A Cosmological Solution to the Doublet-Triplet Splitting Problem

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based on ***hep-ph:240#.#####*** in collaboration with:
Csaba Csaki, **Raffaele Tito D'Agnolo** and **Eric Kuflik**

The D/T splitting problem

Embedding of the Higgs doublet H in a representation of $SU(5)$

$$\mathbf{5}_H \longrightarrow (\mathbf{3},1)_{-1/3} \oplus (1,\mathbf{2})_{1/2} = T \oplus H$$

New color triplet T with mass $\gtrsim M_{GUT} \simeq 10^{16}$ GeV from bounds on proton lifetime

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What keeps the doublet <u>light</u> while the triplet is <u>heavy</u>?

The D/T splitting problem

In minimal $SU(5)$ the masses of H and T come from the potential

$$V(\mathbf{5}_H, \mathbf{24}_\Sigma) = -\mu_5^2 \mathbf{5}_H^\dagger \mathbf{5}_H + \frac{\lambda}{4}(\mathbf{5}_H^\dagger \mathbf{5}_H)^2 + \alpha \mathbf{5}_H^\dagger \mathbf{5}_H \mathrm{Tr}\left[\mathbf{24}_\Sigma^2\right] + \beta \mathbf{5}_H^\dagger \mathbf{24}_\Sigma^2 \mathbf{5}_H + \delta \mathbf{5}_H^\dagger \mathbf{24}_\Sigma \mathbf{5}_H$$

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After SSB of $SU(5)$ to the SM gauge group by the vev

$$\langle \mathbf{24}_\Sigma \rangle = v_\Sigma \,\mathrm{diag}(-2, -2, -2, 3, 3), \quad v_\Sigma \sim M_{GUT}$$

we obtain the potential

$$V_{eff}(T, H) = \left[-\mu_5^2 + v_\Sigma^2 \left(-30\alpha + 4\beta - \frac{2\delta}{v_\Sigma}\right)\right] H^\dagger H + \left[-\mu_5^2 + v_\Sigma^2 \left(-30\alpha + 9\beta + \frac{3\delta}{v_\Sigma}\right)\right] T^\dagger T + \frac{\lambda}{4}\left(T^\dagger T + H^\dagger H\right)^2$$

The D/T splitting problem

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from which we can read the masses

$$m_H^2 \equiv -\mu_5^2 + v_\Sigma^2\left(-30\alpha + 4\beta - \frac{2\delta}{v_\Sigma}\right) \qquad m_T^2 \equiv -\mu_5^2 + v_\Sigma^2\left(-30\alpha + 9\beta + \frac{3\delta}{v_\Sigma}\right)$$

The D/T splitting problem

To have a light electroweak scale while keeping the triplet near M_{GUT} we need

$$m_H^2 \equiv -\mu_5^2 + v_\Sigma^2\left(-30\alpha + 4\beta - \frac{2\delta}{v_\Sigma}\right) = \mathcal{O}(m_h^2) \qquad m_T^2 \equiv -\mu_5^2 + v_\Sigma^2\left(-30\alpha + 9\beta + \frac{3\delta}{v_\Sigma}\right) = \mathcal{O}(M_{GUT}^2)$$

Technically natural choice but large tuning between μ_5^2 and coefficients of v_Σ^2

The D/T splitting problem: A cosmological solution?

Can this tuning be a result of cosmological dynamics similar to cosmological solutions to the weak scale hierarchy problem? (e.g. *Sliding Naturalness* **[D'Agnolo, Teresi, 21']**)

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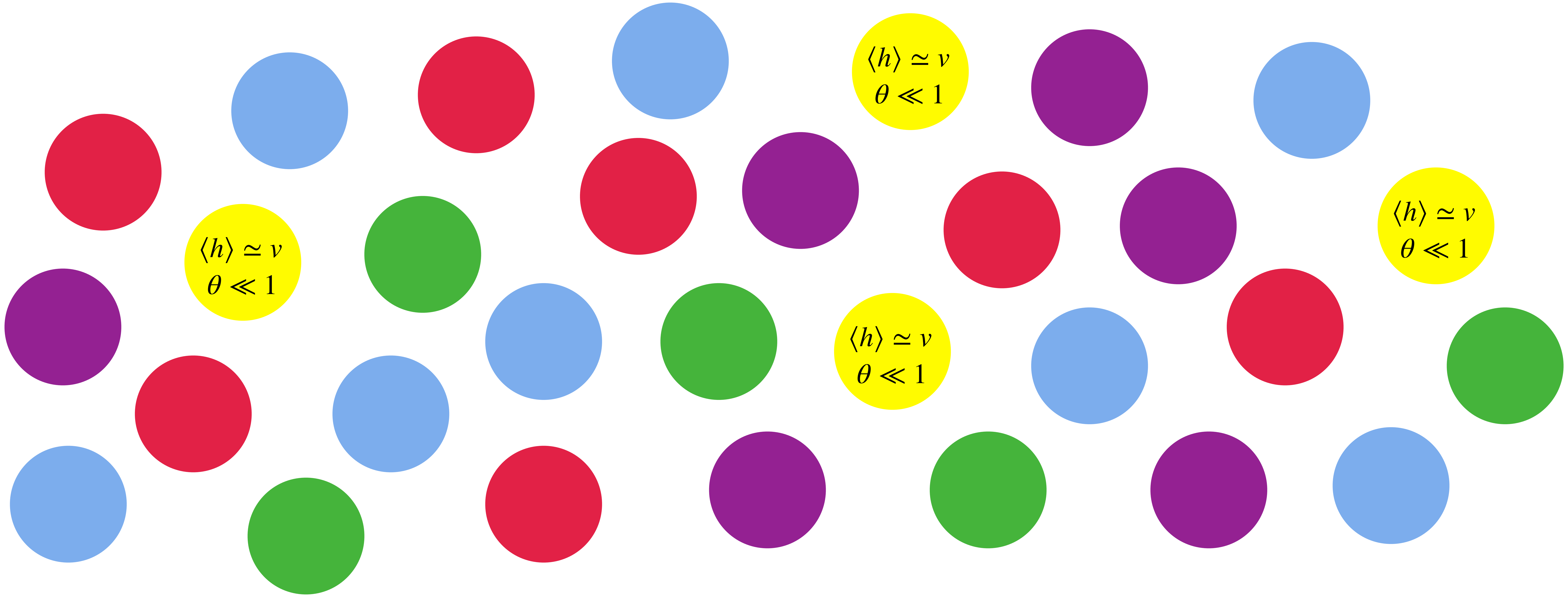
Sliding Naturalness relies on the Higgs vev to trigger a low energy potential

$\Longrightarrow$ Naturally gives a solution to the D/T splitting problem

Sliding Naturalness: A quick summary

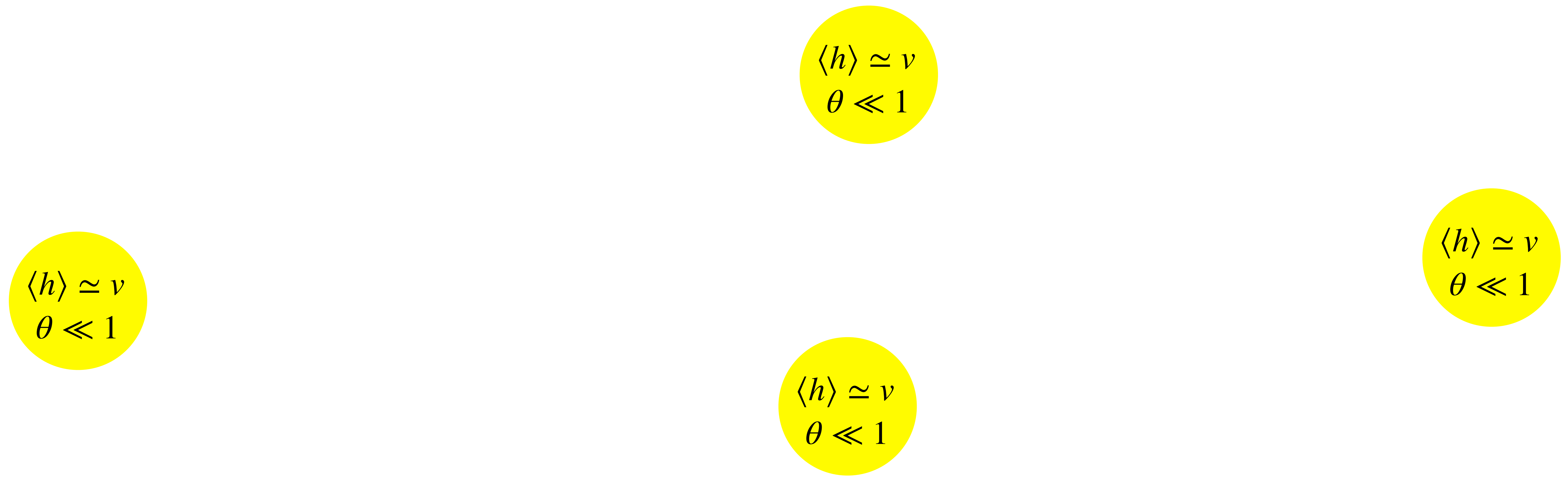
Landscape of Higgs masses, θ and CC values



[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

After reheating and a time $t_c \sim 1/H(\Lambda_{QCD}) \sim 10^{-5}s$



[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

Only universes with the observed values of $\langle h \rangle$ and θ can live cosmologically long times

$\langle h \rangle \simeq v$

$\theta \ll 1$

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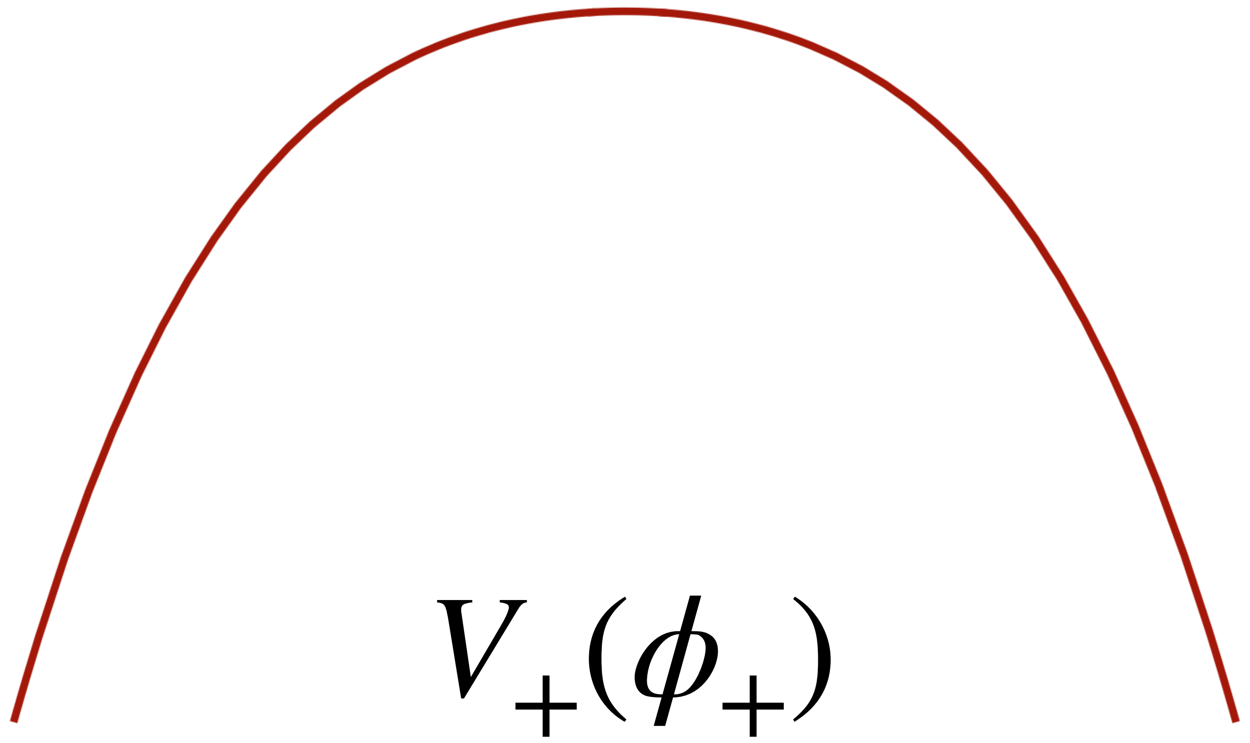
[D'Agnolo, Teresi, 21']

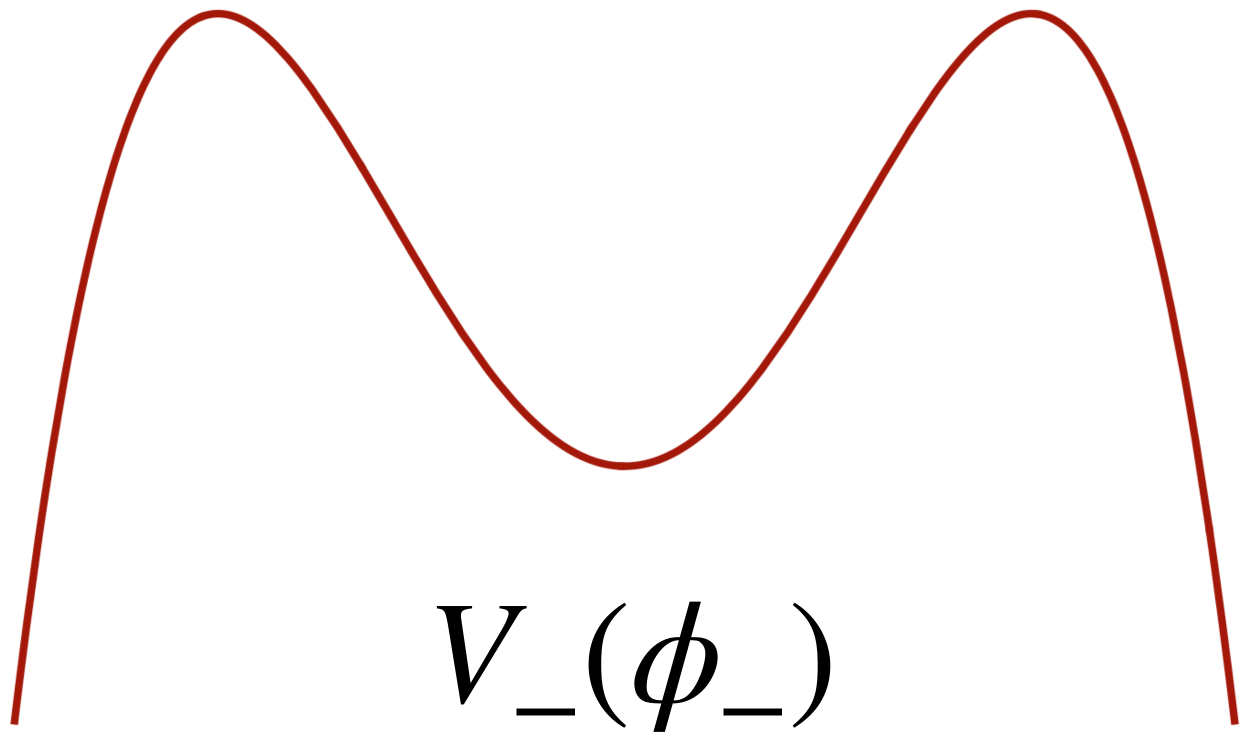
Sliding Naturalness: A quick summary

A pedagogical model (zoom in on shallow minimum)

We introduce two scalar fields with potential:

$$V_\pm = m_{\phi_\pm}^2 M_\pm^2 \left(\mp \frac{\phi_\pm^2}{2M_\pm^2} - \frac{\phi_\pm^4}{4M_\pm^4} \right)$$





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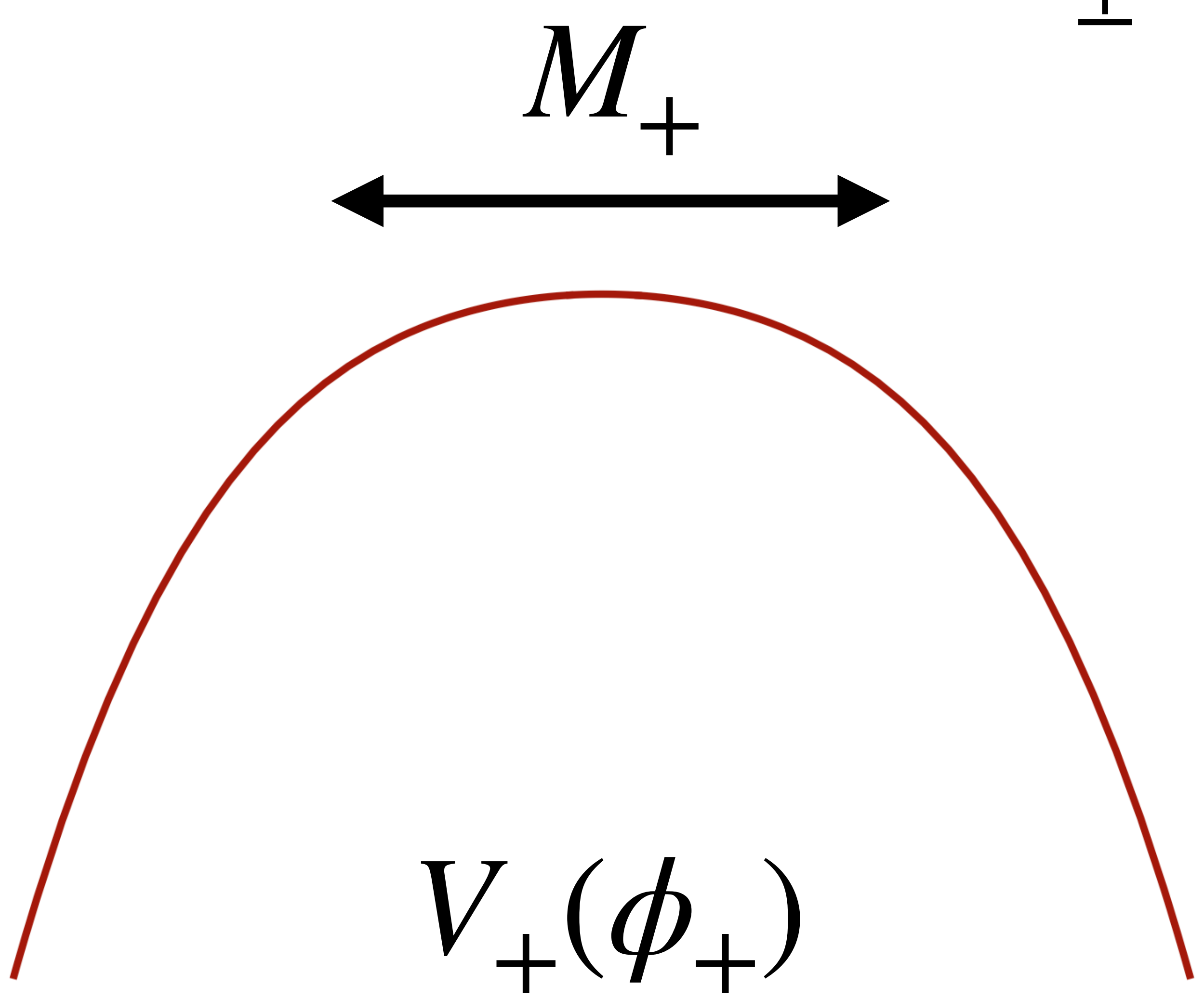
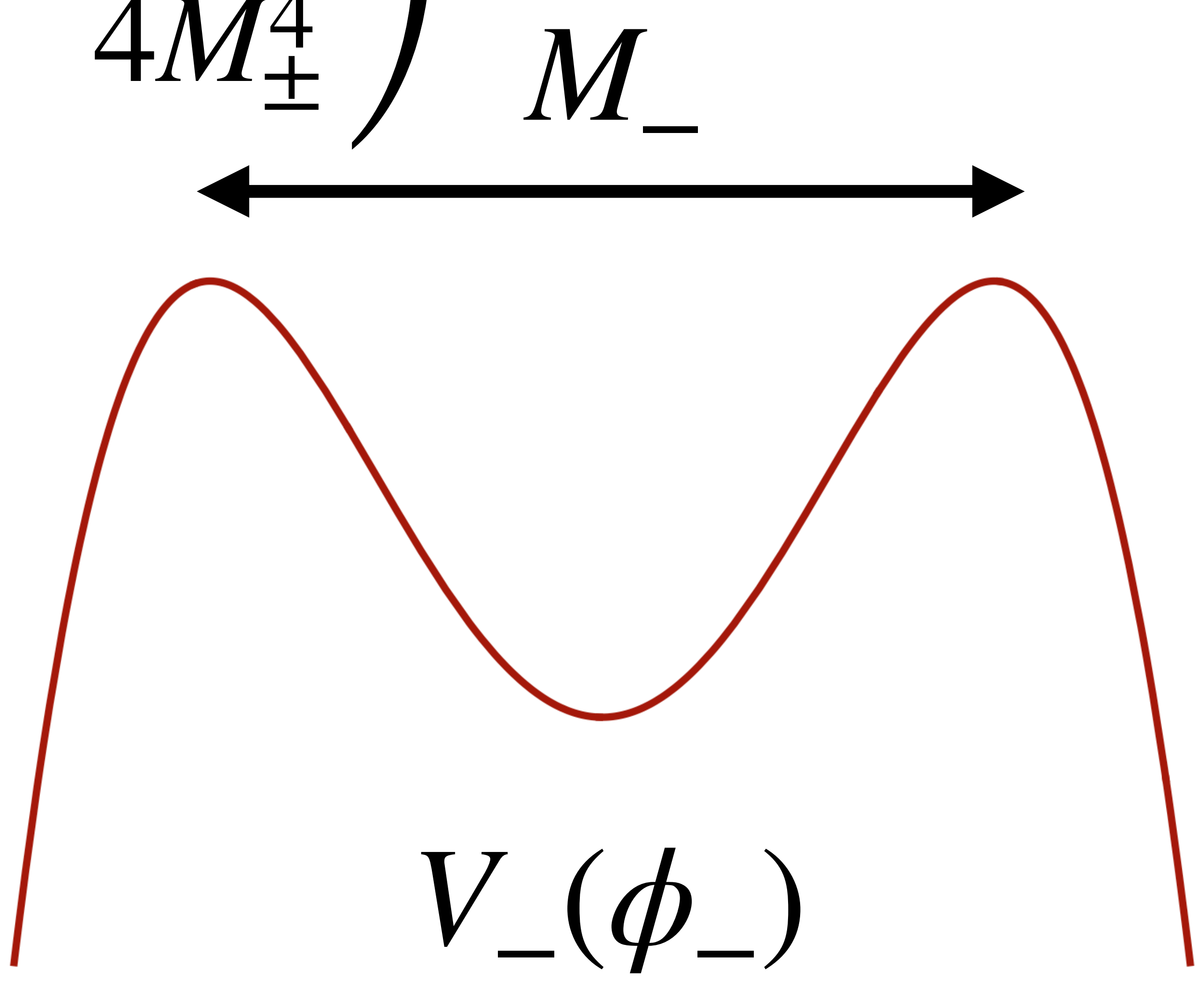




[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

A pedagogical model

The universes crunch for $|\phi_\pm| \gtrsim M_\pm$ due to a large contribution to the CC from $V_\pm(\phi_\pm)$

[Strumia, Teresi, 19']

[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

A pedagogical model

We add the axion-like coupling to gluons:

$$V_{\phi_\pm H} = -\frac{\alpha_s}{8\pi}\left(\theta + \frac{\phi_+}{F_+} + \frac{\phi_-}{F_-}\right)\mathrm{Tr}\left[G\tilde{G}\right]$$

[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

A pedagogical model

After QCD phase transition $V_{\phi_\pm H}$ becomes:

$$V_{\phi_\pm H} \simeq \frac{\Lambda^4(\langle h \rangle)}{2}\left(\theta + \frac{\phi_+}{F_+} + \frac{\phi_-}{F_-}\right)^2$$

where

$$\Lambda^4(\langle h \rangle) = m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \qquad \text{for} \qquad m_{u,d} \lesssim 4\pi f_\pi$$

is a **monotonic function** of $\langle h \rangle$

[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

Assumptions

A technically natural choice:

$$M_{\pm}/F_{\pm} \ll 1$$

[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

A technically natural choice: $M_\pm / F_\pm \ll 1$

Pedagogical assumptions (do not affect the conclusions):

$$m_{\phi_-} \gg m_{\phi_+} \quad \Longrightarrow \quad \phi_- \text{ starts rolling before } \phi_+$$

$$\frac{M_-}{F_-} \lesssim \theta + \frac{M_+}{F_+} \text{ to neglect the cross-term } \frac{\phi_- \phi_+}{F_- F_+} \text{ in } V_{\phi_\pm H}$$

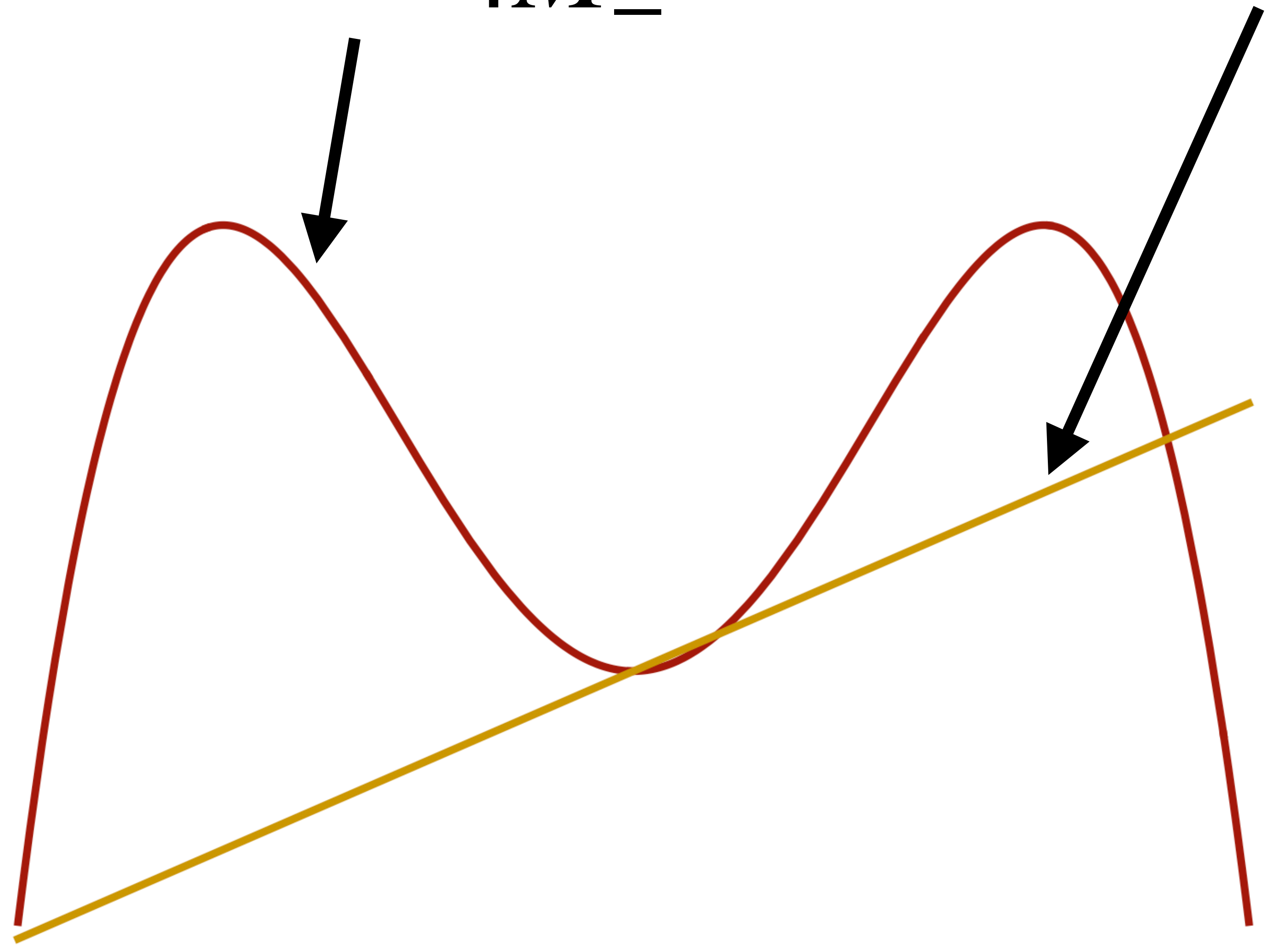
$$\Longrightarrow \text{ two separate minimization problems for } \phi_+ \text{ and } \phi_-$$

[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

With these assumptions the potential of ϕ_- looks like:

$$V_- = \frac{1}{2} m_{\phi_-}^2 \phi_-^2 - \frac{m_{\phi_-}}{4M_-^2}\phi_-^4 + \Lambda^4(\langle h \rangle)\theta_{eff}^- \frac{\phi_-}{F_-}$$



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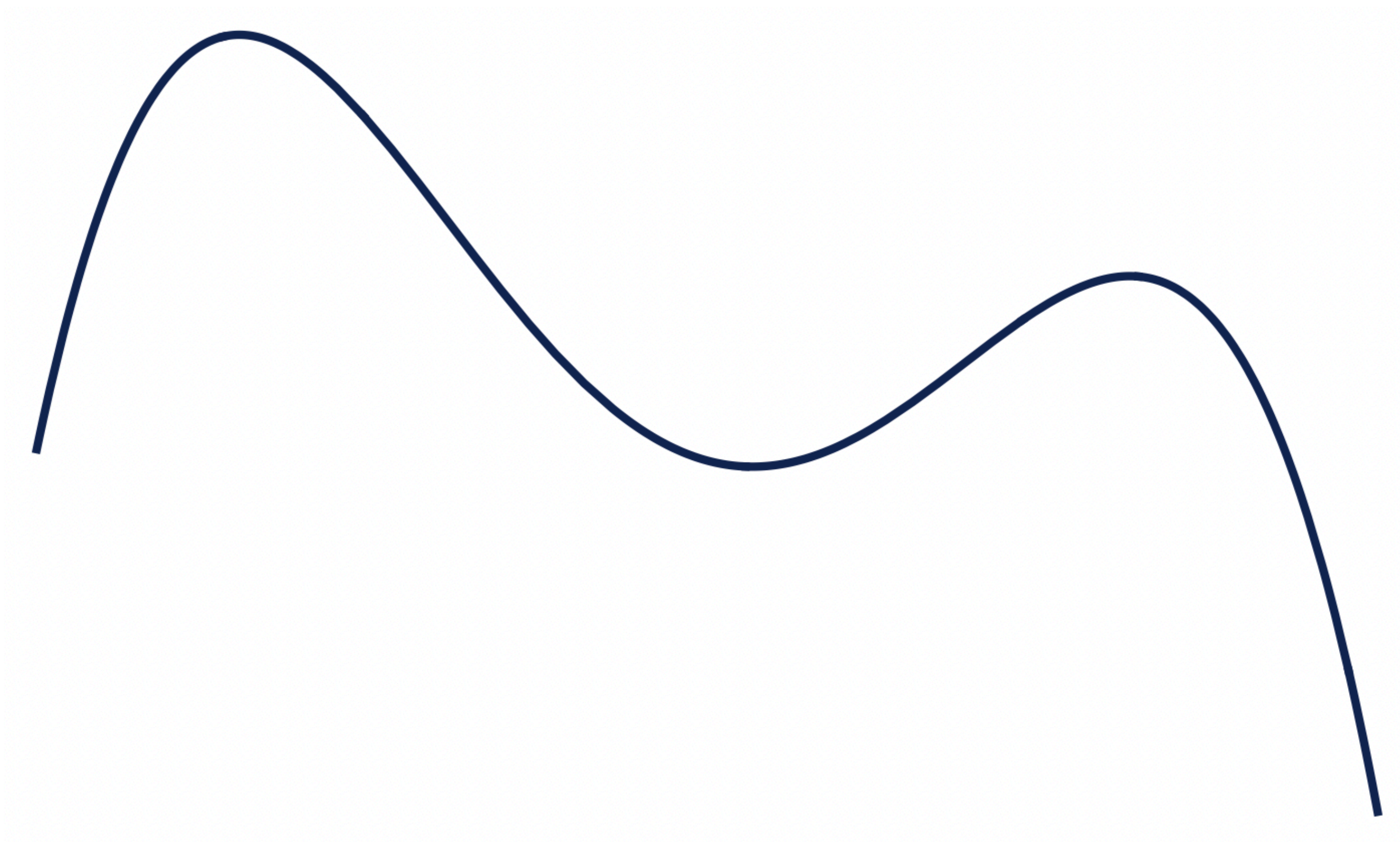

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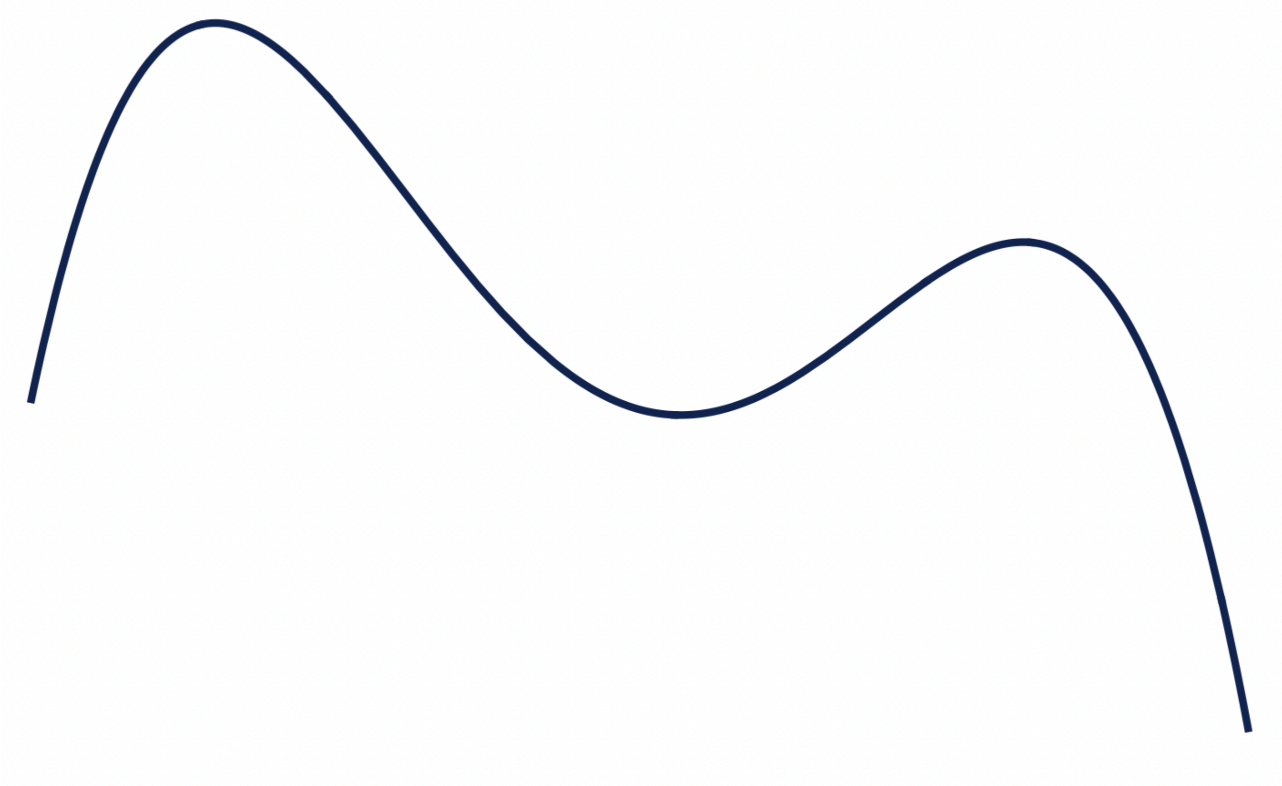


The local minimum around the origin is safe if:

$$\Lambda^4(\langle h \rangle) \lesssim \frac{m_{\phi_-}^2}{\theta_{eff}^-} M\_F\_$$

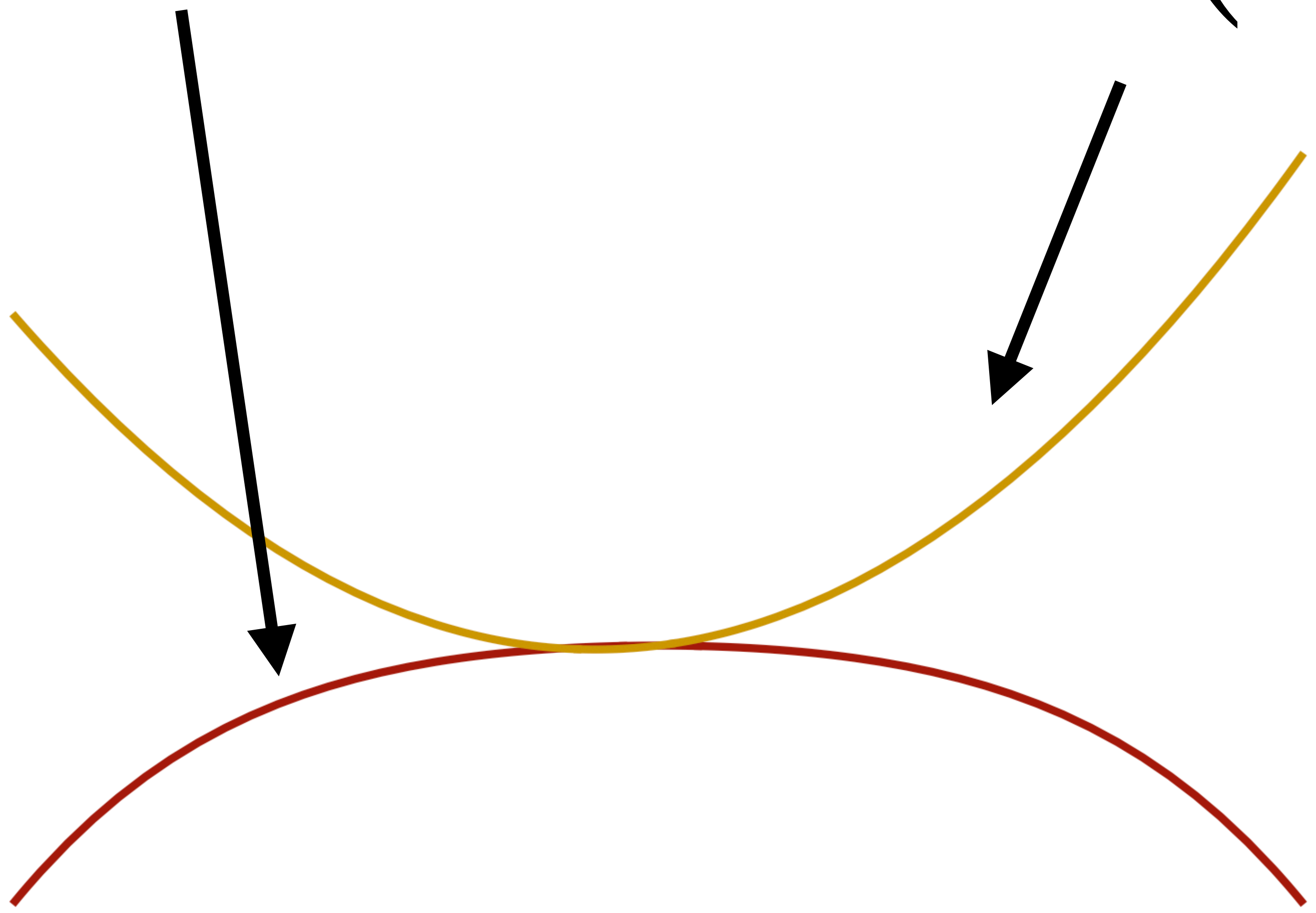
i.e. the mass term dominates the tadpole

[D'Agnolo, Teresi, 21']

Sliding Naturalness: A quick summary

With these assumptions the potential of ϕ_+ looks like:

$$V_+ = -\frac{1}{2}m_{\phi_+}^2\phi_+^2 - \frac{m_{\phi_+}}{4M_+^2}\phi_+^4 + \Lambda^4(\langle h\rangle)\left(\theta_{eff}^+\frac{\phi_+}{F_+} + \frac{\phi_+^2}{2F_+^2}\right)$$



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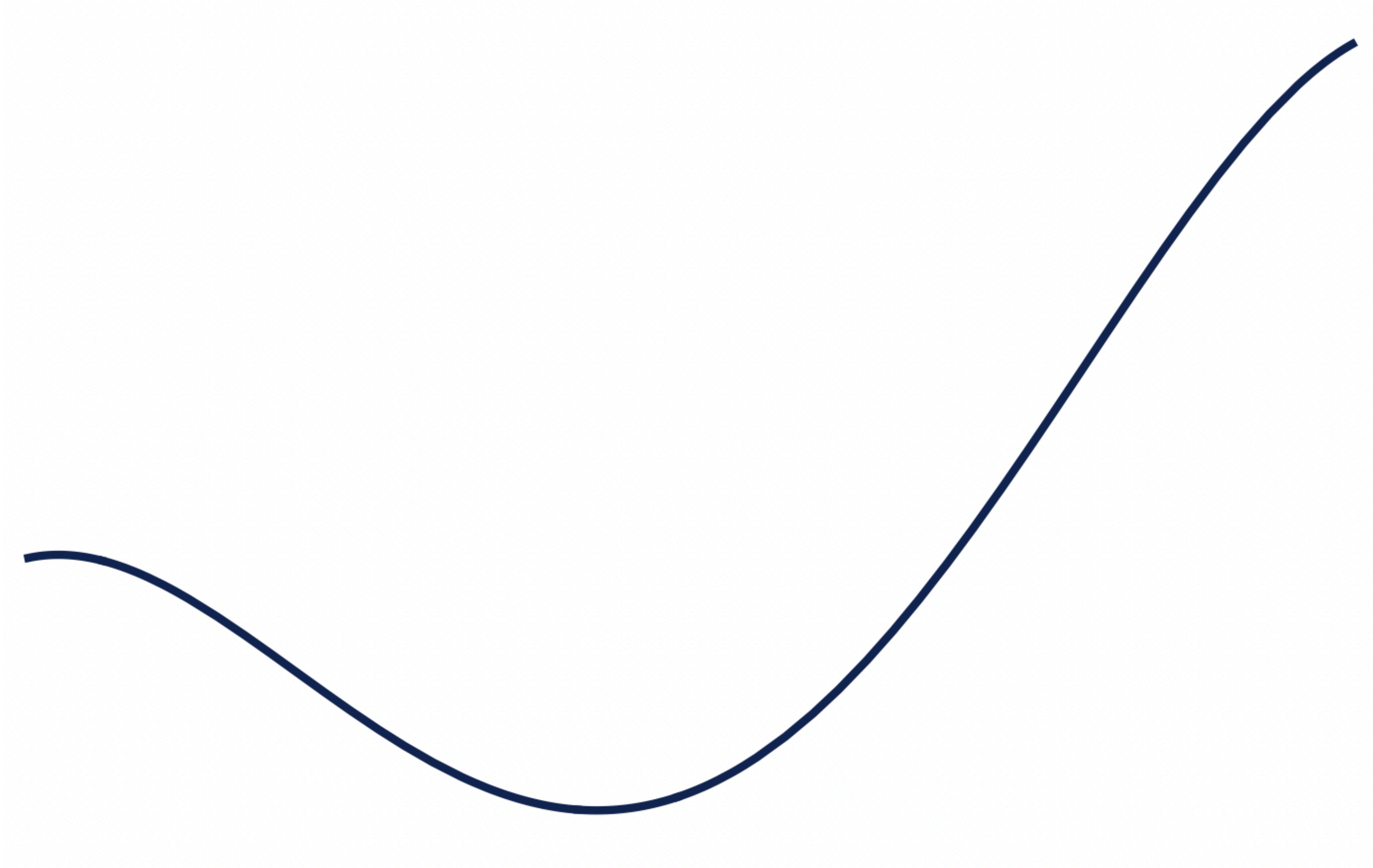

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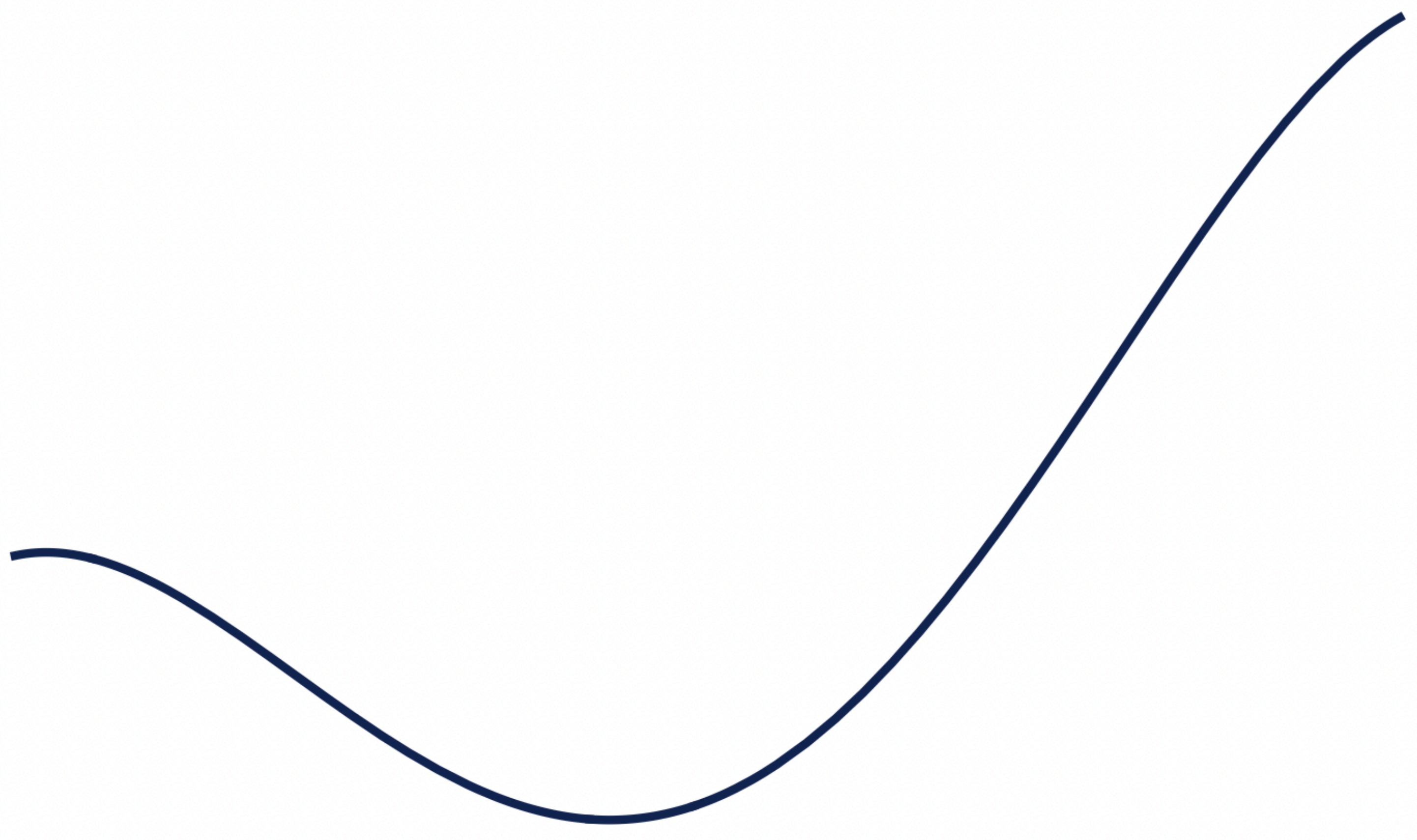

A safe local minimum around the origin is created if:

$$\Lambda^4(\langle h \rangle) \gtrsim m_{\phi_+}^2 F_+ \qquad \underline{\text{and}} \qquad \theta_{eff}^+ \lesssim \frac{M_+}{F_+}$$

i.e. the QCD term dominates the negative mass term of ϕ_+ at the origin

[D'Agnolo, Teresi, 21']

The D/T splitting problem in the Multiverse

Assumptions

There is some physics that scans the tuning in m_H^2 and m_T^2 with $v_\Sigma \simeq M_{GUT}$ fixed

The unified gauge group and particle content at M_{GUT} are the same in all universes
(not crucial for the discussion but allows us to make it more concrete)

The D/T splitting problem in the Multiverse

If μ_5 and δ are scanned over there will be contributions to $V(\phi_\pm)$ depending on μ_5 and δ

We can classify different values of μ_5 and δ by four different criteria:

1- $\langle H \rangle = 0$ or $\langle H \rangle > 0$
2 - $\langle T \rangle = 0$ or $\langle T \rangle > 0$
3 - $m_H \simeq M_{GUT}$ or $m_H \ll M_{GUT}$
4 - $m_T \simeq M_{GUT}$ or $m_T \ll M_{GUT}$

$$m_H^2 \equiv -\mu_5^2 + v_\Sigma^2 \left(-30\alpha + 4\beta - \frac{2\delta}{v_\Sigma}\right)$$

$$m_T^2 \equiv -\mu_5^2 + v_\Sigma^2 \left(-30\alpha + 9\beta + \frac{3\delta}{v_\Sigma}\right)$$

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4 - $m_T \simeq M_{GUT}$ or $m_T \ll M_{GUT}$

$$m_H^2 \equiv -\mu_5^2 + v_\Sigma^2 \left(-30\alpha + 4\beta - \frac{2\delta}{v_\Sigma}\right)$$

$$m_T^2 \equiv -\mu_5^2 + v_\Sigma^2 \left(-30\alpha + 9\beta + \frac{3\delta}{v_\Sigma}\right)$$

$\Longrightarrow$ We have to consider 16 qualitatively different Universes

Crunching the D/T splitting problem

We add to the minimal $SU(5)$ GUT two singlet scalars with the following potential

$$V_\phi = V_+ + V_- - \frac{\alpha_5}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta_5\right) \mathrm{Tr}\left[F_5 \tilde{F}_5\right]$$

with $F_\pm \gtrsim M_{GUT}$

Crunching the D/T splitting problem

At low energies the potential $V_{H\phi}$ becomes

$$V_{H\phi} \supset -N_3 \frac{\alpha_s}{8\pi}\left(\frac{\phi_+}{F_+}+\frac{\phi_-}{F_-}+\theta\right)\mathrm{Tr}\left[G\tilde{G}\right] - N_2 \frac{\alpha_w}{8\pi}\left(\frac{\phi_+}{F_+}+\frac{\phi_-}{F_-}+\theta_w\right)\mathrm{Tr}\left[W\tilde{W}\right]$$

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In the SM the second term can be removed by a $U(1)_{B+L}$ field redefinition, but in $SU(5)$ Yukawa interactions mediated by T explicitly break this anomalous symmetry

$\Longrightarrow$ We need to consider the contributions of both operators to $V(\phi_\pm)$!

Crunching the D/T splitting problem

At low energies the potential $V_{H\phi}$ becomes

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$\Longrightarrow$ We need to consider the contributions of both operators to $V(\phi_\pm)$

We have to evaluate UV instanton contributions to $V(\phi_\pm)$
from $SU(3)_c$ and $SU(2)_L$ in all the universes!

Crunching the D/T splitting problem

Instantons generate a potential for axion-like particles

In the UV, where the gauge coupling is small, instantons give the leading contribution to the potential

In the IR, where the gauge coupling increases, the strong dynamics give the leading contribution to the potential

Crunching the D/T splitting problem

Instantons generate a potential for axion-like particles

In the UV, where the gauge coupling is small, instantons give the leading contribution to the potential

In the IR, where the gauge coupling increases, the strong dynamics give the leading contribution to the potential

$\Longrightarrow$ We need to compute these contributions in each universe and compare them

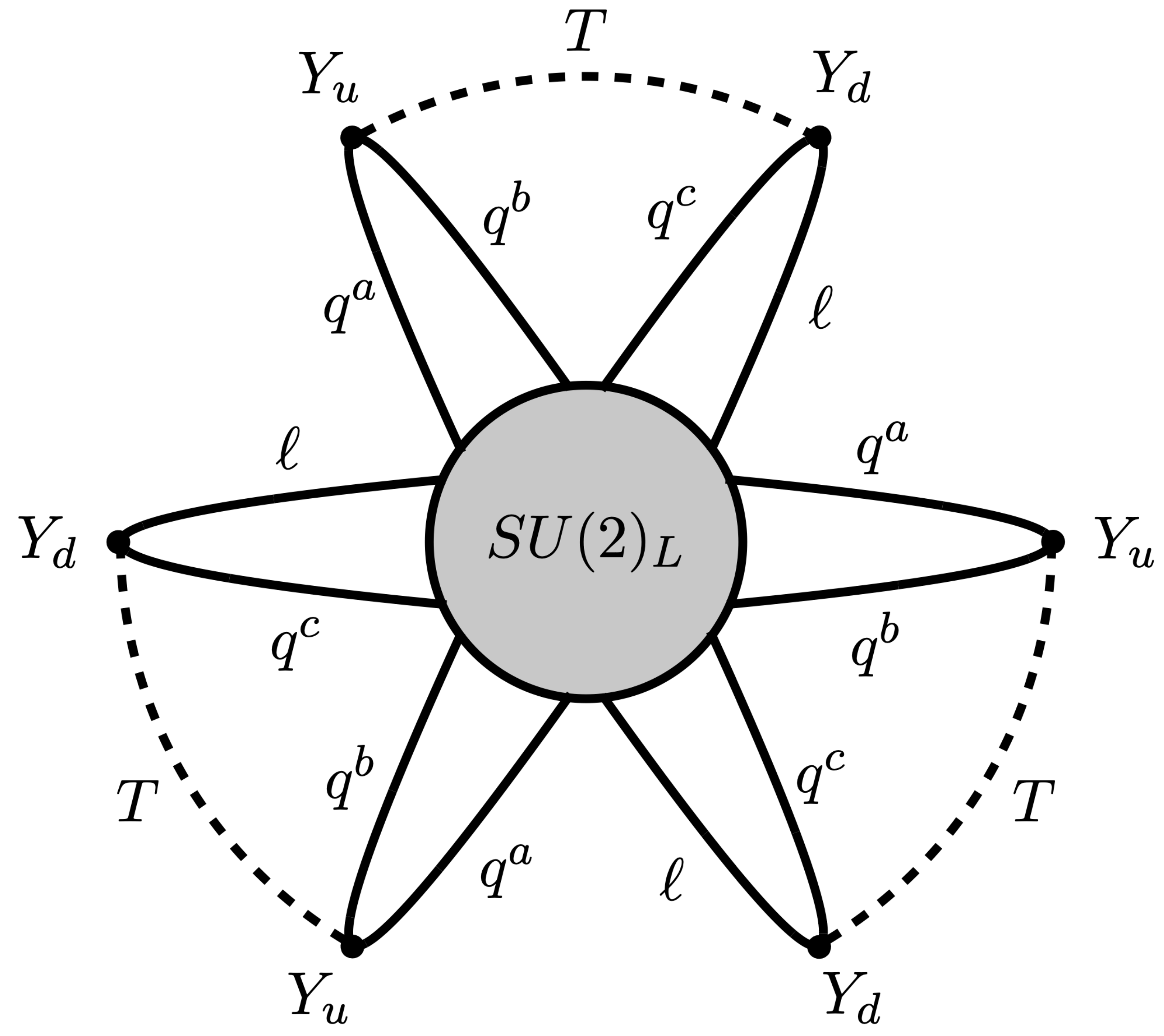
Crunching the D/T splitting problem

Example: $SU(2)_L$ (constrained) instantons

$SM : Tr\left[W\tilde{W}\right]$ is not observable

$B+L$ violating operator

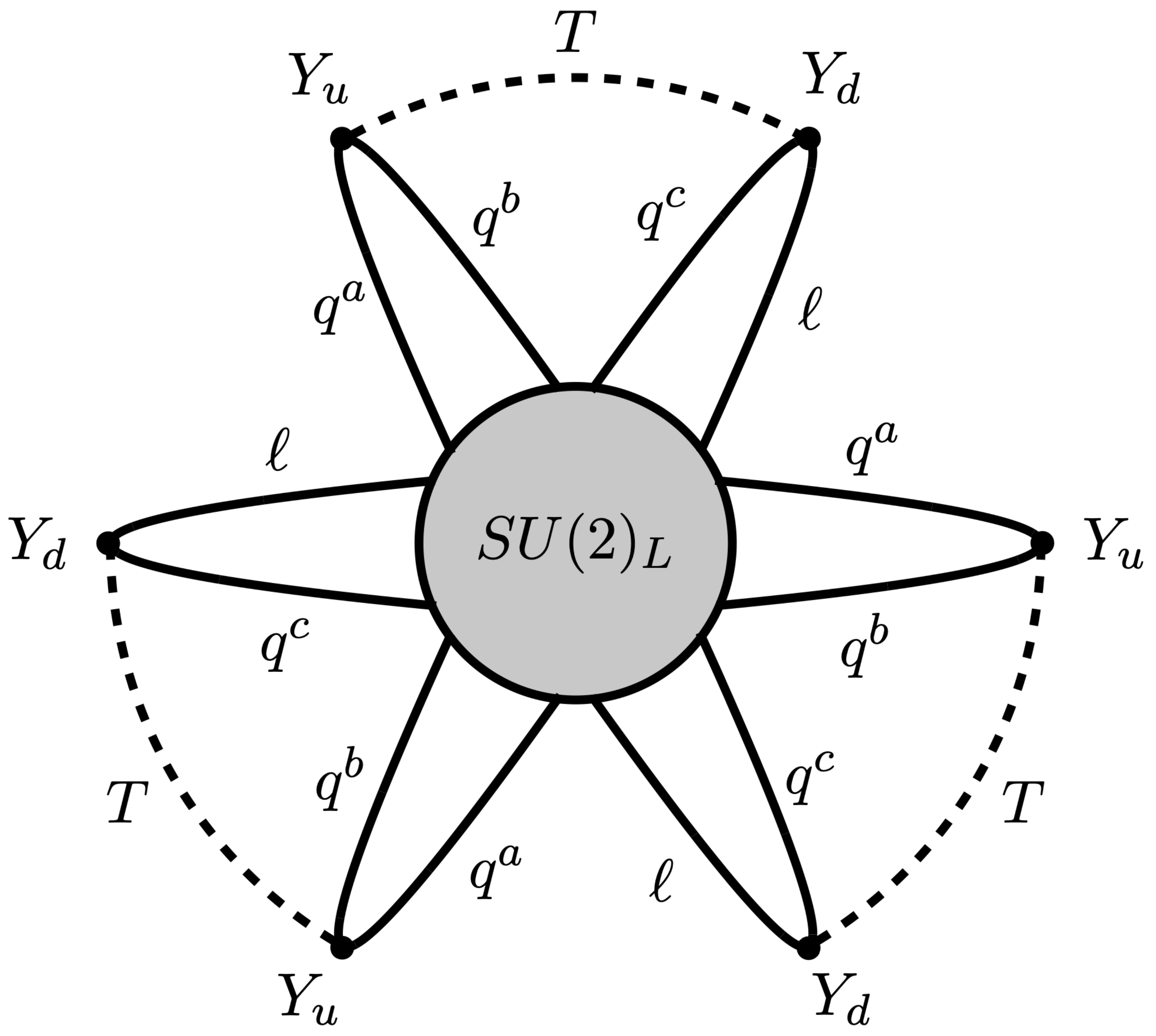
$$SU(5) \to SM : Tr\left[W\tilde{W}\right] + \frac{QQQL}{m_T^2}$$



Crunching the D/T splitting problem

Example: $SU(2)$ (constrained) instantons

$$\frac{V_{SU(2)_L}(\phi_\pm)}{\cos\left(\frac{\phi_+}{F_+}+\frac{\phi_-}{F_-}+\theta_w\right)} \sim \det(Y_u Y_d)\left[M_{GUT}^4\left(\frac{2\pi}{\alpha_{GUT}}\right)^4 e^{-\frac{2\pi}{\alpha_{GUT}}} + \frac{v_H^{10}}{M_{GUT}^6}\left(\frac{2\pi}{\alpha_w(v_H)}\right)^4 e^{-\frac{2\pi}{\alpha_W(v_H)}}\right]$$



Crunching the D/T splitting problem

The QCD scale Λ_{QCD}

In our universe the size of the $\phi_\pm$ potential is

$$\Lambda^4_{QCD} \equiv m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \simeq (80 \text{ MeV})^4$$

In practice this number is inaccessible in the other universes

In other universes we approximate Λ_{QCD} by the scale where $\alpha_s = 1 \Longrightarrow \Lambda_{SU(3)}$

Crunching the D/T splitting problem

The QCD scale Λ_{QCD}

All we need to check is that the $\phi_\pm$ potentials in other universes are very different from $\left(\Lambda^{us}_{SU(3)}\right)^4$

Crunching the D/T splitting problem

Our Universe has $\langle H \rangle > 0$, $\langle T \rangle = 0$, $m_H \ll M_{GUT}$ and $m_T \simeq M_{GUT}$

In this case the QCD IR contribution is

$$V_{\phi_\pm H} \simeq \frac{\Lambda^4(\langle h \rangle)}{2} \left(\theta + \frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} \right)^2$$

$$\Lambda^4(\langle h \rangle) = m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \qquad \text{for} \qquad m_{u,d} \lesssim 4\pi f_\pi$$

Crunching the D/T splitting problem

Our Universe has $\langle H \rangle > 0$, $\langle T \rangle = 0$, $m_H \ll M_{GUT}$ and $m_T \simeq M_{GUT}$

While the $SU(5)$ instanton contribution to $\phi_\pm$ potentials are:

$$\frac{V(\phi_\pm)}{\cos\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta_5\right)} \simeq \frac{\det(Y_{10}Y_5)}{(12\pi^2)^3} \times (2{,}10 \times 10^{-10}\,\text{GeV})^4$$

which gives a much smaller contribution than $\Lambda^4(\langle h \rangle) \simeq (80\,\text{GeV})^4$

Crunching the D/T splitting problem

Our Universe has $\langle H \rangle > 0$, $\langle T \rangle = 0$, $m_H \ll M_{GUT}$ and $m_T \simeq M_{GUT}$

The instantons contributions are negligible compared to the QCD strong dynamics contributions: **the mechanism works as expected here**

Crunching the D/T splitting problem

Doublet Higgs light Triplet Higgs Heavy			
$\langle H \rangle > 0$ $\langle T \rangle = 0$	$\langle H \rangle > 0$ $\langle T \rangle > 0$	$\langle H \rangle = 0$ $\langle T \rangle = 0$	$\langle H \rangle = 0$ $\langle T \rangle > 0$
Our universe: $\Lambda^4 \sim \Lambda^4_{\rm QCD}$	QCD broken @ GUT: $\Lambda^4 \ll \Lambda^4_{\rm QCD}$	Massless fermions $\Lambda^4 \ll \Lambda^4_{\rm QCD}$	QCD broken @ GUT: $\Lambda^4 \ll \Lambda^4_{\rm QCD}$
Doublet Higgs heavy Triplet Higgs light			
$\langle H \rangle > 0$ $\langle T \rangle = 0$	$\langle H \rangle > 0$ $\langle T \rangle > 0$	$\langle H \rangle = 0$ $\langle T \rangle = 0$	$\langle H \rangle = 0$ $\langle T \rangle > 0$
All fermions heavy. Confinement is earlier. $\Lambda^4 \gg \Lambda^4_{\rm QCD}$	All fermions heavy. Confinement earlier or Broken QCD $\Lambda^4 \gg \Lambda^4_{\rm QCD}$	Massless fermions $\Lambda^4 \ll \Lambda^4_{\rm QCD}$	EW confinement $\Lambda^4 \ll \Lambda^4_{\rm QCD}$
Doublet Higgs heavy Triplet Higgs heavy			
$\langle H \rangle > 0$ $\langle T \rangle = 0$	$\langle H \rangle > 0$ $\langle T \rangle > 0$	$\langle H \rangle = 0$ $\langle T \rangle = 0$	$\langle H \rangle = 0$ $\langle T \rangle > 0$
All fermions heavy. Confinement is earlier. $\Lambda^4 \gg \Lambda^4_{\rm QCD}$	QCD broken @ GUT: $\Lambda^4 \ll \Lambda^4_{\rm QCD}$	Massless fermions $\Lambda^4 \ll \Lambda^4_{\rm QCD}$	QCD broken @ GUT: EW confinement $\Lambda^4 \ll \Lambda^4_{\rm QCD}$
Doublet Higgs light Triplet Higgs light			
$\langle H \rangle > 0$ $\langle T \rangle = 0$	$\langle H \rangle > 0$ $\langle T \rangle > 0$	$\langle H \rangle = 0$ $\langle T \rangle = 0$	$\langle H \rangle = 0$ $\langle T \rangle > 0$
Confinement is later. $\Lambda^4 < \Lambda^4_{\rm QCD}$	QCD broken @ Triplet scale: $\Lambda^4 \ll \Lambda^4_{\rm QCD}$	Massless fermions $\Lambda^4 \ll \Lambda^4_{\rm QCD}$	QCD broken @ Triplet scale: EW confinement $\Lambda^4 \ll \Lambda^4_{\rm QCD}$

Conclusion