

# Top mass uncertainties in double Higgs production

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Frascati , April 15-17<sup>th</sup>, 2024

Based on: E. Bagnaschi, R. Groeber, G.D. EPJ C83, (2023), 11

See also: Bonciani, Giardino, Groeber, G.D. (18 ), Bellafrente et al. (22)



# Outline

- Status of the Higgs sector of the SM
- The trilinear Higgs self-coupling from Higgs pair production: a way to compute analytically the virtual NLO corrections.
- A new Monte Carlo code for Higgs pair production, flexible in the input parameters and in the choice of the top mass renormalizations scheme
- A study of the top mass scheme dependence with this new MC
- Conclusions

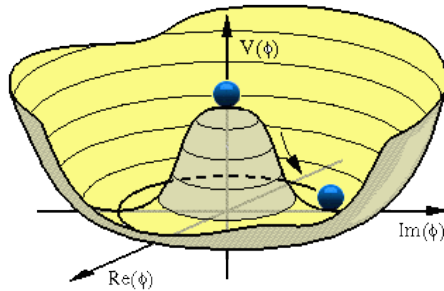
# The Higgs sector, what we know

$$\mathcal{L}_{Higgs} = (\lambda_{ij} \bar{\psi}_i \psi_j \phi + h.c.) + |D^\mu \phi|^2 - V(\phi)$$

EWSB:  $m_f, \bar{h}ff$

$m_{W,Z}, HVV, HHVV$

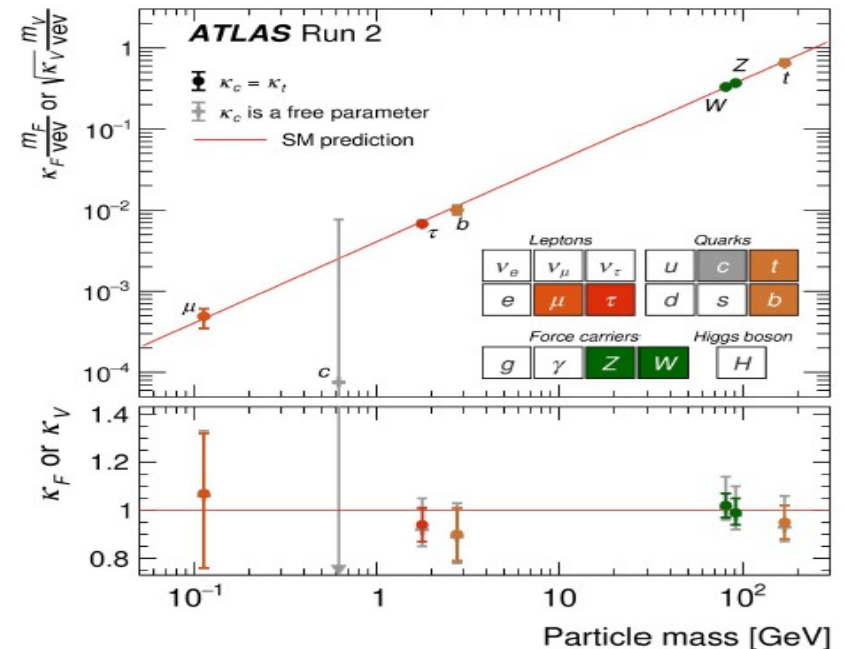
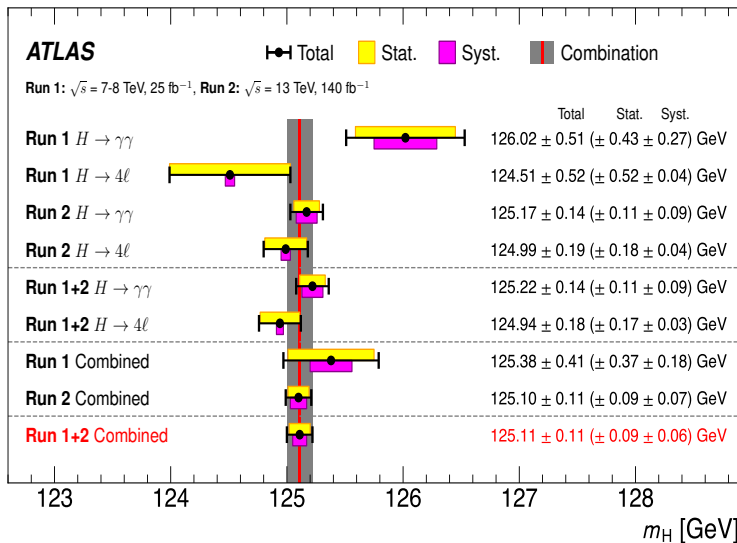
$m_H, HHH, HHHH, \dots$



The ground state of the potential known since long time

$$G_\mu = \frac{1}{2v^2}$$

$$v = \langle \phi^\dagger \phi \rangle^{1/2} \sim 246 \text{ GeV}$$



# Testing the shape of $V(H)$ : Higgs self couplings

$$V(H) = \lambda_2 v^2 H^2 + \lambda_3 v H^3 + \frac{1}{4} \lambda_4 H^4 + \dots$$

n-Higgs production probes (n+1)-Higgs self-coupling

single Higgs production  $\rightarrow \lambda_2 \equiv m_H^2 / (2 v^2)$

$$\sigma(pp \rightarrow H)_{SM} \sim 50 \text{ pb}$$

double Higgs production  $\rightarrow \lambda_3$

$$\sigma(pp \rightarrow HH)_{SM} \sim 30 \text{ fb}$$

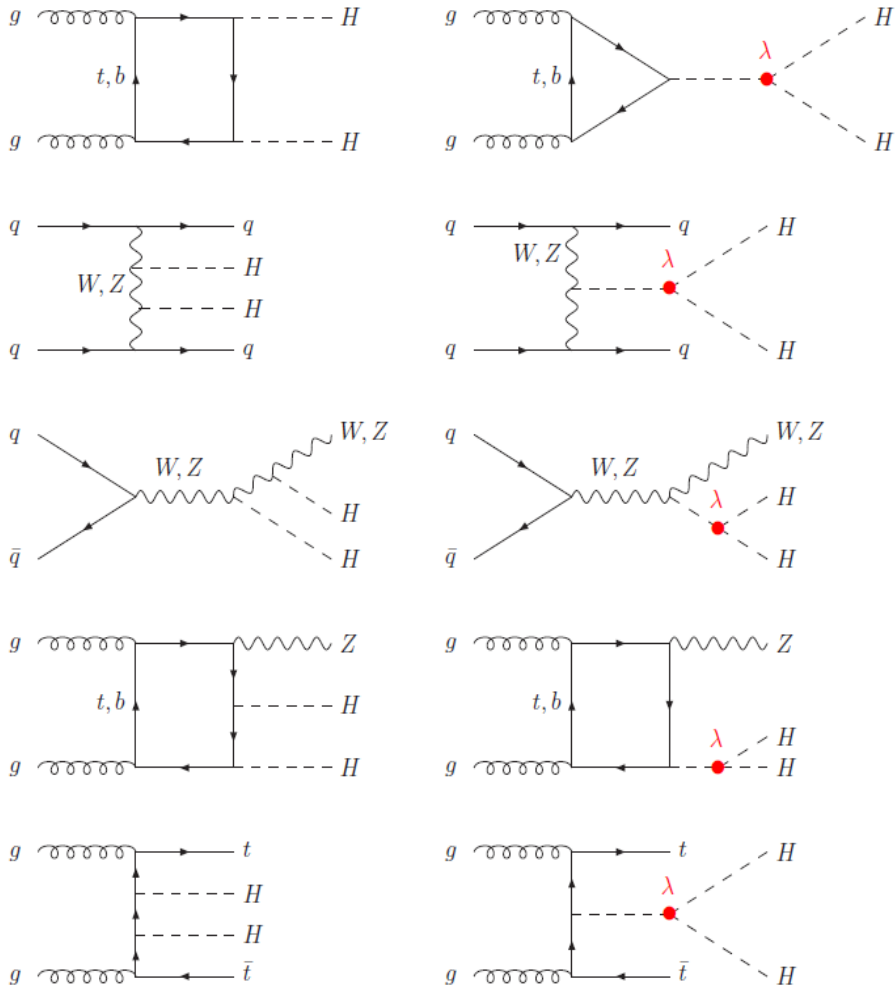
triple Higgs production  $\rightarrow \lambda_4$

$$\sigma(pp \rightarrow HHH)_{SM} \sim 0.1 \text{ fb}$$

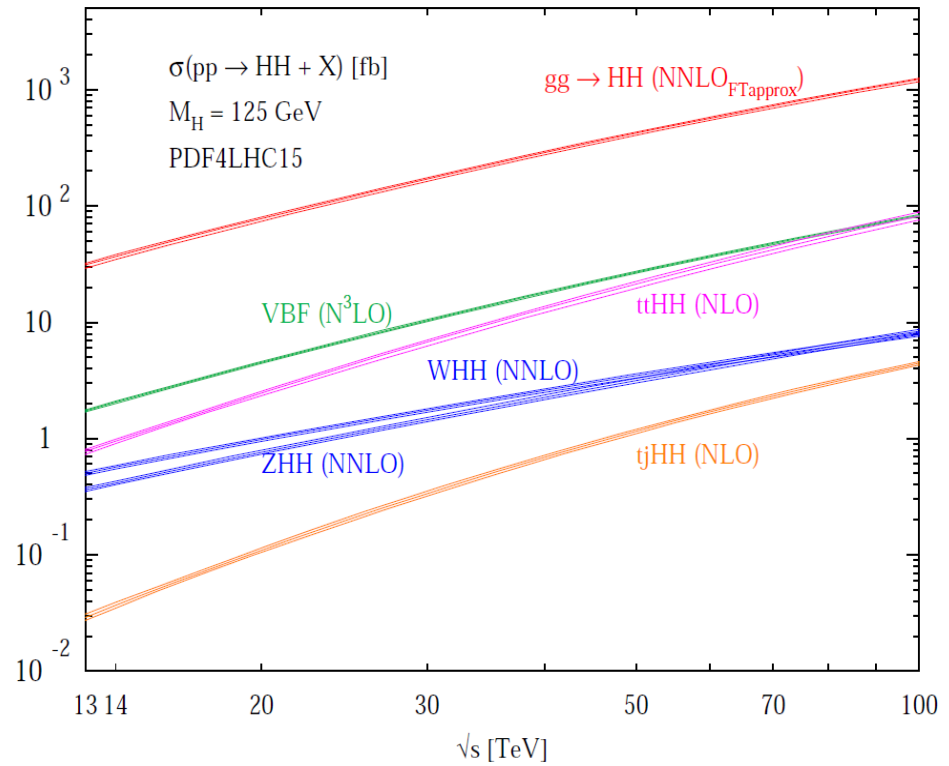
SM: at tree-level only  $\lambda_3$  and  $\lambda_4$ , fixed in terms of  $\lambda_2$

$$\lambda_3^{SM} = \lambda_4^{SM} = \lambda_2 = m_H^2 / (2v^2), \quad v = (\sqrt{2}G_\mu)^{-1/2}$$

# Probing $\lambda_3$ : double Higgs production @LHC



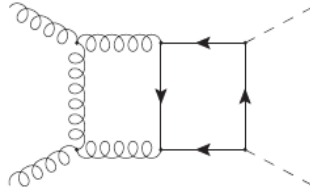
destructive interference between signal ( $\lambda$ ) and background diagrams



# Theoretical status of $gg \rightarrow hh$

LO: exact analytic [Glover, van der Bij \(88\)](#)

## NLO QCD:



3 mass scales:  $s/m_t^2, t/m_t^2, m_H^2/m_t^2$

### Approximate analytical:

- Born-improved HTL ( $m_t \rightarrow \infty$ )  
[Dawson, Dittmaier, Spira \(98\)](#)  
+  $(\dots/m_t^2)^n$  corrections  
[Grigo et al. \(13\)](#); [Giardino, Groeber, G.D. \(16\)](#)
- $FT_{\text{approx}}$ ,  $FT'_{\text{approx}}$  (real exact, virtual HTL)  
[Maltoni, Vryonidou, Zaro \(14\)](#)
- Pade' approximation using large  $m_t$  and threshold expansion  
[Groeber, Maier, Rauh \(18\)](#)
- High-energy (HE) expansion  
[Davies, Mishima, Steinhauser, Wellmann \(18\)](#)
- Transverse momentum ( $p_T$ ) expansion  
[Bonciani, Giardino, Groeber, G.D. \(18\)](#)
- Merging  $p_T$  and HE expansions  
[Bellafronte et al. \(22\)](#)

### Bottleneck of the calculation

### exact numerical:

quite demanding calculation from a computational point of view  
[Borowka et al. \(16\)](#), [Baglio et al. \(19,20\)](#)

NNLO + NLL QCD: HEFT  
[De Florian, Mazzitelli \(13\)](#)  
[Grigo, Melnikov, Steinhauser \(14\)](#),  
[Grigo, Hoff, Steinhauser \(15\)](#)  
.....

### Small mass expansion

[Xu et al. \(19\)](#), [Want et al. \(21\)](#)

# Looking for an analytic result, why?

**Analytic result:** a result expressed in terms of “*functions*” that can be computed with a (public) code in a reasonable (very short) amount of time (ex. Log → HPL, GHPL ...)

**Virtues** (with respect to a numerical result): flexibility in the input parameters and in modifications of the setup (introduction of kappa parameters), coverage of any phase-space point (no interpolating functions needed). Good features for constructing a MonteCarlo code.

**Problem:** more energy scales in the diagrams less available “*known*” *functions*.

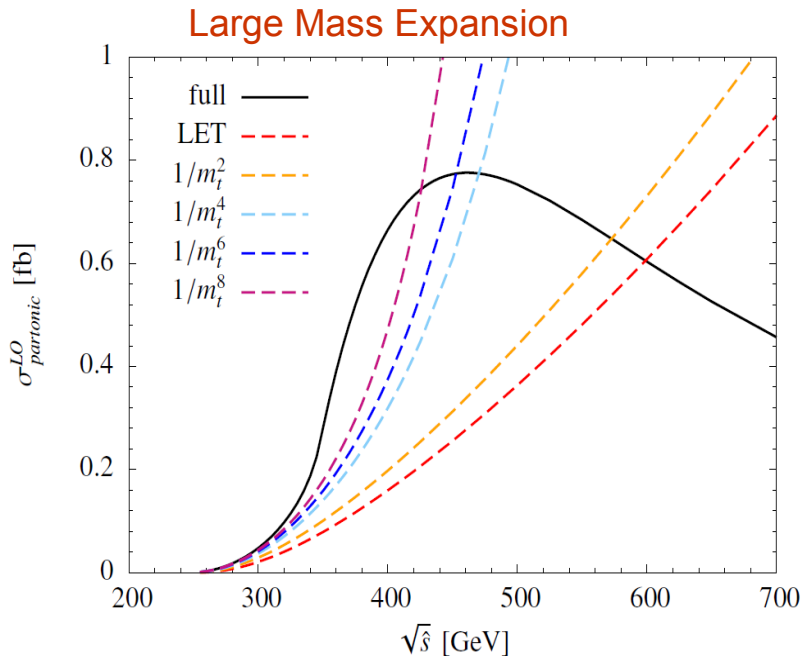
**Solution A:** reduce the numbers of scales in the problem. Look for an “*approximate*” result obtained by expanding the diagrams in terms of the ratio of small energy scales v.s. large energy scales. The dependence of the result by the large energy scales is kept exact. The result is valid in specific regions of the phase-space where the energy hierarchy is realized.

**N.B.** more scales are reduced, more available “*known*” *functions*. But more restricted region of validity of the result (compromise).

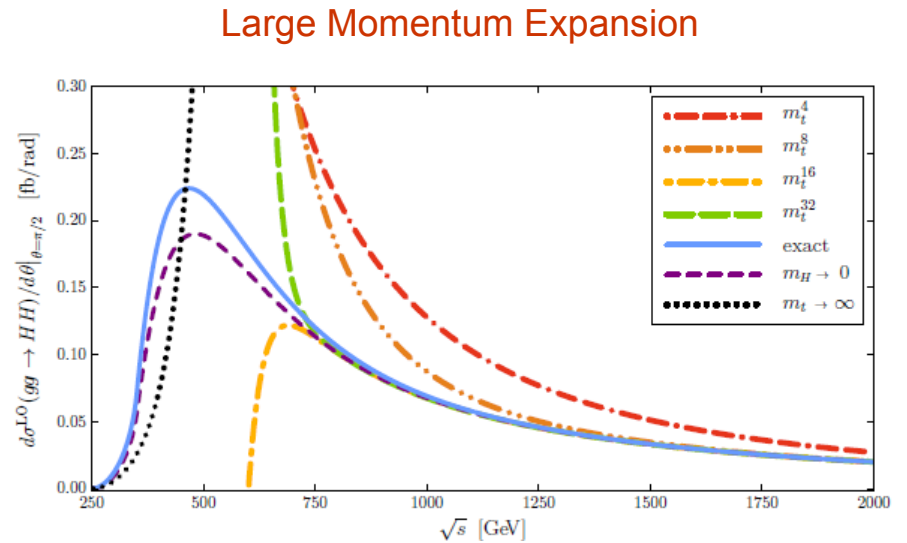
**Solution B:** combine together different “*approximate*” results that cover complementary regions of the phase-space in order to have a full coverage of it.

- HTL: covers well the threshold region (validity  $s/(4m_t^2) \lesssim 1$ )
- p<sub>T</sub>-expansion: covers well the region up to  $\sqrt{s} \lesssim 750$  GeV (validity  $|t|/(4m_t^2) \lesssim 1$ )
- HE-expansion: covers well the region  $\sqrt{s} \gtrsim 700$  GeV (validity  $|t|/(4m_t^2) \gtrsim 1$ )

# Judging the approximations from the LO



HTL: Ok threshold



Davies, Mishima, Steinhauser, Wellmann (18)

High-Energy expansion: Ok tail

None of these approximations cover the important C.M. energy region  $\sqrt{s} \lesssim 700$  GeV

At NLO to try to cure the bad behavior of the approximations in the “wrong” region one can use the reweighting

$$d\sigma_{NLO} = d\sigma_{LO} \left( \frac{d\sigma_{NLO}}{d\sigma_{LO}} \right)$$

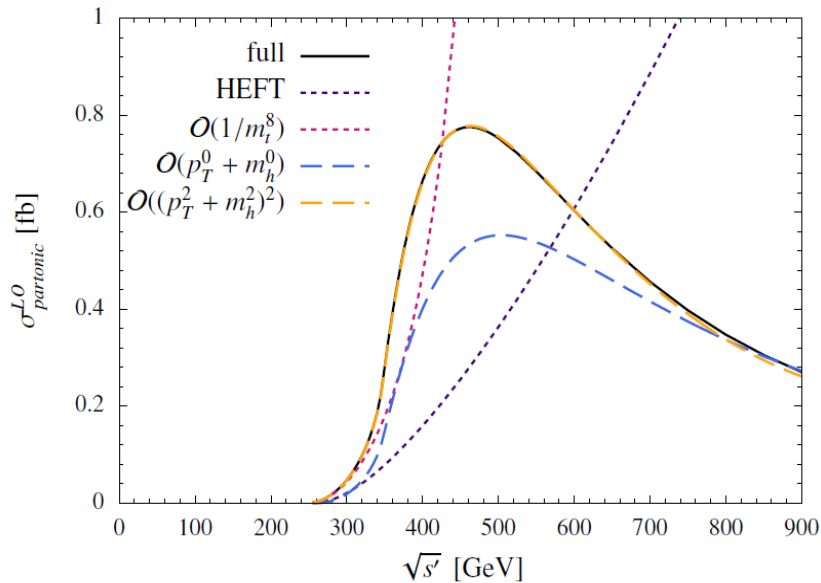
exact  $\swarrow$   $\nwarrow$

$\frac{\text{approx}}{\text{approx}} \sim \frac{\text{bad}}{\text{bad}} \sim \text{good}$



# Judging the approximation from the LO

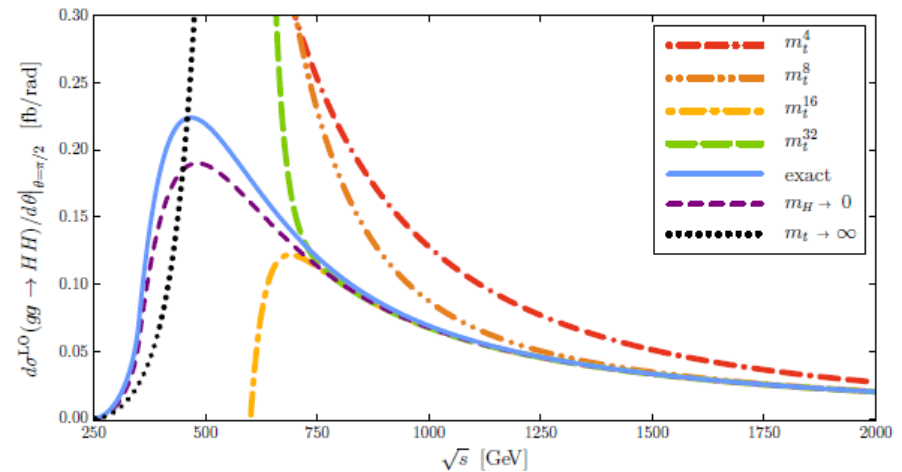
## Transverse Momentum Expansion



Bonciani, Giardino, Groeber, G.D. (18)

The important C.M. energy region  $\sqrt{s} \lesssim 700$  is perfectly covered

## Large Momentum Expansion



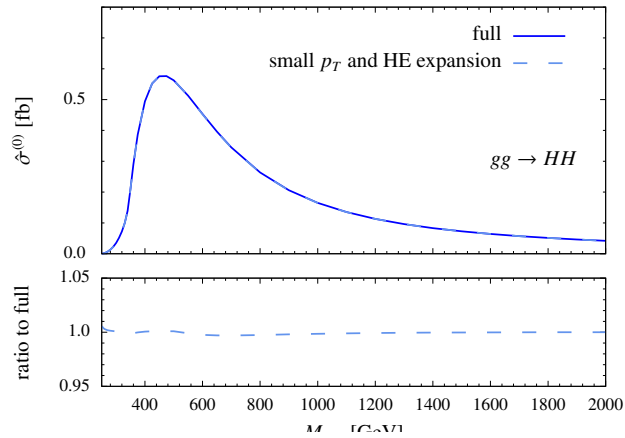
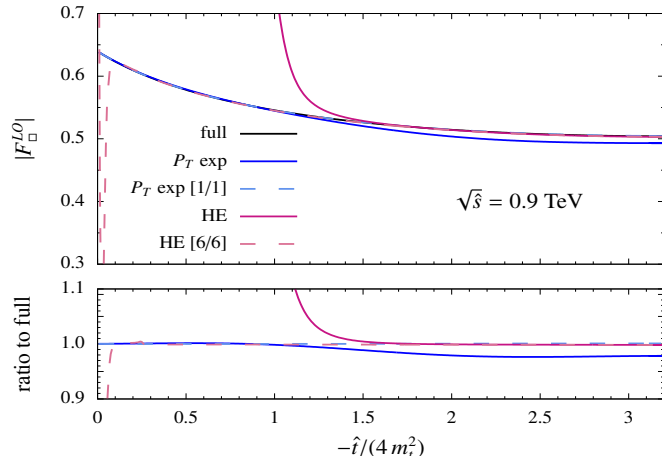
Davies, Mishima, Steihauser, Wellmann (18)

High-Energy expansion: Ok tail

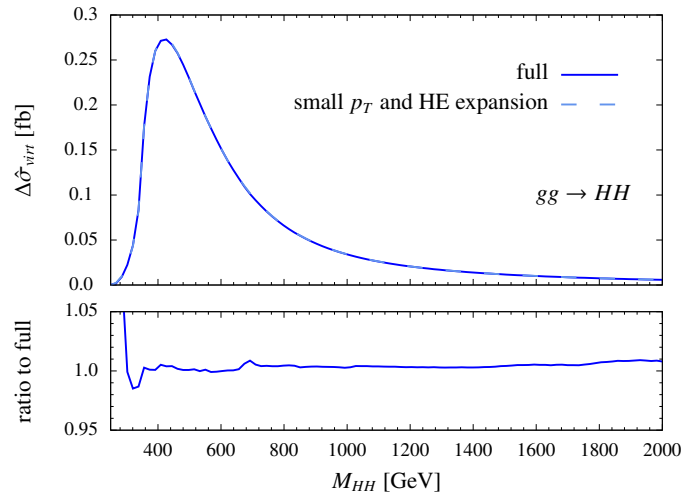
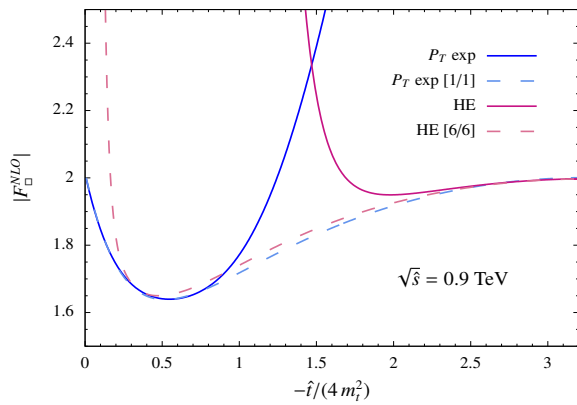
The two expansions cover complementary regions of the phase-space

# Merging the $p_T$ and HE expansions

Extend the range of validity of each expansion up to or beyond his border using Pade' approximants. Construct a [1,1]  $p_T$ -Pade' and a [6,6] HE-Pade'



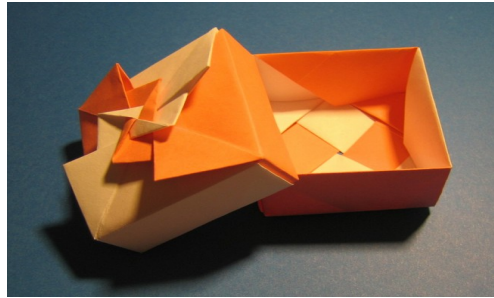
LO



NLO

# A new Monte Carlo on the market

- Currently in the POWHEG-BOX there is a Monte Carlo generator ( $G_{GHH}$ ) for Higgs boson pair production at NLO ( [Heinrich et al. \(17\)](#), [Jones et al. \(18\)](#), [Heinrich et al. \(20\)](#) ).  
The MC is based around the two-loop numerical results of Borowka et al. (16) which are implemented via a series of interpolating grids (to account for modified trilinear couplings etc..) matched with the HE-expansion results for large values of the center-of-mass energy.
- Inputs are fixed, no possibility to change the renormalization scheme for the top mass.



- We developed a new code Monte Carlo code, always based on the POWHEG-BOX MC framework, based on our analytic evaluation of the two-loop contribution.
- Features:
  - a) freedom in the assignment of all input parameters including the trilinear Higgs self-coupling ( $\kappa_\lambda$  rescaling).
  - b) possibility of varying the renormalization scheme employed for the top mass
- Possible future features:
  - i) rescaling of the Yukawa coupling ( $\kappa_t$ ).
  - ii) resonant production.
  - iii) .....

# Results and comparison with previous works

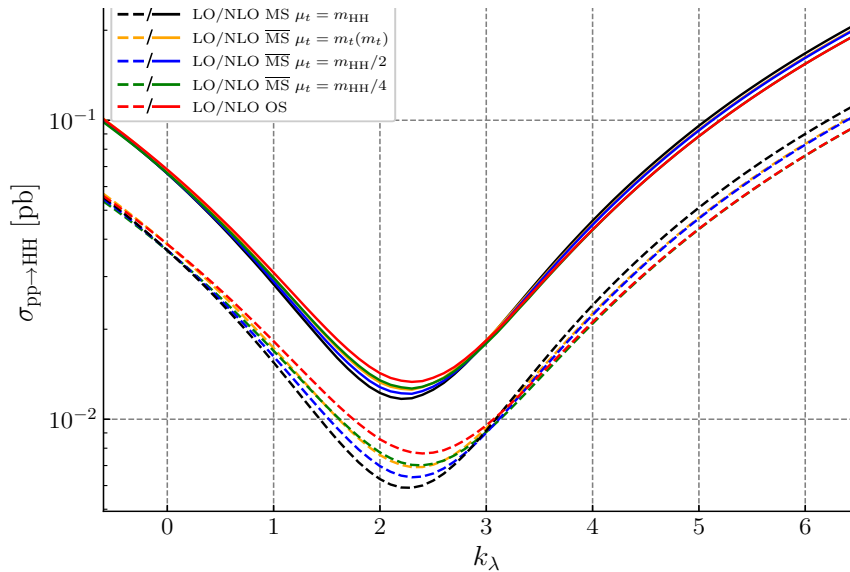
## Setup:

- $\sqrt{s} = 13.6$  TeV
- PDF: NNPDF31\_nlo\_as\_0118
- SHOWER: Pythia 8
- $\mu_R = \mu_F = M_{HH}/2$
- $\alpha_s$  taken from the PDF ( $\alpha_s(M_Z) = 0.118$ )
- $m_t^{\text{OS}} = 172.5$  GeV,  $m_W = 80.385$  GeV,  $m_H = 125$  GeV,  
 $G_\mu = 1.1663787 \times 10^{-5}$  GeV<sup>-2</sup>
- Both OS and  $\overline{MS}$  top mass employed.  
 $m_t^{\overline{MS}}(\mu_t = m_t^{\overline{MS}}, M_{HH}/4, M_{HH}/2, M_{HH})$

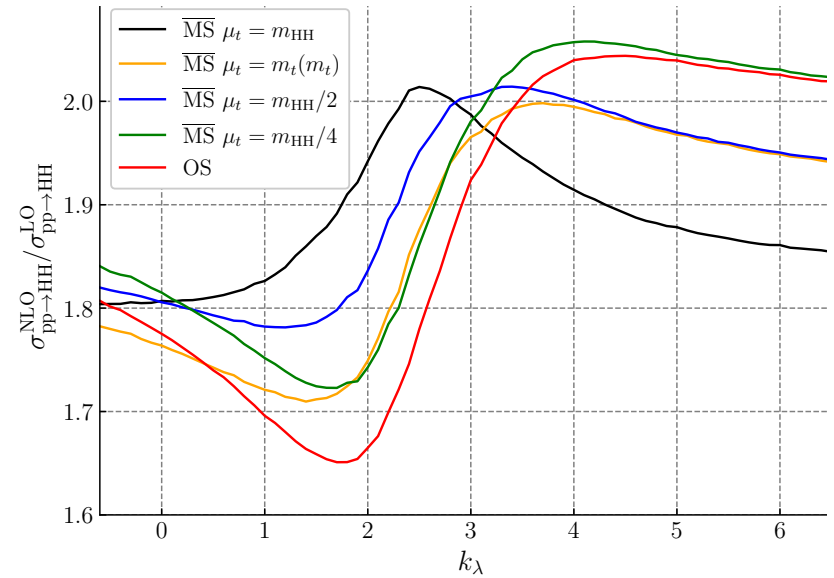
Scale uncertainty estimated from the envelop of a 7 points rescaling of  $\mu_R, \mu_F$   
LO:  $\sim +30\%/(\sim -20\%) \rightarrow$  NLO:  $\sim +15\%/(\sim -15\%)$  regardless  $\lambda_3$  and the top mass scheme

What about scheme dependence?

# Include cross sections and $\kappa$ factors



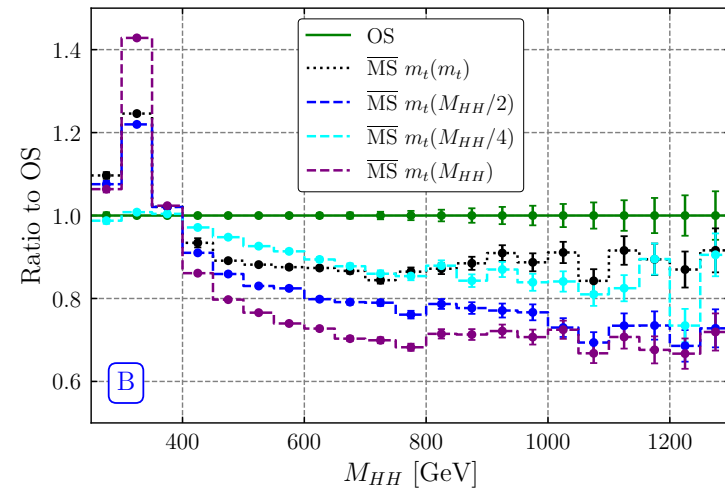
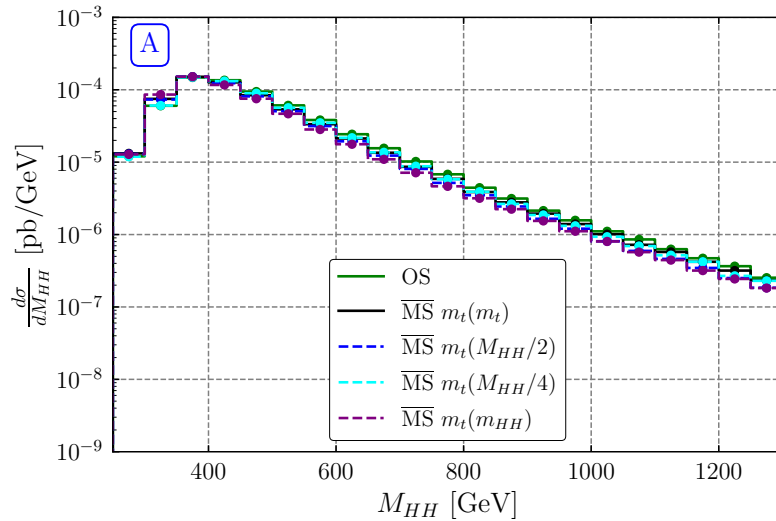
Inclusive cross section at LO and NLO as a function of  $\kappa_\lambda$  for different top-mass renormalization schemes



K factors for different top-mass renormalization schemes

- Minimum of the cross section depends on the top scheme.
- LO  $\rightarrow$  NLO curves get closer, K factors accordingly.
- Initial discrepancy with the `gGHH` MC for  $\kappa_\lambda \neq 1$  resolved after a bug in `gGHH` was fixed by the authors.
- Agreement with the fixed-order calculation of [Baglio et al. \(19\)](#) for  $\kappa_\lambda \leq 1$ , some discrepancy for higher values of  $\kappa_\lambda$ . (Probably their numerical integration is not sufficiently accurate in regions of parameter space where there are strong cancellations).

# SM differential distributions: top mass scheme dependence in $M_{HH}$



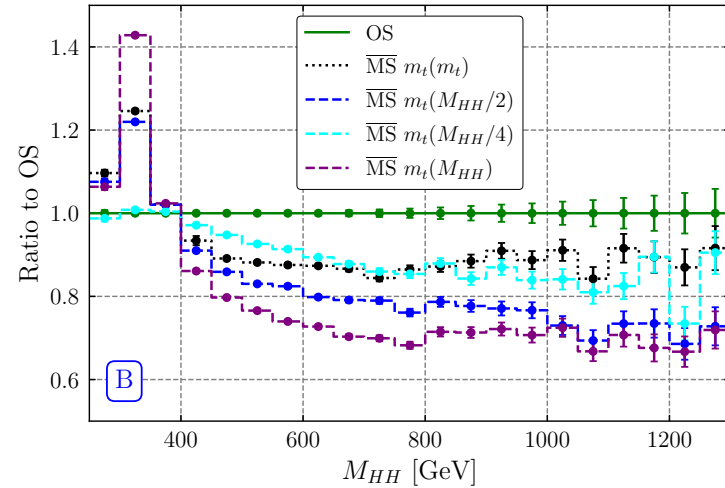
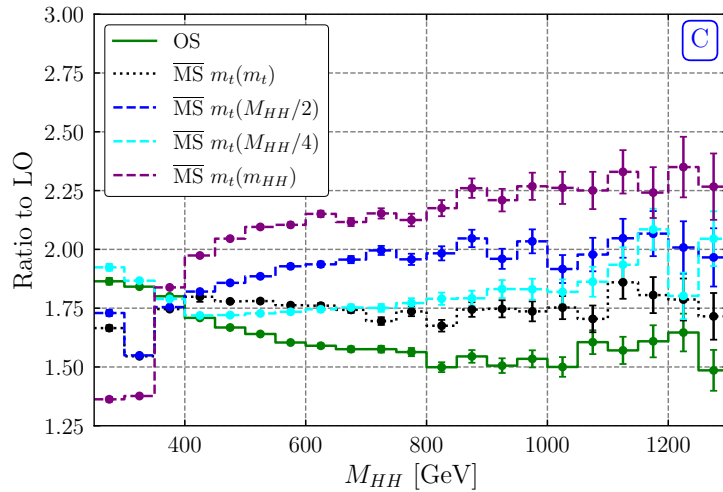
The invariant mass distribution of the two Higgs system for different choices of the top-mass renormalization scheme.

A) absolute distribution at NLO + PS

B) ratio between the  $\overline{\text{MS}}$  predictions and the OS one

- Position of the peak depends on the top mass scheme
- Ratio is quite constant for  $M_{HH} \geq 600$  GeV. For  $M_{HH} \leq 400$  GeV large deviations in the ratio (influence of the position of the peak).

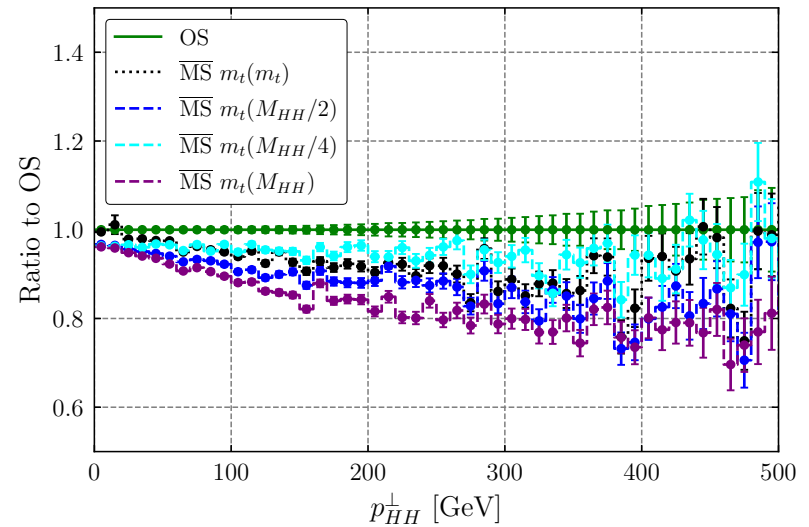
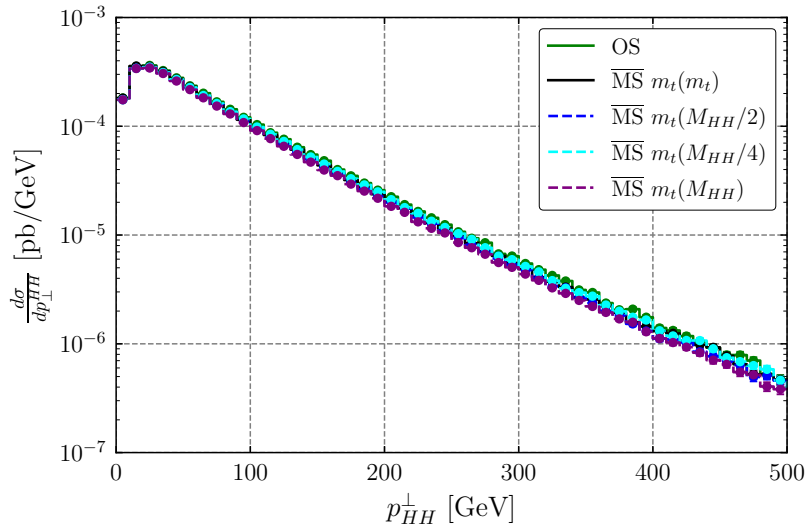
# SM differential distributions: top mass scheme dependence in $M_{HH}$



The invariant mass distribution of the two Higgs system for different choices of the top-mass renormalization scheme.  
 C) K factors  
 B) ratio between the  $\overline{\text{MS}}$  predictions and the OS one

- Position of the peak depends on the top mass scheme
- Ratio is quite constant for  $M_{HH} \geq 600$  GeV. For  $M_{HH} \leq 400$  GeV large deviations in the ratio (influence of the position of the peak).
- K factors imply the reduction of the scheme dependence LO  $\rightarrow$  NLO

# SM differential distributions: transverse momentum of the two higgs system



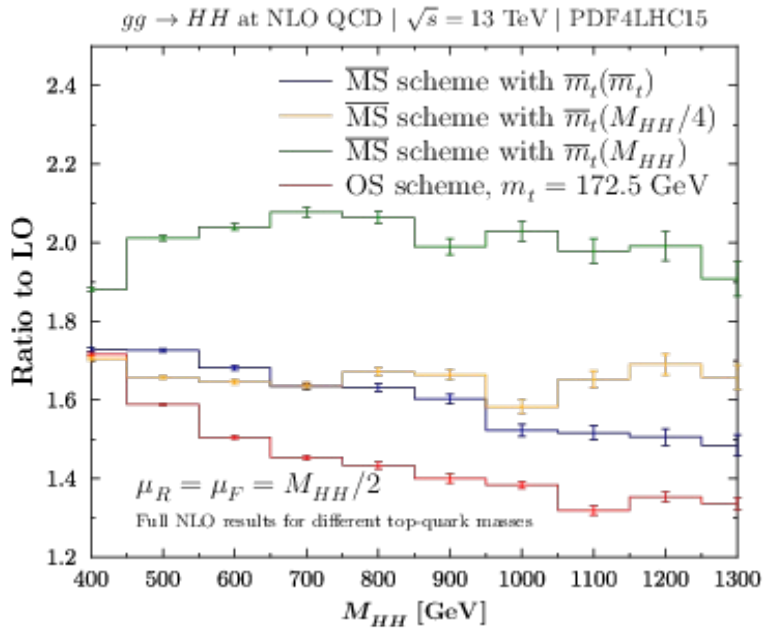
The transverse momentum distribution of the two Higgs system for different choices of the top-mass renormalization scheme.  
 absolute distribution at NLO + PS (left plot)      ratio between the  $\overline{\text{MS}}$  predictions and the OS one (right plot)

- The slope of  $p_{HH}$  depends on the top mass scheme
- $\overline{\text{MS}}$  results are always smaller than the OS one
- In the small  $p_{HH}$  region results are all quite close while there is larger spread for high values of  $p_{HH}$

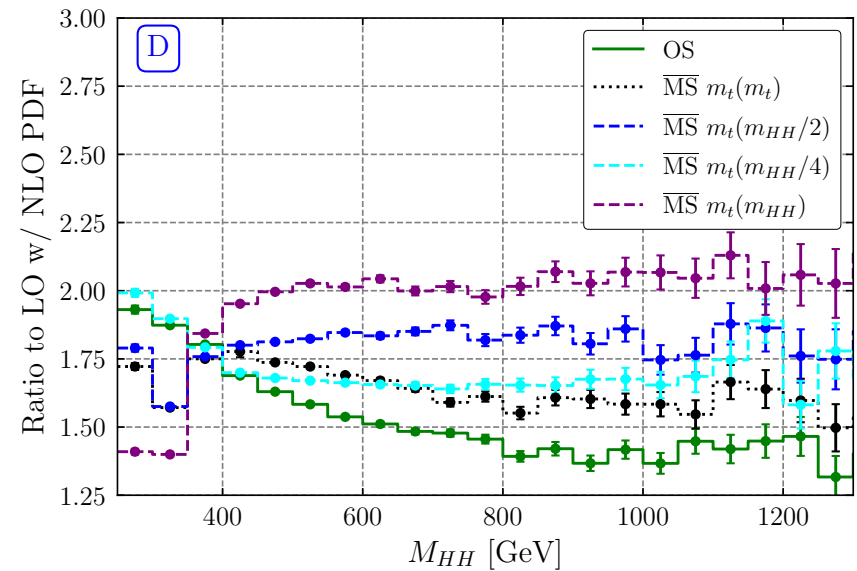


# SM differential distributions: top mass scheme dependence in $M_{HH}$

Comparison with [Baglio et al. \(21\)](#)



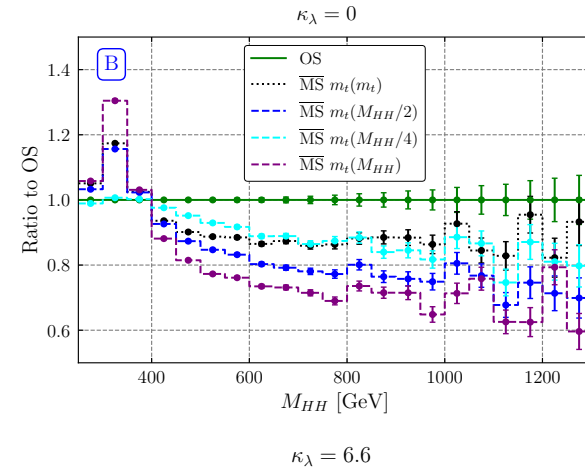
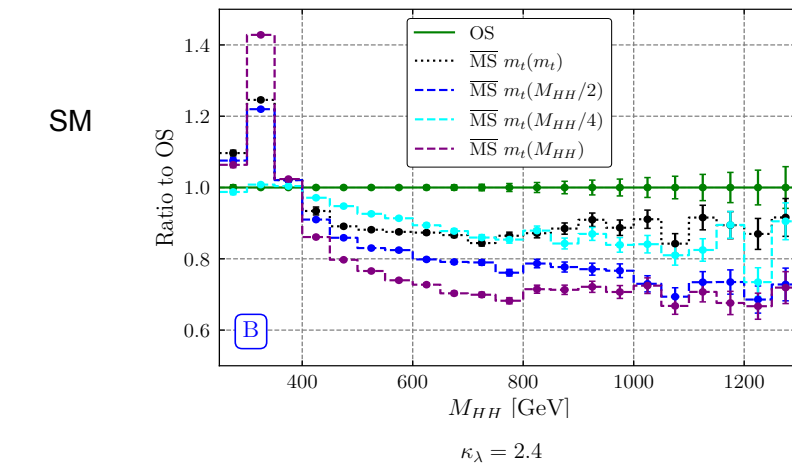
NLO



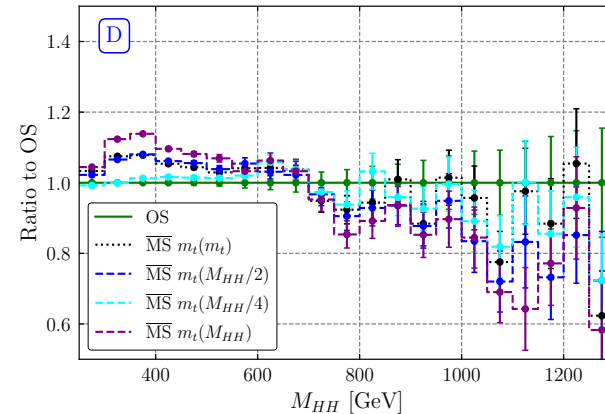
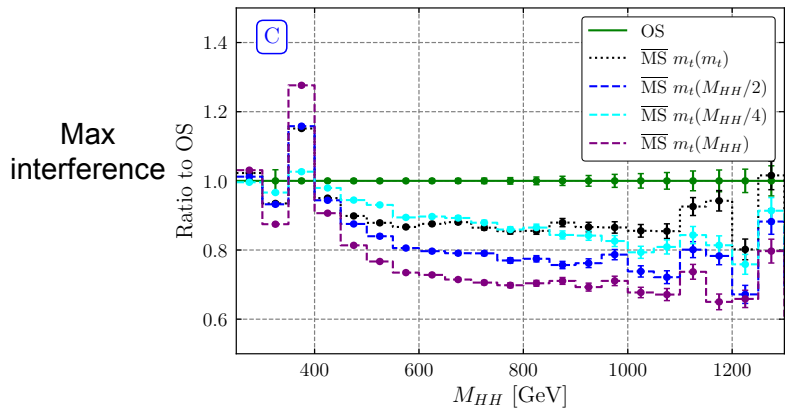
NLO+PS

Good qualitative agreement although the setups were different

# $\lambda_3$ differential distributions: top mass scheme dependence in $M_{HH}$



No signal



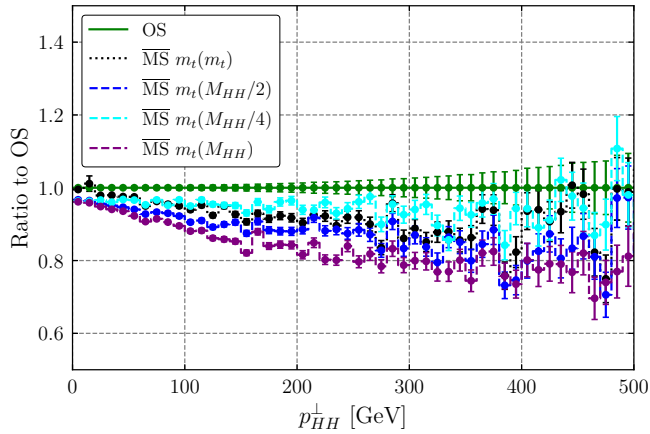
More signal

The invariant mass distribution of the two Higgs system for different choices of the top-mass renormalization scheme.

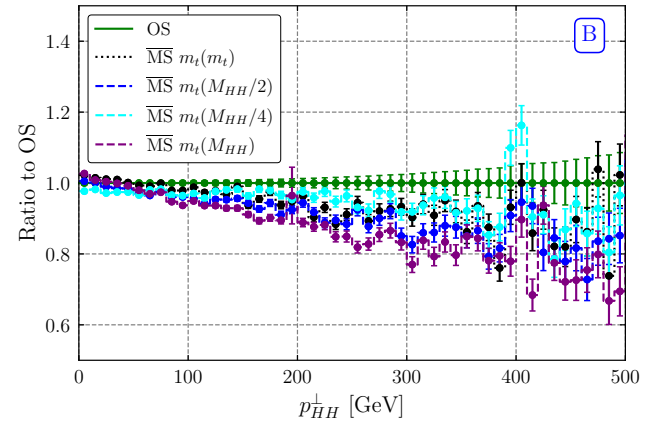
- $\kappa_\lambda = 0$ : very similar to SM. Scheme dependence of the signal milder than that of the background.
- $\kappa_\lambda = 2.4$ : the region around the  $2 m_t$  threshold has a large scheme dependence
- $\kappa_\lambda = 6.6$ : mild scheme dependence

# $\lambda_3$ differential distributions: transverse momentum of the two higgs system

SM

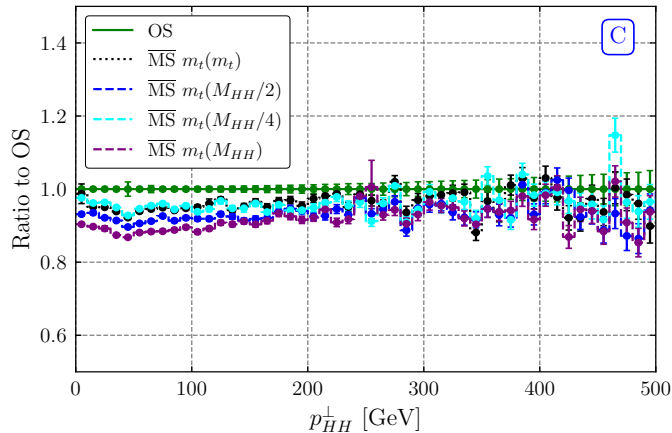


$\kappa_\lambda = 0$



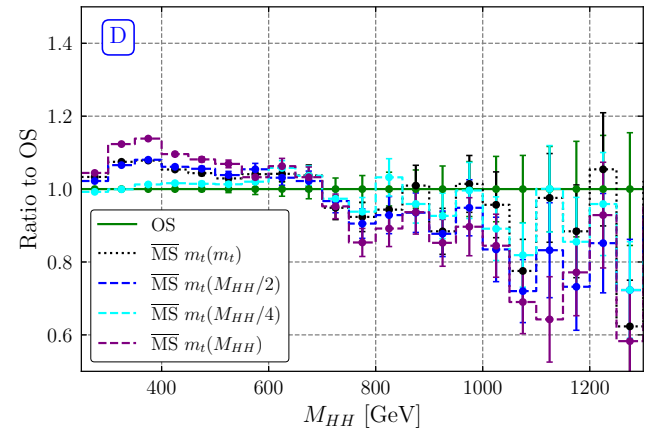
No signal

$\kappa_\lambda = 2.4$



Max interference

$\kappa_\lambda = 6.6$



More signal

The transverse momentum distribution of the two Higgs system for different choices of the top-mass renormalization scheme.

- $\kappa_\lambda = 0$ : very similar to SM although with less spread
- $\kappa_\lambda = 2.4$ : quite small scheme dependence and very similar for any  $p_{HH}$
- $\kappa_\lambda = 6.6$ : similar to  $\kappa_\lambda = 2.4$  but with more spread

# Conclusions

- The shape of the Higgs potential is presently very poorly known. Determining the trilinear self couplings from double Higgs production is the new challenge. Accurate predictions are needed.
- I discussed a way to compute analytically the virtual NLO corrections via the merging of the  $p_T$  and HE-expansion results that is very efficient from a computational point of view
- I presented a new Monte Carlo code for Higgs pair productions based on this analytic evaluation of the virtual corrections whose main feature is flexibility in the input parameters and choice of the renormalization scheme for the top mass.
- Going from LO to NLO the top mass scheme dependence is reduced but in the SM for large  $M_{HH}$  or large  $p_T$  can reach up to 20%.
- Modified trilinear coupling: signal contribution shows a milder scheme dependence than the background one.