(2) Istituto Nazionale di Fisica Nucleare

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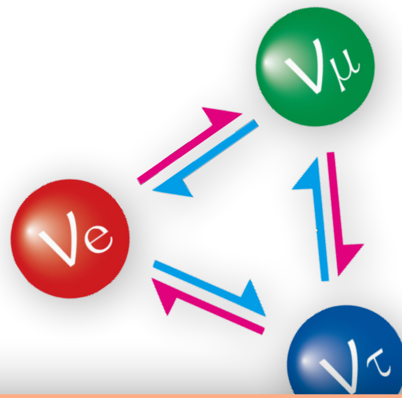


Modular Symmetry: a new perspective on the Flavor Puzzle

Matteo Parriciatu (1)(2)

Based on: JHEP09(2023)043, in collaboration with D. Meloni (1)(2)

IRN Terascale @ LNF, Frascati
April 15-17th, 2024

$3\,\nu$ Paradigm

$$\underbrace{|\nu_\alpha\rangle}_{\text{Interaction basis}} = \sum_{i=1}^{3} U^*_{\alpha i}\, \underbrace{|\nu_i\rangle}_{\text{Mass basis}}$$

$\theta_{12}\ \theta_{13}\ \theta_{23}$

δ_{CP} $\boxed{\alpha_1\ , \alpha_2}$

Majorana

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \times \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$$

$c_{ij} \equiv \cos\theta_{ij}\ ,\ s_{ij} \equiv \sin\theta_{ij}$ Pontecorvo - Maki - Nakagawa - Sakata

$$\boxed{P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3}\sum_{j=1}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \exp\left(-i\frac{\Delta m^2_{ij} L}{2E}\right)}$$

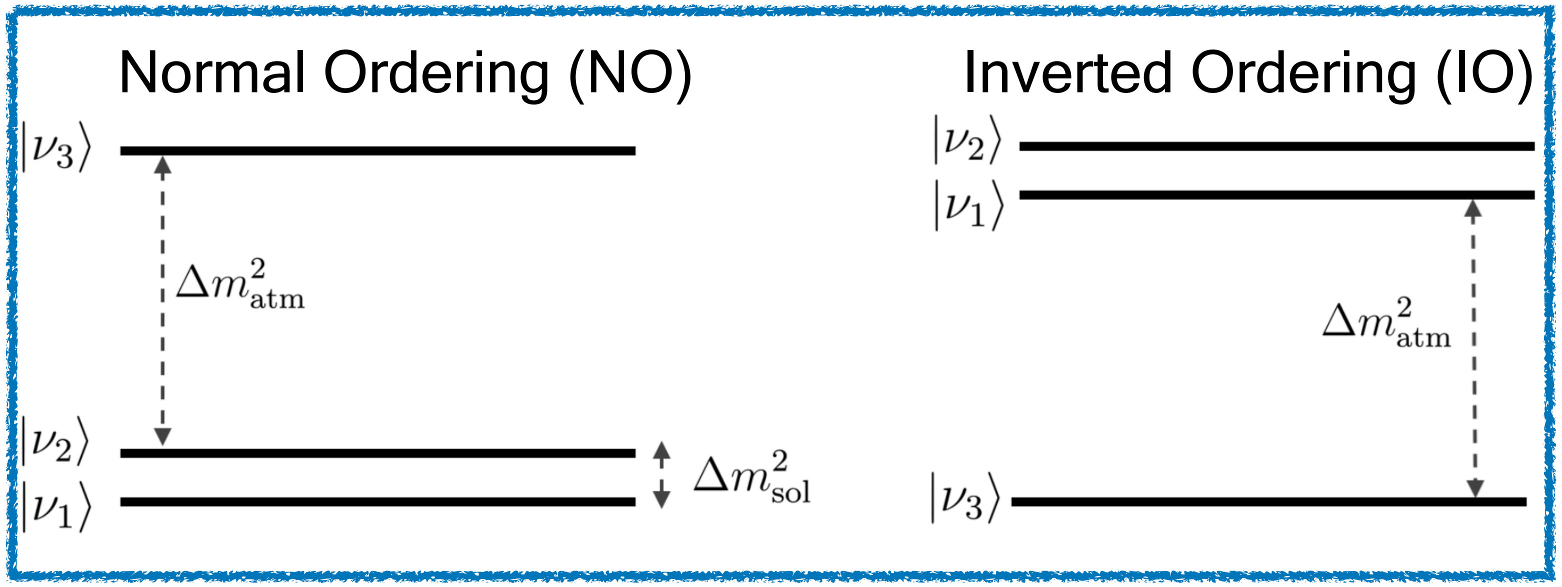
Sign determined from MSW

$$|\Delta m^2_{\text{sol}}| \equiv |m_2^2 - m_1^2| \qquad |\Delta m^2_{\text{atm}}| \equiv |m_3^2 - m_{1,2}^2|$$

$$r \equiv \frac{\Delta m^2_{\text{sol}}}{|\Delta m^2_{\text{atm}}|} \approx 1/30$$

Undetermined sign

?

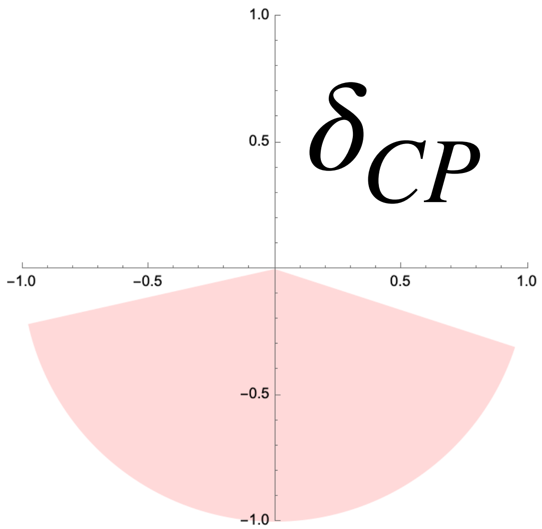


Best fit values

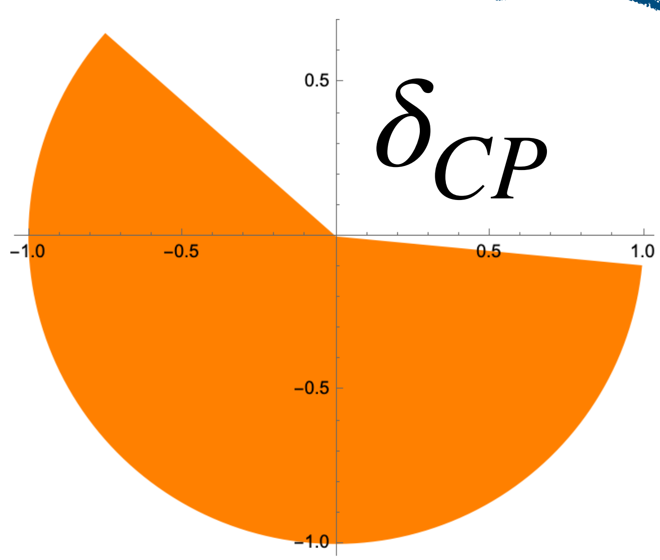
$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 42.4^\circ \qquad (\mathbf{NO})$$
$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 48.9^\circ \qquad (\mathbf{IO})$$

F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

Two big and one small angles



NO
IO



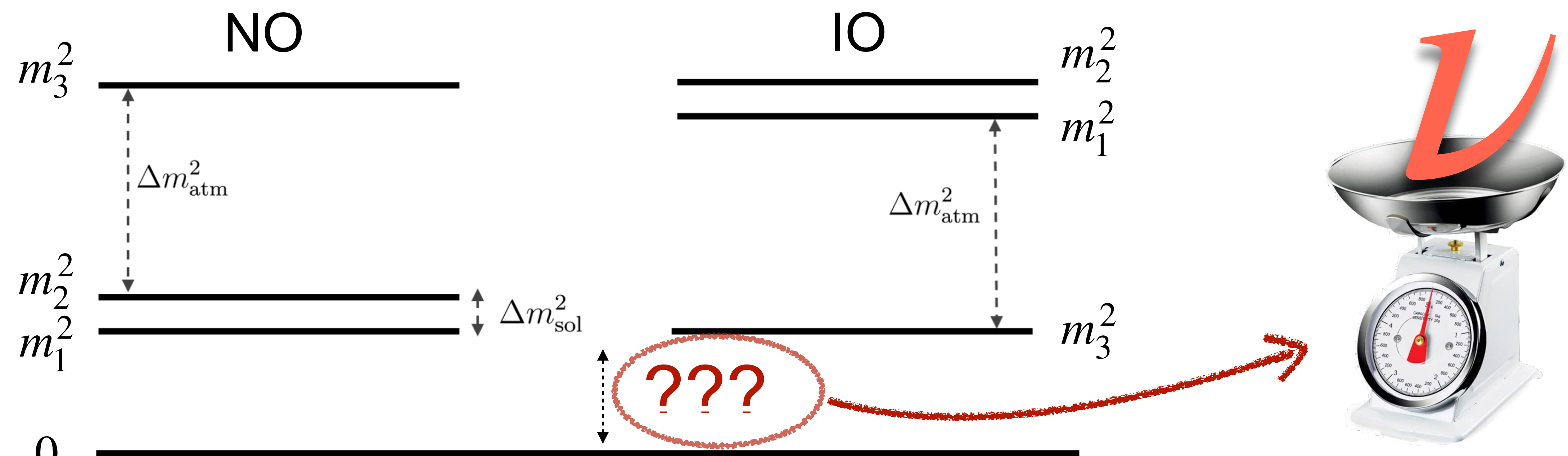
Violation if

$$\frac{\delta_{CP}}{\pi} \neq \{0,1\}$$

CP-conserving still allowed at 2σ (NO)

3σ ranges from
F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

Flavor puzzle and neutrino mixing



$$\sum_i^3 m_i < 0.115 \text{ eV} \qquad (95\% \text{ C.L.})$$

A. Shadab et al.

Phys. Rev. D **103** (Apr, 2021)

Cosmology

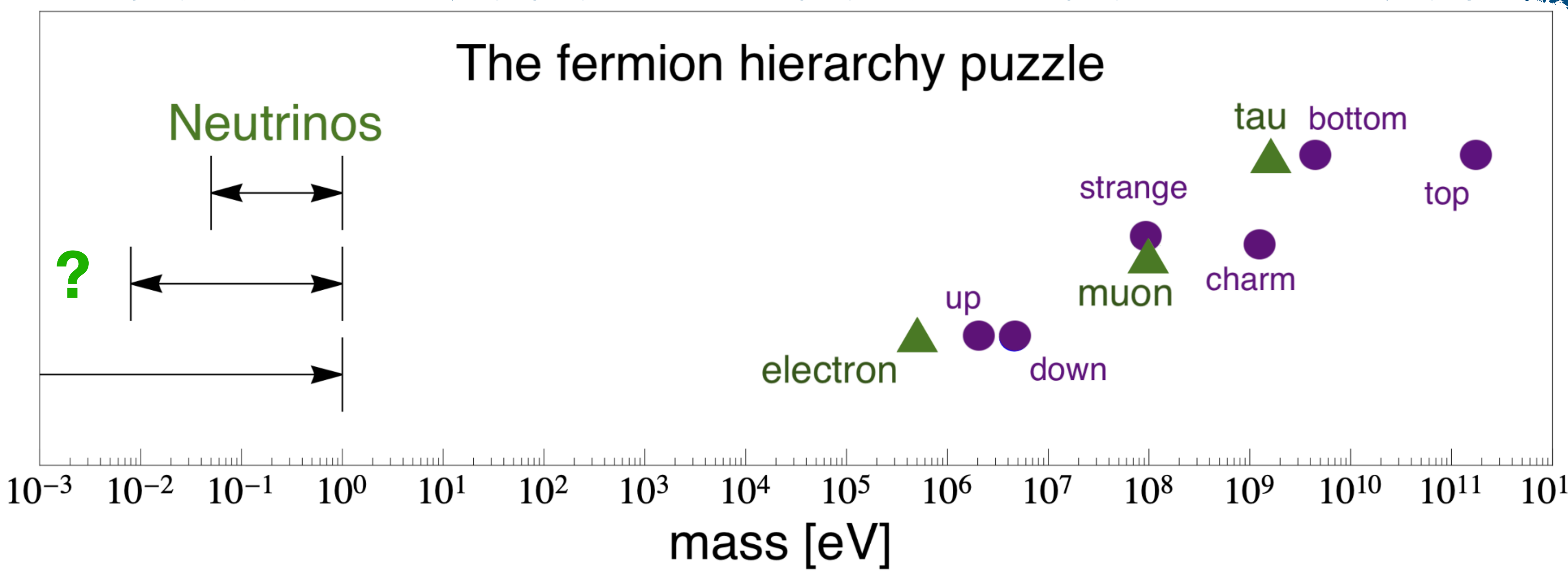
$$\sqrt{\sum_i m_i^2 |U_{ei}|^2} < 0.8 \text{ eV}$$

Nat. Phys. **18** (Feb, 2022)

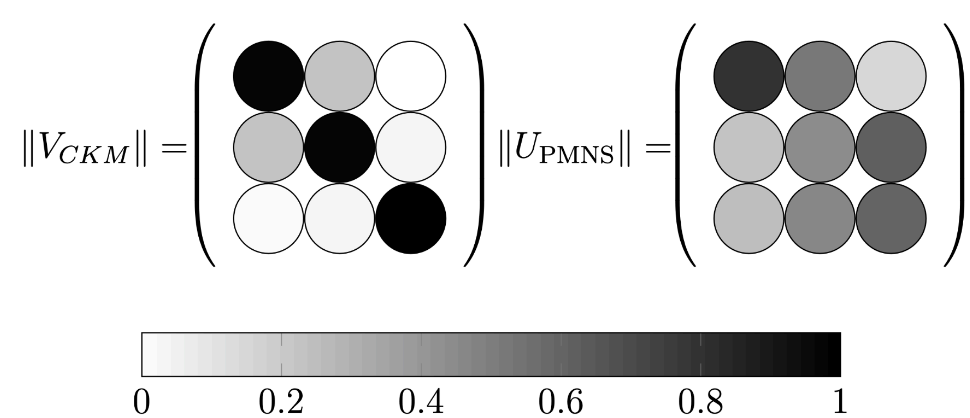
KATRIN

▲ leptons

● quarks



$$m_e/m_\mu \simeq \frac{1}{200} \qquad m_\mu/m_\tau \simeq \frac{1}{17}$$



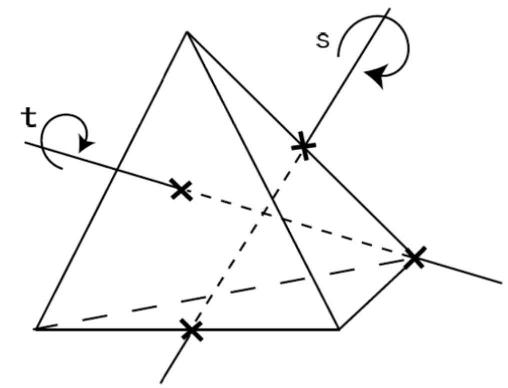
D. Meloni [1709.02662v2]

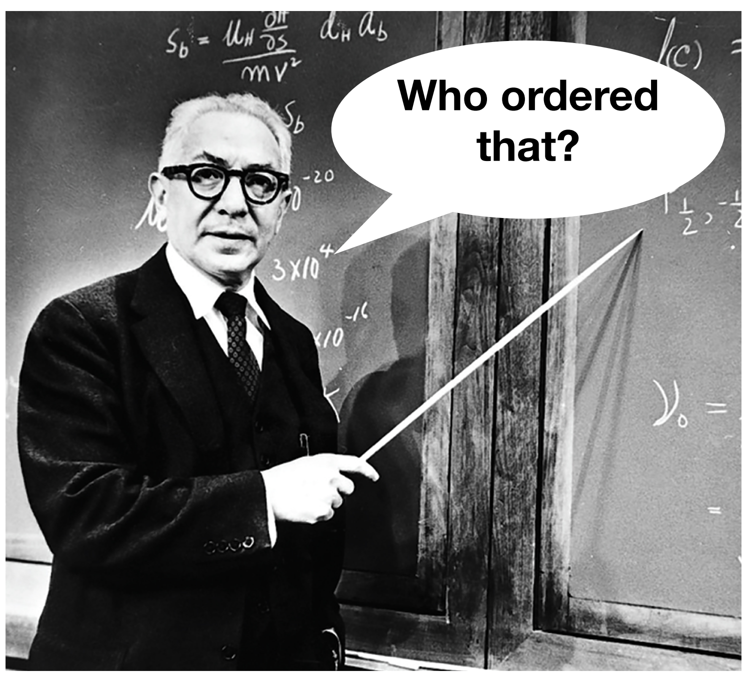
Three "copies", different masses

ν_e	ν_μ	ν_τ
e	μ	τ

non-abelian discrete symmetries

$S_3 \quad A_4 \quad S_4 \quad A_5 \ldots\ldots$





Isidor I. Rabi

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal compatible with data until 2012

$$\theta_{13} = 0 \quad \theta_{12} \simeq 35^\circ \quad \theta_{23} = 45^\circ$$

Not zero

F. P. An *et al.*, "Observation of electron-antineutrino disappearance at daya bay," *Phys. Rev. Lett.* **108** (Apr, 2012)

Shortcomings of the traditional approach

EFT with scalar "flavons" ϕ_i

$$\mathscr{W}_{Yukawa} \supset \frac{\alpha}{\Lambda} E^c (L\phi_i)_{\mathbf{1}} H_d$$

✗ $U_{\text{PMNS}} = U^0_{\text{TBM}} + \ldots$ corrections

$\downarrow$ $\theta_{13} = 0$ Automatic

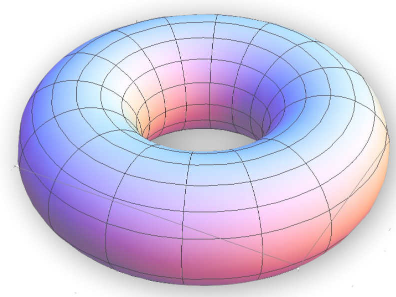
$\downarrow$ $\theta_{13} \approx 8.5^\circ$?

$\delta\theta_{23}\ ,\ \delta\theta_{12} \approx 8.5^\circ$

✗ α_i γ_i β_i ς_i ϱ_i ς_i η_i γ_i ϱ_i α_i β_i $\ldots$

✗ $V(\phi_i) \to$ Mess!

F. Feruglio
[1706.08749]



$\tau \equiv$ modulus

$$(y_{jk}\bar{L}_j H \ell_{Rk} + \text{h.c.})$$

Free structureless parameters in the SM

Modular Invariance

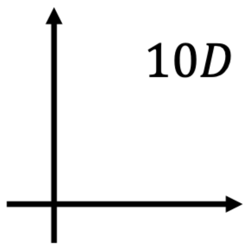
$Y(\tau)$

Modular forms: less free parameters

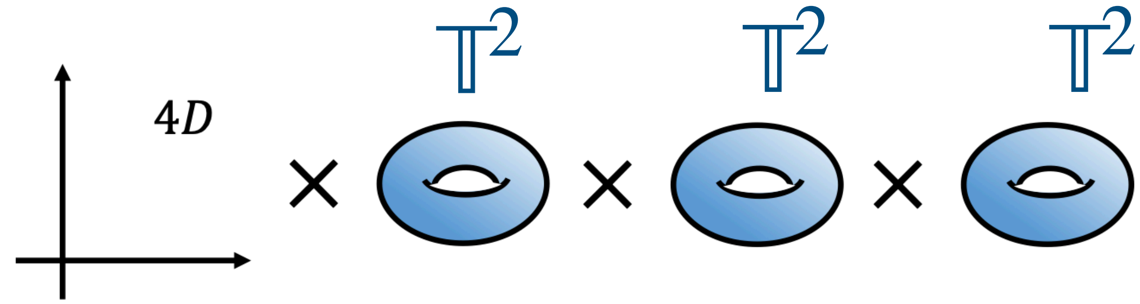
Predictivity ✓

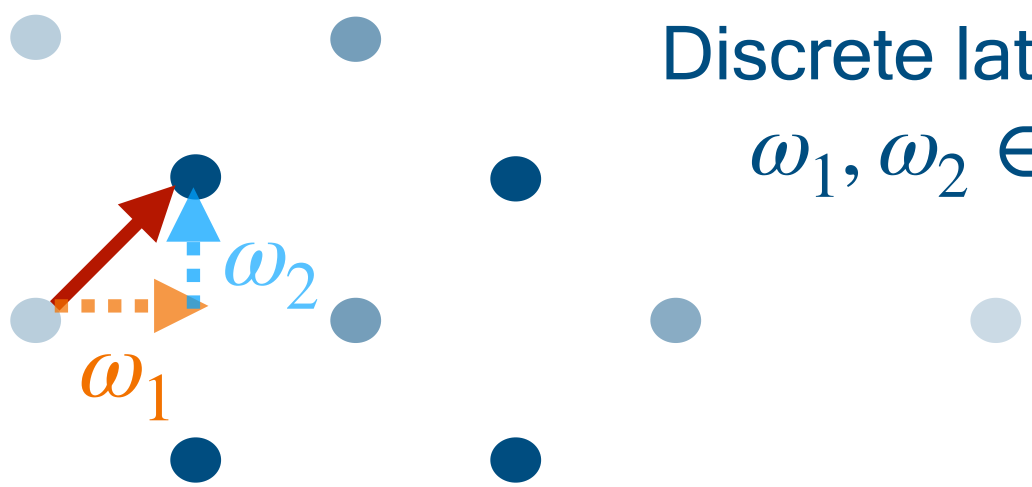
User-friendly ✗

Modular invariance: a step forward?



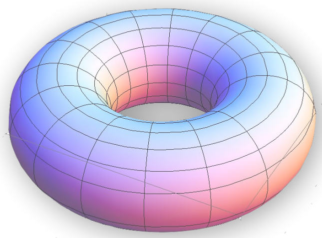
Compactification
SUSY-conserving





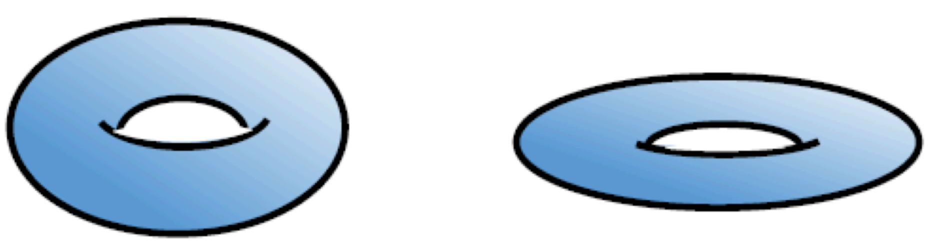
Discrete lattice Λ

$\omega_1, \omega_2 \in \mathbb{C}$

$\mathbb{C}/\Lambda \equiv$ 

$$\tau \equiv \frac{\omega_2}{\omega_1}$$

Modulus



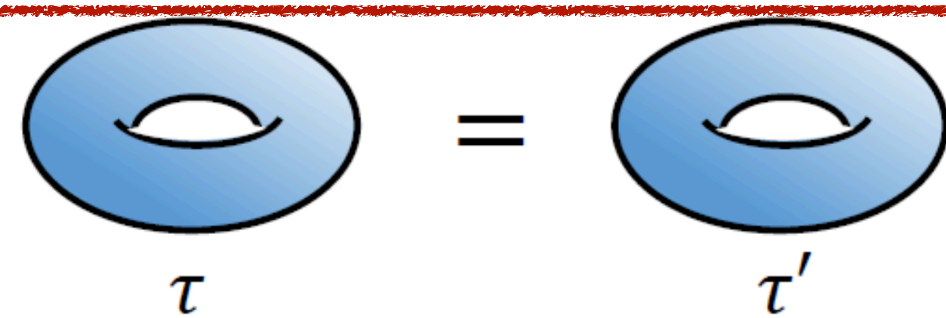
Change basis? $a, b, c, d \in \mathbb{Z}$

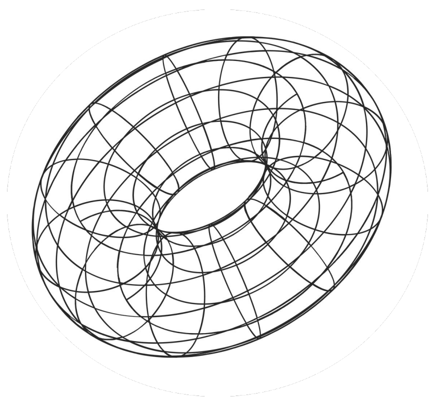
$$\begin{pmatrix} \omega_2' \\ \omega_1' \end{pmatrix} = \gamma \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix}$$

$$\boxed{\gamma \in \mathrm{SL}(2,\mathbb{Z}) \equiv \text{Modular group} \equiv \Gamma}$$

$$\tau \equiv \frac{\omega_2}{\omega_1} \xrightarrow{\mathrm{SL}(2,\mathbb{Z})} \tau' = \frac{a\,\omega_2 + b\,\omega_1}{c\,\omega_2 + d\,\omega_1} = \boxed{\frac{a\tau + b}{c\tau + d}}$$

$\tau \to \tau'$





$$\int d^4x\, d^6y\, \mathscr{L}_{10D} \Longrightarrow \int d^4x\, \mathscr{L}_{EFT}$$

What is a modular form? $Y(\tau)$

- $Y(\gamma(\tau)) = (c\tau + d)^k\, Y(\tau)$

$c, d \in \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \gamma$

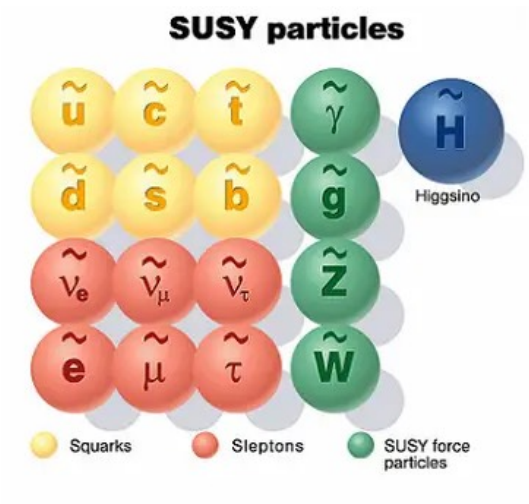
$a, b, c, d \in \mathbb{Z} \quad , \quad ad - bc = 1$

Holomorphic in: $\{\tau \in \mathbb{C} \mid \mathrm{Im}(\tau) > 0\}$

- "Weight" k>0

Very constraining!

Superfields transformations



$$
\begin{cases}
\tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} & \gamma \in \Gamma \\[2em]
\underbrace{\varphi^{(I)}}_{\text{Usual matter fields}} \to (c\tau + d)^{-k_I} \underbrace{\rho^{(I)}(\gamma)}_{\text{Unitary irrep. of } \Gamma_N \subset \Gamma} \varphi^{(I)}
\end{cases}
$$

$N = 1, 2, 3, \ldots$ "Level"

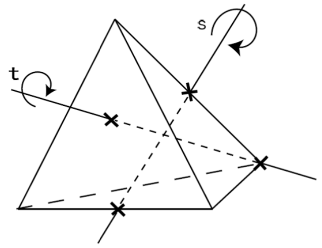
Finite modular group

$$\Gamma_N$$

for $N \leq 5$ isomorphic to

$N = 2$	$N = 3$	$N = 4$	$N = 5$
S_3	A_4	S_4	A_5

non-abelian discrete groups



- $\mathcal{W}(\Phi) = \sum (Y_{I_1 \ldots I_n}(\tau)\, \varphi^{(I_1)} \ldots \varphi^{(I_n)})_{\mathbf{1}}$

$\mathscr{W}(\Phi)$ modular invariant if:

$$\begin{cases} \rho \otimes \rho_{I_1} \otimes \rho_{I_2} \ldots \otimes \rho_{I_n} \supset \mathbf{1} & \longrightarrow \text{Usual} \\ k_Y = k_{I_1} + k_{I_2} + \ldots + k_{I_n} & \longrightarrow \text{Novelty} \end{cases}$$

$$Y_{I_1 \ldots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \ldots I_n}(\tau)$$

Yukawa: modular forms of weight k_Y

$$\varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)}$$

Superfields with modular charges $-k_I$

$$(c\tau + d)^{k_Y} (c\tau + d)^{-\sum k_{I_n}} = 1$$

Symmetry breaking

$$\Gamma$$

$$< \tau > \equiv \text{VEV}$$

$$\Gamma$$

$$Y(\tau)$$

uniquely determined by τ

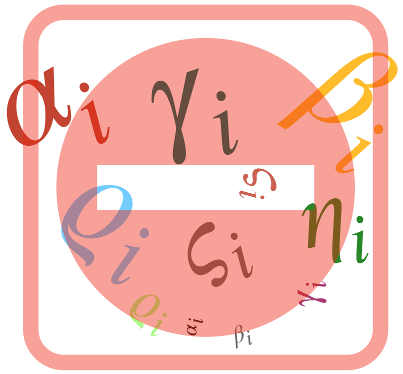
$$\text{Fourier} \sim \sum_{n}^{\infty} a_n q^n$$

$$q \equiv e^{2\pi i \tau}$$

Lepton mass matrices

$$M_e \sim \sum_i \alpha_i \begin{pmatrix} f_{11}(\tau) & f_{12}(\tau) & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & f_{33}(\tau) \end{pmatrix}$$

$f_{ij} \equiv$ pre-determined functions of τ



$\alpha_i \equiv$ limited number of free parameters

Let's choose $N = 3\ (\Gamma_3 \cong A_4)$

- Neutrino sector: Weinberg $\mathscr{W}_\nu \supset \frac{1}{\Lambda}(H_u H_u L L Y)_{\mathbf{1}}$
- Low weights: $k_L = +1\ ,\ k_u = 0$ F. Feruglio [1706.08749]
- $L \sim \mathbf{3} \equiv (L_e, L_\mu, L_\tau)$ $L_\alpha \sim \mathbf{2}_{-1/2}$ under $SU(2)_L \times U(1)_Y$

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \sim \mathbf{3}$$

Weight: $k_Y = 2$

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3}_S + \mathbf{3}_A$$

$$L \otimes L \otimes Y^{(2)}_{\mathbf{3}} \sim (\mathbf{3} \otimes \mathbf{3}) \otimes \mathbf{3} \supset \mathbf{1}$$

Weights: $-k_L - k_L + k_Y = 0 \to -1 - 1 + 2 = 0$ ✓

- Neutrino sector: Weinberg

$$\mathscr{W}_\nu \supset \frac{1}{\Lambda}(H_u H_u L L Y)_{\mathbf{1}}$$

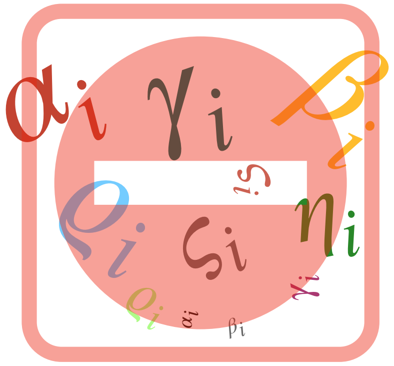
$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \sim \mathbf{3}$$

Weight: $k_Y = 2$

$$\boxed{m_\nu = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \frac{v_u^2}{\Lambda}}$$

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + \ldots \\ Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + \ldots) \\ Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + \ldots) \end{aligned}$$

$$q \equiv e^{2\pi i \tau}$$



But poor fit, too constrained

Non-Abelian discrete flavour symmetry

$$\mathscr{W}_L \supset \frac{\alpha}{\Lambda} E_i^c (L_j \varphi_k)_{\mathbf{1}} H_d$$

$$\varphi \sim \mathbf{2} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \text{Flavon}$$

Free parameters

Structure dependent on the symmetry breaking sector

Modular flavour symmetry

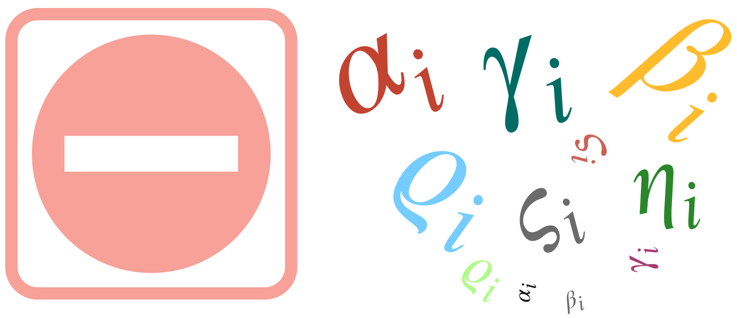
$$\mathscr{W}_L \supset \alpha E_i^c [L_j (Y_{\mathbf{2}}(\tau))_k]_{\mathbf{1}} H_d$$

$$Y_{\mathbf{2}}(\tau) \sim \mathbf{2} = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}$$

Completely fixed

The only unknown is the complex VEV of τ

i, j, k , flavour indices



$$\tau = \mathrm{Re}\,\tau + i\,\mathrm{Im}\,\tau$$

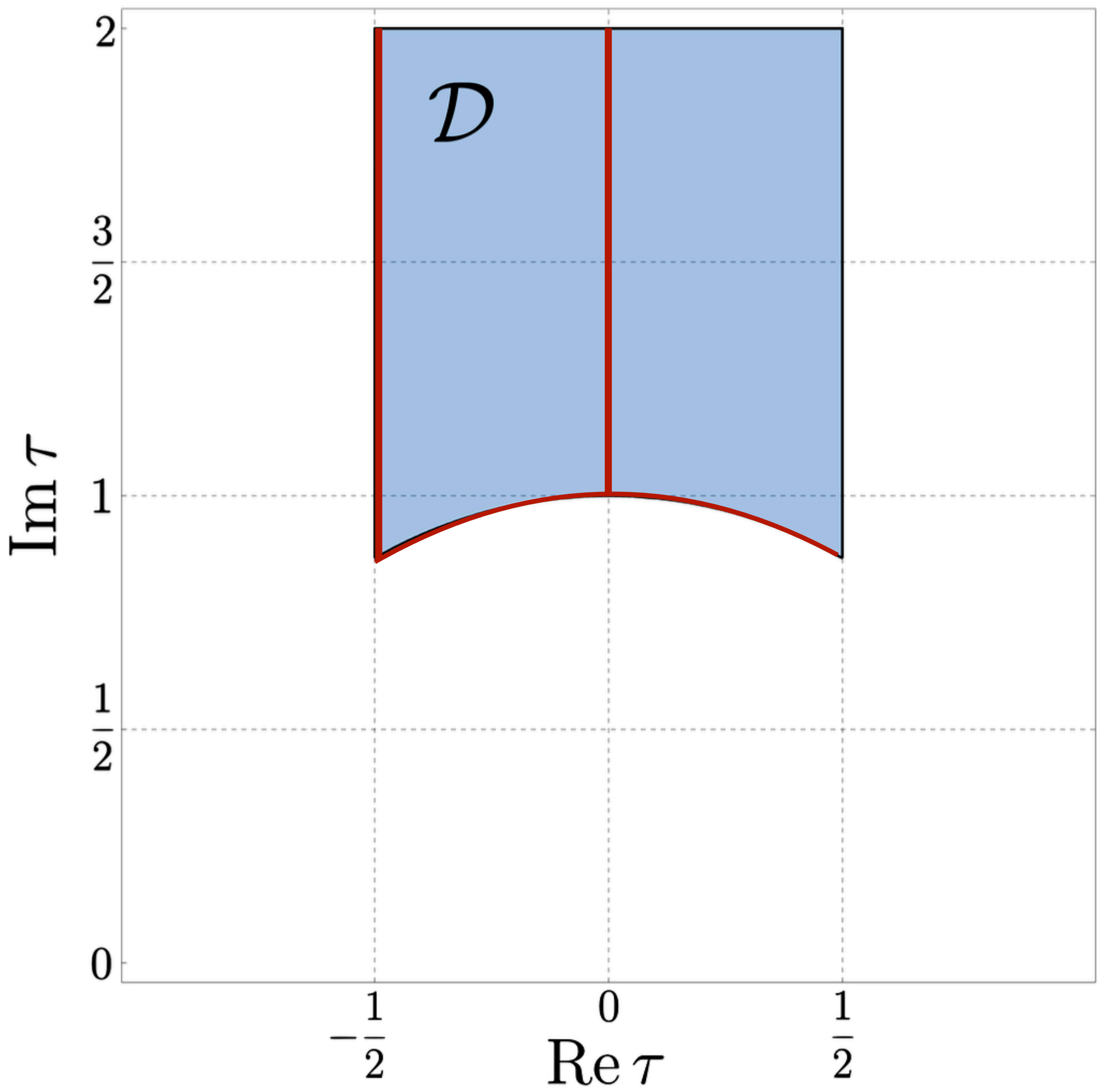
Impose CP symmetry on the model

- gCP $\Rightarrow$ $\alpha_i \in \mathbb{R}$

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

Only source of CPV in the model is the VEV of τ



$$\mathcal{D} = \left\{ \tau \in \mathbb{C} \,:\, \mathrm{Im}\,\tau > 0\,,\ \ |\mathrm{Re}\,\tau| \leq \frac{1}{2}\,,\ \ |\tau| \geq 1 \right\}$$

- Every $\tau \notin \mathscr{D}$ can be mapped in $\tau' \in \mathscr{D}$ through Γ transformation
- CP conserving values

Only source of CPV in the model is the VEV of τ

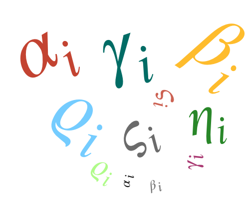
Finite modular groups: searching for simplicity

Substantial number of <u>successful</u> models since 2018

$N = 2$

S_3 ✗ A_4 ✓ S_4 ✓ A_5 ✓ $\ldots$

T. Kobayashi, K. Tanaka, T.H. Tatsuishi
Phys. Rev. D **98** (Jul, 2018)

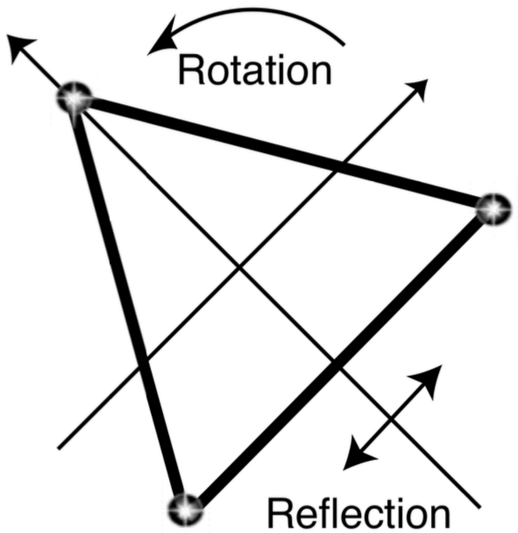
- 

- Extra flavons
- Charged-leptons hierarchy by "hand"

$N = 2$

S_3

Permutation group of 3 objects



- Smallest non-abelian discrete group ✓

- Three irreducible representations

$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$
singlet	pseudo-singlet	doublet

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \to \notin \mathbf{2}\ (S_3)$$

A basis for modular forms of weight k

- $$\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \longrightarrow Y_1(\tau), Y_2(\tau) \sim \mathbf{2}$$

Dedekind's eta function $(q \equiv e^{2\pi i\, \tau})$

Lowest weight: k=2

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} \to (c\tau + d)^2 \rho(\gamma)_{\mathbf{2}} \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$$

Very limited number! ✓

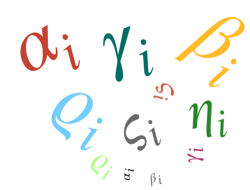
-

N	$d_k(\Gamma(N))$	Γ_N
2	$\boxed{k/2+1}$	S_3

$$\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1} \quad , \quad \mathbf{1}' \otimes \mathbf{2} = \mathbf{2} \quad , \quad \mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$$

Guiding principles

Extra flavons besides τ (modulus)



$$\frac{y_e}{y_\mu} \sim \mathcal{O}(10^{-3})$$

$$Y_1(\tau) = \frac{7}{100} + \frac{42}{25}q + \frac{42}{25}q^2 + \frac{168}{25}q^3 + \dots$$

$$Y_2(\tau) = \frac{14\sqrt{3}}{25}q^{1/2}(1 + 4q + 6q^2 + \dots)$$

Fourier expansion $(q \equiv e^{2\pi i \tau})$

$$\longrightarrow \quad |Y_2(\tau)| \lesssim |Y_1(\tau)|$$

$$\mathrm{Im}\,\tau \gtrsim 1$$

Following guiding principles...

$D_\ell \equiv \begin{pmatrix} \text{electron} \\ \text{muon} \end{pmatrix} \sim \mathbf{2}$ $\ell_3 \equiv \text{tau} \sim \mathbf{1}'$

		E_1^c	E_2^c	E_3^c	D_ℓ	ℓ_3	$H_{d,u}$
	$SU(2)_L \times U(1)_Y$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, \mp 1/2)$
Irreps	$\Gamma_2 \cong S_3$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}$
Weights	k_I	4	0	-2	2	2	0

Charged-leptons

$$\boxed{\mathcal{W}_e^H = \alpha E_1^c H_d (D_\ell Y_{\mathbf{2}}^{(3)})_{\mathbf{1}} + \beta E_2^c H_d (D_\ell Y_{\mathbf{2}})_{\mathbf{1}'} + \gamma E_3^c H_d \ell_3 + \alpha_D E_1^c H_d \ell_3 Y_{\mathbf{1}'}^{(3)}}$$

$(m_\tau, m_\mu, m_e) \sim m_\tau (1, |Y_1|, |Y_1^3|)$ $|Y_1| \sim \mathcal{O}(10^{-2})$

Following guiding principles...

$$\boxed{\mathcal{W}_e^H = \alpha E_1^c H_d (D_\ell Y_{\mathbf{2}}^{(3)})_{\mathbf{1}} + \beta E_2^c H_d (D_\ell Y_{\mathbf{2}})_{\mathbf{1'}} + \gamma E_3^c H_d \ell_3 + \alpha_D E_1^c H_d \ell_3 Y_{\mathbf{1'}}^{(3)}}$$

$$(m_\tau, m_\mu, m_e) \sim m_\tau(1, |Y_1|, |Y_1^3|)$$

$$|Y_1| \sim \mathcal{O}(10^{-2})$$

$$M_\ell^\dagger = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(3)})_1 & \alpha(Y_{\mathbf{2}}^{(3)})_2 & \alpha_D Y_{\mathbf{1'}}^{(3)} \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix} v_d$$

Charged-leptons masses reproduced with:

$$\frac{\beta}{\alpha} \sim \frac{\gamma}{\alpha} \sim \frac{\alpha_D}{\alpha} \approx \mathcal{O}(1)$$

Modular invariance: texture zeros!

Neutrino sector from Weinberg operators

$$\mathcal{W}_\nu^{k_\ell=2} \supset \frac{g}{\Lambda} H_u H_u (D_\ell D_\ell)_{\mathbf{2}} Y_{\mathbf{2}}^{(2)} + \frac{g'}{\Lambda} H_u H_u D_\ell \ell_3 (Y_{\mathbf{2}}^{(2)}) + \\ + \frac{g''}{\Lambda} H_u H_u (D_\ell D_\ell)_{\mathbf{1}} Y_{\mathbf{1}}^{(2)} + \frac{g_p}{\Lambda} H_u H_u \ell_3 \ell_3 (Y_{\mathbf{1}}^{(2)}) \,.$$

- $\Lambda \to$ Scale of new physics
- $\{g'/g\,, g''/g\,, g_p/g\} \in \mathbb{R}$ free dimensionless parameters
- Reproduces mixing and Δm_{ij}^2 ✔

JHEP 09 (2023) 043
D. Meloni, M.Parriciatu

- Fit VS 6 dimensionless observables (no CP)

$$q_j = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, m_e/m_\mu, m_\mu/m_\tau, r\}$$

$$\langle\tau\rangle = \boxed{\mathrm{Re}\,\tau} + i\,\mathrm{Im}\tau$$

Only source of CPV

$$\boxed{q \equiv e^{2\pi i\tau}}$$

Parameter	Best-fit value and 1σ range	
$\Delta m^2_{\rm sol}/(10^{-5}\ {\rm eV}^2)$	$7.36^{+0.16}_{-0.15}$	
	NO	**IO**
$\lvert\Delta m^2_{\rm atm}\rvert/(10^{-3}\ {\rm eV}^2)$	$2.485^{+0.023}_{-0.031}$	$2.455^{+0.030}_{-0.025}$
$r \equiv \Delta m^2_{\rm sol}/\lvert\Delta m^2_{\rm atm}\rvert$	0.0296 ± 0.0008	0.0299 ± 0.0008
$\sin^2\theta_{12}$	$0.303^{+0.013}_{-0.013}$	$0.303^{+0.013}_{-0.013}$
$\sin^2\theta_{13}$	$0.0223^{+0.0007}_{-0.0006}$	$0.0223^{+0.0006}_{-0.0006}$
$\sin^2\theta_{23}$	$0.455^{+0.018}_{-0.015}$	$0.569^{+0.013}_{-0.021}$
$\delta_{\rm CP}/\pi$	$1.24^{+0.18}_{-0.13}$	$1.52^{+0.14}_{-0.15}$
m_e/m_μ	0.0048 ± 0.0002	
m_μ/m_τ	0.0565 ± 0.0045	

F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

$\sin^2\theta_{12}$ ✓

$\sin^2\theta_{13}$ ✓

$\sin^2\theta_{23}$ ✓

$$r = \frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}$$ ✓

All reproduced within 1σ range

10 (6) parameters

VS

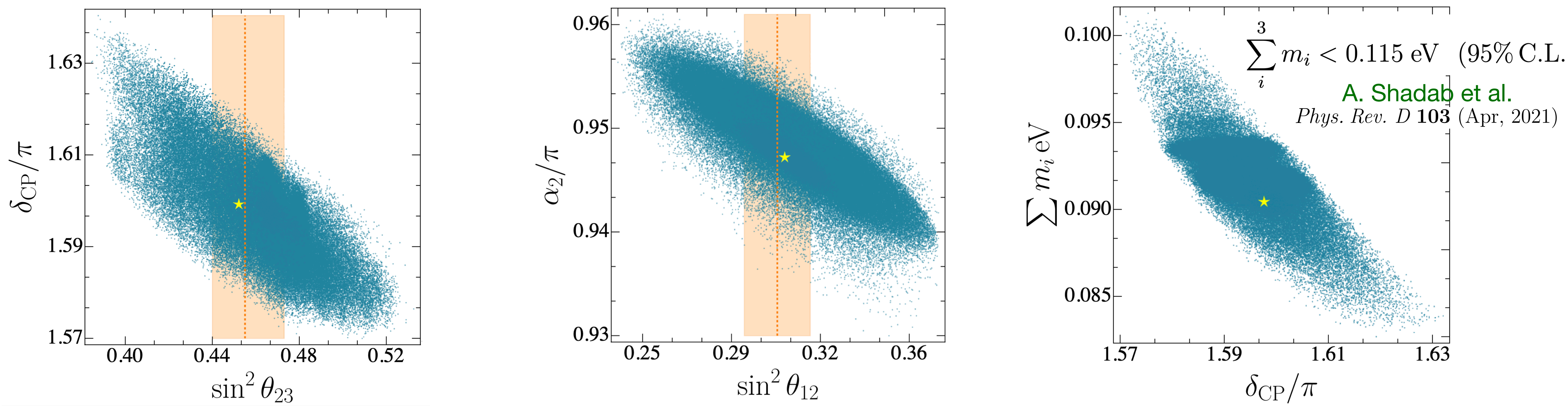
12 (9) observables

Model predictions

- Normal ordering for neutrinos
- Dirac CP-phase near maximal violation $\delta_{CP} \sim 1.6\pi$

Modulus VEV
$\tau \sim \pm 0.09 + 1.7i$

- Correlation plots



★ Minimum χ^2

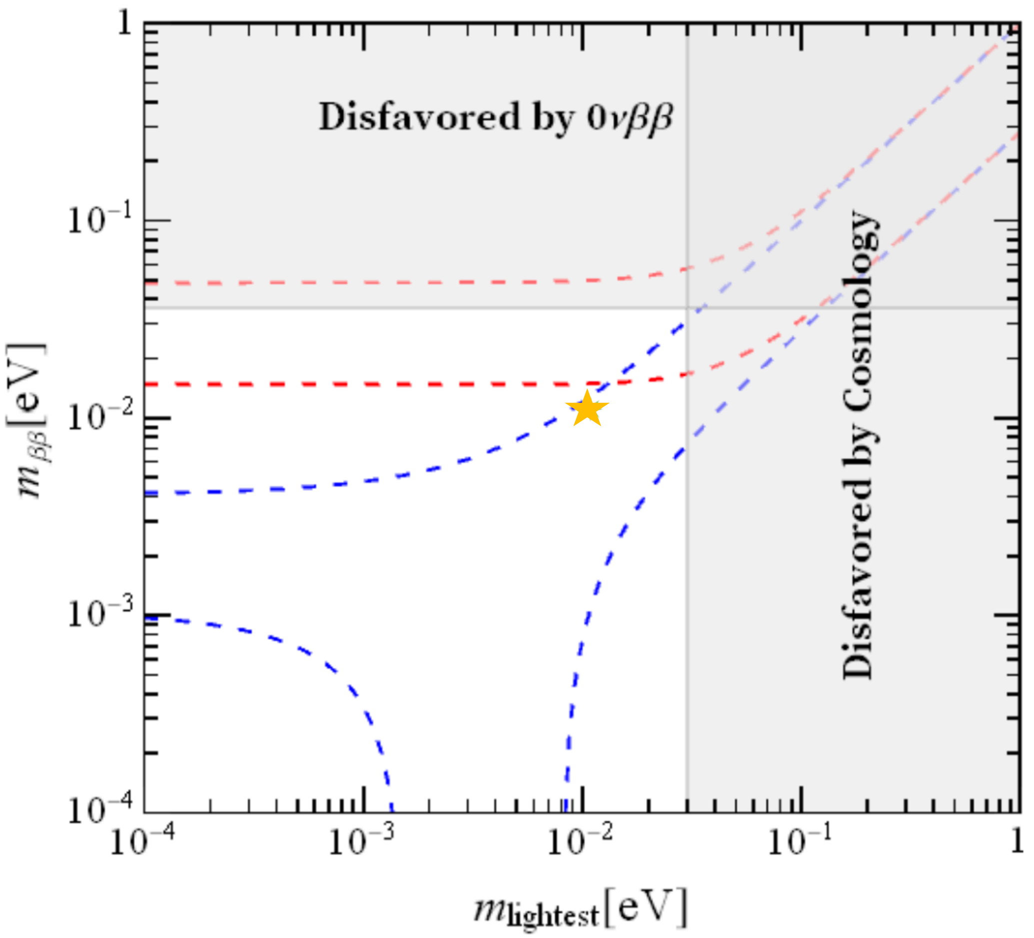
Model predictions

▶ Precise predictions on neutrino masses and observables of exp. interest!

	Best fit $\pm 1\sigma$
m_1 [eV]	$0.0174^{+0.0011}_{-0.0014}$
m_2 [eV]	$0.0194^{+0.0010}_{-0.0012}$
m_3 [eV]	$0.0535^{+0.0004}_{-0.0004}$
$\sum_i m_i$ [eV]	$0.090^{+0.002}_{-0.003}$
$\boxed{\lvert m_{\beta\beta}\rvert\ [\text{meV}]}$	$18.14^{+1.17}_{-1.48}$
$\boxed{m_\beta^{\text{eff}}\ [\text{meV}]}$	$19.60^{+1.02}_{-1.25}$
α_1/π	$\pm 1.129^{+0.019}_{-0.013}$
α_2/π	$\pm 0.946^{+0.004}_{-0.004}$

Neutrinoless double beta decay

Single beta decay (KATRIN)

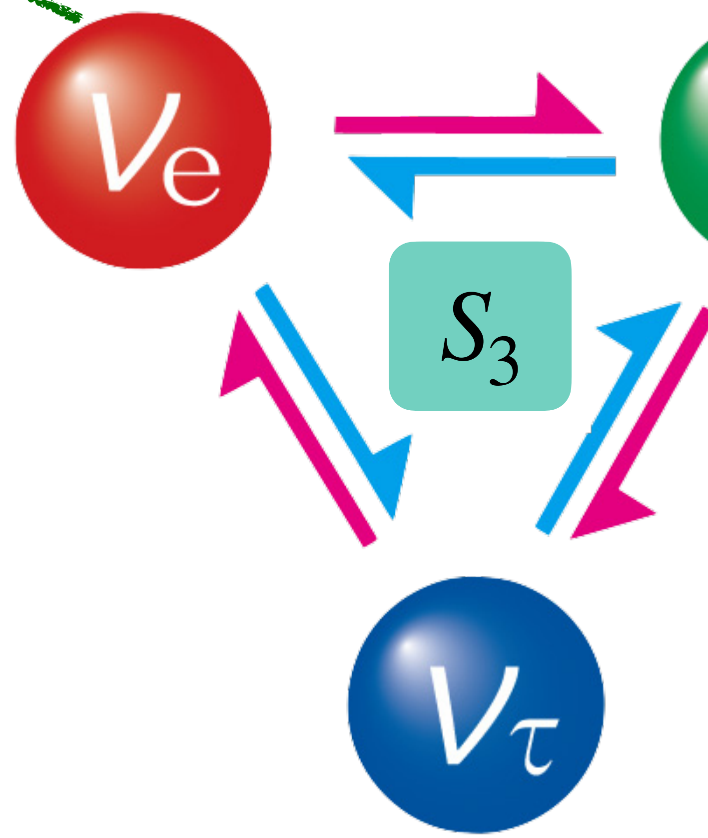


$\langle m_{\beta\beta}\rangle < (36-156)\,\text{meV}$ KamLAND-Zen
Phys. Rev. Lett. **130** (Jan, 2023)

★ Minimum χ^2

What S_3 provides in the literature

Quarks with S_3?



Simple model:
$\Gamma_2 \to$ few Yukawa forms

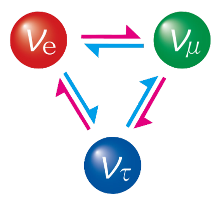
Most predictive model
with $\Gamma_2 \cong S_3$

Conclusions and potential shortcomings

Bottom-up problems

- Kähler potential
- Normalisation of the modular forms
- Modulus stabilization

SUSY-dependent theory



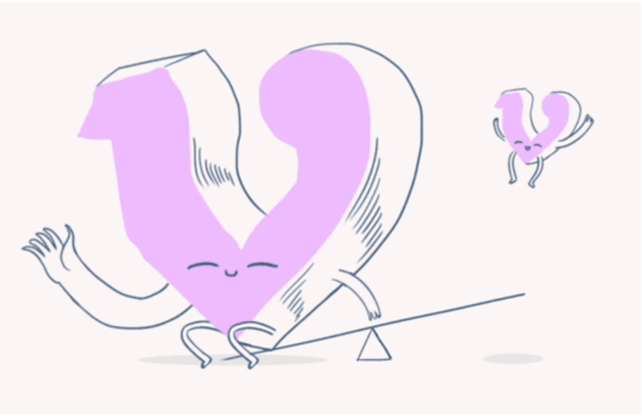
Matteo Parriciatu, Dipartimento di Matematica e Fisica Roma Tre & INFN sezione Roma Tre

Thank you for the attention

BACKUP SLIDES

But what about a $\Gamma_2 \simeq S_3$ seesaw version?

Recent work with S.Marciano, D.Meloni: arxiv:2402.18547



- Reproduces low-energy CP-violation and matter-antimatter asymmetry of the Universe through Leptogenesis

Introduce Minimal seesaw scenario with only 2 RHN transforming as $\sim \mathbf{2}$ under S_3 with weight 2

$$\mathcal{W}_\nu = gH_u N^c D_\ell Y_{\mathbf{2}}^{(2)} + g' H_u (N^c Y_{\mathbf{2}}^{(2)})_{\mathbf{1}'} \ell_3 + g'' H_u (N^c D_\ell)_{\mathbf{1}} Y_{\mathbf{1}}^{(2)} + \\ + \Lambda[(N^c N^c)_{\mathbf{2}} Y_{\mathbf{2}}^{(2)} + \lambda (N^c N^c)_{\mathbf{1}} Y_{\mathbf{1}}^{(2)}]\,,$$

$$M_D = g v_u \begin{pmatrix} -(Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & 2Y_1Y_2 & \frac{g'}{g}(2Y_1Y_2) \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \frac{g''}{g}(Y_1^2 + Y_2^2) & -\frac{g'}{g}(Y_2^2 - Y_1^2) \end{pmatrix}_{\mathrm{RL}}$$

$$\boxed{m_\nu = -M_D^T \mathcal{M}_R^{-1} M_D\,.}$$

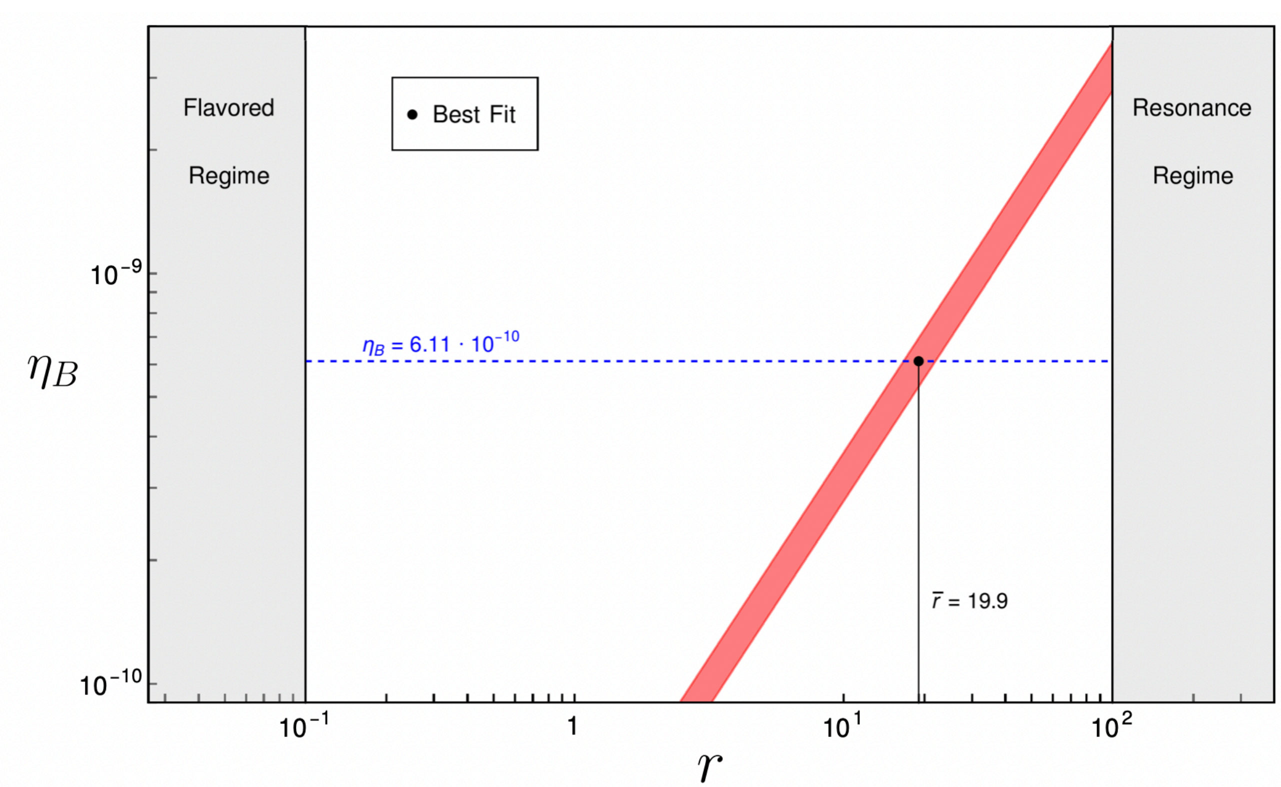
$$\mathcal{M}_R = \Lambda \begin{pmatrix} -(Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) & 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) + \lambda(Y_1^2 + Y_2^2) \end{pmatrix}_{\mathrm{RR}}\,.$$

- Massless neutrino in the spectrum, Normal Ordering
- Excellent fit: $\chi^2 \sim 0.98$ (but now δ_{CP} is fitted)

But what about a $\Gamma_2 \simeq S_3$ seesaw version?

Recent work with S.Marciano, D.Meloni: arxiv:2402.18547

- Reproduces low-energy CP-violation and matter-antimatter asymmetry of the Universe through Leptogenesis



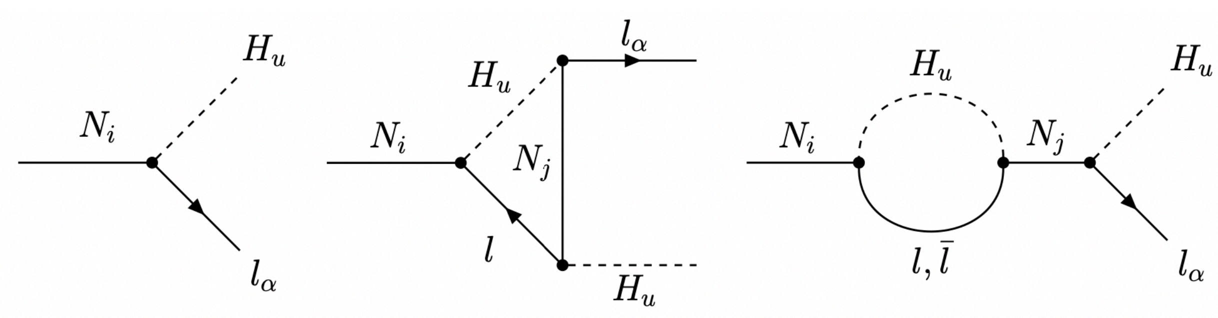


Figure 2: *CP-violating N_i decay.*

$\Lambda \to r\Lambda$ Majorana mass scale not completely fixed by low-energy

Not trivial!

Only source of CP-violation is Re τ

Backup slides: Clebsch-Gordan

- Clebsch-Gordan coefficients for S_3

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \otimes \mathbf{1}' \otimes \mathbf{2} \begin{cases} \mathbf{1} & \sim \quad \psi_1\varphi_1 + \psi_2\varphi_2 \\ \mathbf{1}' & \sim \quad \psi_1\varphi_2 - \psi_2\varphi_1 \\ \mathbf{2} & \sim \quad \begin{pmatrix} \psi_2\varphi_2 - \psi_1\varphi_1 \\ \psi_1\varphi_2 + \psi_2\varphi_1 \end{pmatrix} \end{cases}$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_{\mathbf{2}} \qquad \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}_{\mathbf{2}}$$

S_3 doublets

$$\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1} \quad \sim \quad y_1 y_2$$

$$\mathbf{1}' \otimes \mathbf{2} = \mathbf{2} \quad \sim \quad \begin{pmatrix} -y_1\psi_2 \\ y_1\psi_1 \end{pmatrix}$$

$y_1, y_2 \equiv$ pseudo-singlets $(\mathbf{1}')$

e.g. $(D_\ell Y_{\mathbf{2}}(\tau))_{\mathbf{1}'} = (D_\ell)_1 (Y_{\mathbf{2}}(\tau))_2 - (D_\ell)_2 (Y_{\mathbf{2}}(\tau))_1 \sim \mathbf{1}'$

The Modular symmetry approach

Modular-invariant SUSY action

$$\mathcal{S} = \int d^4x \int d^2\theta d^2\bar{\theta}\, K(\Phi, \bar{\Phi}) + \left[\int d^4x \int d^2\theta\, \mathcal{W}(\Phi) + \text{h.c.}\right]$$

$$\sigma \equiv \Lambda_\tau \tau$$

Kähler potential

- Gives the kinetic terms after the modulus acquires a VEV
- A minimalistic form is chosen

Superpotential

- Holomorphic function of superfields
- Encodes the Higgs Yukawa interactions

The superfields transform as:

$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}, \quad \text{with} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_N$$

Action is invariant if, under Γ_N:

$$\begin{cases} \mathcal{W}(\Phi) \to \mathcal{W}(\Phi) \\ K(\Phi, \bar{\Phi}) \to \underbrace{K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})}_{\text{Kähler transformation}} \end{cases}$$

$\theta, \bar{\theta}$ Grassmann spinor coordinates

$\Phi = (\tau, \varphi)$ Chiral superfields

φ Usual matter supermultiplets

$\rho(\gamma)$ Unitary representation of Γ_N

The Kähler potential...

- Minimalist choice for the Kähler $h \equiv$ positive constant $\Lambda_\tau \equiv$ dimensions of mass

$$K(\Phi, \bar{\Phi}) = -h\Lambda_\tau^2 \log(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\varphi^{(I)}|^2$$

Satisfies $K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})$ under Γ_N

- In a bottom-up approach, this is unjustified

- Corrections of the Kähler potential can spoil the predictivity of the model

M.-C. Chen, S. Ramos-Sánchez, and M. Ratz, "A note on the predictions of models with modular flavor symmetries," *Physics Letters B* **801** (Feb, 2020) 135153.

- This question is an open one

The Modular symmetry approach

The group generators

- Finite modular group can be defined: $\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) \,\middle|\, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod N \right\}$$

subgroups of Γ N=1,2,3…called “level”

$$\overline{\Gamma} \equiv \Gamma/\{\pm\mathbb{I}\}$$
$$\overline{\Gamma}(N) \equiv \Gamma(N)/\{\pm\mathbb{I}\}$$

- Generators S and T of the modular group Γ_N

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1 \qquad S^2 = T^N = (ST)^3 = \mathbb{I}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

- S_3 Generators S and T satisfy:

$$S^2 = T^2 = (ST)^3 = \mathbb{I}$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

The Modular S_3 model: lowest weights

Level 2 modular forms of lowest weight (2) constructed from Dedekind's Eta "seed function"

$$\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \quad , \quad q \equiv e^{2\pi i \tau}$$

$$\longrightarrow \left\{ \eta(\tau/2), \eta\left(\frac{\tau+1}{2}\right), \eta(2\tau) \right\}$$

Closed set under the modular group

$$\alpha + \beta + \gamma = 0$$

$$Y(\alpha, \beta, \gamma | \tau) = \frac{d}{d\tau} [\alpha \log \eta(\tau/2) + \beta \log \eta((\tau+1)/2) + \gamma \log \eta(2\tau)]$$

Impose transformation properties under S_3 generators

$$\rho(S) = \frac{1}{2} \begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

This fixes the constants

$$\downarrow$$

$$\begin{cases} Y_1(\tau) = \dfrac{C}{2} \left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} + \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)} - \dfrac{8\eta'(2\tau)}{\eta(2\tau)} \right] \\[2ex] Y_2(\tau) = \dfrac{C}{2} \sqrt{3} \left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} - \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)} \right] \end{cases}$$

"C" arbitrary

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} \to (c\tau + d)^2 \rho(\gamma)_{\mathbf{2}} \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$$

The Modular S_3 model: the normalisation

Level 2 modular forms of lowest weight (2) constructed from Dedekind's Eta "seed function"

$$\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \quad , \quad q \equiv e^{2\pi i \tau} \longrightarrow \left\{\eta(\tau/2), \eta\left(\frac{\tau+1}{2}\right), \eta(2\tau)\right\}$$

Closed set under the modular group

"C" arbitrary

$$\begin{cases} Y_1(\tau) = \dfrac{C}{2}\left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} + \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)} - \dfrac{8\eta'(2\tau)}{\eta(2\tau)}\right] \\ Y_2(\tau) = \dfrac{C}{2}\sqrt{3}\left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} - \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)}\right] \end{cases}$$

Impose CP symmetry on the model

- Superpotential parameters must be real: less free parameters

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

$$Y(\tau) \xrightarrow{\text{CP}} Y(-\tau^*) = Y^*(\tau)$$

- In our case, this is true if C is purely imaginary

$$C = \frac{7i}{25\pi}$$

The choice made in this work

Only source of CPV is the VEV of modulus τ

The Modular S_3 model: charged-leptons sector

Found two viable choices for modular charges and weights

$$k_D = k_\ell$$

$$\begin{cases} k_{E_1} = 6 - k_\ell \\ k_{E_2} = 2 - k_\ell \\ k_{E_3} = -k_\ell \end{cases} \qquad\qquad \begin{cases} k_{E_1} = 4 - k_\ell \\ k_{E_2} = 2 - k_\ell \\ k_{E_3} = -k_\ell \end{cases}$$

"Hierarchical"

$$M_\ell^\dagger = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(3)})_1 & \alpha(Y_{\mathbf{2}}^{(3)})_2 & \alpha_D Y_{\mathbf{1}'}^{(3)} \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix} v_d$$

Charged-leptons masses reproduced with:

$$\frac{\beta}{\alpha} \sim \frac{\gamma}{\alpha} \sim \frac{\alpha_D}{\alpha} \approx \mathcal{O}(1)$$

"Minimal"

$$M_\ell^\dagger = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(2)})_1 & \alpha(Y_{\mathbf{2}}^{(2)})_2 & 0 \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix} v_d$$

Charged-leptons masses reproduced with:

$$\frac{\beta}{\alpha} \sim \frac{\gamma}{\alpha} \approx \mathcal{O}(10)$$

$$\begin{pmatrix} (Y_{\mathbf{2}}^{(3)})_1 \\ (Y_{\mathbf{2}}^{(3)})_2 \end{pmatrix}_{\mathbf{2}} = \begin{pmatrix} Y_1(Y_1^2 + Y_2^2) \\ Y_2(Y_1^2 + Y_2^2) \end{pmatrix}_{\mathbf{2}} \quad , \quad Y_{\mathbf{1}'}^{(3)} = (Y_2^3 - 3Y_1^2 Y_2)_{\mathbf{1}'}$$

$v_d \equiv$ VEV of H_d

Backup slides

Numerical procedure

- Define a "figure of merit", i.e. chi-square for every set of parameters $l(p_i) \equiv \sqrt{\chi^2(p_i)}$

$$\chi^2(p_i) = \sum_{j=1}^{6} \left(\frac{q_j(p_i) - q_j^{\text{b-f}}}{\sigma_j} \right)^2$$

$$p_i = \{\tau, \beta/\alpha, \gamma/\alpha, ..., g'/g, g_p/g, ...\}$$

$$q_j = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, m_e/m_\mu, m_\mu/m_\tau, r\}$$

- Define a "potential" with a given temperature T and a threshold

$$V(p_i) = \begin{cases} l(p_i) & , \quad l(p_i) \leq l_{\max} \\ +\infty & , \quad \text{otherwise} \end{cases}$$

P. P. Novichkov, J. T. Penedo, S. T. Petcov, and A. V. Titov, "Modular S_4 models of lepton masses and mixing," (2019)

- At iteration "t", generate a new point from a Gaussian centred on the previous one ⟶
- Accept the new point with a probability given by:

$$P_\alpha = \min[1, \exp(V(p_i^{(t)}) - V(p_i'))/T]$$

Backup slides

A measure of fine-tuning:
Altarelli-Blankenburg

$$\text{Fine-tuning} = \frac{\sum_i \left| \frac{\text{par}_i}{\delta\text{par}_i} \right|}{\sum_i \left| \frac{\text{obs}_i}{\sigma_i} \right|}$$

- δpar_i The shift of the parameter from the point of minimum, which increases the chi-square by one unit, while keeping all other parameters fixed.
- σ_i Experimental errors for each observable as extracted from F. Capozzi et al. *Phys. Rev. D* **104** (Oct, 2021)
- The model with less fine-tuning, in our case, seems to be the one with the least number of free parameters (model I)

Backup slides

Model I [7]

	Best-fit and 1σ range
$\mathrm{Re}\,\tau$	$\pm 0.0895^{+0.0032}_{-0.0055}$
$\mathrm{Im}\,\tau$	$1.697^{+0.023}_{-0.016}$
β/α	$14.33^{+0.58}_{-0.38}$
γ/α	$17.39^{+1.38}_{-0.87}$
g'/g	$31.57^{+27.59}_{-10.29}$
g''/g	$7.17^{+6.36}_{-2.36}$
g_p/g	$8.51^{+7.99}_{-3.03}$
$v_d\,\alpha$ [MeV]	102.14
$v_u^2\,g/\Lambda$ [eV]	0.47
$\sin^2\theta_{12}$	$0.300^{+0.013}_{-0.006}$
$\sin^2\theta_{13}$	$0.0223^{+0.0004}_{-0.0006}$
$\sin^2\theta_{23}$	$0.452^{+0.015}_{-0.009}$
r	$0.0295^{+0.0007}_{-0.0006}$
m_e/m_μ	$0.0048^{+0.0001}_{-0.0002}$
m_μ/m_τ	$0.0578^{+0.0023}_{-0.0040}$
Ordering	**NO**
δ/π	$\pm 1.594^{+0.007}_{-0.010}$
m_1 [eV]	$0.0182^{+0.0018}_{-0.0014}$
m_2 [eV]	$0.0201^{+0.0017}_{-0.0013}$
m_3 [eV]	$0.0537^{+0.0006}_{-0.0005}$
$\sum_i m_i$ [eV]	$0.092^{+0.004}_{-0.003}$
$\langle m_{\beta\beta}\rangle$ [meV]	$18.89^{+1.90}_{-1.47}$
m_β^{eff} [meV]	$20.26^{+1.69}_{-1.30}$
α_1/π	$\pm 1.124^{+0.014}_{-0.017}$
α_2/π	$\pm 0.949^{+0.005}_{-0.005}$
Fine-tuning	12.2
χ^2_{min}	0.16

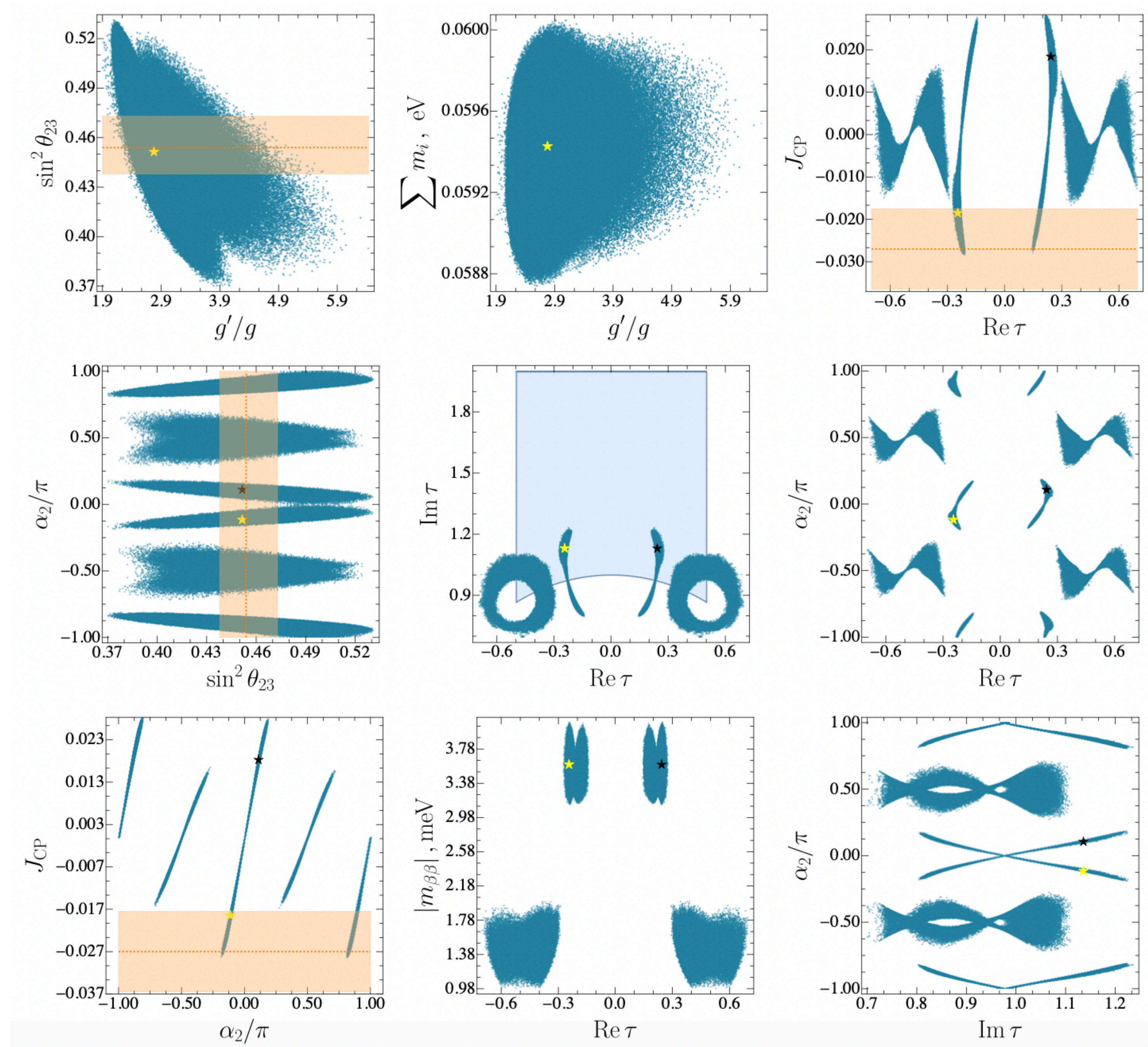
Model II [8]

	Best-fit and 1σ range
$\mathrm{Re}\,\tau$	$\pm 0.090^{+0.004}_{-0.004}$
$\mathrm{Im}\,\tau$	$1.688^{+0.015}_{-0.018}$
β/α	$1.03^{+0.04}_{-0.04}$
γ/α	$1.26^{+0.12}_{-0.08}$
α_D/α	$1.33^{+1.51}_{-1.05}$
g'/g	$41.9^{+73.7}_{-12.8}$
g''/g	$9.55^{+16.81}_{-2.91}$
g_p/g	$11.5^{+21.2}_{-3.8}$
$v_d\,\alpha$ [MeV]	1404.6
$v_u^2\,g/\Lambda$ [eV]	0.35
$\sin^2\theta_{12}$	$0.305^{+0.009}_{-0.015}$
$\sin^2\theta_{13}$	$0.0222^{+0.0007}_{-0.0006}$
$\sin^2\theta_{23}$	$0.454^{+0.007}_{-0.008}$
r	$0.0295^{+0.0007}_{-0.0007}$
m_e/m_μ	$0.0048^{+0.0002}_{-0.0002}$
m_μ/m_τ	$0.0570^{+0.0034}_{-0.0048}$
Ordering	**NO**
δ/π	$\pm 1.597^{+0.009}_{-0.006}$
m_1 [eV]	$0.0174^{+0.0011}_{-0.0014}$
m_2 [eV]	$0.0194^{+0.0010}_{-0.0012}$
m_3 [eV]	$0.0535^{+0.0004}_{-0.0004}$
$\sum_i m_i$ [eV]	$0.090^{+0.002}_{-0.003}$
$\langle m_{\beta\beta}\rangle$ [meV]	$18.14^{+1.17}_{-1.48}$
m_β^{eff} [meV]	$19.60^{+1.02}_{-1.25}$
α_1/π	$\pm 1.129^{+0.019}_{-0.013}$
α_2/π	$\pm 0.946^{+0.004}_{-0.004}$
Fine-tuning	11.2
χ^2_{min}	0.074

Backup slides

	Best-fit and 1σ range
$\mathrm{Re}\,\tau$	$\pm 0.244^{+0.012}_{-0.067}$
$\mathrm{Im}\,\tau$	$1.132^{+0.027}_{-0.297}$
β/α	$0.92^{+0.85}_{-0.03}$
γ/α	$-1.20^{+0.06}_{-2.14}$
$\log_{10}(\alpha_D/\alpha)$	$-13.4^{+13.2}_{-76.3}$
g'/g	$2.76^{+0.21}_{-0.23}$
g''/g	$-2.53^{+0.13}_{-0.03}$
$\log_{10}(\lvert\lambda\rvert)$	$-12.2^{+10.9}_{-59.2}$
$v_d\,\alpha$, [GeV]	$1.08^{+0.06}_{-0.69}$
$v_u^2\,g^2/\Lambda$ [eV]	$3.46^{+0.55}_{-1.65}$
$\sin^2\theta_{12}$	$0.305^{+0.011}_{-0.011}$
$\sin^2\theta_{13}$	$0.0221^{+0.0006}_{-0.0005}$
$\sin^2\theta_{23}$	$0.448^{+0.014}_{-0.016}$
r	$0.0296^{+0.0006}_{-0.0008}$
m_e/m_μ	$0.0048^{+0.0001}_{-0.0002}$
m_μ/m_τ	$0.0574^{+0.0032}_{-0.0050}$
Ordering	**NO**
J_{CP}	$-0.018^{+0.002}_{-0.002}$
α_1/π	0
α_2/π	$\pm 0.112^{+0.792}_{-0.014}$
m_1 [meV]	0
m_2 [meV]	$8.620^{+0.095}_{-0.123}$
m_3 [meV]	$50.806^{+0.016}_{-0.021}$
$\sum_i m_i$ [eV]	$0.0594^{+0.0001}_{-0.0001}$
$\lvert m_{\beta\beta}\rvert$ [meV]	$3.61^{+0.09}_{-0.09}$
m_β^{eff} [meV]	$8.90^{+0.10}_{-0.09}$
d_{FT}	3.03
χ^2_{min}	0.98

Minimal seesaw model: arxiv:2402.18547
with S.Marciano, D.Meloni



Non-standard interactions

tests of modulus couplings

G-J. Ding, FF, 2003.13448

non standard neutrino interactions

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \bar{f}\,\bar{\sigma}^\mu \partial_\mu f + \frac{1}{2}\partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M_\alpha^2 \varphi_\alpha^2$$
$$- (m_e + \mathcal{Z}_\alpha^e \varphi_\alpha) e^c e - \frac{1}{2}\nu (m_\nu + \mathcal{Z}_\alpha^\nu \varphi_\alpha)\nu + h.c. + \ldots .$$

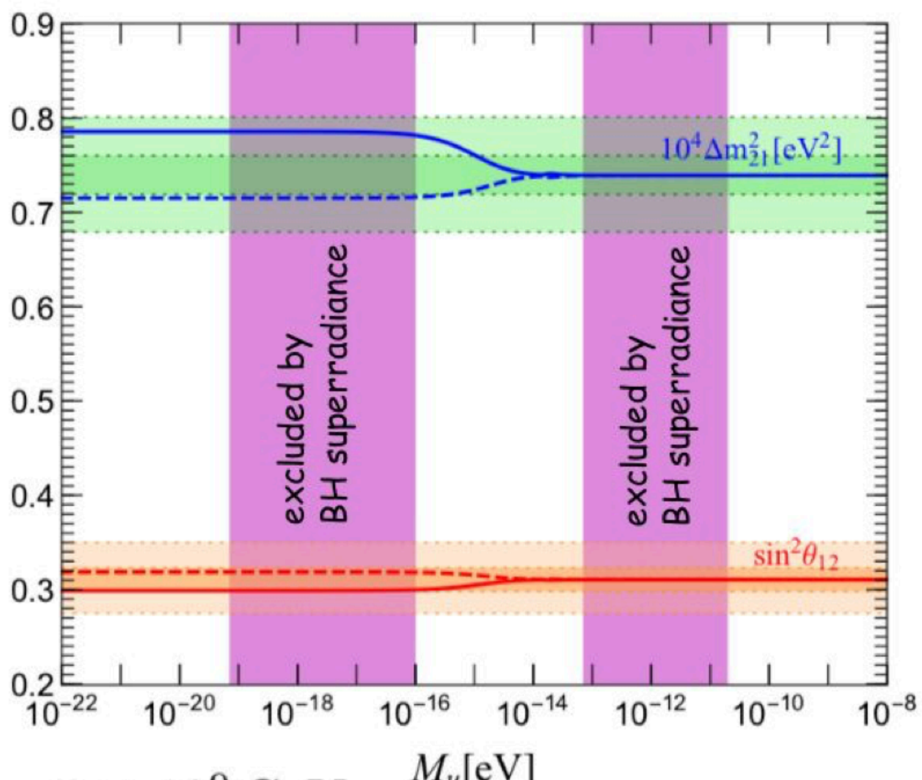
$$\tau = \langle \tau \rangle + \frac{\varphi_u + i\,\varphi_v}{\sqrt{2}}$$

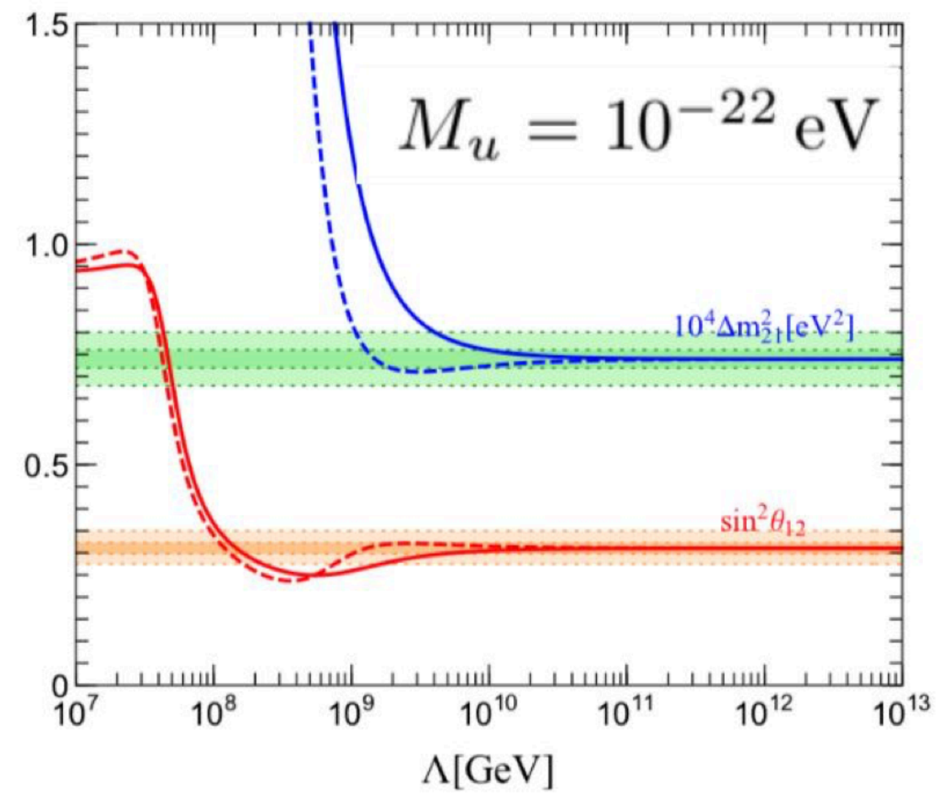
in medium with non-zero electron number density

small, unless the modulus is very light

$$\delta m_\nu(0) = -n_e^0 \frac{\mathrm{Re}(\mathcal{Z}^e)\mathcal{Z}^\nu}{M^2(R)} ,$$

in the sun:





$\Lambda = 5 \times 10^9$ GeV

[modulus VEV]

from Feruglio's slides at Mod. Symmetry Bethe Workshop