Novel Weinberg-like operators from new scalar multiplets

A. Giarnetti, J. Herrero-Garcia, SM, D. Meloni and D. Vatsyayan,
"Neutrino masses from new Weinberg-like operators: Phenomenology of TeV scalar multiplets", arXiv:2312.13356 [hep-ph]

A. Giarnetti, J. Herrero-Garcia, SM, D. Meloni and D. Vatsyayan,
"Neutrino masses from new seesaw models: Low-scale variants and phenomenological implications", arXiv:2312.14119 [hep-ph]

IRN Terascale @ LNF
Auditorium B. Touschek

Laboratori Nazionali di Frascati, 15/04/2024

S. Marciano

Outline of the talk

- Neutrino masses: who ordered that?
- The Weinberg operator with higher SU(2) representations
- List of *genuine* models
- Bounds from the custodial symmetry
- Tree level vs 1-loop neutrino masses
- Phenomenology

Neutrinos change flavor (=oscillate) during their propagation

Neutrinos must have non-zero masses!

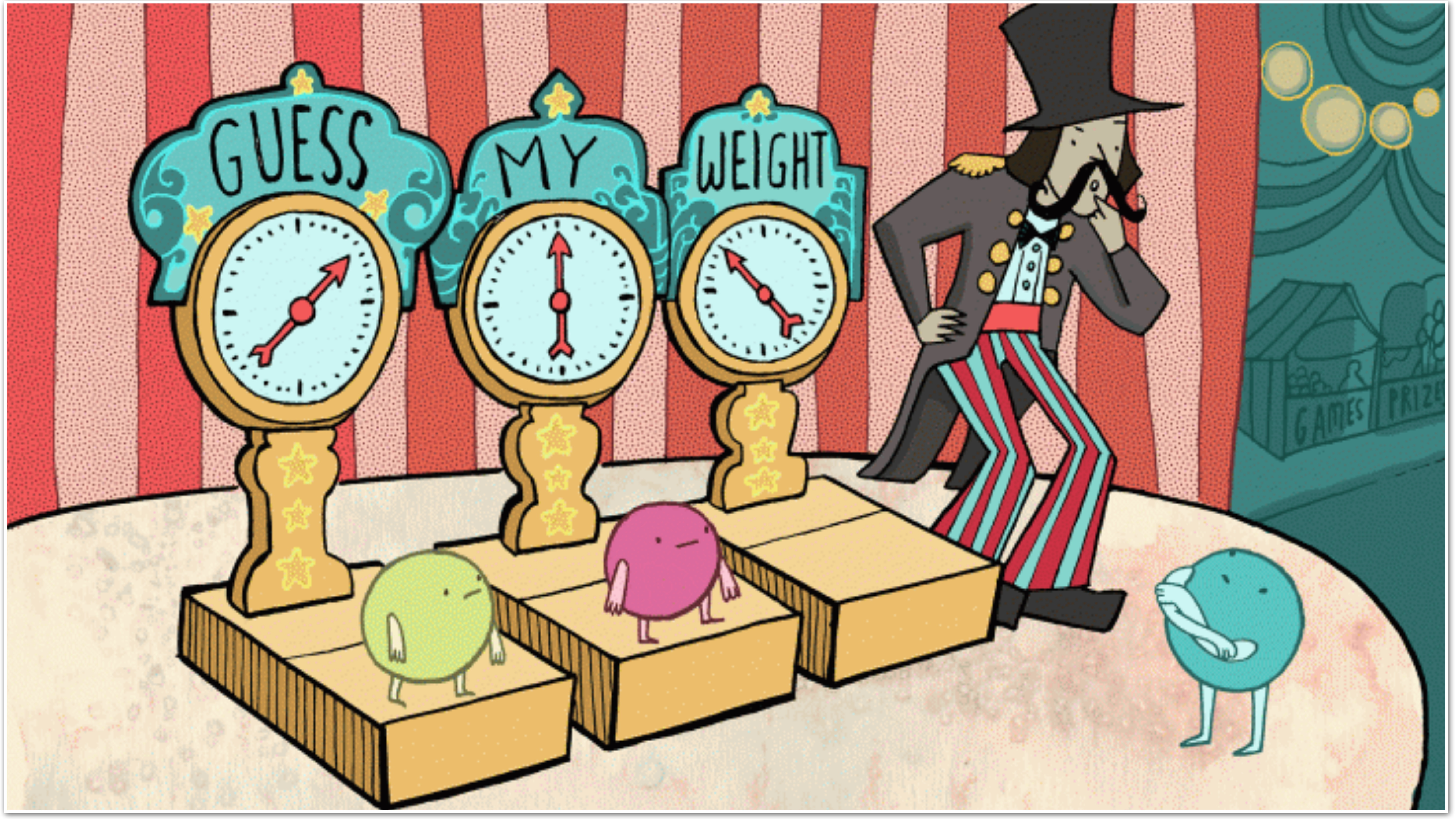
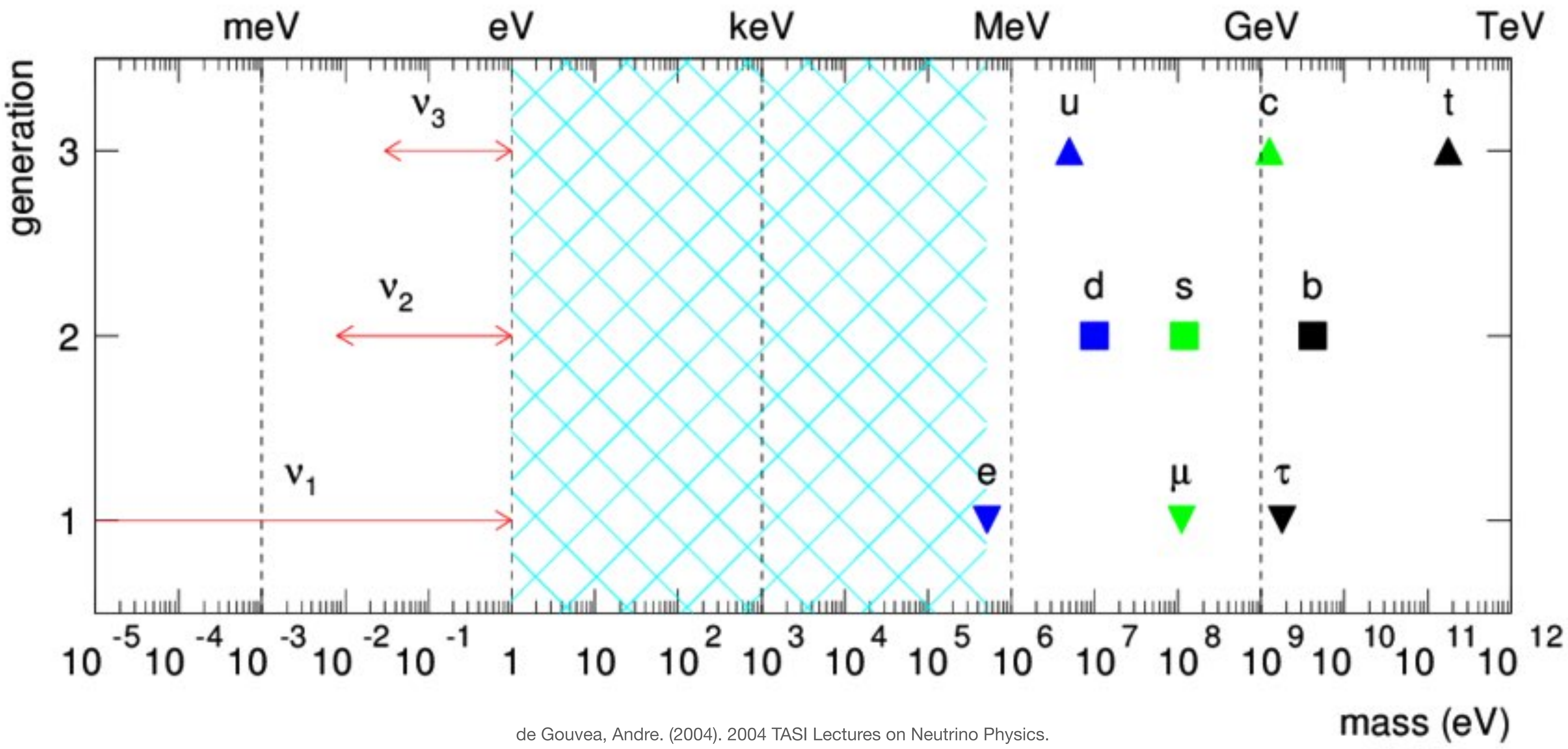


Illustration by Sandbox Studio, Chicago

Neutrino masses



de Gouvea, Andre. (2004). 2004 TASI Lectures on Neutrino Physics.

Effective Lagrangian

Adding to the Standard Model (SM) the most general higher-dimensional Lagrangian respecting the gauge symmetries of the SM is

$$\mathscr{L}_{eft} = \mathscr{L}_{SM} + \sum_{d>4,i} \frac{c_i}{\Lambda^{d-4}} \mathcal{O}^{(d)}$$

Each operator is suppressed by inverse powers of Λ, *i.e.* the New Physics (NP) energy scale.
Notice that we are "agnostic" about NP sources in a purely Effective Field Theory approach. Then, one can question how these higher dimensional operators can be obtained from the full theory.

Only one $d = 5$ operator (Weinberg operator) can be written and it is related to the neutrino Majorana mass term:

$$\mathcal{L}_5 = \frac{c}{\Lambda} LLHH$$

It generates a Majorana mass:

$$m_\nu \simeq c\frac{v^2}{\Lambda}.$$

Just to have an idea of the NP scale we can assume $c \sim 1,\ v \sim 100$ GeV and $m_\nu \sim 1$ eV, obtaining:

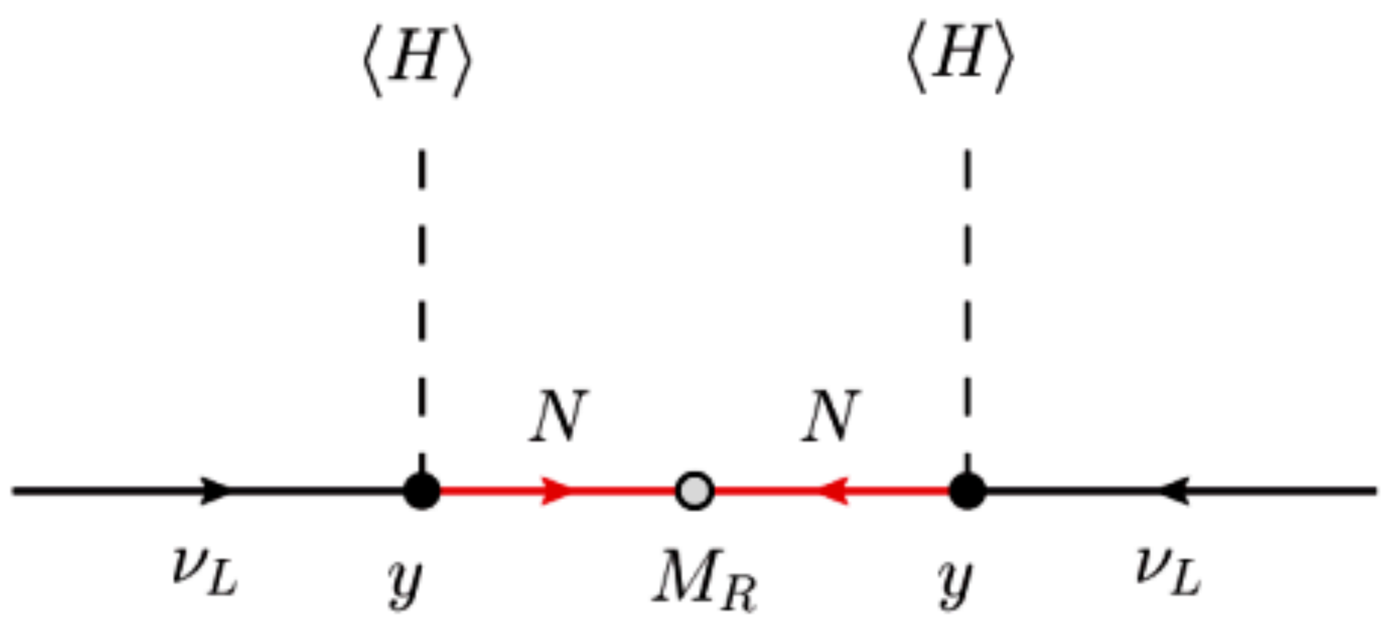
$$\Lambda \simeq \frac{v^2}{m_\nu} \simeq 10^{13}\ \text{GeV}$$

This is the typical scale of NP

(unless $c \ll 1$)

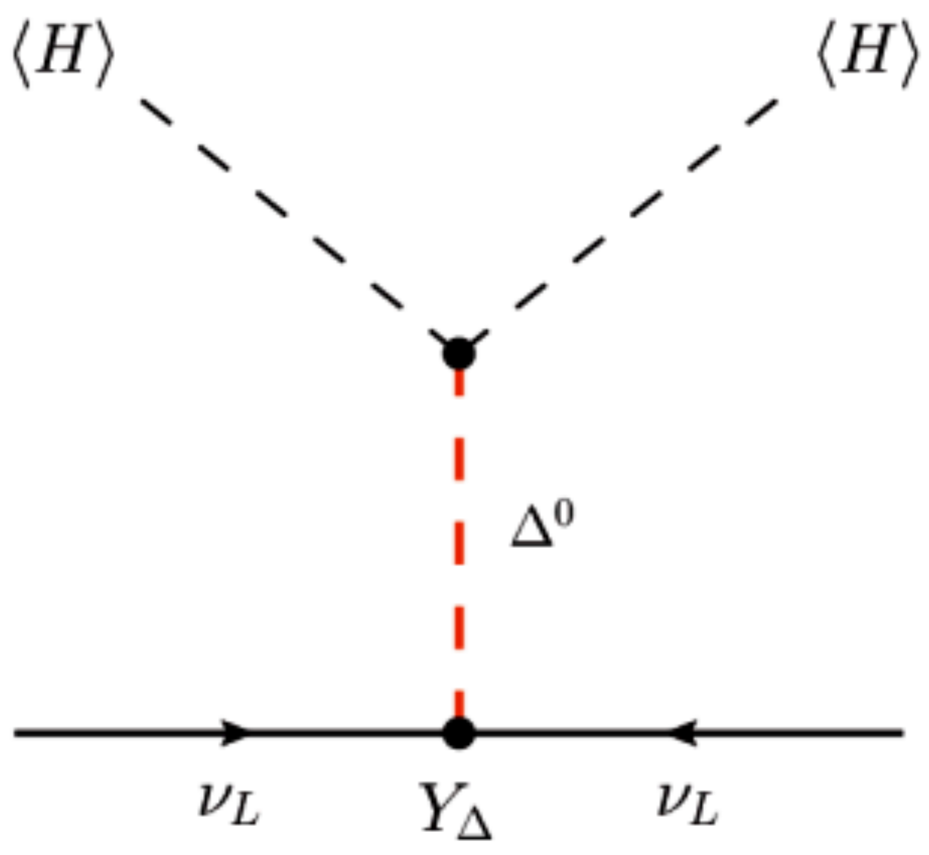
We can obtain the effective dimension-5 operator by integrating out the heavy degrees of freedom. Usually, the UV completions that lead to the Weinberg operator go by the name of *seesaw models* (Type-I, -II and -III)

Type- I



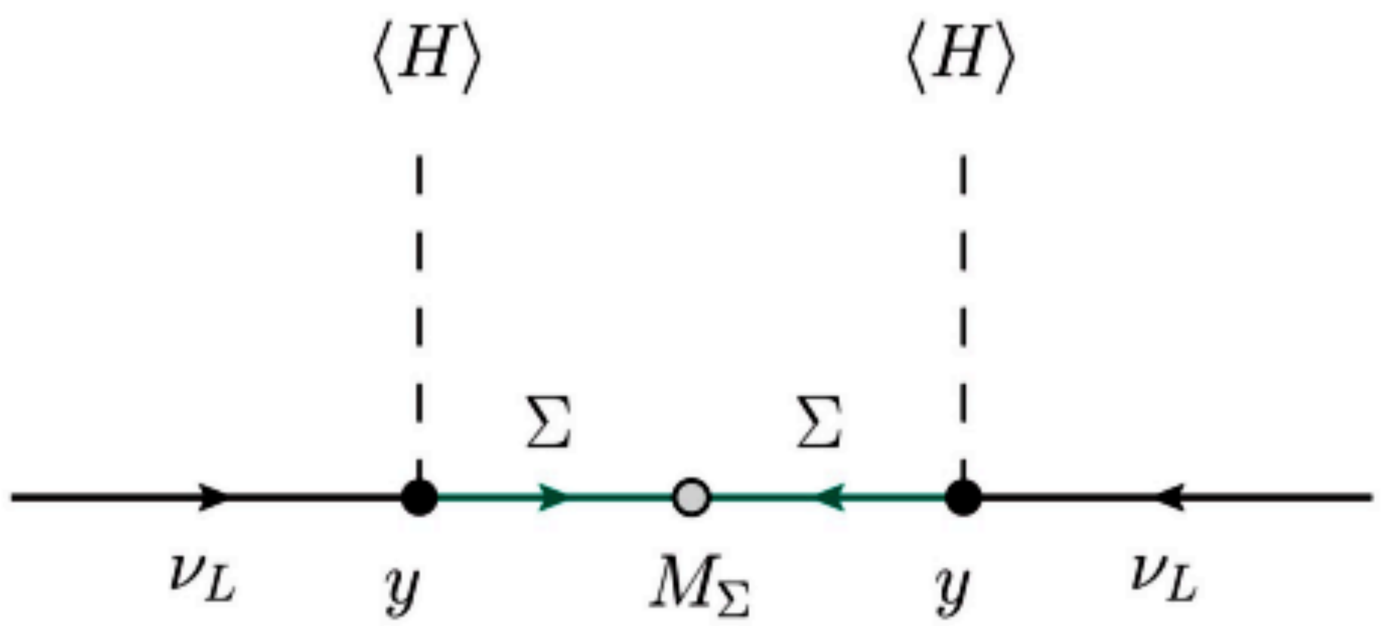
Minkowski, Yanagida, Mohapatra, Senjanovic, Gell-Mann, Ramond, Slansky, Schechter, Valle

Type- II



Schechter, Valle, Mohapatra, Senjanovic

Type- III



Foot, Lew, He, Joshi

What if new Higgs-like particles come into play?

Higher SU(2) representations

BSM Higgs-like scalars

$$\phi_i = (N_i, Y_i) \quad i = 1,2$$

NP degrees of freedom up to representations 5

$\rho(N_i, Y_i, v_{\phi_i}) \simeq 1$ $\longrightarrow$

$$v_{\phi_i} \ll v_\phi$$

Higher SU(2) representations

BSM Higgs-like scalars

$$\phi_i = (N_i, Y_i) \quad i = 1,2$$

$\xrightarrow{\rho(N_i, Y_i, v_{\phi_i}) \simeq 1}$

$$v_{\phi_i} \ll v_\phi$$

$$\mathcal{L}^5_{eft} = \frac{c_{0i}}{\Lambda}\phi\phi_i LL + \frac{c_{ii}}{\Lambda}\phi_i\phi_i LL + \frac{c_{ij}}{\Lambda}\phi_i\phi_j LL$$

The choice for the new Higgs-like scalars and for the heavy mediators will be done in order to avoid the 2HDM (widely studied in literature) and the usual Type-I, -II, -III seesaws (=the BSM contribution would be just a sub-leading correction)

Higher SU(2) representations

BSM Higgs-like scalars

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$$v_{\phi_i} \ll v_\phi$$

$$\mathscr{L}^5_{eft} = \frac{c_{0i}}{\Lambda}\phi\phi_i LL + \frac{c_{ii}}{\Lambda}\phi_i\phi_i LL + \frac{c_{ij}}{\Lambda}\phi_i\phi_j LL$$

The singlet can be obtained only if the new scalar transforms as a *quadruplet* under SU(2), seen that $2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2'$. Also $Y_i = \pm(1/2, 3/2)$.

- $(\phi\phi_i)_{1,3,5}(LL)_{1,3}$. In this case, the UV completion must contain a scalar singlet or triplet. However, the case with the scalar singlet does not provide any contribution to the neutrino masses.
- $(\phi L)_{1,3}(\phi_i L)_{3,5}$. Only the fermion triplets can mediate this process.

Higher SU(2) representations

BSM Higgs-like scalars

$$\phi_i = (N_i, Y_i) \quad i = 1,2$$

$\rho(N_i, Y_i, v_{\phi_i}) \simeq 1$ ⟶ $v_{\phi_i} \ll v_\phi$

$$\mathscr{L}^5_{eft} = \frac{c_{0i}}{\Lambda}\phi\phi_i LL + \frac{c_{ii}}{\Lambda}\phi_i\phi_i LL + \frac{c_{ij}}{\Lambda}\phi_i\phi_j LL$$

In this case $\phi_i = (2N, \pm|1/2|)$ with $N > 1$. Again, the interesting case is the *quadruplet.*

- $(\phi_i\phi_i)_{1,3,5,7}(LL)_{1,3}$. In this case, either a scalar singlet (which does not provide neutrino masses), or a scalar triplet (leading to sub-leading BSM contributions) is needed.
- $(\phi_i L)_{3,5}(\phi_i L)_{3,5}$. The possible fermion mediators are triplets or pentuplets.

Higher SU(2) representations

BSM Higgs-like scalars

$\phi_i = (N_i, Y_i) \quad i = 1{,}2$ $\xrightarrow{\rho(N_i, Y_i, v_{\phi_i}) \simeq 1}$ $v_{\phi_i} \ll v_\phi$

$$\mathscr{L}^5_{eft} = \frac{c_{0i}}{\Lambda}\phi\phi_i LL + \frac{c_{ii}}{\Lambda}\phi_i\phi_i LL + \frac{c_{ij}}{\Lambda}\phi_i\phi_j LL$$

Let's consider $\phi_i = (N_i, Y_i)$ and $\phi_j = (N_j, Y_j)$ with $N_{i,j} > 2$. Also $|Y_i + Y_j| = 1$.

- $(\phi_i\phi_i)_{N_i \otimes N_j}(LL)_{1,3}$. As already discussed, this case is not interesting.
- $(\phi_i L)_{N_i \otimes 2}(\phi_j L)_{N_j \otimes 2}$. Notice that $N \otimes 2 = (N-1) \oplus (N+1)$, therefore either $N_i = N_j + 2$ or $N_i = N_j$ is needed in order to build a singlet from such a contraction.

Viable models

EFT

Models	New Scalars
$\mathbf{A_I}$	$\Phi_1 = \mathbf{4}^S_{-1/2}$
$\mathbf{A_{II}}$	$\Phi_1 = \mathbf{4}^S_{-3/2}$
$\mathbf{B_I}$	$\Phi_1 = \mathbf{4}^S_{1/2} \quad \Phi_2 = \mathbf{4}^S_{-3/2}$
$\mathbf{B_{II}}$	$\Phi_1 = \mathbf{3}^S_0 \quad \Phi_2 = \mathbf{5}^S_{-1}$
$\mathbf{B_{III}}$	$\Phi_1 = \mathbf{5}^S_1 \quad \Phi_2 = \mathbf{5}^S_{-2}$
$\mathbf{B_{IV}}$	$\Phi_1 = \mathbf{5}^S_0 \quad \Phi_2 = \mathbf{5}^S_{-1}$
$\mathbf{B_V}$	$\Phi_1 = \mathbf{3}^S_0 \quad \Phi_2 = \mathbf{3}^S_{-1}$
$\mathbf{B_{VI}}$	$\Phi_1 = \mathbf{3}^S_{-1} \quad \Phi_2 = \mathbf{5}^S_0$

Viable models

UV completions

Models	New Scalars	Mediator	Op.	Wilson Coeffients
$\mathbf{A_I}$	$\Phi_1 = \mathbf{4}^S_{-1/2}$	$\Sigma = \mathbf{5}^F_0$	$\mathcal{O}_5^{(2)}$	$C_5^{(2)} = y_1 M_\Sigma^{-1} y_1^T$
$\mathbf{A_{II}}$	$\Phi_1 = \mathbf{4}^S_{-3/2}$	$\mathcal{F} = \mathbf{3}^F_{-1}$	$\mathcal{O}_5^{(1)}$	$C_5^{(1)} = y_1 M_\mathcal{F}^{-1} y_H^T + y_H M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_I}$	$\Phi_1 = \mathbf{4}^S_{1/2}$ $\Phi_2 = \mathbf{4}^S_{-3/2}$	$\mathcal{F} = \mathbf{5}^F_{-1}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_{II}}$	$\Phi_1 = \mathbf{3}^S_0$ $\Phi_2 = \mathbf{5}^S_{-1}$	$\mathcal{F} = \mathbf{4}^F_{-1/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_{III}}$	$\Phi_1 = \mathbf{5}^S_1$ $\Phi_2 = \mathbf{5}^S_{-2}$	$\mathcal{F} = \mathbf{4}^F_{3/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_{IV}}$	$\Phi_1 = \mathbf{5}^S_0$ $\Phi_2 = \mathbf{5}^S_{-1}$	$\mathcal{F} = \mathbf{4}^F_{1/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_V}$	$\Phi_1 = \mathbf{3}^S_0$ $\Phi_2 = \mathbf{3}^S_{-1}$	$\sim$	$\sim$	$\sim$
$\mathbf{B_{VI}}$	$\Phi_1 = \mathbf{3}^S_{-1}$ $\Phi_2 = \mathbf{5}^S_0$	$\sim$	$\sim$	$\sim$

Type-II

Viable models

UV completions

Models	New Scalars	Mediator	Op.	Wilson Coeffients
$\mathbf{A_I}$	$\Phi_1 = \mathbf{4}^S_{-1/2}$	$\Sigma = \mathbf{5}^F_0$	$\mathcal{O}_5^{(2)}$	$C_5^{(2)} = y_1 M_\Sigma^{-1} y_1^T$
$\mathbf{A_{II}}$	$\Phi_1 = \mathbf{4}^S_{-3/2}$	$\mathcal{F} = \mathbf{3}^F_{-1}$	$\mathcal{O}_5^{(1)}$	$C_5^{(1)} = y_1 M_\mathcal{F}^{-1} y_H^T + y_H M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_I}$	$\Phi_1 = \mathbf{4}^S_{1/2}$ $\Phi_2 = \mathbf{4}^S_{-3/2}$	$\mathcal{F} = \mathbf{5}^F_{-1}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_{II}}$	$\Phi_1 = \mathbf{3}^S_0$ $\Phi_2 = \mathbf{5}^S_{-1}$	$\mathcal{F} = \mathbf{4}^F_{-1/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_{III}}$	$\Phi_1 = \mathbf{5}^S_1$ $\Phi_2 = \mathbf{5}^S_{-2}$	$\mathcal{F} = \mathbf{4}^F_{3/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_{IV}}$	$\Phi_1 = \mathbf{5}^S_0$ $\Phi_2 = \mathbf{5}^S_{-1}$	$\mathcal{F} = \mathbf{4}^F_{1/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$
$\mathbf{B_V}$	$\Phi_1 = \mathbf{3}^S_0$ $\Phi_2 = \mathbf{3}^S_{-1}$	$\sim$	$\sim$	$\sim$
$\mathbf{B_{VI}}$	$\Phi_1 = \mathbf{3}^S_{-1}$ $\Phi_2 = \mathbf{5}^S_0$	$\sim$	$\sim$	$\sim$

Type-II

$$-\mathcal{L}_5 = \frac{1}{2}\sum_i C_5^{(i)}\,\mathcal{O}_5^{(i)} + \text{H.c.}$$

$$\mathcal{O}_5^{(0)} = (LH)_\mathbf{1}(LH)_\mathbf{1}\,, \qquad \mathcal{O}_5^{(1)} = (LH)_\mathbf{N}(L\Phi_1)_\mathbf{N}$$

$$\mathcal{O}_5^{(2)} = (L\Phi_1)_\mathbf{N}(L\Phi_1)_\mathbf{N}\,, \qquad \mathcal{O}_5^{(3)} = (L\Phi_1)_\mathbf{N}(L\Phi_2)_\mathbf{N}$$

Bounds on the v_{bsm}

$$\phi_1 = (N_1, Y_1) \qquad \phi_2 = (N_2, Y_2) \qquad \rho = m_W^2 / \left(c_w^2 m_Z^2\right)$$

$$\rho - 1 = \Delta\rho\left(N_i, Y_i, v_i\right) = \frac{\left[\left(\frac{N_1^2-1}{4}\right) - 3Y_1^2\right] v_1^2 + \left[\left(\frac{N_2^2-1}{4}\right) - 3Y_2^2\right] v_2^2}{\left(\sqrt{2}G_F\right)^{-1} - \left[\left(\frac{N_1^2-1}{4}\right) - 3Y_1^2\right] v_1^2 - \left[\left(\frac{N_2^2-1}{4}\right) - 3Y_2^2\right] v_2^2}$$

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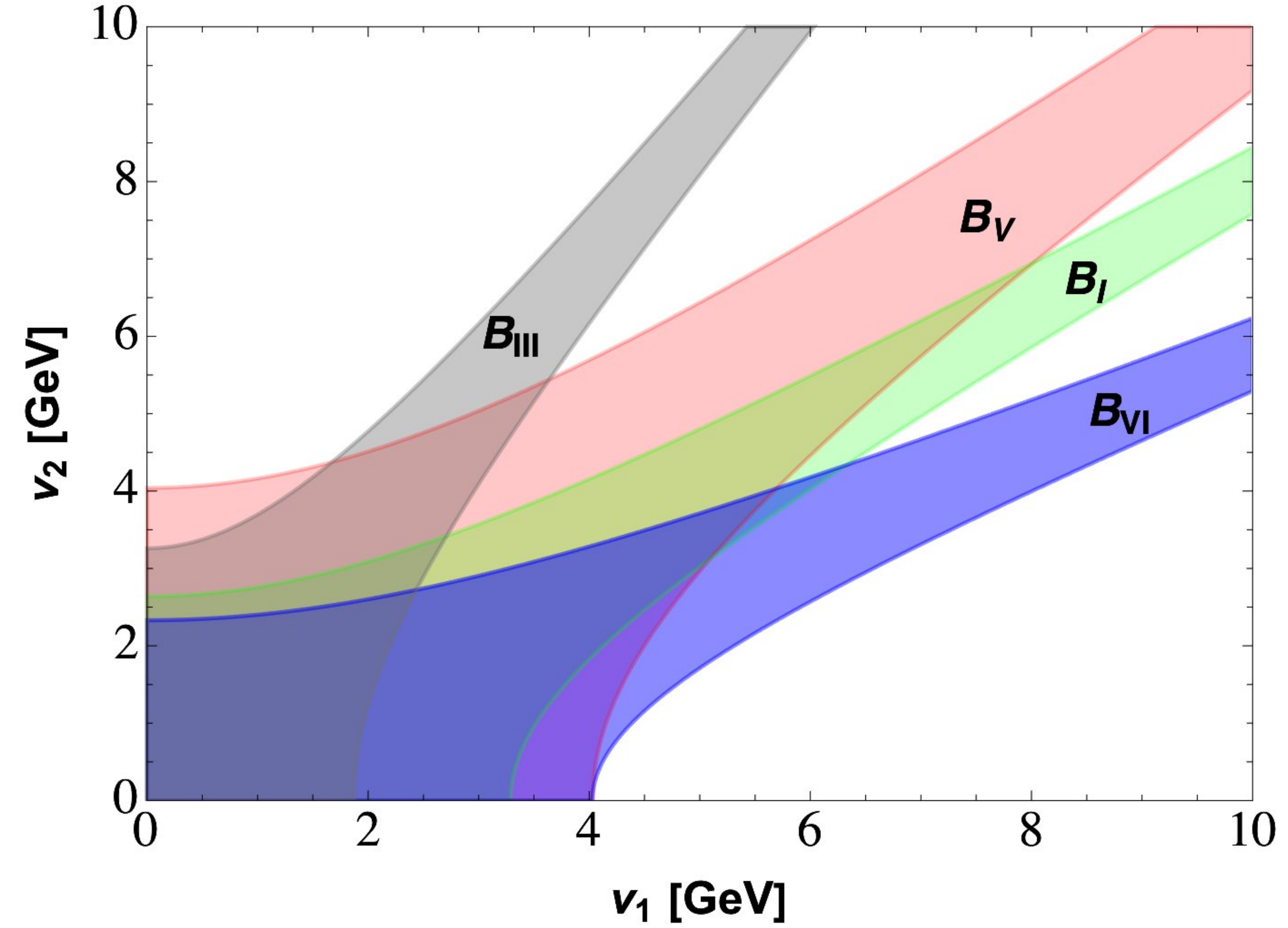
From the global fit of electroweak precision data

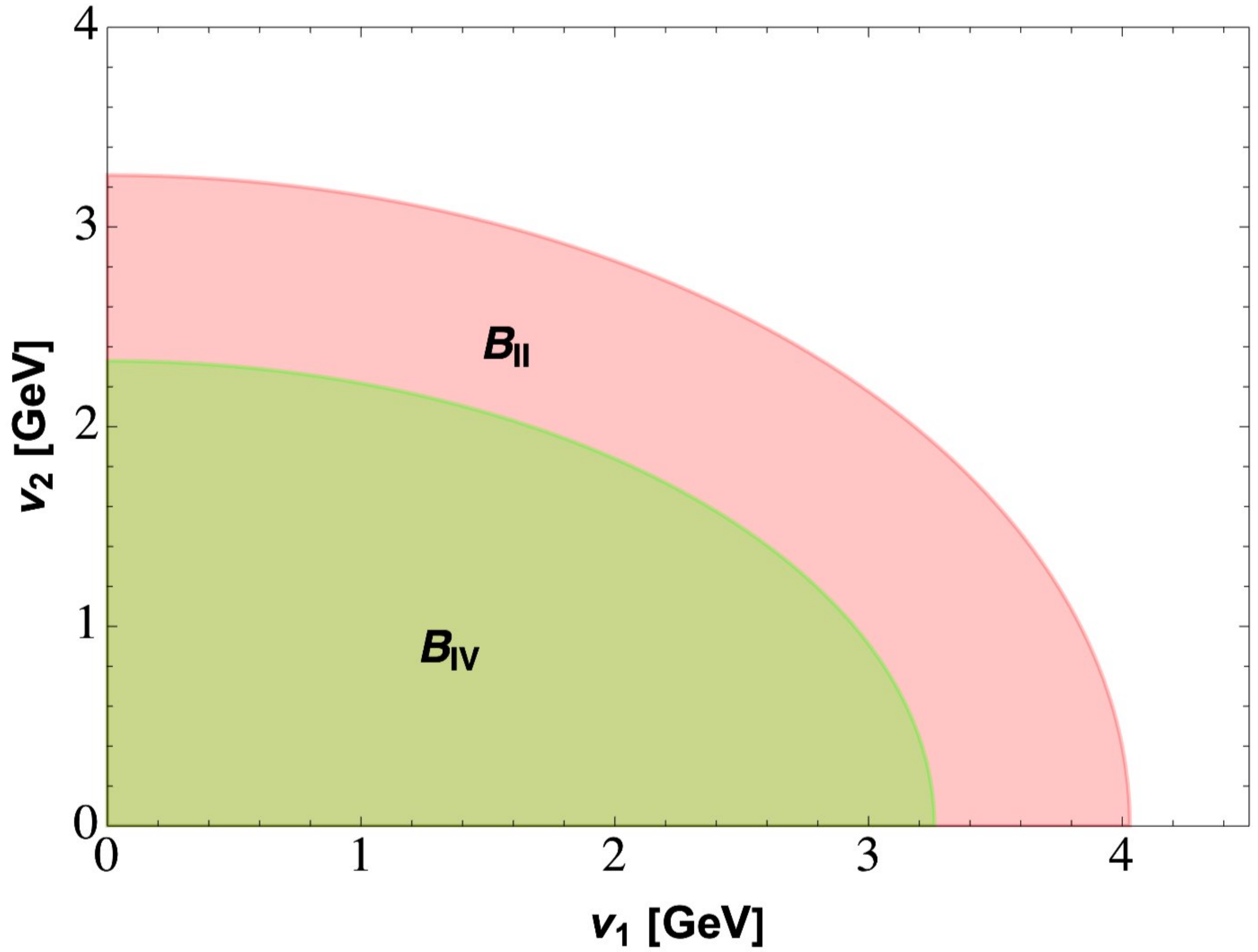
$\rho = 1 + \alpha T$ PDG 2022: $T = 0.04 \pm 0.06$

Scenario	Region	a_1(GeV)	a_2(GeV)
$\mathbf{B_I}$	Hyperbola	3.3	2.6
$\mathbf{B_{II}}$	Ellipse	4.0	3.3
$\mathbf{B_{III}}$	Hyperbola	1.9	3.3
$\mathbf{B_{IV}}$	Ellipse	3.3	2.3
$\mathbf{B_V}$	Hyperbola	4.0	4.0
$\mathbf{B_{VI}}$	Hyperbola	4.0	2.3

scenario	$v_1^{\max}$(GeV)
$\mathbf{A_I}$	3.3
$\mathbf{A_{II}}$	2.6

C.L. 95%





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scenario	$v_1^{\max}$(GeV)
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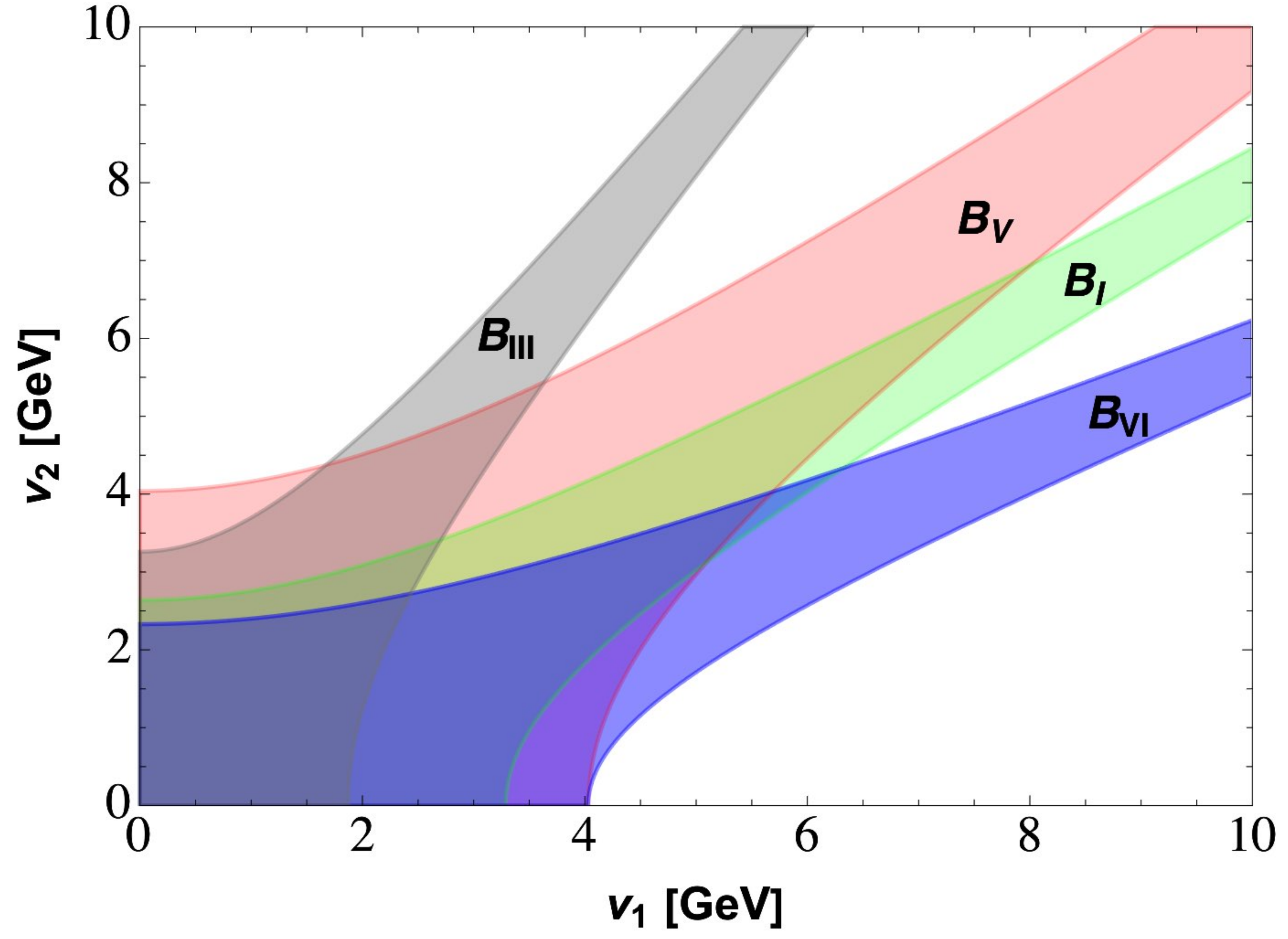
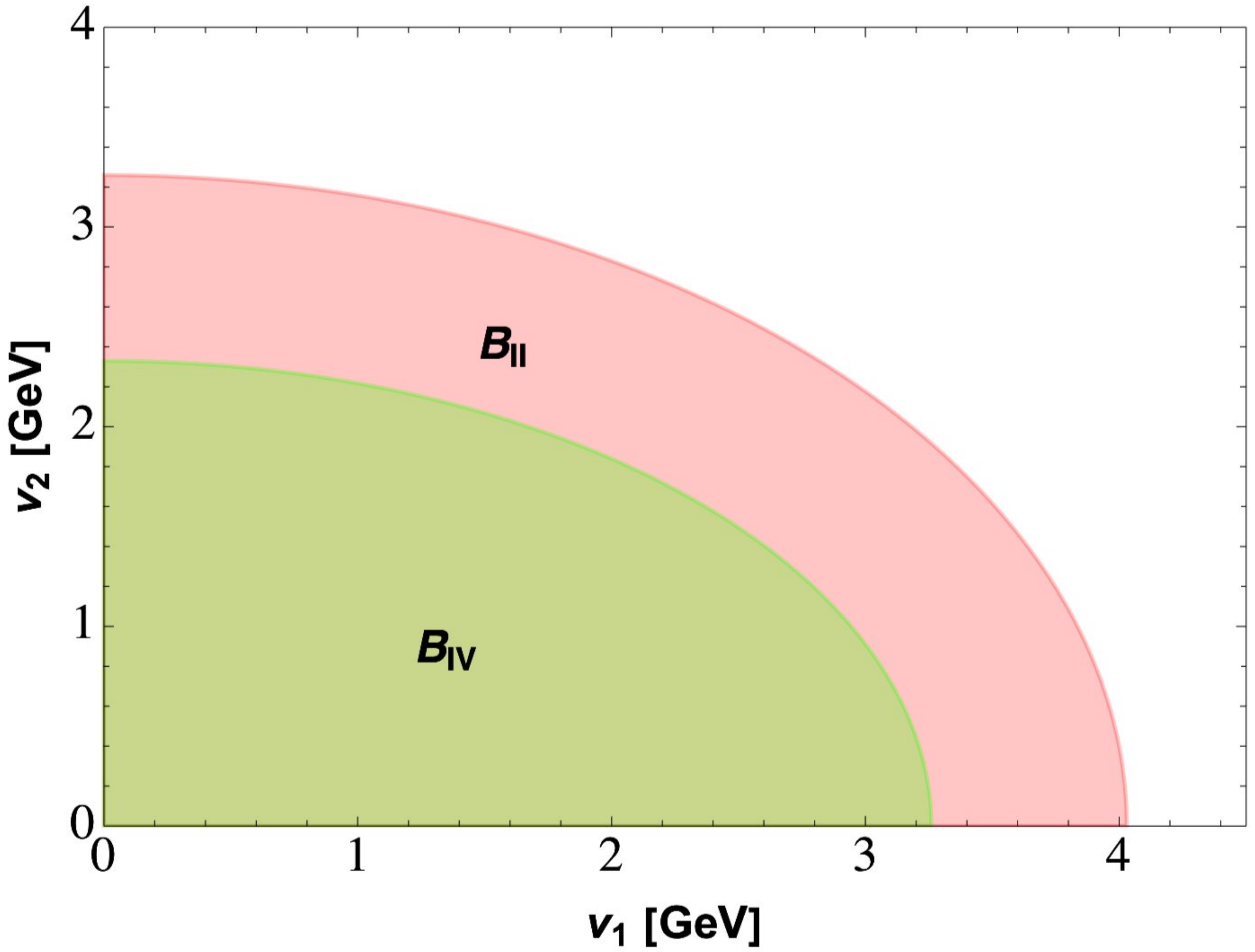
$\mathbf{A_{II}}$	2.6

C.L. 95%

Usual Seesaws **new BSM scalars**

$\Lambda \sim 10^{13}$ GeV $\Lambda \lesssim 10^{11}$ GeV





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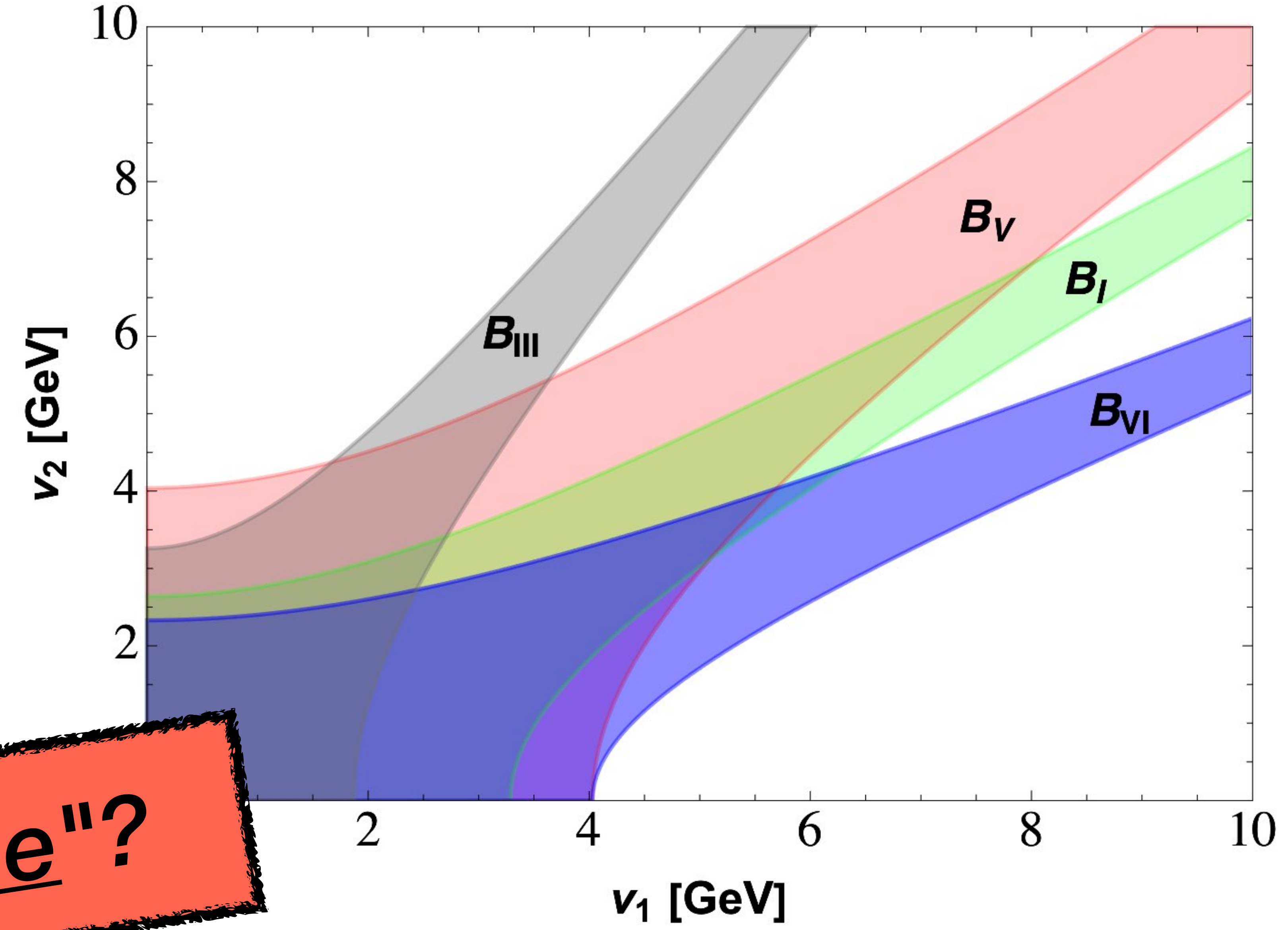
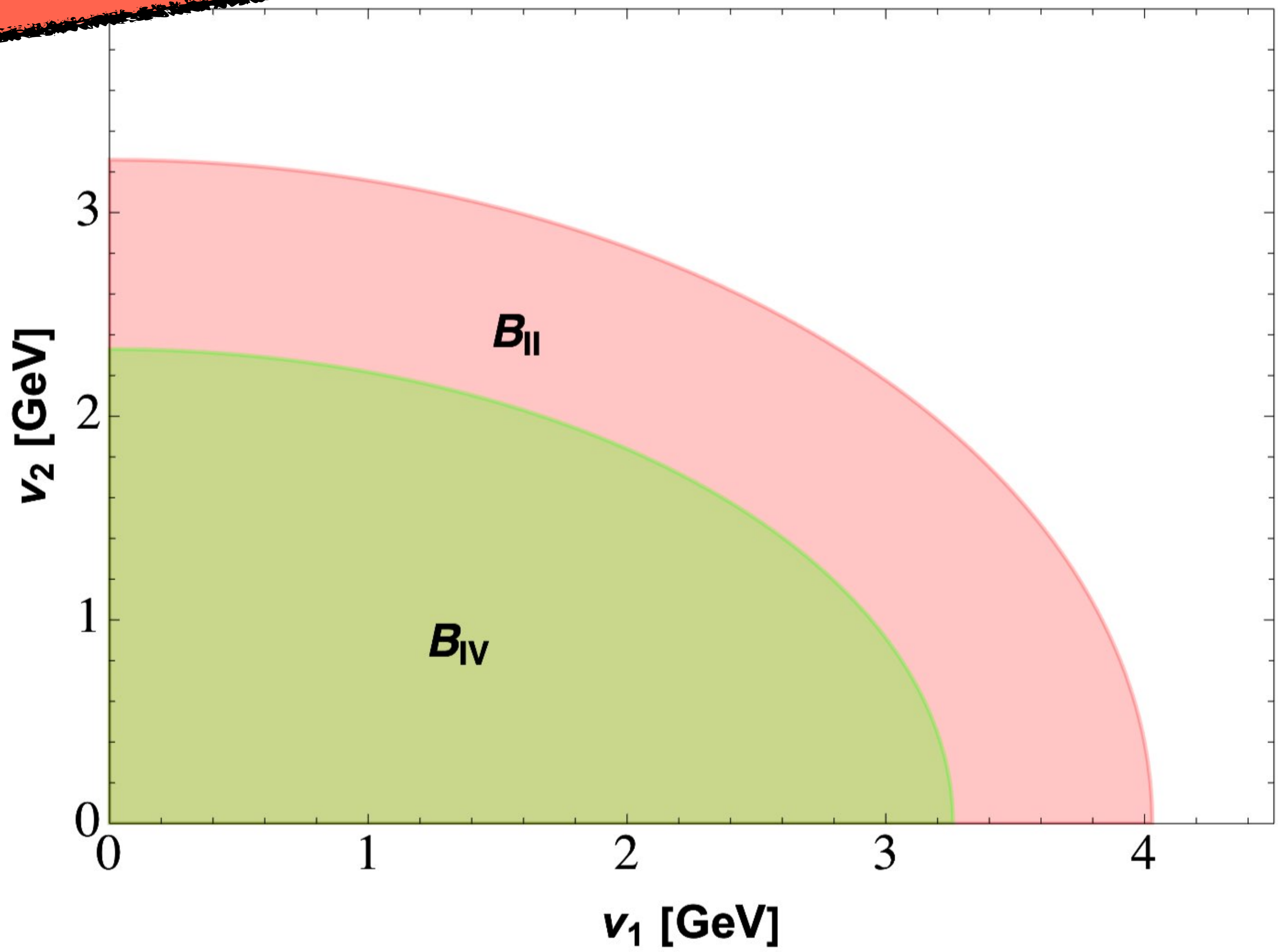
$\mathbf{B_{VI}}$	Hyperbola	4.0	2.3

[illegible]	3.3
$\mathbf{A_{II}}$	2.6

Are these models "<u>genuine</u>"?

C.L. 95%





Usual Seesaws

$\Lambda \sim 10^{13}$ GeV

new BSM scalars

$\Lambda \leqslant 10^{11}$ GeV

Road to "genuineness": induced vevs

$$V(\phi, \phi_i, \phi_j) \supset \lambda_i \phi_i \phi^2 + \mu_i \phi_i \phi^3 + \kappa \phi_i \phi_j \phi^2$$

Road to "genuineness": induced vevs

$$V(\phi, \phi_i, \phi_j) \supset \lambda_i \phi_i \phi^2 + \mu_i \phi_i \phi^3 + \kappa \phi_i \phi_j \phi^2$$

Under the assumption $m_{\phi_i} \gg v_{sm}$ a small induced vev is produced:

$$v_{\phi_i} \simeq \lambda_i \frac{v_{sm}^2}{m_{\phi_i}^2}$$

λ_i as a dimensionful parameter

$$\phi^2 \sim 2 \otimes 2 = 3 \oplus 1$$

TRIPLET / SINGLET

Road to "genuineness": induced vevs

$$V(\phi, \phi_i, \phi_j) \supset \lambda_i \phi_i \phi^2 + \mu_i \phi_i \phi^3 + \kappa \phi_i \phi_j \phi^2$$

Under the assumption $m_{\phi_i} \gg v_{sm}$ a small induced vev is produced:

$$v_{\phi_i} \simeq \mu_i \frac{v_{sm}^3}{m_{\phi_i}^2}$$

μ_i as a dimensionless parameter

$$\phi^3 \sim 2 \otimes 2 \otimes 2 = 4 \oplus 2 + 2'$$

QUADRUPLET / DOUBLET

Road to "genuineness": induced vevs

$$V(\phi, \phi_i, \phi_j) \supset \lambda_i \phi_i \phi^2 + \mu_i \phi_i \phi^3 + \kappa \phi_i \phi_j \phi^2$$

Under the assumption $m_{\phi_i} \gg v_{sm}$ a relation among the vevs is implied:

$$v_{\phi_i} \simeq \kappa\, v_{\phi_j} \frac{v_{sm}^2}{m_{\phi_i}^2}$$

κ as a dimensionless parameter

$$\phi^2 \sim 2 \otimes 2 = 3 \oplus 1$$

$$\phi_i \sim n, \quad \phi_j \sim n\,, n \otimes n \supset 1$$

$$\phi_i \sim 2n, \quad \phi_j \sim 2(n+1),$$
$$2n \otimes 2(n+1) = 3 \oplus 5 \oplus \ldots \oplus 4n+1$$

$$\phi_i \sim 2n+1, \quad \phi_j \sim 2n+3,$$
$$2n+1 \otimes 2n+3 = 3 \oplus 5 \oplus \ldots \oplus 4n+3$$

Road to "genuineness": induced vevs

$$V(\phi, \phi_i, \phi_j) \supset \lambda_i \phi_i \phi^2 + \mu_i \phi_i \phi^3 + \kappa \phi_i \phi_j \phi^2$$

Models	New Scalars	Mediator	Op.	Wilson Coeffients	$\mathbf{v}_{bsm} \ll \mathbf{v}_{sm}$
$\mathbf{A_I}$	$\Phi_1 = \mathbf{4}^S_{-1/2}$	$\Sigma = \mathbf{5}^F_0$	$\mathcal{O}_5^{(2)}$	$C_5^{(2)} = y_1 M_\Sigma^{-1} y_1^T$	✔
$\mathbf{A_{II}}$	$\Phi_1 = \mathbf{4}^S_{-3/2}$	$\mathcal{F} = \mathbf{3}^F_{-1}$	$\mathcal{O}_5^{(1)}$	$C_5^{(1)} = y_1 M_\mathcal{F}^{-1} y_H^T + y_H M_\mathcal{F}^{-1} y_1^T$	✔
$\mathbf{B_I}$	$\Phi_1 = \mathbf{4}^S_{1/2}$ $\Phi_2 = \mathbf{4}^S_{-3/2}$	$\mathcal{F} = \mathbf{5}^F_{-1}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$	✔
$\mathbf{B_{II}}$	$\Phi_1 = \mathbf{3}^S_0$ $\Phi_2 = \mathbf{5}^S_{-1}$	$\mathcal{F} = \mathbf{4}^F_{-1/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$	✔
$\mathbf{B_{III}}$	$\Phi_1 = \mathbf{5}^S_1$ $\Phi_2 = \mathbf{5}^S_{-2}$	$\mathcal{F} = \mathbf{4}^F_{3/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$	✘
$\mathbf{B_{IV}}$	$\Phi_1 = \mathbf{5}^S_0$ $\Phi_2 = \mathbf{5}^S_{-1}$	$\mathcal{F} = \mathbf{4}^F_{1/2}$	$\mathcal{O}_5^{(3)}$	$C_5^{(3)} = y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T$	✘

Effective operators leading to neutrino masses

d=5	**Tree level**
Model	ω
$\mathbf{A_1}$	$1/2$
$\mathbf{A_2}$	-1
$\mathbf{B_1}$	$-\sqrt{3}/4$
$\mathbf{B_2}$	$-1/\sqrt{2}$
$\mathbf{B_3}$	2
$\mathbf{B_4}$	$-\sqrt{6}$

$$(m_\nu)_{\alpha\beta} = \omega v_1^2 \left(y_1 M_\Sigma^{-1} y_1^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{A_1}\,,$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v \left(y_H M_\mathcal{F}^{-1} y_1^T + y_1 M_\mathcal{F}^{-1} y_H^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{A_2}\,,$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v_2 \left(y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{B_i}\,,$$

Effective operators leading to neutrino masses

$d \geq 5$	Tree level	Tree level with induced VEVs	
Model	ω	ξ	n
$\mathbf{A_1}$	$1/2$	$1/2\sqrt{3}$	9
$\mathbf{A_2}$	-1	1	7
$\mathbf{B_1}$	$-\sqrt{3}/4$	$1/4$	9
		$-1/12\ (-1/4)$	11
$\mathbf{B_2}$	$-1/\sqrt{2}$	$1/4$	9
$\mathbf{B_3}$	2	-1	7^*
$\mathbf{B_4}$	$-\sqrt{6}$	$-3/2$	7^*

$$(m_\nu)_{\alpha\beta} = \omega v_1^2 \left(y_1 M_\Sigma^{-1} y_1^T\right)_{\alpha\beta} \quad \text{for } \mathbf{A_1},$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v \left(y_H M_\mathcal{F}^{-1} y_1^T + y_1 M_\mathcal{F}^{-1} y_H^T\right)_{\alpha\beta} \quad \text{for } \mathbf{A_2},$$
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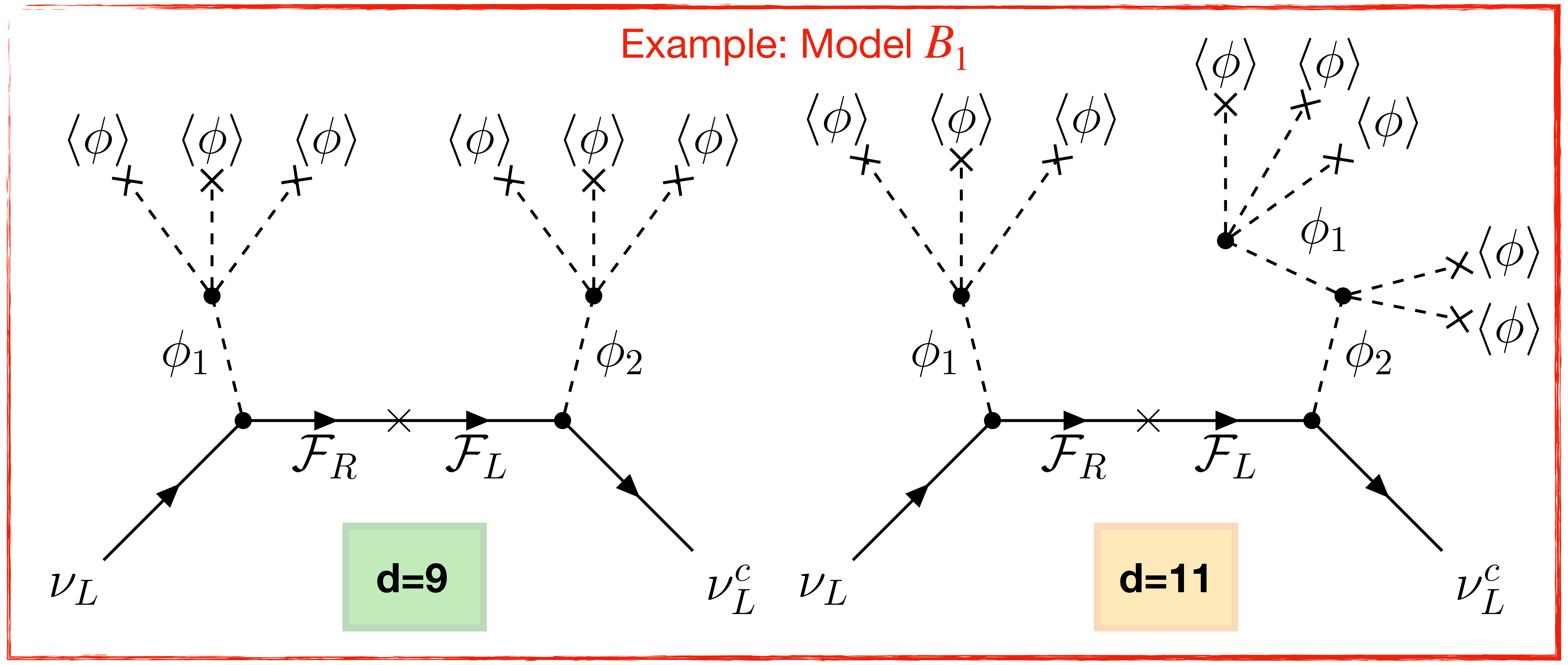

$$\mathcal{O}_n^{(0)} = \xi \frac{c_n^{(0)}}{\Lambda^{n-4}} LLHH(H^\dagger H)^{\frac{n-5}{2}}$$

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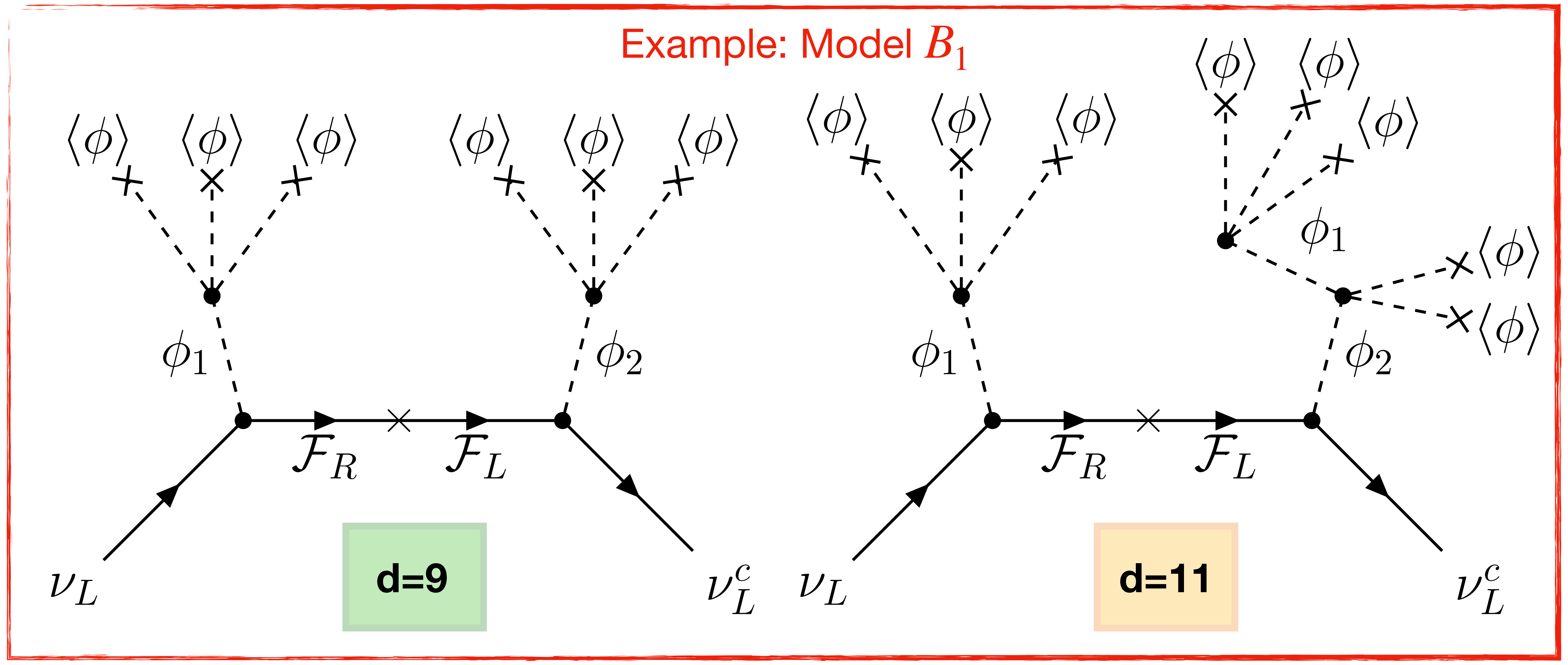
Effective operators leading to neutrino masses

$d\geq 5$	**Tree level**	**Tree level with induced VEVs**	
Model	ω	ξ	n
$\mathbf{A_1}$	$1/2$	$1/2\sqrt{3}$	9
$\mathbf{A_2}$	-1	1	7
$\mathbf{B_1}$	$-\sqrt{3}/4$	$1/4$	9
		$-1/12\ (-1/4)$	11
$\mathbf{B_2}$	$-1/\sqrt{2}$	$1/4$	9
$\mathbf{B_3}$	2	-1	7^*
$\mathbf{B_4}$	$-\sqrt{6}$	$-3/2$	7^*

$\mathcal{O}_7 = (L\Phi_i)^2(H^\dagger H)$

$$(m_\nu)_{\alpha\beta} = \omega v_1^2 \left(y_1 M_\Sigma^{-1} y_1^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{A_1},$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v \left(y_H M_\mathcal{F}^{-1} y_1^T + y_1 M_\mathcal{F}^{-1} y_H^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{A_2},$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v_2 \left(y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{B_i},$$



$$\mathcal{O}_n^{(0)} = \xi \frac{c_n^{(0)}}{\Lambda^{n-4}} LLHH(H^\dagger H)^{\frac{n-5}{2}}$$

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Effective operators leading to neutrino masses

$d \geq 5$	Tree level	Tree level with induced VEVs	
Model	ω	ξ	n
$\mathbf{A_1}$	$1/2$	$1/2\sqrt{3}$	9
$\mathbf{A_2}$	-1	1	7
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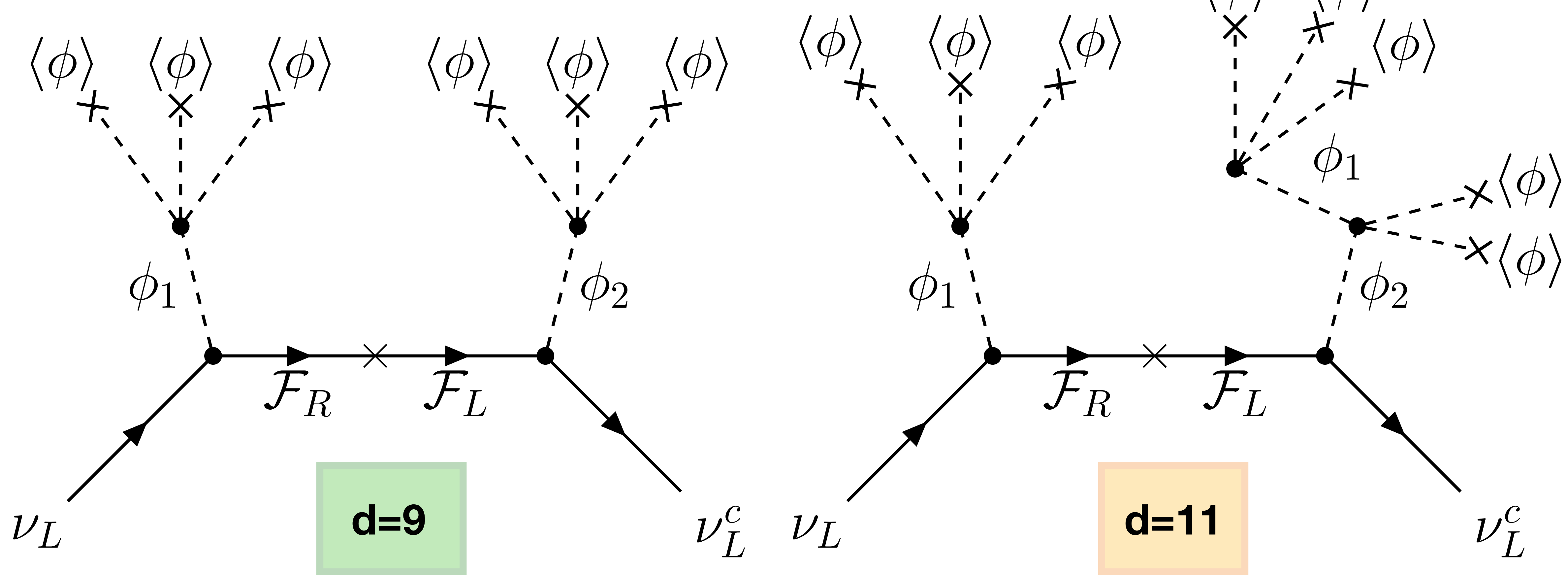
$$(m_\nu)_{\alpha\beta} = \omega v_1^2 \left(y_1 M_\Sigma^{-1} y_1^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{A_1},$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v \left(y_H M_\mathcal{F}^{-1} y_1^T + y_1 M_\mathcal{F}^{-1} y_H^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{A_2},$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v_2 \left(y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T\right)_{\alpha\beta} \qquad \text{for } \mathbf{B_i},$$

Example: Model B_1



$$\mathcal{O}_\nu \simeq \frac{LL<\phi>^6}{M_\mathcal{F}\, m_{\phi_1}^2\, m_{\phi_2}^2}$$

$$\mathcal{O}_\nu \simeq \frac{LL<\phi>^8}{M_\mathcal{F}\, m_{\phi_1}^4\, m_{\phi_2}^2}$$

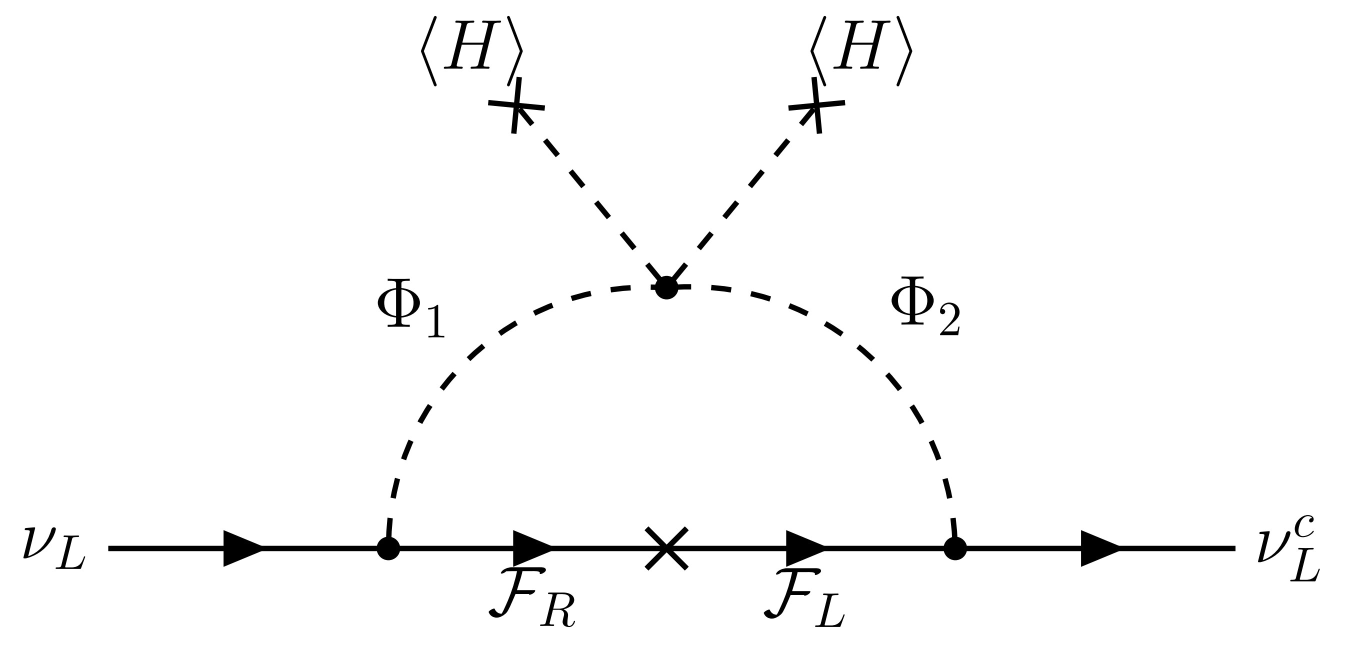
In some cases, one can assume $M_\mathcal{F} \sim m_{\phi_i} \to \mathcal{M}$ so that the low energy effective operators read:

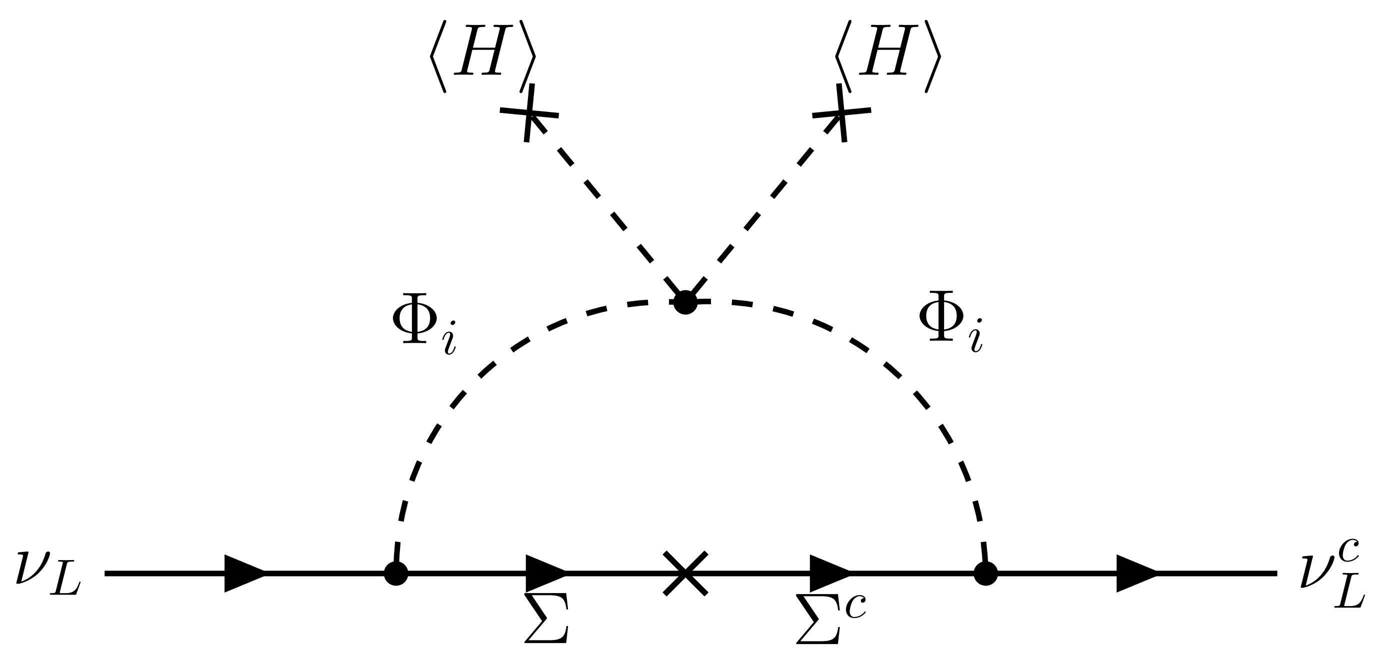
$$\mathcal{O}_\nu^{(d)} \sim \frac{LL<\phi>^{d-3}}{\mathcal{M}^{d-4}}$$

$\mathcal{M} \sim \mathcal{O}(\text{TeV})$

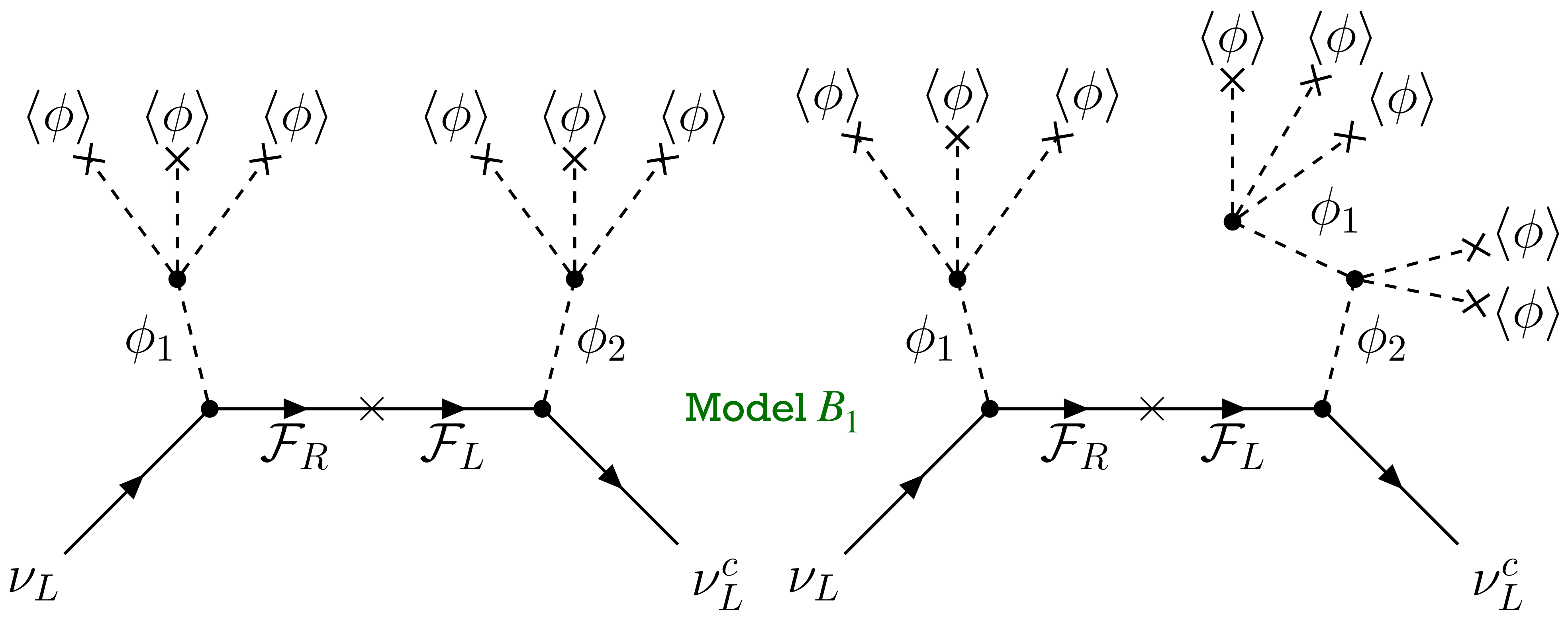
$d\geq 5$	Tree level	Tree level with induced VEVs		Loop level
Model	ω	ξ	n	η
$\mathbf{A_1}$	$1/2$	$1/2\sqrt{3}$	9	$-5/6$
$\mathbf{A_2}$	-1	1	7	2
$\mathbf{B_1}$	$-\sqrt{3}/4$	$1/4$	9	$5/6$
		$-1/12\ (-1/4)$	11	
$\mathbf{B_2}$	$-1/\sqrt{2}$	$1/4$	9	$5/3$
$\mathbf{B_3}$	2	-1	7^*	-5
$\mathbf{B_4}$	$-\sqrt{6}$	$-3/2$	7^*	-5

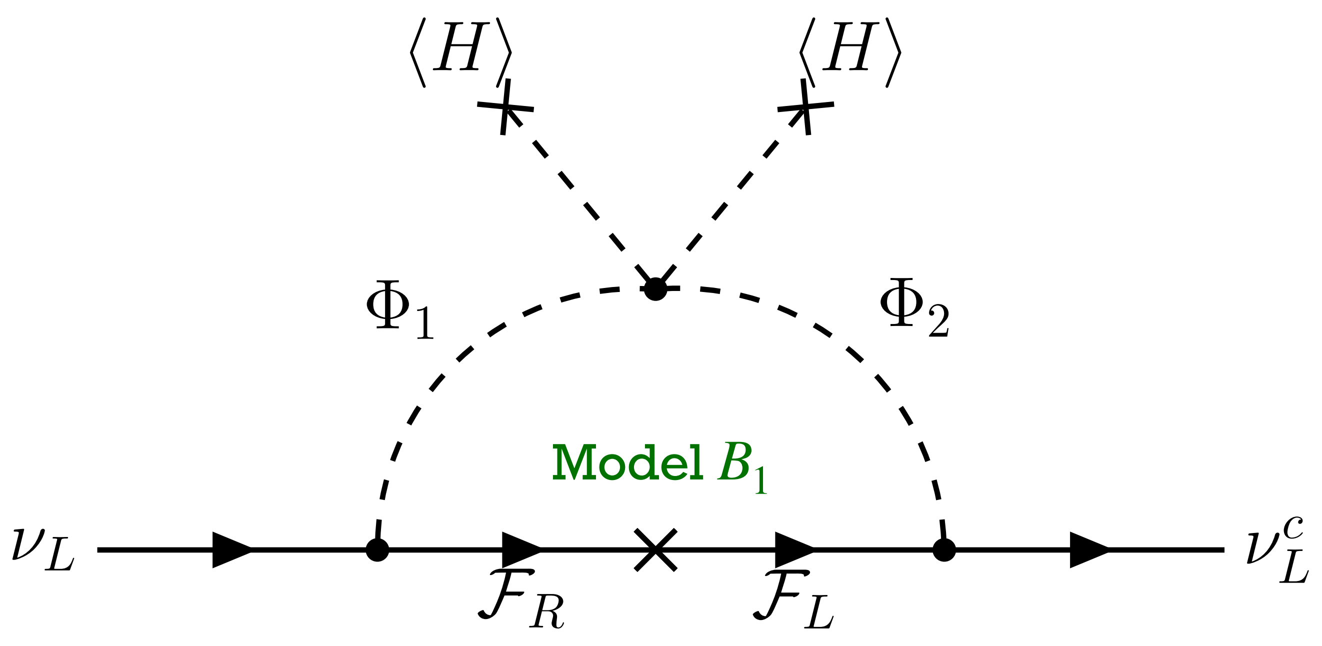
$$(m_\nu)_{\alpha\beta} = \omega v_1^2 \left(y_1 M_\Sigma^{-1} y_1^T\right)_{\alpha\beta} \quad \text{for } \mathbf{A_1},$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v \left(y_H M_\mathcal{F}^{-1} y_1^T + y_1 M_\mathcal{F}^{-1} y_H^T\right)_{\alpha\beta} \quad \text{for } \mathbf{A_2},$$
$$(m_\nu)_{\alpha\beta} = \omega v_1 v_2 \left(y_1 M_\mathcal{F}^{-1} y_2^T + y_2 M_\mathcal{F}^{-1} y_1^T\right)_{\alpha\beta} \quad \text{for } \mathbf{B_i},$$





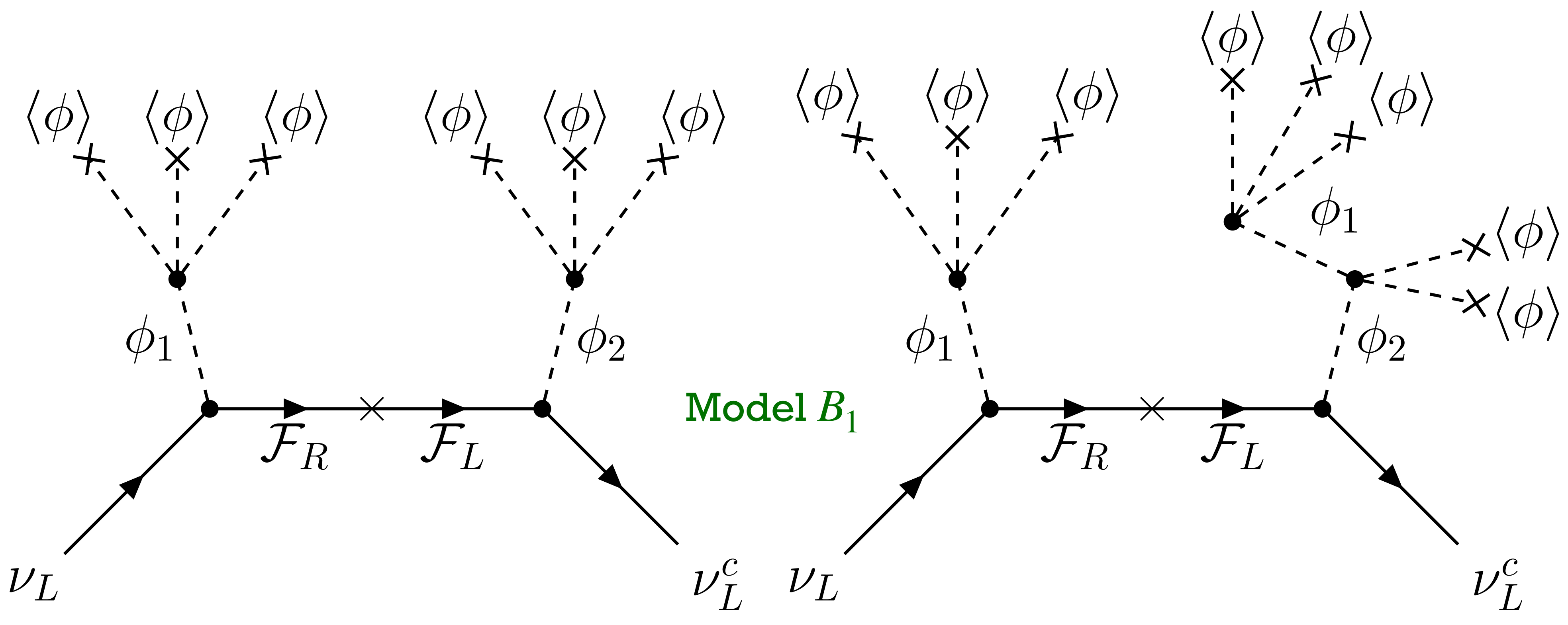
M_{loop} vs M_{tree}

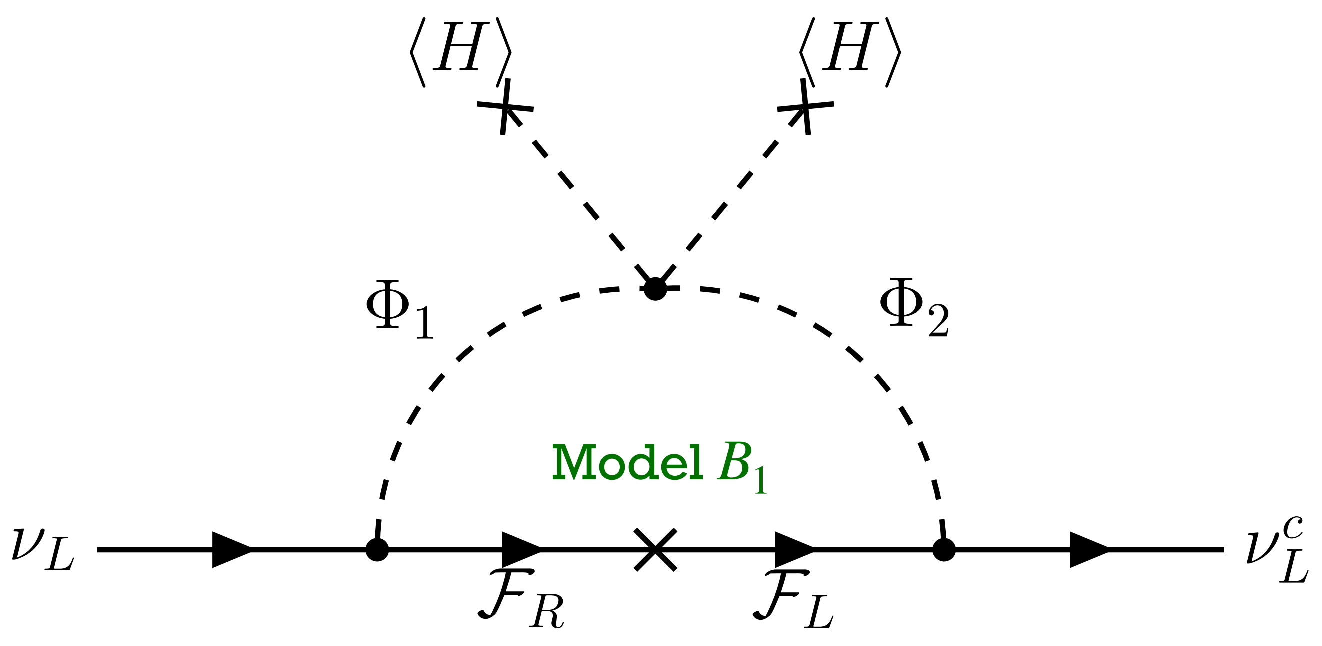




$D > 5$ Weinberg operators $\frac{c_n^{(0)}}{\Lambda^{n-4}} LLHH(H^\dagger H)^{\frac{n-5}{2}}$

M_{loop} vs M_{tree}





$D > 5$ Weinberg operators $\dfrac{c_n^{(0)}}{\Lambda^{n-4}} LLHH(H^\dagger H)^{\frac{n-5}{2}}$

$$m_\nu^{\text{loop}} \simeq \frac{1}{8\pi^2}\left(\frac{\eta\,\bar{\lambda}}{\xi\lambda_1^2}\right)\left(\frac{4m_{\Phi_1}^4}{v^4}\right) m_\nu^{\text{tree}} \quad \text{for } \mathbf{A_1},$$

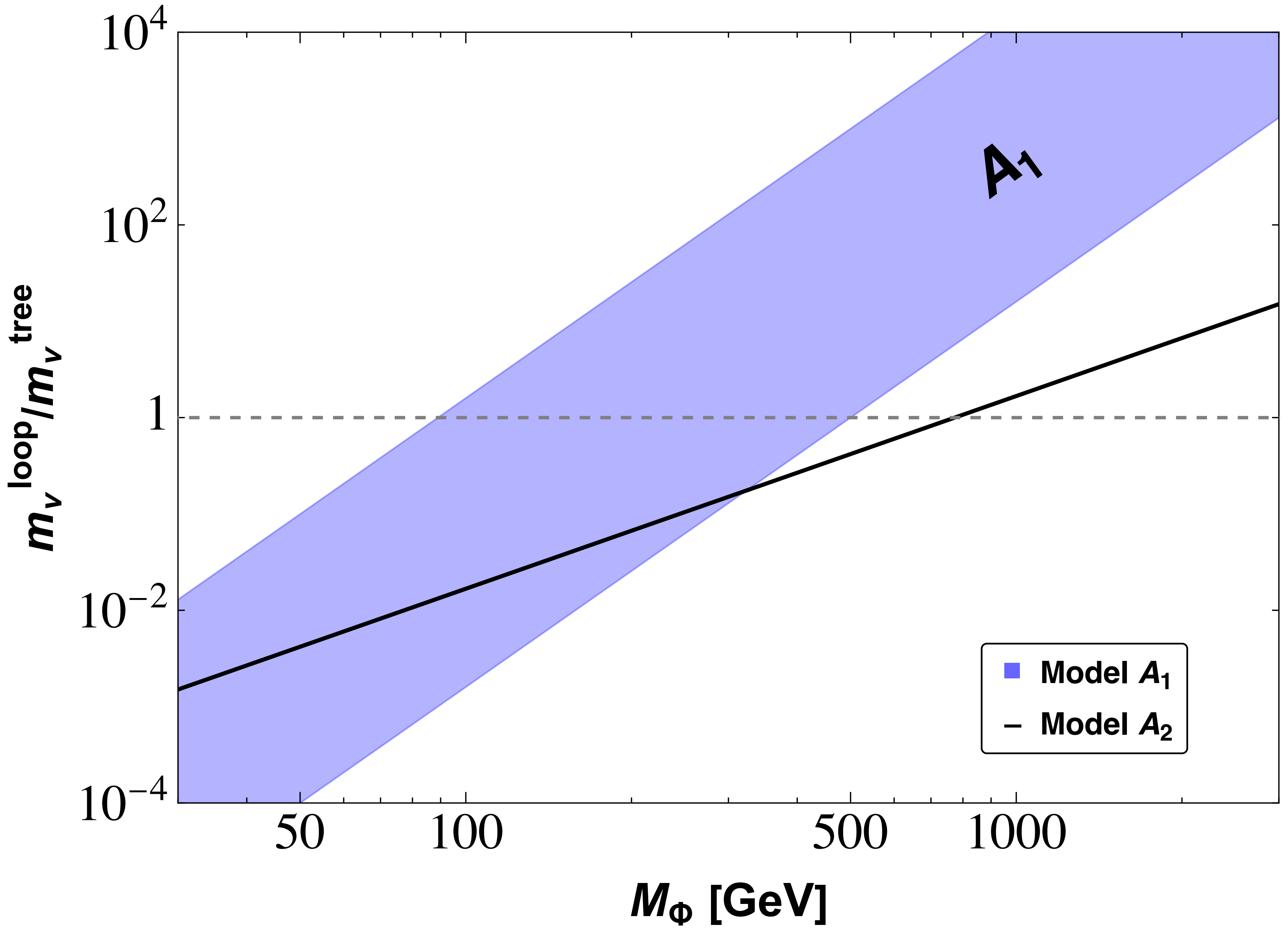
$$m_\nu^{\text{loop}} \simeq \frac{1}{8\pi^2}\left(\frac{\eta}{\xi}\right)\left(\frac{2m_{\Phi_1}^2}{v^2}\right) m_\nu^{\text{tree}} \quad \text{for } \mathbf{A_2},$$

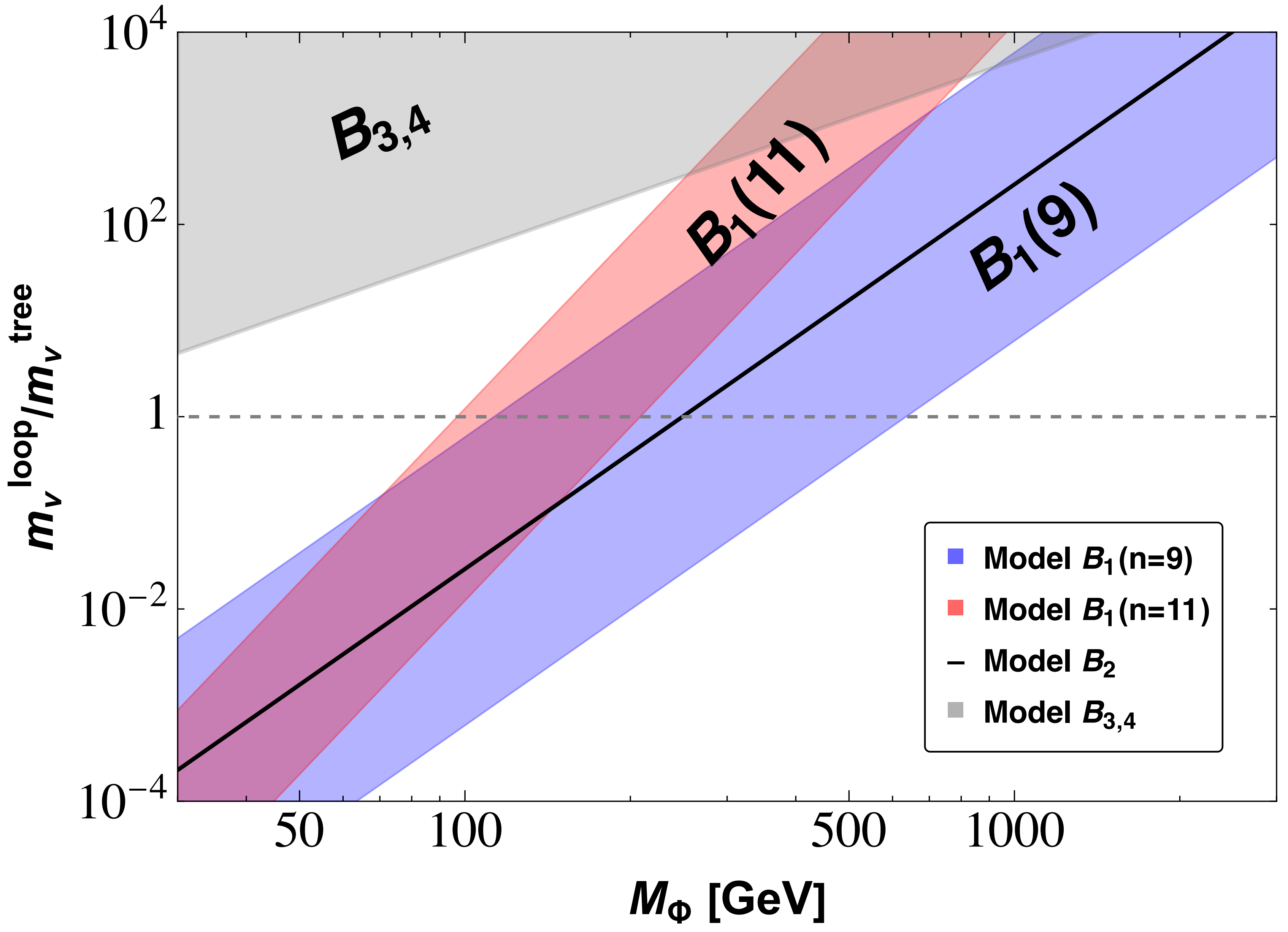
$$m_\nu^{\text{loop}} \simeq \frac{1}{8\pi^2}\left(\frac{\eta\,\lambda_{12}}{\xi\lambda_1\lambda_2}\right)\left(\frac{4m_{\Phi_1}^2 m_{\Phi_2}^2}{v^4}\right) m_\nu^{\text{tree}} \quad \text{for } \mathbf{B_1}\ (\text{n}=9),$$

$$m_\nu^{\text{loop}} \simeq \frac{1}{8\pi^2}\left(\frac{\eta}{\xi\lambda_{1(2)}^2}\right)\left(\frac{8m_{\Phi_{1(2)}}^4 m_{\Phi_{2(1)}}^2}{v^6}\right) m_\nu^{\text{tree}} \quad \text{for } \mathbf{B_1}\ (\text{n}=11),$$

$$m_\nu^{\text{loop}} \simeq \frac{1}{8\pi^2}\left(\frac{\eta}{\xi}\right)\left(\frac{8m_{\Phi_1}^4 m_{\Phi_2}^2}{v^4\mu_1^2}\right) m_\nu^{\text{tree}} \quad \text{for } \mathbf{B_2},$$

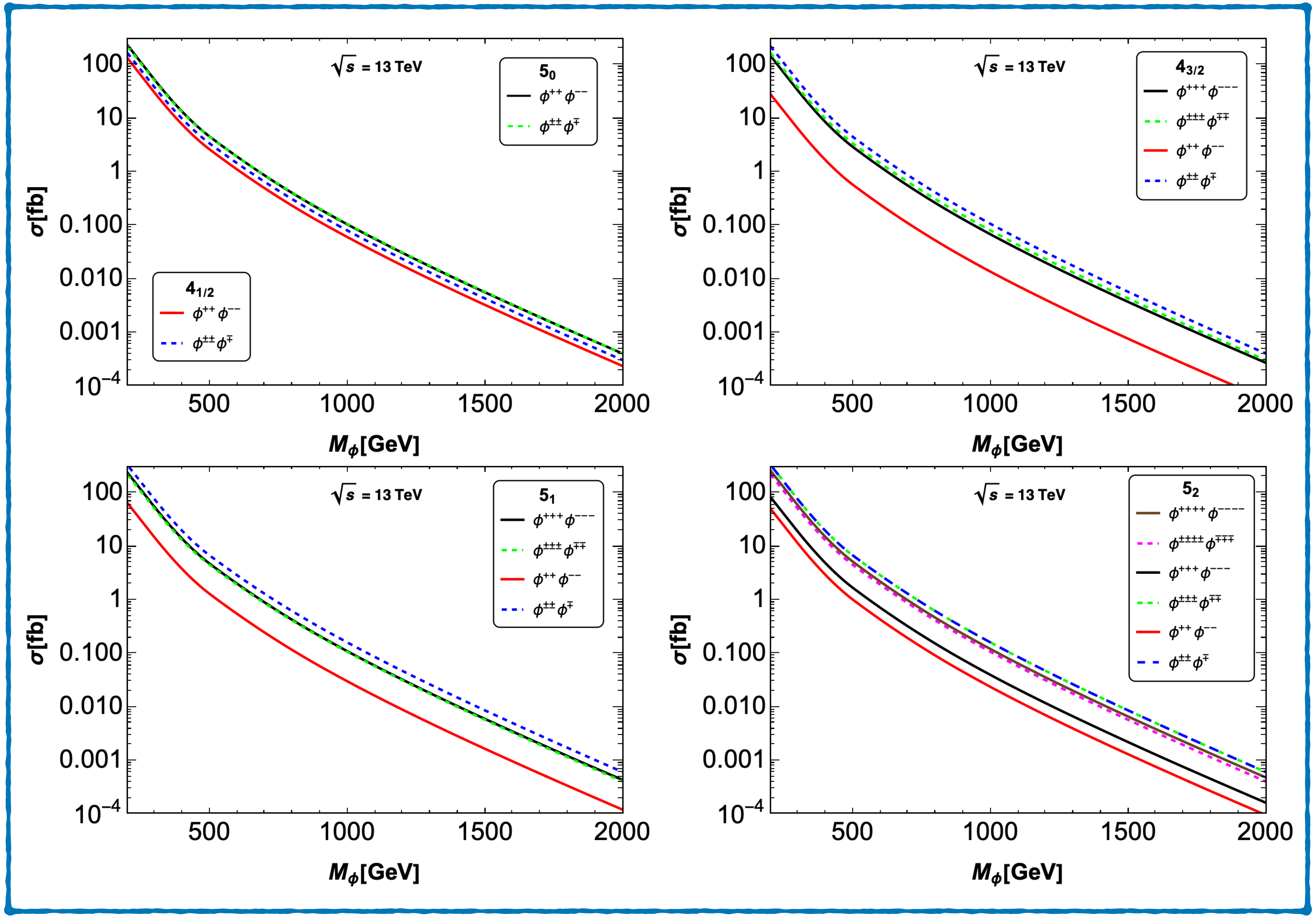
$$m_\nu^{\text{loop}} \simeq \frac{1}{8\pi^2}\left(\frac{\eta}{\xi}\right)\left(\frac{2m_{\Phi_{2(1)}}^2}{v_{1(2)}^2}\right) m_\nu^{\text{tree}} \quad \text{for } \mathbf{B_{3\,(4)}}.$$





PHENOMENOLOGY

Pair production



$$q\bar{q} \to \gamma, Z \to \Phi^{\pm\pm\pm\pm}\Phi^{\mp\mp\mp\mp}\,, \Phi^{\pm\pm\pm}\Phi^{\mp\mp\mp}\,, \Phi^{\pm\pm}\Phi^{\mp\mp}\,, \Phi^{\pm}\Phi^{\mp}\,,$$
$$q\bar{q}' \to W^{\pm} \to \Phi^{\pm\pm\pm\pm}\Phi^{\mp\mp\mp}\,, \Phi^{\pm\pm\pm}\Phi^{\mp\mp}\,, \Phi^{\pm\pm}\Phi^{\mp}\,.$$

$\sigma(pp \to \Phi^{n\pm}\Phi^{n\pm}) > \sigma(pp \to \Phi^{m\pm}\Phi^{m\pm})$ for $n > m \geq 2$

$\sigma(pp \to \Phi^{n\pm}\Phi^{(n-1)\mp}) > \sigma(pp \to \Phi^{m\pm}\Phi^{(m-1)\mp})$ for $n < m$

with $(n, m) \geq 2$

Decays

doubly-charged heavy scalars

$$\Gamma(\Phi^{\pm\pm} \to W^\pm W^\pm) = S^2_{2W^\pm} \frac{g^4 v_\Phi^2 M^3_{\Phi^{\pm\pm}}}{16\pi M_W^4} \left(\frac{3M_W^4}{M^4_{\Phi^{\pm\pm}}} - \frac{M_W^2}{M^2_{\Phi^{\pm\pm}}} + \frac{1}{4} \right) \beta \left(\frac{M_W^2}{M^2_{\Phi^{\pm\pm}}} \right) ,$$

$$\Gamma(\Phi^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta) = \frac{|h_{\alpha\beta}|^2 M_{\Phi^{\pm\pm}}}{4\pi\,(1+\delta_{\alpha\beta})} , \qquad \sum_{\alpha,\beta} \Gamma(\Phi^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta) = \kappa^2 \frac{M_{\Phi^{\pm\pm}}}{8\pi\, v_\Phi^2} \sum_{k=1}^{3} m_k^2 ,$$

cascade decays

$$\Gamma(\Phi^{\pm\pm} \to \Phi^\pm \pi^\pm) = S^2_{\Phi^\pm W^\pm} \frac{g^4 |V_{ud}|^2 \Delta M^3 f_\pi^2}{16\pi M_W^4} ,$$

$$\Gamma(\Phi^{\pm\pm} \to \Phi^\pm l^\pm \nu_l) = S^2_{\Phi^\pm W^\pm} \frac{g^4 \Delta M^5}{240\pi^3 M_W^4} ,$$

$$\Gamma(\Phi^{\pm\pm} \to \Phi^\pm q\bar{q}') = 3\Gamma(\Phi^{\pm\pm} \to \Phi^\pm l^\pm \nu_l) ,$$

thriply-charged heavy scalars

$$\Gamma(\Phi^{\pm\pm\pm} \to W^\pm W^\pm W^\pm) = \frac{3g^6}{2048\pi^3} \frac{v_\Phi^2 M^5_{\Phi^{\pm\pm\pm}}}{M_W^6} ,$$

$$\Gamma(\Phi^{\pm\pm\pm} \to W^\pm l^\pm l^\pm) = \frac{g^2}{3072\pi^3} \frac{M^3_{\Phi^{\pm\pm\pm}} \sum_i m_i^2}{v_\Phi^2 M_W^2} .$$

doubly-charged heavy scalars

$$\Gamma(\Phi^{\pm\pm} \to W^\pm W^\pm) = S^2_{2W^\pm} \frac{g^4 v_\Phi^2 M^3_{\Phi^{\pm\pm}}}{16\pi M_W^4} \left(\frac{3M_W^4}{M^4_{\Phi^{\pm\pm}}} - \frac{M_W^2}{M^2_{\Phi^{\pm\pm}}} + \frac{1}{4} \right) \beta \left(\frac{M_W^2}{M^2_{\Phi^{\pm\pm}}} \right),$$

$$\Gamma(\Phi^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta) = \frac{|h_{\alpha\beta}|^2 M_{\Phi^{\pm\pm}}}{4\pi\,(1+\delta_{\alpha\beta})}, \quad \sum_{\alpha,\beta} \Gamma(\Phi^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta) = \kappa^2 \frac{M_{\Phi^{\pm\pm}}}{8\pi\, v_\Phi^2} \sum_{k=1}^{3} m_k^2,$$

cascade decays

$$\Gamma(\Phi^{\pm\pm} \to \Phi^\pm \pi^\pm) = S^2_{\Phi^\pm W^\pm} \frac{g^4 |V_{\dots}}{1\dots}$$

$$\Gamma(\Phi^{\pm\pm} \to \Phi^\pm l^\pm \nu_l) = S^2_{\Phi^\pm W^\pm} \frac{g^4 \Delta\dots}{240\pi\dots}$$

$$\Gamma(\Phi^{\pm\pm} \to \Phi^\pm q\bar{q}') = 3\Gamma(\Phi^{\pm\pm} \to \dots$$

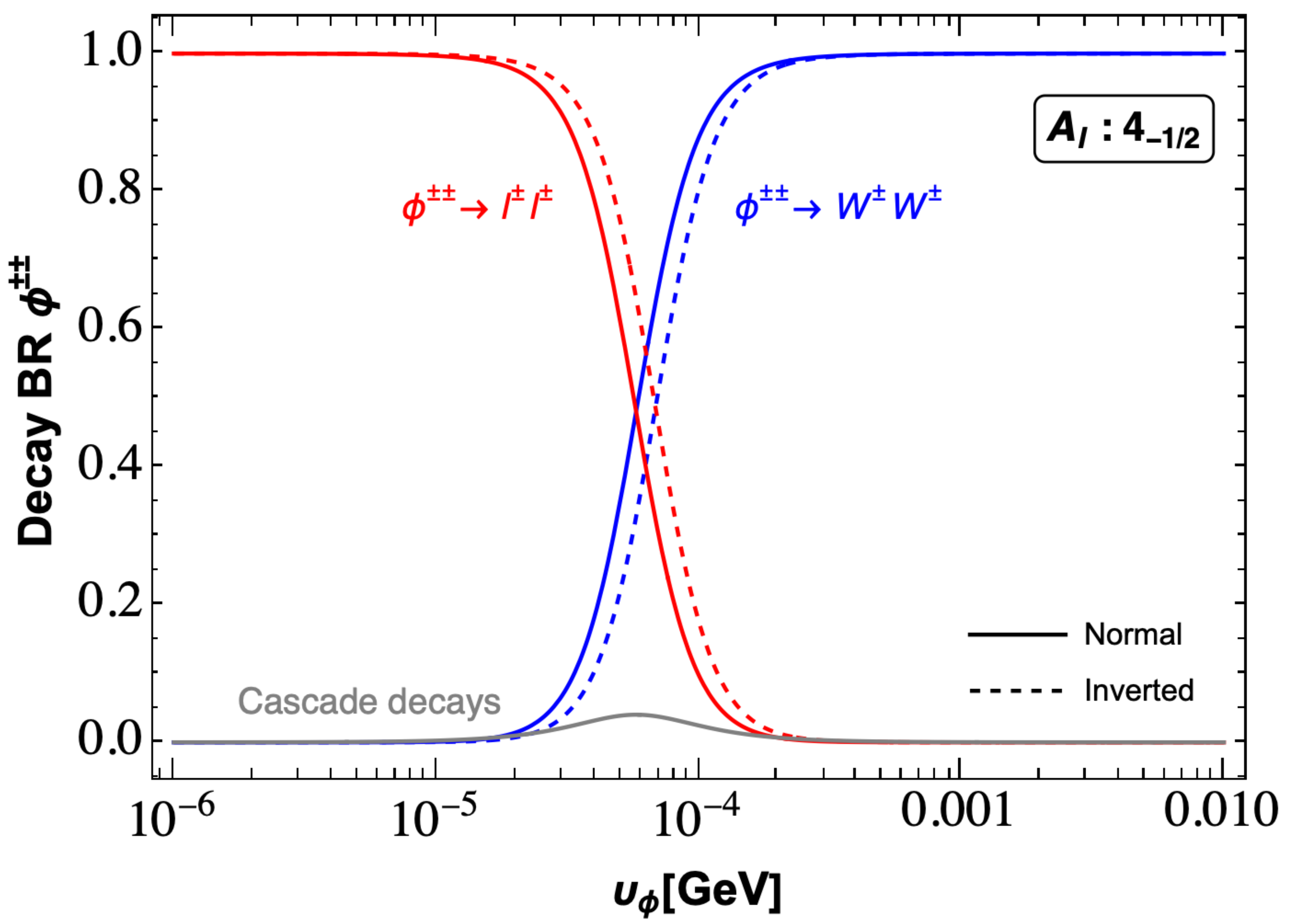
$$h^{\Phi_i}_{\alpha\beta} = \kappa_i \frac{(m_\nu)_{\alpha\beta}}{v_i},$$

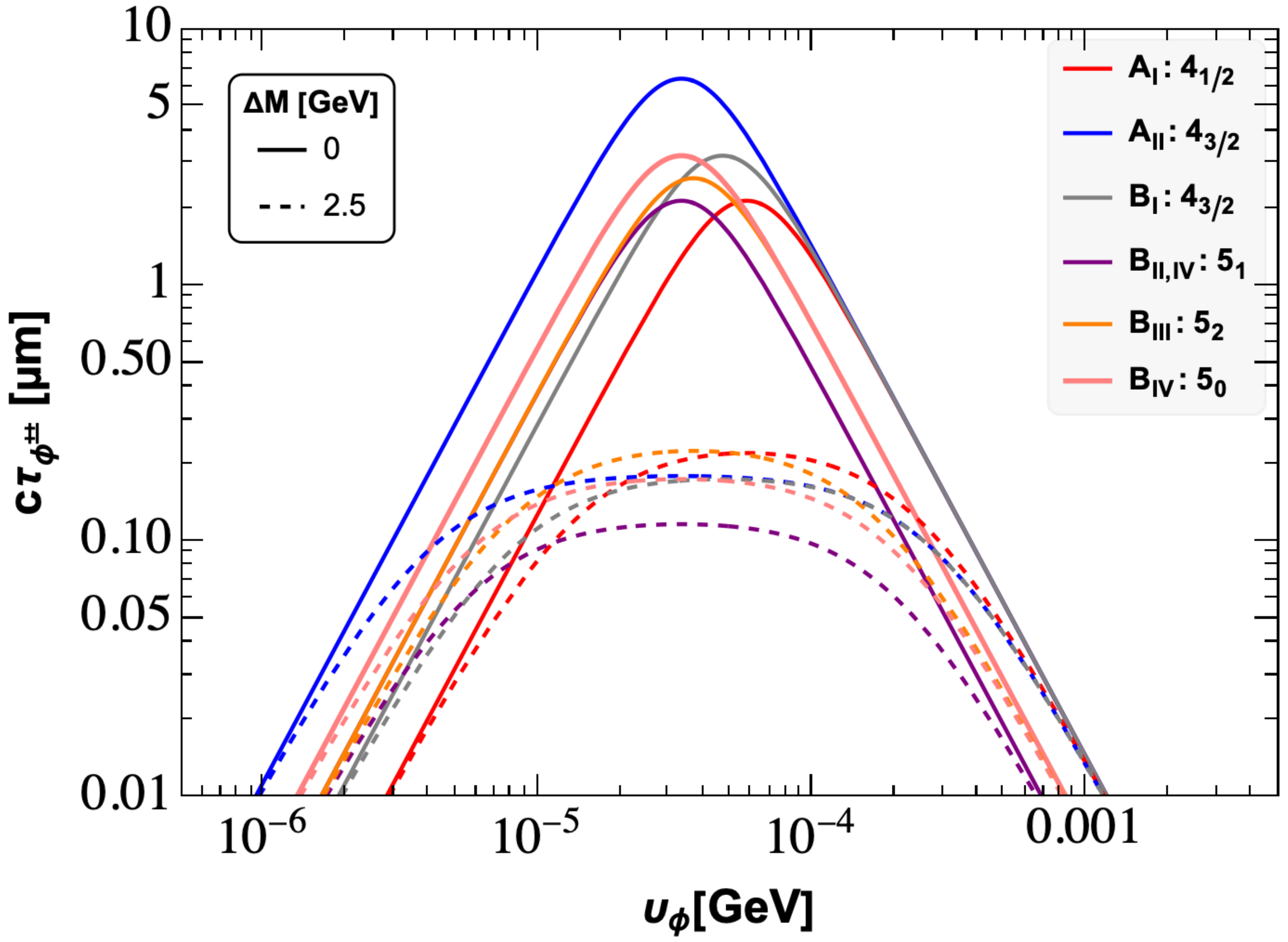
Scenario	Scalar	Couplings and scale factors: $\lvert\kappa\rvert$	$S_{2W^\pm}$	$S_{\Phi^\pm W^\pm}$	$\Phi^{\pm\pm\pm}\Phi^{\mp\mp}W^\mp$	$\Phi^{4\pm}\Phi^{3\mp}W^\mp$
$\mathbf{A_I}$	$\mathbf{4}_{-1/2}$	$\sqrt{3}$	$\sqrt{6}$	$\sqrt{3/2}$	–	–
$\mathbf{A_{II}}$	$\mathbf{4}_{-3/2}$	$1/\sqrt{3}$	$\sqrt{6}$	$\sqrt{2}$	$\sqrt{3/2}$	–
$\mathbf{B_I}$	$\mathbf{4}_{-3/2}$	$2/\sqrt{3}$	$\sqrt{6}$	$\sqrt{2}$	$\sqrt{3/2}$	–
$\mathbf{B_{II}}$	$\mathbf{5}_{-1}$	1	$3\sqrt{2}$	$\sqrt{3}$	$\sqrt{2}$	–
$\mathbf{B_{III}}$	$\mathbf{5}_{-2}$	$\sqrt{3/2}$	$2\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{2}$
$\mathbf{B_{IV}}$	$\mathbf{5}_{-1}$	1	$3\sqrt{2}$	$\sqrt{3}$	$\sqrt{2}$	–
$\mathbf{B_{IV}}$	$\mathbf{5}_{0}$	$\sqrt{2/3}$	$2\sqrt{3}$	$\sqrt{2}$	–	–
$\mathbf{B_{VI}}$	$\mathbf{5}_{0}$	$\sqrt{6}$	$2\sqrt{3}$	$\sqrt{2}$	–	–

...ars

$$\dots \frac{M^5_{\Phi^{\pm\pm\pm}}}{M_W^6},$$

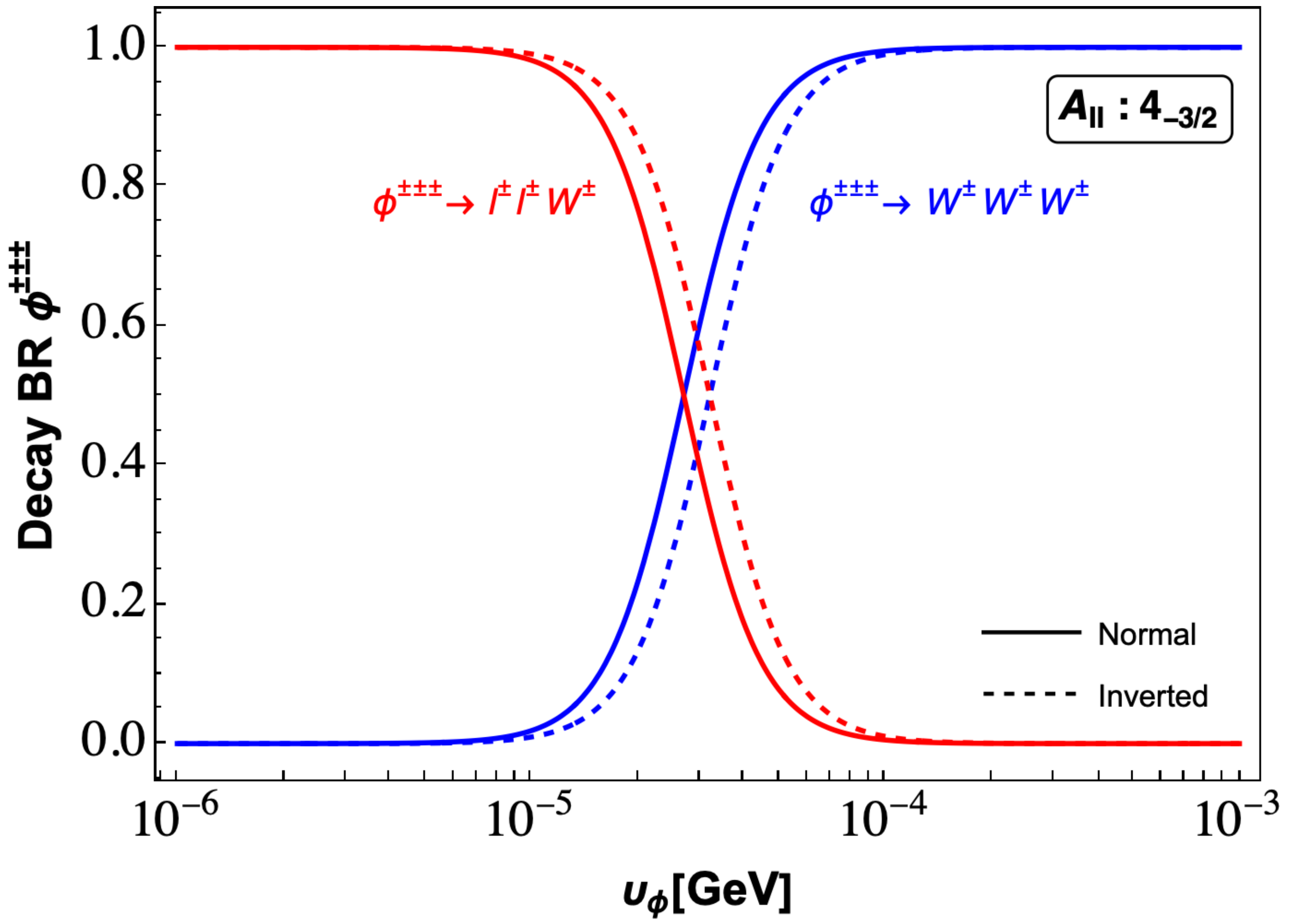
$$\dots \frac{\dots_{\Phi^{\pm\pm\pm}} \sum_i m_i^2}{v_\Phi^2 M_W^2}.$$

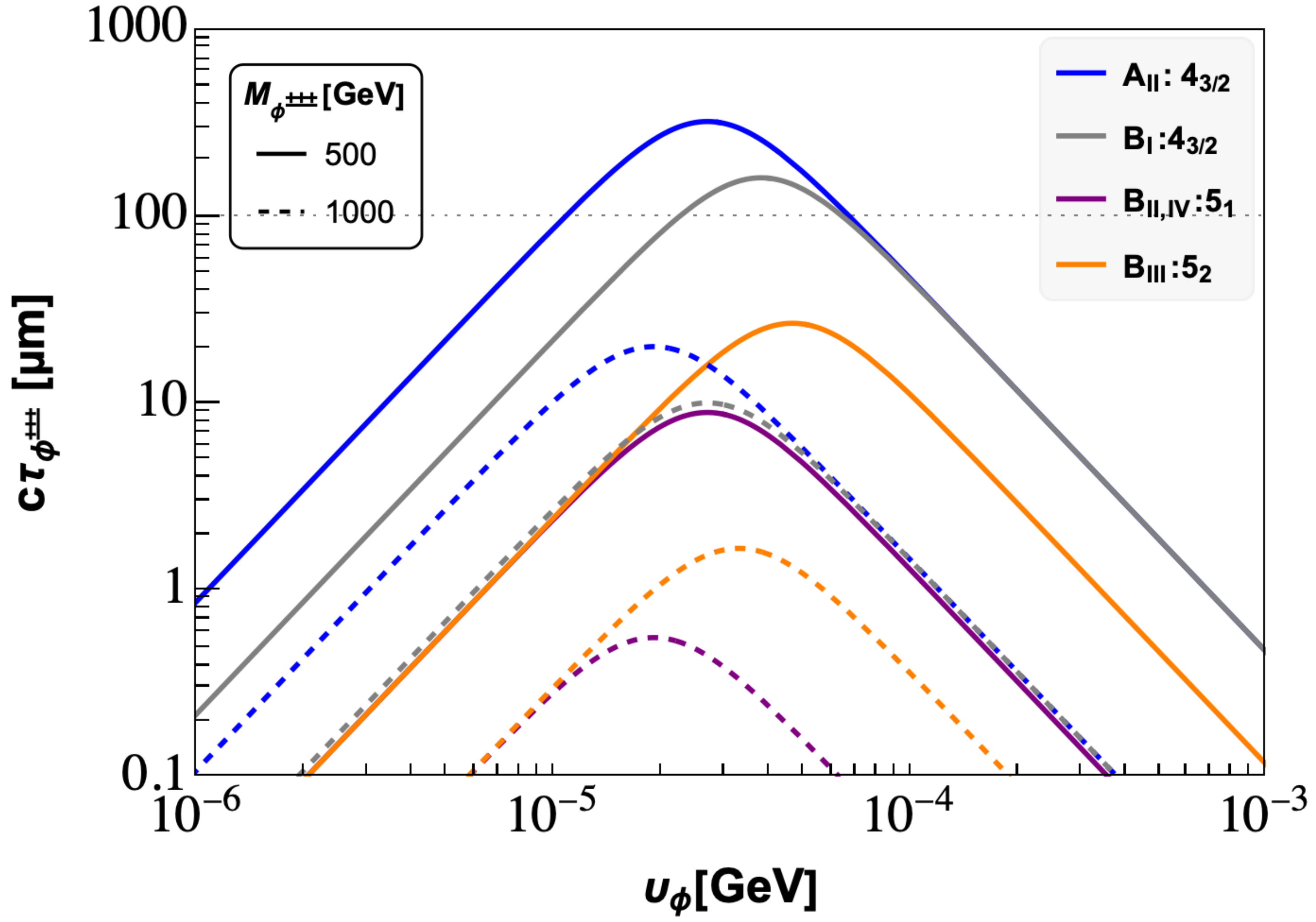




doubly-charged heavy scalars

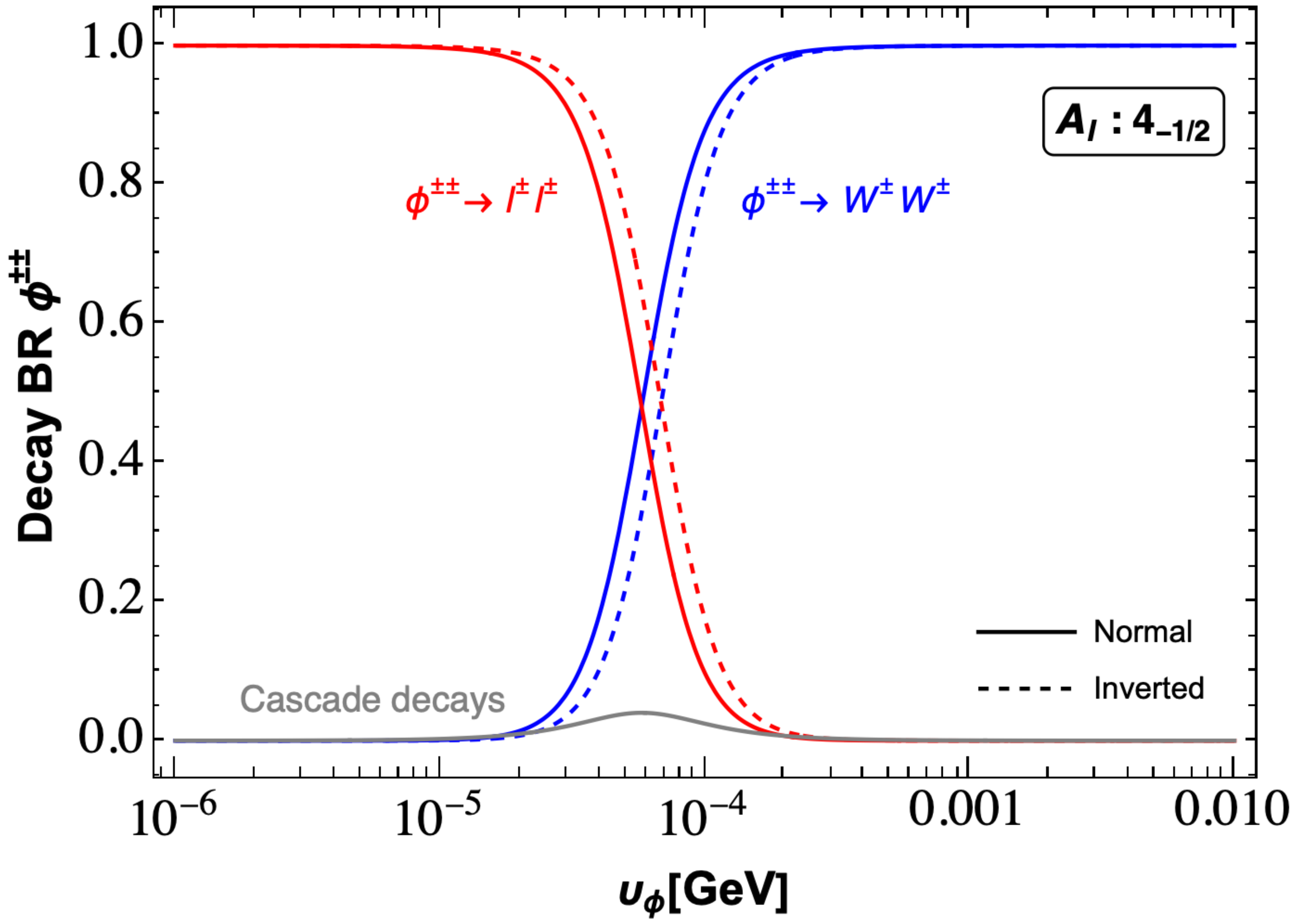
thriply-charged heavy scalars

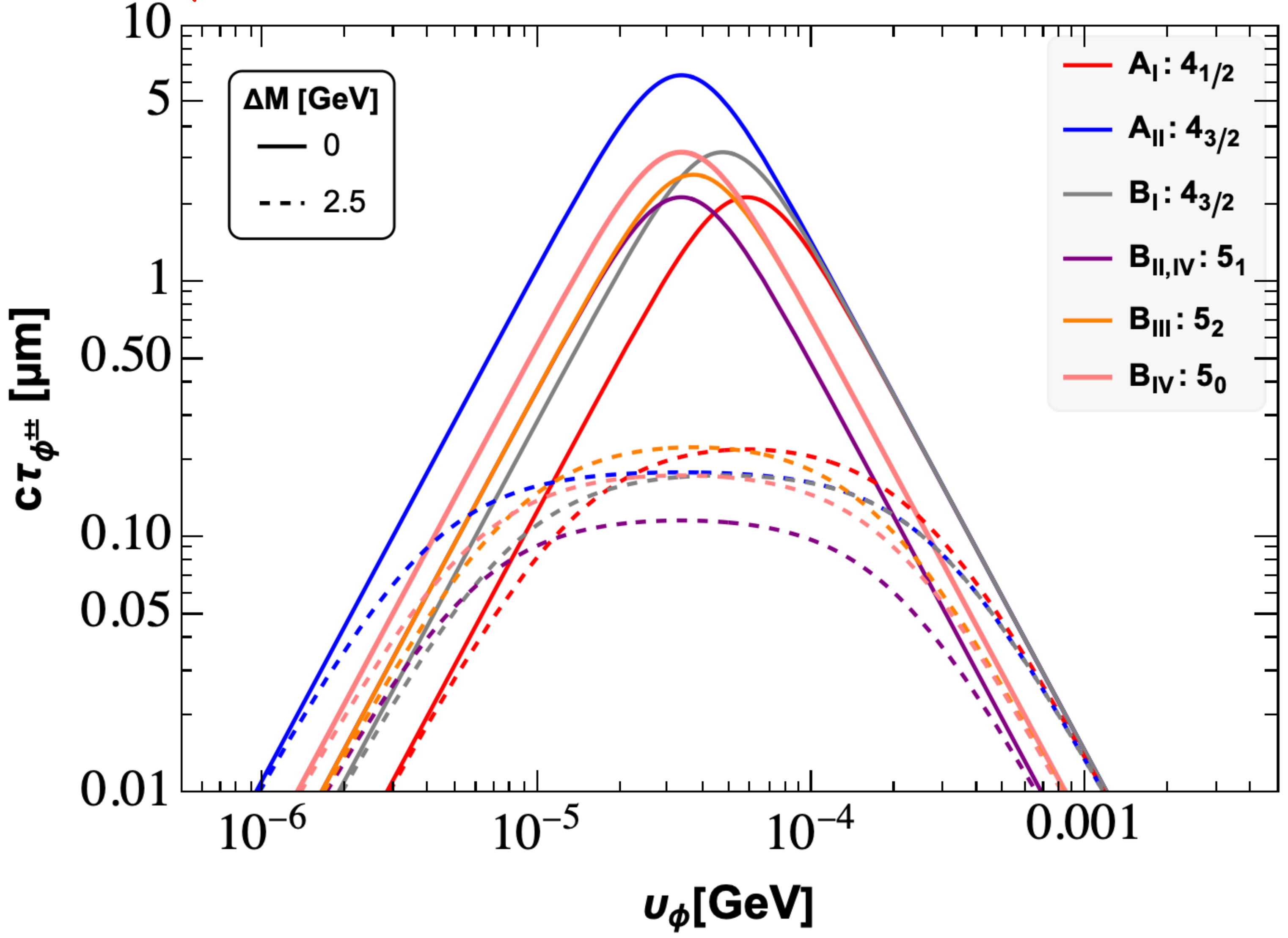




Decays

$M_{\Phi^{\pm\pm}} = 500$ GeV



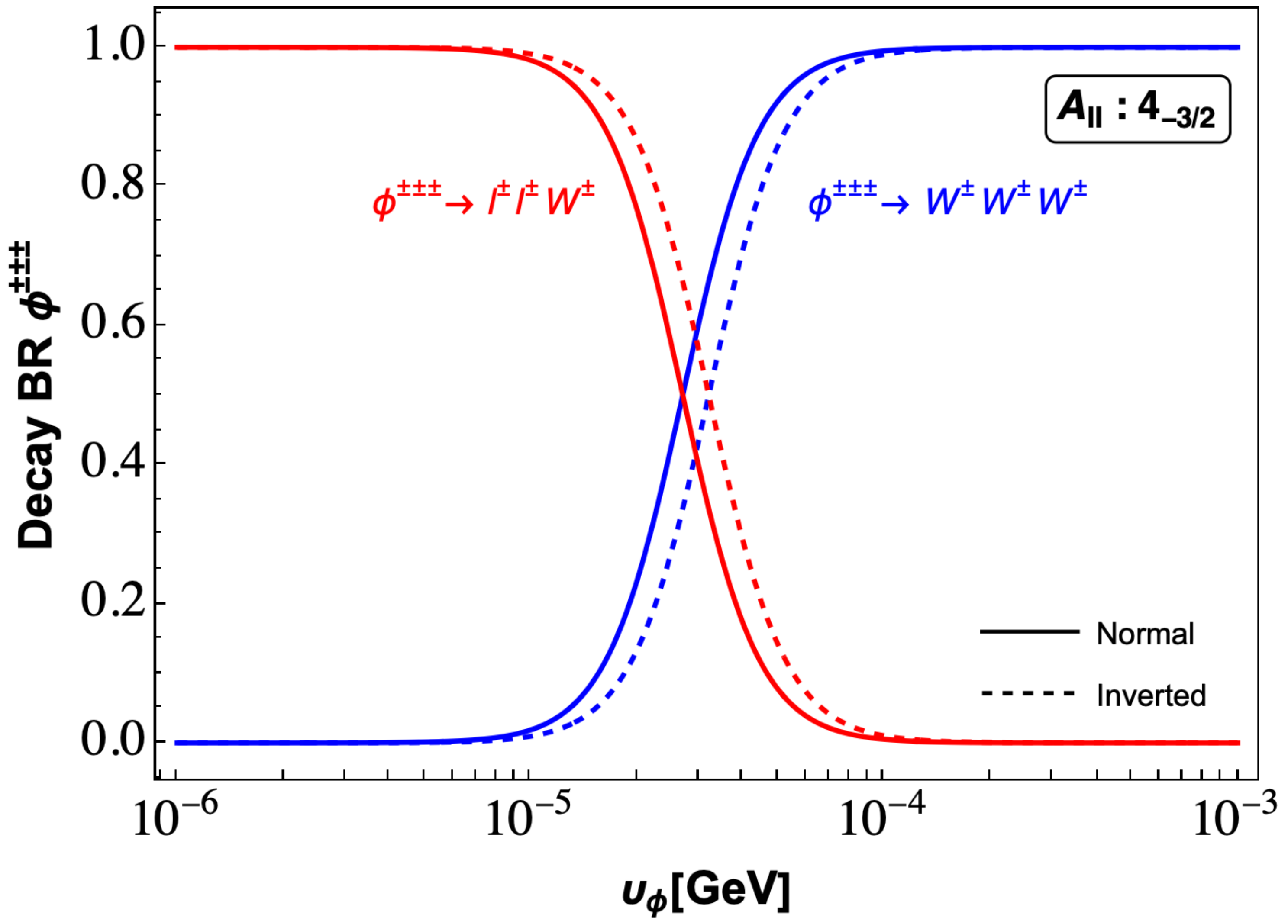


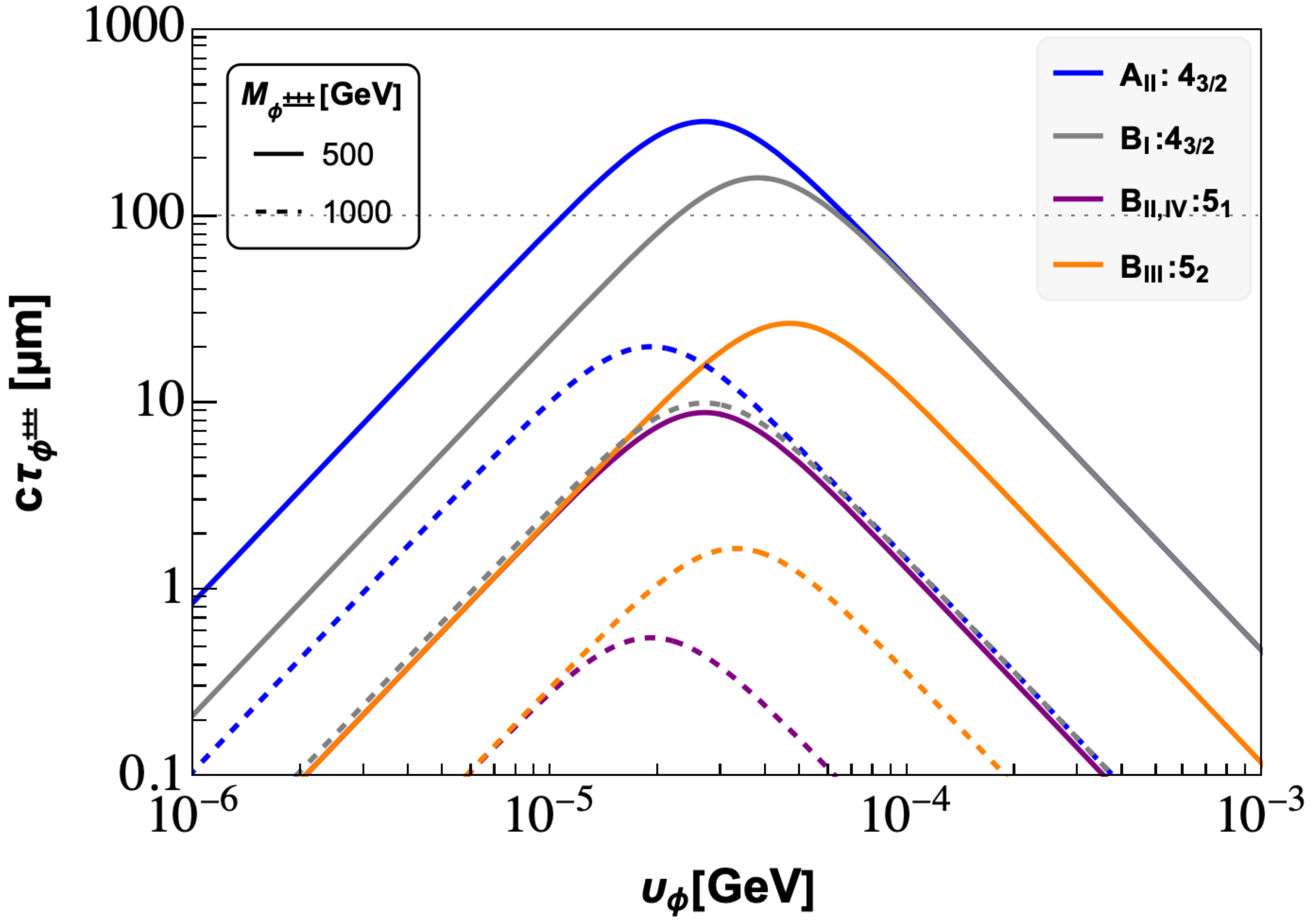
no signals at CMS

proper lenght decay $\mathcal{O}(10\mu m)$

may give signals

proper lenght decay $\mathcal{O}(0.1 - 1\, mm)$





Final Remarks

In this talk:

- ✔ Neutrino masses from higher SU(2) representations:
  Tree level vs 1-loop
- Effective d=6 operators:
  deviation from unitarity and corrections to gauge bosons couplings
- Lepton Number Violation:
  Majorons
- ✔ Phenomenology:
  Production and Decays

... more in

A. Giarnetti, J. Herrero-Garcia, SM, D. Meloni and D. Vatsyayan,
"Neutrino masses from new Weinberg-like operators: Phenomenology of TeV scalar multiplets", arXiv:2312.13356 [hep-ph]

A. Giarnetti, J. Herrero-Garcia, SM, D. Meloni and D. Vatsyayan,
"Neutrino masses from new seesaw models: Low-scale variants and phenomenological implications", arXiv:2312.14119 [hep-ph]

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THANK YOU

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BACKUP SLIDES

A. Giarnetti, J. Herrero-Garcia, SM, D. Meloni and D. Vatsyayan,
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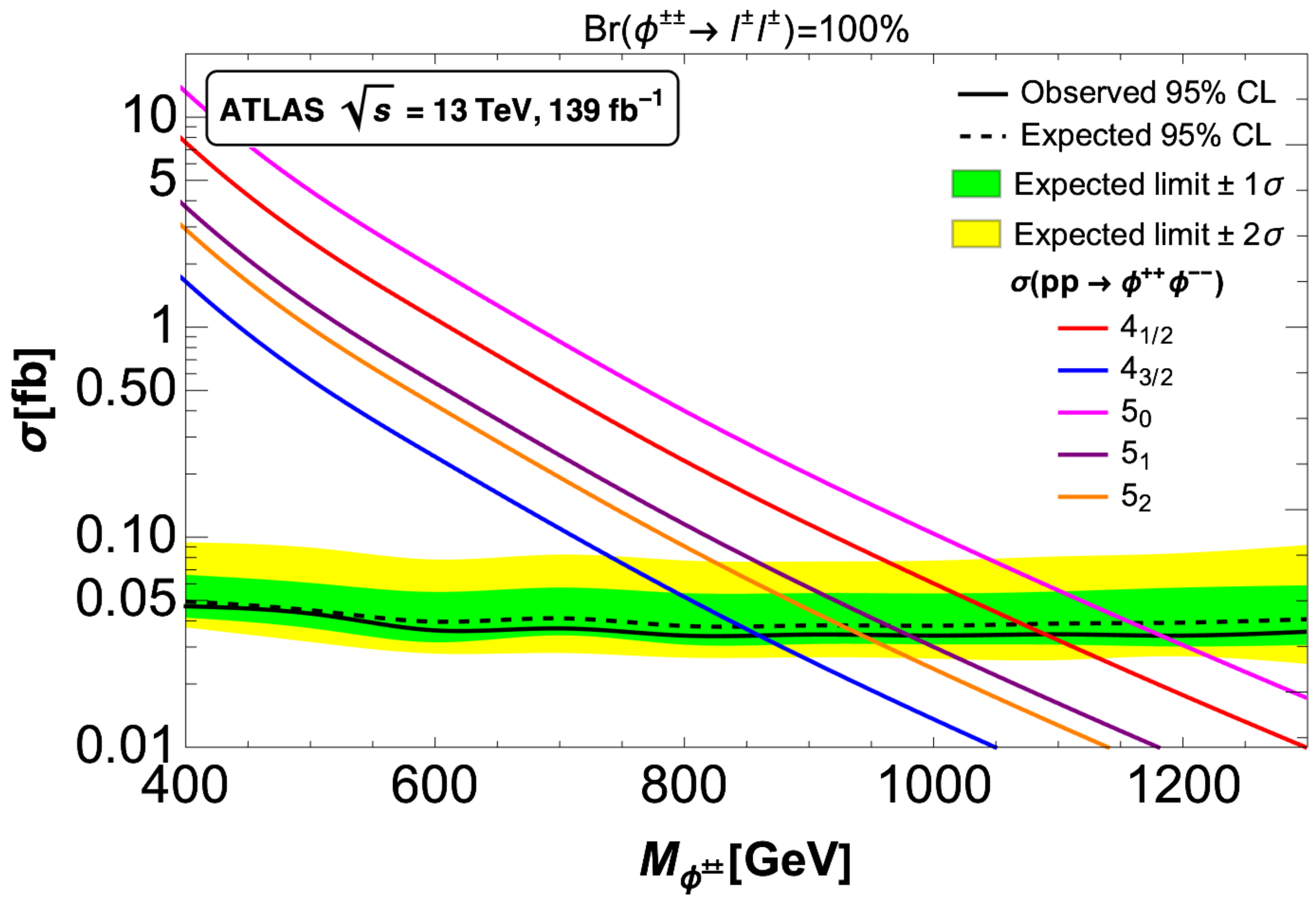
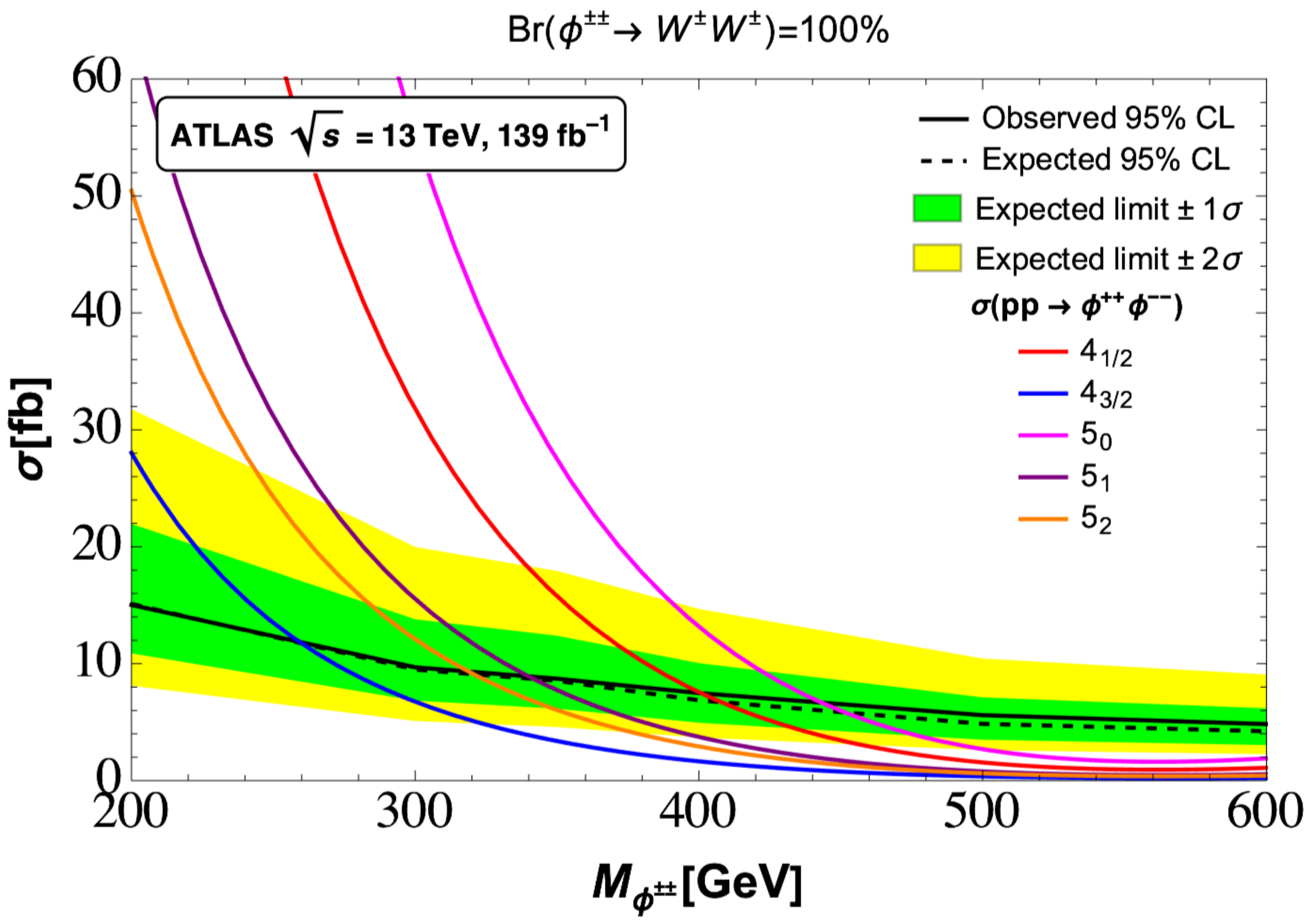
Laboratori Nazionali di Frascati, 15/04/2024

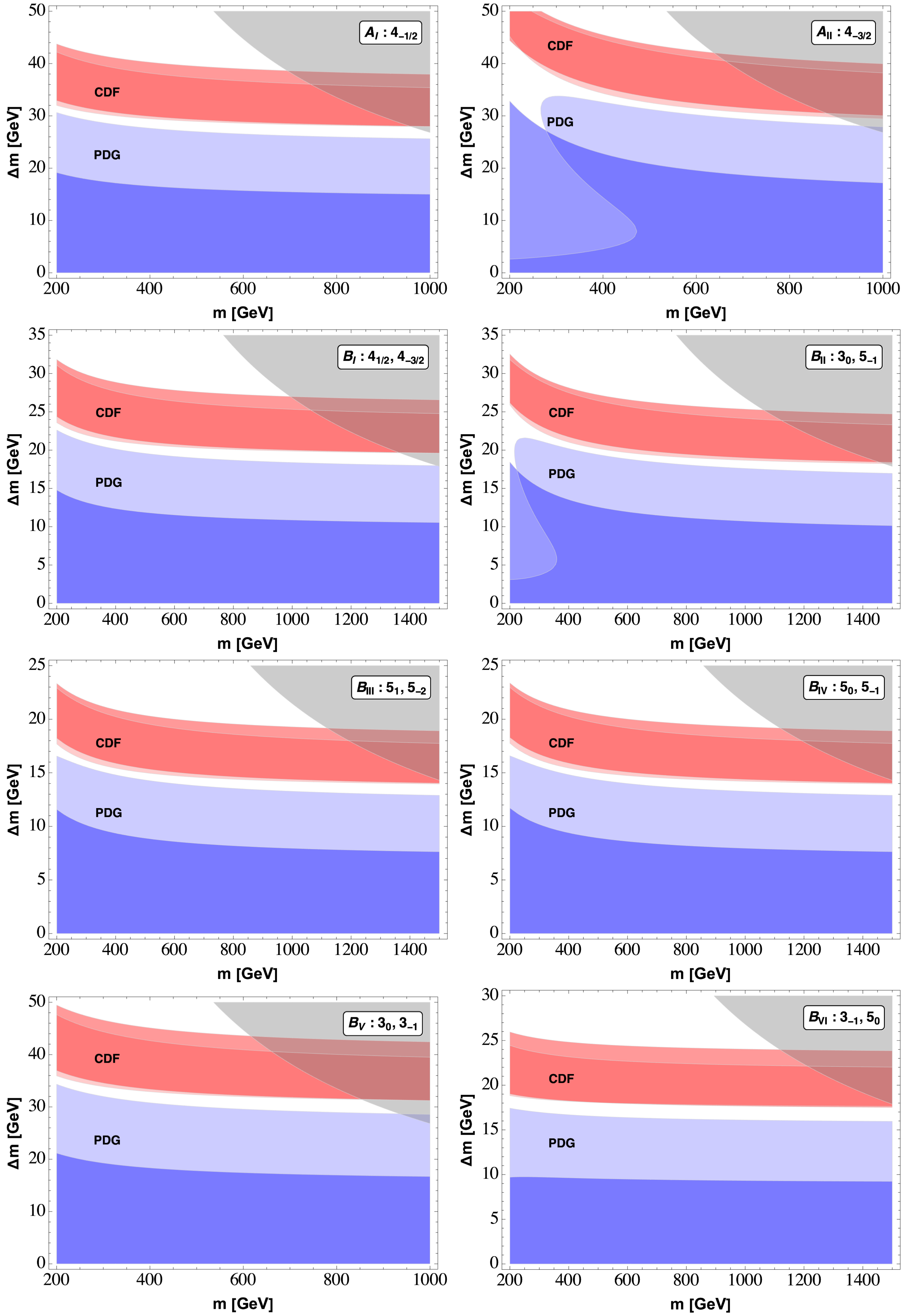
S. Marciano





	Limits on $M_{\Phi^{\pm\pm}}$ [GeV]	
Multiplet	$\mathrm{BR}(\Phi^{\pm\pm}\to l^{\pm}l^{\pm}) = 100\%$	$\mathrm{BR}(\Phi^{\pm\pm}\to W^{\pm}W^{\pm}) = 100\%$
$\mathbf{4}_{1/2}$	1090	400
$\mathbf{4}_{3/2}$	860	260
$\mathbf{5}_{0}$	1180	440
$\mathbf{5}_{1}$	980	340
$\mathbf{5}_{2}$	940	320

Collider signatures



$$T = \frac{1}{16\pi c_W^2 s_W^2 M_Z} \sum_{I_3=-I}^{+I} (I^2 - I_3^2 + I + I_3)\theta_+(M_{I_3}, M_{I_3-1}),$$

$$S = -\frac{Y}{3\pi} \sum_{I_3=-I}^{+I} I_3 \ln\frac{M_{I_3}}{\mu^2} - \frac{2}{\pi} \sum_{I_3=-I}^{+I} (I_3 c_W^2 - Y s_W^2)^2 \xi\left(\frac{M_{I_3}}{M_Z}, \frac{M_{I_3}}{M_Z}\right)$$

$$m_W \simeq m_W^{\rm SM}\left[1 - \frac{\alpha}{4(1-2s_W^2)}(S - 2(1-s_W^2)T)\right]$$

Collider signatures

FCNC radiative decays $l_\alpha \to l_\beta \gamma$

Model	Yukawa combination	Upper limits $\alpha\beta = \mu e$	$\alpha\beta = \tau e$	$\alpha\beta = \tau\mu$
$\mathbf{A_1}$	$\lvert y_1^{\beta^*} y_1^\alpha \rvert (\mathrm{TeV}/M_\Sigma)^2$	< 0.0002	< 0.13	< 0.16
$\mathbf{A_2}$	$\lvert y_1^{\beta^*} y_1^\alpha \rvert (\mathrm{TeV}/M_\mathcal{F})^2$	< 0.0004	< 0.24	< 0.28
$\mathbf{B_1}$	$\lvert y_1^{\beta^*} y_1^\alpha - 0.5\, y_2^{\beta^*} y_2^\alpha \rvert (\mathrm{TeV}/M_\mathcal{F})^2$	< 0.0004	< 0.29	< 0.34
$\mathbf{B_2}$	$\lvert y_1^{\beta^*} y_1^\alpha - 50\, y_2^{\beta^*} y_2^\alpha \rvert (\mathrm{TeV}/M_\mathcal{F})^2$	< 0.0011	< 0.72	< 0.84
$\mathbf{B_3}$	$\lvert y_1^{\beta^*} y_1^\alpha - 2.12\, y_2^{\beta^*} y_2^\alpha \rvert (\mathrm{TeV}/M_\mathcal{F})^2$	< 0.0002	< 0.15	< 0.18
$\mathbf{B_4}$	$\lvert y_1^{\beta^*} y_1^\alpha + 6.6\, y_2^{\beta^*} y_2^\alpha \rvert (\mathrm{TeV}/M_\mathcal{F})^2$	< 0.0004	< 0.24	< 0.28

Model	$\mathcal{O}_6^{(i)}$	Upper limits κ_{ee}	$\kappa_{\mu\mu}$	$\kappa_{\tau\tau}$	$\kappa_{\tau\mu}$	$\kappa_{\tau e}$	$\kappa_{\mu e}$
$\mathbf{A_1}$	$\mathcal{O}_6^{(1)}$	< 0.0013	< 0.0028	< 0.0053	< 0.0005	< 0.0005	$< 1.3 \times 10^{-6}$
$\mathbf{A_2}$	$\mathcal{O}_6^{(0)}$	< 0.0019	< 0.0042	< 0.0079	< 0.0007	< 0.0008	$< 2 \times 10^{-6}$
$\mathbf{B_1}$	$\mathcal{O}_6^{(1)}$	< 0.0036	< 0.0042	< 0.0012	< 0.0007	< 0.0008	$< 2 \times 10^{-6}$
$\mathbf{B_1}$	$\mathcal{O}_6^{(2)}$	< 0.0003	< 0.0007	< 0.0013	< 0.0001	< 0.0001	$< 3.3 \times 10^{-7}$
$\mathbf{B_2}$	$\mathcal{O}_6^{(2)}$	< 0.0038	< 0.0084	< 0.0159	< 0.0014	< 0.0016	$< 4 \times 10^{-6}$
$\mathbf{B_3}$	$\mathcal{O}_6^{(2)}$	< 0.0024	< 0.0028	< 0.0008	< 0.0005	< 0.0005	$< 1.3 \times 10^{-6}$
$\mathbf{B_4}$	$\mathcal{O}_6^{(1)}$	< 0.0038	< 0.0084	< 0.0159	< 0.0014	< 0.0016	$< 4 \times 10^{-6}$

Non-diagonal Z coupling

Deviation from Unitarity

$$\mathcal{L}^6_{eft} = c_i \left(\overline{L}\,\phi_i\right) iD^\mu\gamma_\mu \left(\phi_i^\dagger L\right)$$
$$v^2_{\phi_i} c_i = \epsilon$$

After SSB and disregarding couplings with the Higgses and Goldstone bosons

$\longrightarrow$

$$\mathcal{L}^{d\leq 6}_\nu = i\overline{\nu}_L\gamma^\mu\partial_\mu \left(\mathbb{1}+\epsilon\right)\,\nu_L - \frac{1}{2}\left(\overline{\nu^c}_L\,M_\nu\,\nu_L + \text{h.c.}\right)$$

Deviation from Unitarity

$$\mathscr{L}^6_{eft} = c_i \left(\overline{L}\,\phi_i\right) iD^\mu\gamma_\mu \left(\phi_i^\dagger L\right)$$
$$v^2_{\phi_i} c_i = \epsilon$$

After SSB and disregarding couplings with the Higgses and Goldstone bosons →

$$\mathscr{L}^{d\leq 6}_\nu = i\overline{\nu}_L\gamma^\mu\partial_\mu \left(\mathbb{1}+\epsilon\right)\, \nu_L - \frac{1}{2}\left(\overline{\nu^c}_L M_\nu \nu_L + \text{h.c.}\right)$$

After the field redefinition $\nu_L \to (\mathbb{1}+\epsilon)^{-1/2}\,\nu_L$ and rotating ν_L (l) with a unitary matrix U^ν_L:

$\nu_i = \nu_{Li} + \nu^c_{Li}$

$$\mathscr{L}^{d\leq 6}_{leptons} = \frac{1}{2}\overline{\nu}_i \left(i\,\gamma^\mu\partial_\mu - M^{diag}_{\nu\, i}\right)\, \nu_i + \overline{l}_\alpha \left(i\,\gamma^\mu\partial_\mu - M^{diag}_{l\,\alpha}\right)\, l_\alpha +$$
$$+\mathscr{L}_{CC} + \mathscr{L}_{NC} + \mathscr{L}_{EM}$$

The usual PMNS mixing matrix is replaced by a nonunitary matrix

$$U_{PMNS} \to U \equiv \left(\mathbb{1}-\epsilon\right)\, U^\nu_L$$

Deviation from Unitarity

$$\mathcal{L}^6_{eft} = c_i \left(\overline{L}\,\phi_i\right) iD^\mu\gamma_\mu \left(\phi_i^\dagger L\right)$$
$$v_{\phi_i}^2\, c_i = \epsilon$$

After SSB and disregarding couplings with the Higgses and Goldstone bosons →

$$\mathcal{L}^{d\leq 6}_\nu = i\overline{\nu}_L\gamma^\mu\partial_\mu\left(\mathbb{1}+\epsilon\right)\nu_L - \frac{1}{2}\left(\overline{\nu^c}_L\, M_\nu\,\nu_L + \text{h.c.}\right)$$

After the field redefinition $\nu_L \to \left(\mathbb{1}+\epsilon\right)^{-1/2}\nu_L$ and rotating ν_L (l) with a unitary matrix U^ν_L:

$\nu_i = \nu_{L\,i} + \nu^c_{L\,i}$

$$\mathcal{L}^{d\leq 6}_{leptons} = \frac{1}{2}\overline{\nu}_i\left(i\,\gamma^\mu\partial_\mu - M^{diag}_{\nu\, i}\right)\nu_i + \overline{l_\alpha}\left(i\,\gamma^\mu\partial_\mu - M^{diag}_{l\,\alpha}\right) l_\alpha +$$
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$$U_{PMNS} \to U \equiv \left(\mathbb{1}-\epsilon\right)\, U^\nu_L$$

Deviation from unitarity

Deviation from Unitarity

The usual PMNS mixing matrix is replaced
by a nonunitary matrix

$$U_{PMNS} \to U \equiv (\mathbb{1} - \epsilon)\ U_L^\nu$$

$$\mathcal{L}^6_{eft} = c_i^6 \left(\overline{L}\,\phi_i\right) iD^\mu \gamma_\mu \left(\phi_j^\dagger L\right)$$

$$v^2_{\phi_i} c_i^6 = \epsilon$$

$$c_i^6 = \frac{Y_i Y_i^\dagger}{\Lambda^2}$$

Deviation from Unitarity

The usual PMNS mixing matrix is replaced by a nonunitary matrix

$$U_{PMNS} \to U \equiv (\mathbb{1} - \epsilon)\, U_L^\nu$$

$$\mathcal{L}^6_{eft} = c_i^6 \left(\bar{L}\,\phi_i\right) iD^\mu \gamma_\mu \left(\phi_j^\dagger L\right)$$

$$v_{\phi_i}^2\, c_i^6 = \epsilon$$

$$c_i^6 = \frac{Y_i\, Y_i^\dagger}{\Lambda^2}$$

$$c^6 = \frac{Y\, Y^\dagger}{\Lambda^2}$$

$$c^5 = \frac{Y\, Y^T}{\Lambda}$$

$$c^6 \simeq \frac{c^5}{\Lambda} \to 0$$

In the models under analysis the deviation from unitarity is suppressed by the smallness of the neutrino masses, due to the correlation between c^5 and c^6.

Corrections to the vector boson couplings

$$\mathcal{L}^6_{eft} = c_i \left(\overline{L}\,\phi_i\right) iD^\mu\gamma_\mu \left(\phi_i^\dagger L\right)$$

$$c_i = (Y_i\,\Lambda^{-2}\,Y_i^\dagger)$$

	$\mathcal{O}_{SM}$			$\mathcal{O}_1$		
Model	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$
A_1	✘	✘	✘	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{4}$
A_2	1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{9}{4}$	0	$-\frac{3}{4}$

Z-couplings in units of
$c_i\,e/(2\,c_w\,s_w)$

W-couplings in units of
$c_i\,e/(2\sqrt{2}\,s_w)$

Corrections to the vector boson couplings

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$$c_i = (Y_i \Lambda^{-2} Y_i^\dagger)$$

	$\mathcal{O}_{SM}$			$\mathcal{O}_1$		
Model	$Z\ \nu_\alpha\nu_\beta$	$Z\ e_\alpha e_\beta$	$W\ e\nu$	$Z\ \nu_\alpha\nu_\beta$	$Z\ e_\alpha e_\beta$	$W\ e\nu$
A_1	✘	✘	✘	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{4}$
A_2	1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{9}{4}$	0	$-\frac{3}{4}$

Z-couplings in units of
$c_i\, e/(2\, c_w\, s_w)$

W-couplings in units of
$c_i\, e/(2\sqrt{2}\, s_w)$

Dominant effect ($v_{sm} \gg v_{bsm}$)
$\mathcal{O}_{SM} = c\left(\overline{L}\,\phi\right) iD^\mu \gamma_\mu \left(\phi^\dagger L\right)$

Corrections to the vector boson couplings

$$\mathscr{L}^6_{eft} = c_i \left(\overline{L}\,\phi_i\right) iD^\mu\gamma_\mu \left(\phi_j^\dagger L\right)$$

$$c_i = (Y_i\,\Lambda^{-2}\,Y_i^\dagger)$$

	$\mathcal{O}_{SM}$			$\mathcal{O}_1$		
Model	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$
A_1	✘	✘	✘	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{4}$
A_2	1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{9}{4}$	0	$-\frac{3}{4}$

Z-couplings in units of
$c_i\,e/(2\,c_w\,s_w)$

W-couplings in units of
$c_i\,e/(2\sqrt{2}\,s_w)$

Dominant effect ($v_{sm} \gg v_{bsm}$)
$\mathcal{O}_{SM} = c\left(\overline{L}\,\phi\right) iD^\mu\gamma_\mu \left(\phi^\dagger L\right)$

Sub-leading effect
$\mathcal{O}_1 = c_1\left(\overline{L}\,\phi_1\right) iD^\mu\gamma_\mu \left(\phi_1^\dagger L\right)$

Corrections to the vector boson couplings

$$\mathscr{L}^6_{eft} = c_i \left(\overline{L}\,\phi_i\right) iD^\mu \gamma_\mu \left(\phi_i^\dagger L\right)$$

$$c_i = (Y_i\,\Lambda^{-2}\,Y_i^\dagger)$$

	$\mathcal{O}_{SM}$			$\mathcal{O}_1$		
Model	$Z\ \nu_\alpha\nu_\beta$	$Z\ e_\alpha e_\beta$	$W\ e\nu$	$Z\ \nu_\alpha\nu_\beta$	$Z\ e_\alpha e_\beta$	$W\ e\nu$
A_1	✘	✘	✘	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{4}$
A_2	1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{9}{4}$	0	$-\frac{3}{4}$

Z-couplings in units of $c_i\, e/(2\, c_w\, s_w)$

W-couplings in units of $c_i\, e/(2\sqrt{2}\, s_w)$

	$\mathcal{O}_1$			$\mathcal{O}_2$		
Model	$Z\ \nu_\alpha\nu_\beta$	$Z\ e_\alpha e_\beta$	$W\ e\nu$	$Z\ \nu_\alpha\nu_\beta$	$Z\ e_\alpha e_\beta$	$W\ e\nu$
B_1	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{7}{4}$	$\frac{3}{4}$	-2	$-\frac{3}{4}$
B_2	0	0	$\frac{2}{3}$	$\frac{6}{5}$	$-\frac{2}{5}$	2
B_3	$-\frac{16}{5}$	0	$-\frac{4}{5}$	$-\frac{2}{5}$	$\frac{6}{5}$	2
B_4	$\frac{6}{5}$	$-\frac{2}{5}$	2	0	0	$-\frac{12}{5}$

Corrections to the vector boson couplings

$$\mathscr{L}^6_{eft} = c_i \left(\overline{L}\,\phi_i\right) iD^\mu \gamma_\mu \left(\phi_i^\dagger L\right)$$

$$c_i = (Y_i \Lambda^{-2} Y_i^\dagger)$$

	$\mathcal{O}_{SM}$			$\mathcal{O}_1$		
Model	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$
A_1	✘	✘	✘	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{4}$
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W-couplings in units of $c_i\,e/(2\sqrt{2}\,s_w)$

	$\mathcal{O}_1$			$\mathcal{O}_2$		
Model	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$	$Z\,\nu_\alpha\nu_\beta$	$Z\,e_\alpha e_\beta$	$W\,e\nu$
B_1	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{7}{4}$	$\frac{3}{4}$	-2	$-\frac{3}{4}$
B_2	0	0	$\frac{2}{3}$	$\frac{6}{5}$	$-\frac{2}{5}$	2
B_3	$-\frac{16}{5}$	0	$-\frac{4}{5}$	$-\frac{2}{5}$	$\frac{6}{5}$	2
B_4	$\frac{6}{5}$	$-\frac{2}{5}$	2	0	0	$-\frac{12}{5}$

Notice that the relation $g_w = g_{Z_\nu} + g_{Z_e}$ holds only for the model A_2 for the operator $\mathcal{O}_{SM}$ (custodial symmetry conservation)

Breaking of $U(1)_L$

The accidental global U(1) of lepton number can be broken by the presence of the new Higgses

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SPONTANEOUS SYMMETRY BREAKING

- No explicit lepton number violating (LNV) terms
- The symmetry breaking is realized by the non-zero vevs of the new scalars
- Production of an additional pure massless Goldstone boson

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- The symmetry breaking is realized by the non-zero vevs of the new scalars
- Production of an additional pure massless Goldstone boson

EXPLICIT SYMMETRY BREAKING

- LNV terms in the scalar potential, *e.g.* $H^2\phi_i\phi_j$
- A new pseudo-Goldstone boson with a non-zero mass is provided
- Its mass is proportional to the LNV terms and to the VEVs of the scalars

The Majoron - A-class models

$$V(H,\Phi) = -\mu_H^2\, H^\dagger H + \mu_\Phi^2\, \Phi^\dagger\Phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger\Phi)^2 +$$
$$+\lambda_3\, H^\dagger H \Phi^\dagger \Phi + \lambda_4\, H^* H \Phi^* \Phi + \lambda_5\, \Phi^*\Phi\, \Phi^*\Phi +$$
$$+\left[\lambda_6\, \Phi^* HHH + \lambda_7\, H\Phi H\Phi + \lambda_8\, H^*\Phi HH + \text{h.c.}\right]$$

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$$+\lambda_3\, H^\dagger H \Phi^\dagger \Phi + \lambda_4\, H^* H \Phi^* \Phi + \lambda_5\, \Phi^*\Phi\, \Phi^*\Phi +$$
$$+\left[\lambda_6\, \Phi^* H \Phi\Phi + \lambda_7\, H\Phi H\Phi + \lambda_8\, H^*\Phi H H + \text{h.c.}\right]$$

LNV terms

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$$+\lambda_3 H^\dagger H \Phi^\dagger \Phi + \lambda_4 H^* H \Phi^* \Phi + \lambda_5 \Phi^*\Phi\, \Phi^*\Phi +$$
$$+\left[\lambda_6 \Phi^* H\Phi\Phi + \lambda_7 H\Phi H\Phi + \lambda_8 H^*\Phi HH + \text{h.c.}\right]$$

LNV terms

$$m_z^2 = 0\,, \quad m_J^2 = \frac{v_H (v_H^2 + v_\Phi^2)}{9 v_\Phi}\left[-3\sqrt{3}\lambda_8 + 24\frac{v_\Phi}{v_H}\lambda_7 + 2\sqrt{3}\frac{v_\Phi^2}{v_H^2}\lambda_6\right]$$

$$V(H,\Phi) = -\mu_H^2 H^\dagger H + \mu_\Phi^2 \Phi^\dagger\Phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger \Phi)^2 + $$
$$+\lambda_3 H^\dagger H \Phi^\dagger \Phi + \lambda_4 H^* H \Phi^* \Phi + \lambda_5 \Phi^*\Phi\, \Phi^*\Phi + $$
$$+\left[\lambda_6 \Phi^* H\Phi\Phi + \lambda_7 H\Phi H\Phi + \lambda_8 H^*\Phi HH + \text{h.c.}\right]$$

LNV terms

$$m_z^2 = 0\,, \quad m_J^2 = \frac{v_H (v_H^2 + v_\Phi^2)}{9 v_\Phi}\left[-3\sqrt{3}\lambda_8 + 24\frac{v_\Phi}{v_H}\lambda_7 + 2\sqrt{3}\frac{v_\Phi^2}{v_H^2}\lambda_6\right]$$

Breaking of hypercharge

The Majoron - A-class models

$$V(H,\Phi) = -\mu_H^2\, H^\dagger H + \mu_\Phi^2\, \Phi^\dagger \Phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger\Phi)^2 +$$

$$+\lambda_3\, H^\dagger H \Phi^\dagger \Phi + \lambda_4\, H^* H \Phi^* \Phi + \lambda_5\, \Phi^*\Phi\, \Phi^*\Phi +$$

$$+\left[\lambda_6\, \Phi^* H \Phi\Phi + \lambda_7\, H\Phi H\Phi + \lambda_8\, H^*\Phi H H + \text{h.c.}\right]$$

LNV terms

Breaking of lepton number

$$m_z^2 = 0\,, \quad m_J^2 = \frac{v_H\,(v_H^2 + v_\Phi^2)}{9 v_\Phi}\left[-3\sqrt{3}\lambda_8 + 24\frac{v_\Phi}{v_H}\lambda_7 + 2\sqrt{3}\frac{v_\Phi^2}{v_H^2}\lambda_6\right]$$

Breaking of hypercharge

The Majoron - B-class models

$$V(H, \Phi, \Delta) \subset + \lambda_1 \Delta^* \Phi^* H\, \Delta + \lambda_2 (\Delta^* \Phi^* H\, \Delta)' + \lambda_3 \Phi^* \Phi^* H\, H + \lambda_4 \Phi^* \Phi^* H \Phi +$$
$$+ \lambda_5 \Phi^* H^* H\, H + \lambda_6 H\, H\, H \Delta + \lambda_7 H\, H \Phi \Delta + \lambda_8 H \Phi \Phi \Delta + \lambda_9 \Phi \Phi \Phi \Delta + \text{h.c.}$$

$$V(H,\Phi,\Delta) \subset +\lambda_1 \Delta^* \Phi^* H\, \Delta + \lambda_2 (\Delta^* \Phi^* H\, \Delta)' + \lambda_3 \Phi^* \Phi^* H\, H + \lambda_4 \Phi^* \Phi^* H \Phi +$$
$$+\lambda_5 \Phi^* H^* H\, H + \lambda_6 H\, H\, H \Delta + \lambda_7 H\, H \Phi \Delta + \lambda_8 H \Phi \Phi \Delta + \lambda_9 \Phi \Phi \Phi \Delta + \text{h.c.}$$

Neglecting contributions $\mathcal{O}(v_{\phi,\Delta}/v_H)$

$$m_z^2 = 0\,, \quad m_J^2 = \frac{\lambda_5 v_H^3}{2\sqrt{3}v_\Phi}\,, \quad m_\Omega^2 = \frac{\lambda_6 v_H^3}{2 v_\Delta}$$

The Majoron - B-class models

$$V(H,\Phi,\Delta) \subset + \lambda_1 \Delta^* \Phi^* H\,\Delta + \lambda_2 (\Delta^* \Phi^* H\,\Delta)' + \lambda_3 \Phi^* \Phi^* H\,H + \lambda_4 \Phi^* \Phi^* H \Phi +$$

$$+ \lambda_5 \Phi^* H^* H\,H + \lambda_6 H\,H\,H \Delta + \lambda_7 H\,H \Phi \Delta + \lambda_8 H \Phi \Phi \Delta + \lambda_9 \Phi \Phi \Phi \Delta + \text{h.c.}$$

$U(1)_L$ $\qquad$ $U(1)_X$

Neglecting contributions $\mathcal{O}(v_{\phi,\Delta}/v_H)$

$$m_z^2 = 0\,, \quad m_J^2 = \frac{\lambda_5 v_H^3}{2\sqrt{3} v_\Phi}\,, \quad m_\Omega^2 = \frac{\lambda_6 v_H^3}{2 v_\Delta}$$

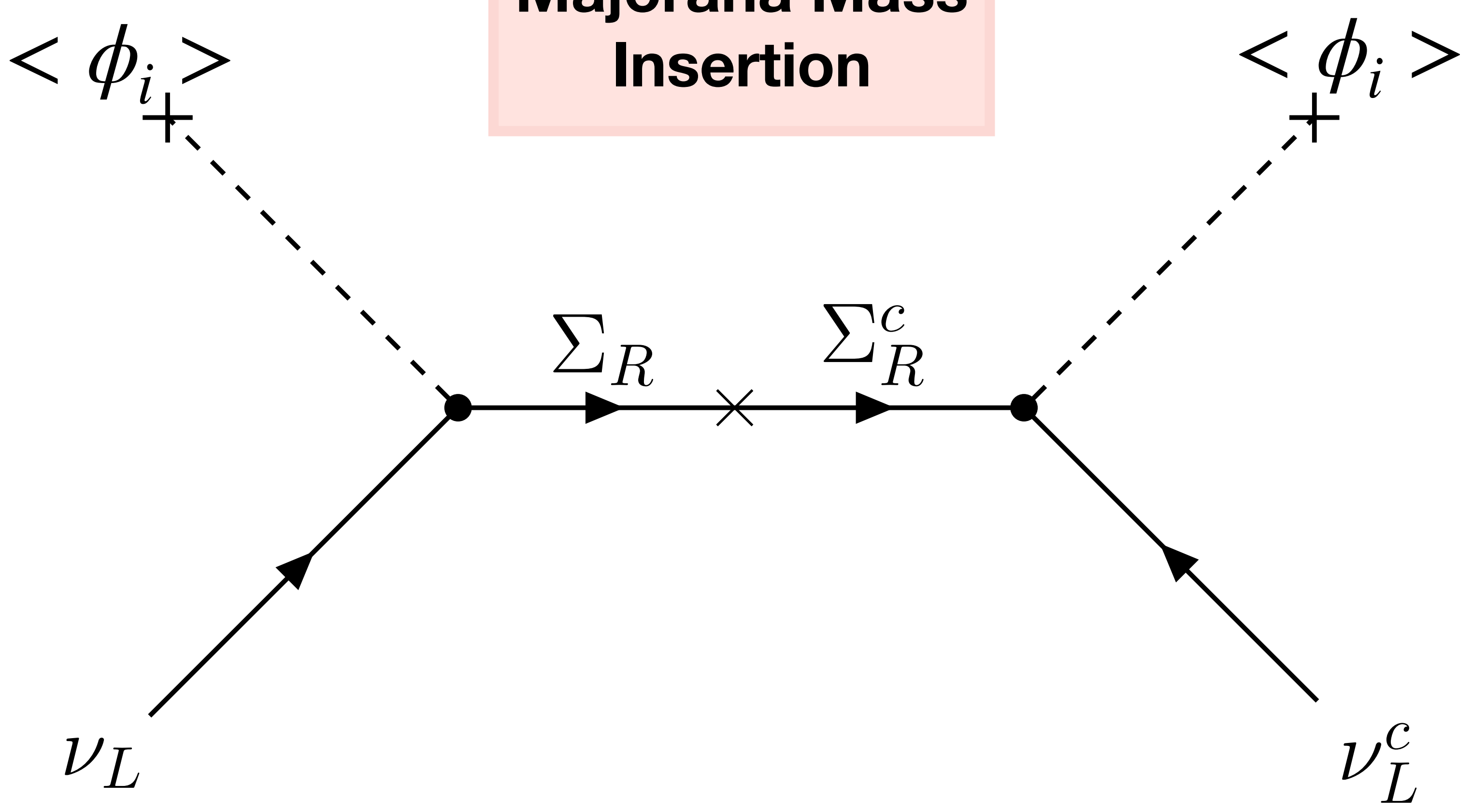
Higher SU(2) representations - UV completions

BSM Higgs-like scalars

$$\phi_i = (N_i, Y_i) \quad i = 1,2 \xrightarrow{\rho(N_i, Y_i, v_{\phi_i}) \simeq 1} v_{\phi_i} \ll v_\phi$$

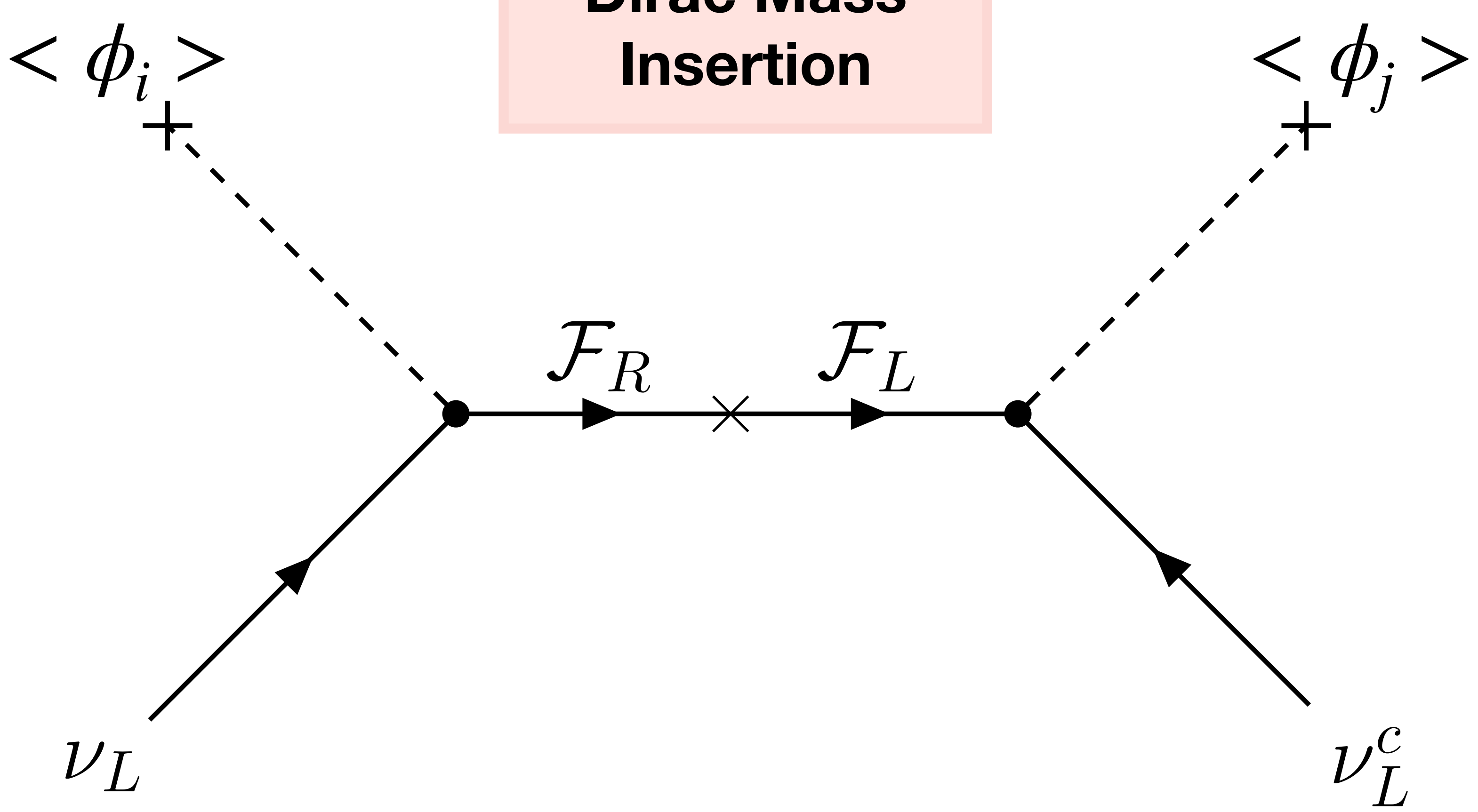
$$\mathcal{L}^5_{eft} = \frac{c_{0i}}{\Lambda}\phi\phi_i LL + \frac{c_{ii}}{\Lambda}\phi_i\phi_i LL + \frac{c_{ij}}{\Lambda}\phi_i\phi_j LL$$

Majorana Mass Insertion



$$\mathcal{L} \supset \overline{\Sigma}_R i D^\mu \gamma_\mu \Sigma_R - (Y_i \overline{L} \phi_i \Sigma_R + \text{h.c}) - \frac{1}{2} M_\Sigma \Sigma_R^T C^\dagger \Sigma_R$$

Dirac Mass Insertion



$$\mathcal{L} \supset -\left(Y_i \overline{L} \phi_i \mathcal{F}_R + \text{h.c.}\right) - \left(Y_j \overline{L} \phi_j \mathcal{F}^c_L + \text{h.c.}\right) + \overline{\mathcal{F}}\left(i D^\mu \gamma_\mu - M_{\mathcal{F}}\right)\mathcal{F}$$

The Majoron - A-class models

$$V(H,\Phi) = -\mu_H^2\, H^\dagger H + \mu_\Phi^2\, \Phi^\dagger\Phi + \lambda_1 (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger\Phi)^2 +$$

$$+\lambda_3\, H^\dagger H \Phi^\dagger\Phi + \lambda_4\, H^* H \Phi^* \Phi + \lambda_5\, \Phi^*\Phi\, \Phi^*\Phi +$$

$$+\left[\lambda_6\, \Phi^* HHH + \lambda_7\, H\Phi H\Phi + \lambda_8\, H^*\Phi HH + \text{h.c.}\right]$$

LNV terms

$$H^0 = v_H + S_H + i\,\chi$$

$$\Phi^0 = v_\Phi + S_\Phi + i\,\eta$$

$$\begin{pmatrix} z \\ J \end{pmatrix} = O \begin{pmatrix} \chi \\ \eta \end{pmatrix}$$