Introduction to the theory of nuclear fission

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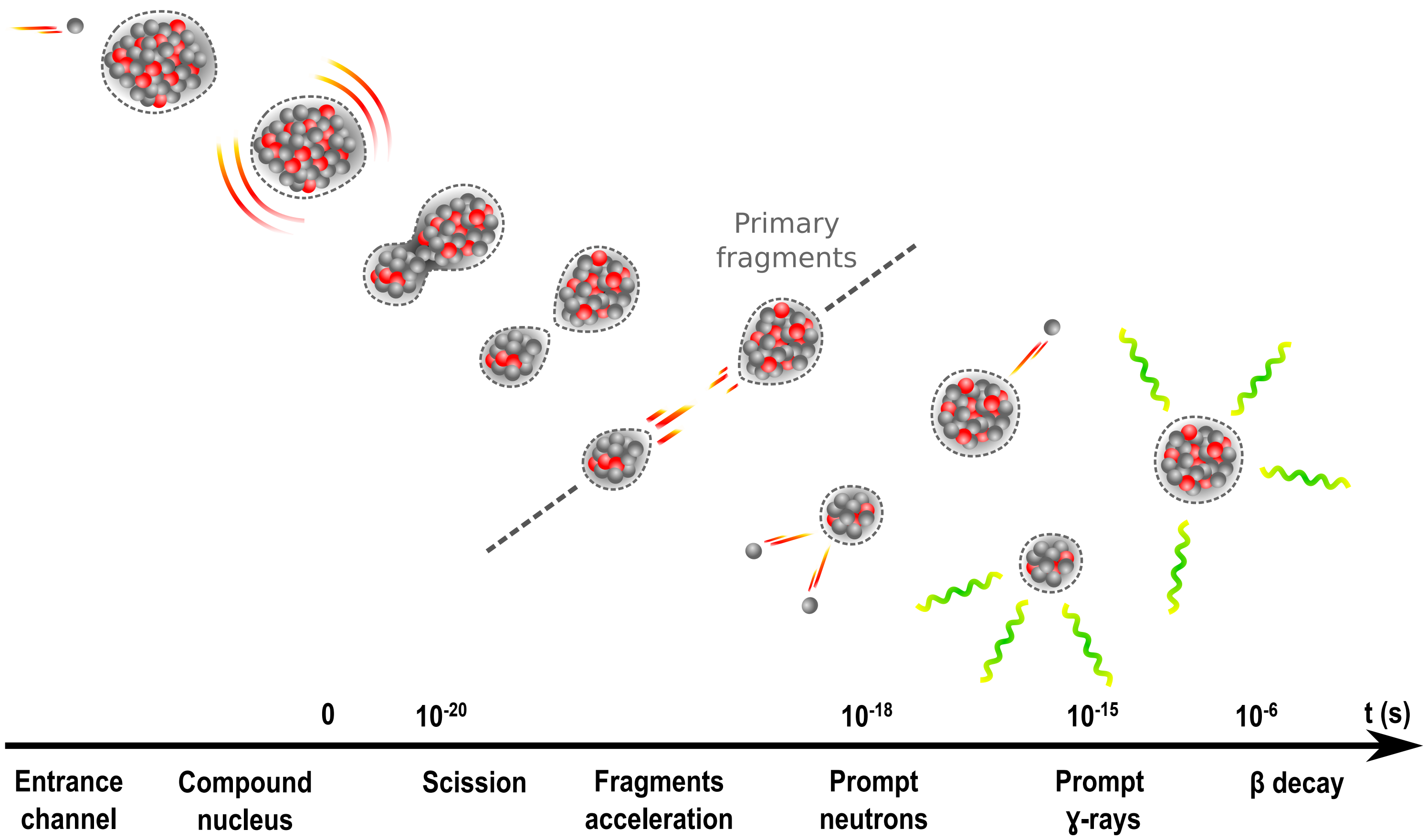
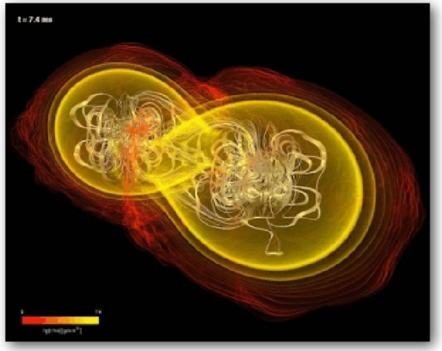
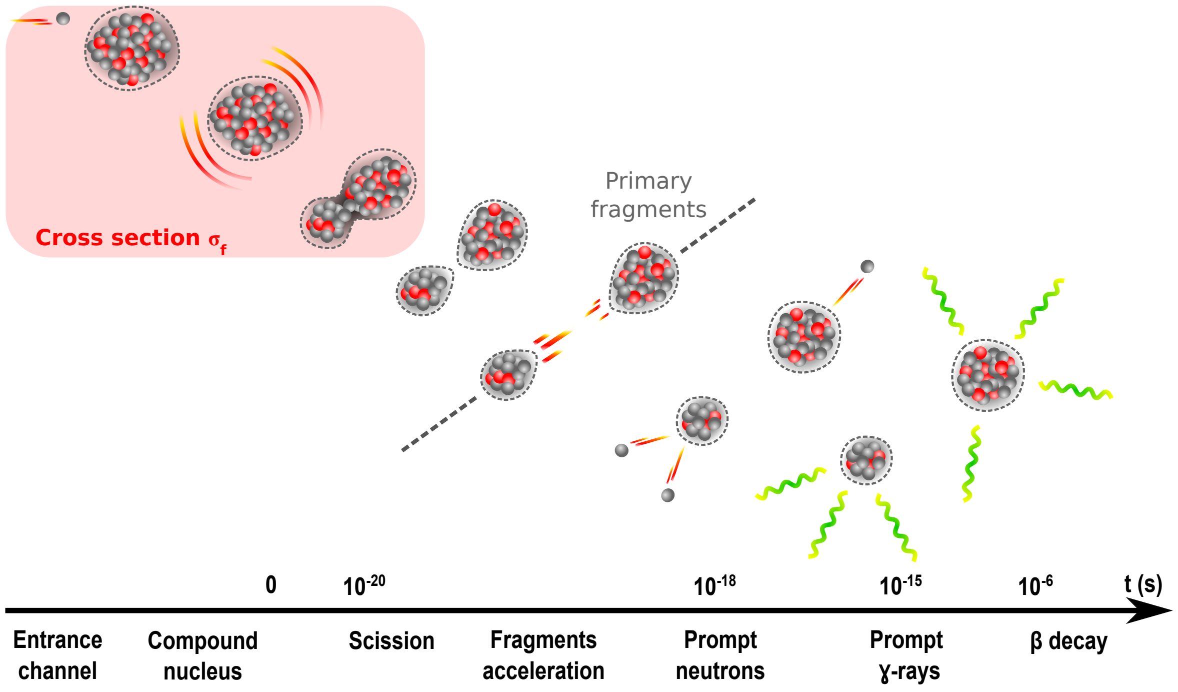
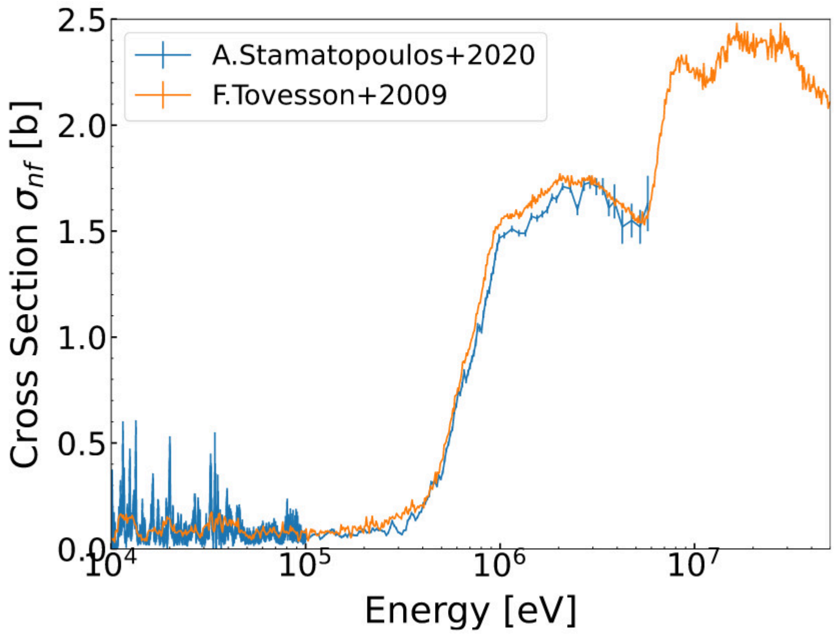
Fission of an atomic nuclei





Observables: cross section or half-life

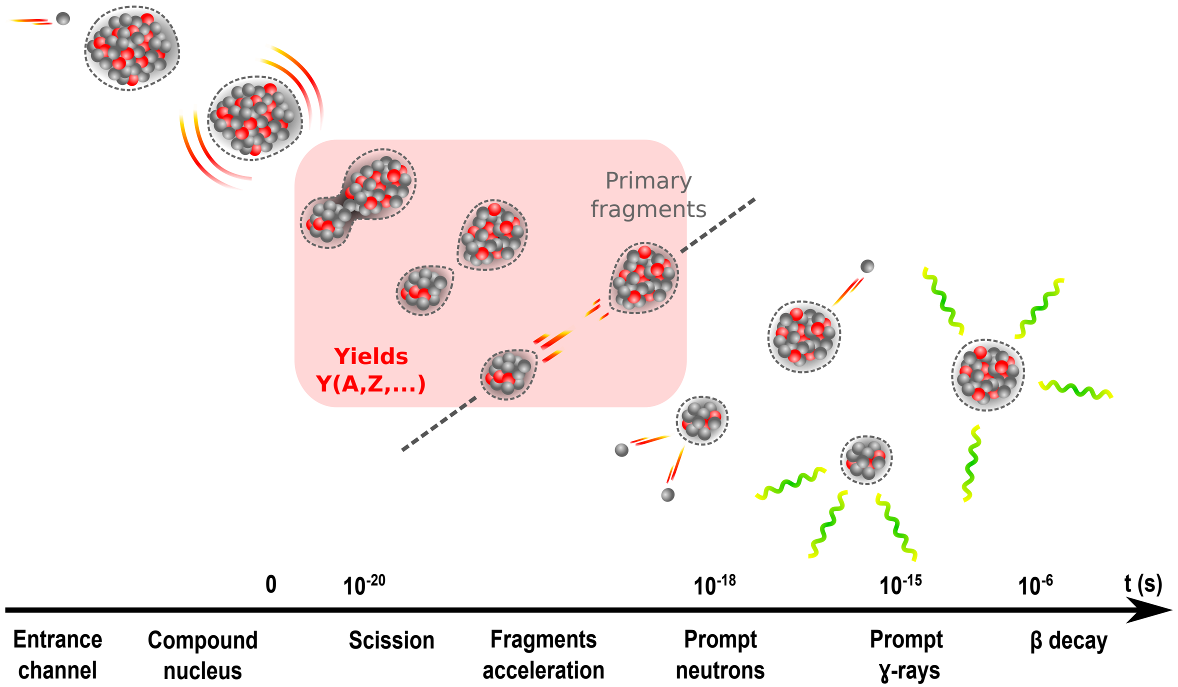


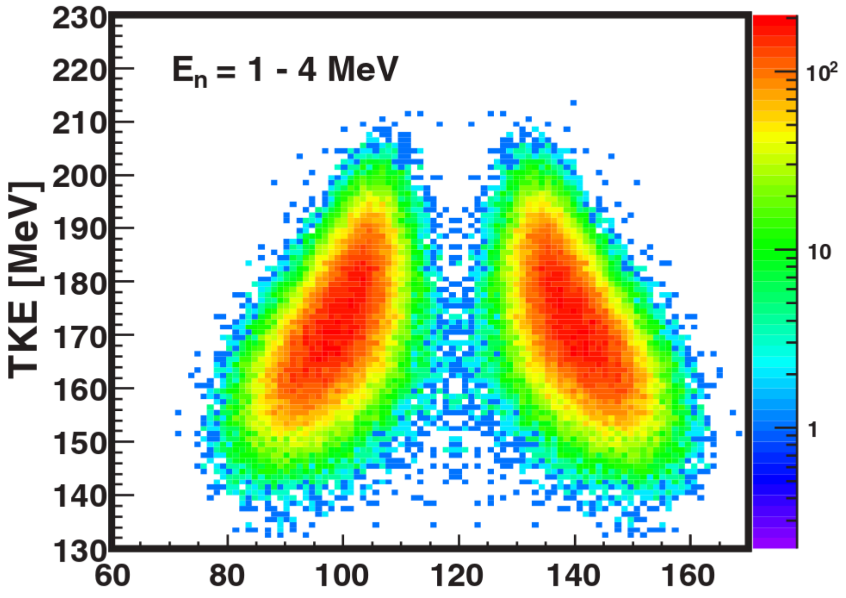


Total cross section for ^{240}Pu(n,f)

N. Schunck *et al.*, Prog. Part. Nucl. Phys. **125** (2022)

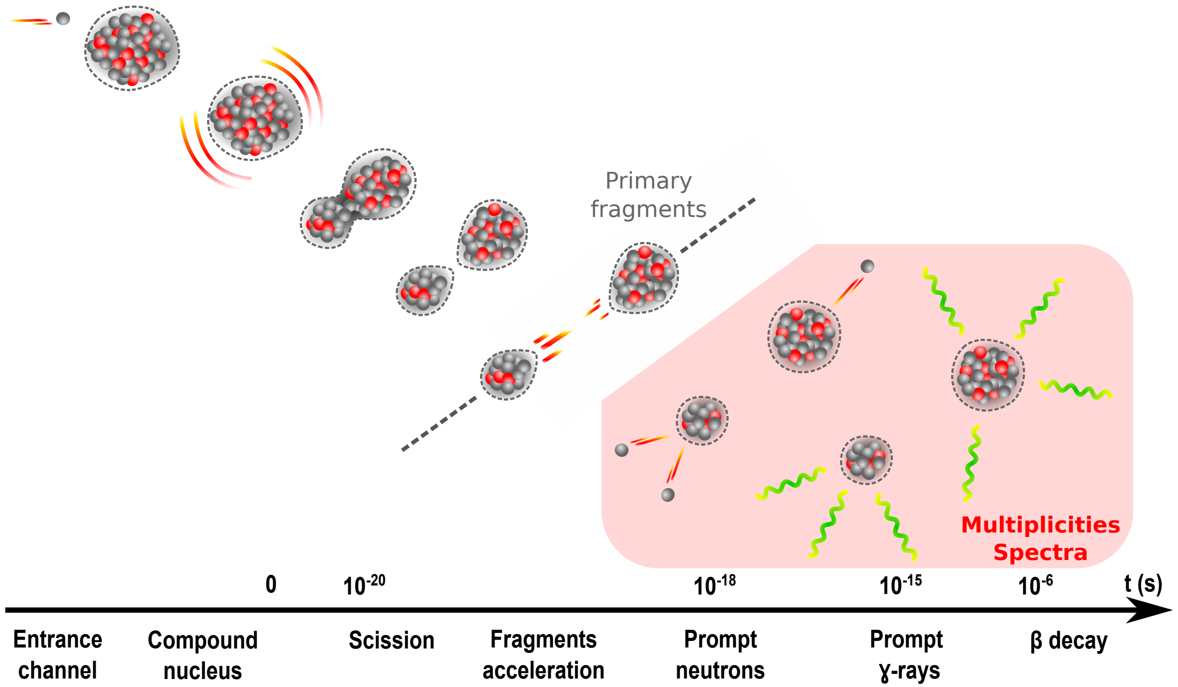
Observables: primary fragments yields

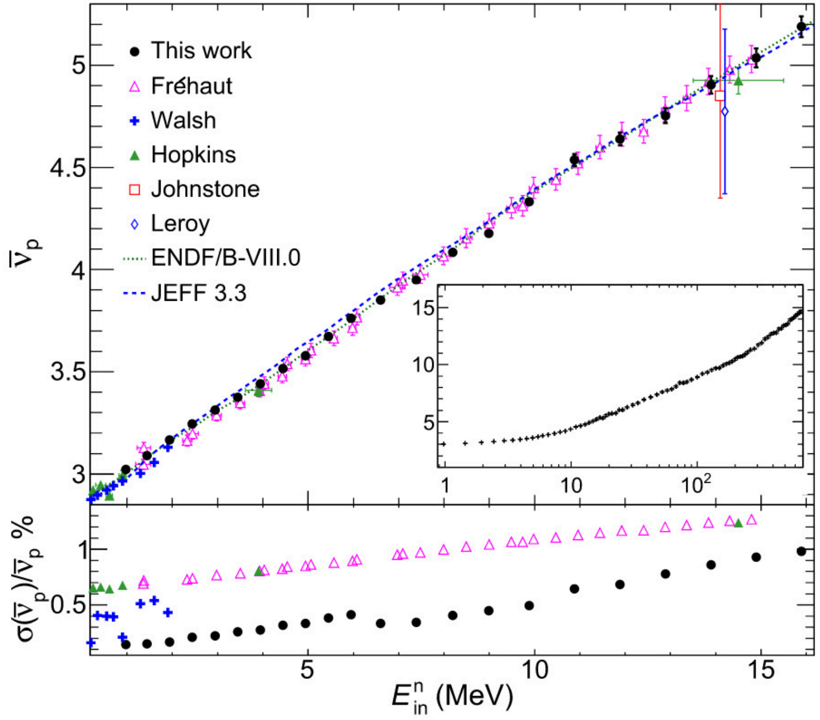




Total kinetic energy (TKE) and mass (A) distribution from ^{238}U(n,f)
D. L. Duke *et al.*, PRC **94** (2016)

Observables: prompt particles properties





Average prompt neutron multiplicity
for ^{239}Pu(n,f)
P. Marini *et al.*, PLB **835** (2022)

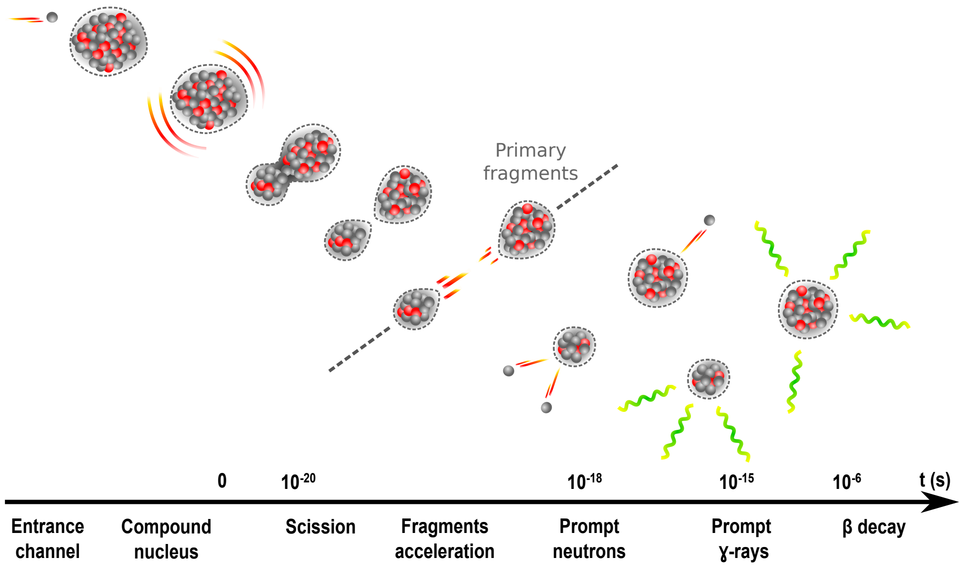
A theoretical description of fission

An ideal fission model would

- be able to predict **any fission observable**
- for **any entrance channel**
- reproduce accurately our current experimental knowledge
- reliably **predict unknown areas**
- link with an underlying theory (e.g. QCD)

Why ?

- For **fun**damental nuclear physics
- For nuclear astrophysics
- For nuclear technologies (nuclear data)



What are the theoretical challenges ?

What are the theoretical challenges ?

- Quantum many-body problem with >200 particles
  $\psi(r_1, \cdots, r_A)$ more than 10^{200} **Bytes to store the wavefunction**
- What is the Hamiltonian ?
  how to treat the **3-body sector** ?
- Various time scales
  From 10 zs (10^{-21} s) up to several μs
- Unbound system
- Reaction with a large number of output channels (>1000)

Different applications: different needs

Fission cross section
σ_f

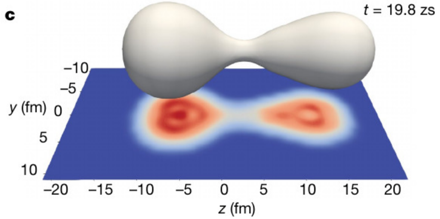
~1%
U, Pu

— Precision required

Primary fragments yields
Y(A,Z,KE,...)

~1% on Y(A,Z)
U, Pu

Actinides
Super-heavy
exotic nuclei

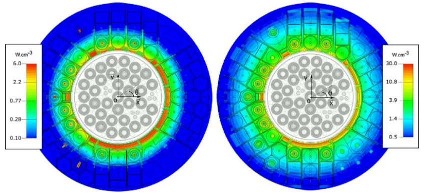


Hundreds of
heavy neutron-rich
nuclei

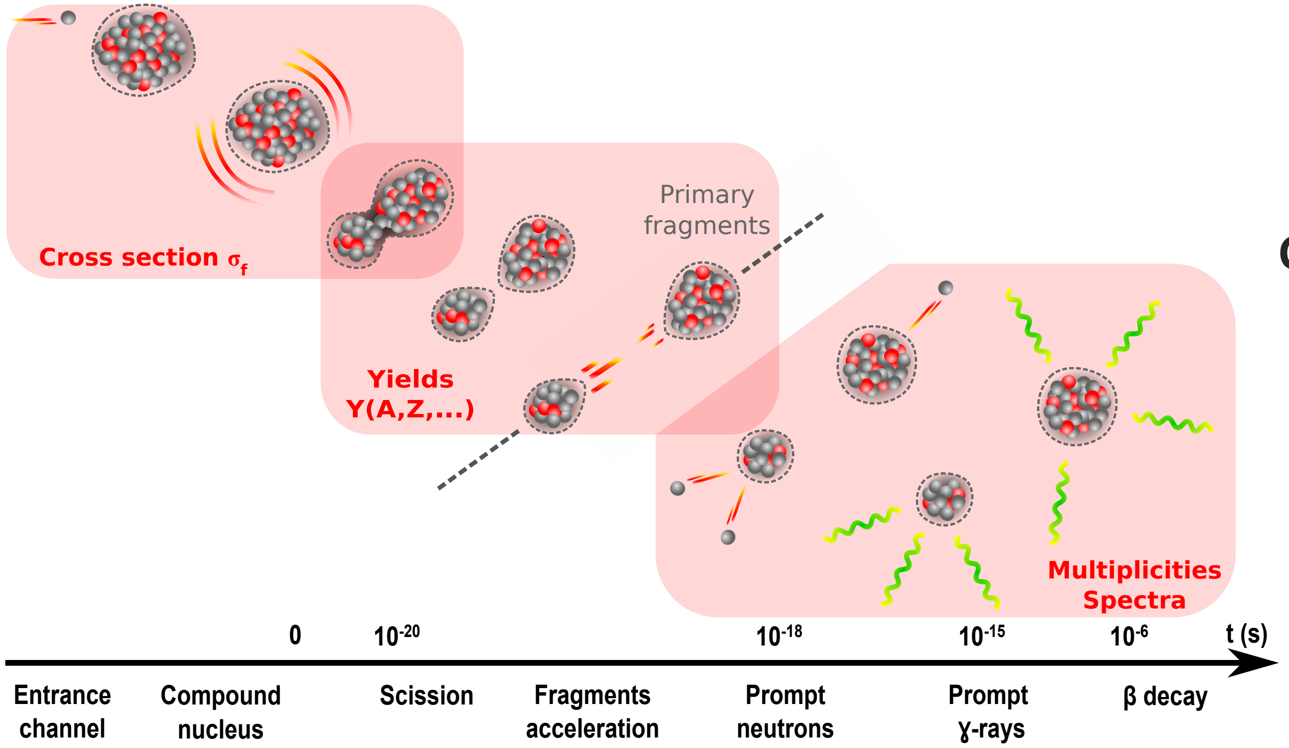
Prompt emission
ν, χ_ν, M_γ, χ_γ

0.1% on ν
U, Pu

~7% on E_γ
U, Pu



Reducing the theoretical difficulty



One model per type of observables

- σ_f: 'One channel' reaction theory
- $Y(A, Z, \ldots)$: Scission point models, many-body dynamics
- $\nu, \chi_\nu, M_\gamma, \chi_\nu$: Statistical deexcitation models

Roadmap of this lecture

I. The one-dimensional fission picture

- Spontaneous fission half-life
- Fission isomers

II. Induced fission cross section

- The Hauser-Feschbach blender

III. Generation of the primary fragments

- Scission point models
- Dynamics of the compound nucleus

IV. (Primary fragments deexcitation)

General fission theory references

Books

- C. Wagemans, *The nuclear fission process*, CRC Press (1991)

  Pedagogical description of the fission phenomenology
- H. J. Krappe & K. Pomorski, *Theory of nuclear fission*, Springer (2012)
- W. Younes & W. D. Loveland, *An introduction to nuclear fission*, (2021)

  Designed for master students

Reviews

- N. Schunck & L. Robledo, *Microscopic theory of nuclar fission: a review*, Rep. Prog. Phys. **79** (2016)

  Energy density functional based approaches only
- M. Bender *et al.*, *Future of nuclear fission theory*, J. Phys. G: Nucl. Part. Phys. **47** (2020)

  Assesses the remaining challenges of fission theory
- N. Schunck & D. Regnier, *Theory of nuclear fission*, Prog. Part. Nucl. Phys. **125** (2022)

Roadmap of this lecture

I. **The one-dimensional fission picture**

- **Spontaneous fission half-life**
- **Fission isomers**

II. Induced fission cross section

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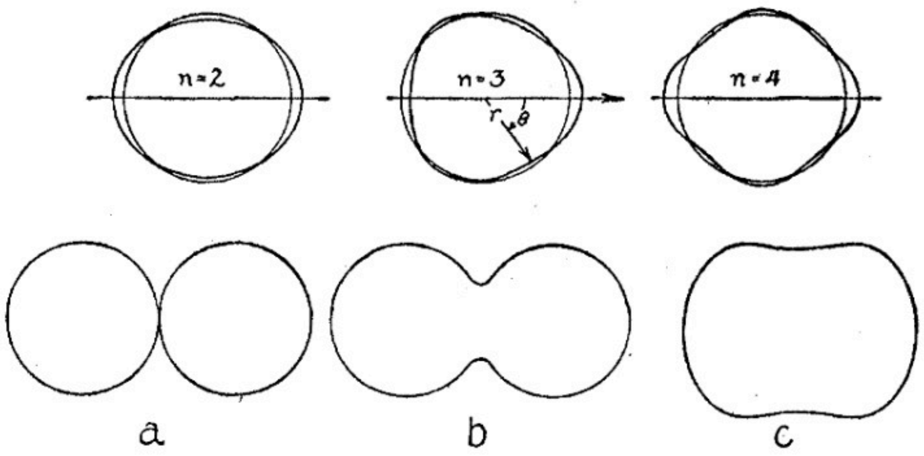
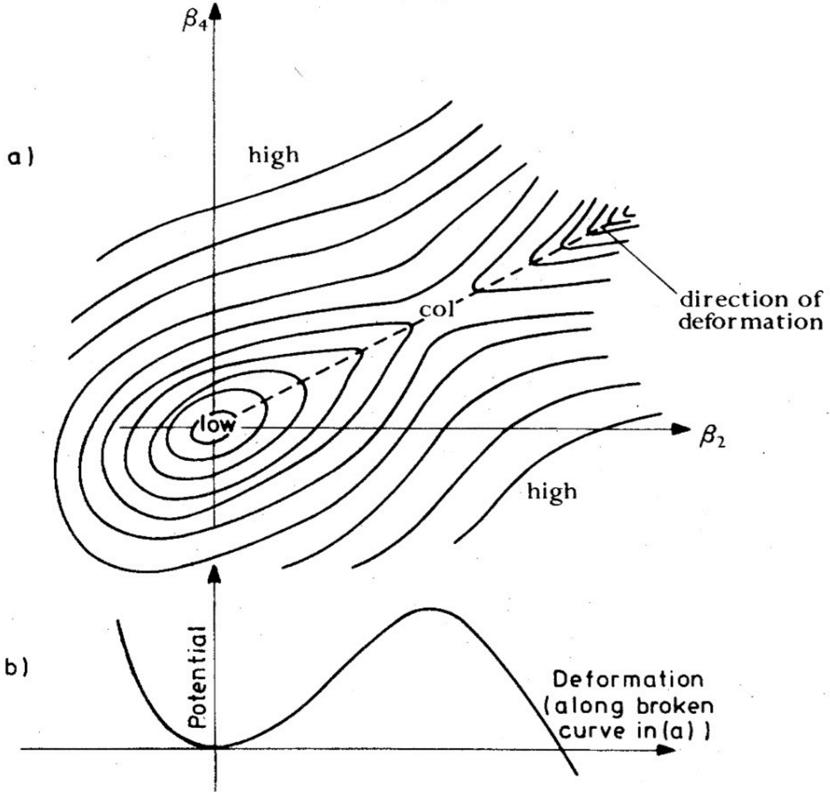
The deformed liquid drop picture



N. Bohr *et al.*, Phys. Rev. **56** (1939)

- Surface shape parameterized by $\beta_2, \beta_4, \cdots$
- Analytical formula for the energy $E(\beta_2, \beta_4, \cdots)$

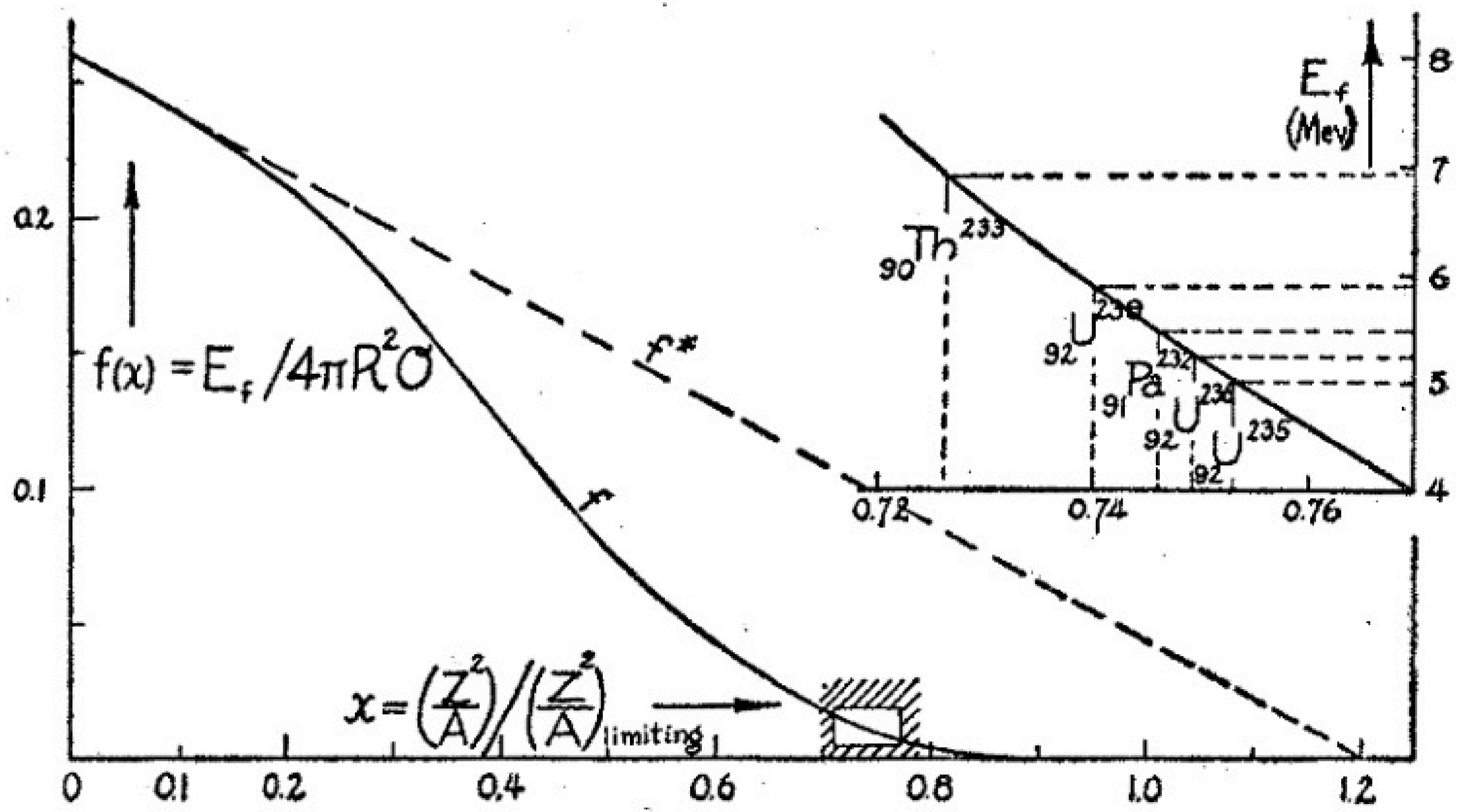


S. Bjornholm *et al.*, Rev. Mod. Phys. **52** (1980)

Competition between Coulomb et nuclear forces
➡️ **Fission barrier**

One path of particular interest
➡️ **The 'elongation' coordinate**

Energy required to fission

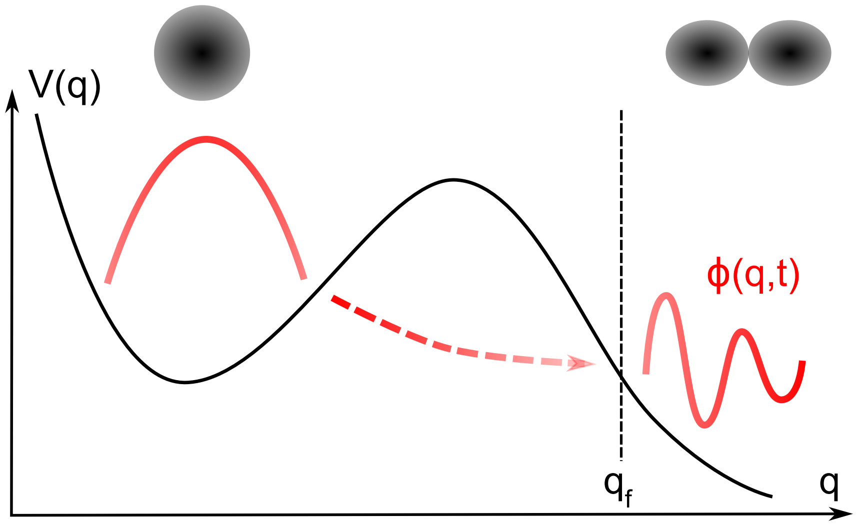


Fission barrier height as a function of a fissility parameter
N. Bohr *et al.*, Phys. Rev. **56** (1939)

Nucleus	B_{max} [MeV]
^{236}U	5.67
^{238}U	6.30
^{233}Th	6.65

Highest fission barrier from RIPL-3
R. Capote *et al.*, Nucl. Dat. Sheet. **110** (2009)

Introducing quantum mechanics



Building a 1D Schrödinger equation

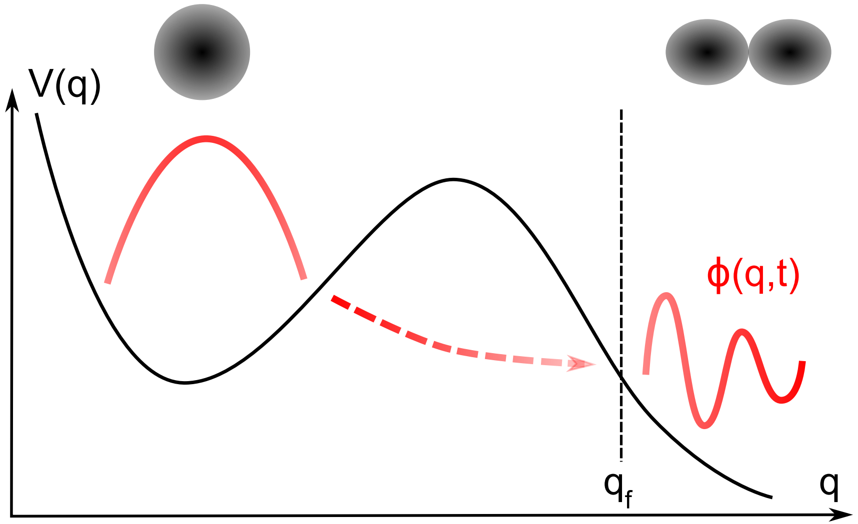
- Hilbert space $\mathcal{H}$ = $Span\{|q\rangle \mid q \in \mathbb{R}\}$
- The wave function $\phi(q,t)$ represents the fissioning nucleus
- It obeys

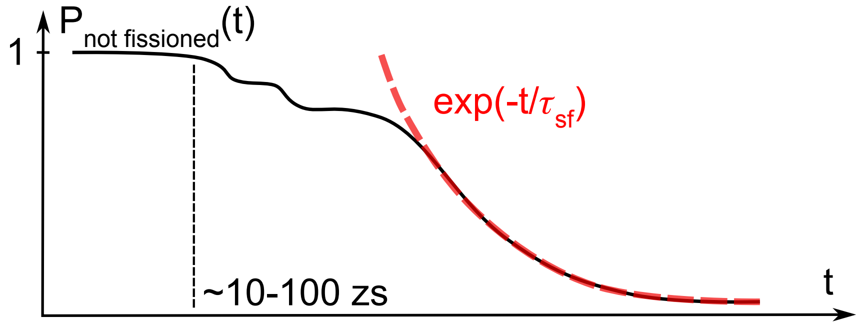
$$i\hbar \frac{\partial}{\partial t}\phi(q,t) = \hat{H}\phi(q,t).$$

- The Hamiltonian is assumed to be

$$\hat{H} = \frac{h^2}{2}\frac{\partial}{\partial q}\frac{1}{M(q)}\frac{\partial}{\partial q} + V(q).$$

Introducing quantum mechanics





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Spontaneous fission half-life

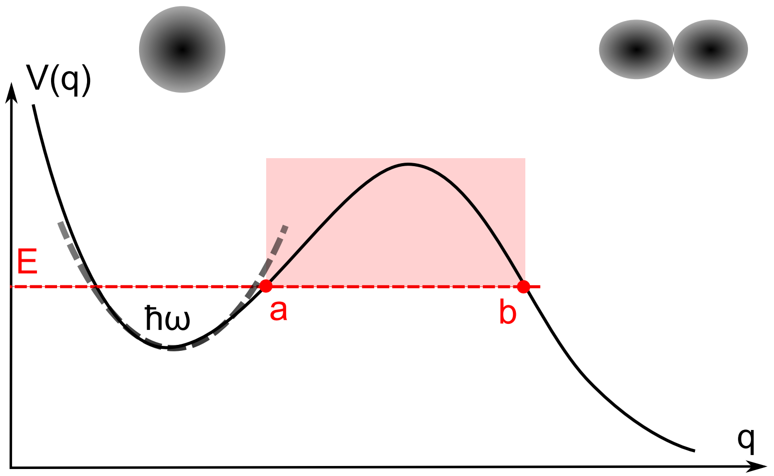
$$P_{\text{not fissioned}}(t) = \int_{q<q_f} |\phi(q,t)|^2 dq$$

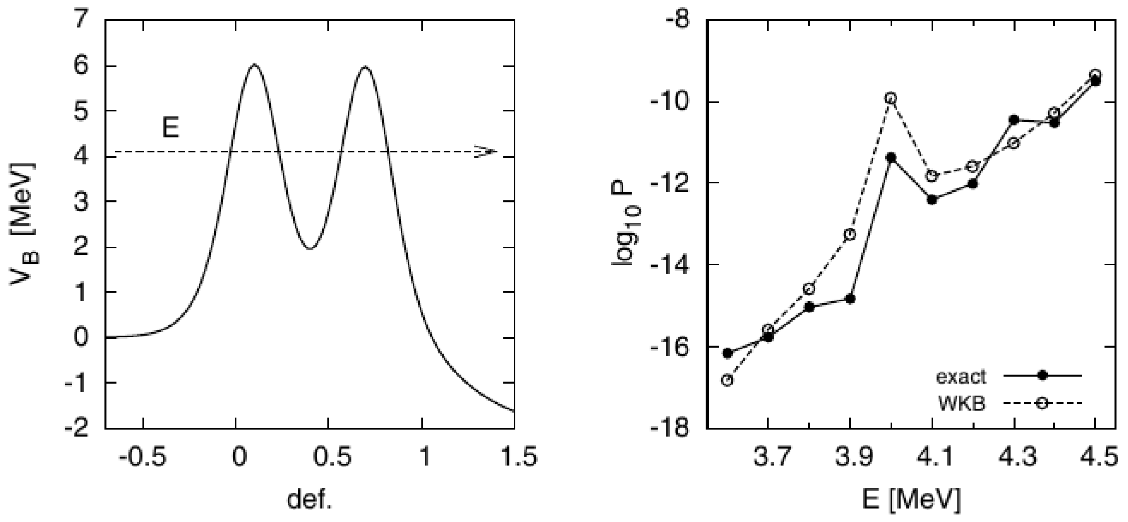
- Behaves as an exponential for long times

Practical half-life calculations

Wentzel-Kramers-Brillouin (WKB) approx.

A. Messiah, Mécanique quantique (1969)





H. J. Krappe *et al.*, *Theory of nuclear fission* (2012)

$$\tau_{sf} = \frac{2\pi}{\omega} exp\left[\frac{2}{\hbar}\int_{q=a}^{b}\sqrt{2M(q)[V(q)-E]}dq\right]$$

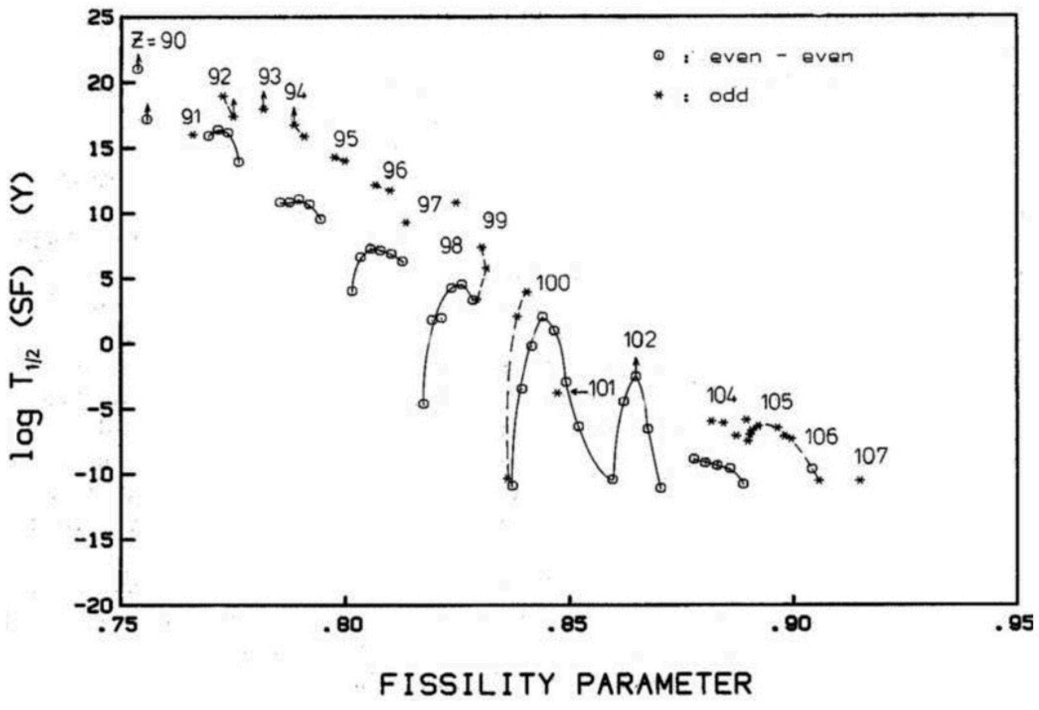
- E: energy of the initial state
- ω: frequency of an oscillator potential that fits the well

⚠ Highly sensitive to all ingredients

Change E by 1MeV

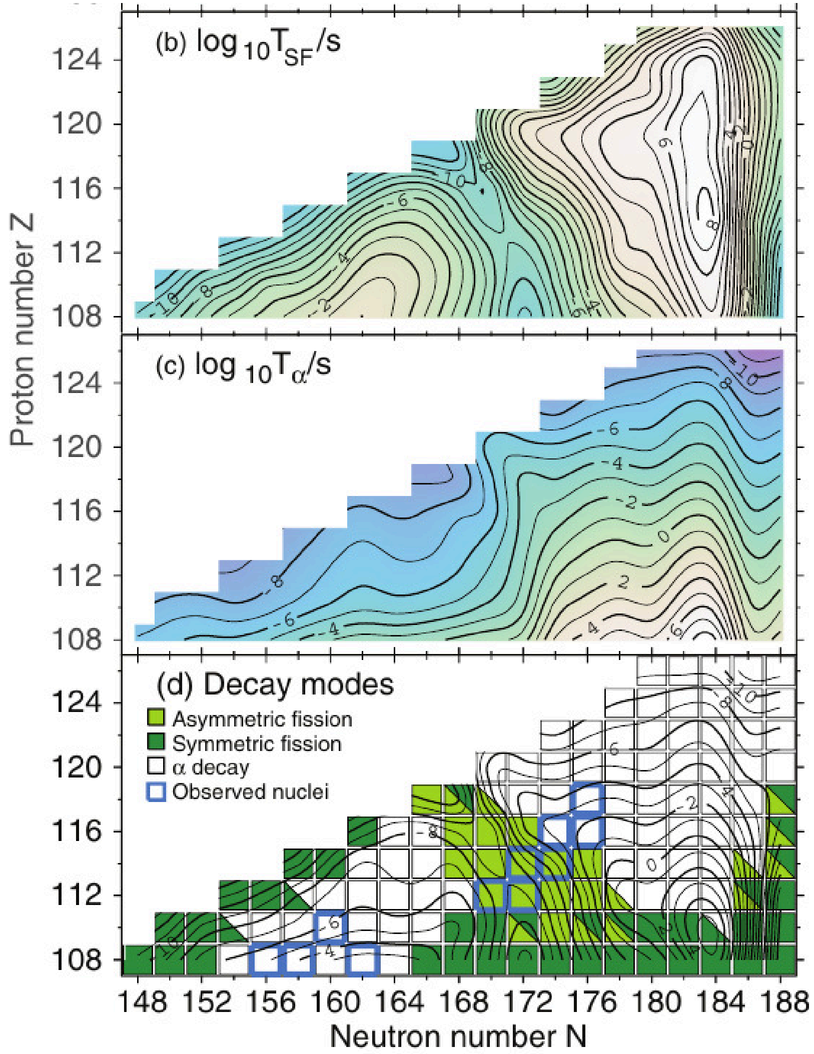
$\Longrightarrow$ 5 orders of magnitude difference on τ_{sf}

Systematics of SF half-lives



Measured spontaneous fission half-lives

C. Wagemans, *The nuclear fission process* (1991)



Theoretical predictions

A. Staszczak, Phys. Rev. C 87 (2013)

Potential and inertia ?

Back to the 1D Schrödinger equation

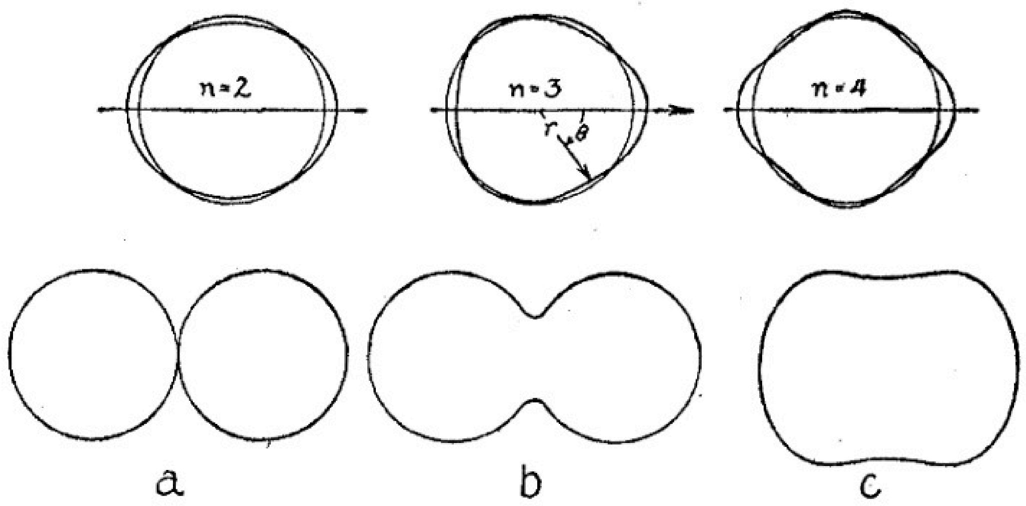
$$i\hbar\frac{\partial}{\partial t}\phi(q,t) = \hat{H}\phi(q,t).$$

with the Hamiltonian

$$\hat{H} = \frac{h^2}{2}\frac{\partial}{\partial q}\frac{1}{M(q)}\frac{\partial}{\partial q} + V(q).$$

- $M(q)$ inertia
- $V(q)$ potential

From the liquid drop picture



N. Bohr *et al.*, Phys. Rev. **56** (1939)

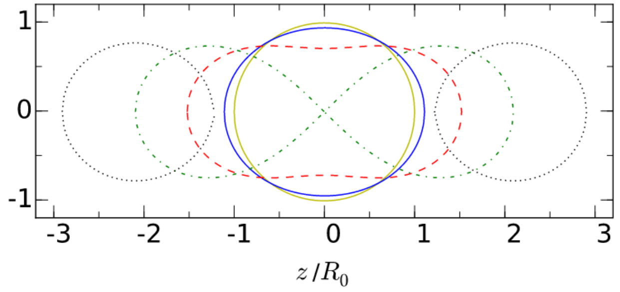
- **Coordinates:** shape of the nuclear surface
- **Potential:** liquid drop formula
- **Inertia:** from hydrodynamics

Potential and inertia ?

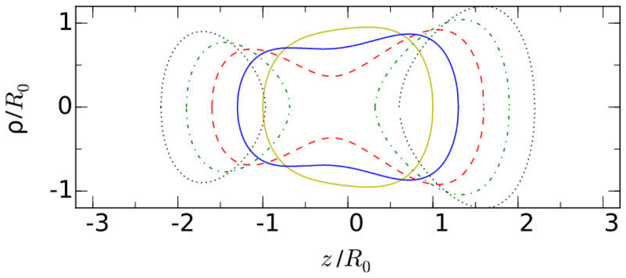
From the 'microscopic-macroscopic' models

- **Coordinates**

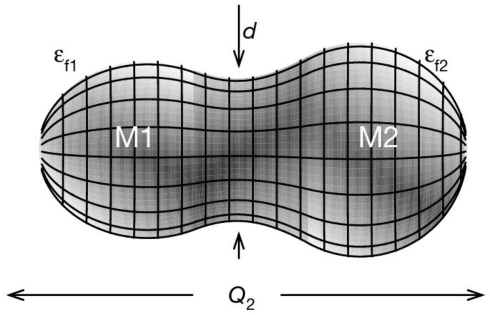
  Shape of the nuclear surface



V. V. Pashkevich, Nucl. Phys. A **169**(1971)



M. Brack *et al.* Rev. Mod. Phys. **44** (1972)



P. Moller *et al.* Nature 409 (2001)

- **Potential energy**

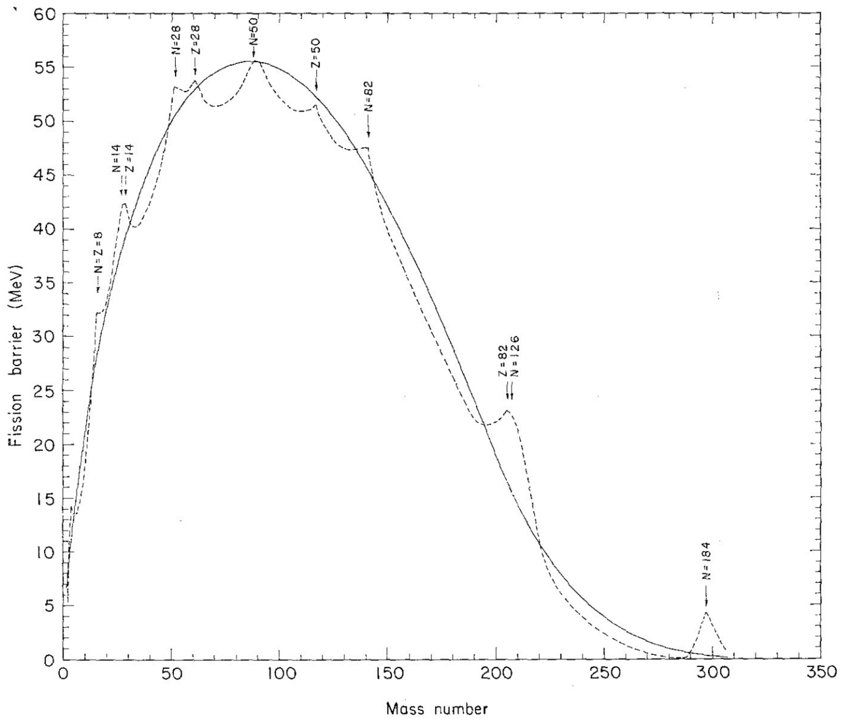
$$V(q) = V_{\text{liquid drop}}(q) + \delta E_{shell}(q) + \delta E_{pair}(q)$$

- **Inertia**

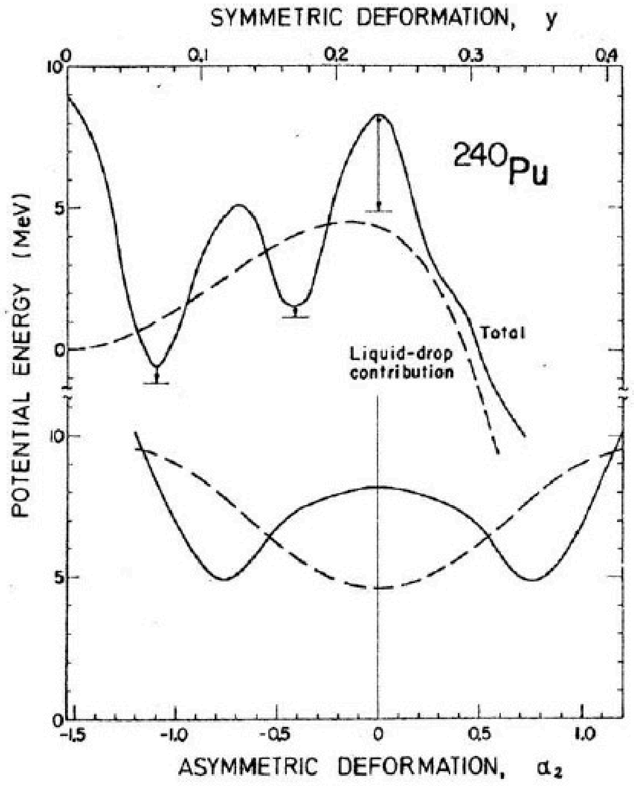
  Werner-Wheeler method: incompressible, nearly irrotational hydrodynamics

  Davies *et al.* Phys. Rev. C **13** (1976)

Taking into account the shell effects I



W. D. Myers *et al.*, Nucl. Phys. 81 (1966)



J. R. Nix, Ann. Rev. Nucl. Sci **22** (1972)

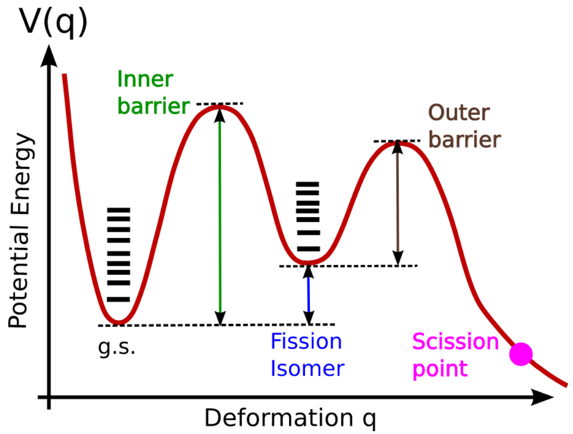
✅ **Structures in $\tau_{sf}(N, Z)$**

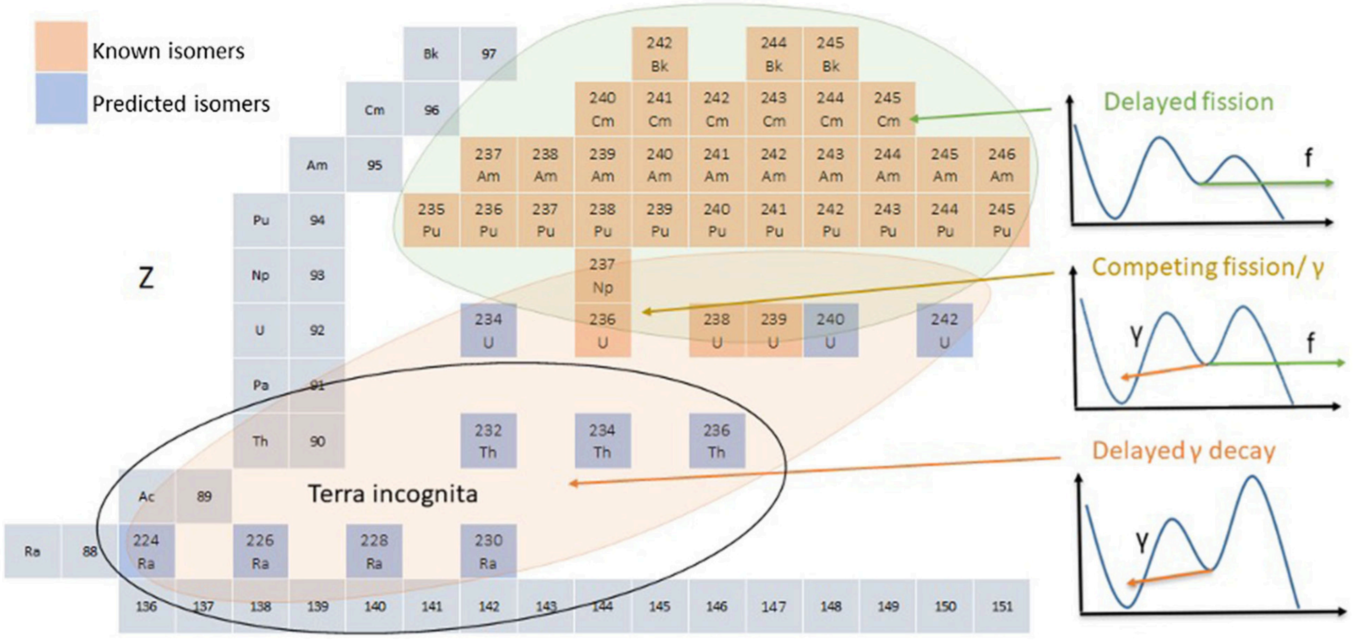
✅ **Double-humped fission barrier**
✅ **Asymmetric fission**

Taking into account the shell effects II

- 14 ms state in ^{242}Am that fission (10^{14} shorter than τ_{sf} from ground state)
  Polikanov *et al.* (1962)
- Double-hamped fission barrier
  Strutinsky (1967)

✅ **Fission isomers**





S. Leoni *et al.* Eur. Phys. J. Spec. Top. (2024)

Potential and inertia ?

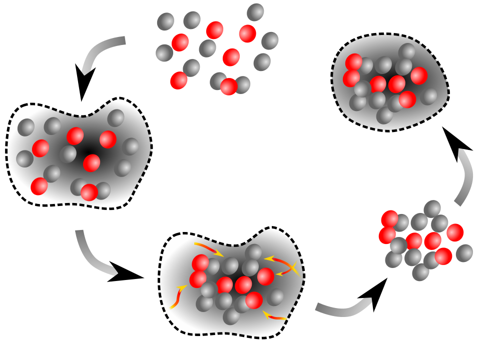
From self-consistent mean field

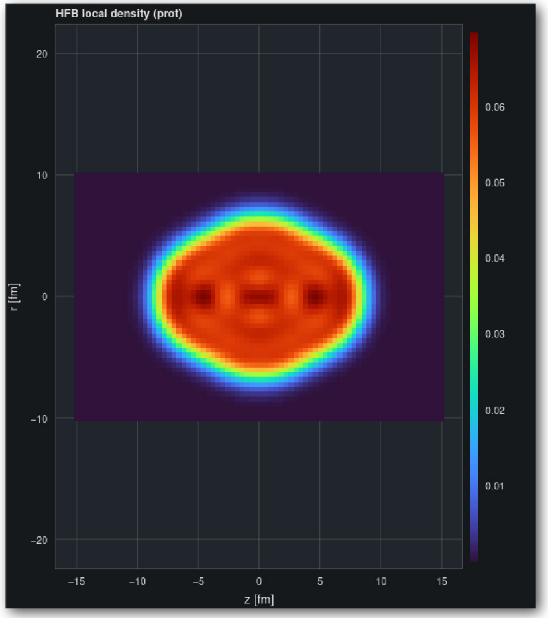
(aka Single-Reference Energy Density Functional Theory)

Hartree-Fock-Bogoliubov for a ground state

- $\psi(r_1, \cdots, r_A)$ describes the fissioning nucleus
- $|\psi\rangle \in \{\text{Bogoliubov vacua}\}$
- Look for

$$|\psi_0\rangle = \min_{|\psi\rangle} \left[\frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle} \right]$$





Proton one-body density of ^{238}U
Numerical cost ~1 h.cpu

- Mean-field + pairing picture
- **Self-consistent mean-field** potential

Potential and inertia ?

From self-consistent mean field

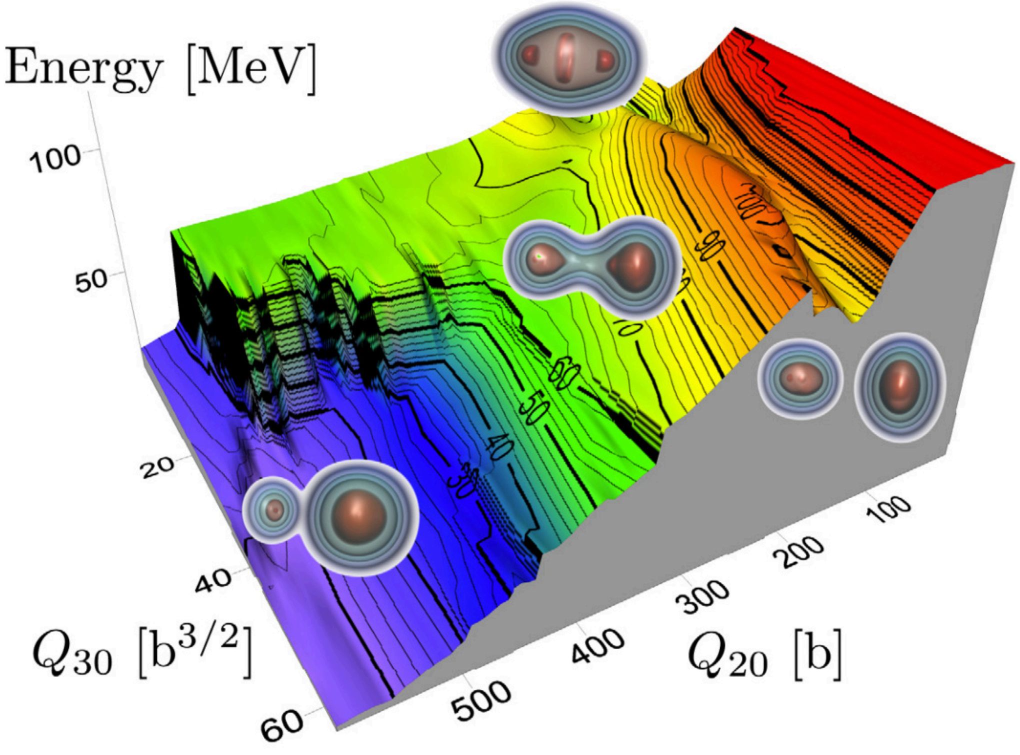
(aka Single-Reference Energy Density Functional Theory)

Constrained Hartree-Fock-Bogoliubov

- $\psi(r_1, \cdots, r_A)$ describes the fissioning nucleus
- $|\psi\rangle \in \{\text{Bogoliubov vacua}\}$
- $\langle\psi|\hat{Q}_{lm}|\psi\rangle = q$
- Look for

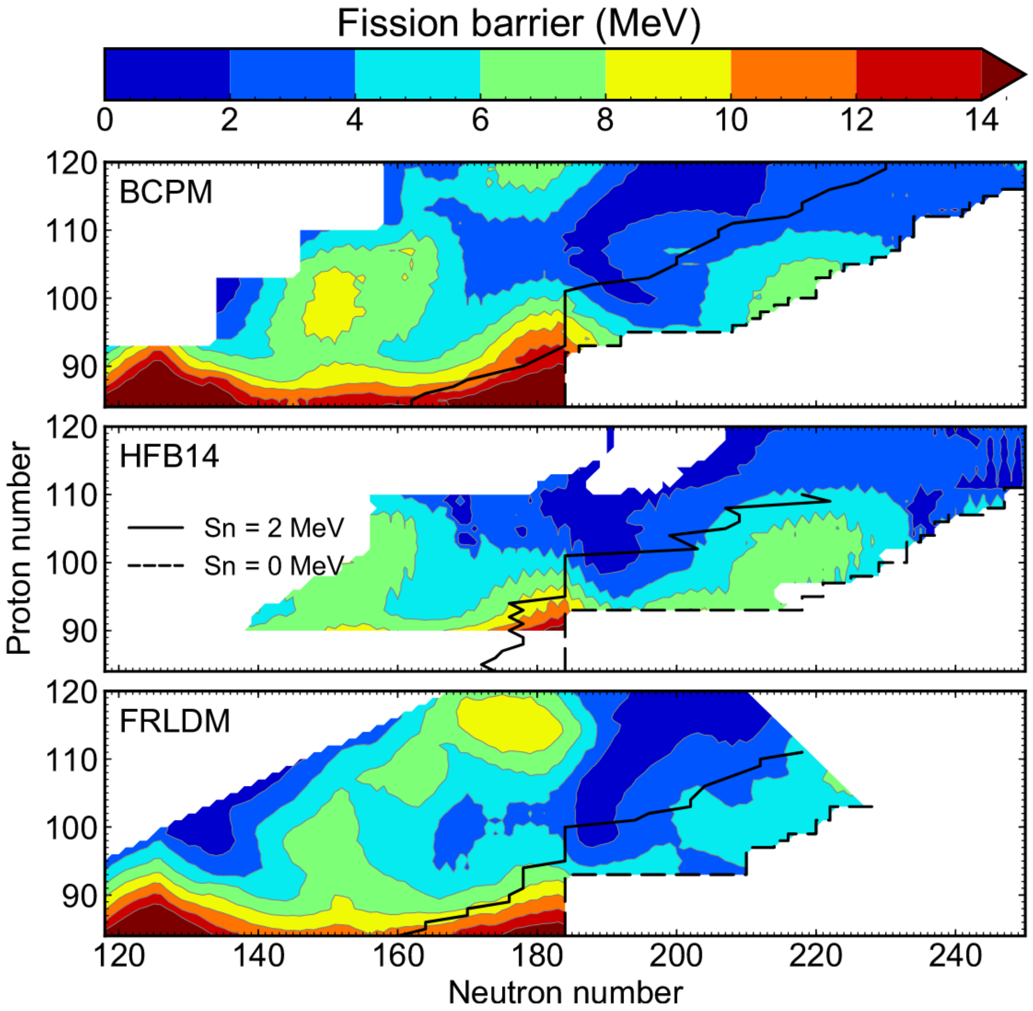
$$|\psi(q)\rangle = \min_{|\psi\rangle} \left[\frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle} \right]$$

- **Coordinates:** expectation values of multipole moments $\hat{Q}_{lm}$
- **Potential:** HFB energy (plus correction)
- **Inertia:** from TDGCM, ATDHFB, etc



N. Schunck *et al.* Prog. Part. Nucl. Phys. (2022)

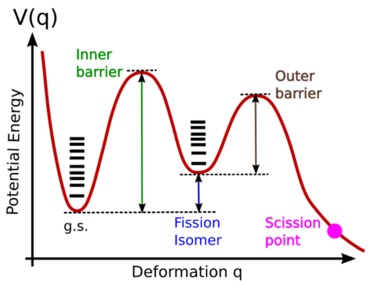
Comparison of potentials



- Microscopic-macroscopic: FRLDM
  From a mean-field potential
- Energy density functional: BCPM, HFB14
  From a nucleon-nucleon effective interaction

Wrapping up

1D fission picture



Potential & inertia

- Liquid drop
- Microscopic-marcroscopic
- Self-consistent mean-field

Phenomelogy

- Spontaneous fission
- Fission isomers
- Asymmetric fission

Questions ?

- Other approaches to get the potential and inertia ?
- **How do we justify the 1D Shrödinger equation ?**

Method 1: Transform and split

S. Bjornhom *et al.*, Rev. Mod. Phys. **52** (1980)

- $\psi(r_1, \cdots, r_A)$ describes the fissioning nucleus
- $\hat{H} = \sum_i^A \hat{K}_{r_i} + V(r_1, \cdots r_A)$

Idea

- Find a one-to-one mapping

$$(r_1, \cdots, r_A) \rightarrow (q, x_1, \cdots, x_{A-1})$$

  - **q** deformation coordinates
  - $x_1, \cdots x_{A-1}$ intrinsic coordinates

- Split the Hamiltonian

$$\hat{H} = \left[\frac{h^2}{2} \frac{\partial}{\partial q} \frac{1}{M(q)} \frac{\partial}{\partial q} + V(q) \right] + \hat{H}_{x_1, \cdots, x_{A-1}} + \hat{H}_{coupling}$$

The fission 'mille-feuille'

J.-F. Feuillette, *Le millefeuille praliné*, (difficulté moyen)

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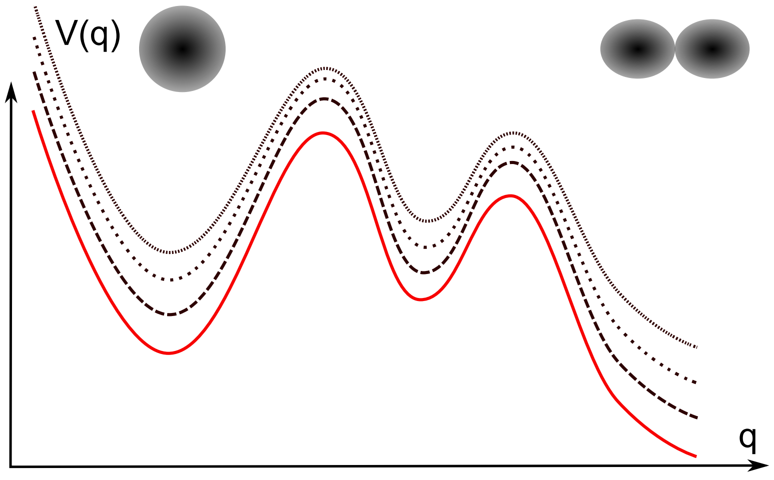

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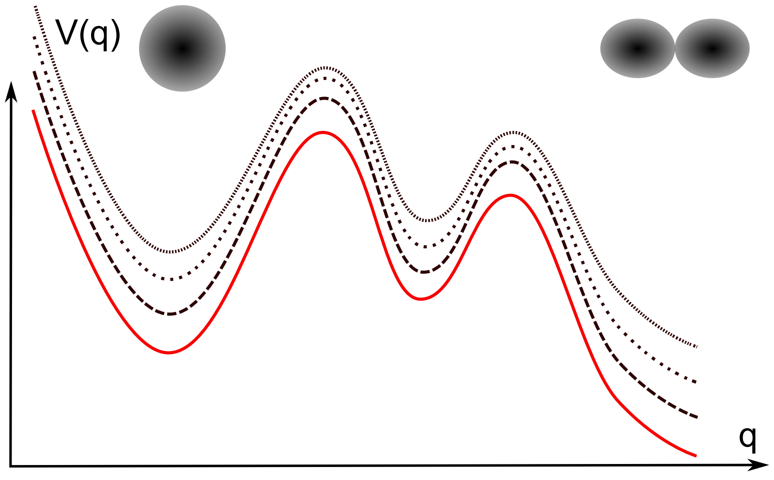


The fission 'mille-feuille'

J.-F. Feuillette, *Le millefeuille praliné*, (difficulté moyen)

✅ Qualitative ideas
❌ No practical application

Method 2: Add and integrate

Time Dependent Generator Coordinate Method (TDGCM)

Core ideas

- Generate an 'interesting' set of states

$$\{\phi_q(r_1, \cdots, r_A) | q \in \mathbb{R}\}$$

  - **q** deformation coordinate
  - with some properties (SME, GOA, etc)

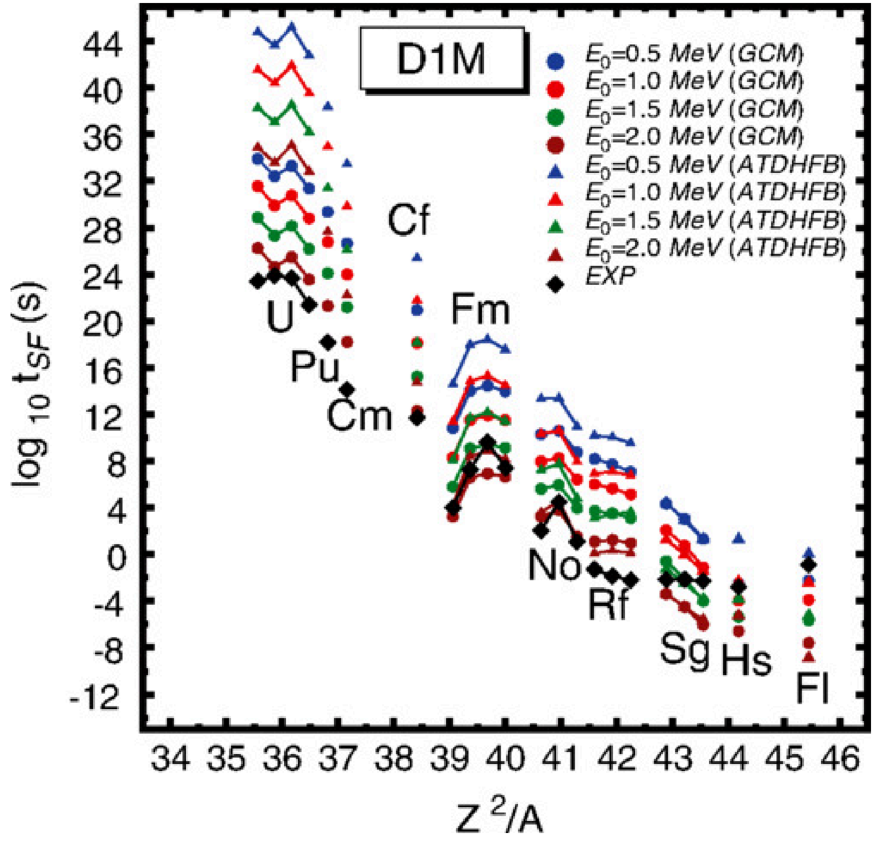
- Assume at any time

$$|\psi(t)\rangle = \int_q f(q,t) |\phi_q\rangle dq$$

- A time dependent variational principle yields

$$i\hbar \frac{\partial}{\partial t} \tilde{f}(q,t) = \left[\frac{h^2}{2} \frac{\partial}{\partial q} \frac{1}{M(q)} \frac{\partial}{\partial q} + V(q) \right] \tilde{f}(q,t)$$

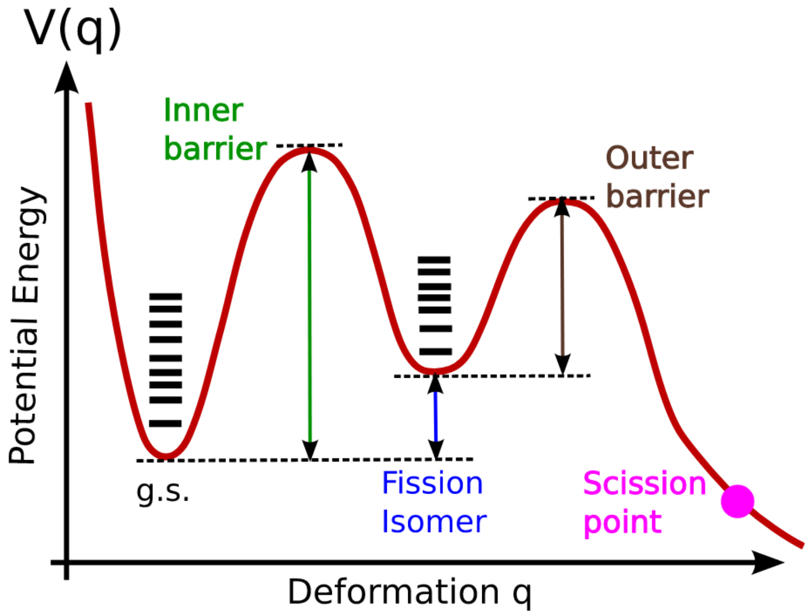
✅ Well compatible with self-consistent mean field
✅ Potential and inertia from $\hat{H}$ and $|\phi(q)\rangle$
❌ Are all the assumptions verified ?



R. Rodriguez-Guzman *et al.*, Phys. Rev. C **89** (2014)

Wrapping up again

1D fission picture



$$i\hbar\frac{\partial}{\partial t}\phi(q,t) = \left[\frac{h^2}{2}\frac{\partial}{\partial q}\frac{1}{M(q)}\frac{\partial}{\partial q} + V(q)\right]\phi(q,t)$$

Potential & inertia

- Liquid drop
- Microscopic-marcroscopic
- Self-consistent mean-field
- etc

Phenomelogy

- Spontaneous fission
- Fission isomers
- Asymmetric fission

Justification

- By splitting the degrees of freedom
- From the Time-Dependent Generator Coordinate Method
- etc

Roadmap of this lecture

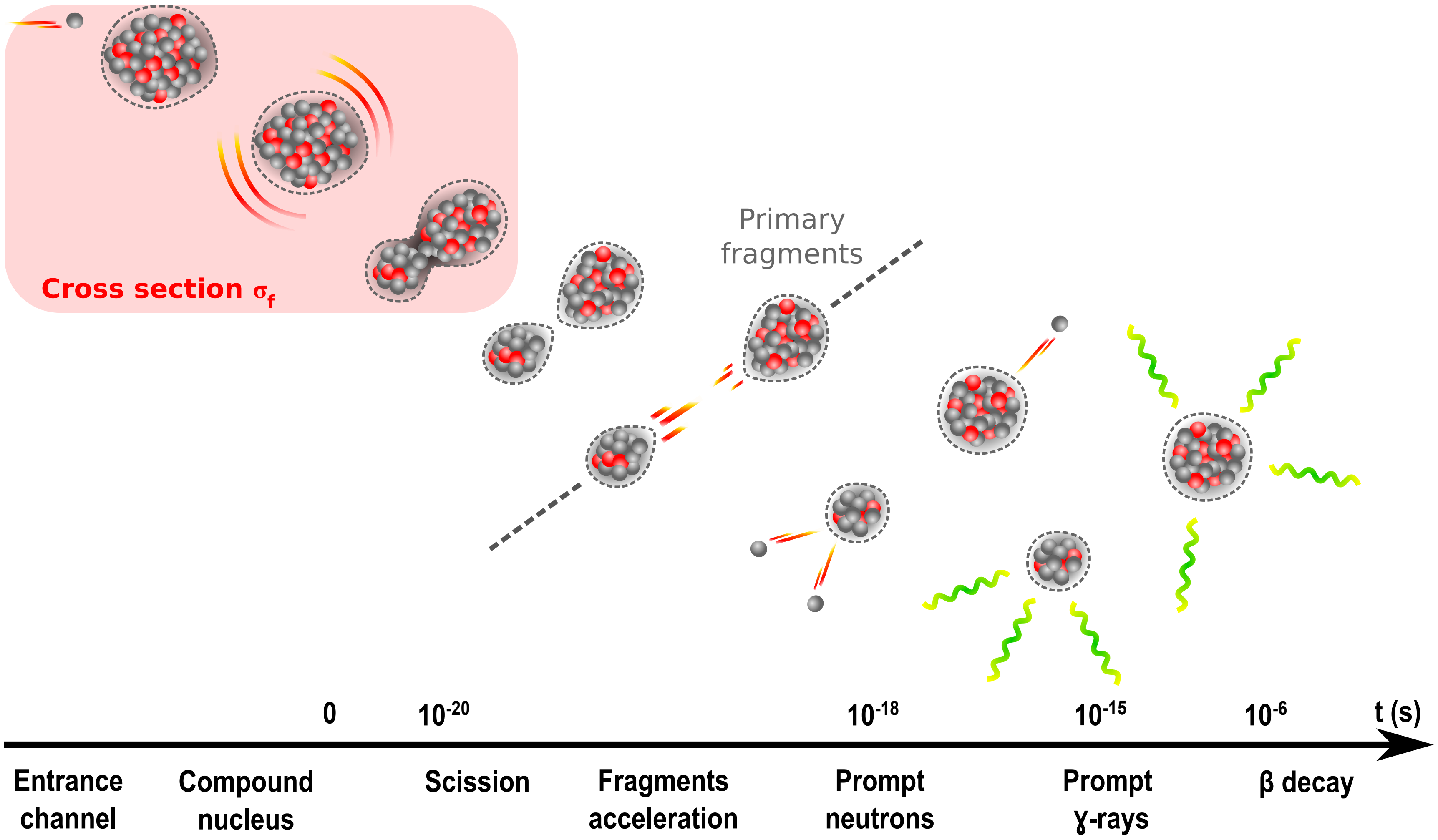
I. The one-dimensional fission picture
- Spontaneous fission half-life
- Fission isomers

II. **Induced fission cross section**
- **The Hauser-Feschbach blender**

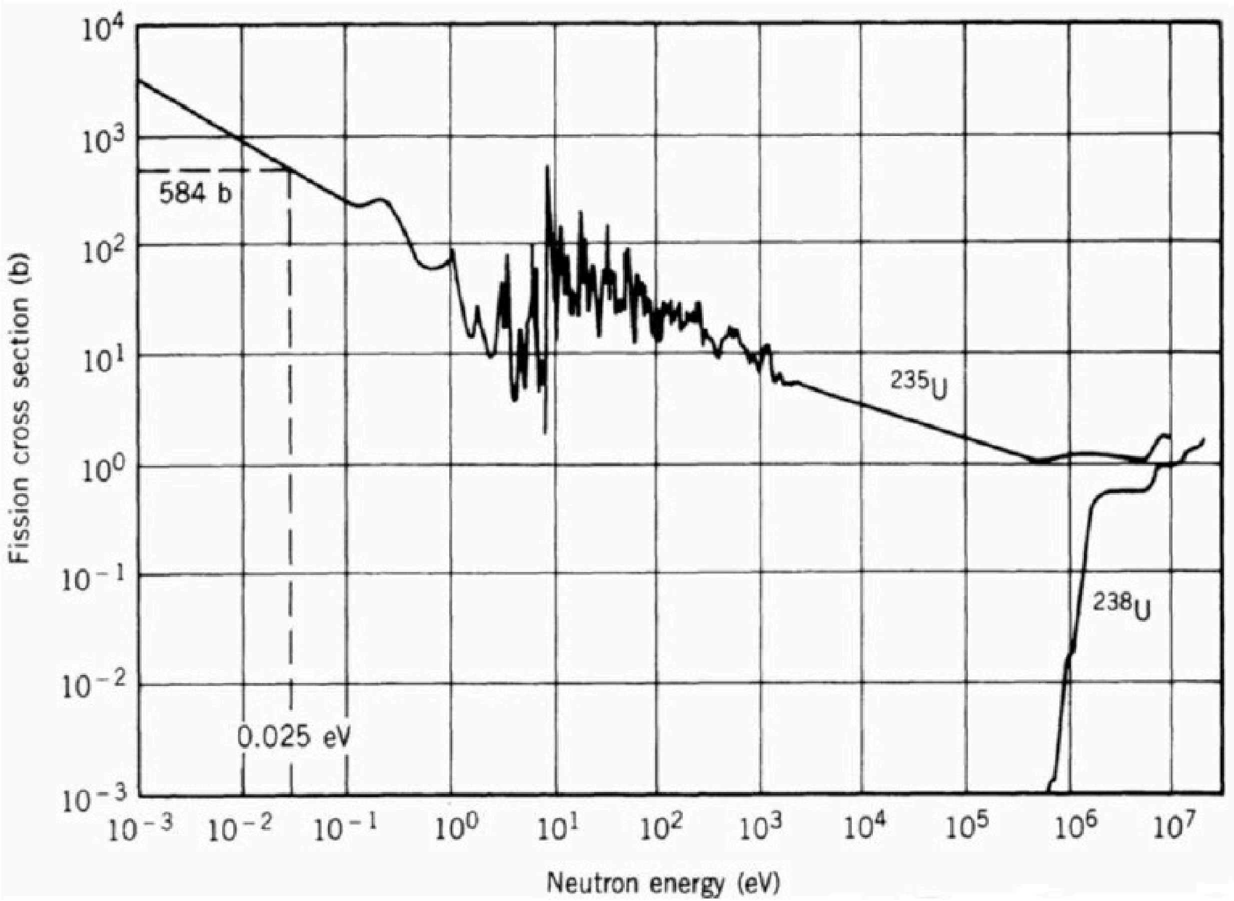
III. Generation of the primary fragments
- Scission point models
- Dynamics of the compound nucleus

IV. (Primary fragments deexcitation)

Where are we ?


Cross section σ_f
Primary fragments
0
10^{-20}
10^{-18}
10^{-15}
10^{-6}
t (s)
Entrance channel
Compound nucleus
Scission
Fragments acceleration
Prompt neutrons
Prompt γ-rays
β decay

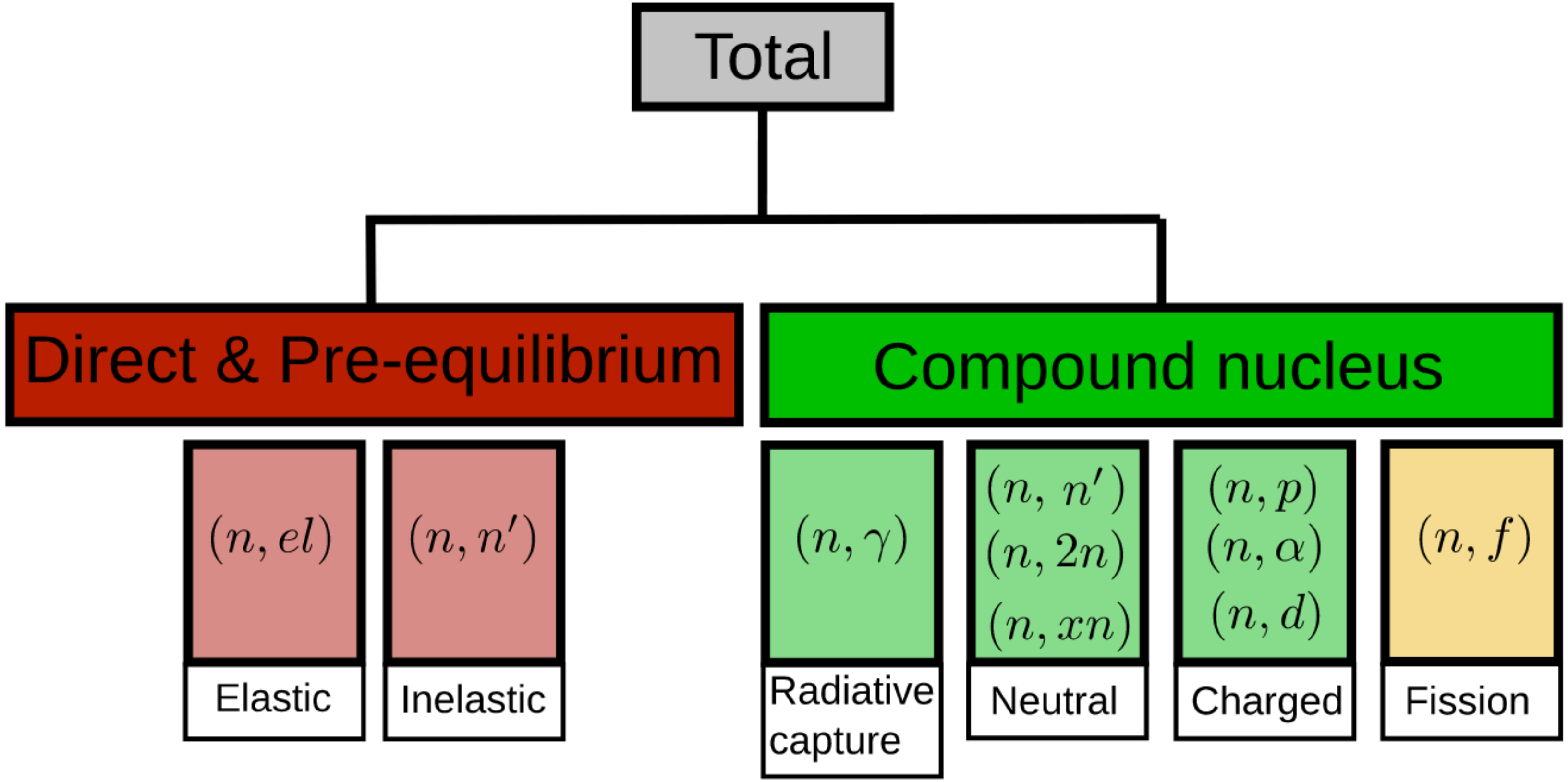
Two fission cross sections



Neutron induced fission cross sections for a fertile and fissile target
K. S. Krane, *Introductory Nuclear Physics* (1988)

- Focus on the continuum part
- See talk of **P. Tamagno**

Fission: 'one' channel among others



The Hauser-Feschbach blender

Core idea

- Bohr independance (compound nucleus)

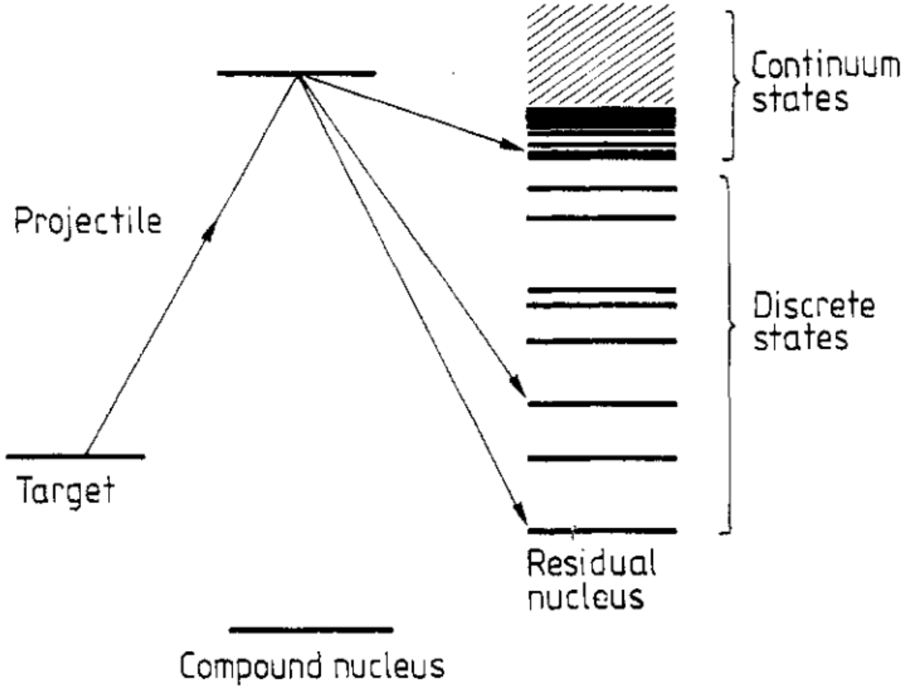
$$\sigma_{\alpha\beta} = \sigma_\alpha P_\beta^{decay}$$

  - α, β input and output channels

- Time reversal symmetry
- Cross section (no spin, no width fluctuations)

$$\sigma_{\alpha\beta}(E^*) \propto \lambda_\alpha^2 \rho(E^*) \frac{\Gamma_\alpha(E^*)\Gamma_\beta(E^*)}{\sum_\gamma \Gamma_\gamma(E^*)}$$

  - $\Gamma_\alpha = \hbar^2/\tau_\alpha$ decay width from compound nuc.



P. E. Hodgson, Rep. Prog. Phys. 50 (1987)

Fission width from... the 1D fission model

Real life

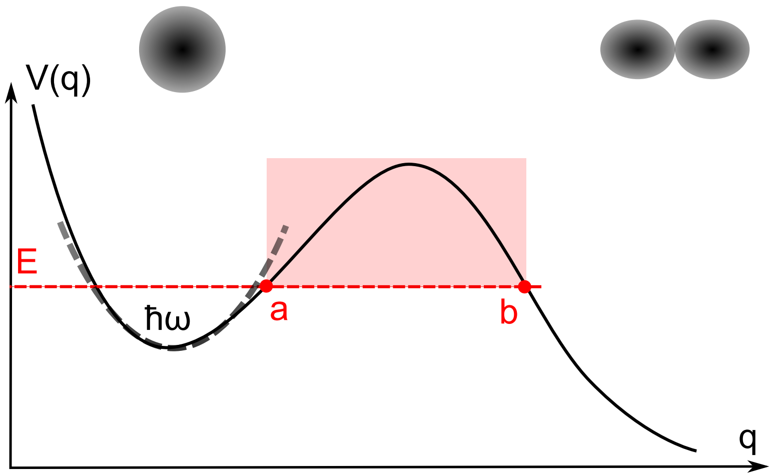
Fission = huge number of channels

Approximation

Fission $\simeq$ one big channel

Wentzel-Kramers-Brillouin (WKB) approx.

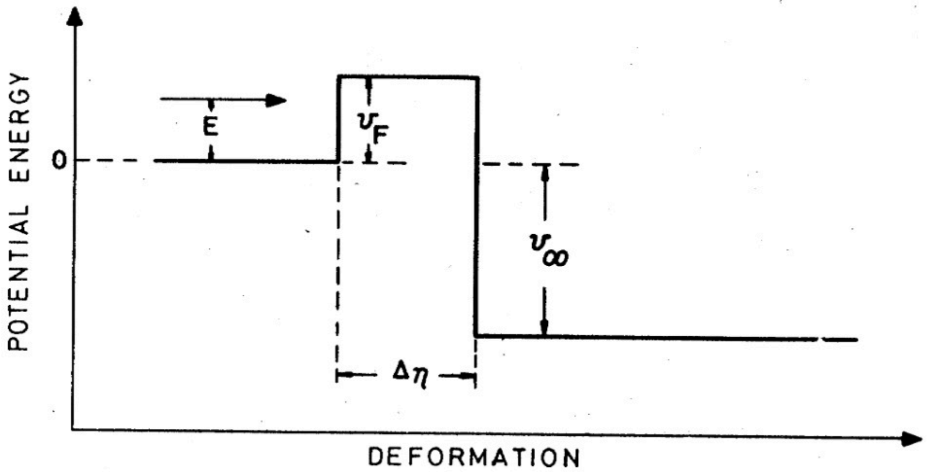
A. Messiah, Mécanique quantique (1969)



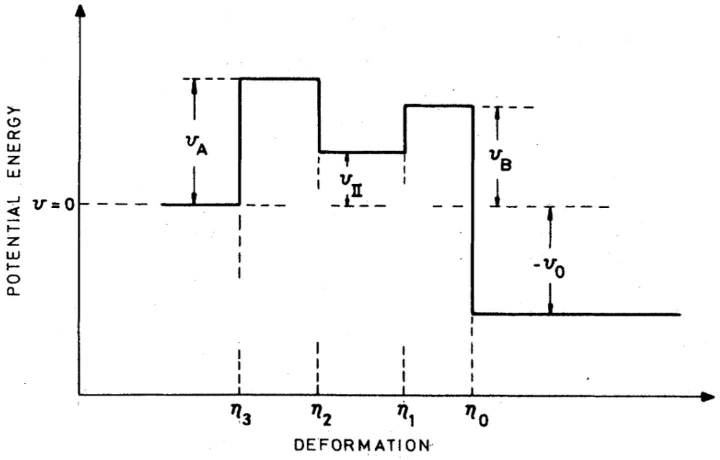
$$\tau_{sf} = \frac{2\pi}{\omega} exp\left[\frac{2}{\hbar}\int_{q=a}^{b}\sqrt{2M(q)[V(q)-E]}dq\right]$$

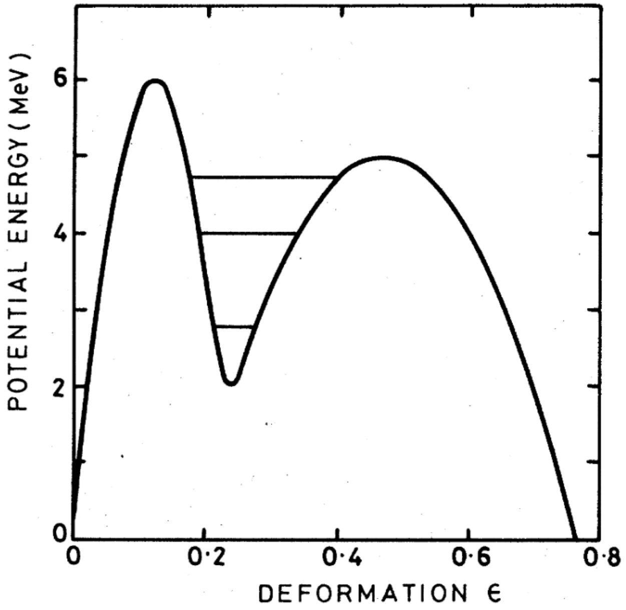
- E: energy of the initial state
- ω: frequency of an oscillator potential that fits the well

In practice: transmission through the barrier



S. Bjornholm *et al.*, Rev. Mod. Phys (1980)





Transmission coefficient `

$$T_f(E^*) \simeq 2\pi\rho(E^*)\Gamma_f(E^*)$$

`

Widespread practices

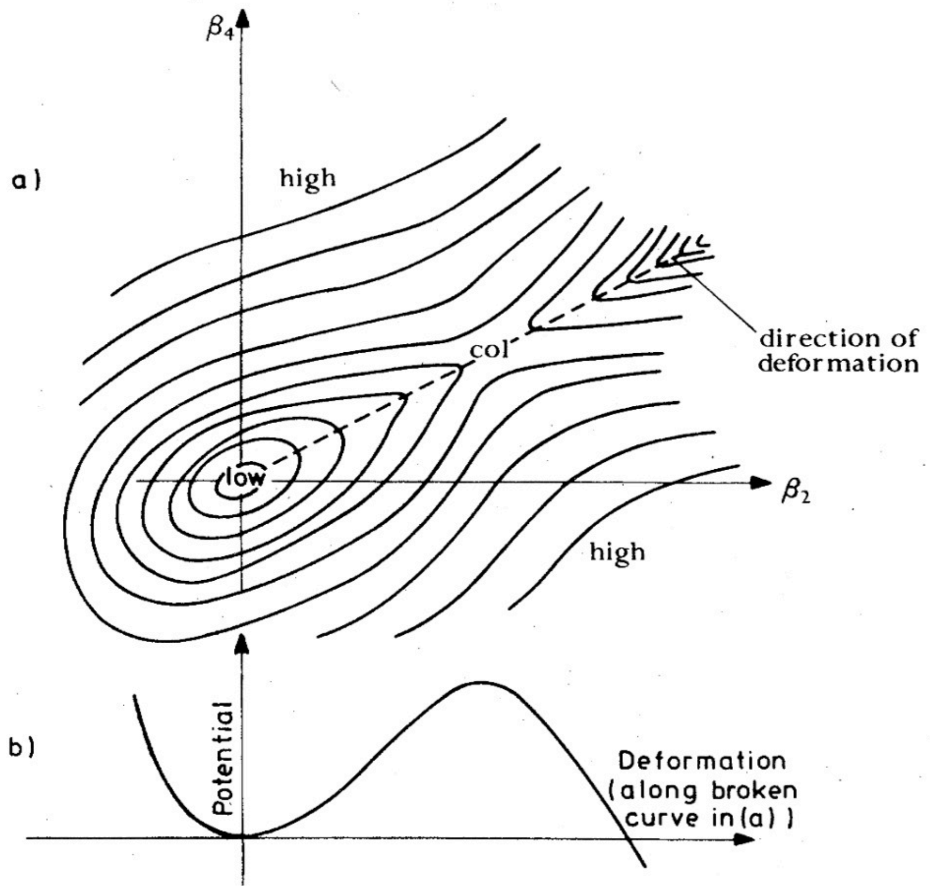
- **Constant inertia** $M(q) = M$
- **Analytical potential** to get analytical $T_f(E^*)$
- Potential fitted to reproduce cross sections

In practice: several fission channels

As energy increases,
new fission channels open up.

Total fission transmission coefficient:

$$T_{f,tot}(E^*) = \sum_{k \in \text{fission channels}} T_f(E - \epsilon_k)$$



The fission 'mille-feuille'
Each energy curve is associated to a different fission channel
J.-F. Feuillette, *Le millefeuille praliné*, (difficulté moyen)

In practice: several fission channels

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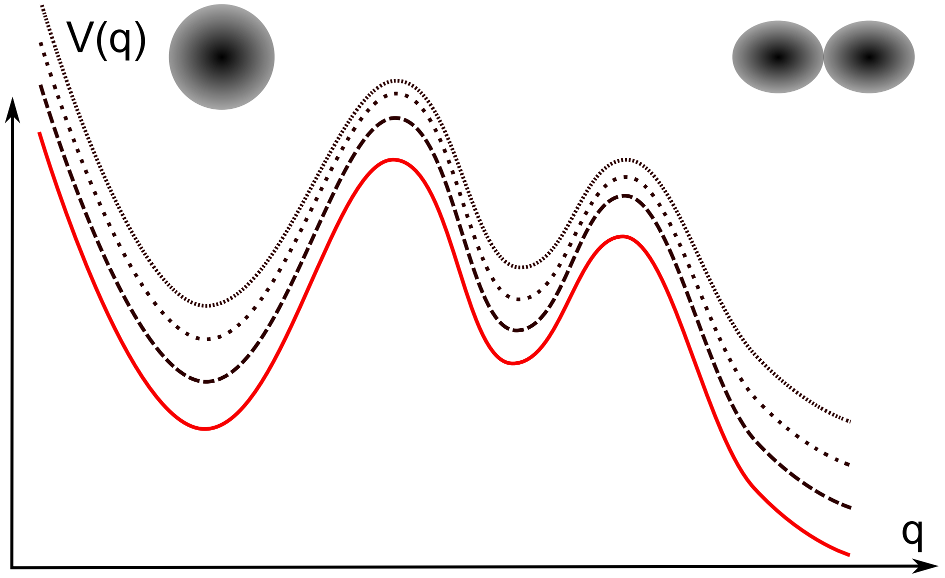
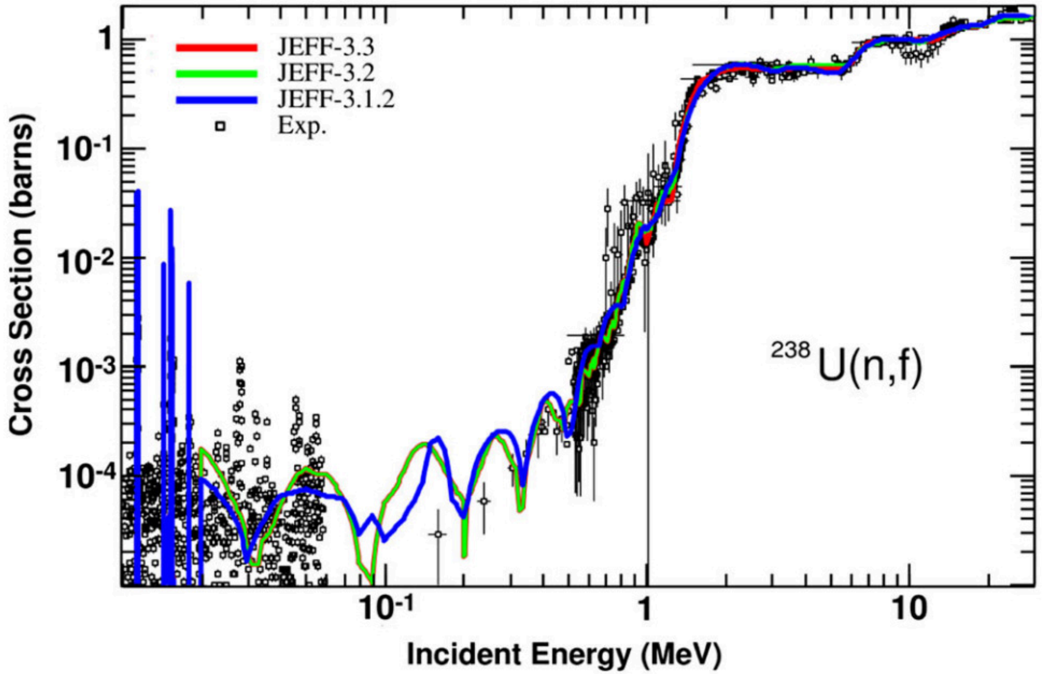
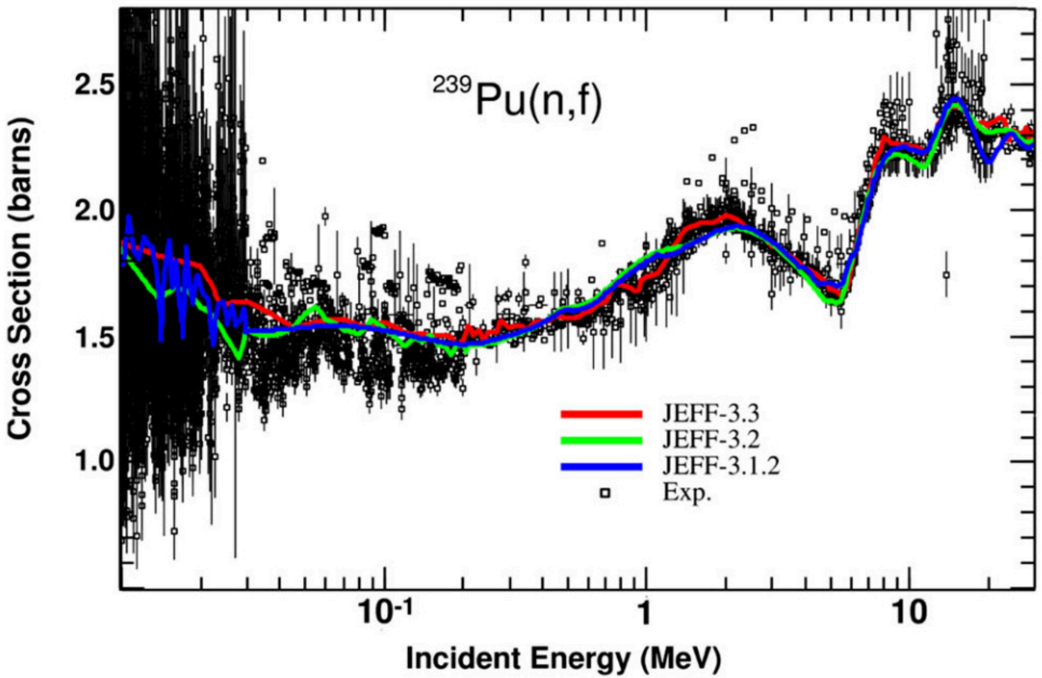


The fission 'mille-feuille'
Each energy curve is associated to a different fission channel
J.-F. Feuillette, *Le millefeuille praliné*, (difficulté moyen)

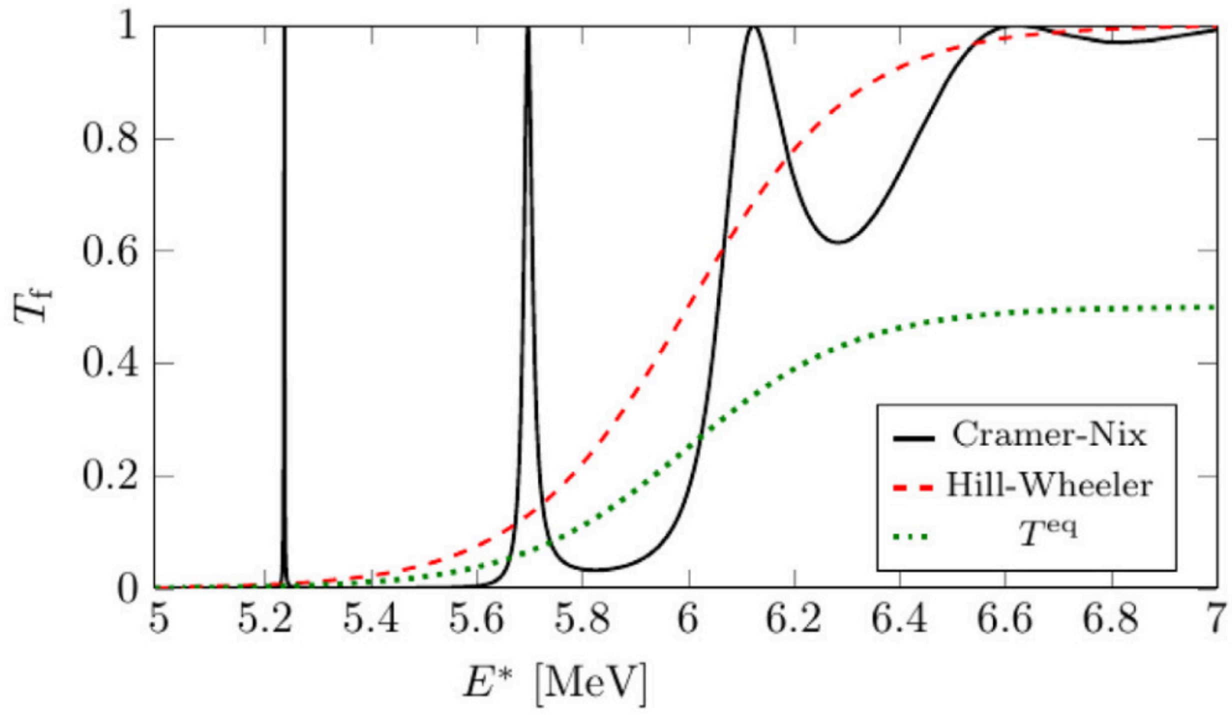
Fine tuned fission cross sections



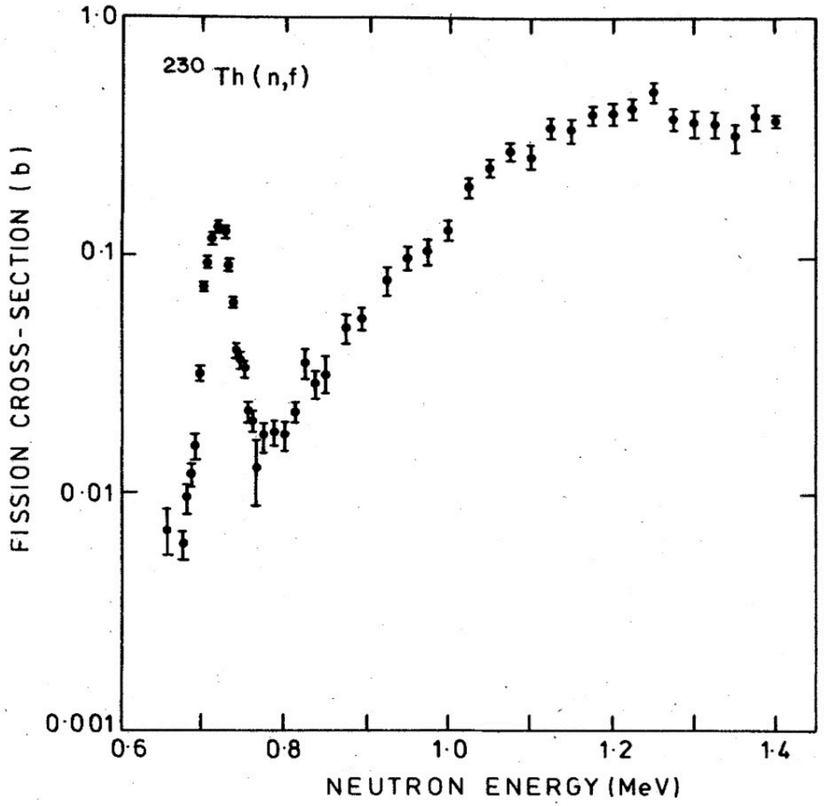


A. Plompen *et al.* EPJA **56** (2020)

Effect of the double-humped barrier



Courtesy of P. Tamagno



James *et al.* (1972)

Wrapping up

Need for all channels

- The **coumpound nucleus** picture
- A few fission channels
- Use the **1D fission picture**

In real life

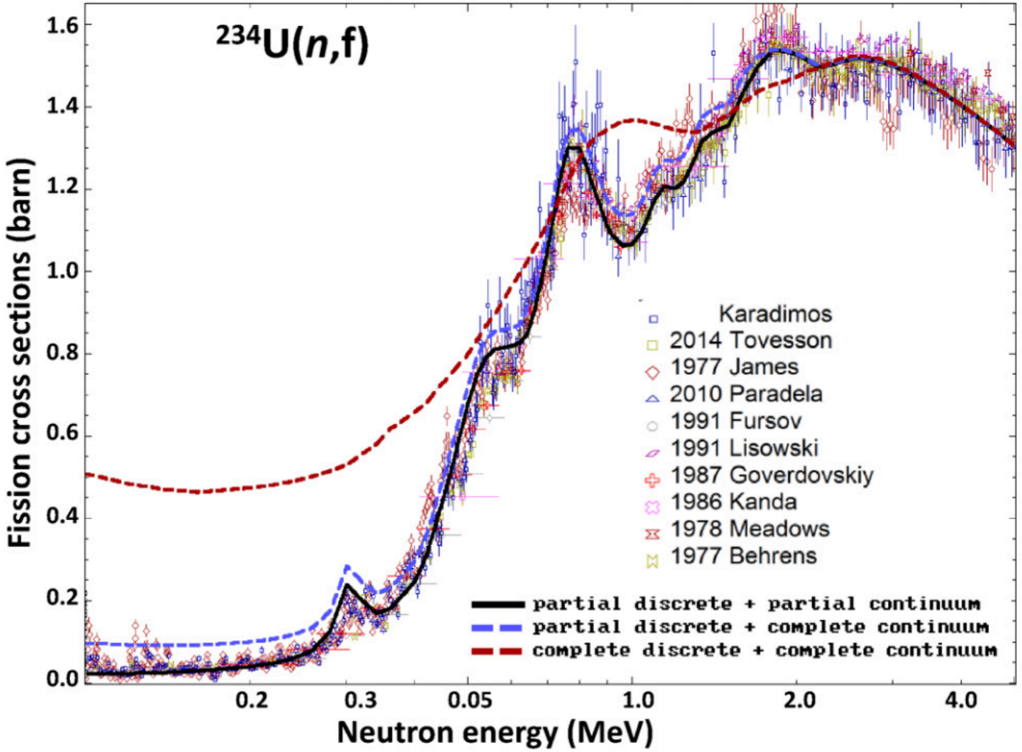
- Highly sensitive observable
- Ingredients **fitted to cross sections**

Going further

- R-matrix theory for low energy resonances
- Efforts to link to self-consistent mean field

  A. Plompen *et al.* EPJA **56** (2020)

  G.F. Bertsch *et al.* Phys. Rev. C **107** (2023)
- Absorption in the second well
- etc



Roadmap of this lecture

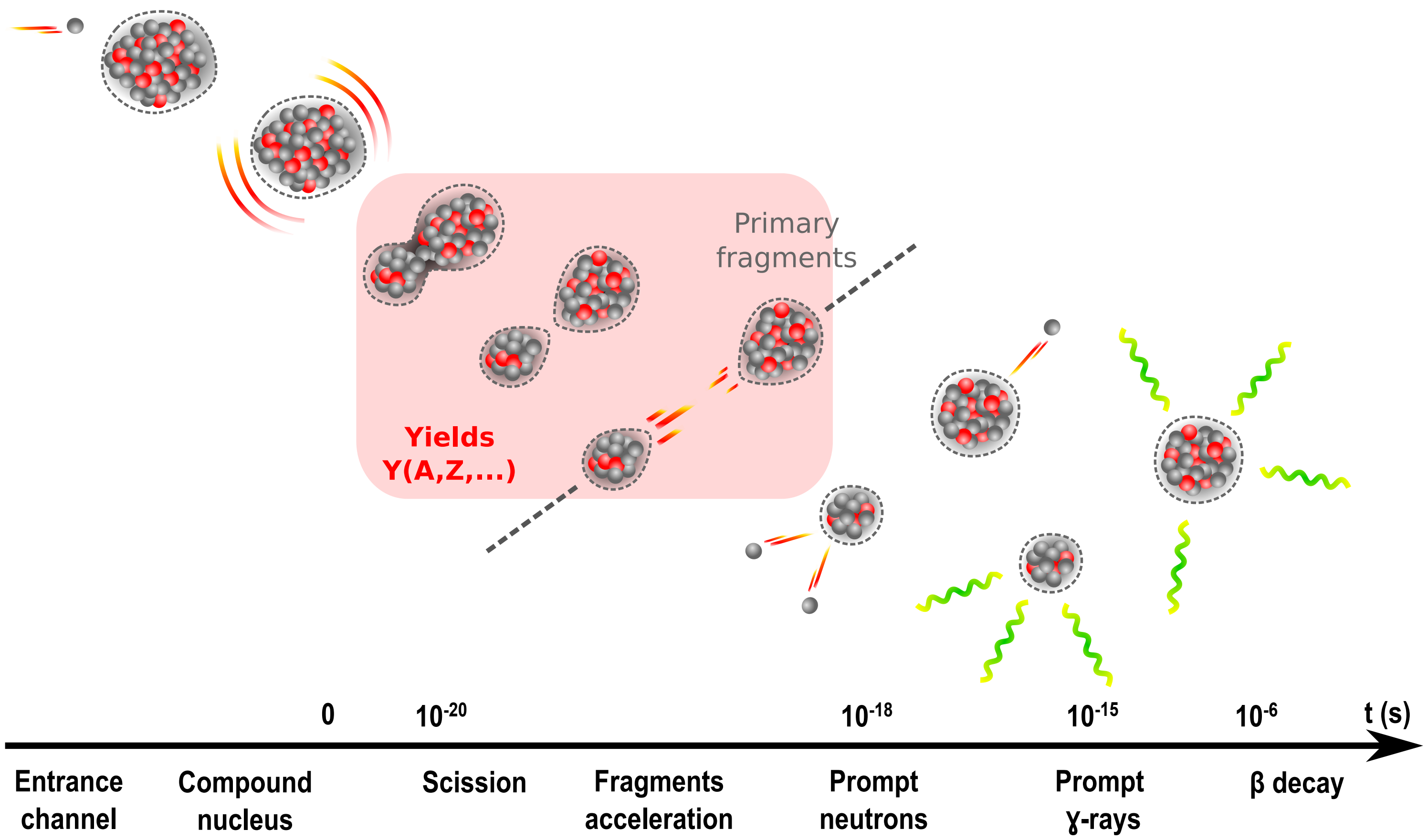
I. The one-dimensional fission picture
- Spontaneous fission half-life
- Fission isomers

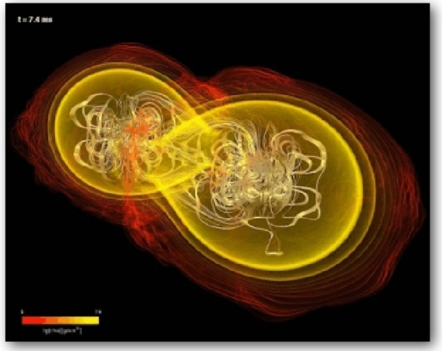
II. Induced fission cross section
- The Hauser-Feschbach blender

III. **Generation of the primary fragments**
- **Scission point models**
- **Dynamics of the compound nucleus**

IV. (Primary fragments deexcitation)

Fission of an atomic nuclei

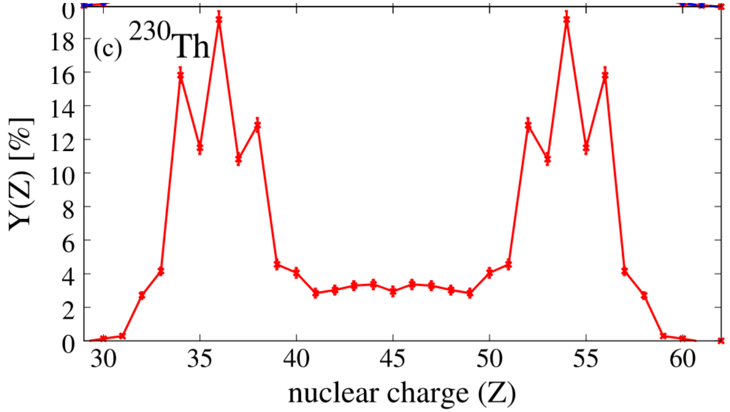




What do we want to know ?

Fragments charge and neutron numbers

When no more nuclear interaction between the fragments

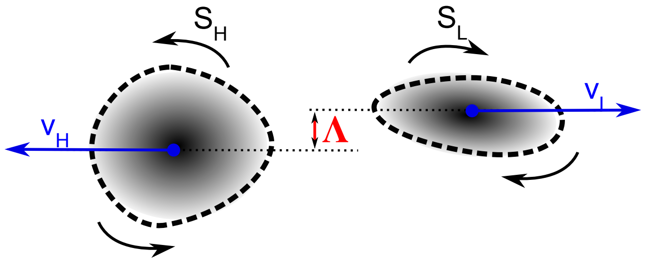


A. Chatillon *et al.* Phys. Rev. C **99** (2019)

Angular momentum content

After complete acceleartion of the fragments

$$S_0 = S_{Light} + S_{Heavy} + \Lambda$$



Energy content

After complete acceleration of the fragments

$$Q_{fission} = TKE + E^*_{Light} + E^*_{Heavy} \simeq 210 \text{ MeV}$$

- High precision required for prompt neutron emission
  - $S_n \simeq 6$ MeV
  - $\bar{\nu} \simeq 3$
  - **2 MeV wrong $\implies$ 10% error on $\bar{\nu}$**
  - Precision required for a reactor $\simeq 0.1\%$

Miscellaneous

- Emission of 'scission neutrons' ?
- Ternary fission (0.1% of events)
- etc

Fragments acceleration

Classical picture

- Two pointwise charges
- Fragments initialy separated a few fm

$$r(t=0) = r_0(A_L^{1/3} + A_H^{1/3}) + 5 \text{ fm}$$

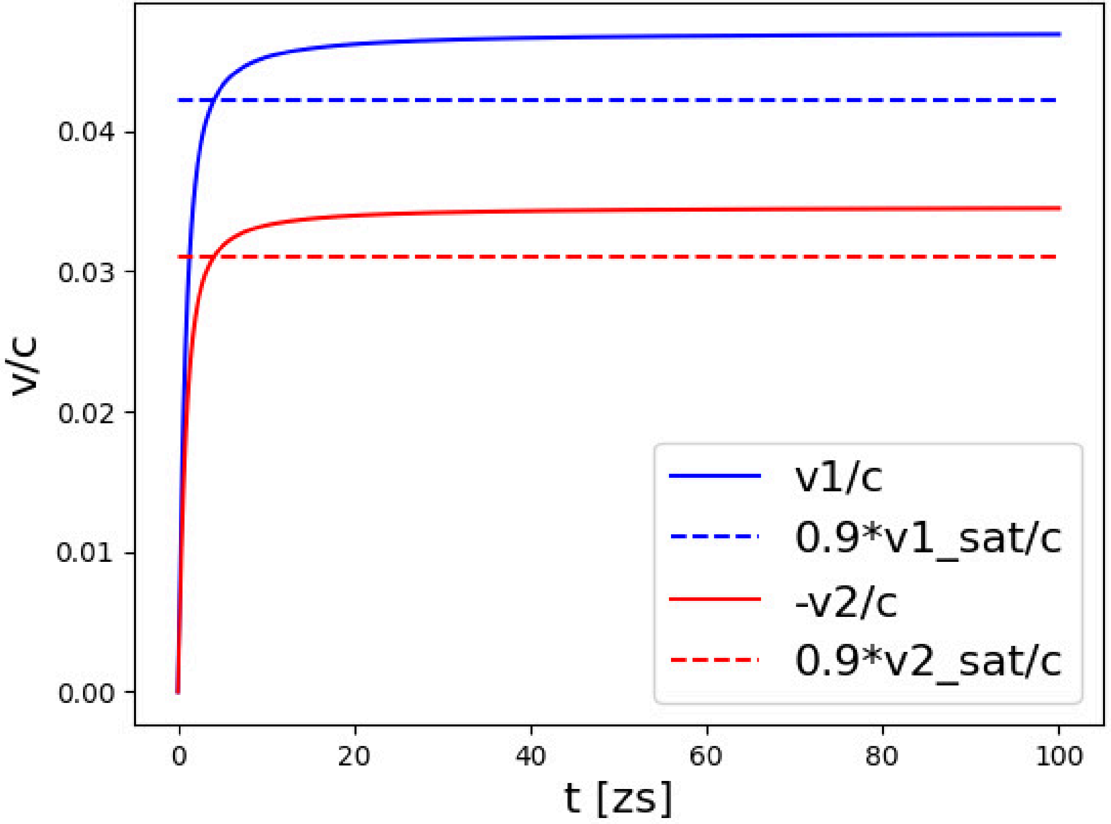
- Equation of motion

$$\mu \ddot{r} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0} \frac{1}{r^2}$$

Application

- Actinide most probable fragments

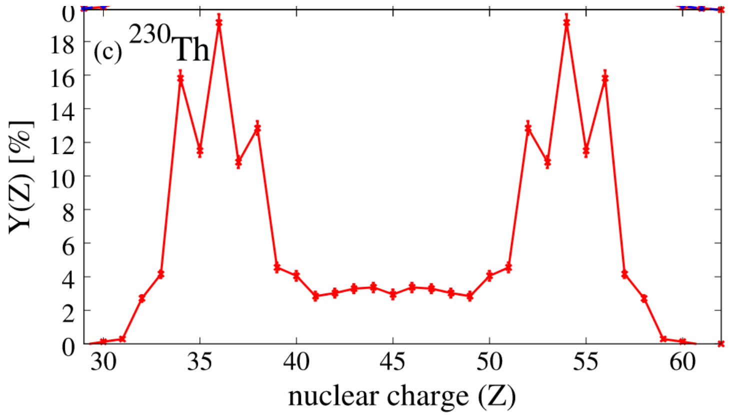
> - **5 zs (10^{-21} s)** to reach 90% of the speed
> - Final speed $\simeq$ **9000 km/h**
> - **100 fm** away at 90% of the speed



What do we want to know ? (revised)

Fragments charge and neutron numbers

When no more nuclear interaction between the fragments



A. Chatillon *et al.* Phys. Rev. C **99** (2019)

Energy content

When no more nuclear interaction between the fragments

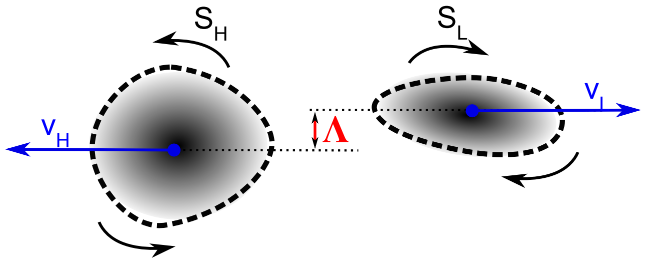
$$Q_{fission} = TKE + E_{Coul.} + E^*_{Light} + E^*_{Heavy} \simeq 210 \text{ MeV}$$

- High precision required for prompt neutron emission
  - $S_n \simeq 6$ MeV
  - $\bar{\nu} \simeq 3$
  - **2 MeV wrong $\Longrightarrow$ 10% error on $\bar{\nu}$**
  - Precision required for a reactor $\simeq 0.1\%$

Angular momentum content

When Coulomb does not change angular momenta anymore

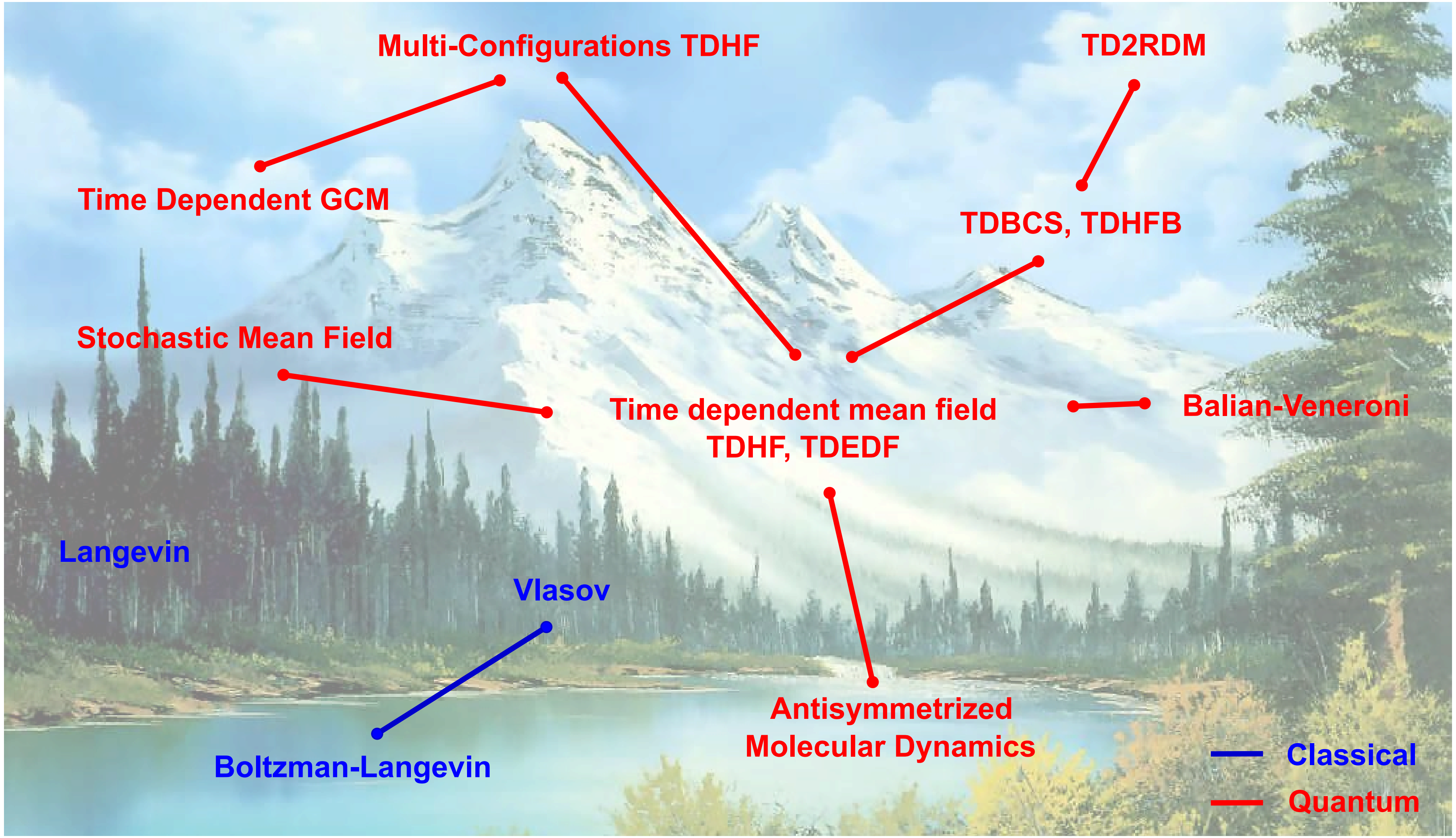
$$S_0 = S_{Light} + S_{Heavy} + \Lambda$$



Miscellaneous

- Emission of 'scission neutrons' ?
- Ternary fission (0.1% of events)
- etc

How to approximate the dynamics ?



Some references

Review papers

- On stochastic approaches of nuclear dynamics

  Y. Abe *et al*, Phys. Rep 275 (1996)
- Time-dependent density-functional description of nuclear dynamics

  T. Nakatsukasa *et al.*, Rev. Mod. Phys, **88** (2016)
- Heavy-ion collisions and fission dynamics with the time-dependent hartree-Fock theory and its extensions

  C. Simenel *et al.*, Prog. Part. Nucl. Phys. **103** (2018)
- The time-dependent generator coordinate method in nuclear physics

  M. Verriere *et al.*, Front. Phys. **8** (2020)

Books

- Quantum theory of finite systems

  J.-P. Blaizot, G. Ripka, MIT Press (1985)
- The nuclear many-body problem

  P. Ring, P. Schuck, Springer science (2004)

Lectures

- Microscopic approaches for nuclear Many-body dynamics

  C. Simenel *et al.*, arXiv:0806.2714v2 (2009)

Scission point models

Let's not do the dynamics after all...

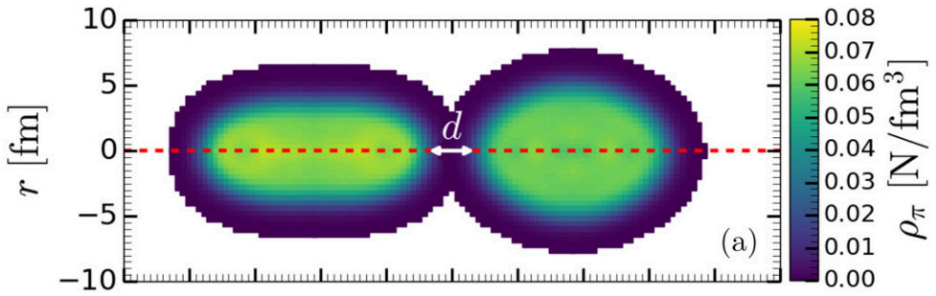
Core ideas

- Generate an 'intersting' set of **final states**

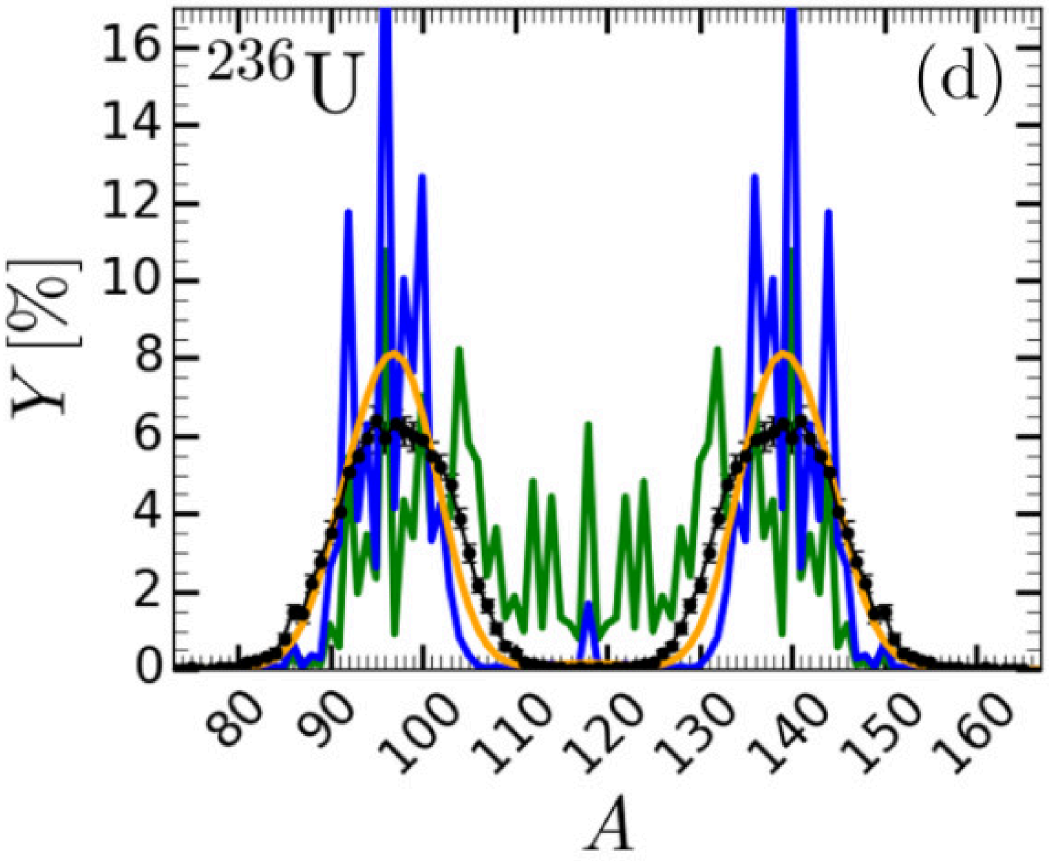
$$\{|\phi_k^f\rangle \mid k \in [1, N]\}$$

- Assume that the dynamics will populate them **statistically**

$$p_f(k) \propto exp(-E(|\phi_k^f\rangle)/T)$$

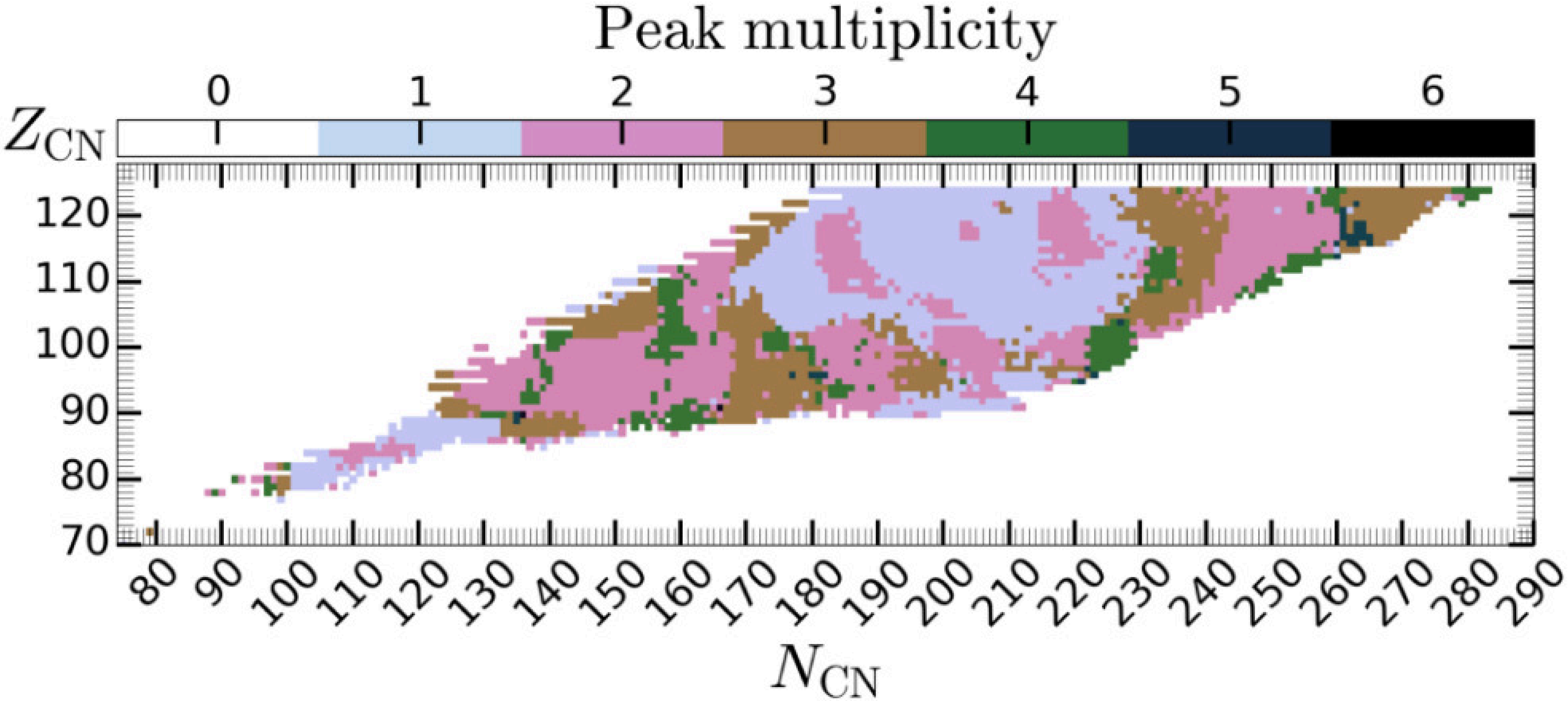


J.-F. Lemaitre *et al.*, Phys. Rev. C **99** (2019)



J.-F. Lemaitre *et al.*, Phys. Rev. C **103** (2021)

Scission point models

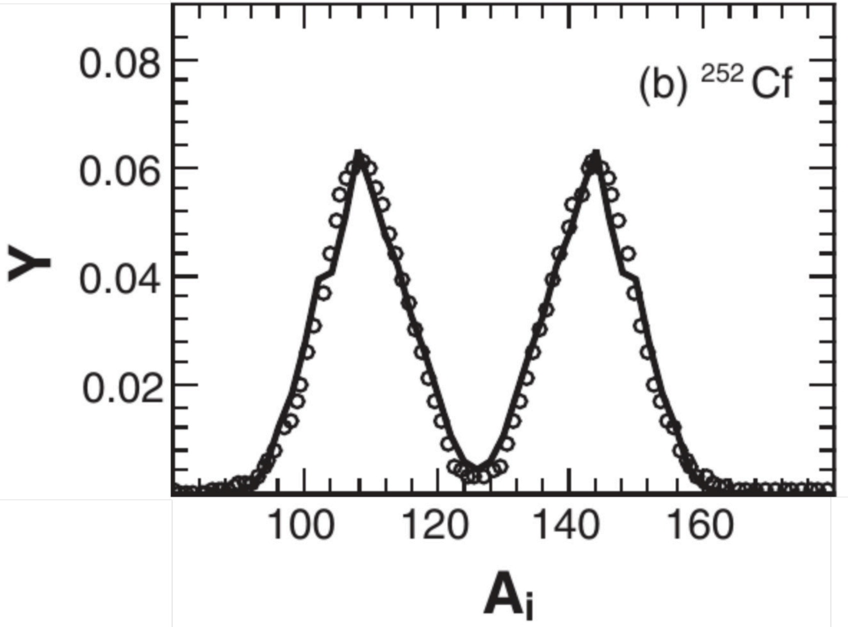


J.-F. Lemaitre *et al.*, Phys. Rev. C **103** (2021)

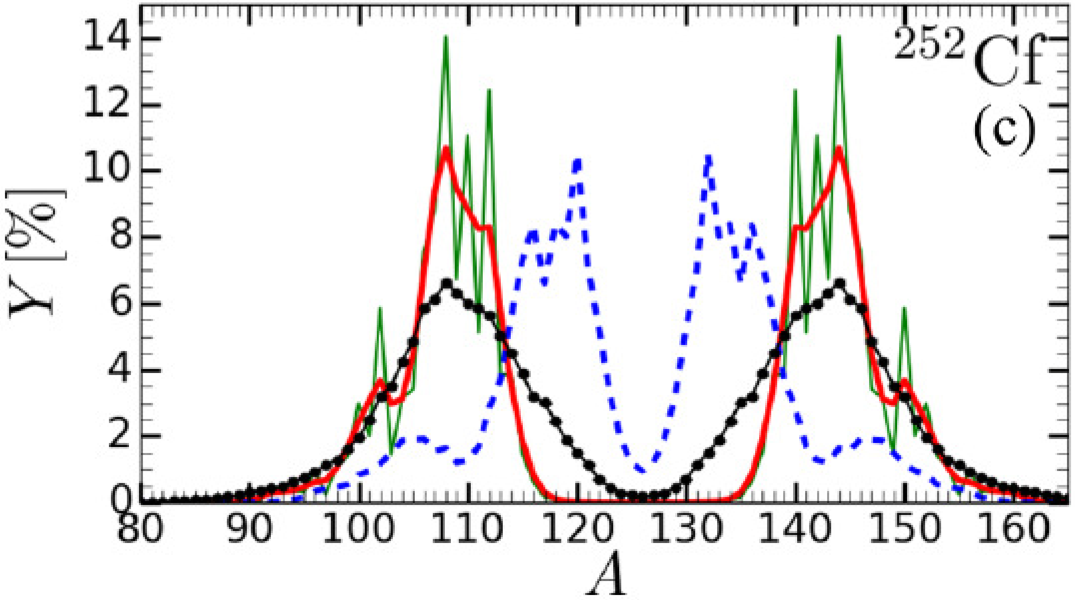
✅ Fast calculations
✅ Systematics predictions

Scission point models

Same nucleus, two scission point models...



H. Pasca *et al.* PRC **99** (2019)



J.-F. Lemaitre *et al.* PRC **99** (2019)

✗ High dependency to the phase-space considered

Langevin dynamics

Let's do dynamics... in a statistical picture

Core ideas

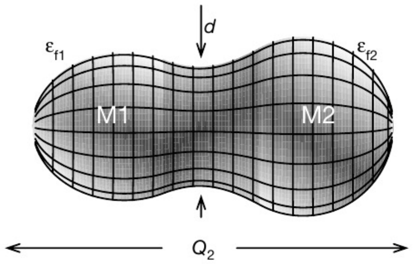
- Split the variables

  $$(r_1, \cdots, r_A) \rightarrow (q_1, q_2, x_1, \cdots, x_{A-2})$$

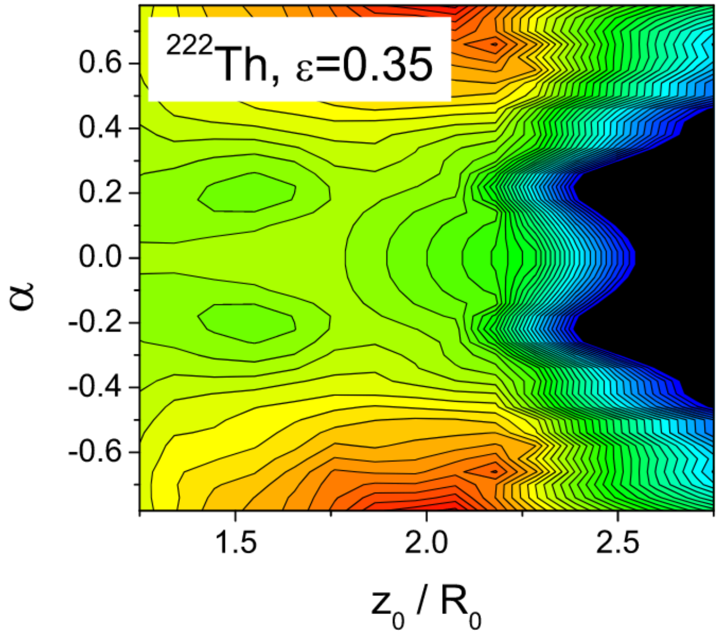
  - q_1, q_2 deformation coordinates
  - $x_1, \cdots x_{A-2}$ intrinsic coordinates
- Quantum state $\rightarrow$ by a statistical distribution
- Assume that at any time:

  $$p(t) = p(q_1, q_2, t) \times exp(-E(x_1, \cdots, x_N)/T)$$
- Use a time dependent variational principle

Need for potential, inertia, viscosity...

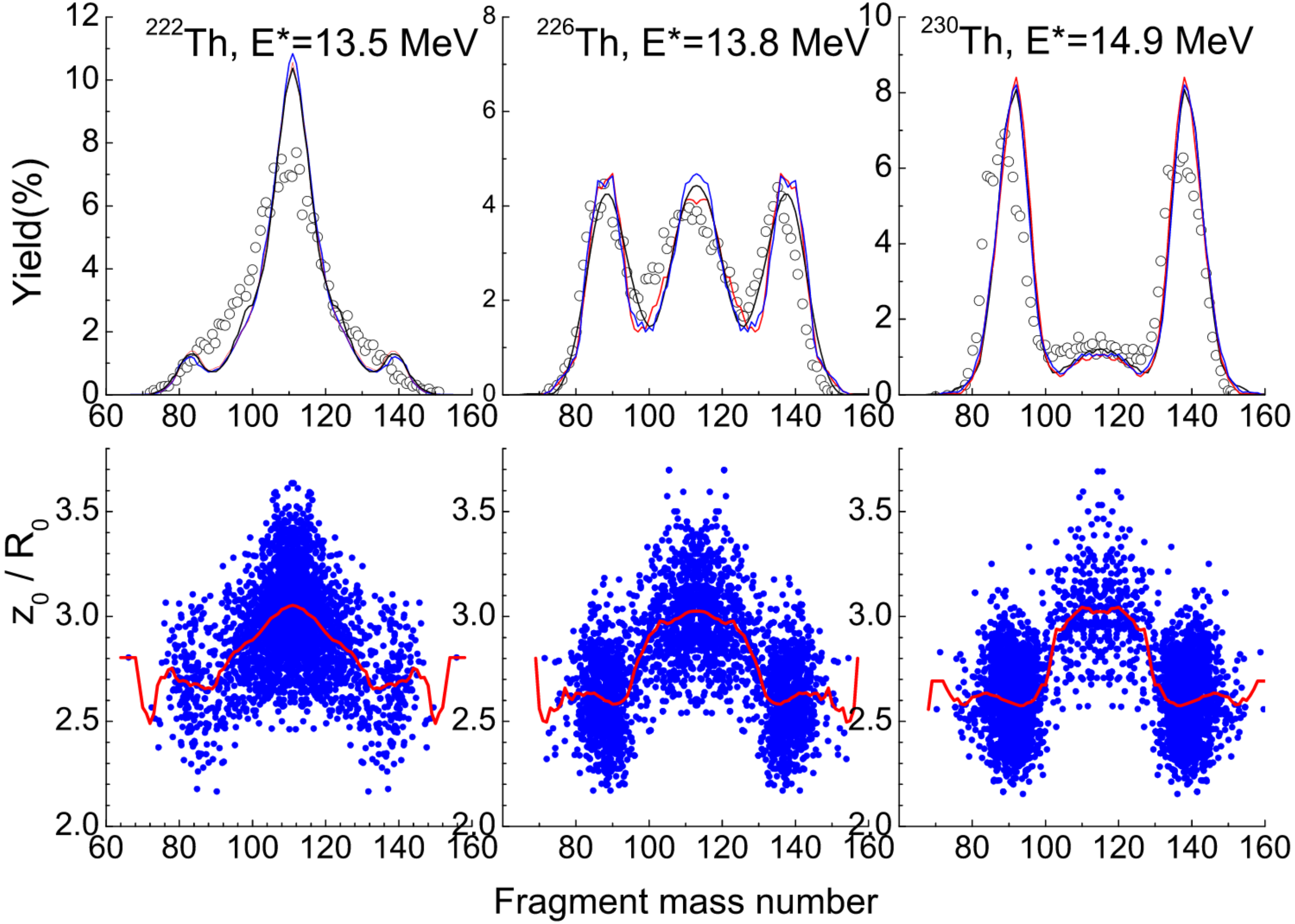


P. Möller et al., Nature **409** (2001)

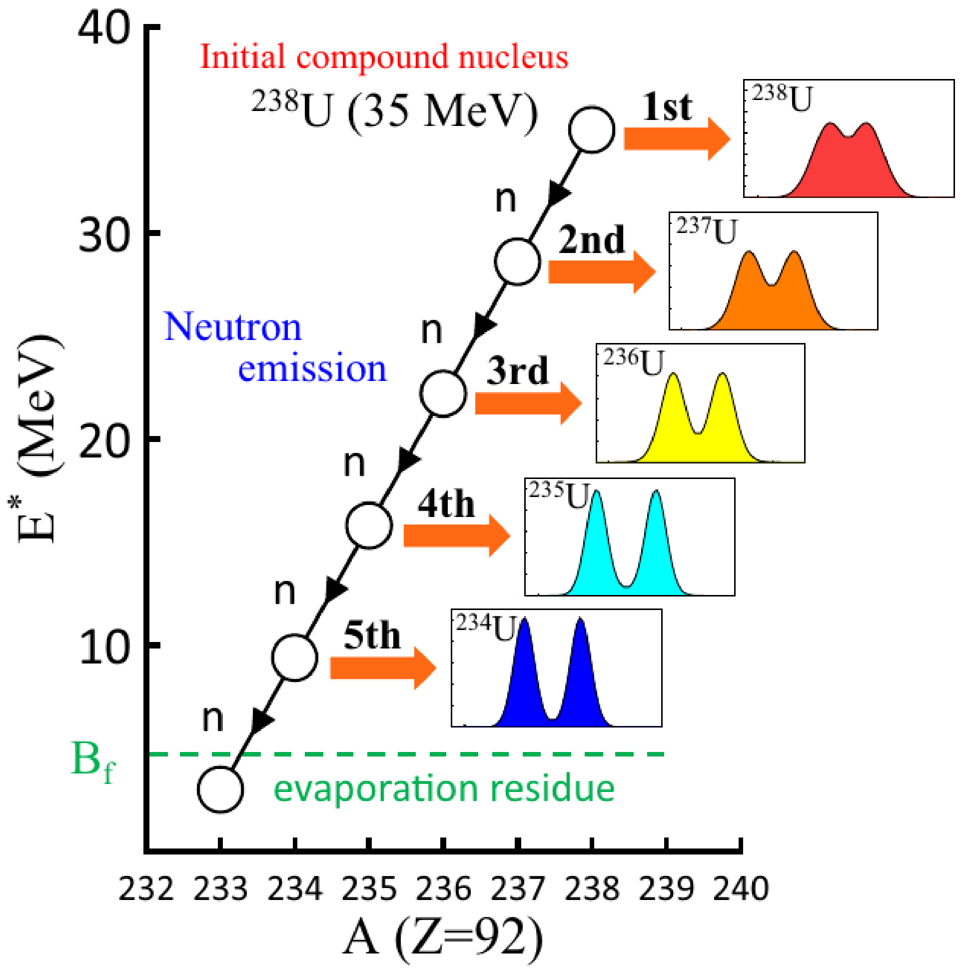


F. A. Ivanyuk Phys. Rev. C **109** (2024)

Langevin dynamics



Langevin dynamics

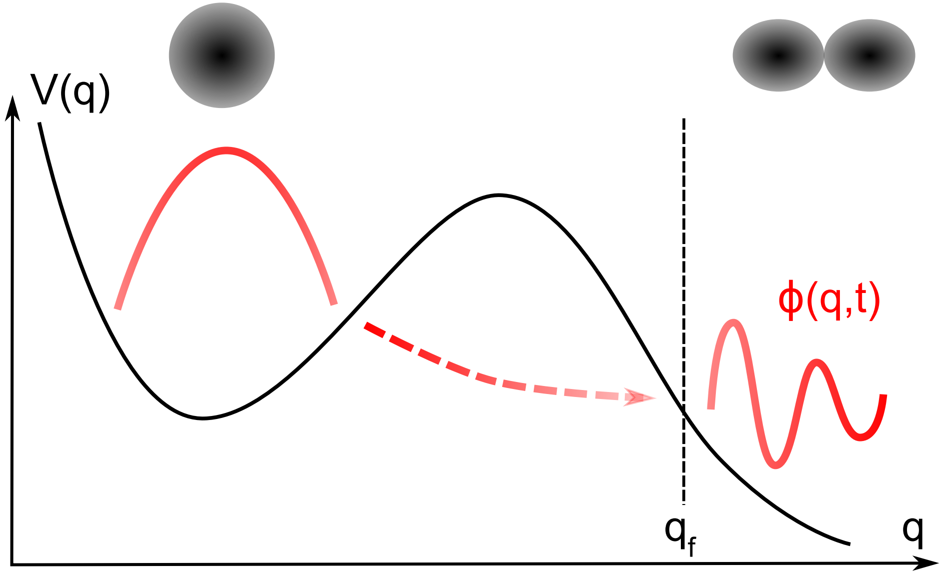


S. Tanaka *et al.* PRC **100** (2021)

Langevin dynamics

Limitations

- Classical picture $\Longrightarrow$ **no quantum tunneling**
- Only predicts the distribution of collective variables
- Stops close to scission
- Difficult to connect with a nucleon-nucleon Hamiltonian



Time-dependent Generator Coordinate Method

Quantum dynamics in a restricted space

Core ideas

- Generate an 'interesting' set of states

$$\{\phi_q(r_1, \cdots, r_A) | q \in \mathbb{R}\}$$

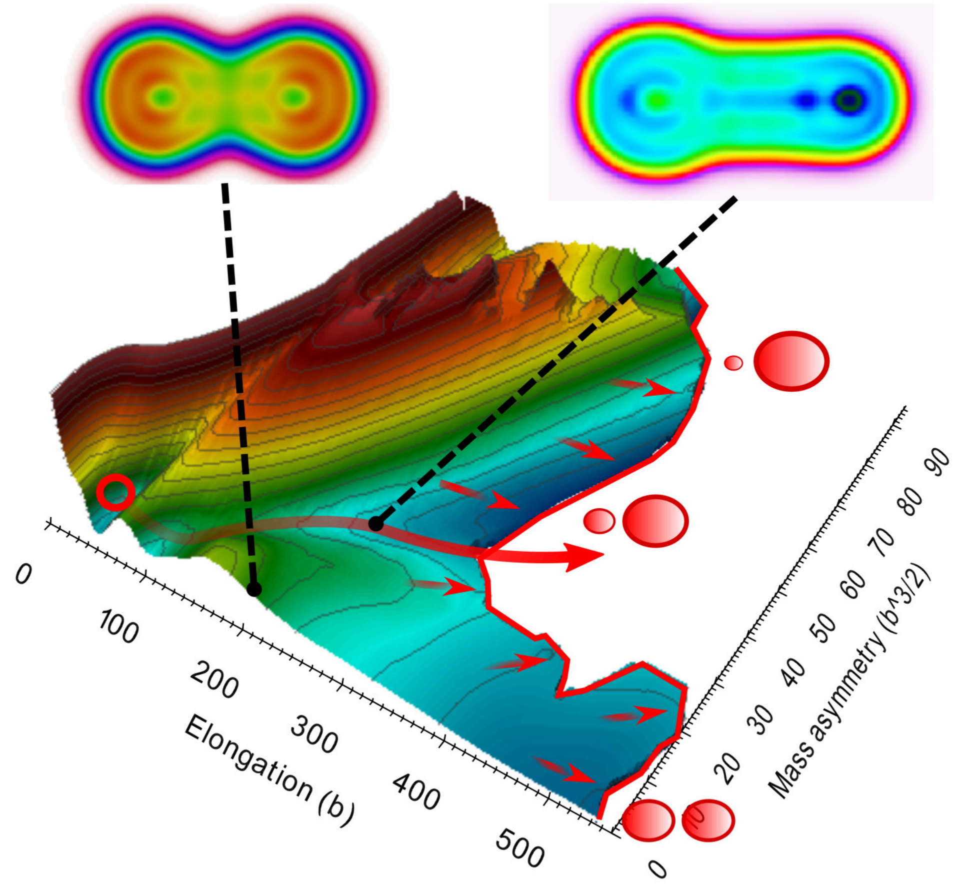
  - **q** deformation coordinate
  - with some properties (SME, GOA, etc)

- Assume at any time

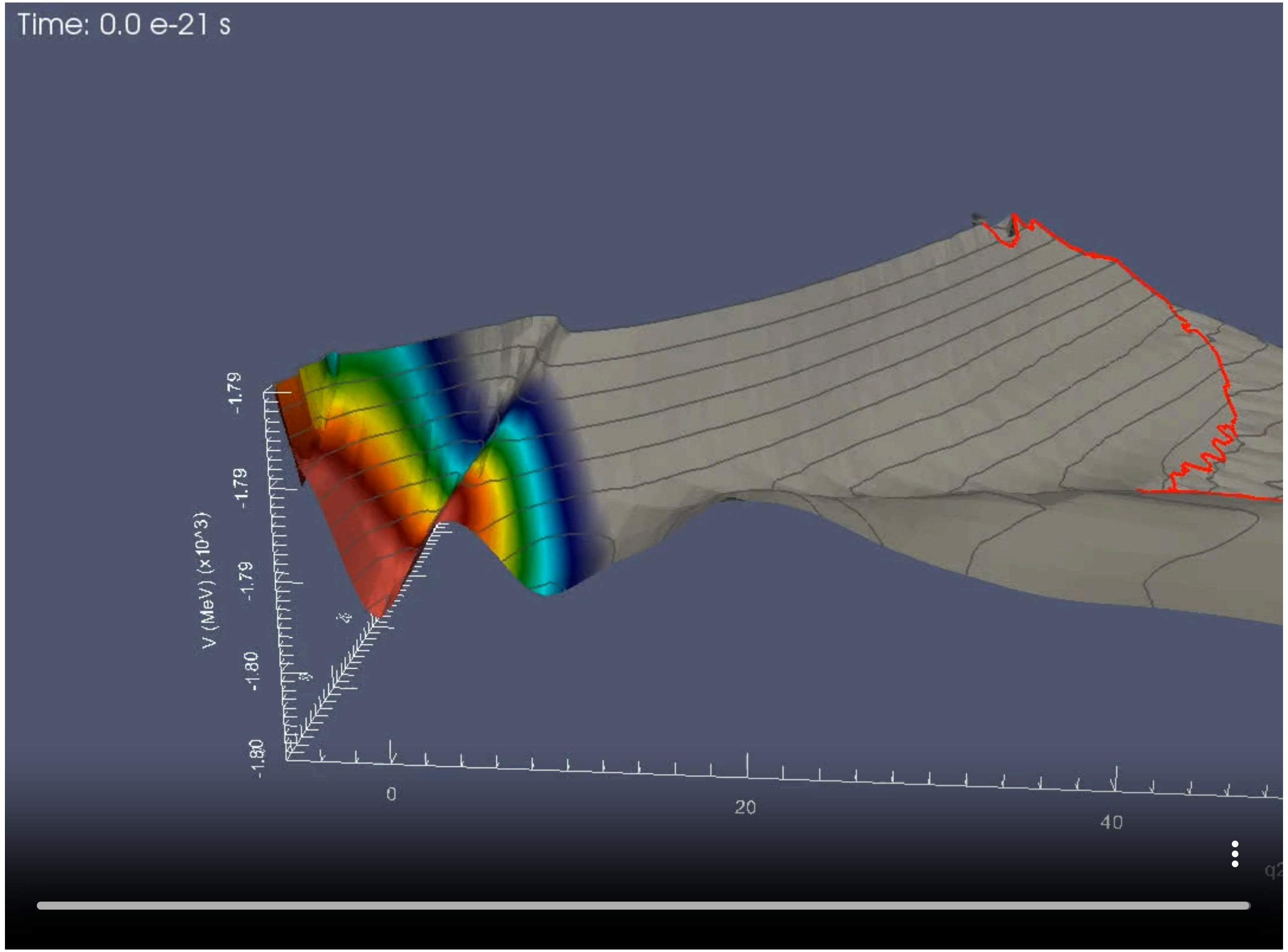
$$|\psi(t)\rangle = \int_q f(q,t)|\phi_q\rangle dq$$

- A time dependent variational principle yields

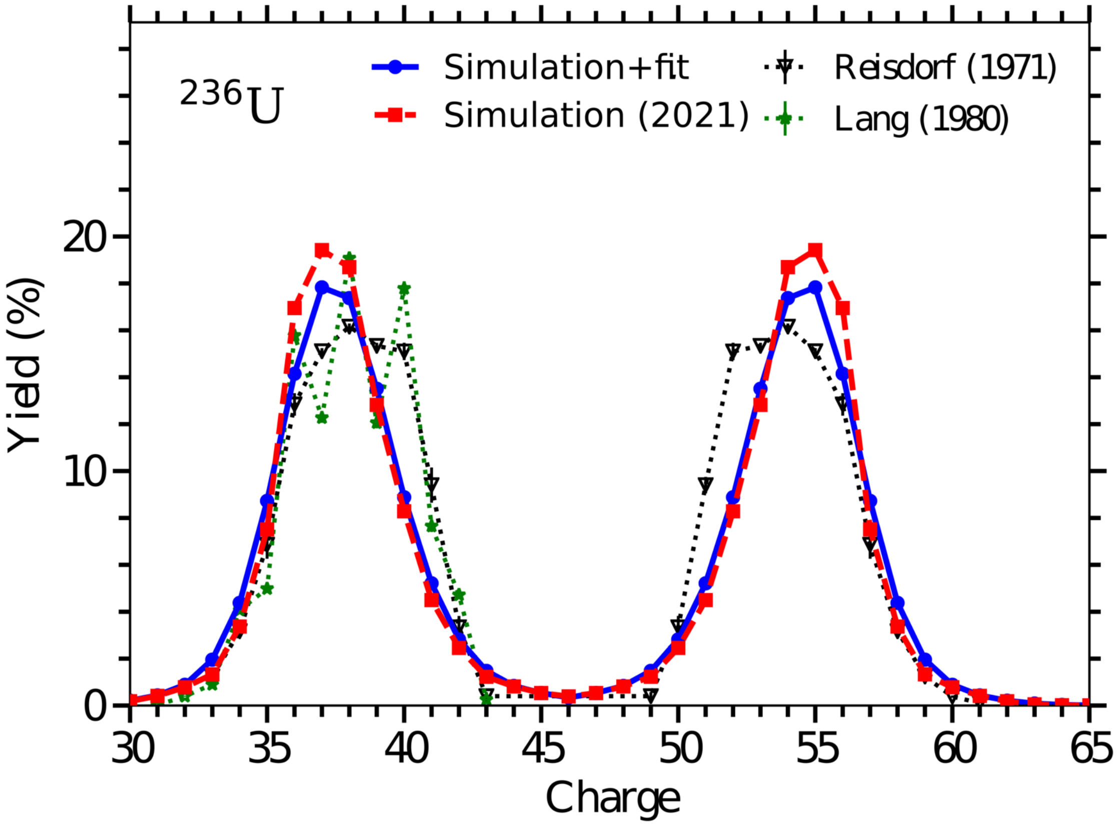
$$i\hbar \frac{\partial}{\partial t}\tilde{f}(q,t) = \left[\frac{h^2}{2}\frac{\partial}{\partial q}\frac{1}{M(q)}\frac{\partial}{\partial q} + V(q)\right]\tilde{f}(q,t)$$



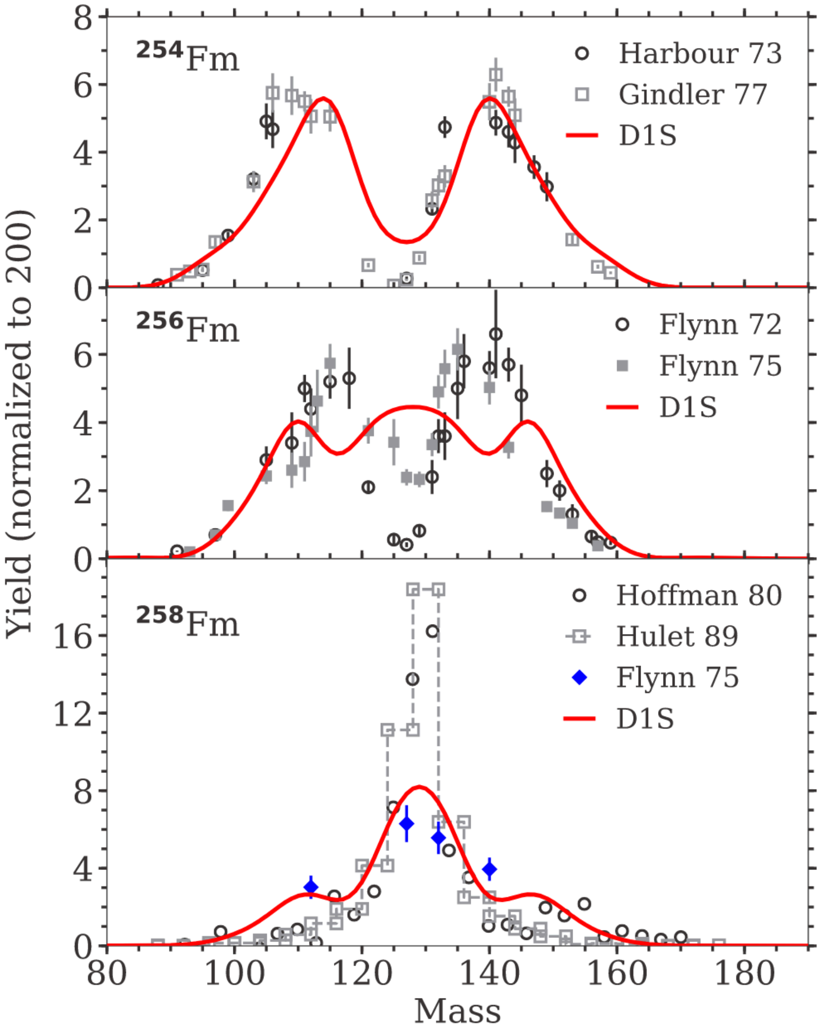
Time-dependent Generator Coordinate Method



Time-dependent Generator Coordinate Method

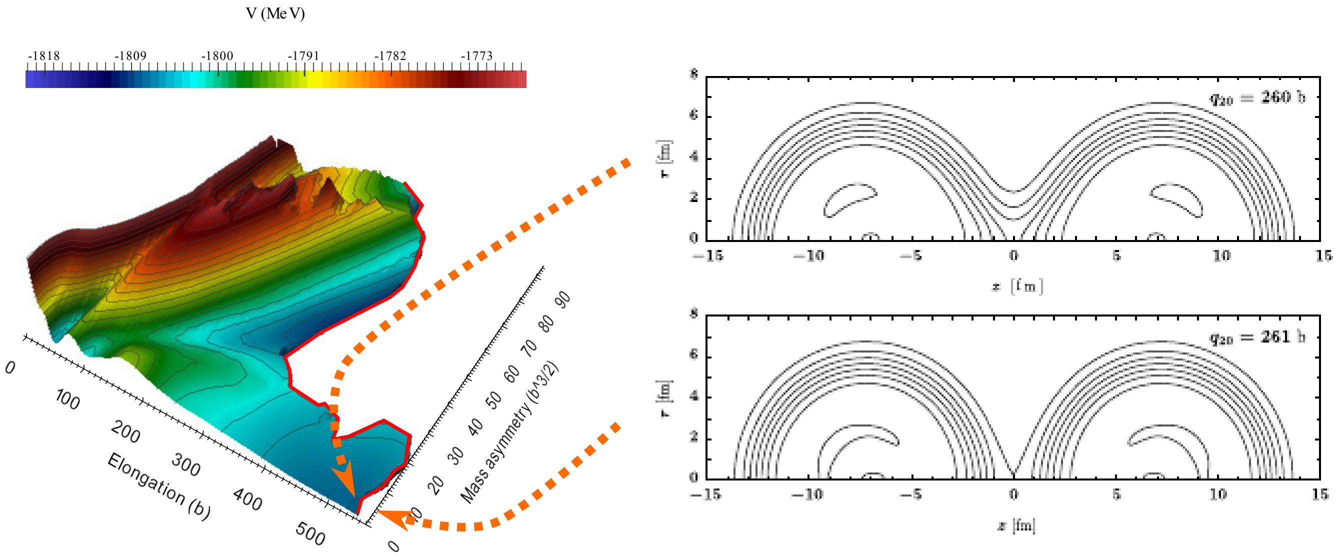


M. Verriere *et al.* PRC **103** (2021)



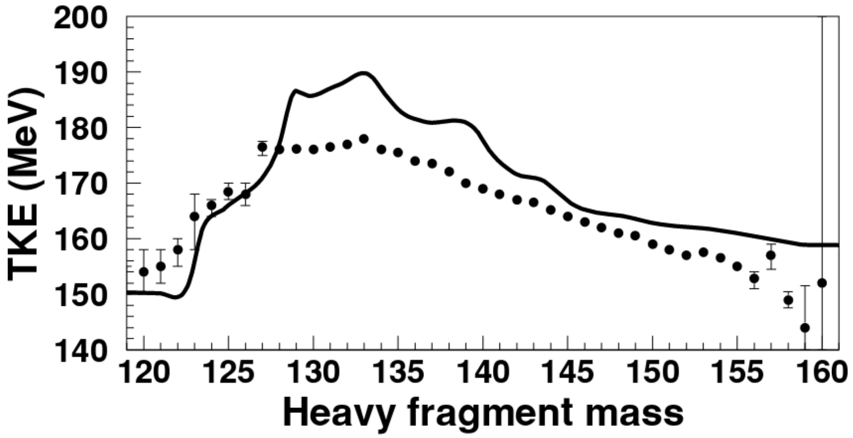
D. Regnier *et al.* Phys. Rev. C **99** (2019)

Time-dependent Generator Coordinate Method



Limitations

- Stops **before scission**
- Other discontinuities
- Bad **energy content**



H. Goutte *et al.* Phys. Rev. C **71** (2005)

Time dependent mean-field approaches

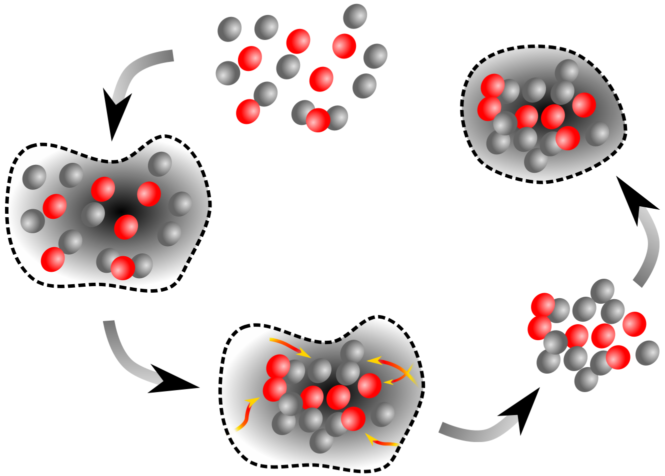
Quantum dynamics in another restricted space

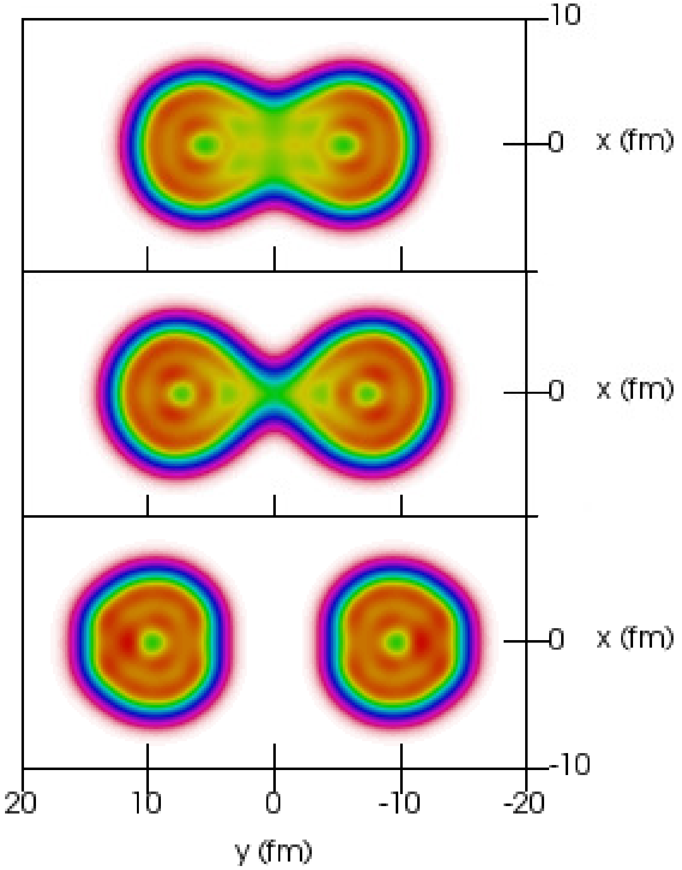
Core ideas

- Assume at any time

$$|\psi(r_1, \cdots, r_A, t)\rangle \in \text{Bogoliubov vaccua}$$

- Use a time-dependent variational principle

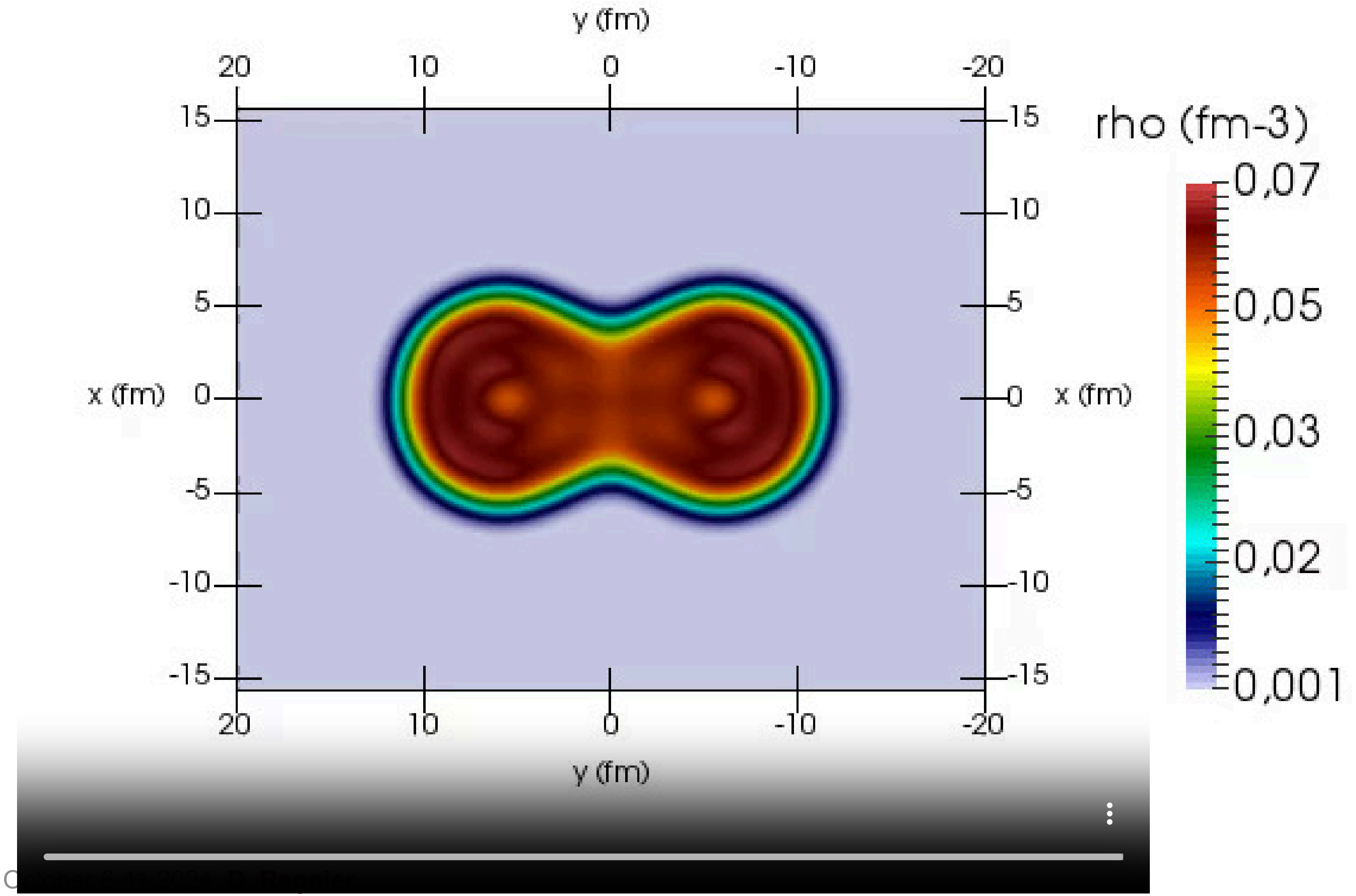




Fission of a ^{258}Fm

It goes through scission !

Time dependent mean-field approaches



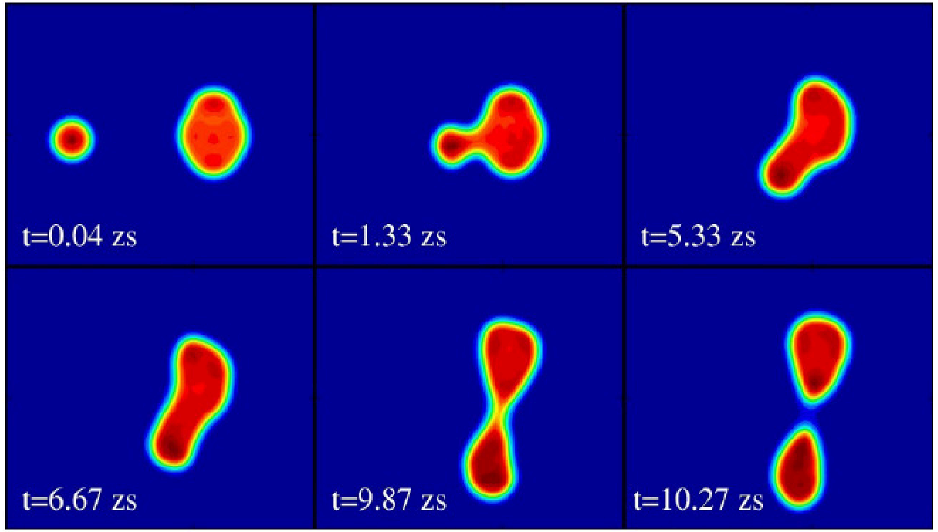
Time dependent mean-field approaches

A game changer in the last decade...

Inclusion of pairing

2014: Fission of ^{258}Fm, ^{264}Fm (no pairing)
C. Simenel *et al.*, PRC **89** (2014)
2015: ^{258}Fm with pairing (TDBCS)
G. Scamps *et al.*, PRC **92** (2015)
2016: ^{240}Pu with pairing (full TDHFB)
A. Bulgac *et al.*, PRL **11** (2016)

Simulation without any symmetry

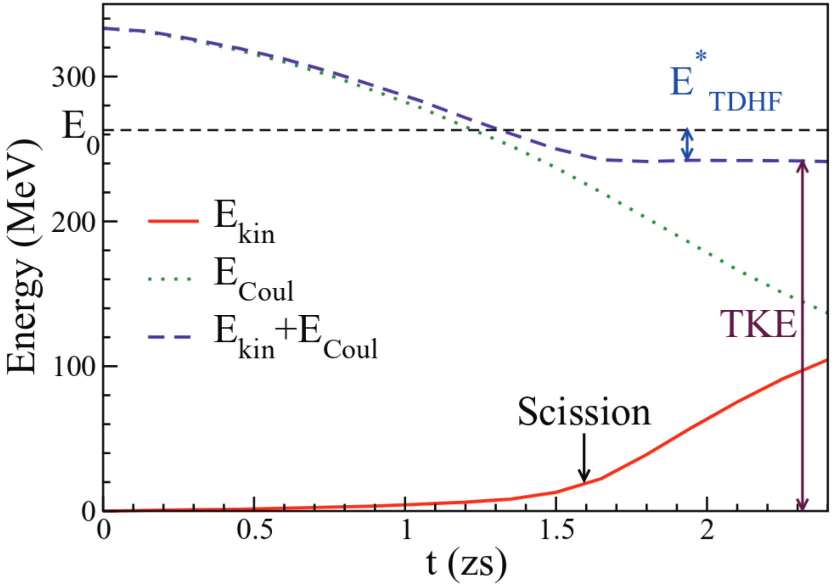


Oberacker *et al.*, Phys. Rev. C **90** (2014)

Leveraging GPUs

Time dependent mean-field approaches

Energy content



C. Simenel *et al.*, Phys. Rev. C (2014)

TKE^{syst}	TKE	A_L^{syst}	A_L	N_L^{syst}	N_L	Z_L^{syst}	Z_L	E_H^*	E_L^*
177.27	182	100.55	104.0	61.10	62.8	39.45	41.2	5.26	17.78
177.32	183	100.56	106.3	60.78	64.0	39.78	42.3	9.94	11.57
177.26	180	100.55	105.5	60.69	63.6	39.81	41.9	3.35	29.73
177.92	181		103.9		62.6		41.3	7.85	9.59

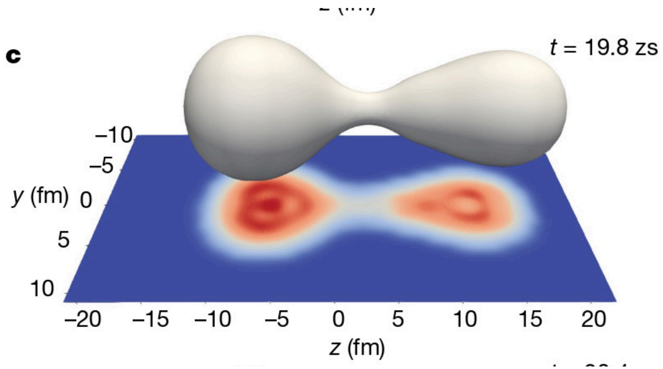
A. Bulgac *et al.*, Phys. Rev. Lett. (2016)

Angular momentum content

Hot topic

See G. Scamps talk...

Deformed shell effects



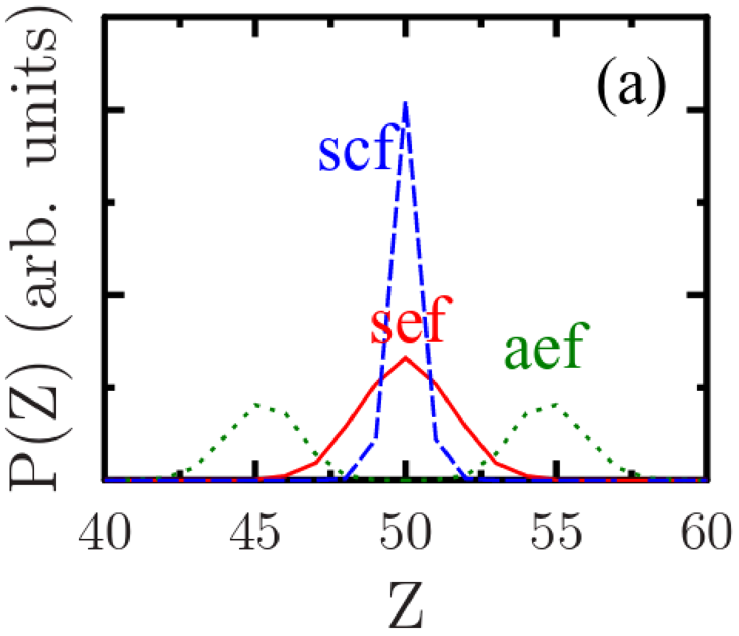
G. Scamps *et al.*, Nature Lett. (2018)

Vibration of the fragments

Time dependent mean-field approaches

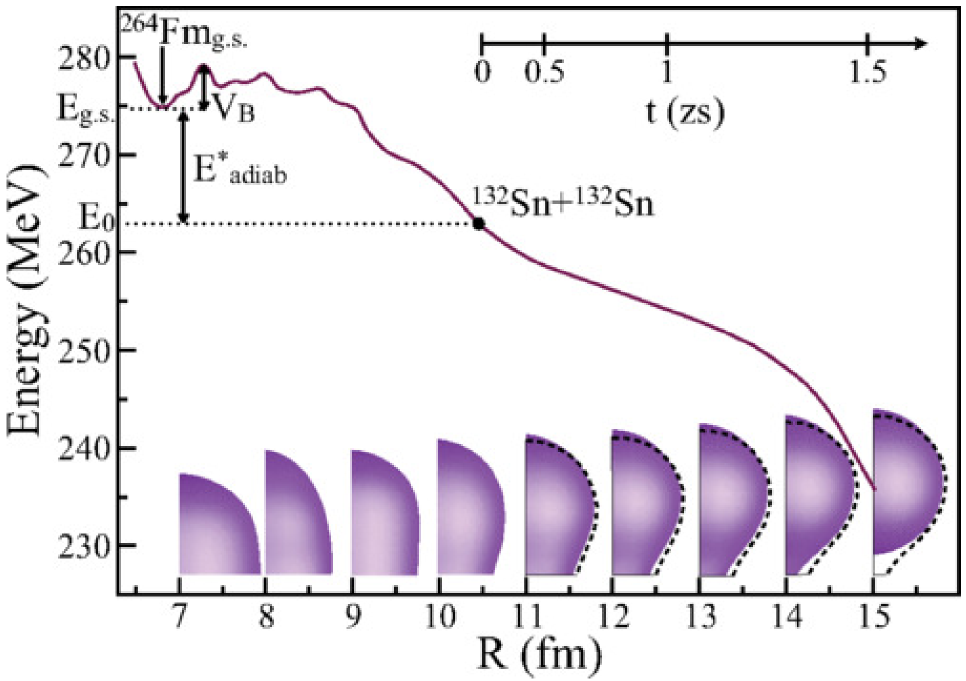
Limitations

Too sharp charge yields



Charge distribution in one fragment.
Three TDBCS simulations of ^{258}Fm fission
G. Scamps *et al.*, PRC **92** (2015)

No collective tunneling



C. Simenel *et al.*, PRC **89** (2014)

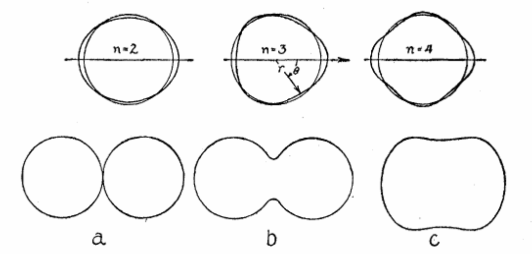
Wrapping up

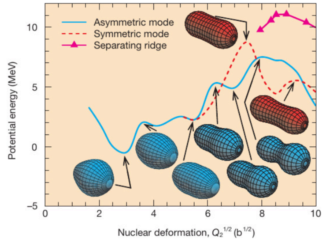
Formation of Primary Fission Fragments

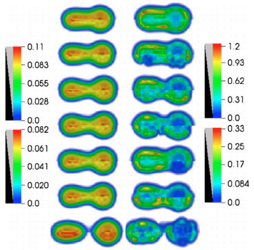
1940 · 1950 · 1960 · 1970 · 1980 · 1990 · 2000 · 2010 · 2020

- Liquid drop picture (1940)
- Explanation of asymmetric Y(A) from shell effects (1960s)
- Y(A) from scission point model (1970s)
- Axially- and reflection-symmetric TDHF dynamics (1980s)
- Y(A) from 2D Langevin dynamics (1980s–1990s)
- Y(Z) from 5D Brownian motion (2000s)
- Y(A) from TDGCM (2000s)
- Symmetry-unrestricted TDHF, TDBCS, TDHFB dynamics (2010s)

N. Schunck *et al.* Prog. Part. Nucl. Phys. **125** (2022)







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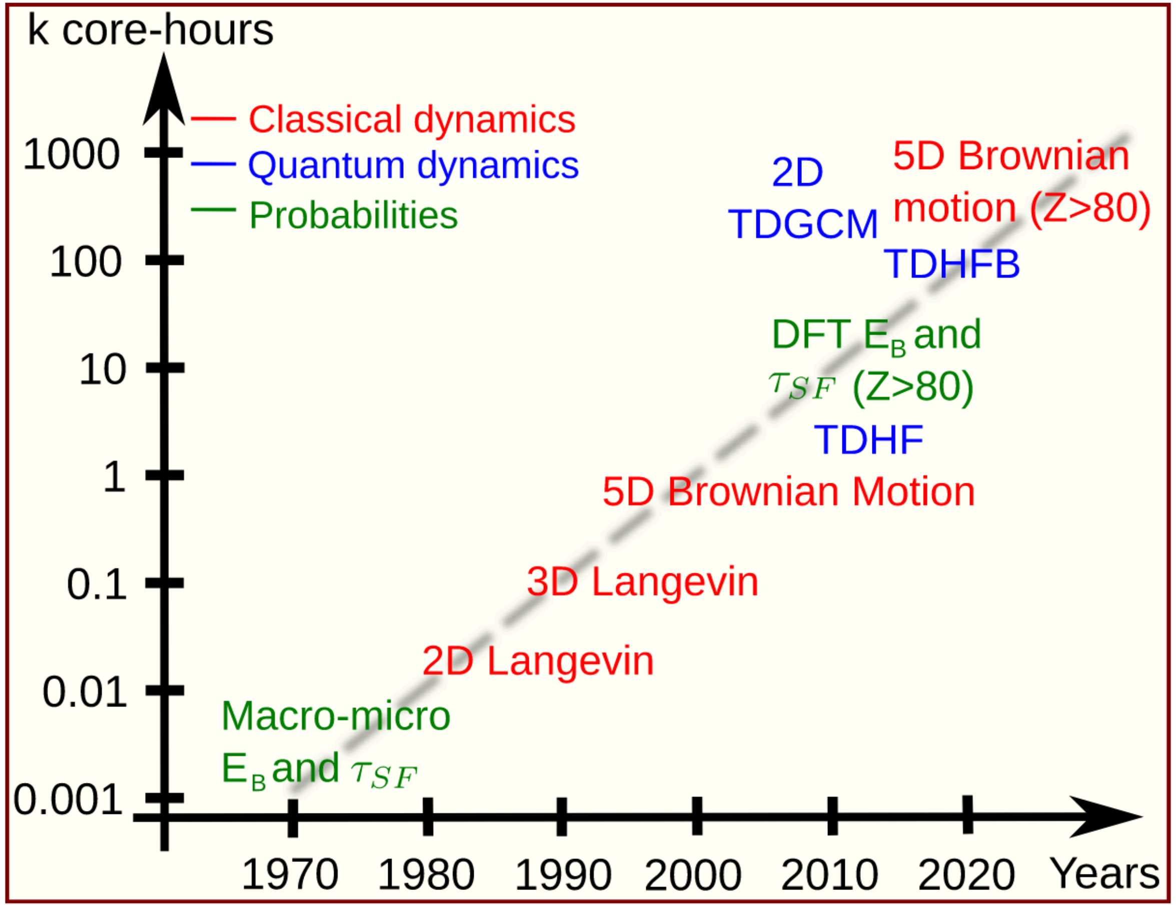
Moore's law in nuclear theory

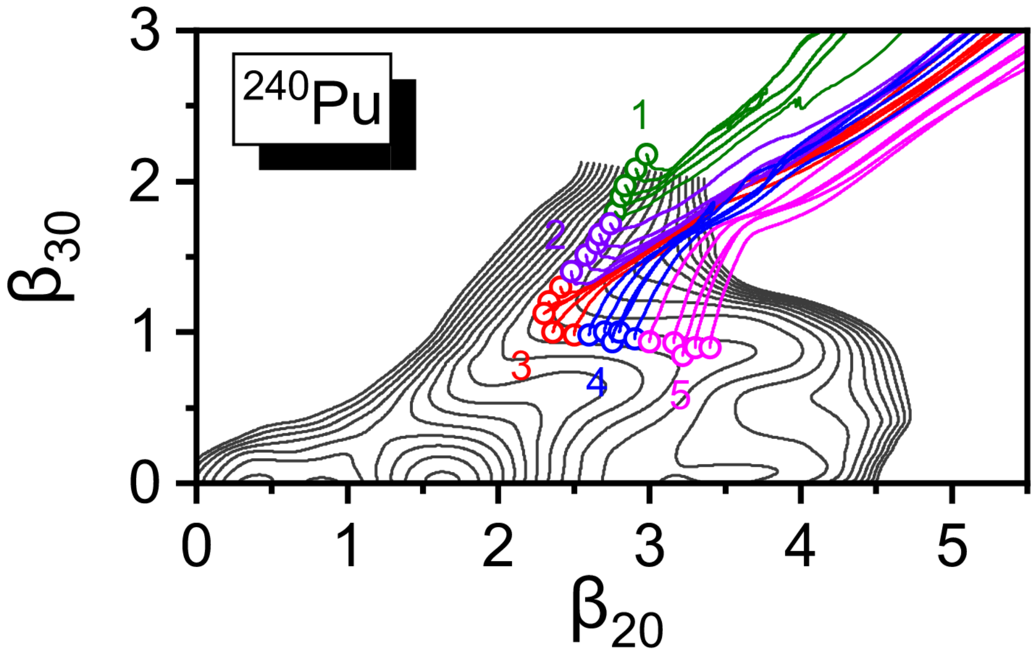


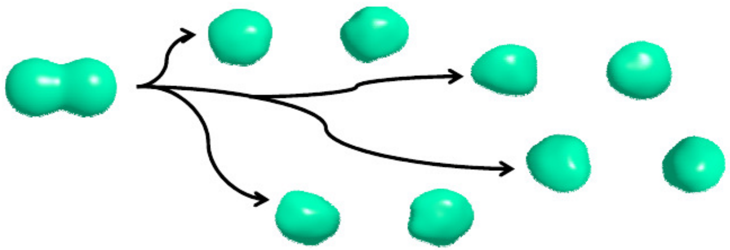
N. Schunck *et al.* Prog. Part. Nucl. Phys. **125** (2022)

Toward many time-dependent mean fields

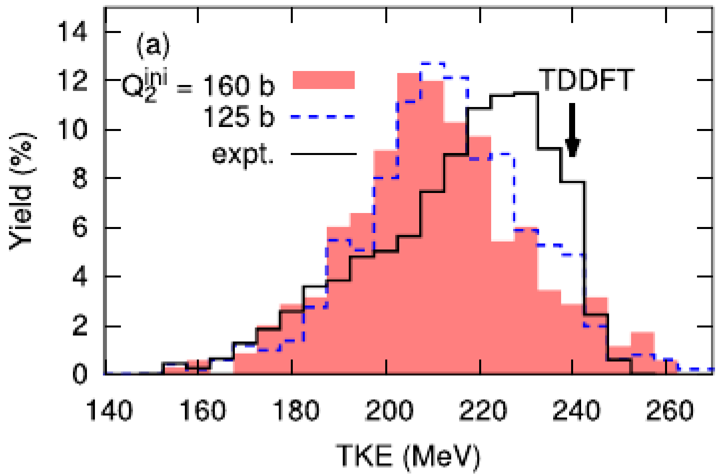
Stochastic mean field

Ensemble of mean-field





D. Lacroix *et al.* EPJA **50** (2014)



Y. Tanimura *et al.* PRL **118** (2017)

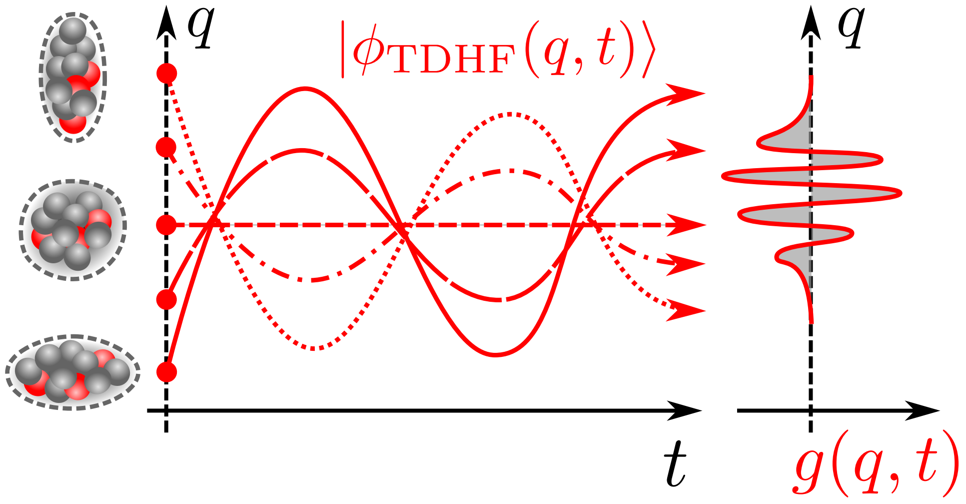
Toward larger variational spaces

Enhanced TDGCM

See N. Pillet talks

Quantum mixing TD mean field states

$$|\psi(t)\rangle = \int_q f(q,t)|\phi(q,t)\rangle$$

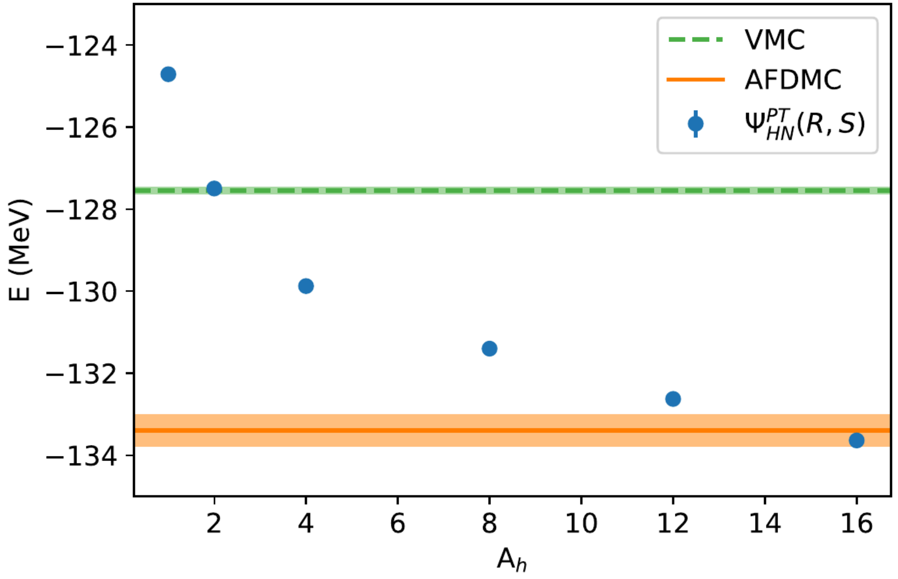


Multi-phonons in 40Ca

P. Marevic *et al.* arXiv:2304.07380 (2023)

Neural network wave functions

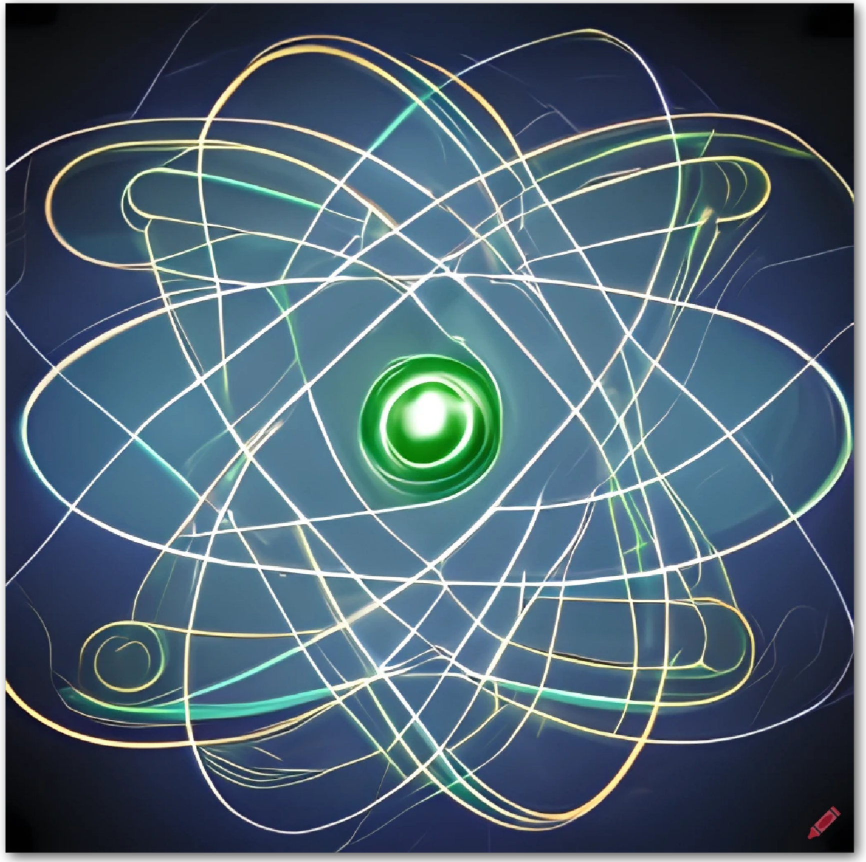
$$\psi(r_1\sigma_1, \cdots r_N\sigma_N) = \mathrm{NN}(r_1\sigma_1, \cdots r_N\sigma_N)$$



Ab-initio ground state energy of ^{16}O

A. Lovato *et al.* PRR **4** (2022)

Thank you for your attention !



CRAIYON: 'Atomic nucleus in the quantum realm'