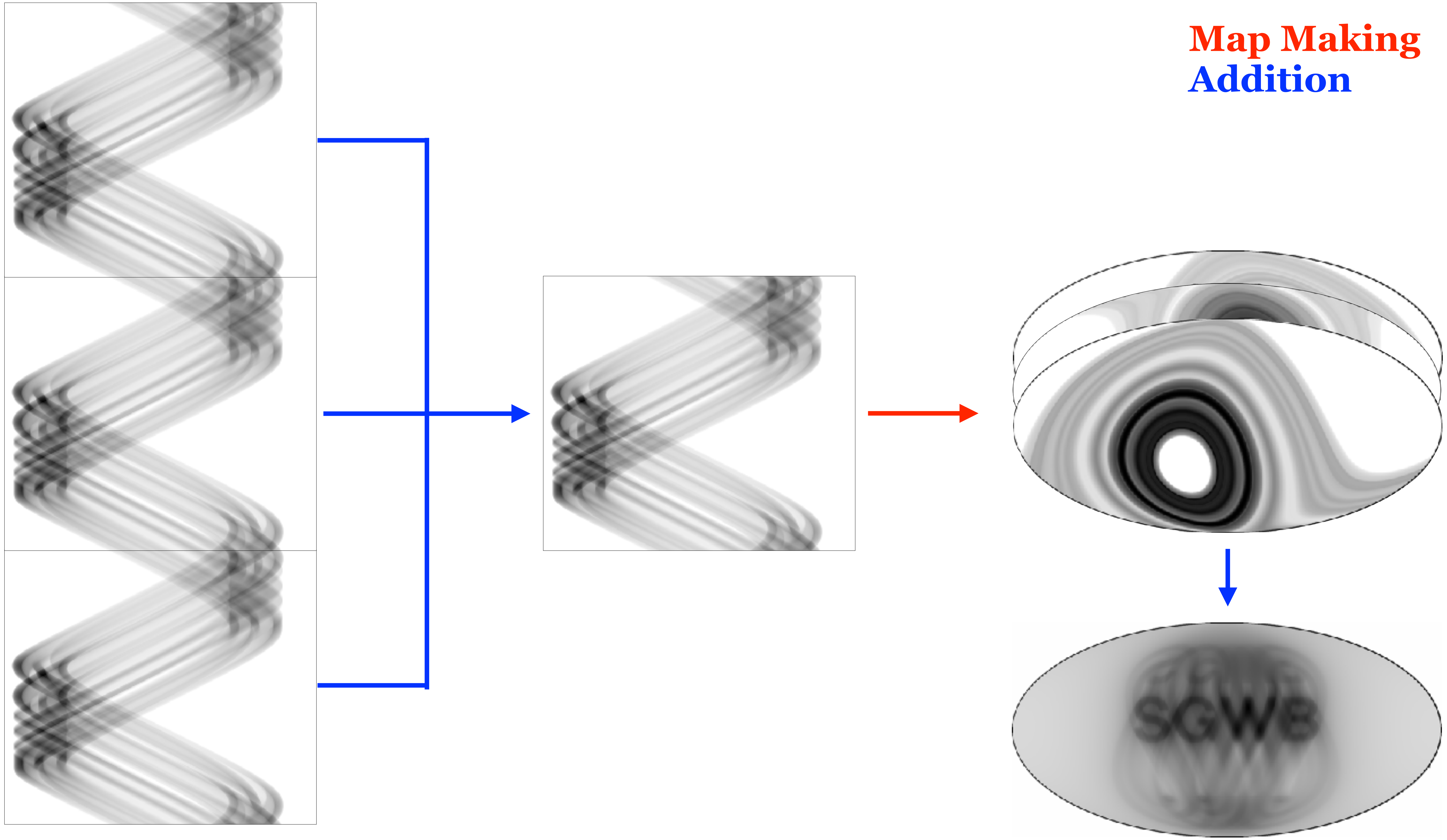
$$\Gamma_{\alpha\alpha'} = \sum_{Ift} K^{I*}_{\alpha,ft}\, \sigma^{-2}_{Ift}\, K^I_{ft,\alpha'}$$
$$= \sum_{I,ft_{sid}} K^{I*}_{\alpha,ft_{sid}}\, K^I_{ft_{sid},\alpha'} \sum_{i_{day}} \sigma^{-2}_{If(i_{day}+t_{sid})}$$

A. Ain et al. PhysRevD.92.022003

Stochastic Pipeline



Stochastic Pipeline (with folding)



Folding

Python Translation

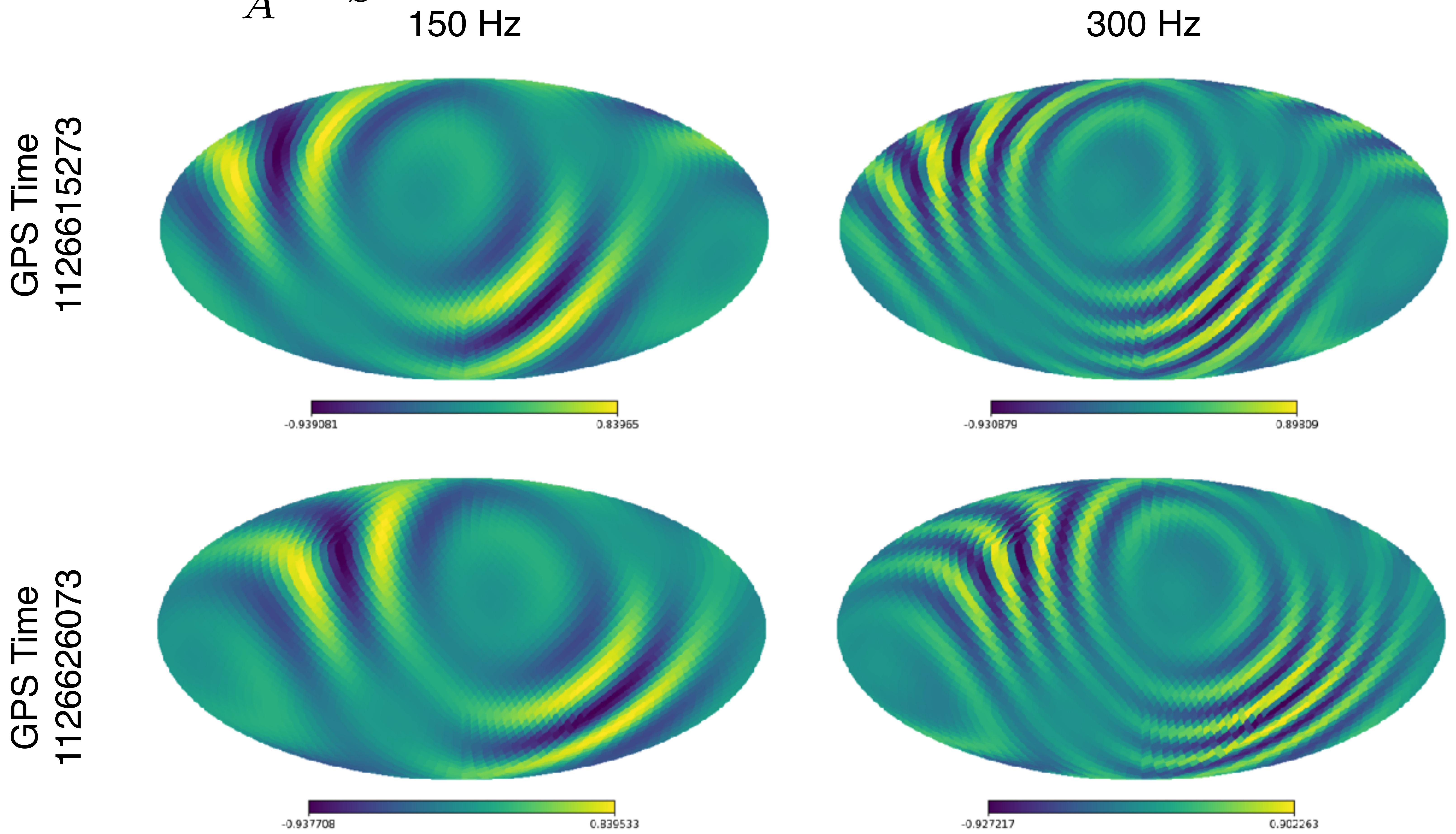
- Matlab based code used so far
- https://git.ligo.org/stochastic-public/stochastic/-/tree/master/Fold
- Paper: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.92.022003
- **Python translation complete by Erik Floden**
- https://git.ligo.org/sanjit.mitra/folding/-/tree/erik_dev?ref_type=heads
- Python version code review ongoing

PyStoch

- Python based pipeline
- Replaces Matlab code stochastic.m
- Takes advantage of folded data
- Efficient ORF calculation
- More optimised than previous codes
- Higher efficiency allows All-Sky All-Frequency search

Breaking Down the ORF

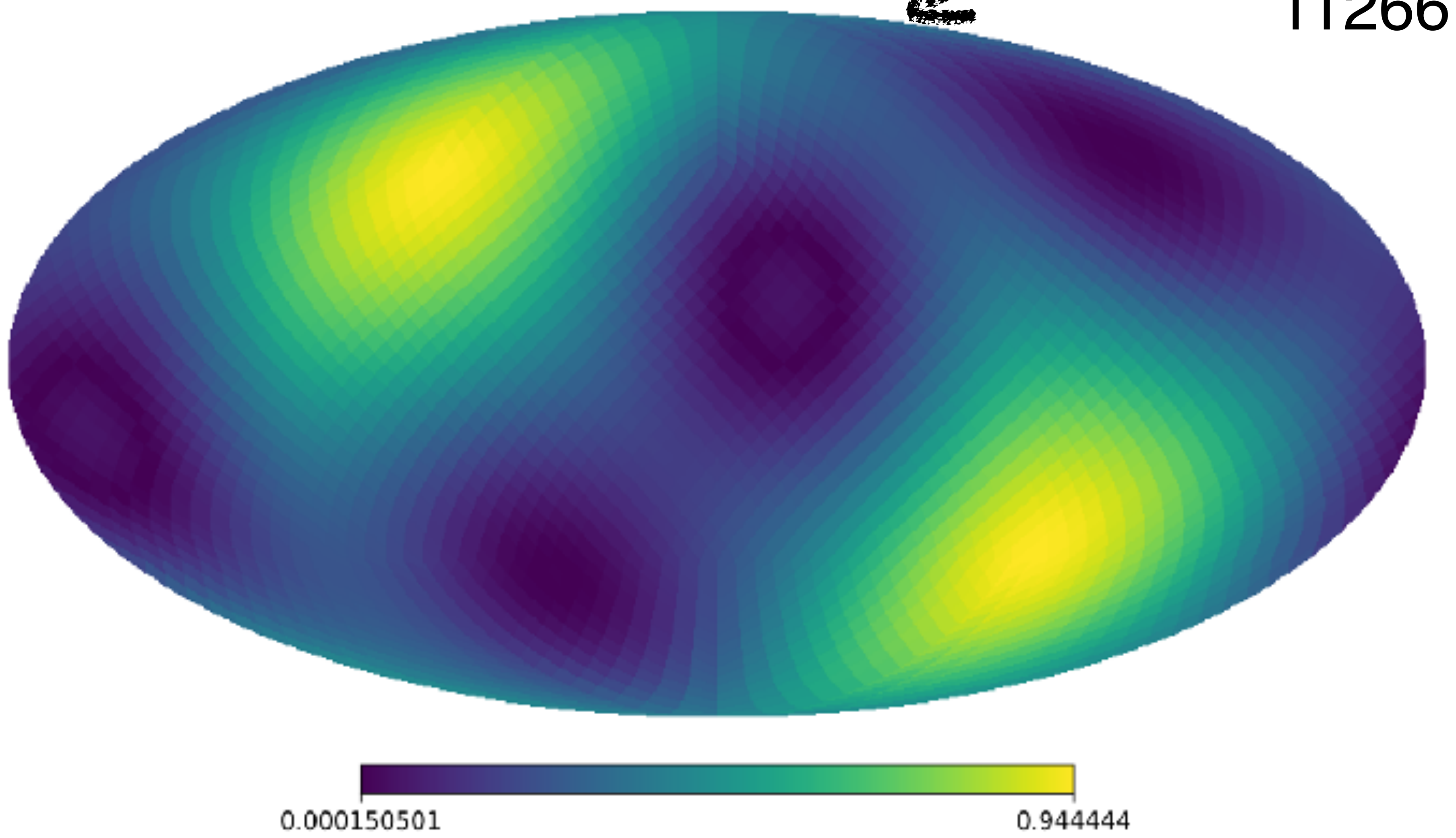
$$\gamma^{I}_{ft,\alpha} := \sum_{A} \int_{S^2} d\hat{\mathbf{\Omega}} F^{A}_{\mathcal{I}_1}(\hat{\mathbf{\Omega}}, t) F^{A}_{\mathcal{I}_2}(\hat{\mathbf{\Omega}}, t) e^{2\pi i f \frac{\hat{\mathbf{\Omega}} \cdot \mathbf{\Delta x}_I(t)}{c}} e_{\alpha}(\hat{\mathbf{\Omega}})$$



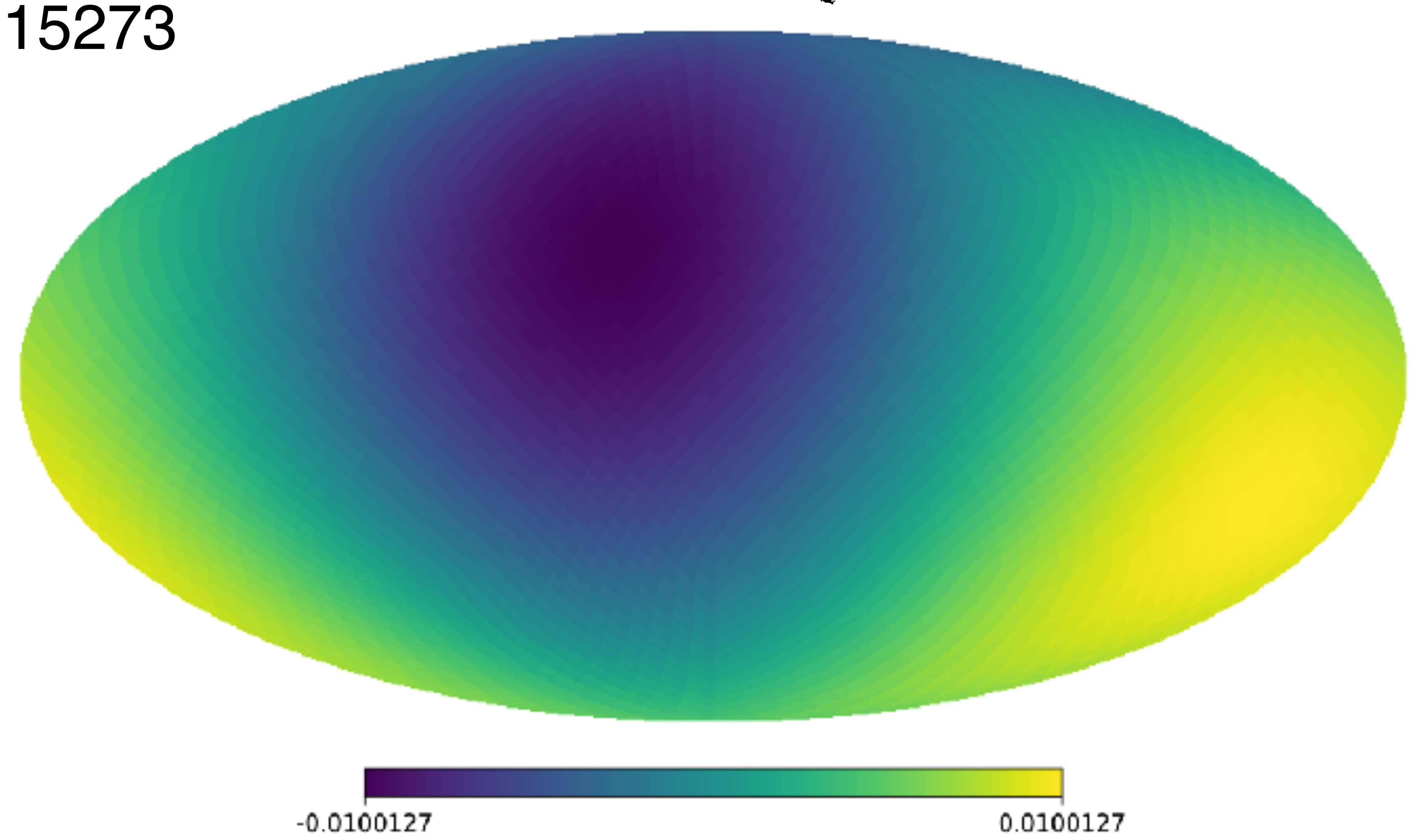
Breaking Down the ORF

$$\gamma^{I}_{ft,\alpha} := \sum_{A} \int_{S^2} d\hat{\mathbf{\Omega}} F^{A}_{\mathcal{I}_1}(\hat{\mathbf{\Omega}}, t) F^{A}_{\mathcal{I}_2}(\hat{\mathbf{\Omega}}, t) e^{2\pi i f \frac{\hat{\mathbf{\Omega}}\cdot\mathbf{\Delta x}_I(t)}{c}} e_{\alpha}(\hat{\mathbf{\Omega}})$$

GPS Time
1126615273



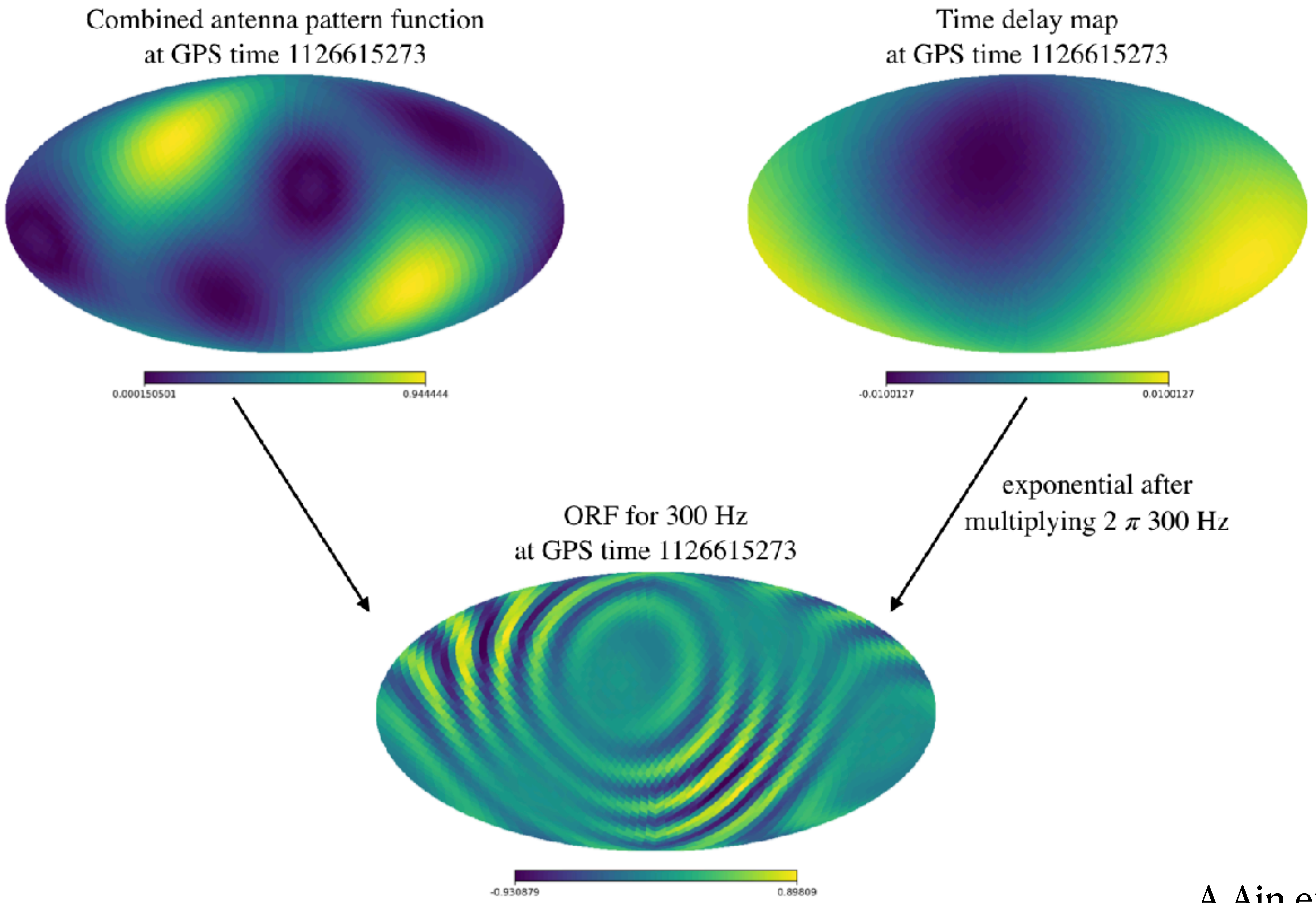
Combined Antenna Response



Time Delay

3314 pairs of seed maps (for one sidereal day) generated in a laptop in 20 seconds.

ORF from ORF Seeds



A.Ain et.al. PhysRevD.98.024001

Narrowband Maps

$$X_\alpha = \sum_{Ift} K^{I*}_{ft,\alpha} \sigma^{-2}_{Ift} C^I_{ft}$$

$$X_\alpha = \tau \sum_f H(f) \sum_{It} \gamma^{I*}_{ft,\alpha} \sigma^{-2}_{Ift} C^I_{ft}$$

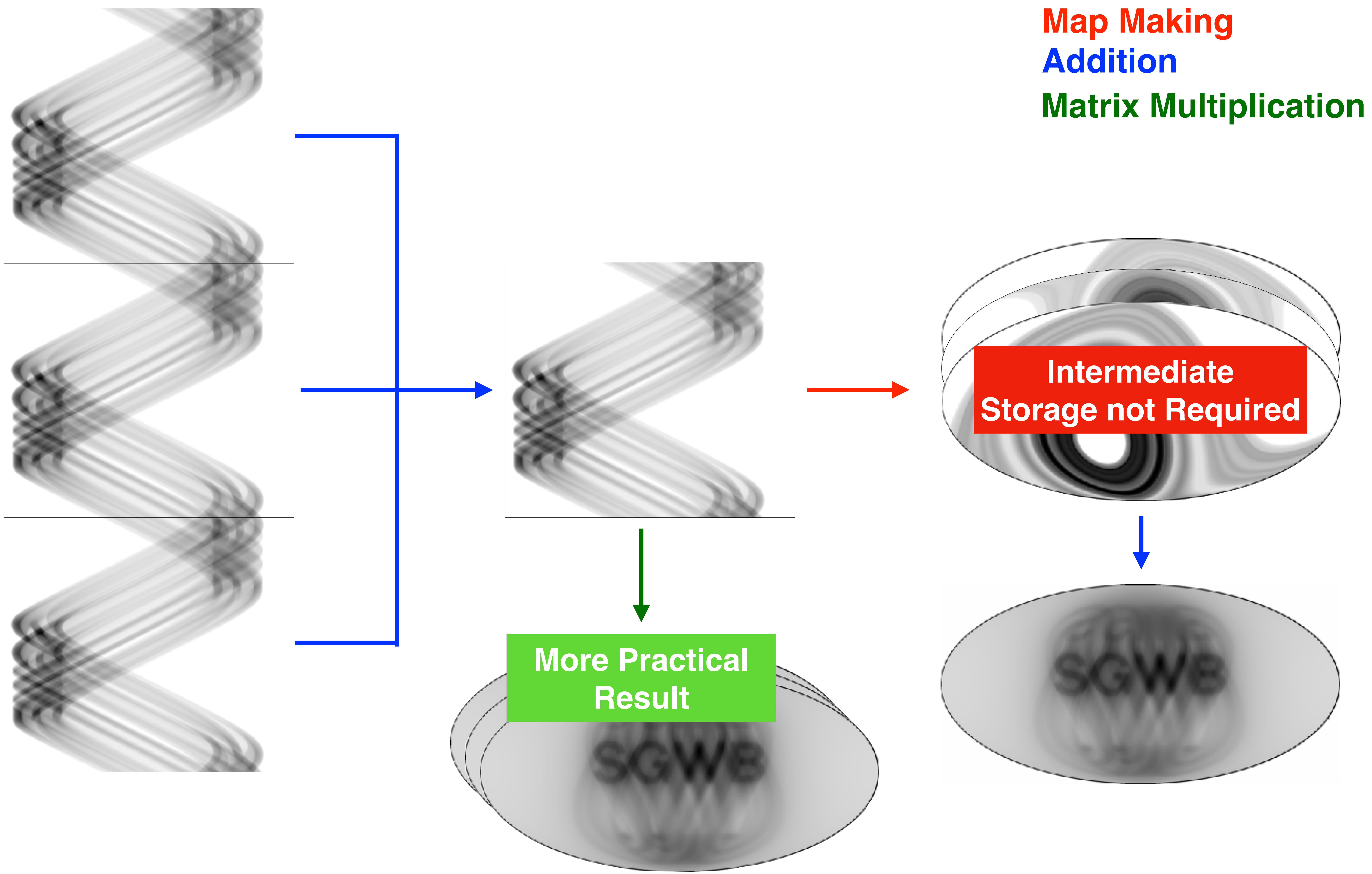
$$X_{f,\alpha} = \tau \sum_{It} \gamma^{I*}_{ft,\alpha} \sigma^{-2}_{Ift} C^I_{ft}$$

Narrowband Dirty Maps

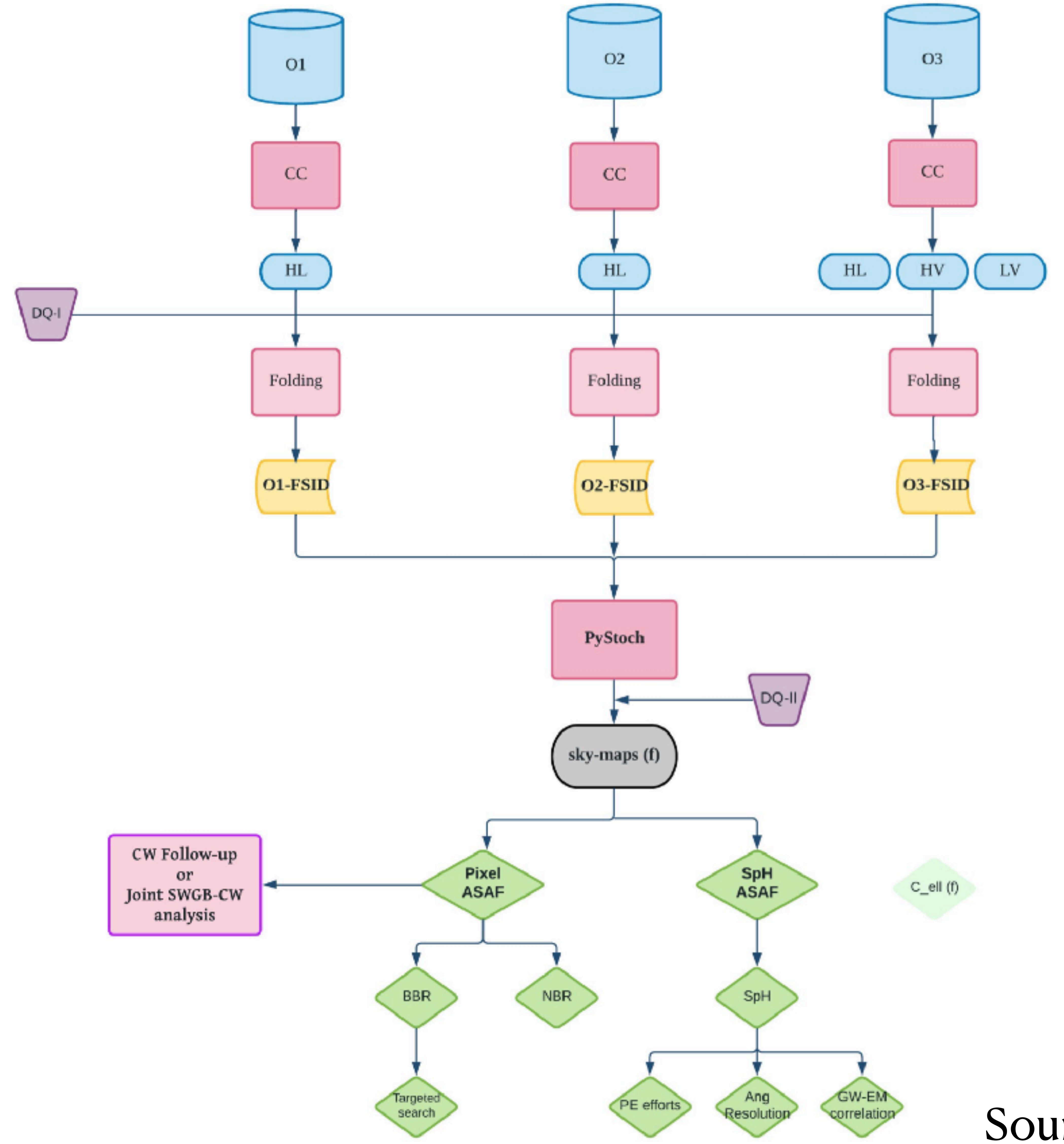
$$X_\alpha = \sum_f H(f) X_{f,\alpha}$$

BBR Dirty Map

Stochastic Pipeline (with folding and PyStoch)



Overall Analysis



CC —> Cross-Correlation
FSID —> Folded Stochastic Intermediate Data
DQ-I —> Data Quality cuts (Time-Domain)
DQ-II —> Data Quality Cuts (Frequency-Domain)
BBR —> BroadBand Radiometer
SpH —> Spherical Harmonic
ASAF —> All-Sky All-Frequency

Source: Jishnu: dcc.ligo.org/LIGO-G2200279

Overall Analysis

- Data quality information from isotropic analysis
- Bad segments and frequency notching lists from isotropic analysis
- Frequency notching
- Lines identified from coherence studies from isotropic analysis
- Calibration lines, power line harmonics (50 Hz and 60 Hz) and known instrumental lines
- Additional notches are added for the ASAF analysis
- Bad time segments
- Identified using traditional 20% delta sigma cut (for power law indices = [-5, 0, 3, 5])
- Further data quality studies will be undertaken as needed.
- Gated data from the pygwb
- Output files from isotropic analysis is the starting point

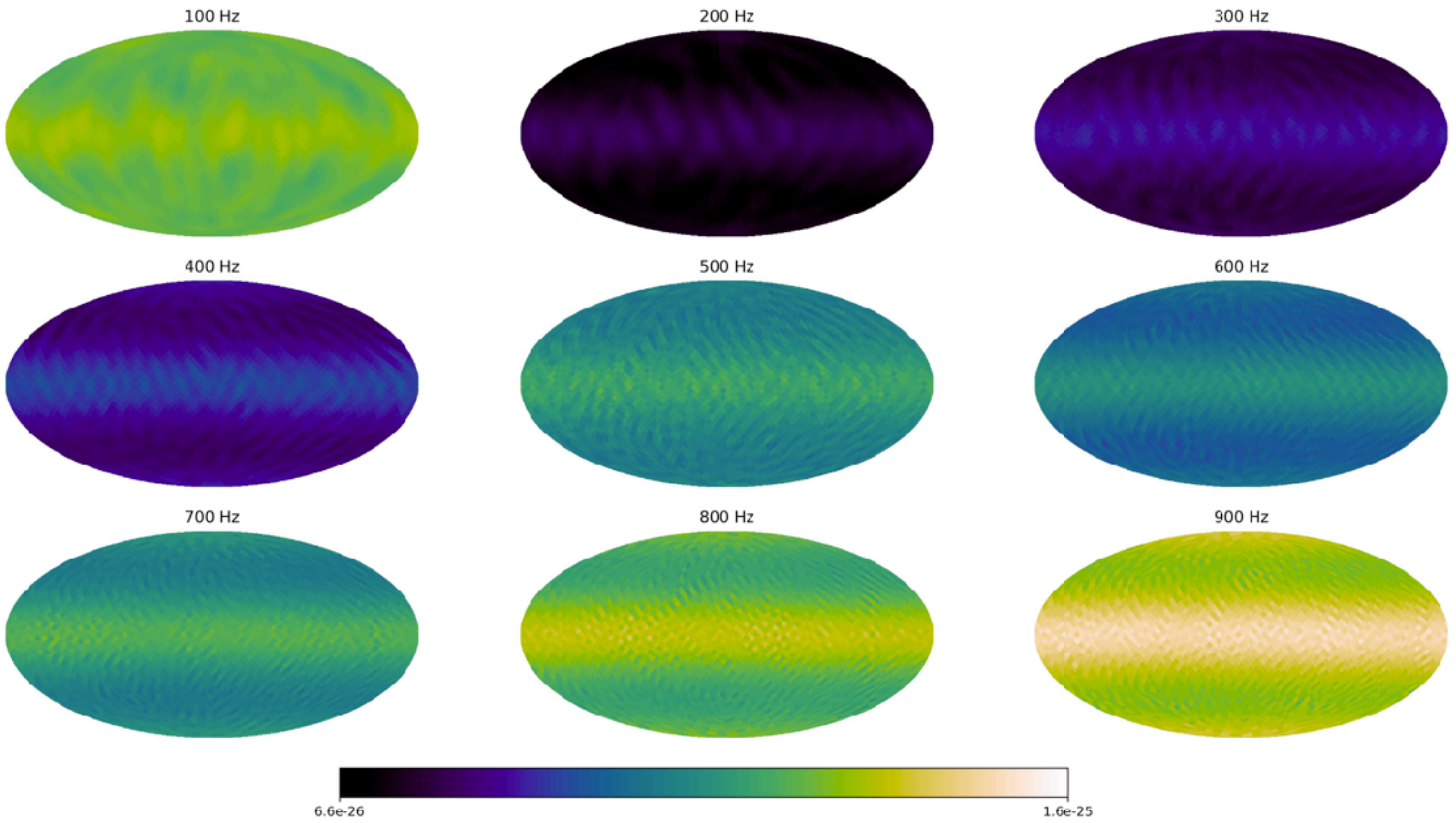
Source: Deepali : dcc.ligo.org/LIGO-G2301128

All-sky all-frequency search

- Analysing sky-maps at every frequency bin.
- Model independent result.
- More strict notch-list.
- https://git.ligo.org/stochastic/PyStoch/-/tree/ASAF
- https://git.ligo.org/stochastic-papers/o3-asaf
- The basis of other anisotropic searches.
- Also identify a set of frequency-pixel pairs candidates which can be followed up with a matched-filtering-based analysis to look for significant outliers.
- We hope that it will become possible to derive many other astrophysical inferences/constraints from the ASAF results.

A few Strain Upper-limit maps

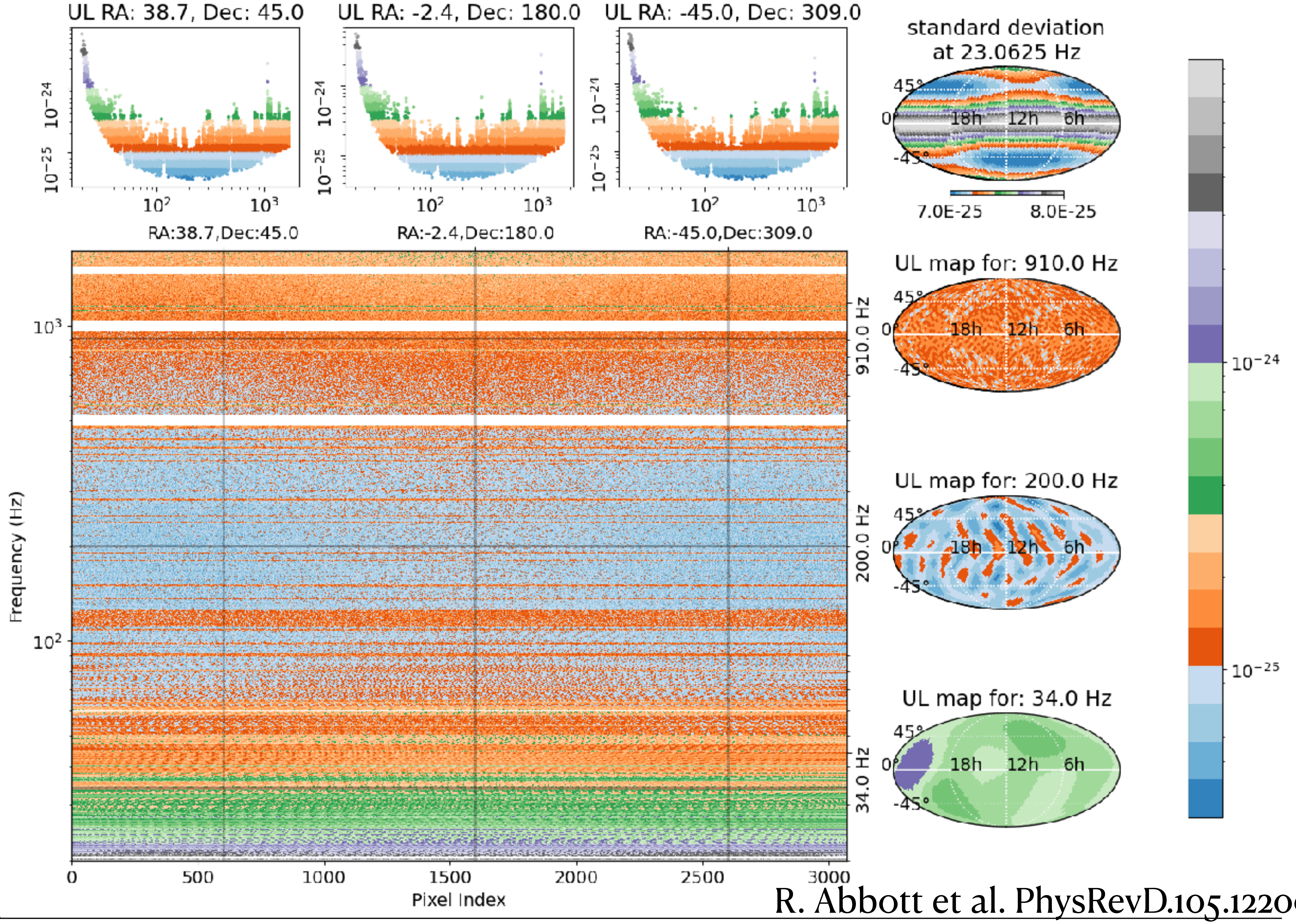
Upper limits of expected gravitational waves strains at different frequencies. The upper limit means the extraterrestrial signal at these frequencies cannot be more than what is shown in these maps. The color-shading indicates the sensitivity of our search in different frequency bands, and for different sky directions. One can see that the search is generally somewhat less sensitive near the equatorial plane. Moreover, the search is most sensitive in the 200-400Hz range.



https://www.ligo.org/science/Publication-O3Radiometer/index.php

The Full ASAF result* with a few cross-sections

Bayesian upper limit on the strain amplitude h for all-sky directions and all-frequency bins. These upper limits are set using the ASAF search performed on the O1+O2+O3 data set. The color bar here denotes the range of upper limit variations. Since the horizontal axis is pixel index, a horizontal line corresponds to a skymap for the frequency shown on the vertical axis. The variation of upper limits along the horizontal axis depicts the variation of the variance of the radiometer map along the latitude (the HEALPix 'ring' pixel numbering used here increases along the co-latitude). Notched frequencies in a baseline appear as horizontal bands in the plot.



*95% bayesian upper limits on strain amplitude

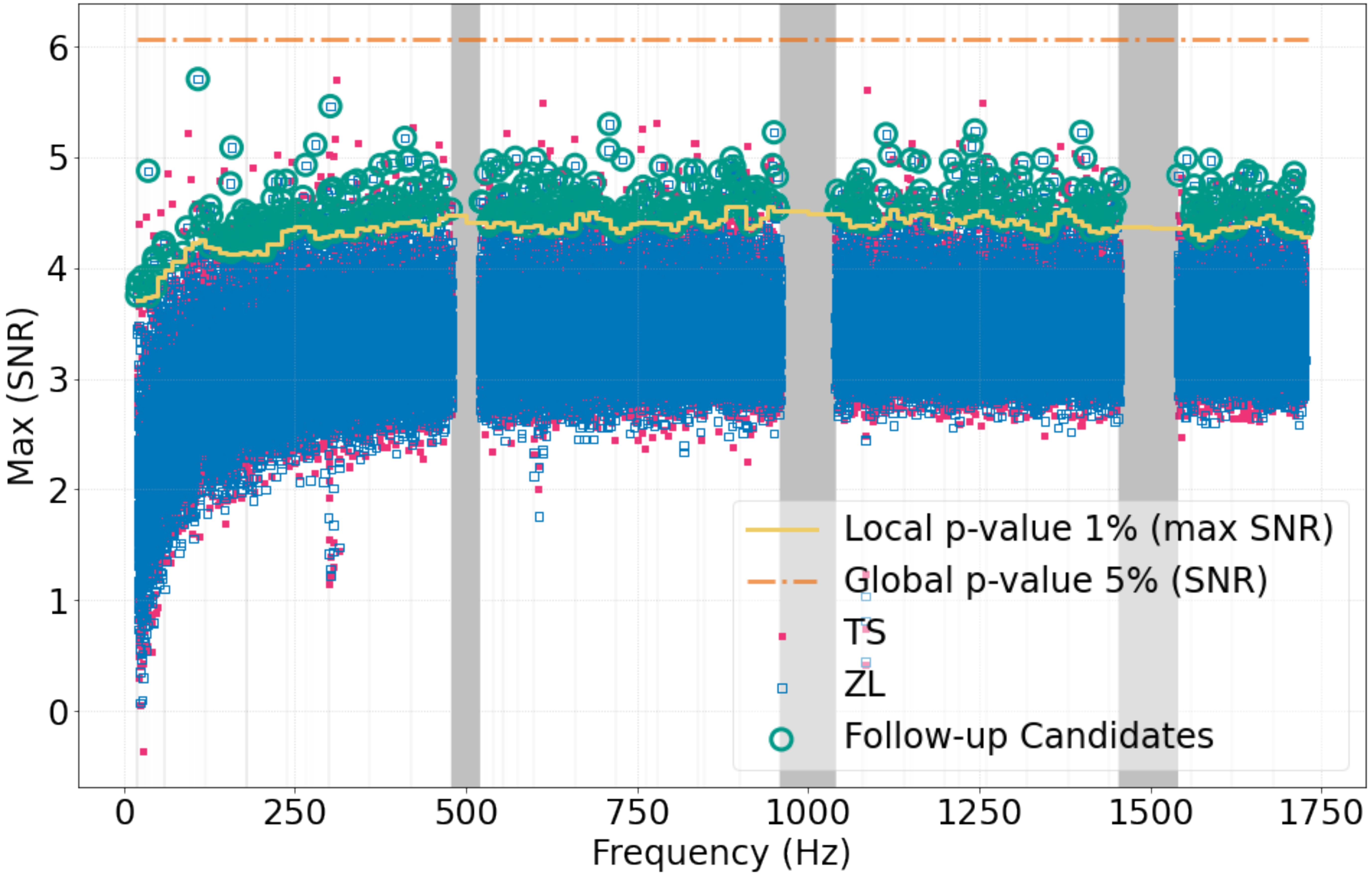
R. Abbott et al. PhysRevD.105.122001

Points of higher SNR (insignificant candidates)

Potential Continuous sources?

https://git.ligo.org/stochastic-papers/o3-asaf/-/blob/master/runs/Plots/freq_maxSNR_scatter/all_combined_followUp_Cand_details.csv

Distribution of maximum SNRs performed over the combined baselines and all three observing runs. The yellow curve shows the threshold which contains 99% of the noise background, smoothed over 3 neighboring 10 Hz bins. Though we do not find any outliers significantly above the noise background, the teal circles represent 515 candidates which may be followed up by a more sensitive matched-filtering-based analysis. The grey solid lines represent the frequencies that are excluded.



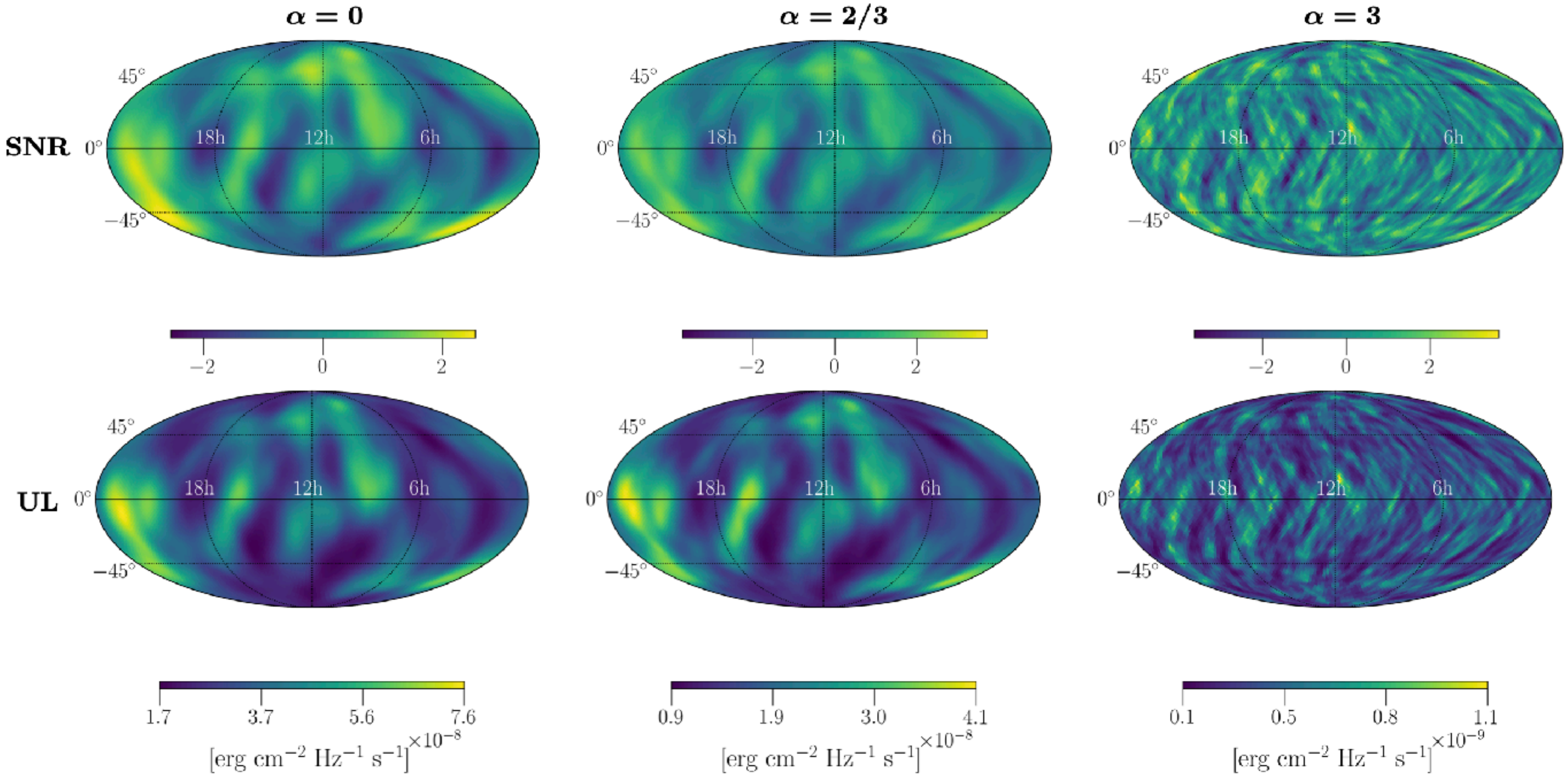
R. Abbott et al. PhysRevD.105.122001

Broadband radiometer

- Point sources with different power-law spectra
- Start from ASAF outputs and combine the narrow-band maps to the broad-band ones with appropriate weights for α = 0, 2/3, 3
- Looking for outliers (SNR>4) with focus on the α=3 case (flat strain, the only one for which full broadband fisher from ASAF is available for the time being)
- Check by eye of BBR ptEst and SNR distributions
- Evaluation of outliers statistical significance using p-value statistics.

Source: Federico De Lillo

Broadband radiometer

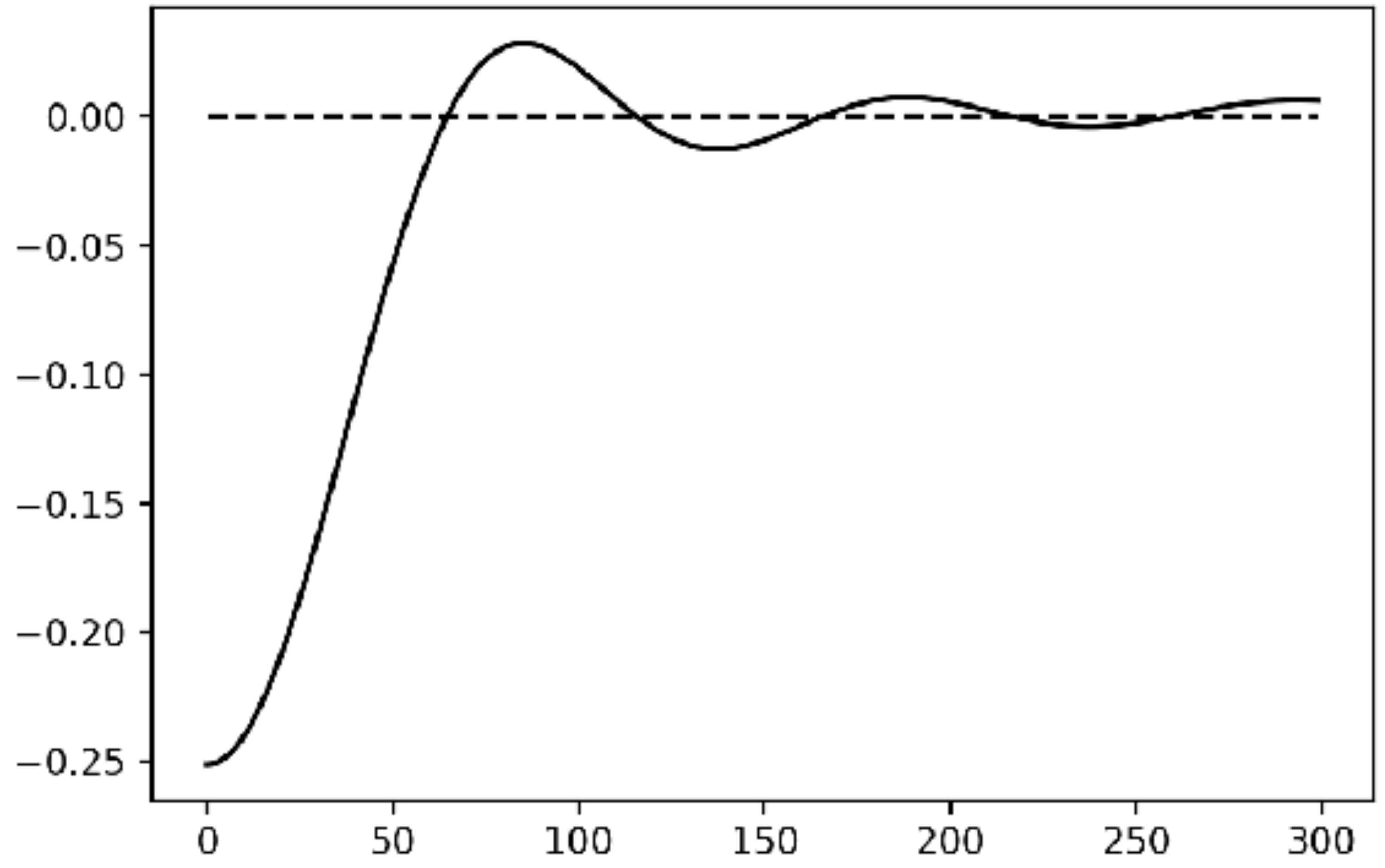


SNR maps from a BBR search for pointlike sources. Bottom row: upper limit (UL) sky maps of the gravitational-wave energy flux. Both sets of maps, presented in equatorial coordinate system, are derived by combining all three LIGO observing runs and the Virgo O3 data.

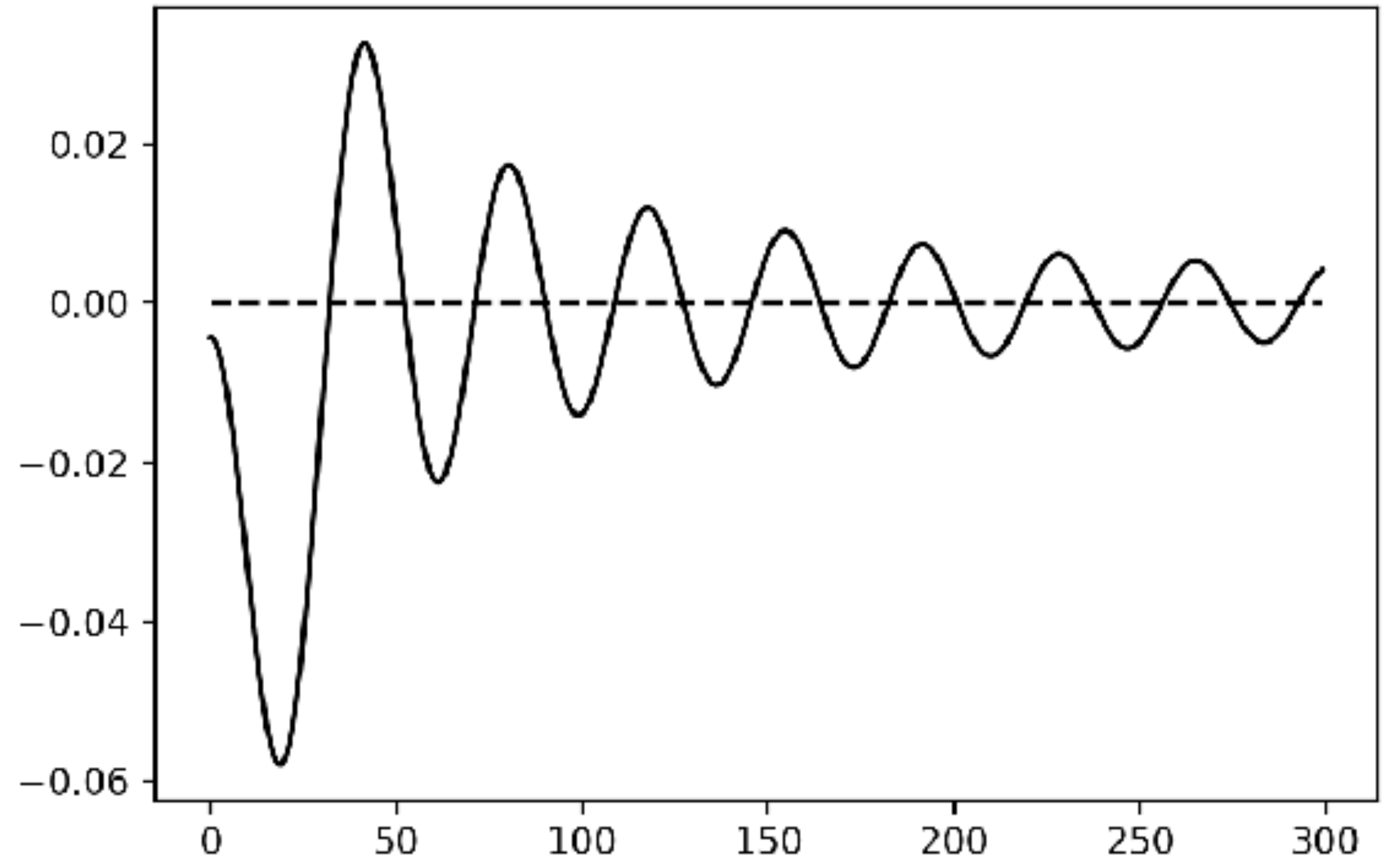
R. Abbott et al. PhysRevD.104.022005

Spherical harmonics search

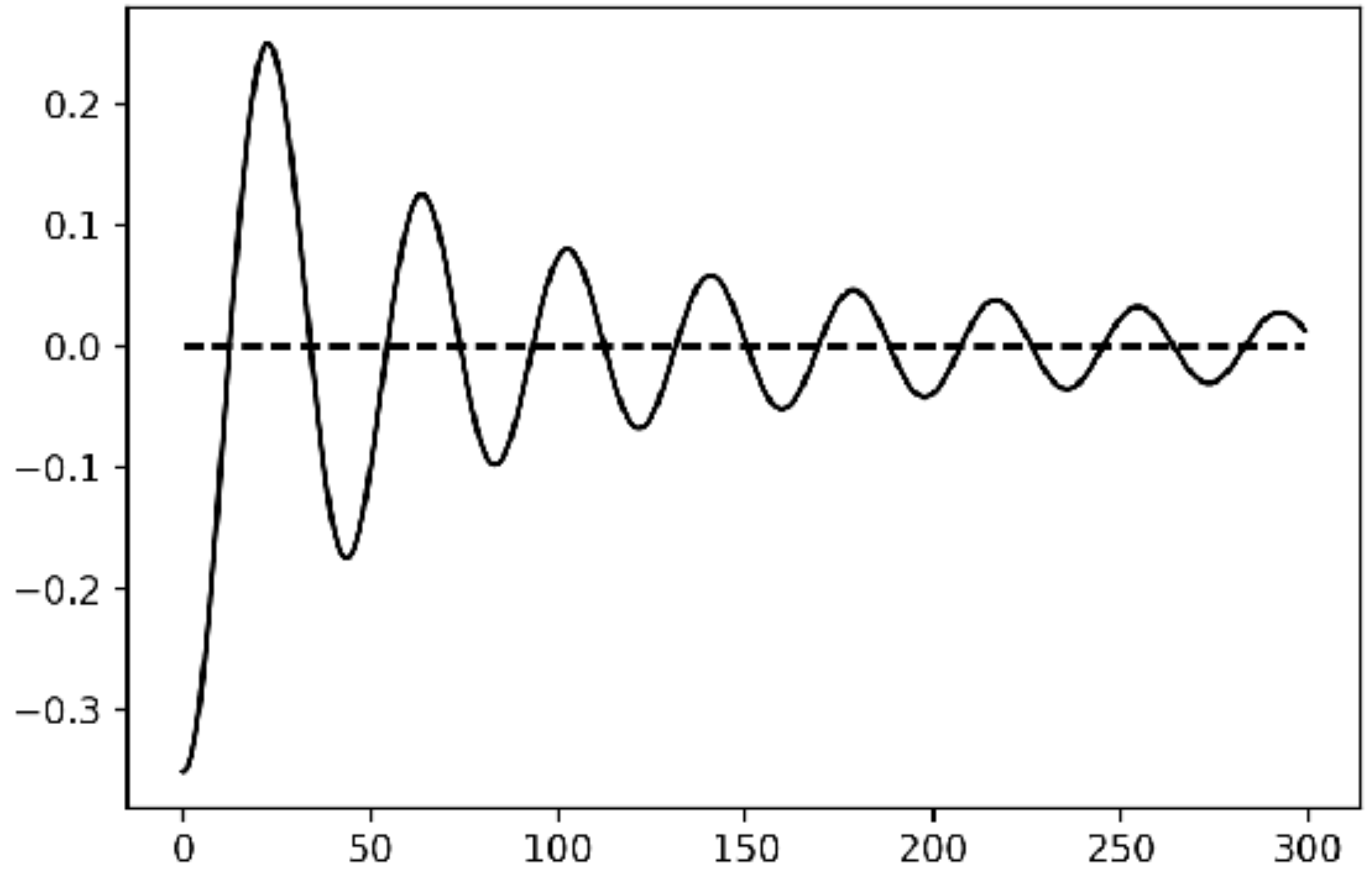
Using PyStoch



Hanford-Livingston



Hanford-Virgo



Livingston-Virgo

- The first spherical harmonic term in the overlap reduction function as calculated by PyStoch for the 3 possible baselines in LIGO Virgo detector network The solid line is for the real part of the function, dashed line is complex.

$$\gamma^I_{ft,lm} = \gamma^I_{f0,lm}\, e^{im2\pi t/T}$$

J. Suresh et al. PhysRevD.103.083024

Spherical harmonics search

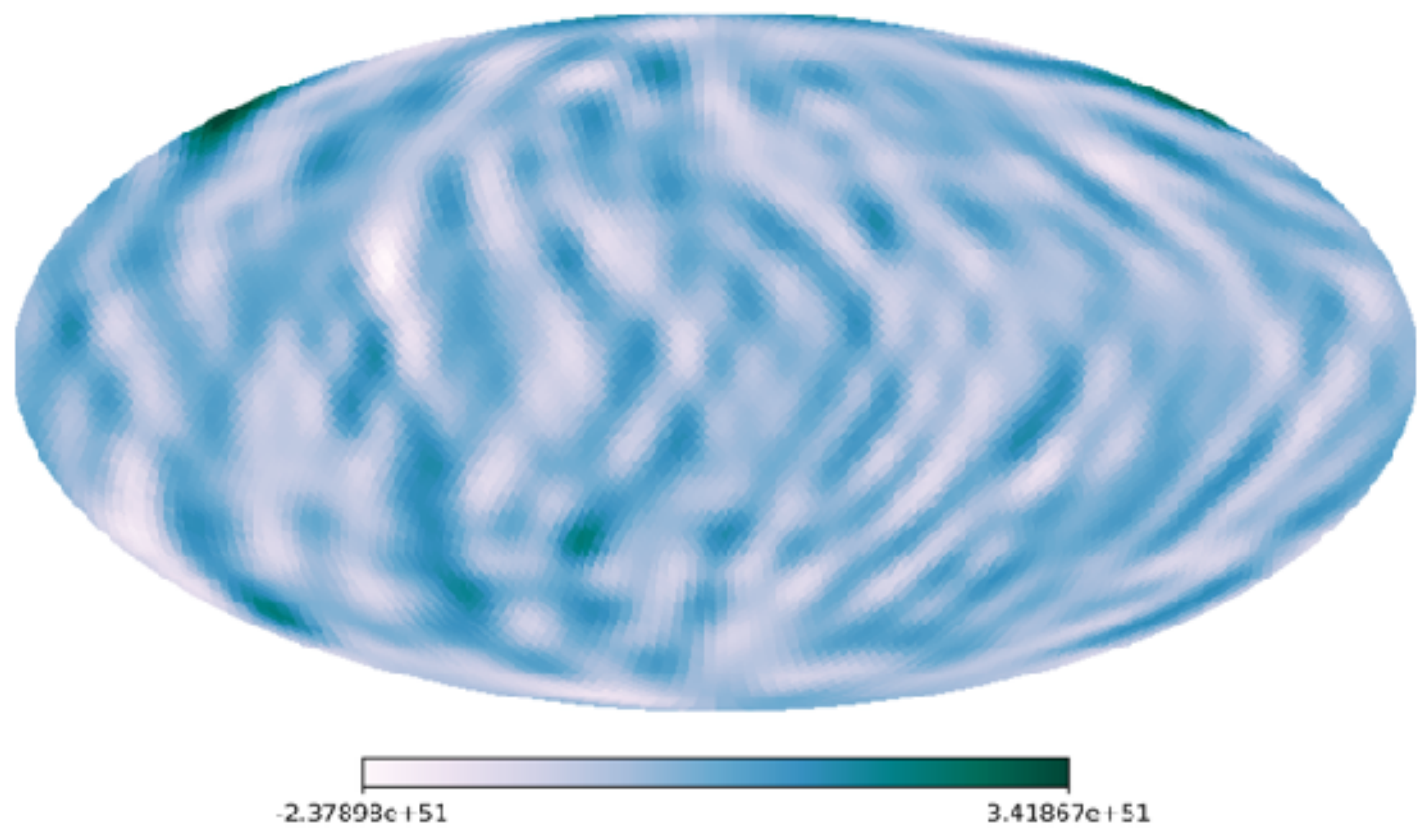
$$\mathcal{P}(\hat{\Omega}) = \mathcal{P}_{\hat{\Omega}'}\delta(\hat{\Omega}, \hat{\Omega}')$$

SGWB in Pixel basis

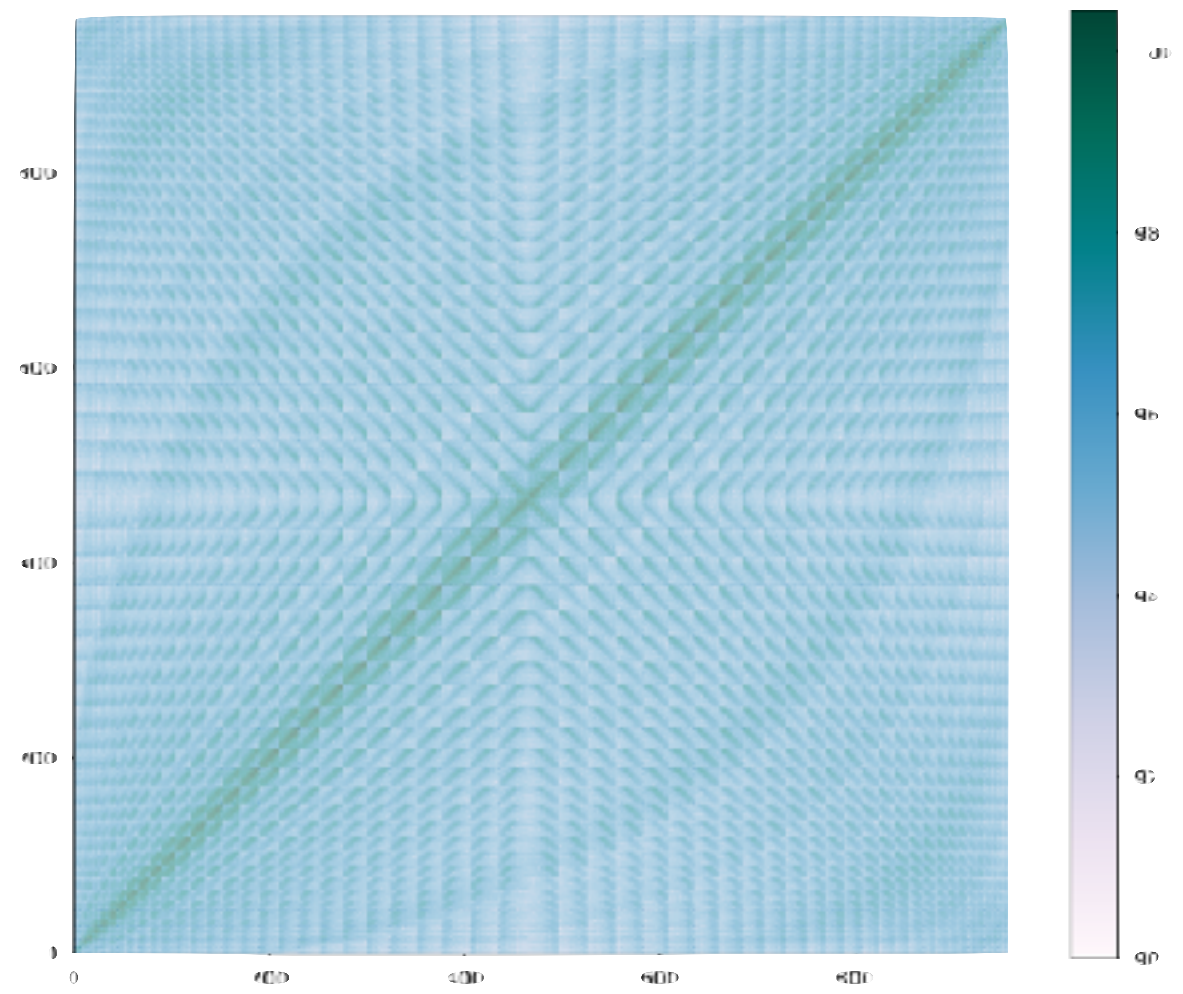
$$\mathcal{P}(\hat{\Omega}) = \mathcal{P}_{lm}Y_{lm}(\hat{\Omega})$$

SGWB in SpH basis

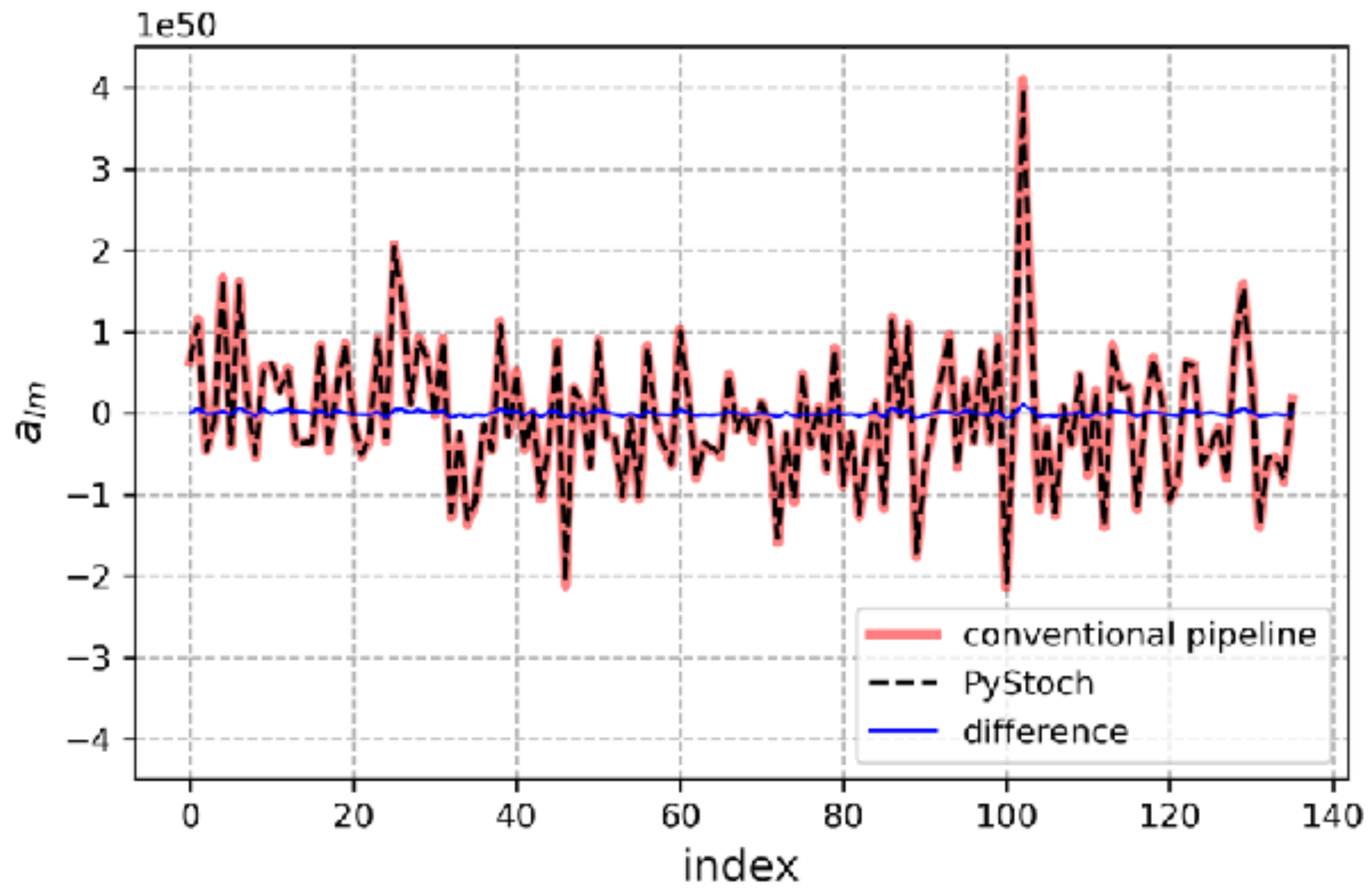
- Extended or defused sources, measure angular power spectra (similar to CMB)



Map from SpH



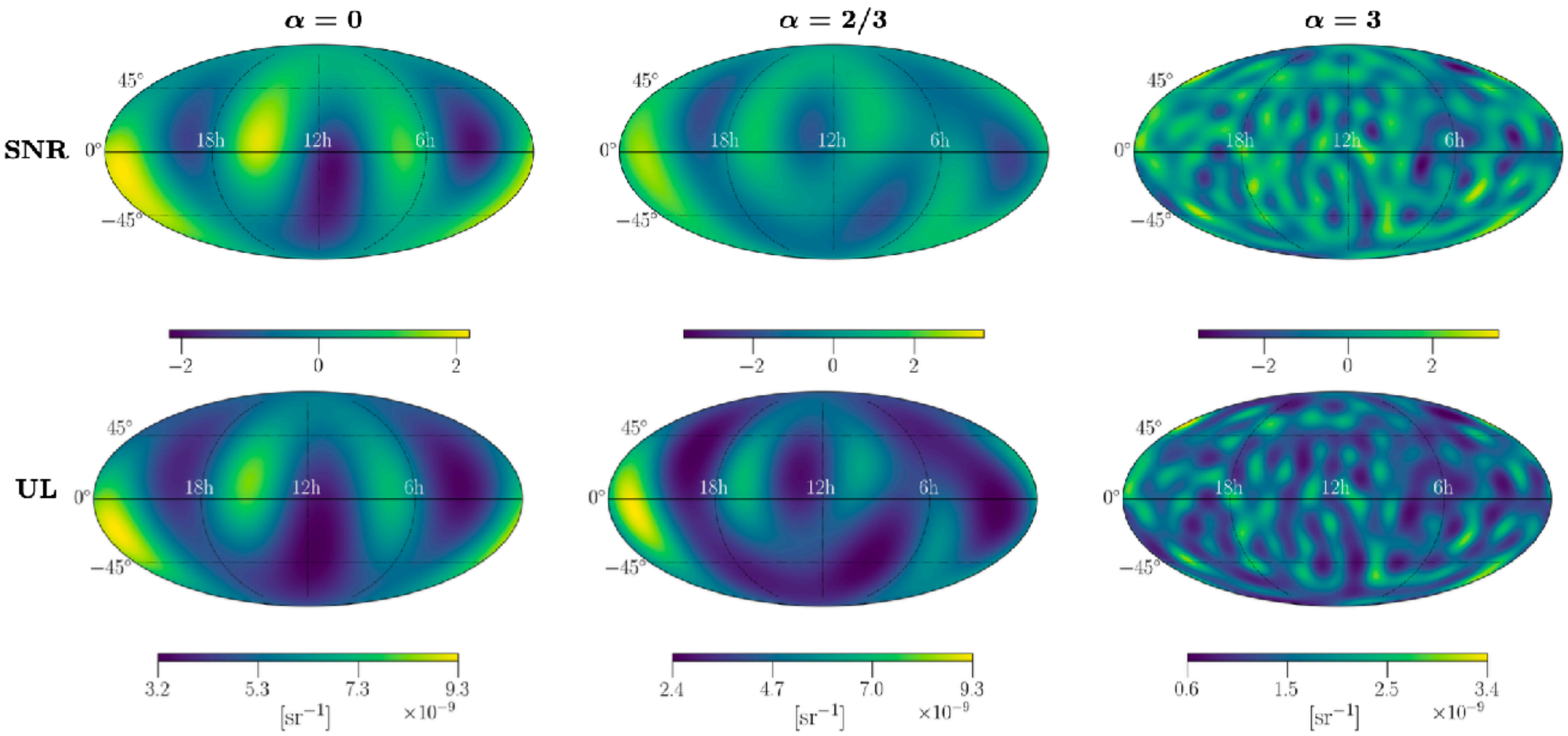
Full Fisher Matrix



Difference between PyStoch and Older pipeline

J. Suresh et al. PhysRevD.103.083024

Spherical Harmonic Decomposition

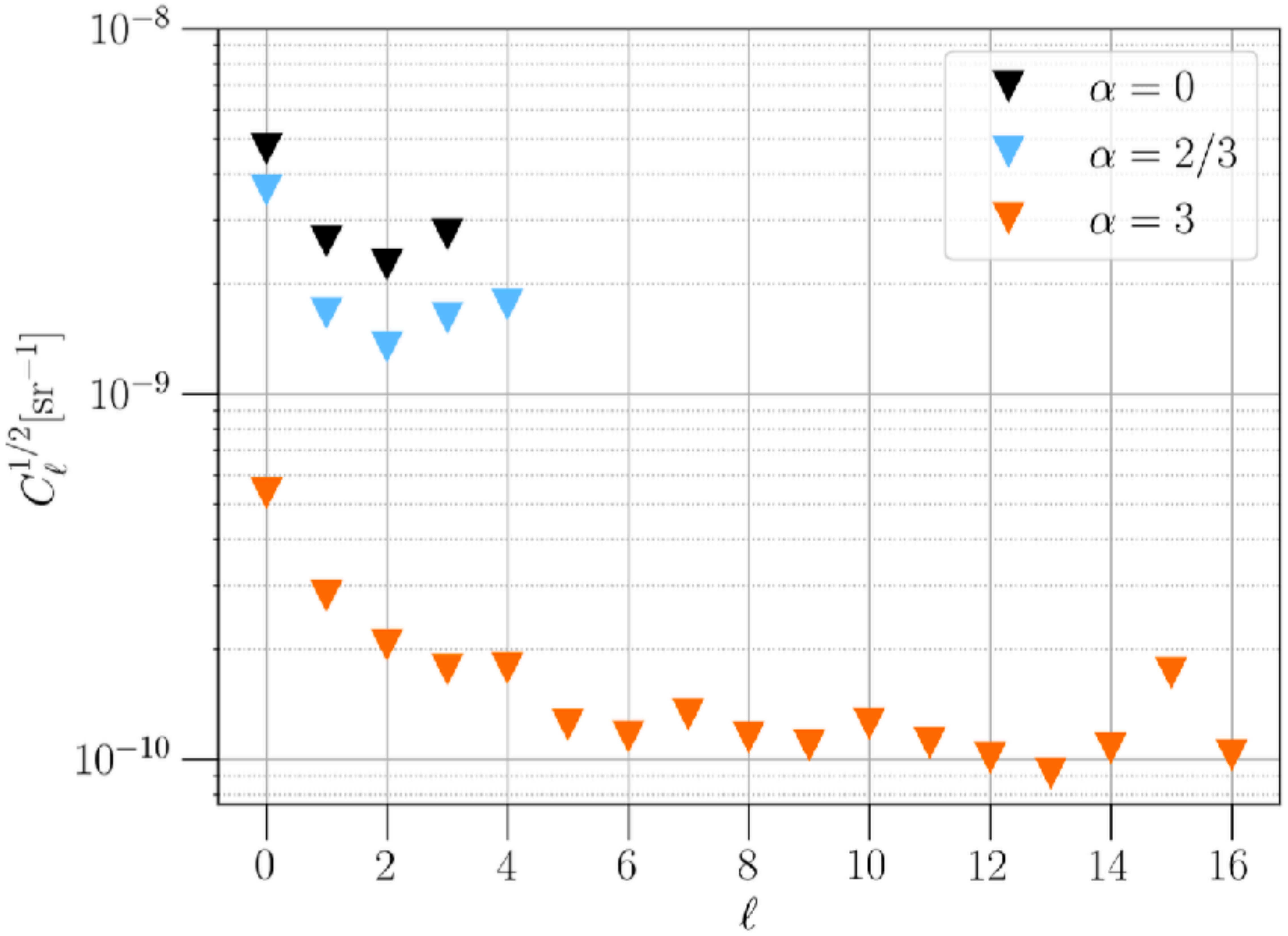


SNR maps from the SHD search for extended sources. Bottom row: sky maps representing 95% upper limit on the normalized gravitational-wave energy density Ωα(Θ)[sr−1].

R. Abbott et al. PhysRevD.104.022005

Spherical Harmonic Decomposition

- l_max is determined by the distance D between detectors and the most sensitive frequency f in the analysis band
- C_l Calculated by simulation because its analytical form is not trivial.

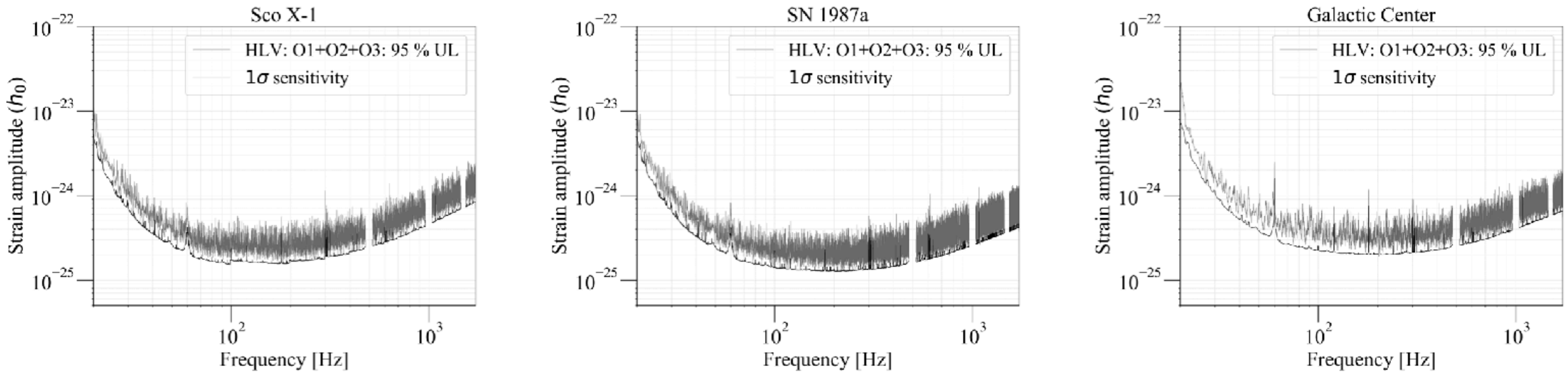


95% upper limits on Cl for different α using combined O1+O2+O3 data.

R. Abbott et al. PhysRevD.104.022005

Narrowband radiometer

- Suitable for point sources having narrow GW frequency band.
- For example SN 1987A, Sci X-1, Galactic Center.
- NBR analysis is run at higher angular resolution (nside = 256).



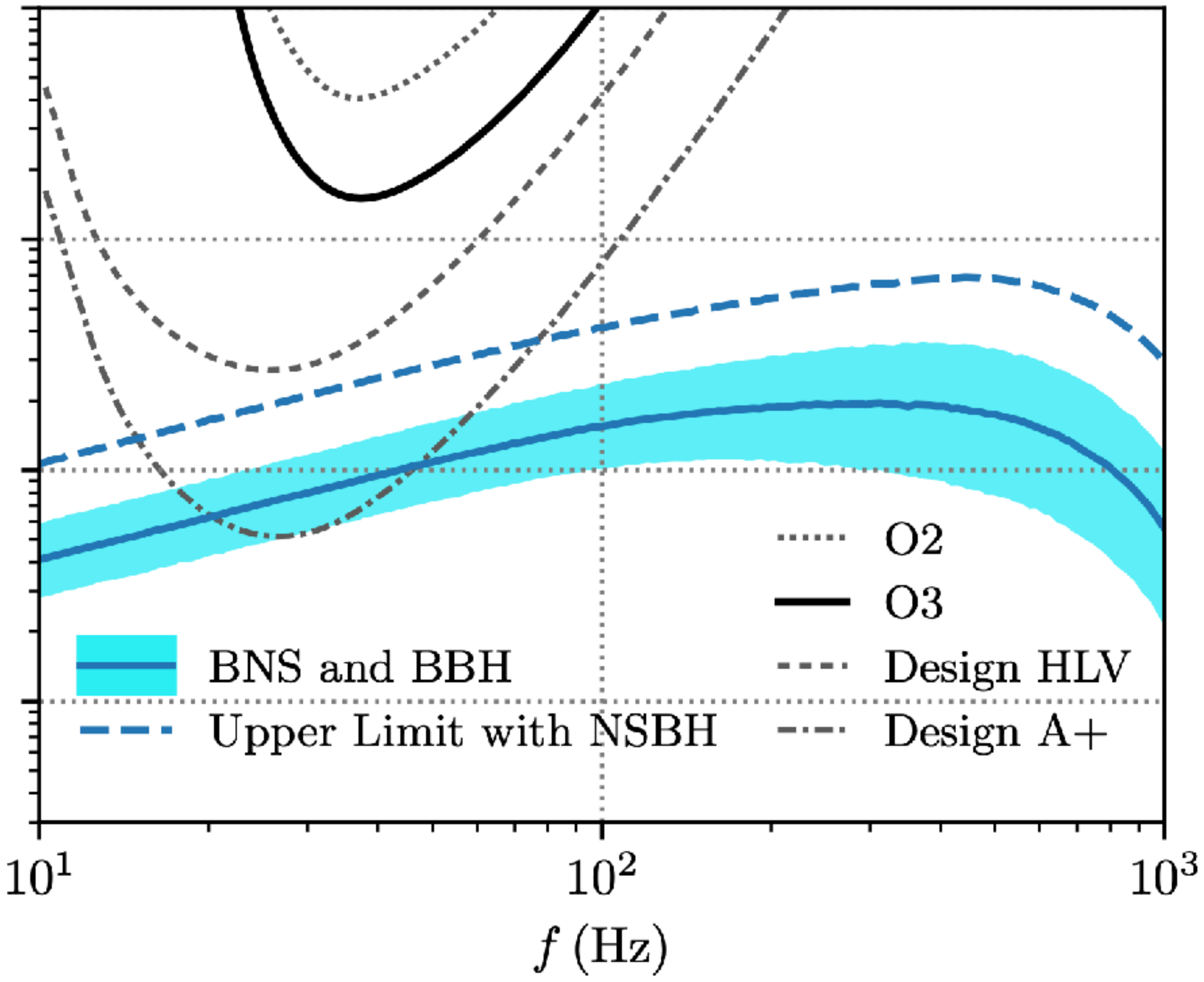
Upper limits on the dimensionless strain amplitude ho, using the data from three observing runs of LIGO-Virgo detectors, at the 95% confidence level for the narrow band radiometer search are indicated by the grey bands. The dark line shows the 1σ sensitivity of the search for each direction.

R. Abbott et al. PhysRevD.104.022005

Future : Detection?

Our current best estimates

- combined BBH and BNS energy density spectra.
- solid blue line shows the median estimate as a function of frequency, while the shaded blue band illustrates 90% credible uncertainties
- 2σ PI curves for O2, O3
- projections for 2 years of the Advanced LIGO-Virgo network at design sensitivity
- envisioned A+ design sensitivity after 2 years
- These curves indicate that by the time the detectors reach the A+ design sensitivity, much of the expected parameter space of the compact binary GWB will be accessible by ground-based detectors.



R. Abbott et al. PhysRevD.104.022004

Thanks!