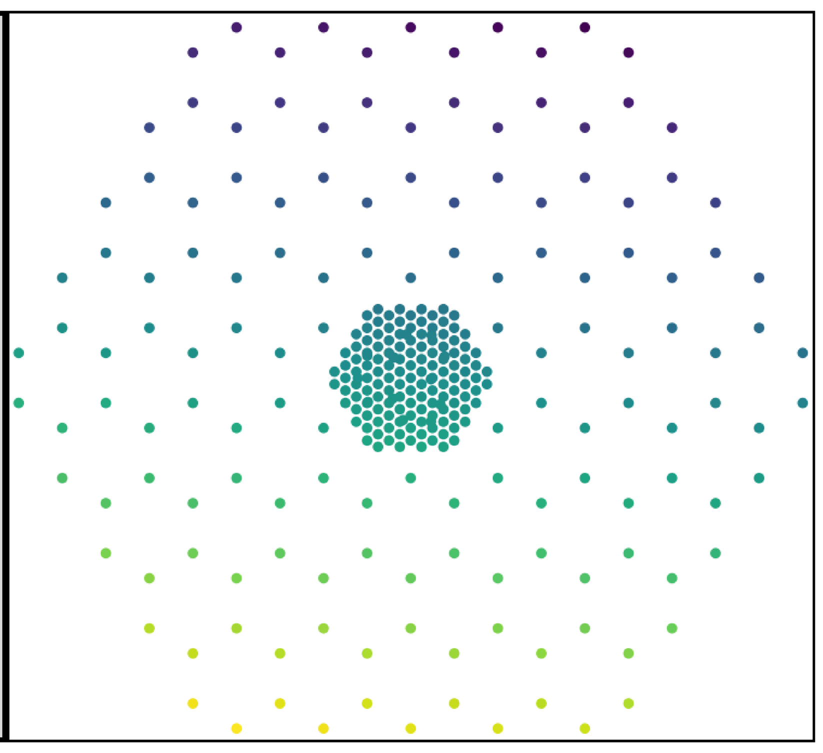
Analytical plane wave reconstruction and uncertainty estimation

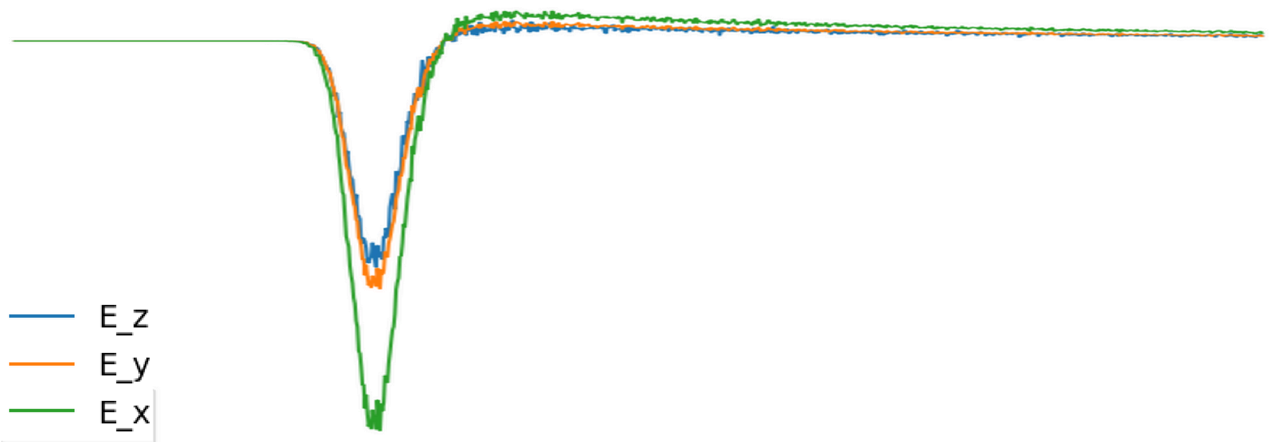
Arsène Ferrière (LPNHE/CEA-LIST)

Introduction

We want to get the arrival direction of a cosmic ray from measured timings on antennas. We also need to estimate the uncertainty of our predictions.

Working on 849 Monte-Carlo simulations. Extracting times from the peak of the radio signal.





Example of a radio signal

Approximating the wavefront shape:

Hyperbolic → Spherical → Planar

For inclined airshowers

As a first approximation <1°

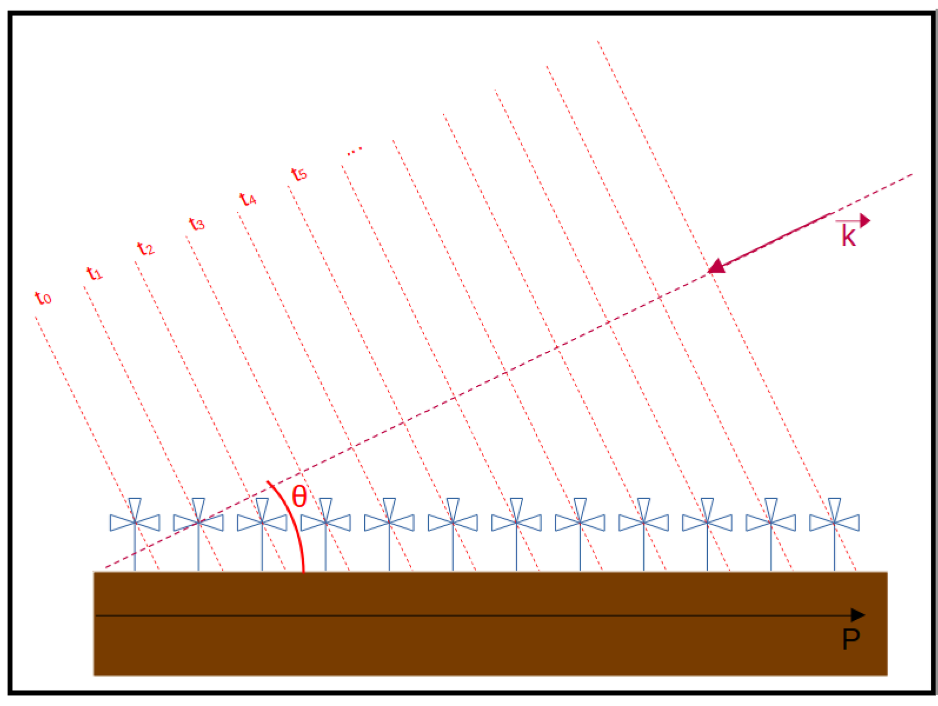
<u>Under planar approximation:</u>

Modeling the wavefront:

$$t_i = \frac{1}{c} P_i^T \underbrace{k}_{\text{unknown}} + t_0 + \overbrace{\epsilon_i}^{\text{Gaussian Noise}}$$

Likelihood function:

$$L_{\mathbf{P},\mathbf{T}}(k) = \frac{1}{2}(c\mathbf{T} - \mathbf{P}k)^T(c\mathbf{T} - \mathbf{P}k)$$



We have a linear regression task.

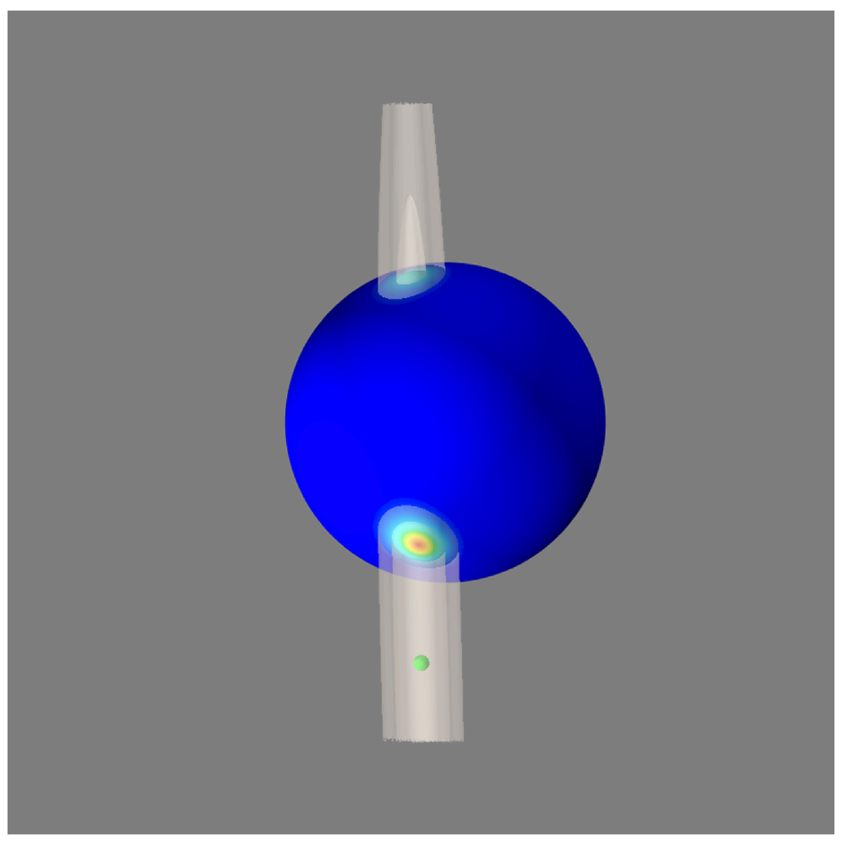
Linear regression under constraint: $\|\mathbf{k}\| = 1$

$$k \sim \mathcal{N}(k^*,\ \Sigma) \text{ with: } \begin{cases} k^* = c(P^T P)^{-1} P^T T \\ \Sigma = (c\sigma)^2 (P^T P)^{-1} \end{cases}$$

The distribution is stretch vertically because the antenna layout is almost flat

Adding the constraint afterward

To add the constraint, we have to find the maximum of the distribution on the blue sphere: Vertical projection



We don't want the distribution of $\mathbf{k}$

We want the distribution of $\mathbf{k} \mid \|\mathbf{k}\| = 1$

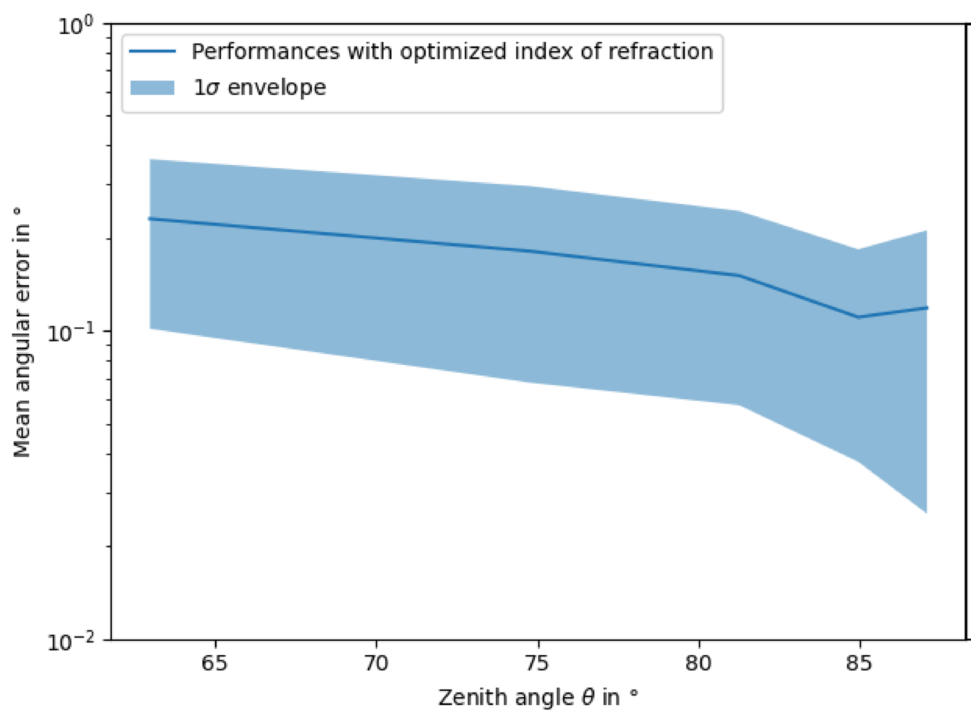
Graphically : It is intersection of white envelope with the blue sphere. (colored distribution)

We can't directly compute $\mathbf{k} \mid \|\mathbf{k}\| = 1$

However : we can consider that the sphere is locally equal to the tangent plane at the best fit value.

$$\mathbf{k} \mid \|\mathbf{k}\| = 1 \approx \mathbf{k} \mid \mathbf{k} \in \mathcal{T}$$

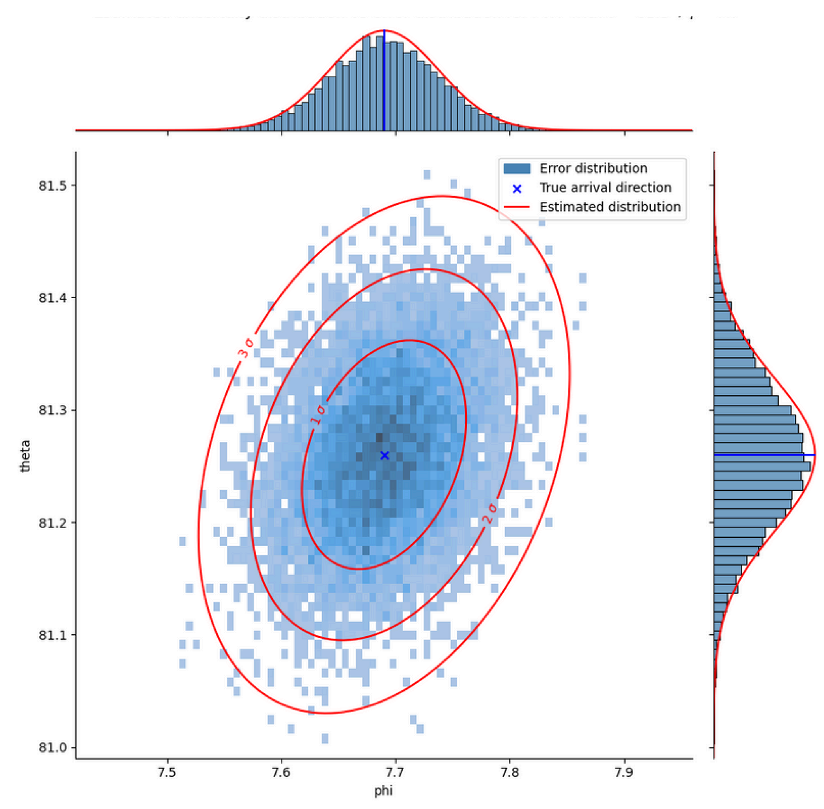
We then have $\begin{pmatrix}\theta \\ \phi\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}\theta_s \\ \phi_s\end{pmatrix}, \bar{\Sigma}\right)$ with $\begin{cases} \bar{\mathbf{\Sigma}} = \left[\mathbf{R_a}^T \mathbf{\Sigma}^{-1} \mathbf{R_a}\right]^{-1} \\ \mathbf{R_a} = \begin{pmatrix} -\cos(\theta_s)\cos(\phi_s) & \sin(\theta_s)\sin(\phi_s) \\ -\cos(\theta_s)\sin(\phi_s) & -\sin(\theta_s)\cos(\phi_s) \\ \sin(\theta_s) & 0 \end{pmatrix} \end{cases}$



Noisy data : Extracted times + 5e-9s noise.

Sub degree precision,
High robustness
More precise for high inclination:
Larger number of triggered antennas

How accurately do we predict the uncertainty ?



Applied to ideal data:
PWF data with 1e-8s noise

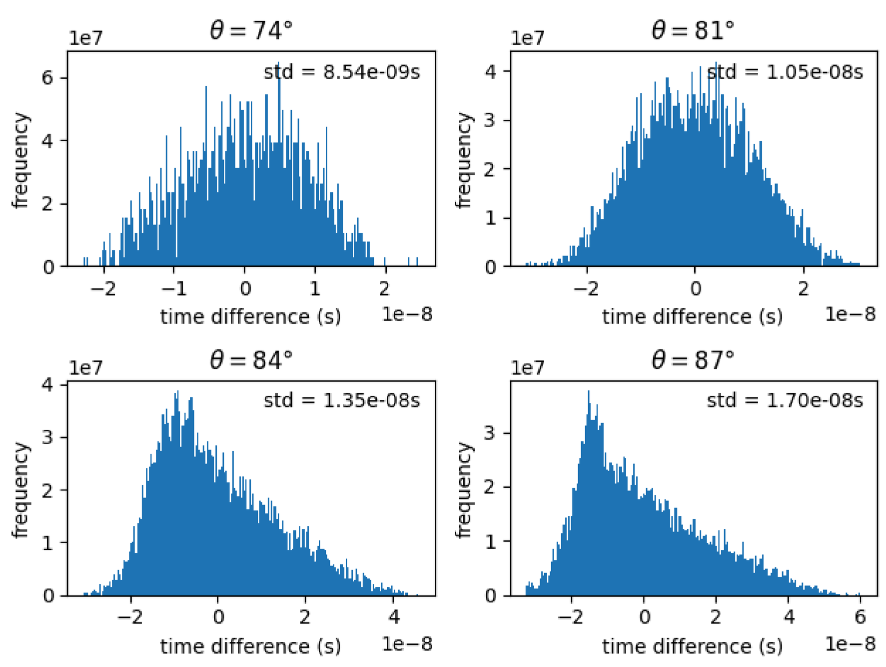
Actual error :
Blue histograms

Predicted uncertainties:
Red isolines.

We have confidence regions

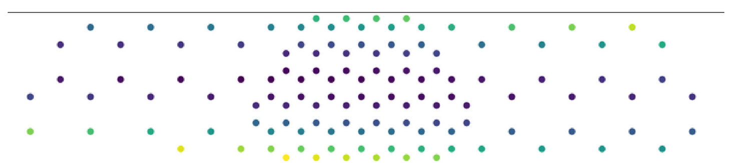
The uncertainty estimations are well calibrated

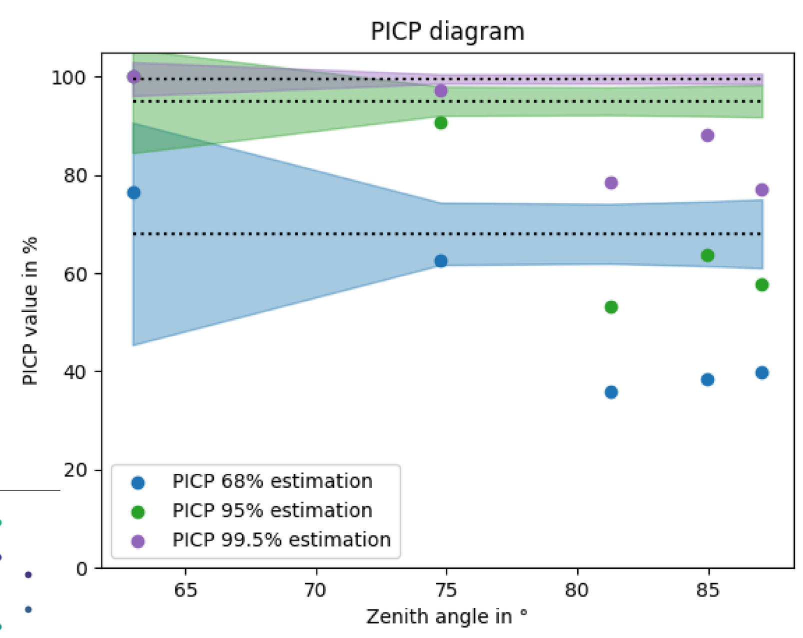
On simulations:



Even if the simulations are not noisy, we look at the distribution of the difference of timings between a perfect plane wave and the measured wavefront.

We will consider this residual as our Gaussian noise





Need to add a correcting term for model error