Quantum computing for high-energy physics simulations

Mathieu PELLEN

University of Freiburg

LAPTh, Annecy, France

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Outline:

→ Introduction:
Why quantum computing and high-energy physics?

- Basics of high-energy physics
  → How to compute a cross section

- Basics of quantum computing
  → How to build a quantum circuit

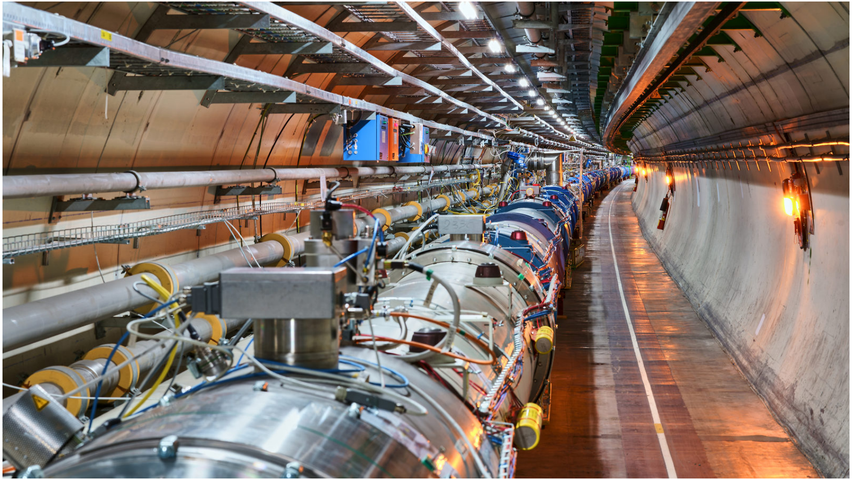
- Applications
  → Quantum integration **[Agliardi, Grossi, MP, Prati; 2201.01547]**
  → Colour amplitudes **[Chawdhry, MP; 2303.04818]**

[source: MatiasEnElMundo/Getty Images]

LHC @ CERN, Geneva (Switzerland)

→ 27km-long tunnel where protons are collided at high-energy (13.6 TeV currently)

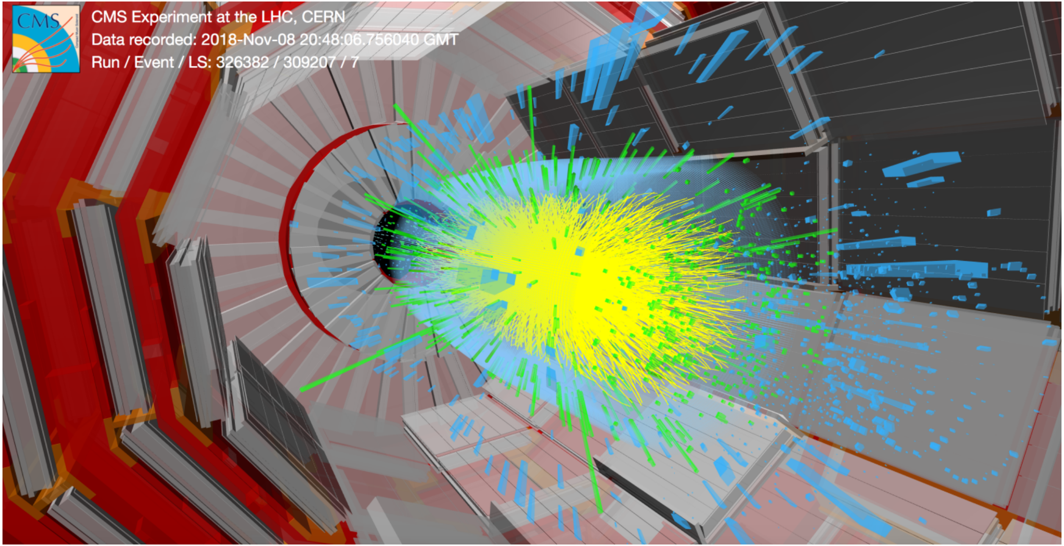
→ Allow to probe fundamental interactions



- Last discovery in high-energy physics (2012 @ LHC, CERN):

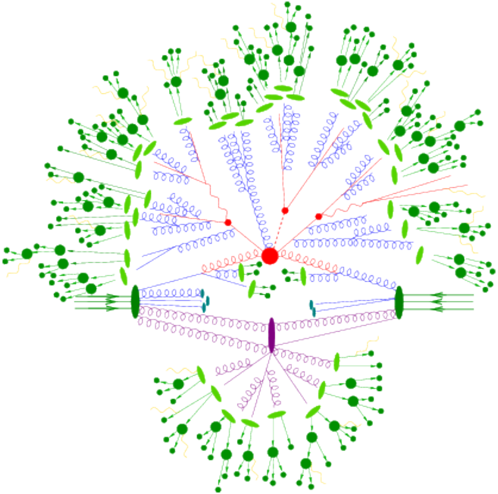
the Higgs boson!

Life at the LHC (in reality)



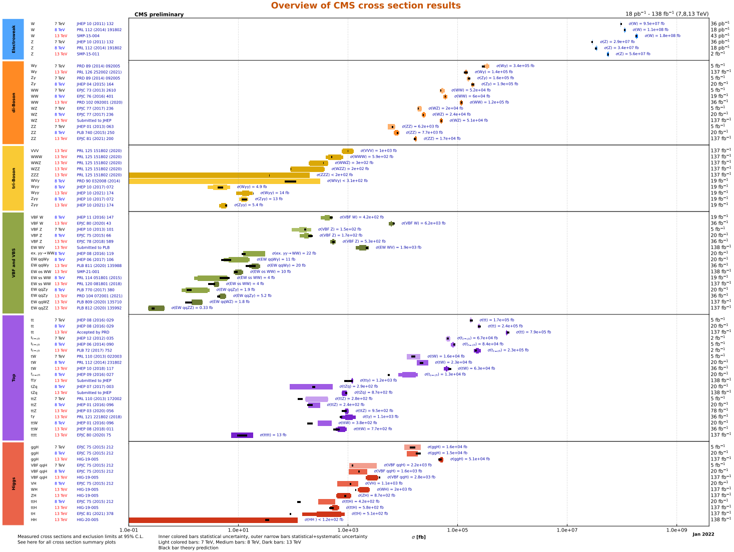
Life at the LHC (on the theory side)

- Factorisation of mechanisms at:
  - high-energy (perturbative)
    $\rightarrow$ Hard-scattering, parton-shower
  - low-energy (non-perturbative)
    $\rightarrow$ parton-distribution function, hadronisation, underlying events, multiple-parton interactions, ...
- All aspects relevant when comparing theory predictions against experimental measurements!
- Monte Carlo method/integration!



[source: Sherpa]

LHC = machine to measure cross sections ...



Computing problem in high-energy physics

$\longrightarrow$ Event generation:

$\sim$ 15% of $\sim$ 3 billion cpuh.y^{-1} for ATLAS

$\longrightarrow$ More in: [Buckley; 1908.00167], [Valassi et al.; 2004.13687]

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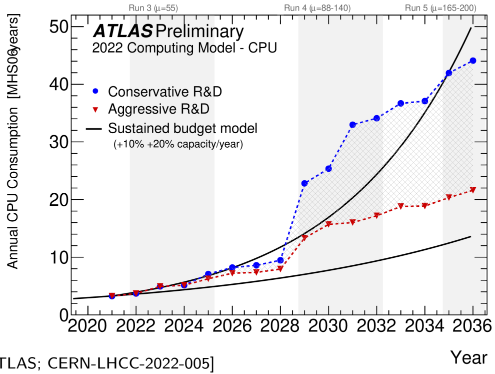

[ATLAS; CERN-LHCC-2022-005]

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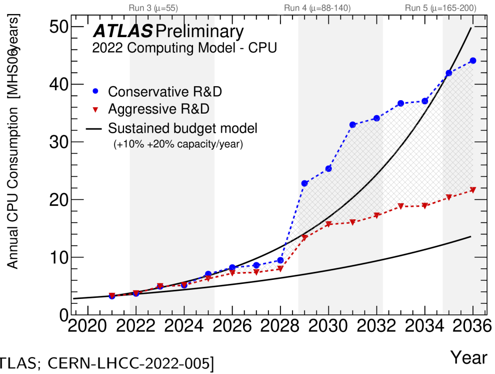

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- One possible solution: GPU

$\rightarrow$ Selected references: [Borowka et al.; 1811.11720], [Carrazza et al.; 2002.12921, 2009.06635, 2106.10279], [Bothmann et al.; 2106.06507] + Talk1 + Talk2

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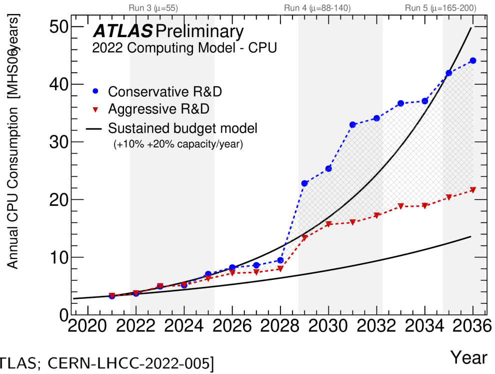

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- Can quantum computing be of any use in HEP?
  → to compute things faster/more efficiently?
  → to compute new things?

Reviews

- [Gray, Terashi; Gray:2022fou] (selected topics)
- [Delgado et al.; 2203.08805] (Snowmass)
- [Klco et al.; 2107.04769] (lattice)

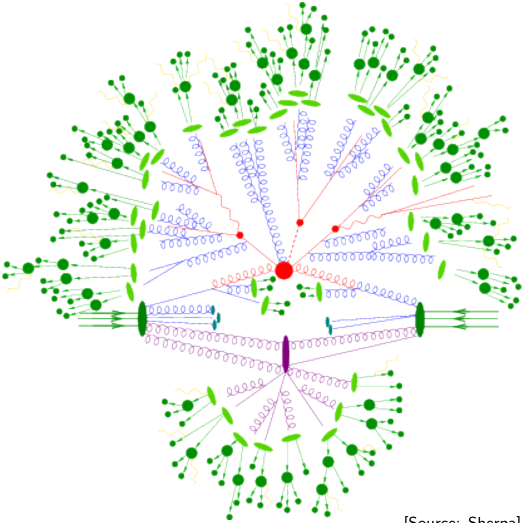
Selected references

- Amplitude/loop integrals: [Ramirez-Uribe et al.; 2105.08703], [Bepari, Malik, Spannowsky, Williams; 2010.00046], **[Chawdhry, MP; 2303.04818]**
- Parton shower: [Bauer, de Jong, Nachman, Provasoli; 1904.03196], [Bepari, Malik, Spannowsky, Williams; 2010.00046], [Williams, Malik, Spannowsky, Bepari; 2109.13975], [Chigusa, Yamazaki; 2204.12500], [Gustafson, Prestel, Spannowsky, Williams; 2207.10694], [Bauer, Chigusa, Yamazaki; 2310.19881]
- Machine learning: [Filipek et al; 2105.04582], [Bravo-Prieto et al; 2110.06933], [Alvi, Bauer, Nachman; 2206.08391] ...
- Others: [Ciavarella; 2007.04447], [Perez-Salinas, Cruz-Martinez, Alhajri, Carrazza: 2011.13934], [Bauer, Freytsis, Nachman; 2102.05044], [Martenez de Lejarza, Cieri, Rodrigo; 2204.06496], **[Agliardi, Grossi, MP, Prati; 2201.01547]**, [Martenez de Lejarza et al.; 2401.03023], [Cruz-Martinez, Robbiati, Carrazza; 2308.05657]

Quantum applications in high-energy physics

→ Quantum applications still in their infancy!

- Is it possible?



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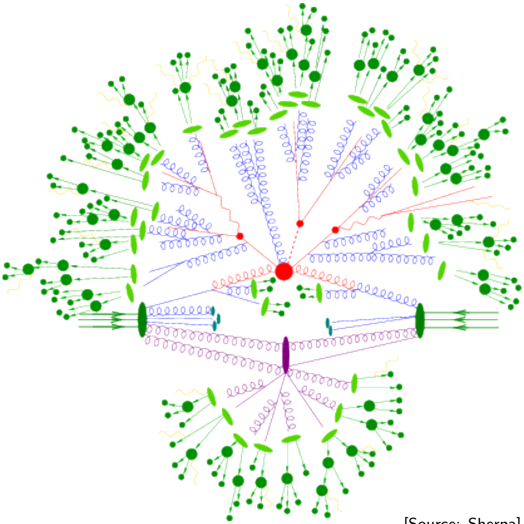
- Is it possible?

- Is there a (theoretical) quantum advantage?



[Source: Sherpa]

Quantum applications in high-energy physics

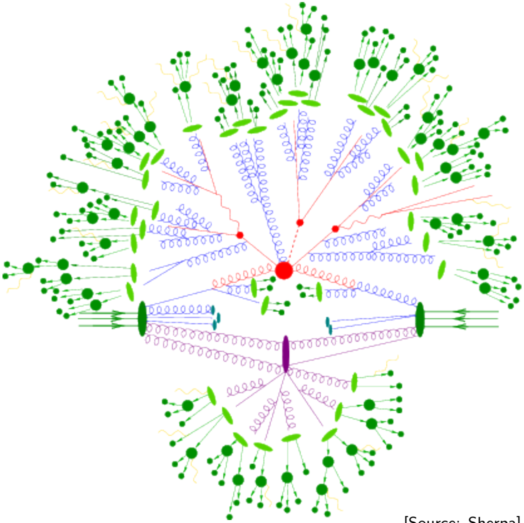
→ Quantum applications still in their infancy!

- Is it possible?

- Is there a (theoretical) quantum advantage?

- Is it more resource efficient than CPU/GPU?

→ Look at the example of quantum simulation/integration in high-energy physics!



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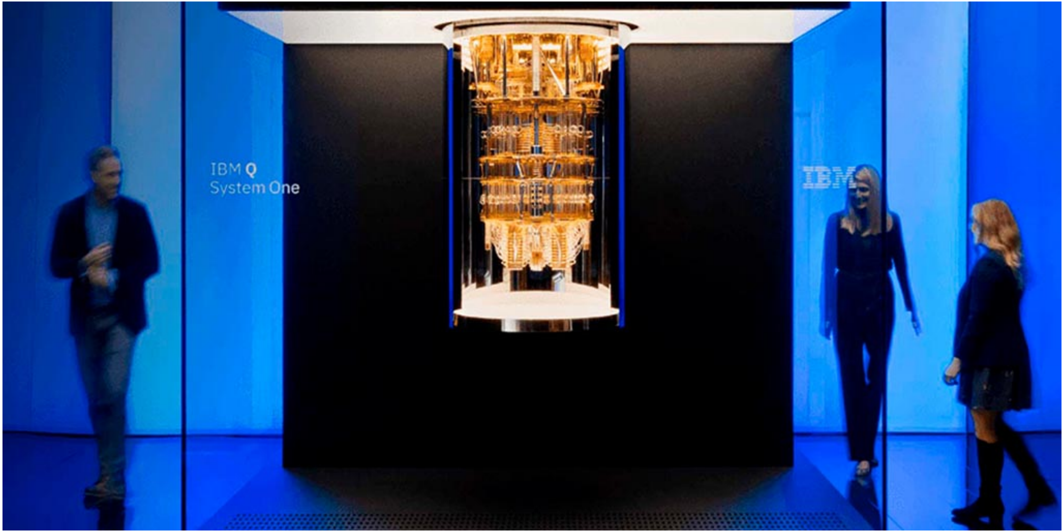
Quantum computers



[IBM]

[Landscape with the worship of the Golden calf, Claude Lorrain, Staatliche Kunsthalle, Karlsruhe (Germany)]

- Basics of high-energy physics
  → How to compute a cross section

Cross section (definition)

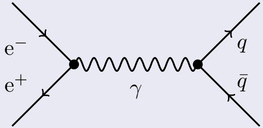
$\rightarrow$ Probability to measure a scattering process
$\rightarrow$ Predictable theoretically and measurable experimentally!

$$\sigma \propto \int \mathrm{d}\Phi |\mathcal{M}|^2$$

- $\mathrm{d}\Phi$: phase-space, depends on final-state particles
  $\rightarrow$ encodes kinematics
- $|\mathcal{M}|^2$: matrix element of the scattering process
  $\rightarrow$ encodes underlying theory and depends on kinematics

Cross section [example] (I)

$$e^+(p_1)\, e^-(p_2) \to q(k_1)\, \bar{q}(k_2) \quad \text{[QED]}$$



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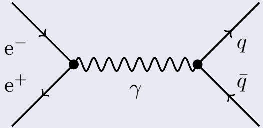
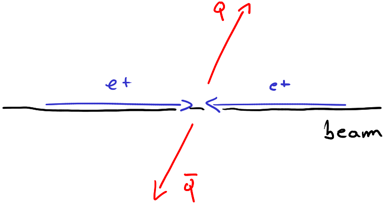


- $\mathrm{d}\Phi \propto \mathrm{d}\phi\,\mathrm{d}\cos\theta$
- $|\mathcal{M}|^2 \propto 1 + \cos^2\theta$

$$\Rightarrow \quad \sigma_{\mathrm{incl}} \propto \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{+1} \mathrm{d}\cos\theta\,(1+\cos^2\theta)$$



→ No dependence on ϕ
(symmetry around z axis)

→ inclusive cross section
(integration over whole phase space)

Cross section [example] (II)

- Experimental measurements performed in parts of the phase space
- Experiments also measure observables
  $\rightarrow$ angles, transverse momentum
- Analytical integration not an option!

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- Experimental measurements performed in parts of the phase space
- Experiments also measure observables
  → angles, transverse momentum
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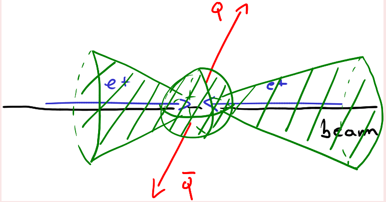
⇒ Solution: **Monte Carlo integration!**

→ $\phi = 2\pi \cdot x_1, \ \cos\theta = 2 \cdot x_2 - 1$

$$\Rightarrow \ \sigma \propto 2\pi \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \, (1+(2x_2-1)^2) \, \Theta(g(x_1,x_2))$$



- $\Theta(g(x_1,x_2))$ encodes experimental selection
- Error scaling as $1/\sqrt{N_{\text{points}}}$

Cross section (Monte Carlo)

→ In general:

$$\sigma \propto \int_0^1 \mathrm{d}x_1 \cdots \int_0^1 \mathrm{d}x_n \, f(x_1, \cdots, x_n) \, \Theta(g(x_1, \cdots, x_n))$$

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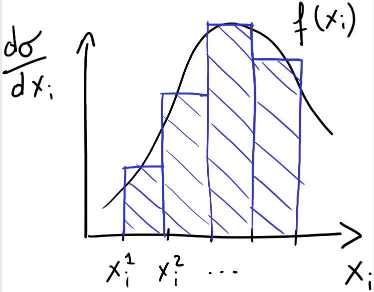
→ In general:

$$\sigma \propto \int_0^1 \mathrm{d}x_1 \cdots \int_0^1 \mathrm{d}x_n \, f(x_1, \cdots, x_n) \, \Theta(g(x_1, \cdots, x_n))$$

- For a given observable $\mathcal{O} = \mathcal{O}(x_1, \cdots, x_n)$:

$$\sigma = \sum_i \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}^i} = \sum_{i,l} c_{il} \frac{\mathrm{d}\sigma}{\mathrm{d}x_l^i}$$

- **Integrating = guessing the values of a function at specific points (Riemann sum)**
- More complex calculations ...
  ... more integration variables ...
  ... more computing resources!

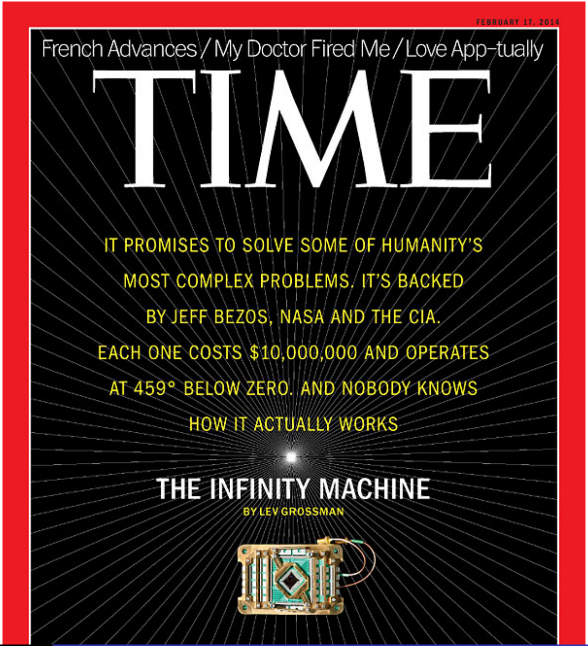


- Basics of quantum computing

  $\rightarrow$ How to build a quantum circuit

<u>Literature:</u>

- *Quantum Computation and Quantum Information*, Nielsen and Chuang
- *Programming Quantum Computers*, Johnston, Harrigan, and Gimeno-Segovia


FEBRUARY 17, 2014
French Advances / My Doctor Fired Me / Love App-tually
TIME
IT PROMISES TO SOLVE SOME OF HUMANITY'S
MOST COMPLEX PROBLEMS. IT'S BACKED
BY JEFF BEZOS, NASA AND THE CIA.
EACH ONE COSTS $10,000,000 AND OPERATES
AT 459° BELOW ZERO. AND NOBODY KNOWS
HOW IT ACTUALLY WORKS
THE INFINITY MACHINE
BY LEV GROSSMAN

$\rightarrow$ Any state can be written as

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix},$$

with $\alpha^2 + \beta^2 = 1$

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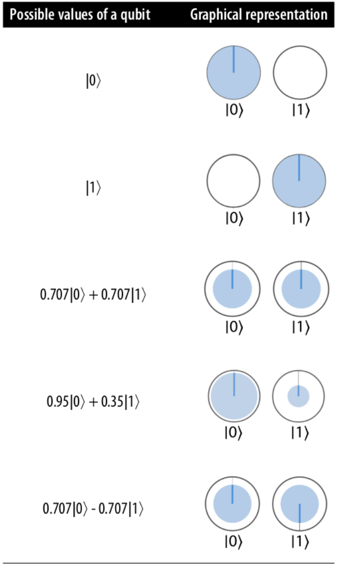
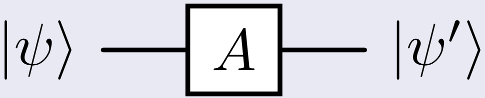
→ Representation



→ In quantum mechanics (and in quantum computing), any operation A is unitary

$$\psi \xrightarrow{A} \psi'$$

$$A\,|\psi\rangle = |\psi'\rangle \;\text{ with } A^*A = AA^* = \mathrm{Id}$$



Example of gates (I)

Pauli-X (X)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi' = \alpha |1\rangle + \beta |0\rangle$$

Example of gates (I)

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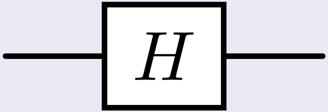
Hadamard (H)

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

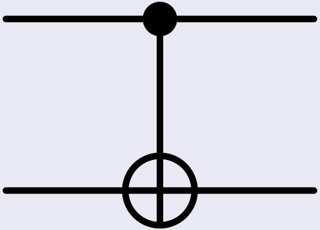


$$\psi = \alpha\,|0\rangle + \beta|1\rangle \rightarrow \psi' = \alpha\frac{|0\rangle + |1\rangle}{\sqrt{2}} + \beta\frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Example of gates (II)

Controlled not (CNOT, CX)

$$CX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$|00\rangle \to |00\rangle \,;\; |01\rangle \to |01\rangle \,;$$
$$|10\rangle \to |11\rangle \,;\; |11\rangle \to |10\rangle \,.$$

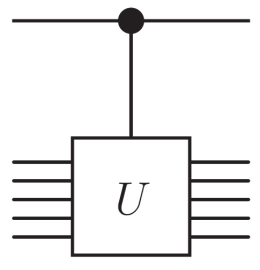
- If 0 $\rightarrow$ nothing happens; If 1 $\rightarrow$ Pauli-X (X)!
- *Control* qubit (top) and *target* qubit (bottom)

Example of gates (III)

Generalised controlled gate (CU)

$$CU = \begin{bmatrix} \mathrm{Id}_2 & 0 \\ 0 & U \end{bmatrix}$$

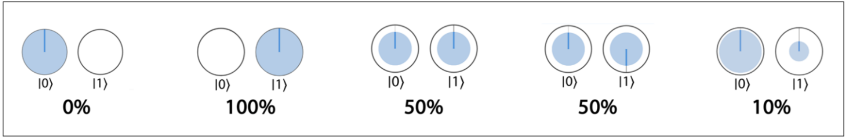
→ One *control* qubit and ...
... many *target* qubits



Measurement process

→ State: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

→ Probabilities: $\begin{cases} \text{for} \quad |0\rangle : \alpha^2 \\ \text{for} \quad |1\rangle : \beta^2 \end{cases}$



NB: Probability obtained after many measurements!

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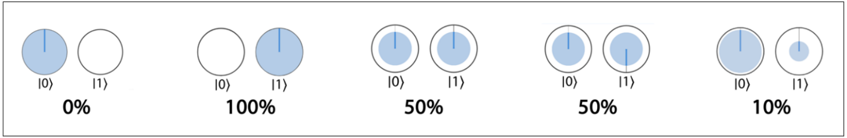


<u>NB:</u> Probability obtained after many measurements!

→ α and β encode information that can be measured/computed!

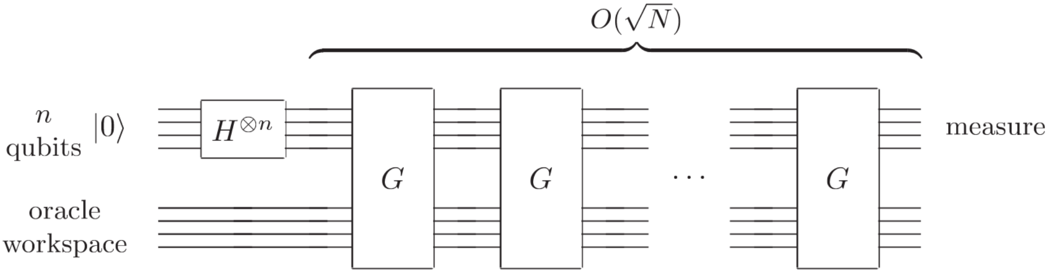
- Applications

  → **Quantum integration**
  [Agliardi, Grossi, MP, Prati; 2201.01547]

  → Colour amplitudes

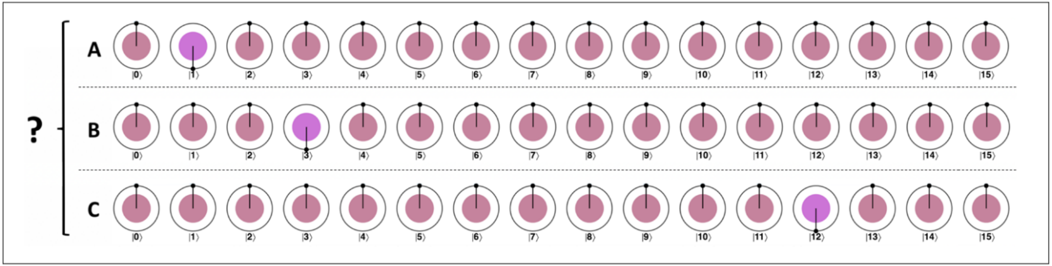
Grover algorithm/iteration

- Very general quantum algorithm
- Quadratic speed up
  $\rightarrow \mathcal{O}(\sqrt{N})$ operations instead of $\mathcal{O}(N)$
- Most famous example: unstructured database search



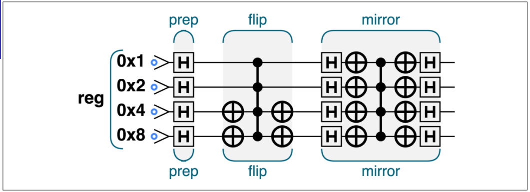
Grover algorithm/iteration

- Example (from [Johnston, Harrigan, Gimeno-Segovia; Programming Quantum Computers])

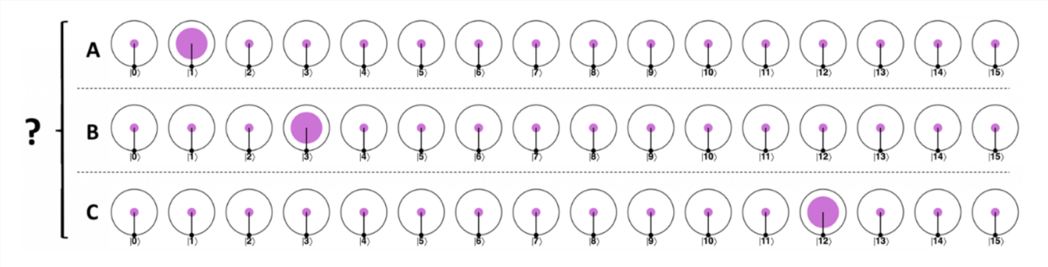


→ What solution is contained in our quantum register?

Grover algorithm/iteration



→ Applying a Grover iteration



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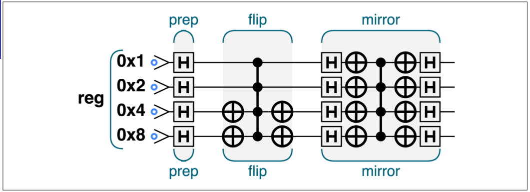

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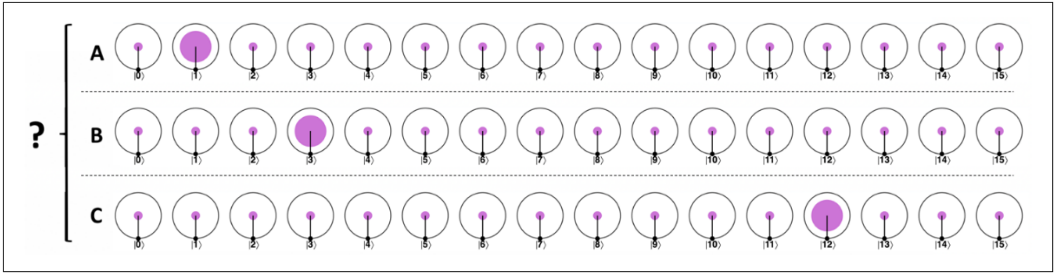
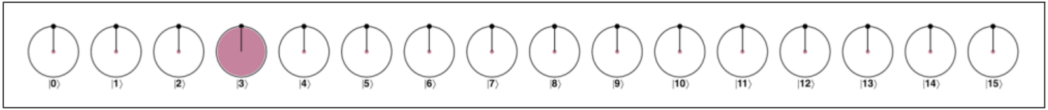


→ Applying it twice (*e.g.* solution B)



Quantum Amplitude Estimate (QAE)

[Brassard, Hoyer, Mosca, Tapp; Quantum Amplitude Amplification and Estimation; quant-ph/0005055]

$$\mathcal{A}|0\rangle = \sqrt{1-a}|\Psi_0\rangle + \sqrt{a}|\Psi_1\rangle$$

QAE estimates a with high probability such that the estimation error scales as $\mathcal{O}(1/M)$ [as opposed to $\mathcal{O}(1/\sqrt{M})$]

M: number of applications of $\mathcal{A}$

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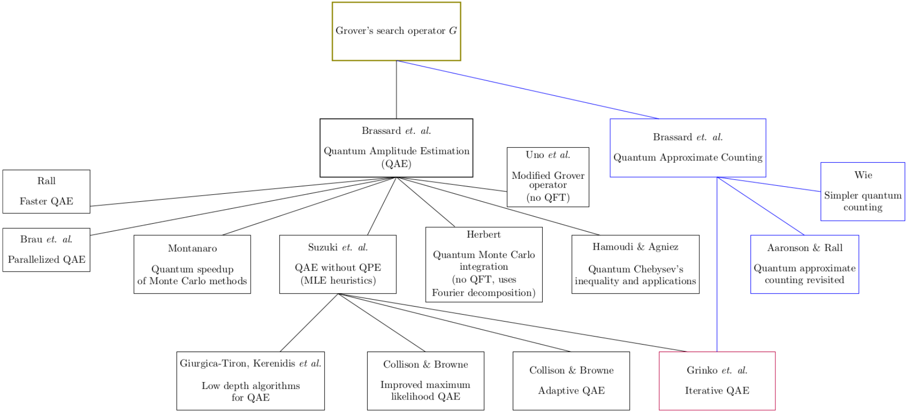
M: number of applications of $\mathcal{A}$

→ What the (orignal) algorithm provides:

- An estimate: $\tilde{a} = \sin^2(\tilde{\theta}_a)$
  with $\tilde{\theta}_a = y\pi/M$, $y \in \{0, ..., M-1\}$, and $M = 2^n$
- A success probability (that can be increased by repeating the algorithm)
- A bound: $|a - \tilde{a}| \leq \mathcal{O}(1/M)$

Quantum Amplitude Estimate (QAE)

→ Basis of quantum Monte Carlo integration and $\mathcal{O}(1/M)$ scaling
→ Various algorithms/implementations available



[Intallura et al.; 2303.04945]

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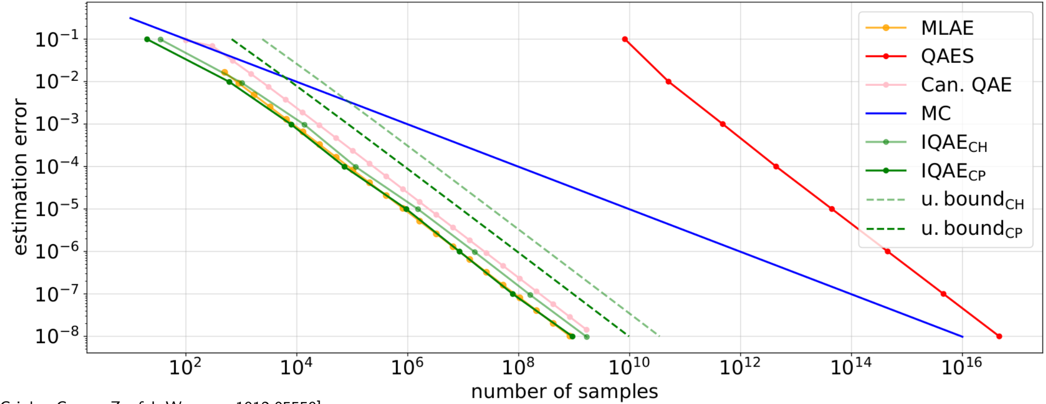

[Grinko, Gacon, Zoufal, Woerner; 1912.05559]

Resulting estimation error for $a = 1/2$ and 95% confidence level with respect to the required total number of oracle queries.

Quantum integration

Extension to

$$\mathcal{A}|0\rangle = \sum_i a_i |\Psi_i\rangle$$

$\rightarrow$ Definition of a piece-wise function with $f(x_i) = a_i$.

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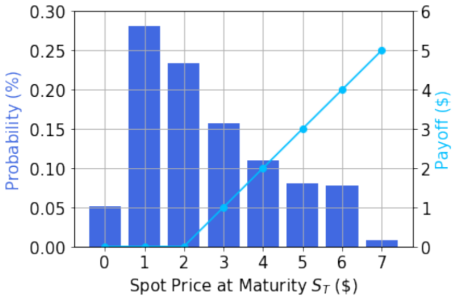
So far used in finance for simple functions in 1D [Zoufal, Lucchi, Woerner; 1904.00043] [Woerner and Egger; 1806.06893], [Stamatopoulos et al.; 1905.02666, 2111.12509], [Rebentrost, Gupt, Bromley; Phys.Rev.A 98 (2018) 022321]

→ Applicable to HEP? What are the limitations?



$$I = \int \mathrm{d}x\, f(x) g(x)$$

Quantum integration

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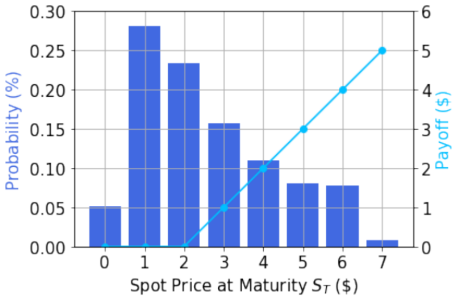
So far used in finance for simple functions in 1D [Zoufal, Lucchi, Woerner; 1904.00043] [Woerner and Egger; 1806.06893], [Stamatopoulos et al.; 1905.02666, 2111.12509], [Rebentrost, Gupt, Bromley; Phys.Rev.A 98 (2018) 022321]

→ Applicable to HEP? What are the limitations?



$$I = \int \mathrm{d}x\, f(x) g(x)$$

- In finance:
  - f: probability
  - g: payoff
- In HEP:
  - f: $|\mathcal{M}|^2$
  - g: $\Theta(\Phi - \Phi_c)$

Applications

- $\mathbf{e^+e^- \to q\bar{q}}$ (in QED) $\to$ 1D problem

$$\sigma \sim \int_{-1}^{1}\int_{0}^{2\pi} \mathrm{d}\cos\theta \mathrm{d}\phi \left(1+\cos^2\theta\right)$$

- $\mathbf{e^+e^- \to q\bar{q}'W}$ $\to$ 2D problem

$$\begin{aligned}\sigma &\sim \int_{M_\mathrm{W}^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}}\int_{-1}^{1}\int_{0}^{2\pi}\int_{0}^{2\pi} \mathrm{d}\Phi_3 \left|\mathcal{M}_{\mathrm{e^+e^-}\to q\bar{q}'\mathrm{W}}\right|^2 \\ &\sim \int_{M_\mathrm{W}^2}^{s}\int_{0}^{s_1^{\mathrm{Max}}} \mathrm{d}\tilde{\Phi}_3 \left|\mathcal{M}'\right|^2\end{aligned}$$

with $\mathcal{M}' = \mathcal{M}_{\mathrm{e^+e^-}\to q\bar{q}'\mathrm{W}}\left(\cos\theta_1 = 0,\ \phi_1 = \pi/2,\ \phi_2 = \pi/2\right)$.

Loading of distribution / encoding into qubits

Encoding the distribution to be integrated into qubits (coefficients of states)

- Exact loading [Shende, Bullock, Markov, quant-ph/0406176] (resource intensive)
- Using quantum machine learning (qGAN) [Zoufal, Lucchi, Woerner; 1904.00043] (not exact)

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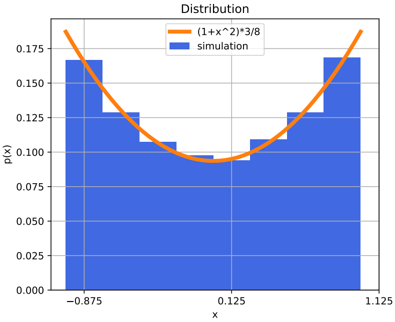
- Example: Exact loading - $1 + x^2$



$\rightarrow$ 3 qubits: $2^3 = 8$ bins

Integration - $1 + x^2$

- Matching boundary of integration (3 qubits $\Rightarrow 2^3$ bins)

Domain	low stat.		high stat.		very high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
$[-0.75; 0]$	0.345	−3.31	0.332	0.706	0.334	0.0331	0.334	-8.31×10^{-3}
$[-0.5; 0]$	0.215	−5.86	0.201	1.15	0.203	0.0986	0.203	−0.0161
$[-0.25; 0]$	0.112	−17.1	0.0939	1.87	0.0960	−0.284	0.0957	−0.0389

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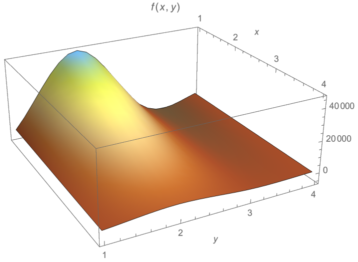
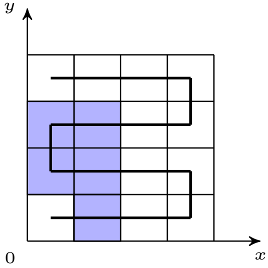
- Non-matching boundary of integration

Qubits number	$[-0.7; 0.6]$				$[-0.625; 0.375]$			
	high stat.		exact		high stat.		exact	
	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$	σ	$\delta[\%]$
3	0.402	-28.0	0.406	-27.1	0.296	-28.1	0.299	-27.5
4	0.463	-17.0	0.468	-16.0	0.408	-1.07	0.412	5.96×10^{-3}
5	0.527	-5.46	0.532	-4.62	0.408	-1.07	0.412	5.96×10^{-3}
6	0.542	-2.76	0.547	-1.81	0.408	-1.07	0.412	5.96×10^{-3}

Integration - 2D [$e^+e^- \to \mathbf{q\bar{q}}'W$ with angles fixed]





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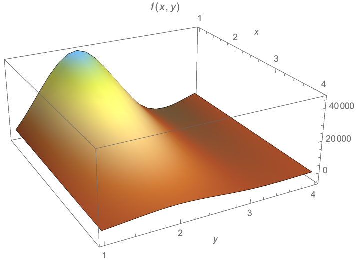

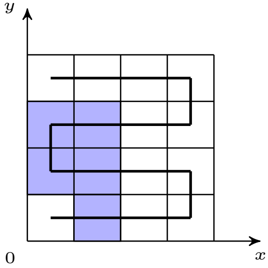

Qubits number	Grid dim.	$\mathcal{S}_1$ σ	$\mathcal{S}_1$ $\delta[\%]$	$\mathcal{S}_2$ σ	$\mathcal{S}_2$ $\delta[\%]$
4	4×4	0.55	0	0.70	-4.1
5	5×5	0.52	-4.92	0.53	-26.6
6	6×6	0.47	-14.1	0.79	9
6	7×7	0.62	-14.4	0.70	-3.0
6	8×8	0.55	0	0.78	7.6

$\mathcal{S}_1$: matching boundary of integration
$\mathcal{S}_2$: no matching boundary of integration

[Agliardi, Grossi, MP, Prati; 2201.01547]

Working but control of uncertainty crucial!

Remarks

- Use Qiskit (IBM python software) subroutines and noiseless quantum simulation (perfect quantum computer)
- For present application, too many qubits for test on real hardware
  $\rightarrow$ 4 qubits for representation $\rightarrow$ 9 total qubits
  $\rightarrow$ 6 qubits for representation $\rightarrow$ 13 total qubits
- Largest quantum computer on IBM quantum experience:
  7 qubits previously (127 qubits now)
  $\rightarrow$ Simulators can go up to 5000 qubits

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Summary

- First application of quantum integration in HEP
- Theoretical quadratic speed-up
- Main challenge: error estimate

- Applications

  → Quantum integration

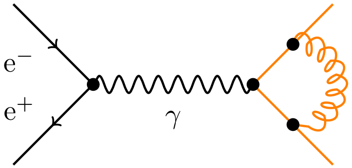
  → **Colour amplitudes**

  [Chawdhry, MP; 2303.04818]

Alternative to loading - [Chawdhry, MP; 2303.04818]

$$\mathcal{M} \sim \sum \mathrm{Tr}\left(T^{a_1} \ldots T^{a_n}\right) \mathcal{K}(1, \ldots, n)$$

- Kinematic part: made of spinors and tensors (and kinematic invariants)
- Colour part: made of SU(3) generators of QCD



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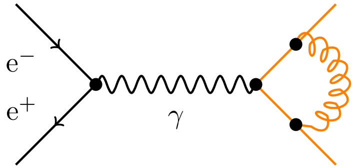

Remarks

- First step towards a full quantum amplitude/Monte Carlo
- Useful for a quantum parton shower

Colour algebra in QCD - crash course

- $[T^a, T^b] = \mathrm{i} f_{abc} T^c$.
- $T^a, T^c, \ldots$: SU(3) generators
- Gell-Mann matrices

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T^4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$T^5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T^6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T^7 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Attention! T^a are not unitary!

$\rightarrow$ In our example, colour factor: $T^a_{ij} T^a_{jk} = C_F \delta_{ik}$

Quantum implementation of colour

- Gluon: 8 colours $\rightarrow$ 3 qubits (2^3)
- Quark: 3 colours $\rightarrow$ 2 qubits (2^2)

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Make non-unitary matrices unitary again!

$\rightarrow$ extend dimension and modify them

$$\overline{T^1} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^2} = \frac{1}{2}\begin{pmatrix} 0 & -\mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^3} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \overline{T^4} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

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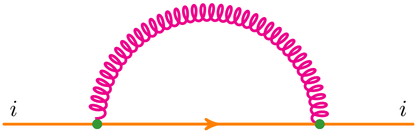
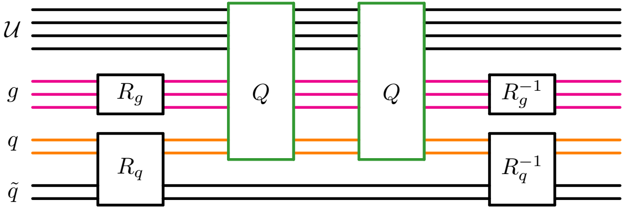
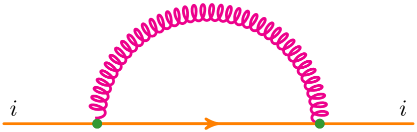
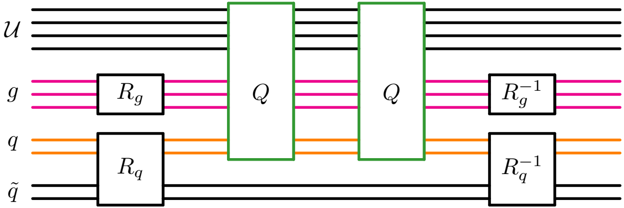
Example





- One-to-one correspondence between Feynman diagram and circuit
- Gates for qqg (Q) and ggg (G) vertices to simulate QCD (colour) interaction





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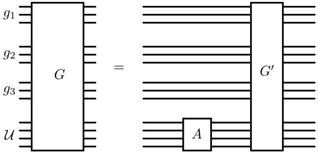

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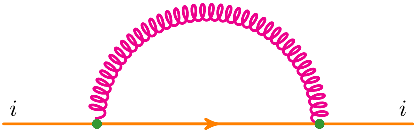

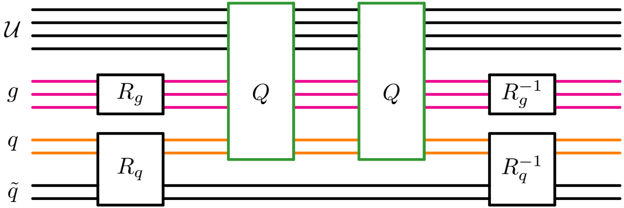

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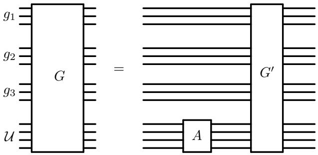

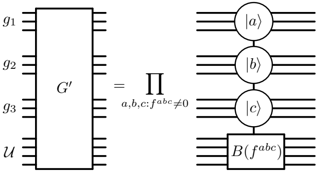

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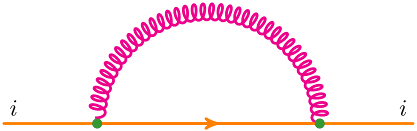
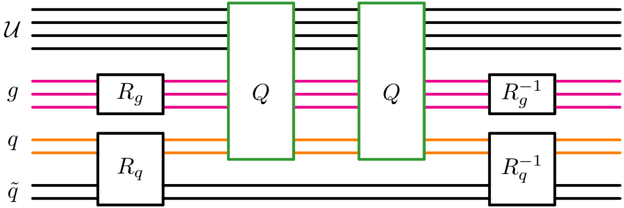
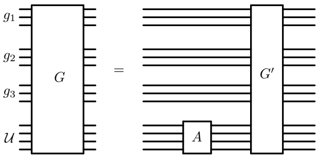
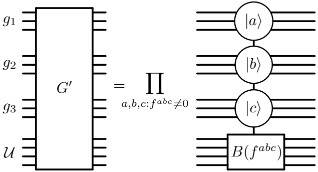
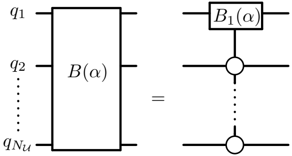




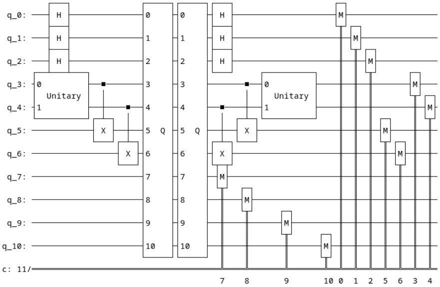
$$\langle\Omega|_{\mathcal{U}} \prod_{i=1}^{N_{ops}} \{B(\alpha_i)A\} |\Omega\rangle_{\mathcal{U}} = \prod_{i=1}^{N_{ops}} \alpha_i$$







(real-life) Example

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Total counts are: {'00000010101': 226, '00010010011': 342, '00000010110': 225, '00000001101': 696, '0001
0010010': 362, '00000010001': 872, '00010001101': 2006, '00000001100': 643, '00010001100': 2006, '000100
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000011': 6983, '00000010011': 815, '00010001011': 7957, '00010010001': 340, '00100010001': 10163, '00010
000001': 1092, '00100000010': 7010}
```

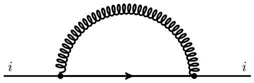
$\rightarrow$ Trace defined in $|00000000000\rangle = |0_{11}\rangle$ state: $|\psi\rangle = \frac{\mathcal{C}}{\mathcal{N}}|0_{11}\rangle + \ldots$

$$\Rightarrow \frac{27415}{N_{\text{shots}}=1000000} \sim \left(\frac{\mathcal{C}}{\mathcal{N}} = \frac{(N_c{=}3)\,C_F}{N_c^{n_q=1}(N_c^2-1)^{n_g=1}}\right)^2$$

$\rightarrow$ Colour factors encoded in one single state (as needed for QAE)

$\rightarrow$ Any colour factor computable

Outlook



- Colour factors as squared
  (done [Chawdhry, MP; 2303.04818])

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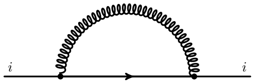

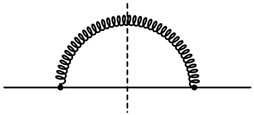

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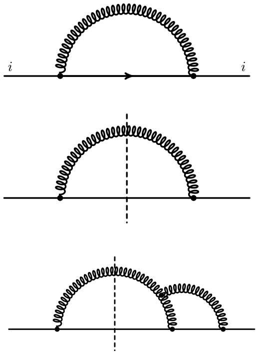
- Colour factor as amplitudes (done)

Outlook



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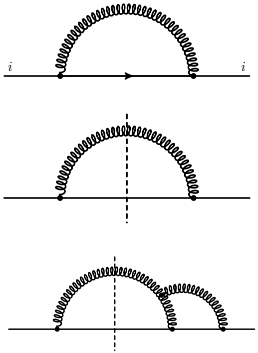

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- First building blocks for full amplitude
- Quantum advantage? (in addition to QAE)
  To compute colour factors for many gluons? (difficult problem)

Road map for quantum Monte Carlo

- Reliable error estimate
  $\rightarrow$ Taking into account binning effects / multi-dimension integrand

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  → On-going collaboration with QUANTINUUM (Cambridge, UK)
- Can there be quantum advantage for event generation? [Bravo-Prieto et al; 2110.06933]

Quantum Monte Carlo in HEP

- Is it possible?
  $\rightarrow$ **Yes.**

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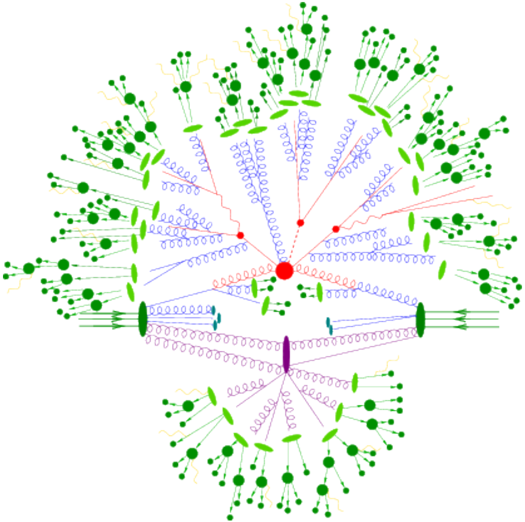
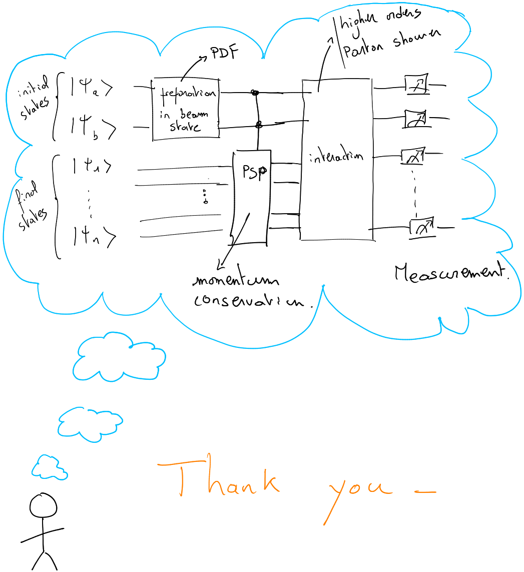
- Is it more resource efficient than CPU/GPU?
  $\rightarrow$ **At the moment, not known.**

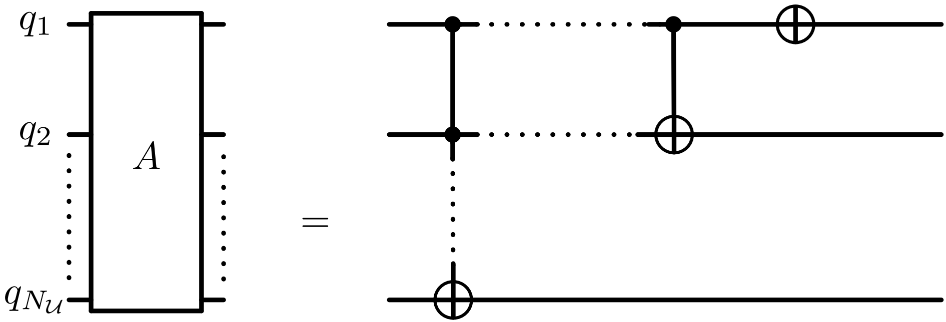
- Are we witnessing a quantum revolution?

- When can we do *useful* quantum computations in the future?




PDF
higher orders
Parton shower
initial states
$|\Psi_a\rangle$
$|\Psi_b\rangle$
preparation in beam state
final states
$|\Psi_1\rangle$
$|\Psi_n\rangle$
PSP
interaction
momentum conservation.
Measurement.
Thank you -

BACK-UP


q_1
q_2
$q_{N_\mathcal{U}}$
A
$=$

$$B_1(\alpha) = \begin{pmatrix} \sqrt{1-|\alpha|^2} & \alpha \\ -\alpha & \sqrt{1-|\alpha|^2} \end{pmatrix} \tag{1}$$

$$B(\alpha)A\,|k\rangle = \begin{cases} \alpha\,|0\rangle + \sqrt{1-|\alpha|^2}\,|1\rangle & \text{if } k=0 \\ |k+1\rangle & \text{if } 0<k<2^{N_\mathcal{U}}-1 \\ \sqrt{1-|\alpha|^2}\,|0\rangle - \alpha\,|1\rangle & \text{if } k=2^{N_\mathcal{U}}-1 \end{cases} \tag{2}$$

$$\langle\Omega|_\mathcal{U} B(\alpha)A|\Omega\rangle_\mathcal{U} = \alpha \tag{3}$$

$$\langle\Omega|_\mathcal{U} \prod_{i=1}^{N_{ops}} \{B(\alpha_i)A\}\,|\Omega\rangle_\mathcal{U} = \prod_{i=1}^{N_{ops}} \alpha_i \tag{4}$$