On the proof of chiral symmetry breaking in QCD-like theories

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Based on:

L. Ciambriello, R. Contino, A. Luzio, MR, L-X. Xu 2212.02930

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Motivations

Strongly-coupled gauge theories in a **confinement phase**

Phenomenologically relevant

- QCD
- Dark matter
- Composite Higgs
- ...

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Lack of theoretical control

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We consider the question

Confinement $\Longrightarrow$ Dynamical symmetry breaking?

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Focus on **QCD-like theories**: $N_c > 2$ colours, N_f massless flavours

- $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \stackrel{?}{\to} SU(N_f)_V \times U(1)_B$
- Need for a **theoretical proof**

Plan

i) Review of AMC and PMC

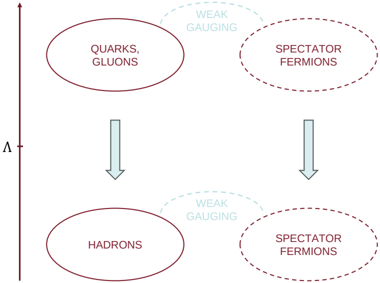
ii) ChSB via N_f-independence

iii) ChSB via downlifting

't Hooft anomaly matching

One of the few non-perturbative analytical tools for strongly-coupled theories ['t Hooft 1980]

- Gauge theory with global symmetry group G_F
- Add weak background gauge fields for G_F
- Cancel the 't Hooft anomalies with spectator fermions



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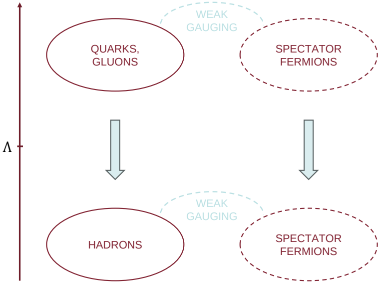

Anomalies must match in the UV and in the IR:

$$\mathcal{A}_{UV} = \mathcal{A}_{IR}$$

Anomaly matching conditions

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Anomaly matching conditions (AMC)

- Assume unbroken symmetry
- Match the anomaly for a generic spectrum of fermions

No solution $\Longrightarrow$ Broken symmetry

Anomaly matching conditions

Assume that **bound states** are interpolated by **gauge-invariant local operators**

- Transform in tensor representations of G_F
- **Equivalent tensors**, corresponding to the **same irreps**, give the same contribution to the anomaly

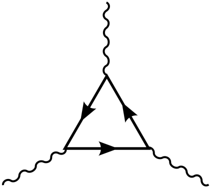
AMC cannot distinguish equivalent tensors

- Each irreps r has an **integer multiplicity** $\ell(r)$ that tells how many times it appears in the spectrum

Anomaly matching conditions

QCD-like gauge theory

$$G_F = G[N_f] = SU(N_f)_L \times SU(N_f)_R \times U(1)_B$$



- 't Hooft anomalies $[SU(N_f)_{L/R}]^3$ and $[SU(N_f)_{L/R}]^2 U(1)_B$

$$\underbrace{\sum_r \ell(r)\mathcal{A}(r)}_{\mathcal{A}_{IR}} = \underbrace{N_c \mathcal{A}(r_q)}_{\mathcal{A}_{UV}}$$

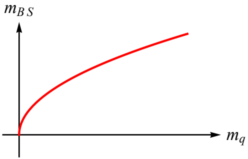
- Denoted as AMC$[N_f]$

Persistent mass conditions

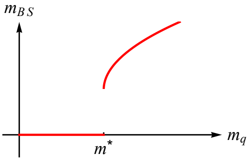
Another tool introduced by ['t Hooft 1980]:

Naive: Bound states "containing" massive quarks are massive

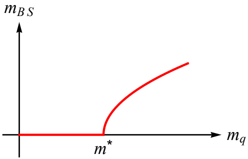
- What does "containing" mean?
- Decoupling applies only in the limit of large masses [Preskill and Weinberg 1981; Dimopoulos and Preskill 1982]



(a)



(b)



(c)

Persistent mass conditions

Give mass to k quarks:

$$G[N_f, k] = SU(N_f - k)_L \times SU(N_f - k)_R \times U(1)_B \times \underbrace{U(1)_{H_1} \times \ldots \times U(1)_{H_k}}_{G_H}$$

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Persistent mass conditions:

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- Can be proved for vector-like gauge theories by developing the approach of [Vafa and Witten 1984]
- For states neutral under G_H we cannot state anything

Persistent mass conditions

One massive quark:

$$G[N_f, 1] = SU(N_f - 1)_L \times SU(N_f - 1)_R \times U(1)_B \times U(1)_H$$

- States with $H \neq 0$ are in vector-like representations

$$0 = \ell(\hat{r}_1) = \sum_r \ell(r) K(r \to \hat{r}_1) \;\; \forall \hat{r}_1 \text{ with } H \neq 0$$

Similar equations for k massive quarks, up to $N_f - 2$

- Overall denoted as PMC$[N_f]$

Strategies for proving ChSB

No integer solutions to AMC$[N_f]$$\cup$PMC$[N_f]$ $\Longrightarrow$ ChSB

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Historical attempts are not general:

- Specialize to only baryonic spectrum [Frishman et al. 1981; Cohen and Frishman 1982; Schwimmer 1982], large N_c [Coleman and Witten 1980]...
- Rely on unproven assumptions, e.g. N_f-independence ['t Hooft 1980; Farrar 1980]

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Our work:

- Prove under which **dynamical assumptions** N_f-independence holds
- Purely* algebraic proof of ChSB based on **downlifting**

ChSB from N_f-independence

Naive: The dynamics behind bound states should not depend on the number of fundamental quarks in the spectrum

- If $\{\ell\}$ solves $\mathrm{AMC}[N_f'] \cup \mathrm{PMC}[N_f']$, then it is a solution for any $N_f > N_f'$

N_f-independence $\Longrightarrow$ ChSB (v1):

- Rewrite AMC as:

$$\sum_{i=0}^{h} a_i(\{\ell\}) N_f^i = 0$$

- Zeroes for infinite values of N_f imply $a_i(\{\ell\}) = 0$
- $a_0(\{\ell\}) = 0$ has no solutions

N_f-independence $\Longrightarrow$ ChSB (v2):

- $\mathrm{AMC}[k \cdot p]$ have no solutions for p prime factor of N_c

ChSB from N_f-independence

The statement of N_f-independence is **ill-defined** in general

- Equivalence between tensor operators changes with N_f
- No well-defined notion of **uplifting** of an irreps from N_f to $N_f + 1$

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Restrict the allowed irreps:

A tensor $T^{\{n\}}_{\{\bar{n}\}}$ of $G[N_f]$ is said **class-A** if:

$$n + \overline{n} < N_f$$

Two different class-A tensors cannot be equivalent

- Class-A irreps uniquely identified by the corresponding class-A tensor

ChSB from N_f-independence

Restricted to class-A irreps **uplifting** is well defined

- $\{\ell(r)\} \to \{\ell(U(r))\}$

Prove N_f-independence as an **algebraic property of AMC and PMC**:

Let $\{\ell(r)\}$ be a solution to $\mathrm{AMC}[N_f'] \cup \mathrm{PMC}[N_f']$ that is **restricted to class A**. Then for any $N_f \geq N_f'$ the set $\{\ell(U(r))\} = \{\ell(r)\}$ is a solution to $\mathrm{AMC}[N_f] \cup \mathrm{PMC}[N_f]$

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Therefore:

Chiral symmetry is spontaneously broken, under the dynamical assumption that the bound states spectrum is restricted to class A

ChSB from downlifting

Let $\{\ell(r)\}$ be a solution to $\mathrm{AMC}[N_f] \cup \mathrm{PMC}[N_f]$

Downlifted spectrum:

$$\bar{\ell}(r') = \sum_r \ell(r) K(r \to r')$$

with r' irreps of $G[N_f - 1]$

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We prove that:

The set $\{\bar{\ell}(r')\}$ is a solution of $\mathrm{AMC}[N_f - 1] \cup \mathrm{PMC}[N_f - 1]$

ChSB from downlifting

Critical values of N_f

- First noticed by Weinberg for baryons with $N_c = 3$
- Values of N_f for which the $[SU(N_f)_L]^2 U(1)_B$ AMC has no integer solutions

$$1 = \sum_r \ell(r) \underbrace{b(r)}_{\text{Baryon number}} \underbrace{d(r_R)}_{\text{Dimension}} \underbrace{T(r_L)}_{\text{Dynkin index}}$$

- LHS is 1, RHS multiple of $p \Longrightarrow$ **No integer solutions**

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In general, we prove:

AMC$[k \cdot p]$ have no integer solutions for p prime factor of N_c

ChSB from downlifting

Combine:

- **Downlifting**: No integer solutions for N'_f implies no integer solutions for $N_f > N'_f$
- **Critical values of N_f**: No integer solutions for $N_f = p_{min}$ smallest prime divider of N_c

Chiral symmetry is spontaneously broken for any $N_f \geq p_{min}$

Assuming the **absence of phase transitions for large quark masses**, we can prove **ChSB also for $N_f < p_{min}$** via a continuity argument

Conclusions

What did we obtain?

- Critical analysis of AMC and PMC
- Proof of chiral symmetry breaking in QCD-like theories
  - N_f-independence
  - Downlifting + critical values of N_f

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Further developments!

- Extension of our results beyond QCD-like theories
- What about chiral gauge theories? [Bolognesi, Konishi, and Luzio 2022]
- New techniques? Generalized symmetries [Gaiotto et al. 2015]

THANKS FOR YOUR ATTENTION!

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Critical values of N_f

There are critical values of N_f, at fixed N_c, for which the $[SU(N_f)_{L/R}]^2 U(1)_B$ AMC has no integer solutions

Example

$N_c = 3,\ N_f = 3 \cdot k$, only $b = 1$ baryons:

- $\ell_1 = \ell\left(\ydiagram{3} \otimes 1\right)$, $\ell_2 = \ell\left(\ydiagram{2,1} \otimes 1\right)$, $\ell_3 = \ell\left(\ydiagram{1,1,1} \otimes 1\right)$,
  $\ell_4 = \ell\left(\ydiagram{2} \otimes \ydiagram{1}\right)$, $\ell_5 = \ell\left(\ydiagram{1,1} \otimes \ydiagram{1}\right)$ + parity conjugates ℓ_6, ℓ_7, ℓ_8, ℓ_9, ℓ_{10}

$$
\begin{aligned}
1 =& \frac{1}{2}(N_f + 2)(N_f + 3)\ell_1 + (N_f^2 - 3)\ell_2 + \frac{1}{2}(N_f - 2)(N_f - 3)\ell_3 + \\
&+ N_f(N_f + 2)\ell_4 + N_f(N_f - 2)\ell_5 + \frac{1}{2}N_f(N_f + 1)\ell_6 + \frac{1}{2}N_f(N_f - 1)\ell_7
\end{aligned} \tag{1}
$$

- LHS is 1, RHS is a multiple of 3 $\Longrightarrow$ No integer solutions