Development of an ultra-fast, likelihood-based, distance inference framework for the next generation of Type Ia Supernova Surveys

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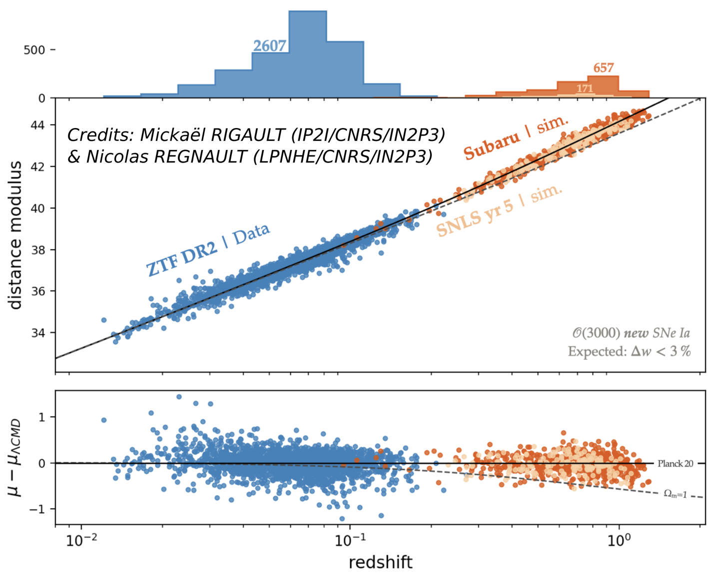
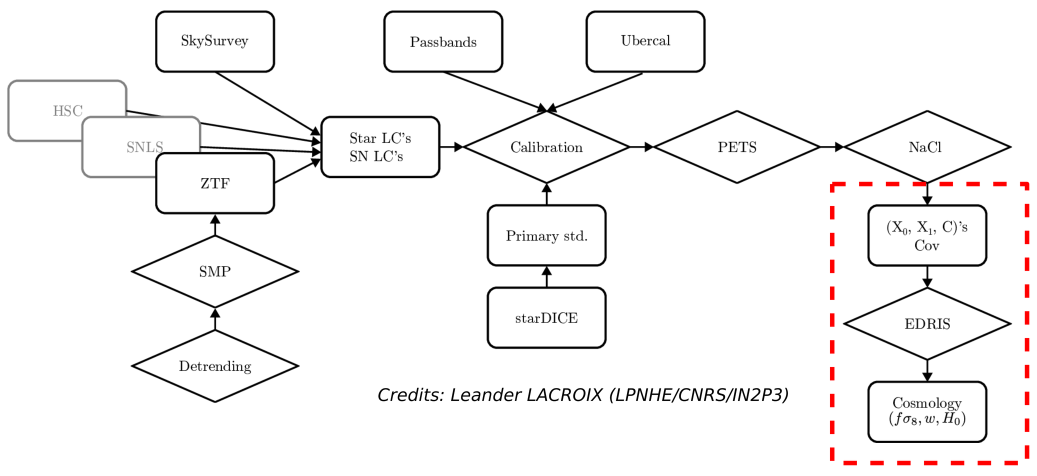
Type Ia supernova surveys and instrumental selection bias

I. Type Ia statistics is multiplied by 5: from O(1000) to O(5000)



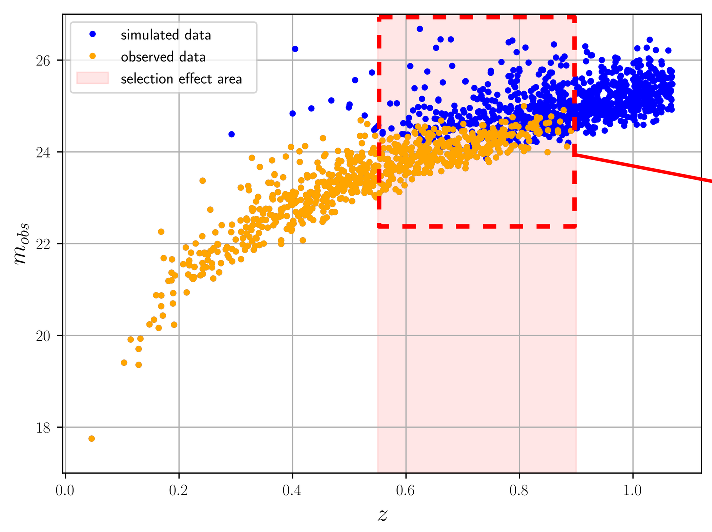


The EDRIS framework is part of a completely independant analysis pipeline called LEMAITRE. LEMAITRE stands for Latest Mapping of the Acceleration with an Independant Trove of Redshifted Explosions.

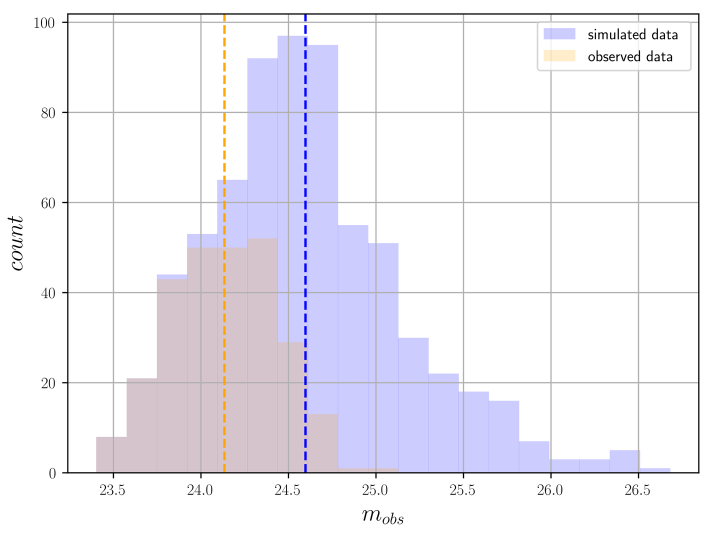
This framework aims to properly handle the upcoming increase of the type Ia supernovae statistics. This increase is allowed by the ZTF, SNLS and HSC/Subaru surveys. Indeed, ZTF will add O(3000) supernovae to the Hubble diagram at low redshifts whereas SNLS and HSC/Subaru will provide O(400) and O(600) supernovae at medium and high redshifts respectively.

II. Selection bias must be taken into account in the likelihood

By adding O(4000) type Ia supernovae to the Hubble diagram, we dramatically decrease the statistical uncertainties on the Dark Energy equation of state 'w'. However, we still have to deal with the computation time of the likelihood function and the systematics effects such as the selection biases. Indeed, each different survey is characterized by its ability to observe type Ia supernovae up to a certain magnitude. This magnitude limitation induces a decrease of the apparent mean magnitude of the population and, therefore, a negative bias on the distance estimator called "Malmquist bias".



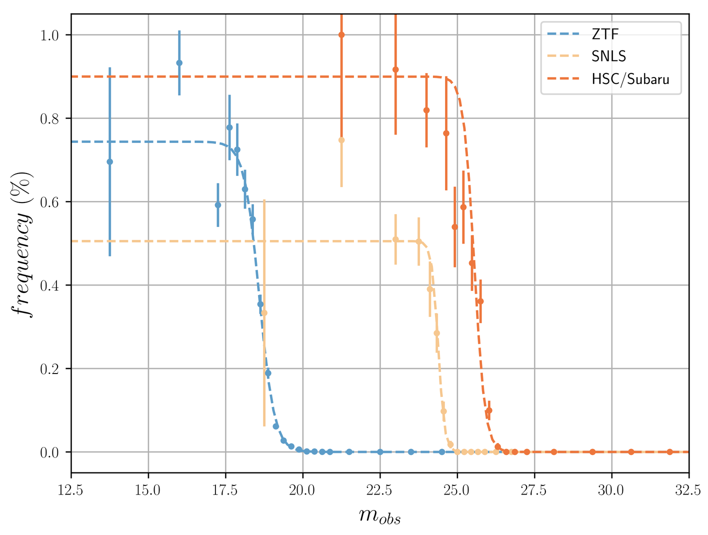
In practice, only the brightest supernovae are detected



1. Modeling of the Malmquist bias

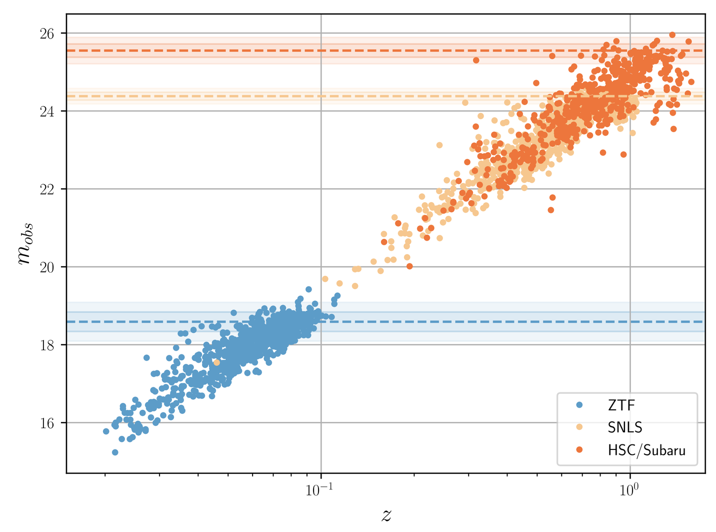
Tripp model (standardization)

$$m_i^* = M^* + \mu_i(z,\theta) + \alpha x^*_{1,i} + \beta c^*_i + \epsilon_i \text{ with } \epsilon_i \sim \mathcal{N}(0,\sigma^2)$$



We model the selection function by a sigmoid...

... then apply this selection to our simulations



$$S(m_{obs}) = \frac{c}{1+e^{\frac{m_{obs}-m_{lim}}{\sigma_d}}} \quad \begin{array}{l} \text{ZTF: } (c=0.74,\ m_{lim}=18.59,\ \sigma_d=0.25) \\ \text{SNLS: } (c=0.51,\ m_{lim}=24.38,\ \sigma_d=0.10) \\ \text{HSC/Subaru: } (c=0.90,\ m_{lim}=25.55,\ \sigma_d=0.17) \end{array}$$

$$m_i = m_i^* + \eta_i \textbf{ if } m_i^* \leqslant m_{lim} + \kappa_i$$
$$\text{with } \eta_i \sim \mathcal{N}(0,C_i) \text{ and } \kappa_i \sim \mathcal{N}(0,\sigma_d{}^2)$$
$$m_i \textbf{ is unobserved otherwise}$$

The truncation effect depends on the unexplained dispersion (or intrinsic dispersion) of the supernovae, the limit magnitude of the surveys and its fluctuation which models the variability of observation conditions.

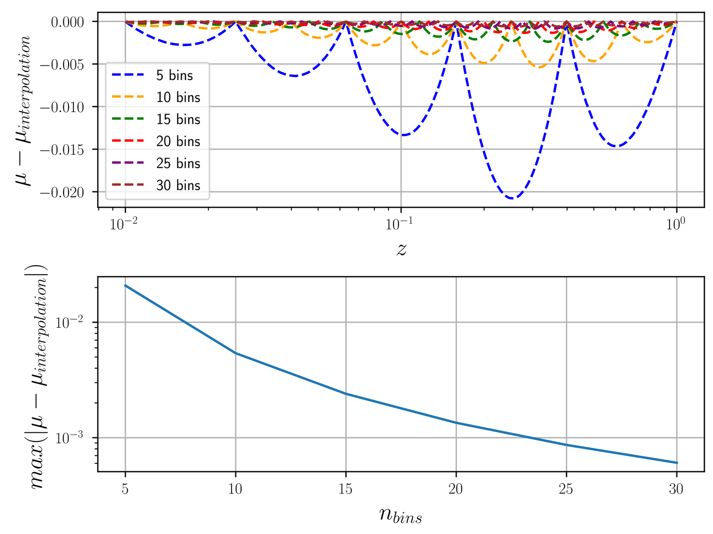
The negative log-likelihood function associated to our model is based on the standard likelihood associated to the multivariate normal distribution. We add two new terms depending on the CDF of the normal distribution to take into account the truncation:

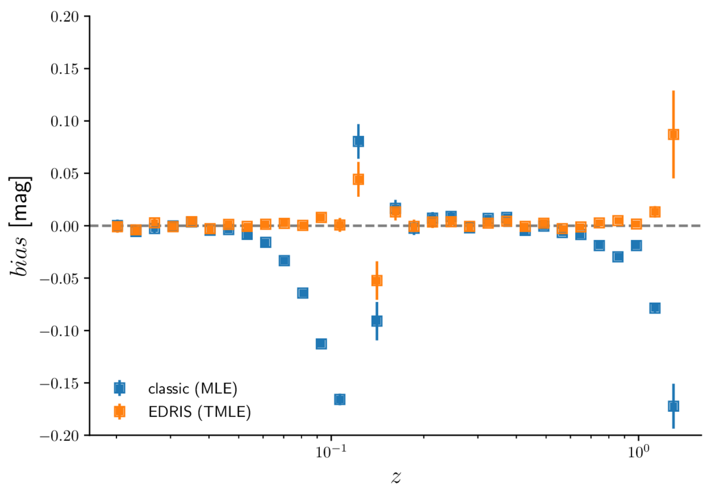
$$\Gamma = -\ln(|W|) + r^\dagger W r + \sum_i 2\ln\left(\Phi\left(\frac{m_{lim} - M^* - \mu_i - \alpha x^*_{1,i} - \beta c^*_i}{\sqrt{\sigma^2 + \sigma_d{}^2}}\right)\right) - 2\ln\left(\Phi\left(\frac{m_{lim} - m_i}{\sqrt{\sigma_d{}^2 + f(C_i)}}\right)\right)$$

$$\text{with } \Phi(z) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right) \text{ and } r = m - M^* - \mu - \alpha x_1 - \beta c$$

2. Estimation of distances and cosmological parameters

We choose to approximate the true cosmology by an order-1 spline. This is a good approximation of most cosmological models as long as we build a sufficient number of bins across the redshift range. In our analysis, we decide to keep 30 bins so that the maximal interpolation error does not exceed 1 mmag.

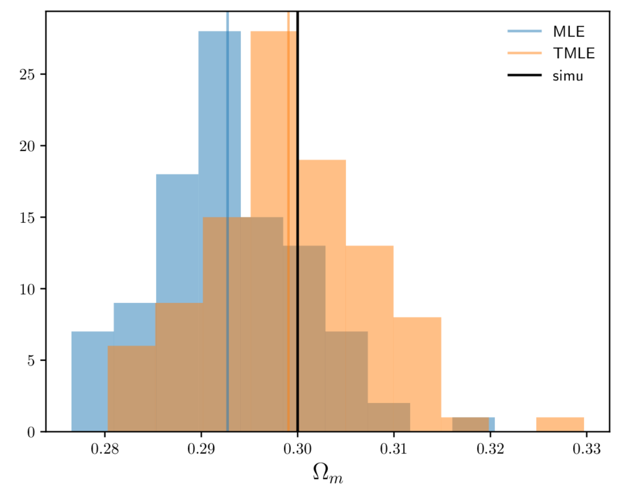




Then, we compare the bias of two distance estimators by running a Monte-Carlo simulation (100 draws):

- the classic Maximum Likelihood Estimator (MLE)
- our new estimator which takes into account the the truncation of the surveys (TMLE)

Here, we study a simplified case where we consider only one standardization parameter (colour) and the covariance matrix of the observations is purely diagonal (no correlation between supernovae and observables). As expected, the TMLE is not biased.



The EDRIS pipeline

French for "Distance Estimator for Incomplete Supernova Surveys"

3. Acceleration of the computation

I- As the covariance matrix of the measurements depends on the intrinsic dispersion, we need to invert it at each step of the minimization.

II- To avoid computation of the likelihood function in O(N³):

- Computation of the invert of the covariance matrix by using the Schur complement technique.
- Diagonalization of the invert of the Schur complement and rewriting of the likelihood parts which depend on it.
- Computation of the likelihood and its gradient using JAX
- Hessian-free optimization

$$W = \begin{pmatrix} C_{mm} + \sigma^2 I_N & C_1 \\ C_1^\dagger & C_2 \end{pmatrix}^{-1}$$

$$S^{-1} = Q(\Lambda + \sigma^2 I_N)^{-1} Q^\dagger$$

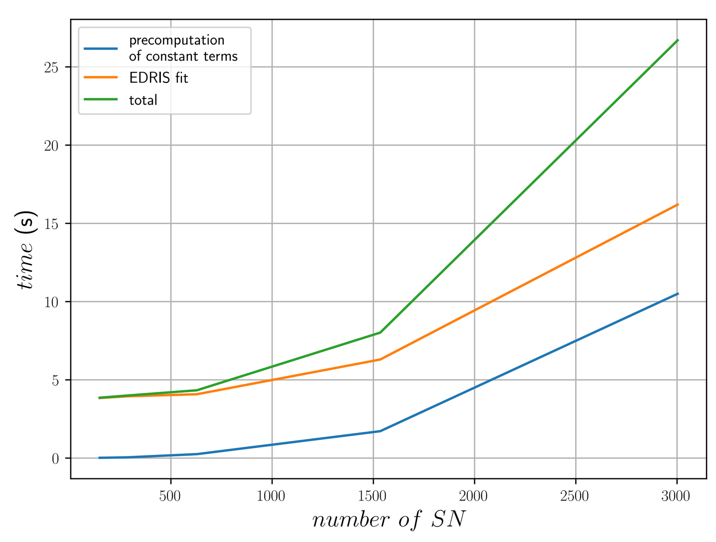
Schur complement of C_2 in $C = W^{-1}$

$$-\ln(|W|) = \sum_i \ln(\Lambda_i + \sigma^2) + \ln(|C_2|)$$

Matrix-vector products → Computation in O(N²)

$$r = (r_1 \quad r_2)$$

$$r^\dagger W r = r_1^\dagger S^{-1} r_1 - 2 r_1^\dagger S^{-1} C_1 C_2^{-1} r_2 + r_2^\dagger C_2^{-1} r_2 + r_2^\dagger C_2^{-1} C_1^\dagger S^{-1} C_1 C_2^{-1} r_2$$



4. What comes next

I. Validation of the selection model

To validate our selection model, we still have to quantify the robustness of our approximations when we deviate from the initial hypothesis. In practice, the selection does not occur on the reconstructed magnitudes but rather on the photometry in observer frame bands. As we do not have access to these quantity directly, what we want to do is, first, provide to EDRIS even more realistic simulations where a cut on the photometry is applied. Then we need to characterize our estimator again using Monte-Carlo simulations to see if our model holds still. If it is not the case, our goal is eventually to include the selection effect and the estimation of distances directly in the likelihood of the light-curve model. This will be a more accurate description of the selection and will reduce the number of error propagations.

II. Application to the LEMAITRE data

To be able to apply EDRIS on the real LEMAITRE data, we first need the result of the Monte-Carlo simulations described above to know if our model is robust enough against realistic selection functions. Also, considering that the intrinsic dispersion of the supernovae, the limit magnitude of the surveys as well as their fluctuations are known is a reasonable assumption. That said, it is needed to propagate their uncertainties correctly in our framework by adding priors in the EDRIS likelihood function. This specific part of the work is yet to be implemented.

References

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