LOCAL ANALYTIC SECTOR SUBTRACTION: A NUMERICAL IMPLEMENTATION

Giovanni Limatola
Università degli Studi di Torino & INFN Sezione di Torino
Work done in collaboration with G. Bertolotti, P. Torrielli, M. Zaro

IRN TERASCALE @ LNF
16 April 2024

OUTLINE

- Motivation
- Local Analytic Sector Subtraction at NLO
- Numerical Implementation: MadNkLO
- Applications: $e^+e^- \to t\bar{t}$ @NLO in α_s
- Conclusions and Outlooks

MOTIVATION

- Entering in a very high precision era for LHC physics (HL-LCH)

- Huge amount of accurate data: increasing accuracy in theoretical computations in QCD (NLO, NNLO and beyond) is needed!

- Ambitious goals for next years: automatisation of NNLO QCD computations
- Many progresses for two loop amplitudes
  - Massive $2 \to 2$ processes
    [Bonciani et al.], [Melnikov et al.], [Dunbar et al.]
  - Massless $2 \to 3$ processes
    ([Badger et al.])

- Cancellation of Infrared divergences for fixed order QCD computations

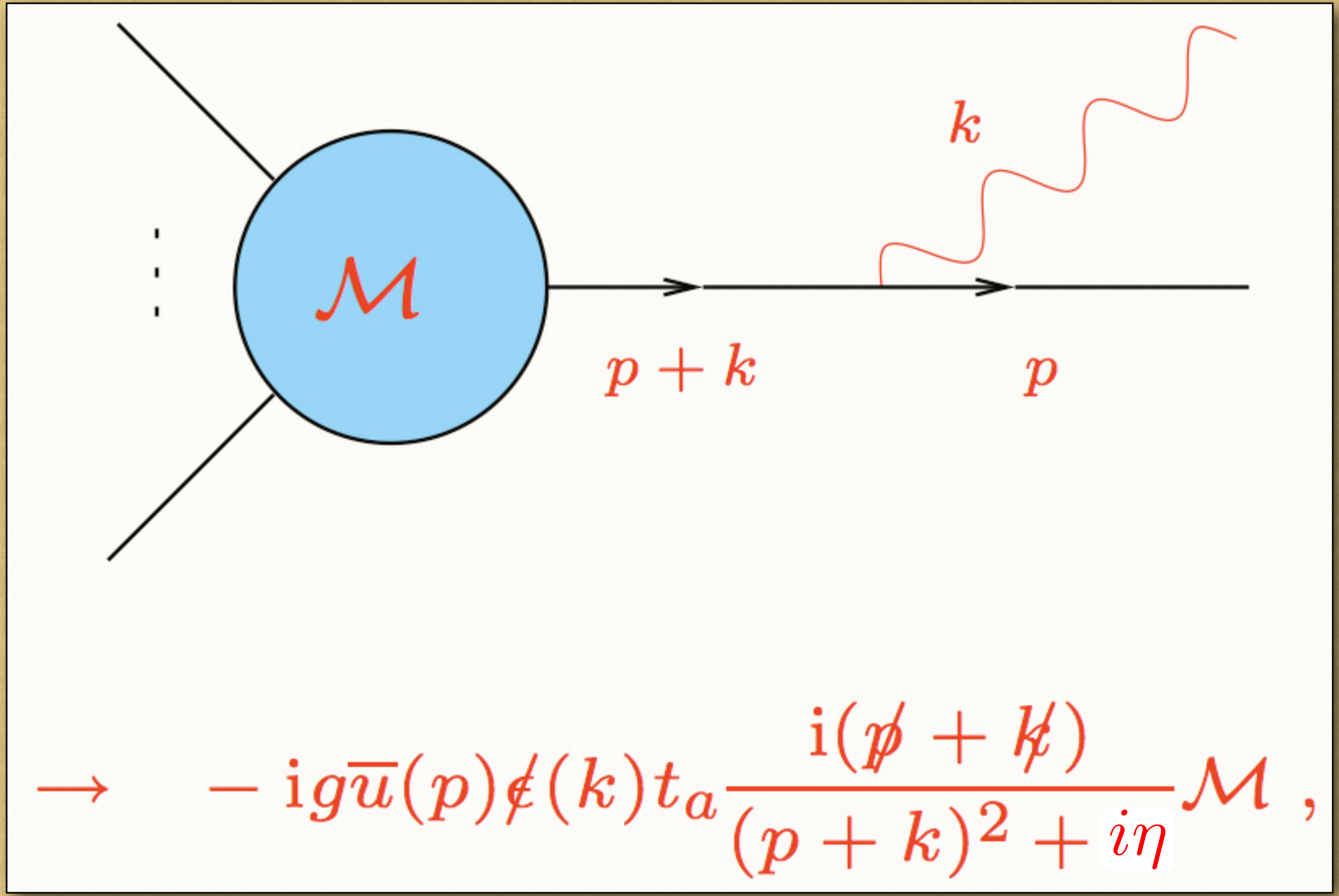
Subtraction at NLO fully automatised

- Frixione-Kunszt-Signer subtraction [Frixione, Kunszt, Signer ('95)]
- Catani-Seymour dipole subtraction [Catani, Seymour ('96), Catani, Dittmaier et al. ('02)]
- Nagy-Soper subtraction [Nagy, Soper ('03), Bevilacqua, Czakon et al. ('13)]

Towards the NNLO automatisation

- Antenna Subtraction [Gehrmann, Gehrmann De Ridder et al. ('95)]
- ColorFul Subtraction [Del Duca, Duhr, Kardos, Somogyi, Troscanyi et al.]
- Sector-improved residue subtraction [Czakon et al.]
- Nested soft-collinear subtraction. [Melnikov et al.]
- Local Analytic Sector Subtraction [Magnea, Maina, Torrielli, Uccirati et al.]
- q-T subtraction [Catani, Grazzini et al.]
- N-jettiness subtraction [Boughezal, Petriello et al.]
- ...many others

LOCAL ANALYTIC SECTOR SUBTRACTION AT NLO



(Thanks to Lorenzo Magnea)

- The emission amplitude diverges as long as

$$k^0p^0(1-\cos\theta_{kp})\to 0$$

- Phase space integration in $d=4-2\epsilon$ dimensions gives poles in ϵ

$$1/\epsilon, 1/\epsilon^2$$

- Same pole structure from IR region of virtual amplitude with opposite sign
- Factorisation of initial state singularities for hadronic collisions
- Final inclusive result is IR finite

A subtraction scheme is needed to compute differential distributions!

For a generic IRC-safe observable X we need to compute $(\delta_i \equiv \delta(X - X(\Phi_i)))$

$d\Phi_{n+1} = d\Phi_n \times d\Phi_{\rm rad}$

$$\frac{d\sigma_{\rm NLO}}{dX} = \int d\Phi_n \left(V\delta_n + \int d\Phi_{\rm rad} \bar{K}\delta_n \right)$$

$$+ \int d\Phi_{n+1} \left(R\delta_{n+1} - \bar{K}\delta_n \right)$$

Add and subtract a local counterterm $\bar{K}$ which **locally** reproduces IR poles of R and can be easily analytically integrated in $d = 4 - 2\epsilon$ dimensions!

The two terms are separately finite and can be numerically evaluated!

LOCAL ANALYTIC SECTOR SUBTRACTION:NLO STRATEGY

- Unitary partition of phase space Φ_{n+1} á la FKS [Frixione, Kunszt, Signer: 9512328]
- Introduction of sector functions $\mathcal{W}_{ij}$ such that
  - $\sum_{i,j\neq i} \mathcal{W}_{ij} = 1$
  - Soft and collinear limits still form a unitary partition

$$\mathbf{S}_i \sum_{k\neq i} \mathcal{W}_{ik} = 1 \qquad\qquad \mathbf{C}_{ij}[\mathcal{W}_{ij} + \mathcal{W}_{ji}] = 1$$

  - Summing over sectors we recover the full singular structure such that $\mathcal{W}_{ij}$ will not appear in counterterms
  - Minimise the number of singular regions of $R\mathcal{W}_{ij}$

LOCAL ANALYTIC SECTOR SUBTRACTION:NLO STRATEGY

- Singularities parametrised in terms of invariants $s_{ab} = 2k_a k_b$
- Factorised expressions for soft and collinear limits

$$\mathbf{S}_i R(\{k\}) = -\mathcal{N}_1 \sum_{l,m} \delta_{f_i g} \frac{s_{lm}}{s_{il} s_{im}} B_{lm}(\{k\}_{\not i})$$

$$\mathbf{C}_{ij} R(\{k\}) = \frac{\mathcal{N}_1}{s_{ij}} P_{ij}^{\mu\nu}(s_{ir}, s_{jr}) B_{\mu\nu}(\{k\}_{\not i \not j}, k)$$

$$\mathbf{S}_i \mathbf{C}_{ij} R(\{k\}) = 2\mathcal{N}_1 C_{f_j} \delta_{f_i g} \frac{s_{jr}}{s_{ij} s_{ir}} B(\{k\}_{i})$$

Born terms need to be evaluated with Born kinematics!

- For a given sector a candidate local counterterm can be defined as

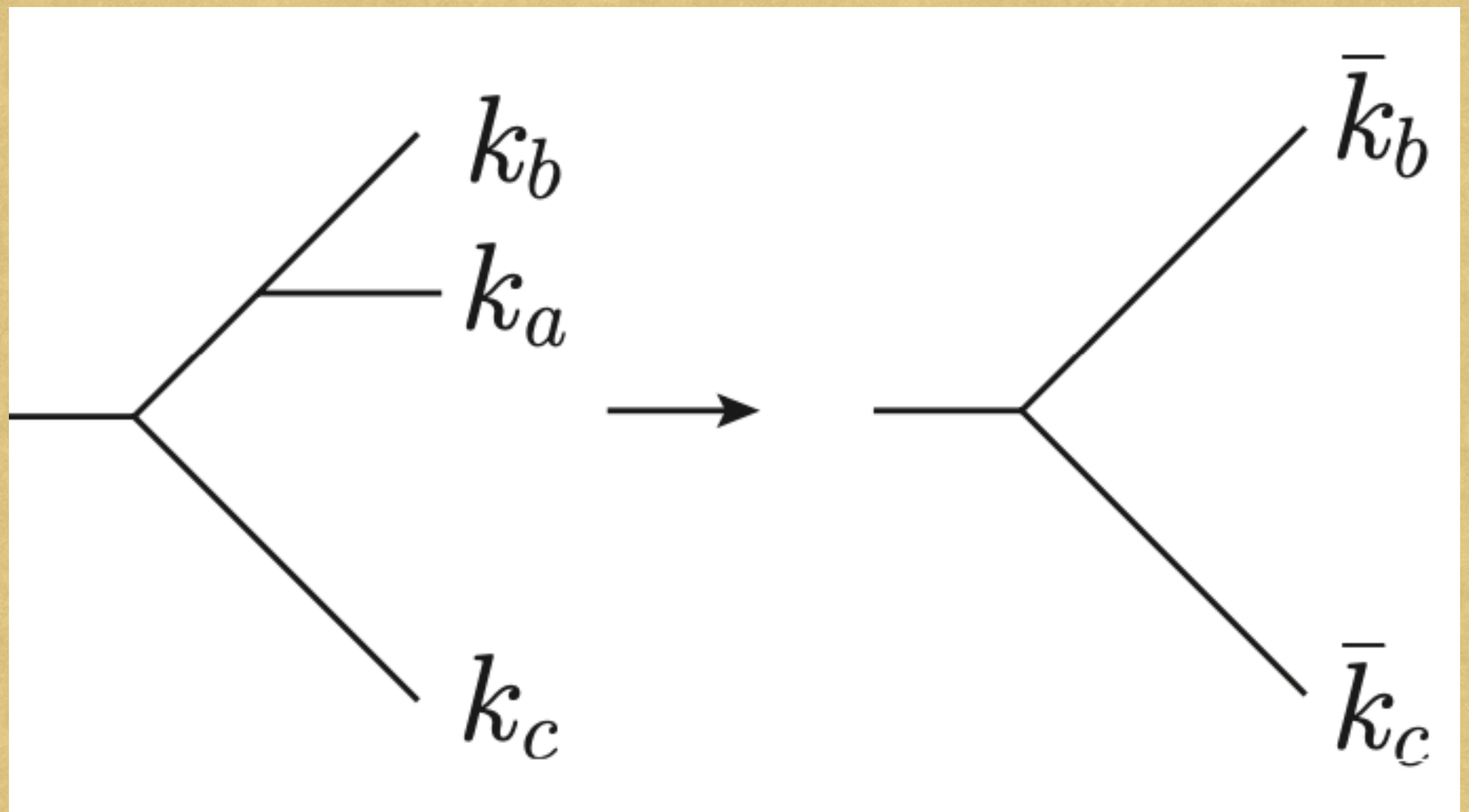
$$K_{ij} = (\mathbf{S}_i + \mathbf{C}_{ij} - \mathbf{S}_i \mathbf{C}_{ij}) R \mathcal{W}_{ij}$$

NLO STRUCTURE: MAPPING

- A mapping from $\Phi_{n+1} = \{k_1, \ldots, k_{n+1}\}$ to $\bar{\Phi}_n = \{\bar{k}_1, \ldots, \bar{k}_n\}$ is required
- Catani-Seymour [Catani, Seymour 9605323] final state dipole mapping $\{k\} \to \{\bar{k}\}^{(abc)}$

$$\bar{k}_b^{(abc)} = k_a + k_b - \frac{s_{ab}}{s_{ac}+s_{bc}} k_c$$

$$\bar{k}_c^{(abc)} = \frac{s_{abc}}{s_{ac}+s_{bc}} k_c$$



- Phase space parametrisation in terms of $y = \dfrac{s_{ab}}{s_{abc}}$ $\quad z = \dfrac{s_{ac}}{s_{ac}+s_{bc}}$

$$d\Phi_{n+1} = d\Phi_n^{(abc)} \times d\Phi_{\mathrm{rad}}^{(\mathrm{abc})}$$

$$d\Phi_{\mathrm{rad}}^{(\mathrm{abc})} \propto (\bar{s}_{bc}^{(abc)})^{(1-\epsilon)} \int_0^{\pi} d\phi \sin^{-2\epsilon}\phi \int_0^1 dy \int_0^1 dz \left[y(1-y)^2 z(1-z)\right]^{-\epsilon}(1-y)$$

NLO STRATEGY:SUMMARY

- Unitary partition of phase space introducing sector functions $\mathcal{W}_{ij}$
- Mapping adapted for each singular contribution (Catani-Seymour mapping)
- **Improved limits** to define counterterms
- **Local cancellation** of IR divergences!
- **Sum Rules** make $\mathcal{W}_{ij}$ disappear from $\bar{K}$

$$\frac{d\sigma_{\rm NLO}}{dX} = \int d\Phi_n \left(V\delta_n + \int d\Phi_{\rm rad} \bar{K} \delta_n \right) + \int d\Phi_{n+1} \left(R\delta_{n+1} - \bar{K}\delta_n \right)$$

→ Finite result for $\epsilon \to 0$

NLO STRATEGY:SUMMARY

- QCD@LHC (ISR & FSR):
  - NLO subtraction implemented for massless final state [Bertolotti et al. 2209.09123]
  - NLO for massive final state [V. Graziani's master thesis work]
- Extensions at NNLO (lepton colliders):
  - NNLO subtraction (massless case) achieved analytically [Bertolotti et al. 2212.11190]
  - Integration of counterterms
  - Checking of locality of the cancellation

Simple structure to be implemented in an informatics framework

NUMERICAL IMPLEMENTATION:

MadNkLO

(Adapted from Paolo Torrielli's slide)

- Numerical implementation in MadNkLO [Hirschi, Deutschmann, Lionetti et al.]
  MG5-inspired python framework to automatically generate all ingredients for NNLO

- Successful validation against MG5_aMC at NLO

Process	aMC LO	MadNkLO LO	aMC NLO corr.	MadNkLO NLO corr.
$e^+e^- \to jj$	0.53209(6)	0.53208(6)	0.019991(7)	0.019991(10)
$e^+e^- \to jjj$	0.4739(3)	0.4740(3)	-0.1461(1)	-0.1463(6)
$pp \to Z$	46361(3)	46362(3)	6810.9(8)	6810.8(4)
$pp \to Zj$	11270(7)	11258(5)	3770(6)	3776(17)
$pp \to W^+W^-j$	42.42(1)	42.39(2)	10.68(5)	10.53(13)

[Bertolotti er al. 2209.09123]

- Validation at inclusive level only: no distributions!

- Cluster resources required!

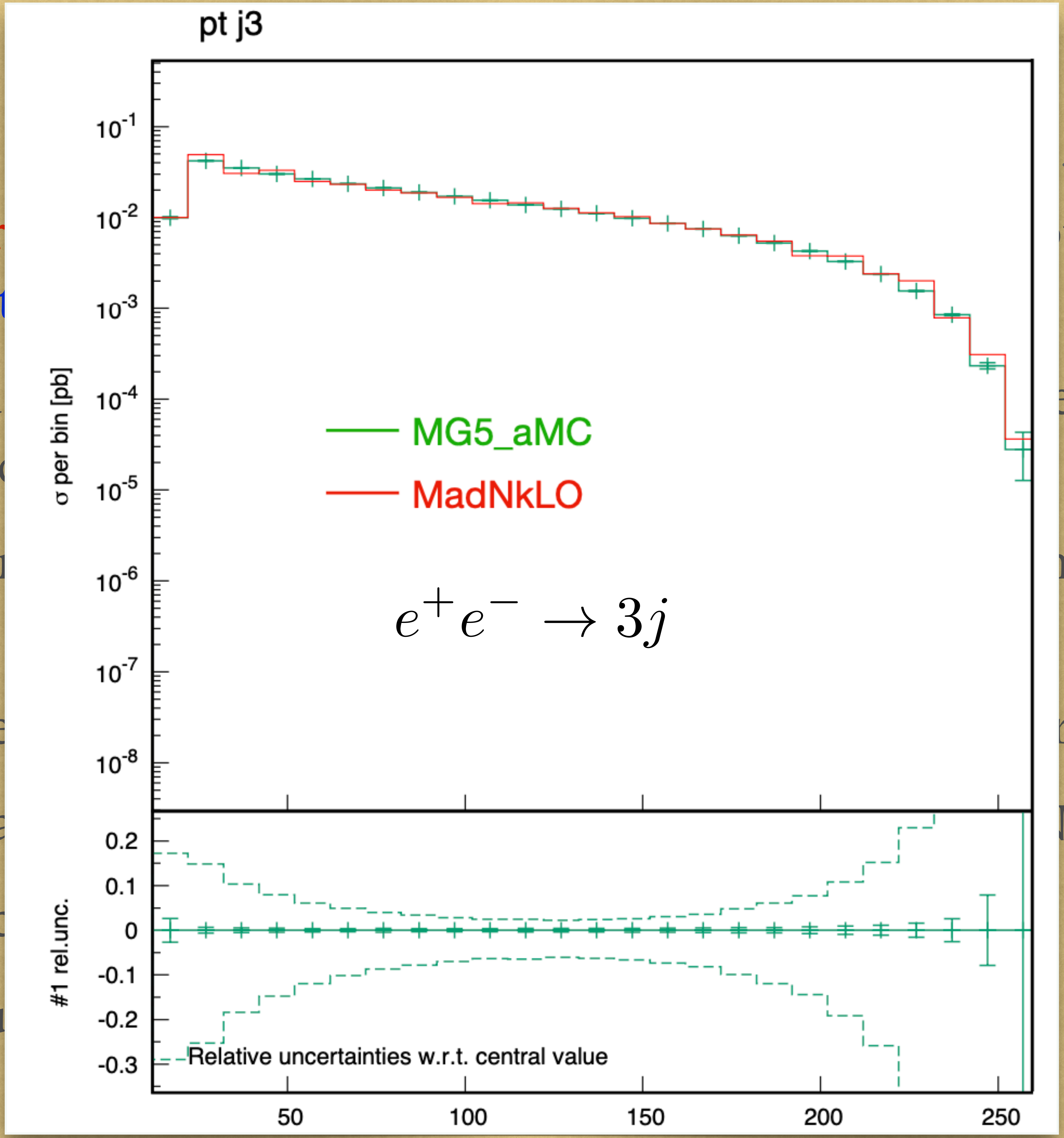
- Very inefficient code →
  - Python used at runtime
  - No efficient phase space integrator

New MadNkLO

- New python to Fortran implementation of Local Analytic Sector Subtraction in MadNkLO [Bertolotti, GL, Torrielli, Zaro]
- Python framework used to fill Fortran templates for process dependent subroutines (matrix elements and counterterms)
- Born Phase space generated with single-diagram multi-channelling method (MadGraph)
- Integration performed contribution by contribution in α_s and sector by sector
- NLO FSR validated against MG5_aMC differentially up to 3 jets at NLO
- Time performances comparable with MG5_aMC
- Further work is required in terms of optimisation…

New MadNkLO

- New python to Fortr… …traction in MadNkLO [Bertolott…
- Python framework u… …ent subroutines (matrix elements and…
- Born Phase space ge… …method (MadGraph)
- Integration performe… …r by sector
- NLO FSR validated a… …NLO
- Time performances c…
- Further work is requ…

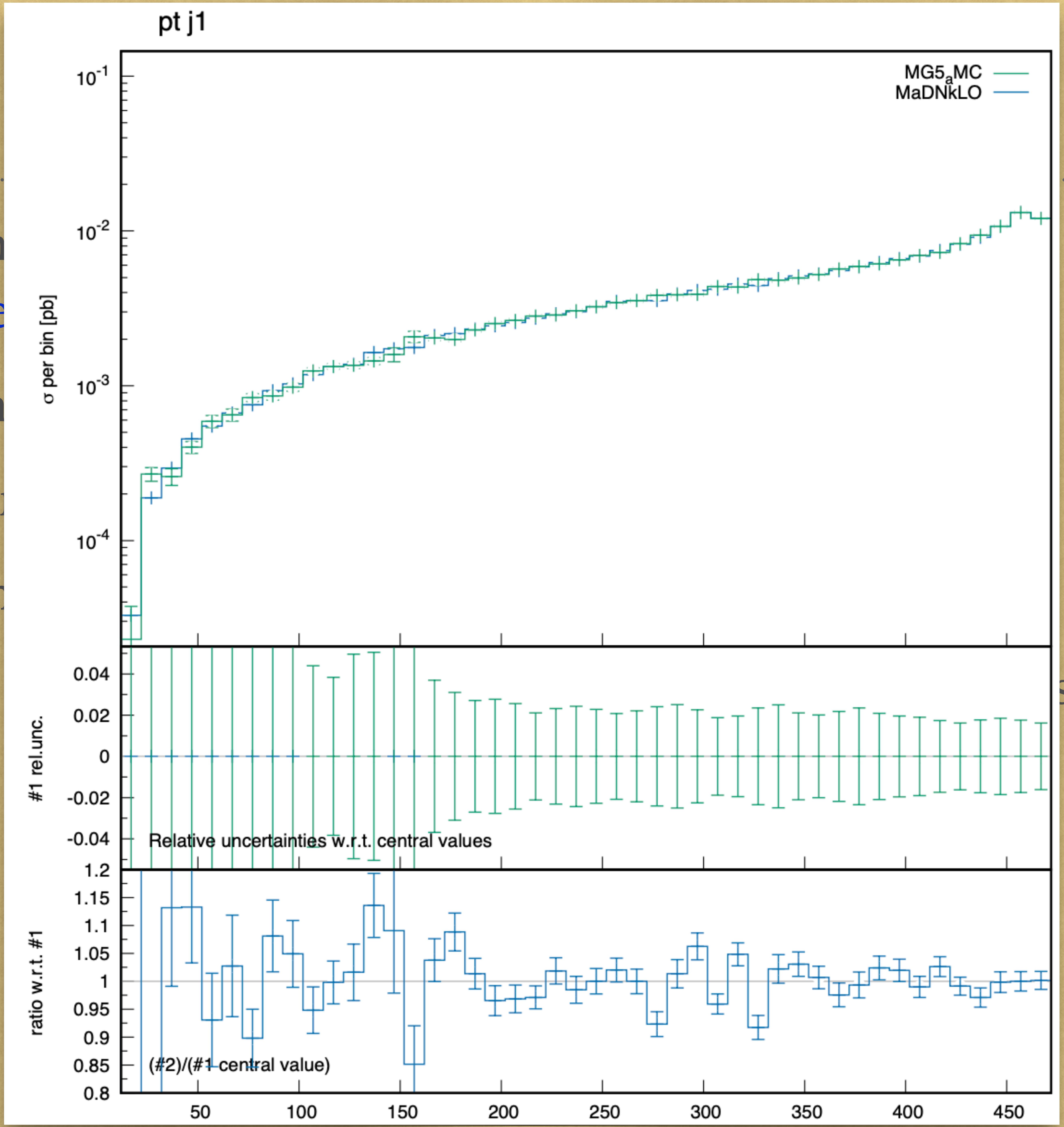


Recent Applications: $e^+e^- \to t\bar{t}$ @ NLO

- Dipole local mapping with massive emitter and massive recoiler [Catani, Dittmaier et al., 0201036]
- Less singular regions but non trivial integrals for counterterms
- Validation at differential level with MG5_aMC
- Very good agreement either at inclusive or at differential level!
- Good starting point to implement NNLO formulas for massive final state!

Recent Application

- Dipole local mappin
  [Catani, Dittmaier e
- Less singular region
- Validation at differe
- Very good agreemen
- Good starting point state!



CONCLUSIONS AND OUTLOOKS

- Local Analytic Sector Subtraction achieved analytically for massless final state at NLO both for lepton and hadron colliders
- Validation with a dedicated numeric framework
- Work required to optimise numerical implementation at NLO
- Extensions of NLO for massive final state (helpful for massive NNLO!)
- Analytic expressions for NNLO FSR massless
- Numerical implementation of NNLO formulas
- Work on NNLO ISR
- Automatisation of NNLO subtraction is our (ambitious) goal!

Thanks for your attention!!!

BACKUP

Dipole local mapping + Factorised expressions of R

Improved limits

$$\bar{\mathbf{S}}_i R(\{k\}) = -\mathcal{N}_1 \sum_{l,m} \delta_{f_i g} \frac{s_{lm}}{s_{il} s_{im}} B_{lm}(\bar{k}^{(ilm)})$$

$$\bar{\mathbf{C}}_{ij} R(\{k\}) = \frac{\mathcal{N}_1}{s_{ij}} P_{ij}^{\mu\nu}(s_{ir}, s_{jr}) B_{\mu\nu}(\{\bar{k}\}^{(ijr)})$$

$$\bar{\mathbf{S}}_i \bar{\mathbf{C}}_{ij} R(\{k\}) = 2\mathcal{N}_1 C_{f_j} \delta_{f_i g} \frac{s_{jr}}{s_{ij} s_{ir}} B(\{\bar{k}\}^{(ijr)})$$

Local counterterm

$$\bar{K}_{ij} = (\bar{\mathbf{S}}_i + \bar{\mathbf{C}}_{ij} - \bar{\mathbf{S}}_i \bar{\mathbf{C}}_{ij}) R \mathcal{W}_{ij}$$

Analytic integration of counterterms over the radiation phase space can be easily performed!

For a generic IRC-safe observable X at lepton collider we need to compute

$(\delta_i \equiv \delta(X - X(\Phi_i)))$

$$\frac{d\sigma_{\mathrm{NLO}}}{dX} = \int d\Phi_n V \delta_n$$ → Explicit poles in ϵ

$$+ \int d\Phi_{n+1} R \delta_{n+1}$$ → Poles in ϵ due to phase space integration

Add and subtract a local counterterm $\bar{K}$ which **locally** reproduces IR poles of R and can be easily analytically integrated in $d = 4 - 2\epsilon$ dimensions!