Improved bounds on the hot QCD axion

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based on:
F. Bianchini, G^2dC, M.Valli, arXiv: 2310.08169

IRN Terascale @ Laboratori Nazionali di Frascati - 16/04/2024

Goal

Set a robust and conservative upper bound on the QCD axion mass using cosmological datasets.

The QCD axion

$$\mathscr{L}_{\text{QCD}} \supset \bar{q}(i\partial_\mu\gamma^\mu - m_q\, e^{i\theta_q\gamma_5})q - \frac{1}{4}G^{a,\mu\nu}G^a_{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G^{a,\mu\nu}\tilde{G}^a_{\mu\nu}$$

$$\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$$

$$\theta \to \bar{\theta} = \theta - \text{Arg}[\text{Det}(\text{M}_\text{u}\text{M}_\text{d})]$$

Neutron EDM

$$|d_n| \simeq 2.4 \cdot 10^{-16}\bar{\theta}\ \text{e}\cdot\text{cm} < 1.8 \cdot 10^{-26}\ \text{e}\cdot\text{cm} \quad \text{implying} \quad |\bar{\theta}| \lesssim 10^{-10}$$

The QCD axion

$$\mathscr{L} \supset \mathscr{L}_{QCD} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{\alpha_s}{8\pi}\frac{a}{f_a} G^{a,\mu\nu}\tilde{G}^a_{\mu\nu} + \mathscr{L}_{\rm int}(\partial_\mu a, q, \ell)$$

The shift symmetry

$$a(x) \to a(x) + \kappa f_a$$

can be used to remove the $\bar{\theta}$ term

$$\mathscr{L} \supset (\bar{\theta} + \kappa)\frac{\alpha_s}{8\pi} G^{a,\mu\nu}\bar{G}^a_{\mu\nu} + \frac{\alpha_s}{8\pi}\frac{a}{f_a} G^{a,\mu\nu}\bar{G}^a_{\mu\nu}$$

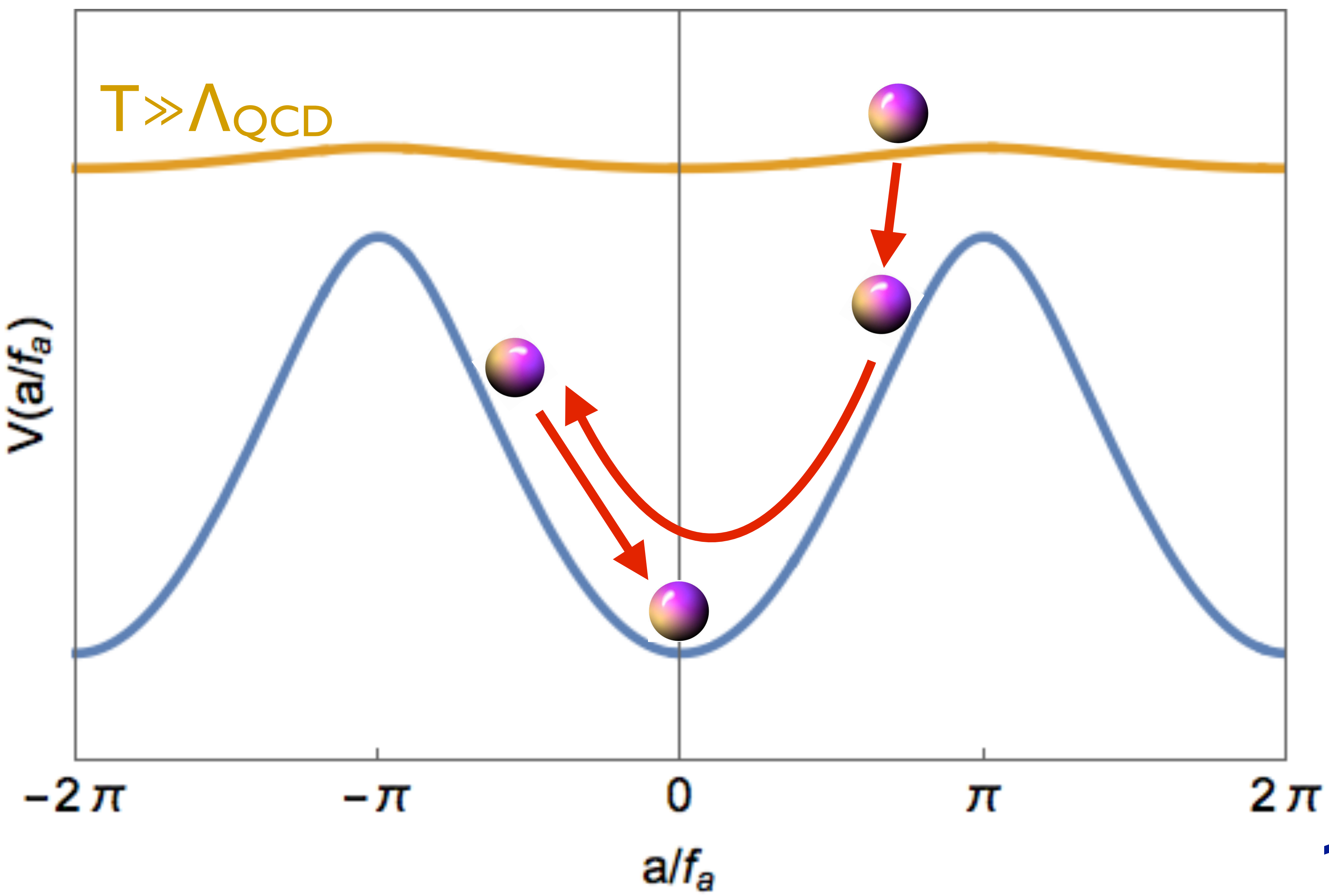
A theorem by Vafa and Witten (1984)

$$e^{-V_4 E(\bar{\theta})} = \int \delta[\phi] e^{-S_0 + i\bar{\theta}Q} = \left| \int \delta[\phi] e^{-S_0 + i\bar{\theta}Q} \right|$$
$$\leq \int \delta[\phi] \left| e^{-S_0 + i\bar{\theta}Q} \right| = e^{-V_4 E(0)}$$

implies that the axion will relax to the minimum of the potential

Early Universe axion production

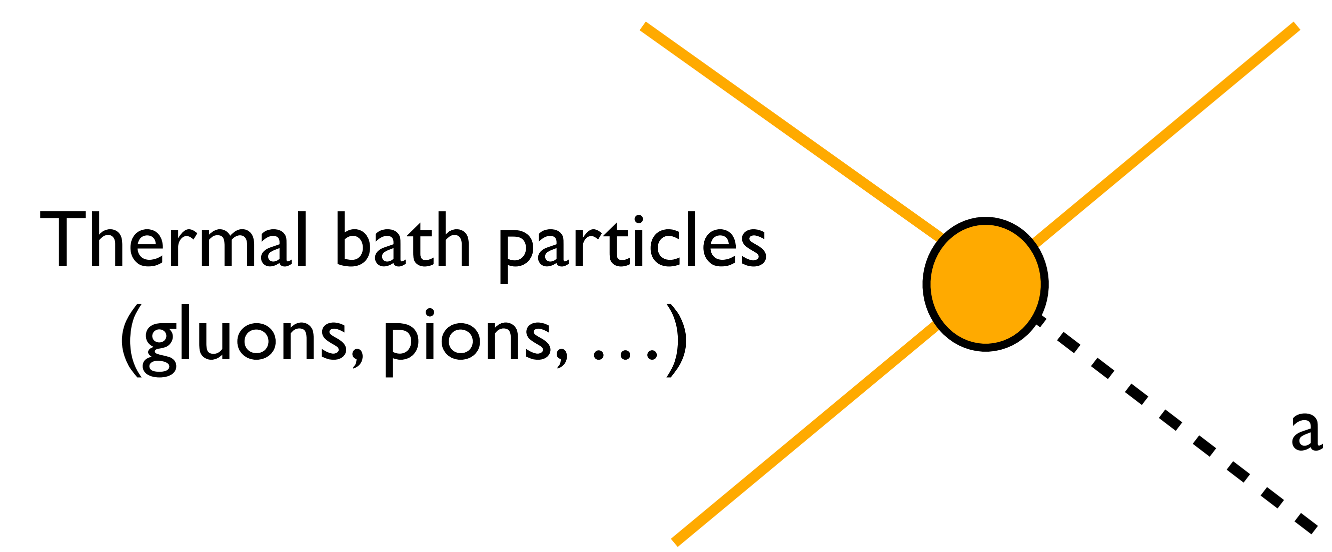
Non-thermal production



$$k_a \ll T$$

$$\ddot{a} + 3H\dot{a} + m_a^2(T) f_a \sin\left(\frac{a}{f_a}\right) = 0$$

Thermal production



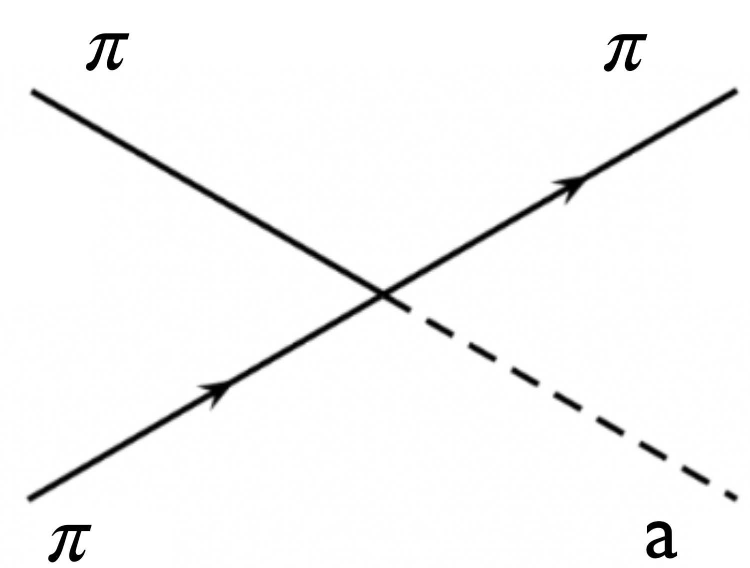
$$k_a \sim T \gg m_a$$

The axion behave similarly to neutrino and contributes to dark radiation. It can only be a small fraction of the dark matter.

Thermal production

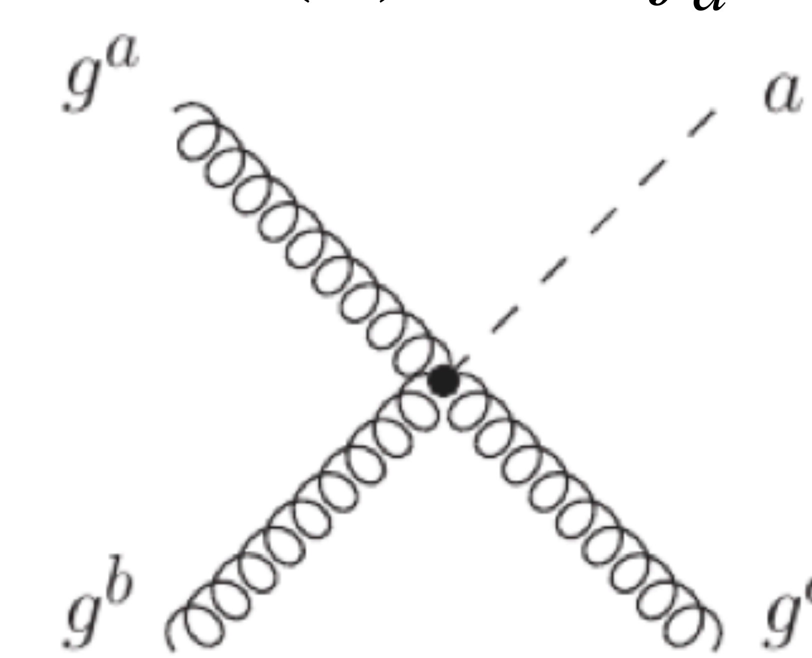
$$\mathscr{L} \supset \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{\alpha_s}{8\pi}\frac{a}{f_a}G^{a,\mu\nu}\tilde{G}^a_{\mu\nu}$$

$\Gamma(T) \sim T^5/(f_a^2 f_\pi^2)$

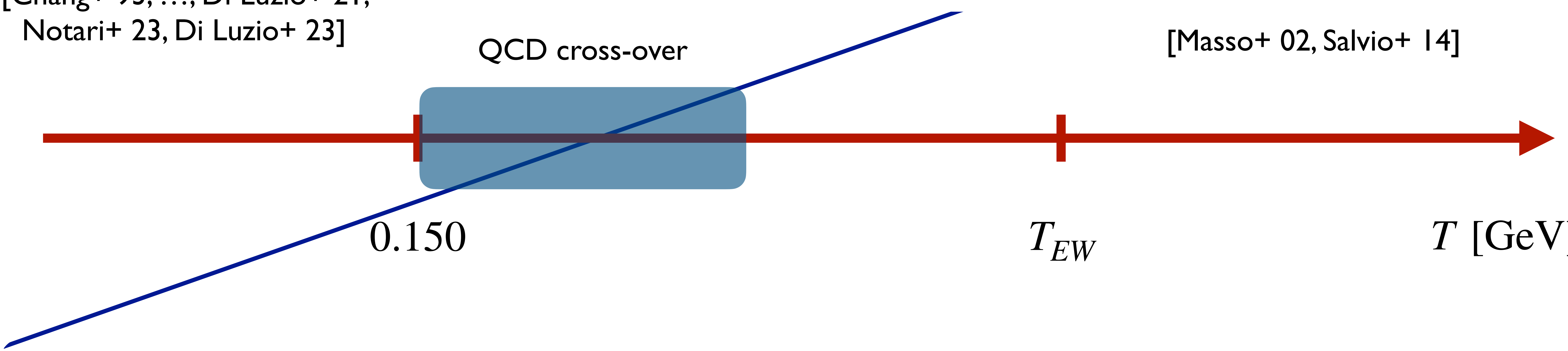


[Chang+ 93, …, Di Luzio+ 21, Notari+ 23, Di Luzio+ 23]

$\Gamma(T) \sim T^3/f_a^2$



[Masso+ 02, Salvio+ 14]



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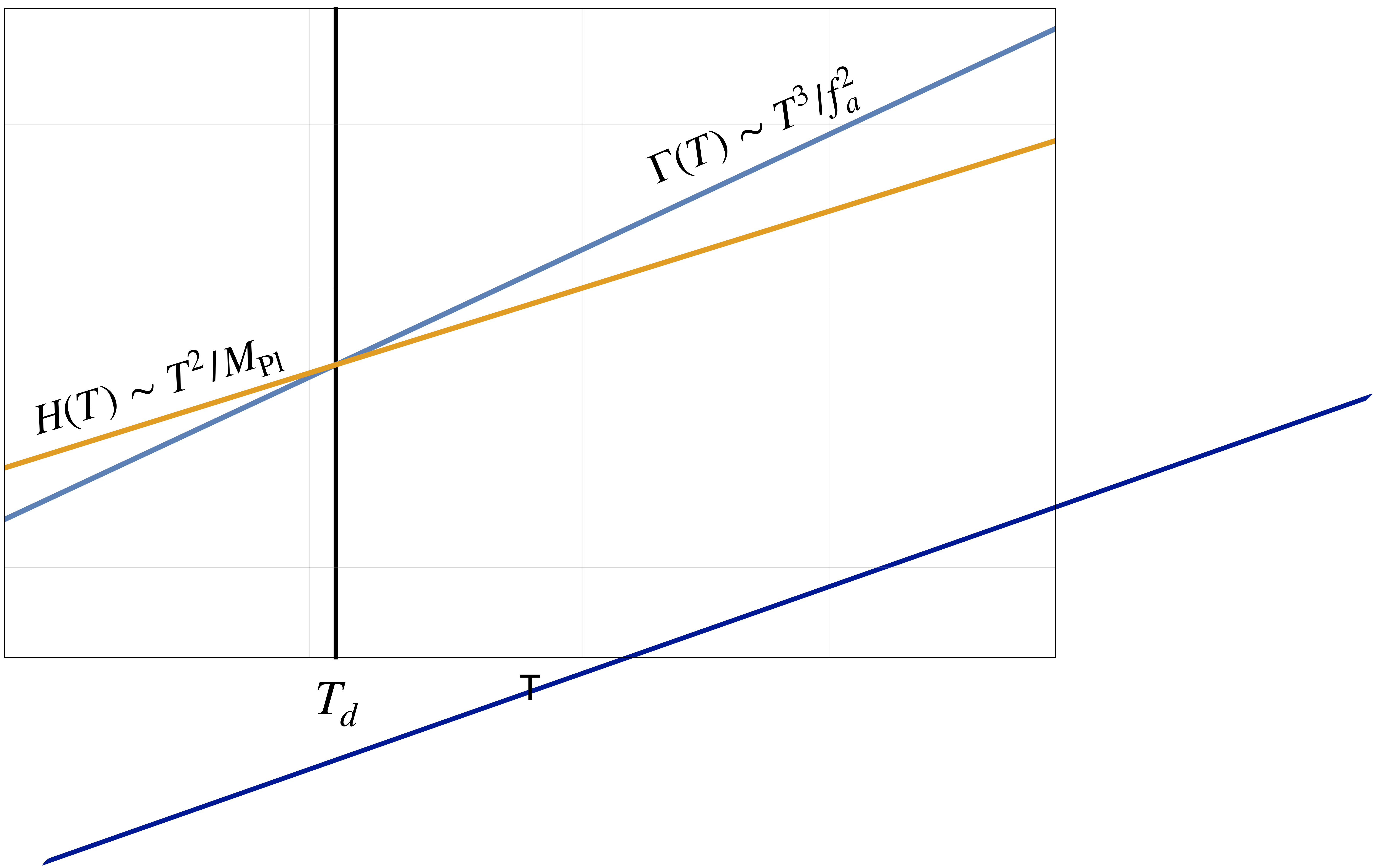

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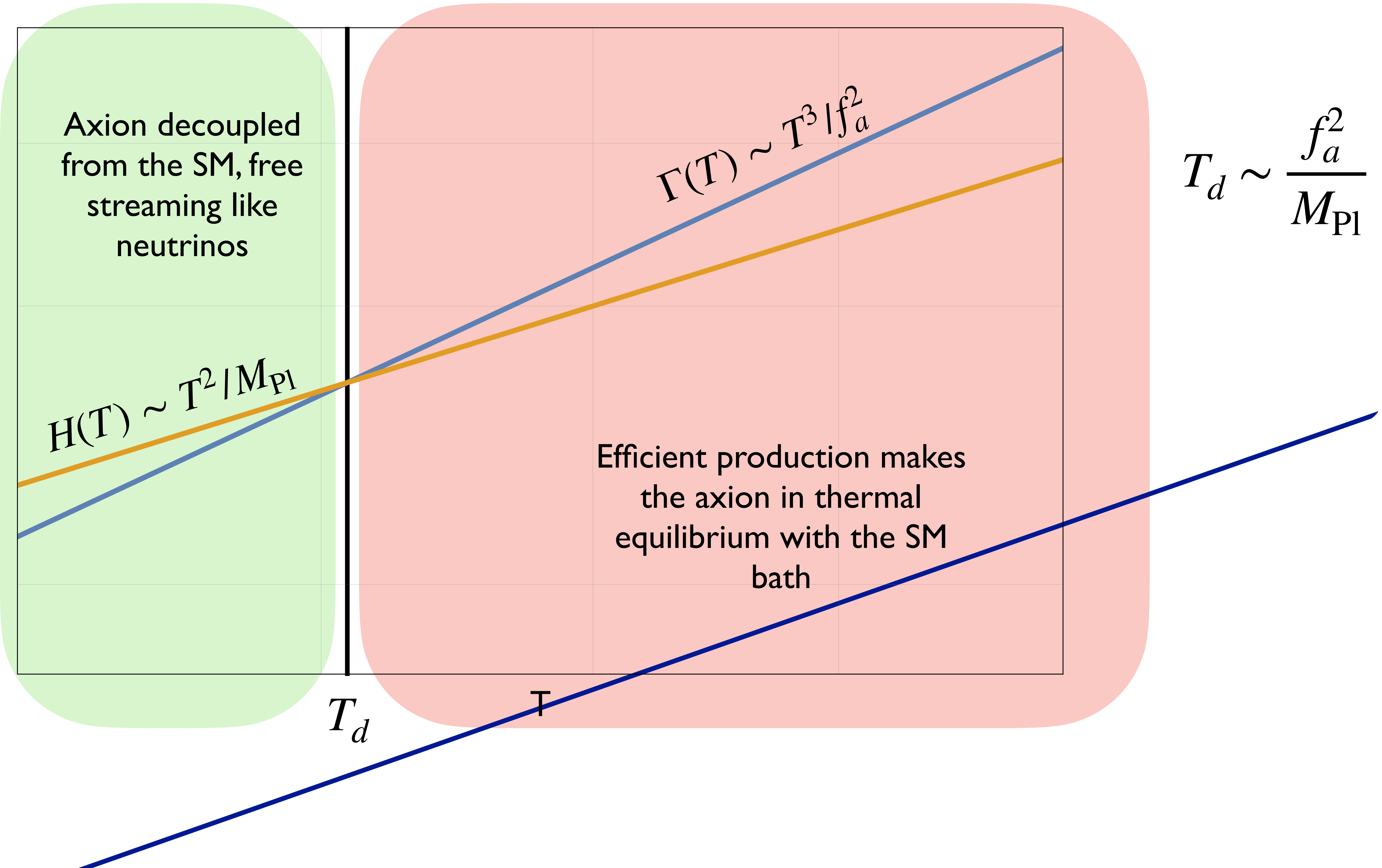
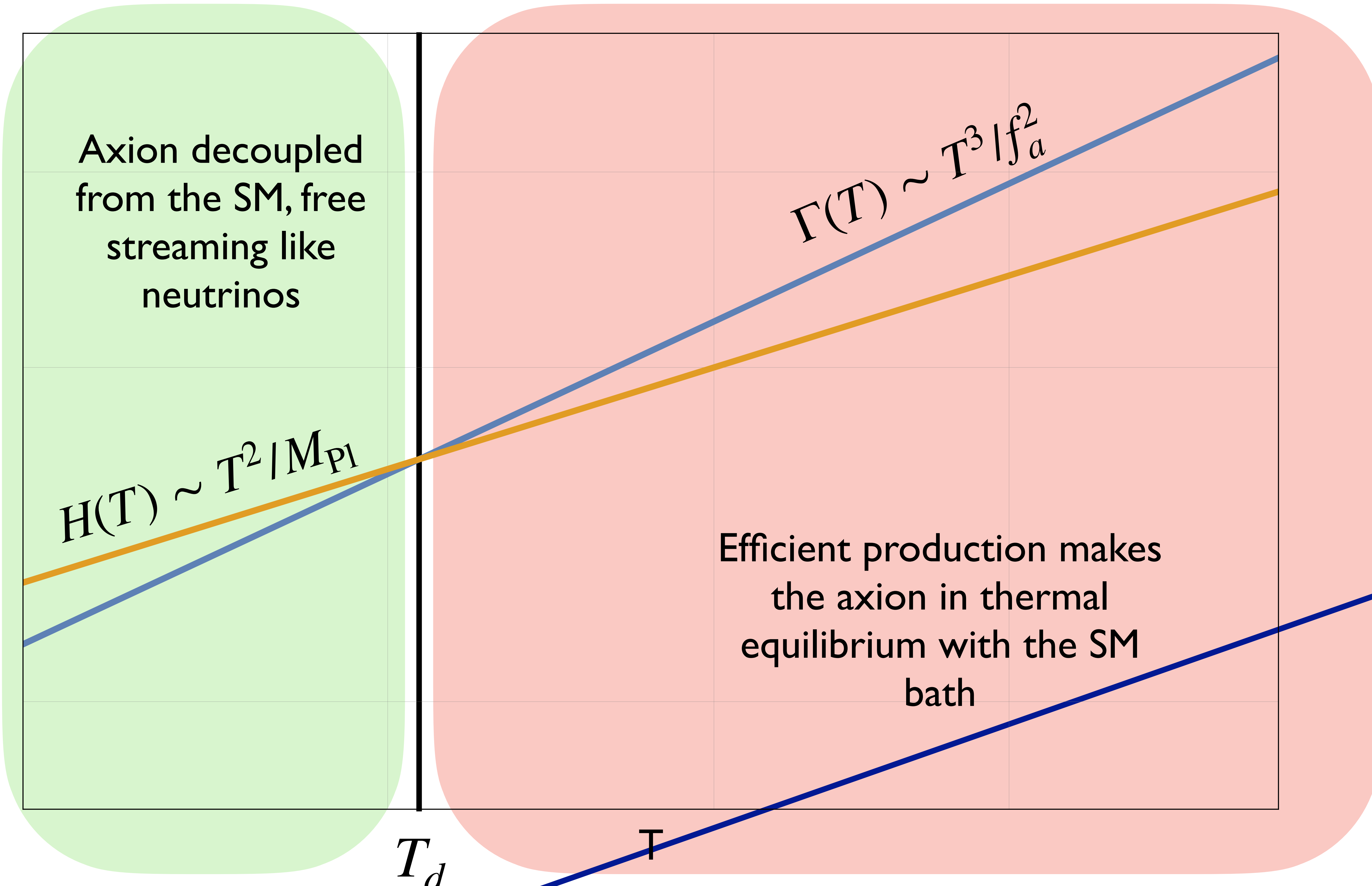


$$T_d \sim \frac{f_a^2}{M_{\rm Pl}} \sim \Lambda_{\rm QCD}\left(\frac{f_a}{10^8\,{\rm GeV}}\right)$$

Thermal production

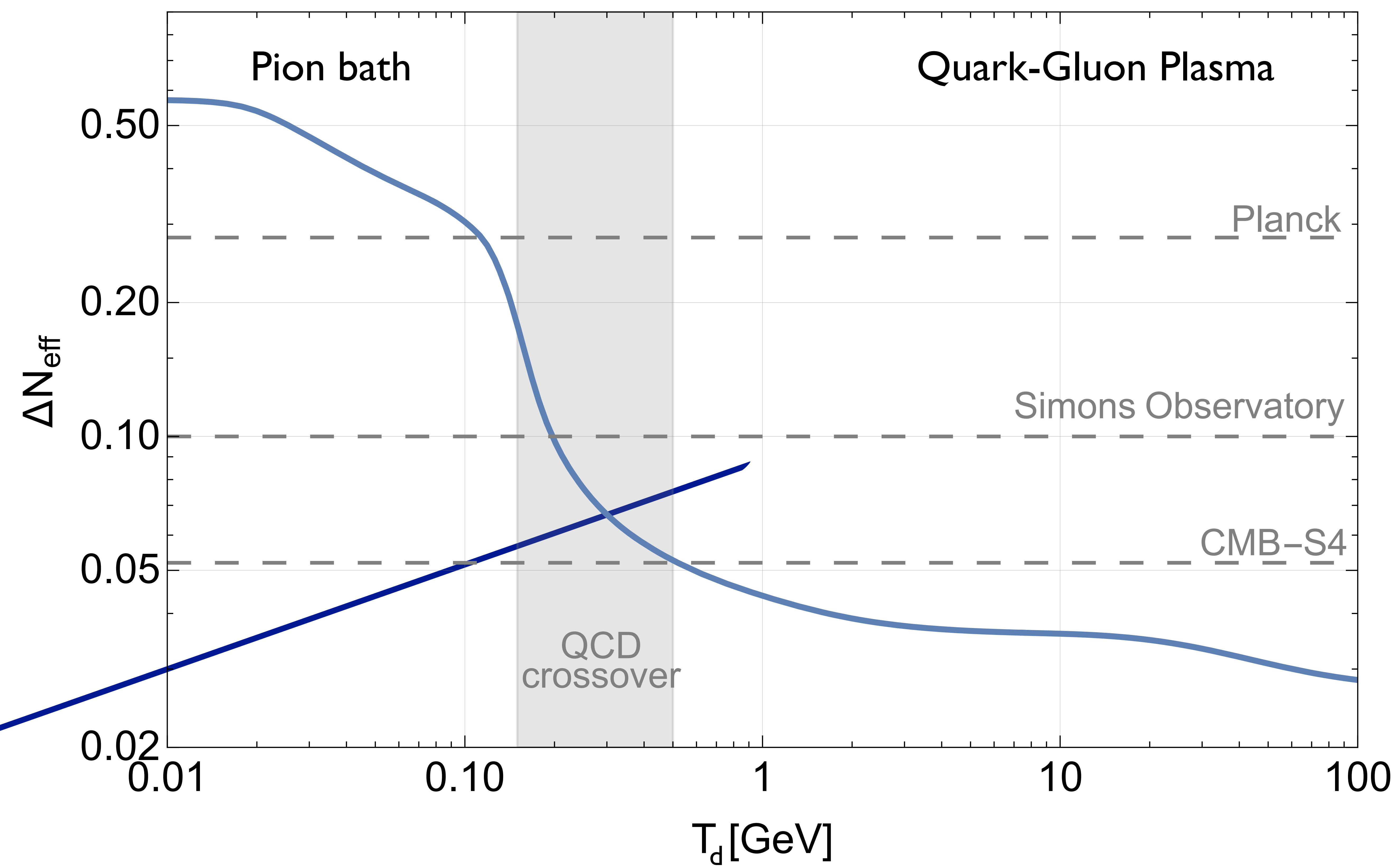


$$T_d \sim \frac{f_a^2}{M_{Pl}} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\,\mathrm{GeV}}\right)$$

$$\Delta N_{eff} = N_{eff} - N_\nu = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}}\left(\frac{\rho_a}{\rho_\gamma}\right)_{CMB}$$

Thermal production

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}}\left(\frac{\rho_a}{\rho_\gamma}\right)_{\rm CMB} \simeq 0.027\left(\frac{106.75}{g_{*,s}(T_d)}\right)^{4/3}$$



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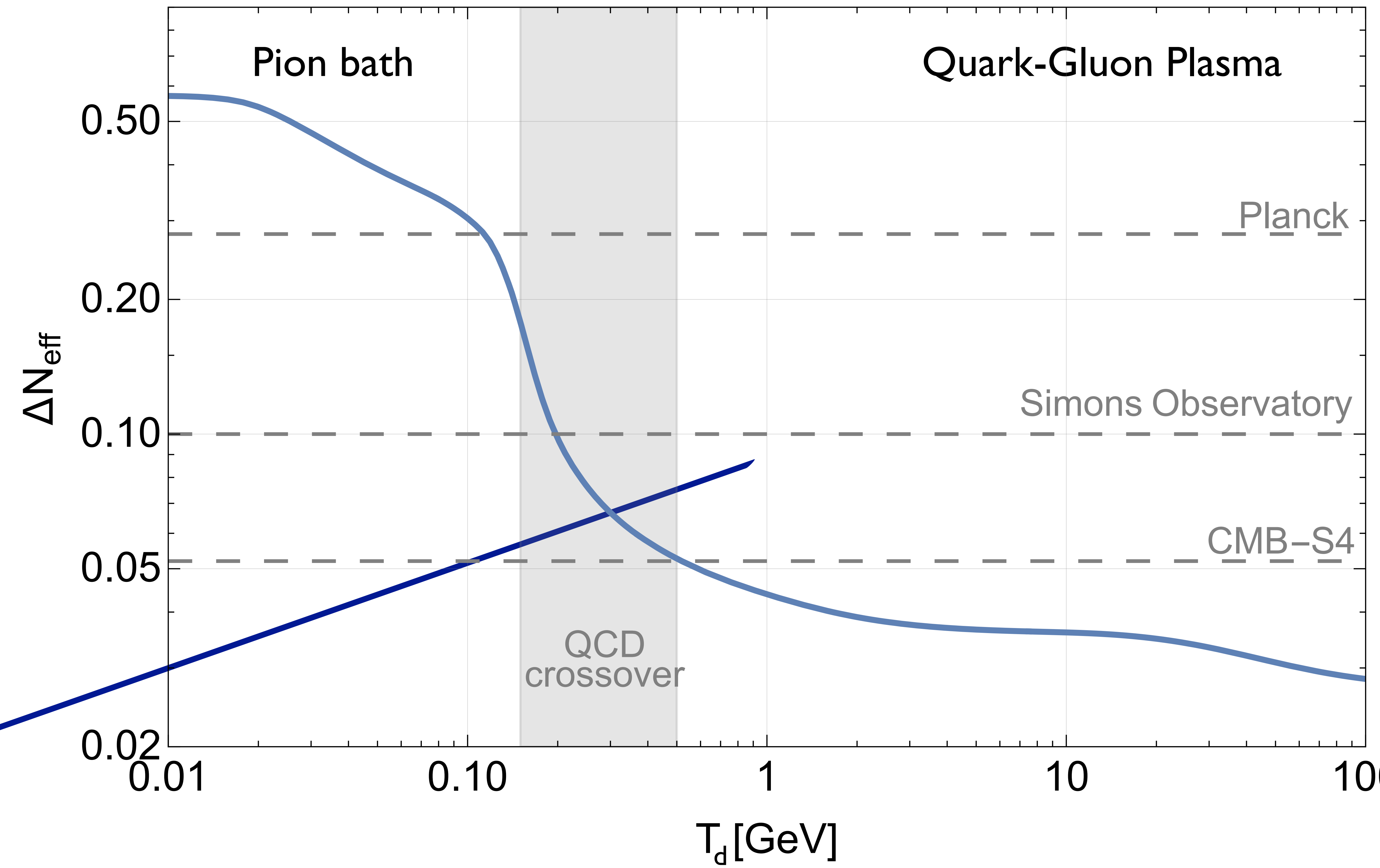
Solve Boltzmann equations

$$\frac{dY}{d\log x} = (Y^{\rm eq} - Y)\frac{\Gamma}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*S}}{d\log x}\right)$$



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Caveats

This formula may not be precise enough:

1. if the cross section depends on momentum, since different momenta will decouple at different times;
2. if the number of degrees of freedom decrease rapidly, higher momenta will be less diluted, leading to spectral distortions;
3. because production may be never in thermal equilibrium.

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Momentum-dependent Boltzmann equation

$$\frac{\partial \mathcal{F}_a}{\partial t} - H\,|\mathbf{k}|\frac{\partial \mathcal{F}_a}{\partial |\mathbf{k}|} = \Gamma_a\left(\mathcal{F}_a^{\rm eq} - \mathcal{F}_a\right)$$

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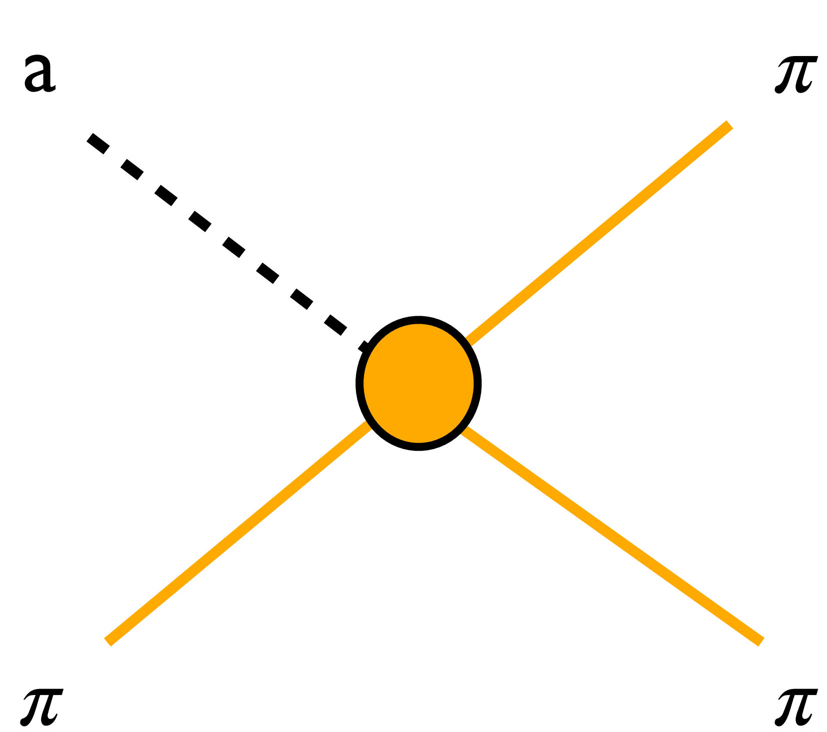
Hot axions from pions

$$\mathscr{L}_{a\pi} \supset \frac{C_{a\pi}}{f_a f_\pi}\partial^\mu a \left(2\partial_\mu \pi^0 \pi^+ \pi^- - \pi^0 \partial_\mu \pi^+ \pi^- - \pi^0 \pi^+ \partial_\mu \pi^-\right)$$



[Chang, Choi 1993]

LO: $\sum |\mathscr{M}_{LO}|^2 = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4}\left(s^2 + t^2 + u^2 - 3m_\pi^4\right)$

[Di Luzio, Martinelli, Piazza 2021]

NLO: breaks down at T~60 MeV

How do you fix this issue?

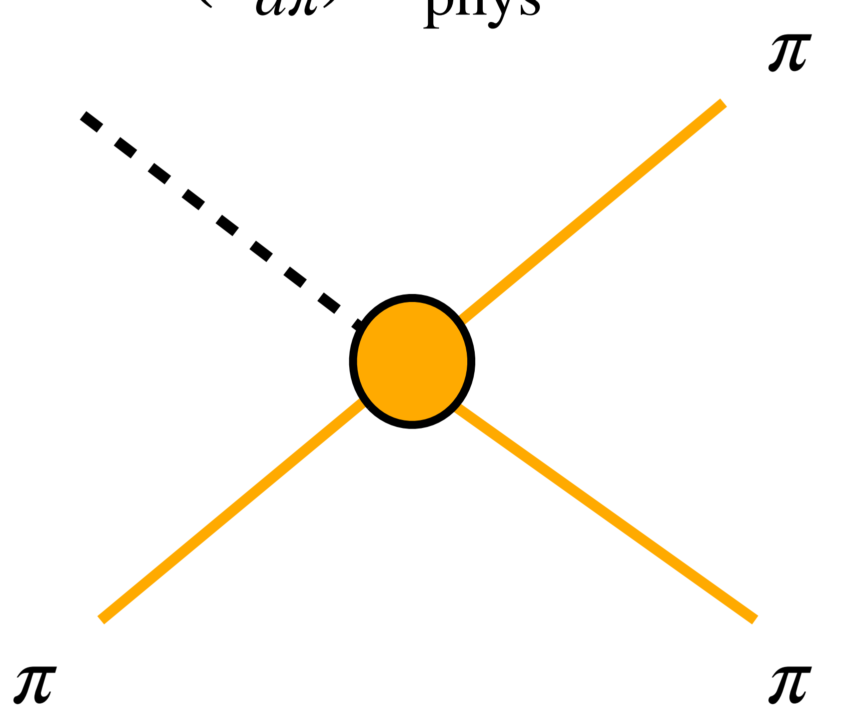
Hot axions from pions

$$\mathscr{L} \supset \bar{q}\left(i\partial_\mu\gamma^\mu + \frac{c_0}{2f_a}\partial_\mu\gamma^\mu a\gamma_5\right)q - \bar{q}_L M_a q_R + \text{h.c.}$$

$$M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}(1+c_3\sigma^3)}$$

$$\pi^0 = \cos(\theta_{a\pi})\ \pi^0_{\text{phys}} + \sin(\theta_{a\pi})\ a_{\text{phys}}$$

$$\simeq \pi^0_{\text{phys}} + \theta_{a\pi}\ a_{\text{phys}}$$



$$\mathscr{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\ \mathscr{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathscr{O}\left(\frac{m_\pi^2}{s}\right)$$

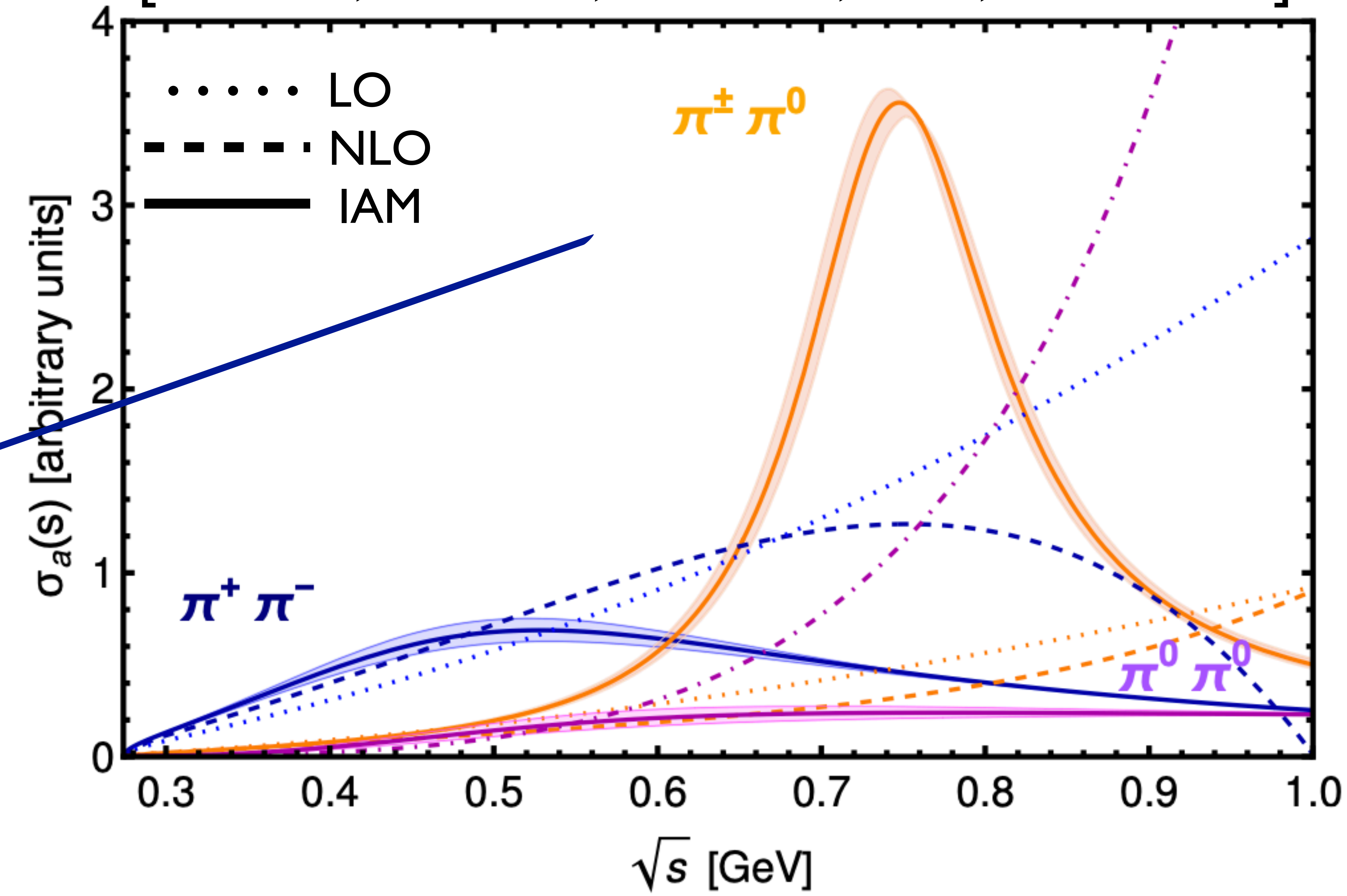
[Notari, Rompineve, Villadoro 2023]

Inverse amplitude method

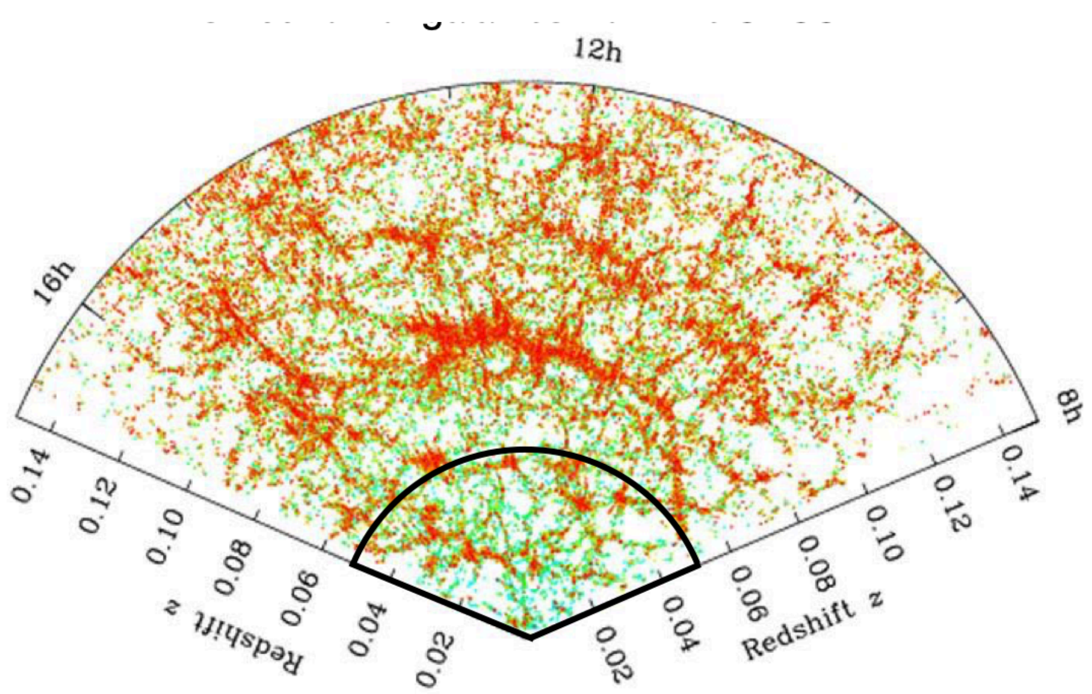
[Truong 1988]

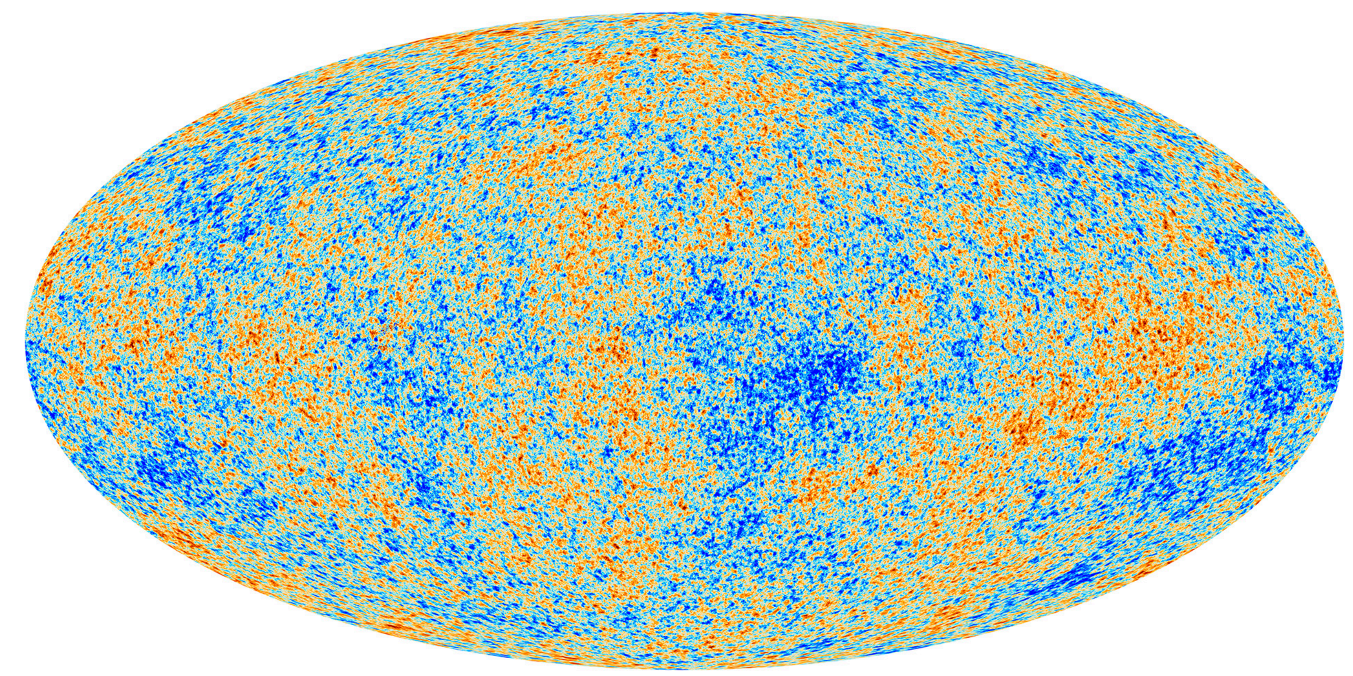
$$t^I_\ell(s) = \frac{t^{I(2)}_\ell(s)}{1 - t^{I(4)}_\ell(s)/t^{I(2)}_\ell(s)}$$

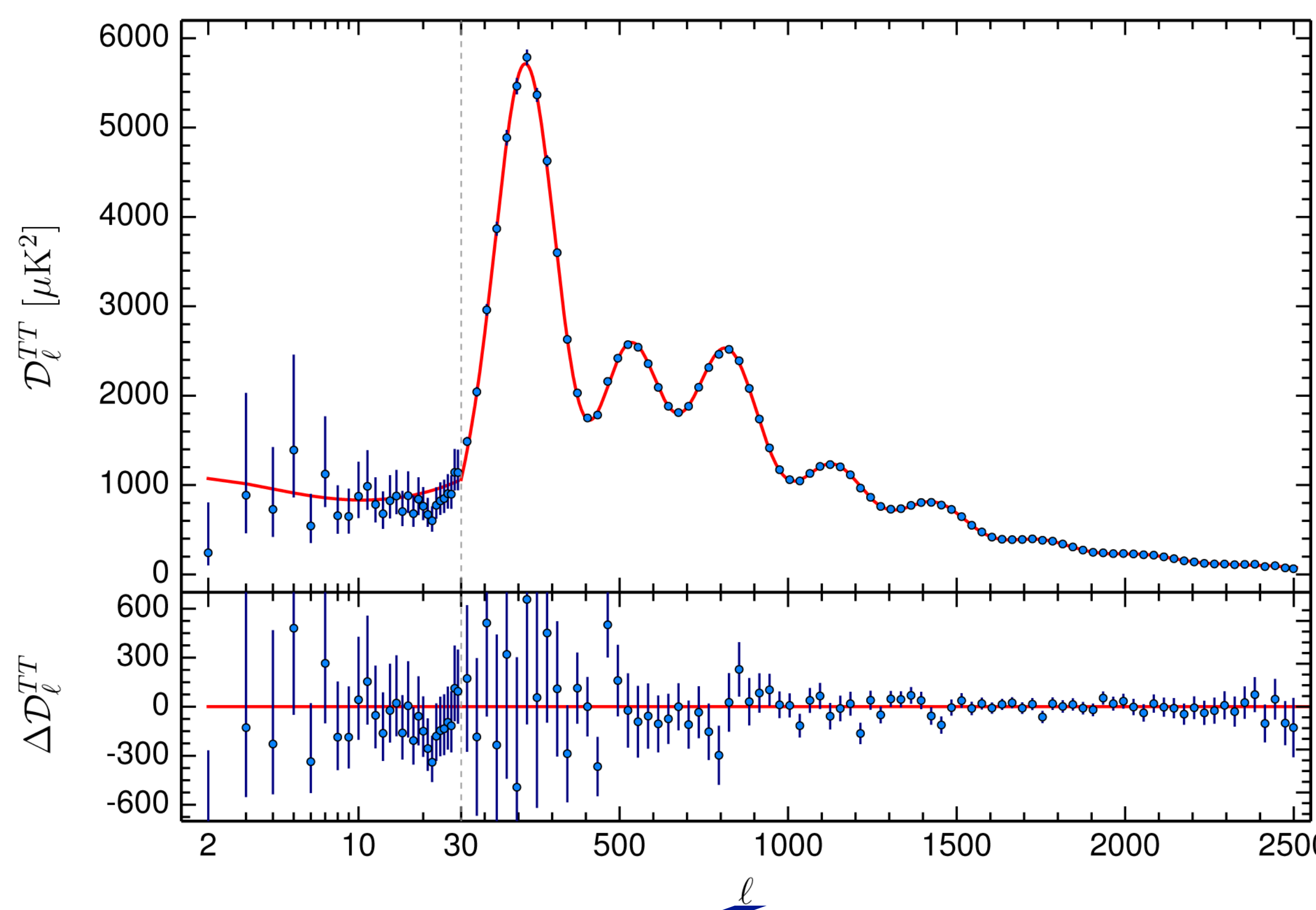
[Di Luzio, Camalich, Martinelli, Oller, Piazza 2023]



How to constrain light axions?







recombination

BBN

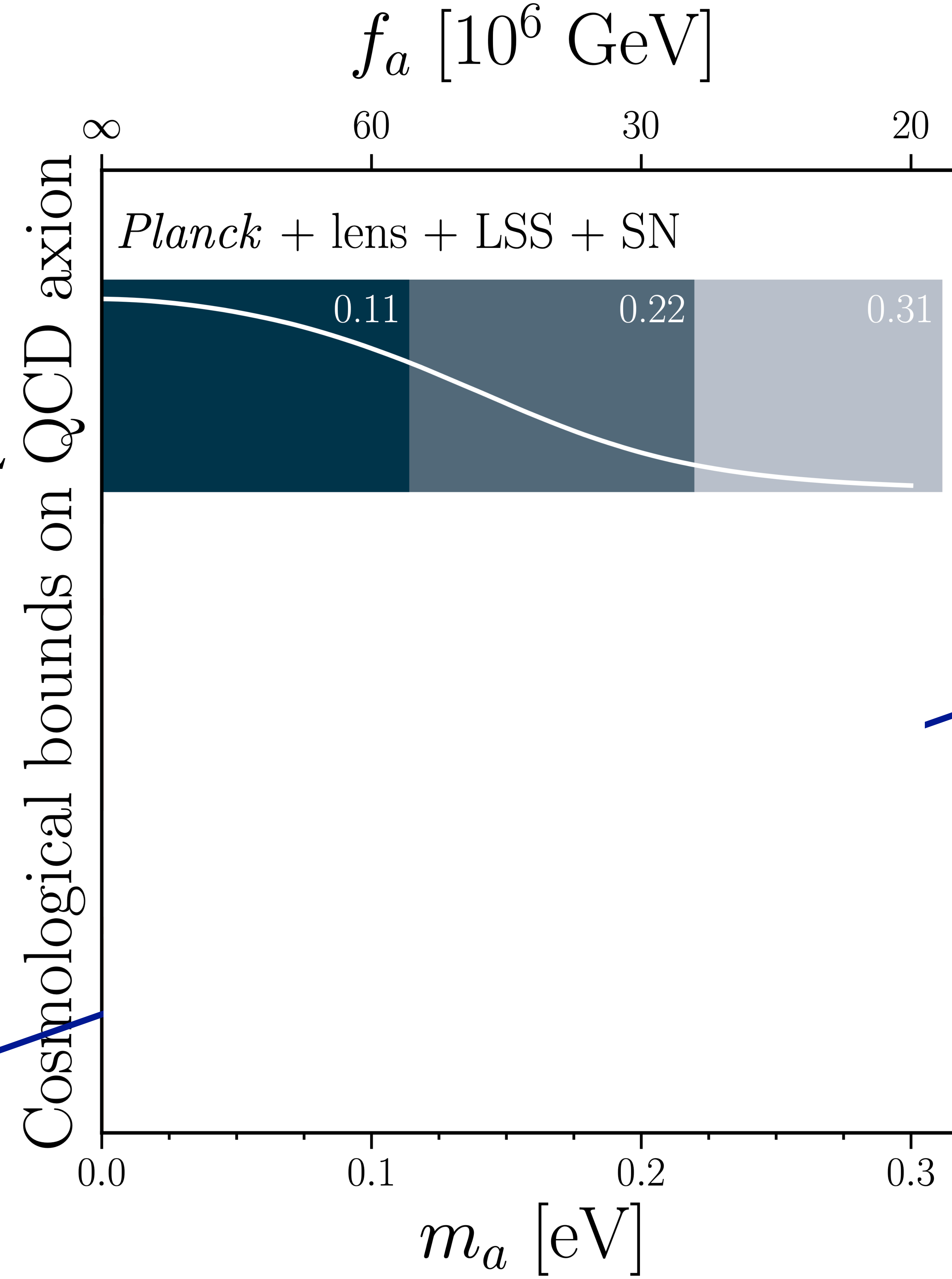
Late Universe 0.3 10^6 Early Universe T [eV]

Results

consistent with [Notari, Rompineve, Villadoro 2023]

$m_a \leq 0.22$ eV at 95 % CL

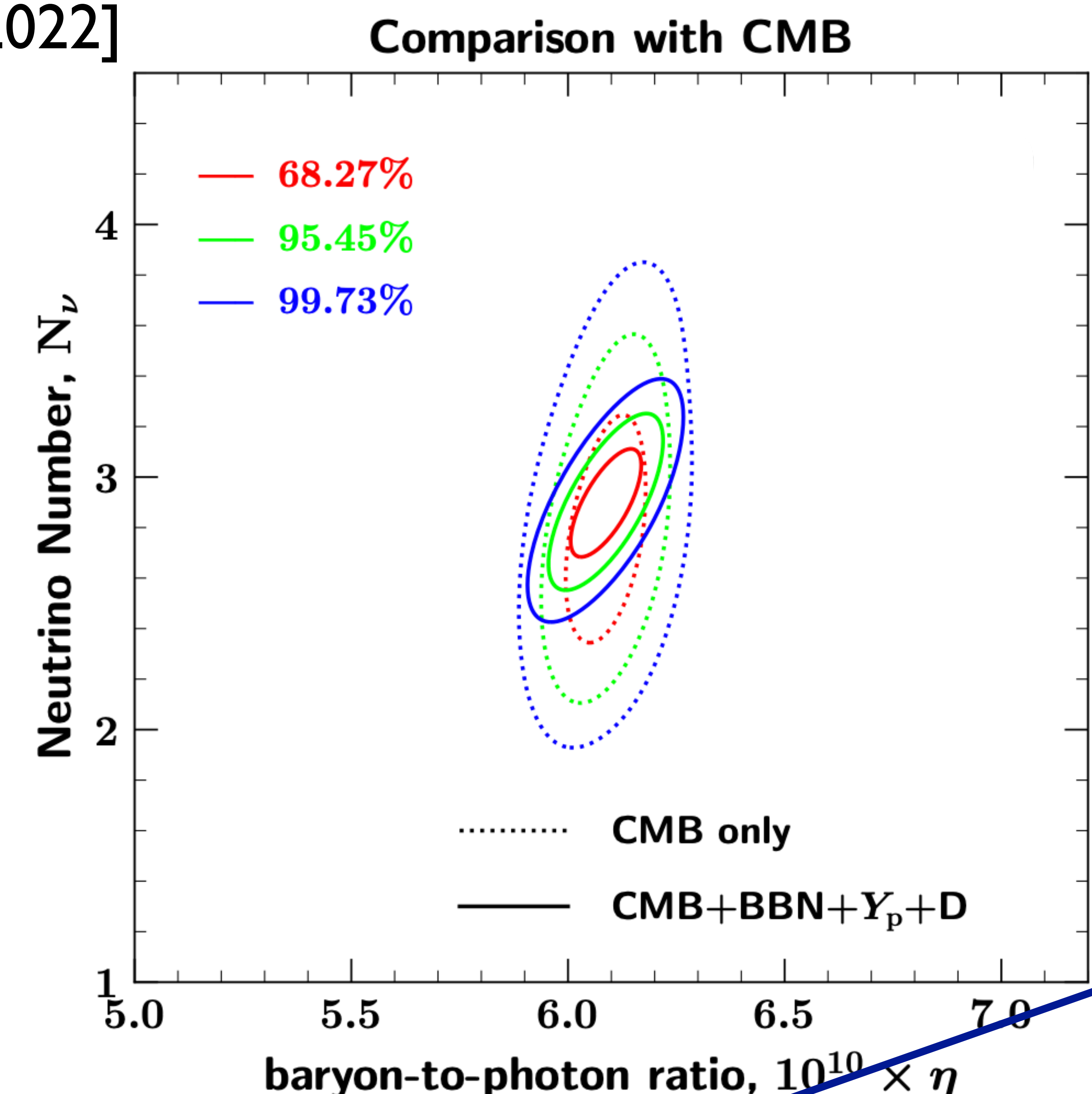
$f_a \geq 2.6 \times 10^7$ GeV at 95 % CL

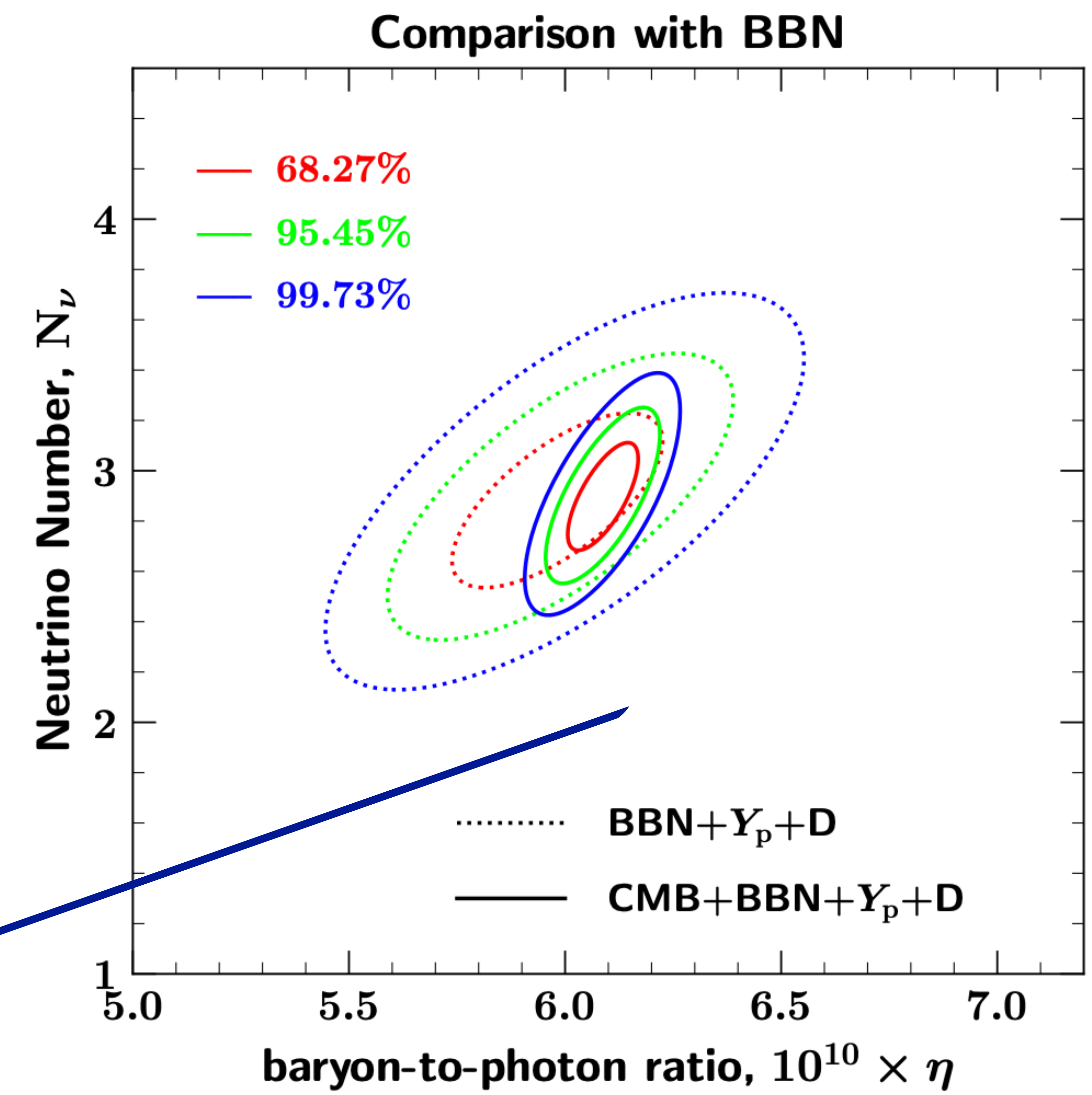


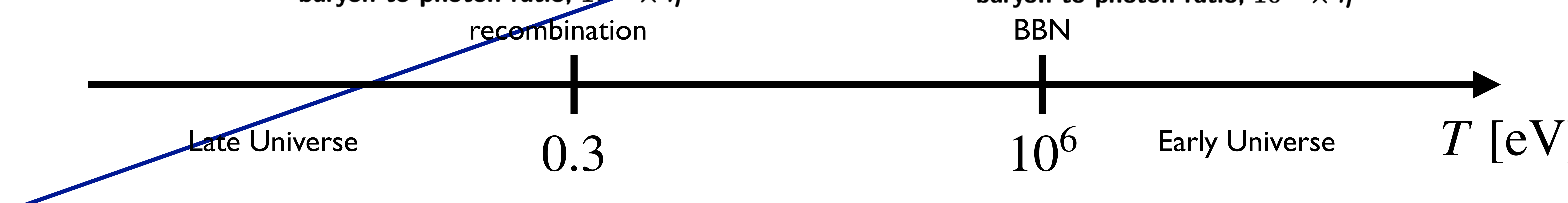
[Bianchini, G^2dC, Valli 2023]

…and BBN?

[Yeh, Shelton, Olive, Fields 2022]

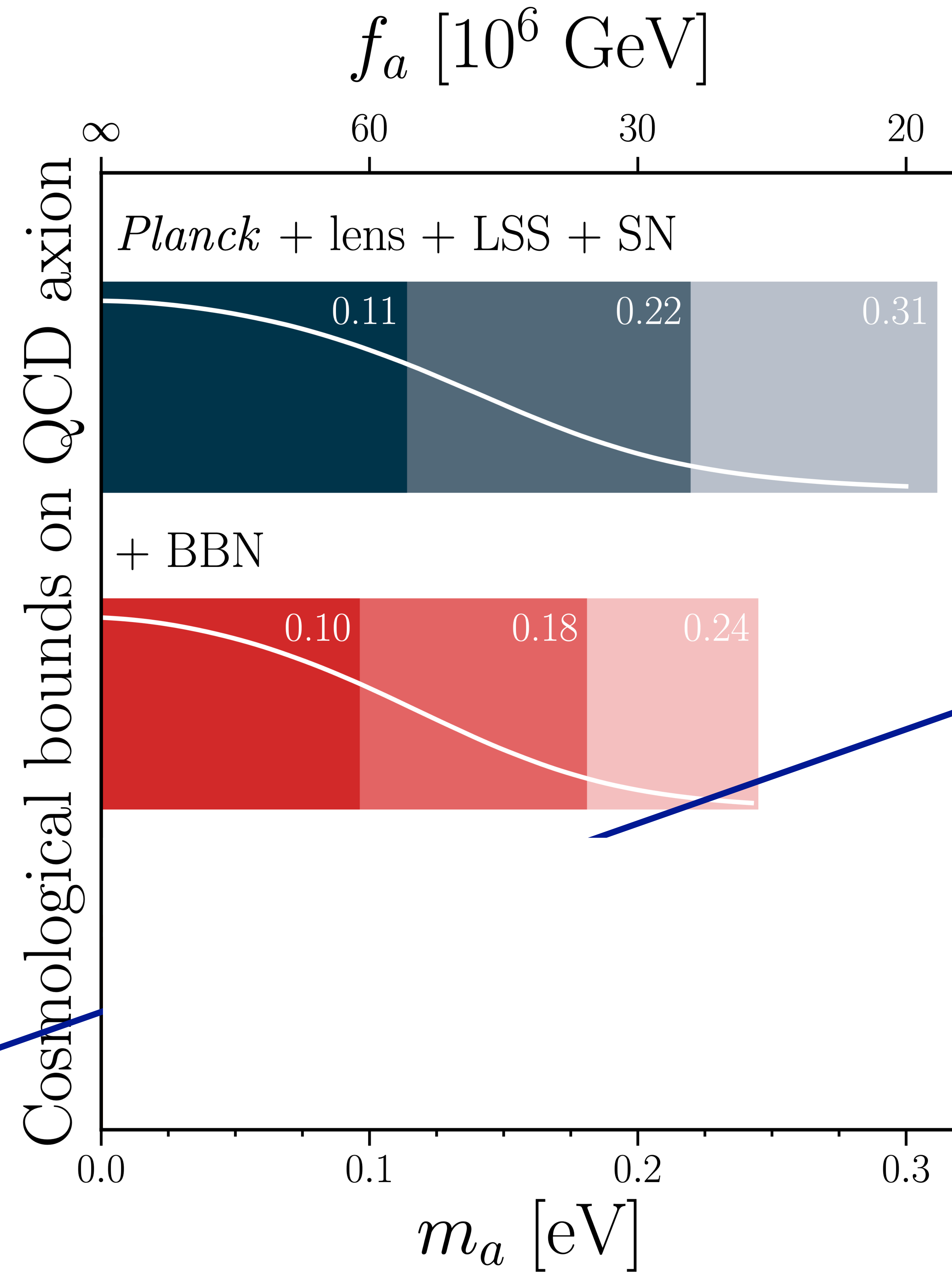






Results

[Bianchini, G^2dC, Valli 2023]

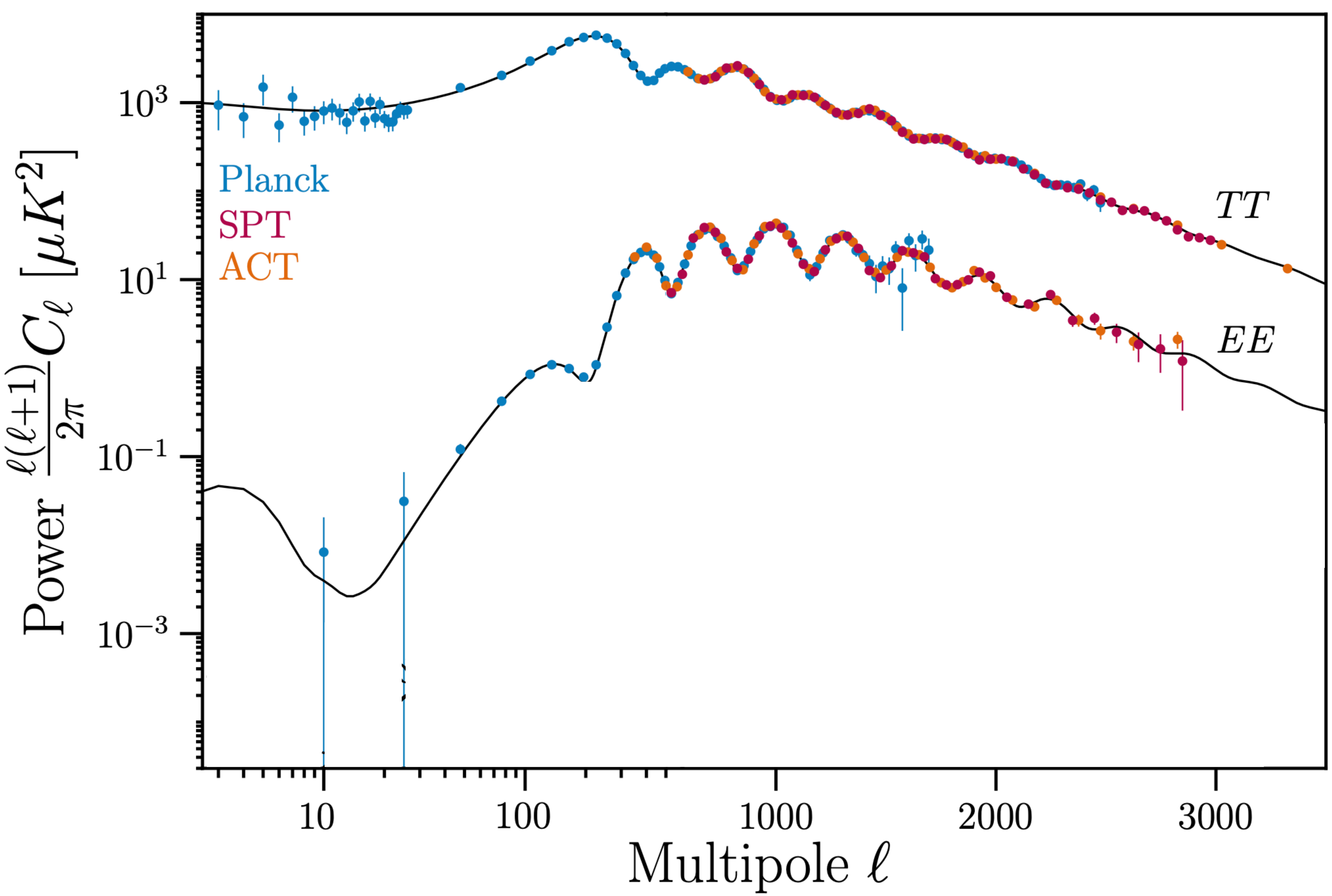


$m_a \leq 0.18$ eV at 95 % HDI

$f_a \geq 3.2 \times 10^7$ GeV at 95 % HDI

We use PRyMordial [Burns, Tait, Valli 2023] to obtain a likelihood for Y_p, D/H and He^3/H as a function of $N_{\rm eff}$ and Ω_B.

…and large multipole?



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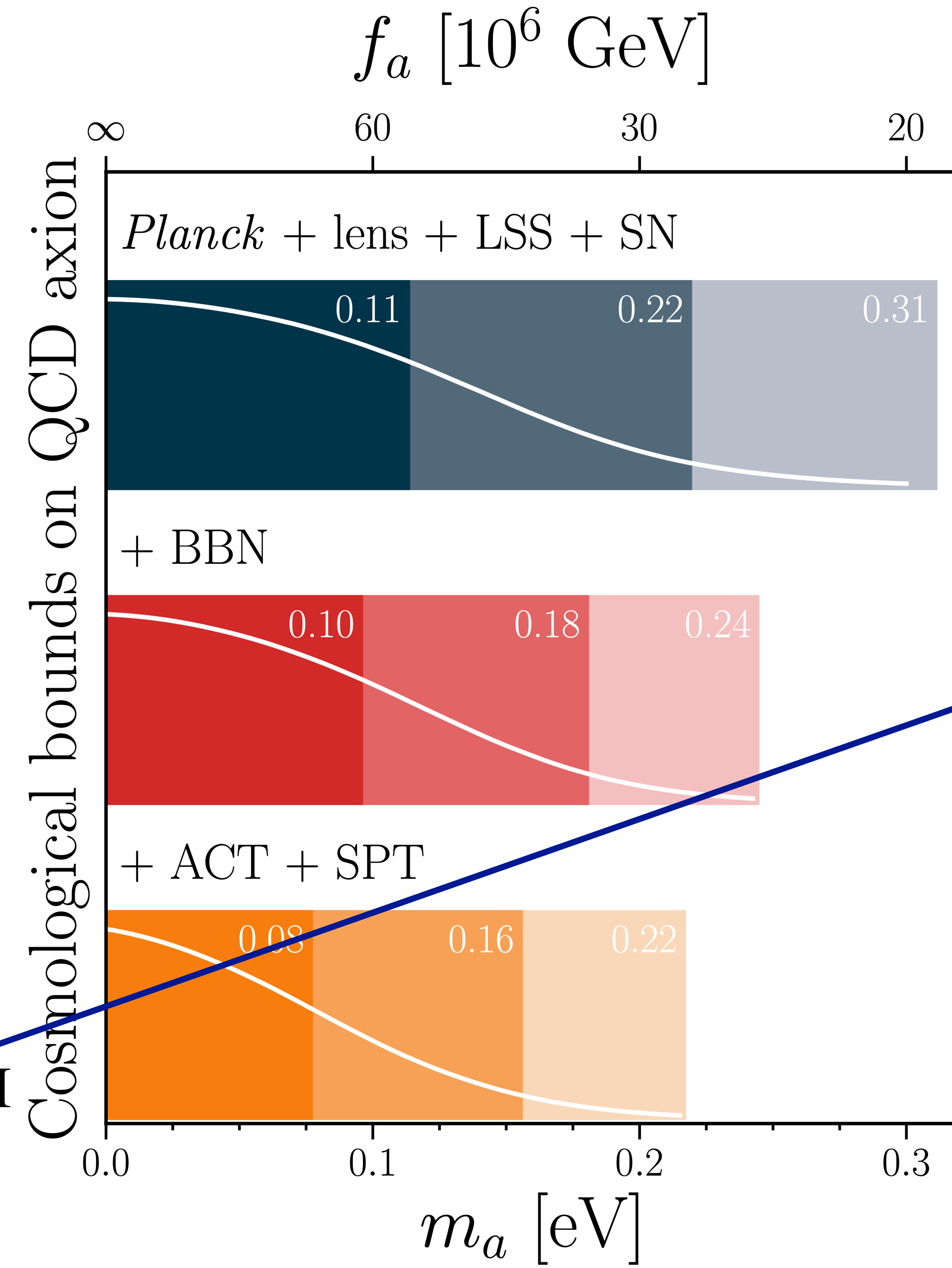


[Bianchini, G^2dC,Valli 2023]

$m_a \leq 0.16$ eV at 95 % HDI

$f_a \geq 3.6 \times 10^7$ GeV at 95 % HDI

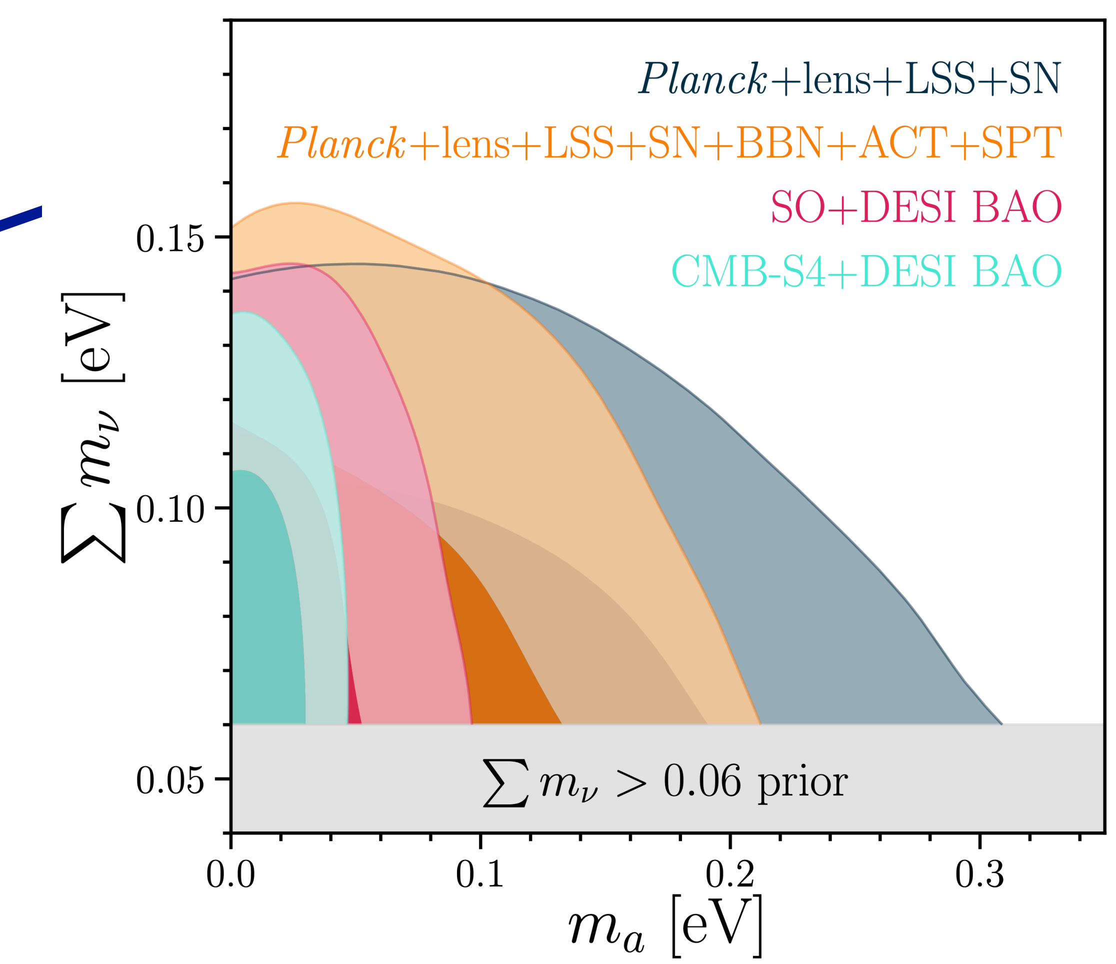
30% improvement with respect to [Notari, Rompineve, Villadoro 2023]

Conclusions

Conservative bound on m_a from up-to-date measurements of CMB, ground-based telescopes and abundances from BBN

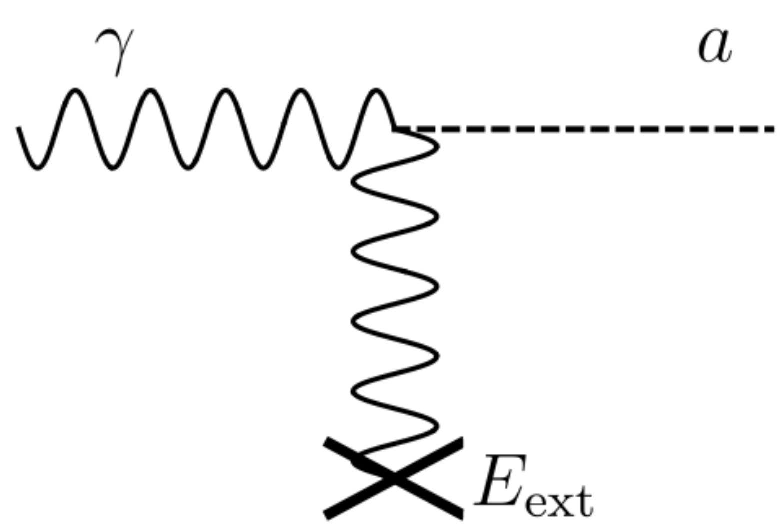
$$m_a \leq 0.16 \text{ eV at } 95\,\% \text{ CL}$$

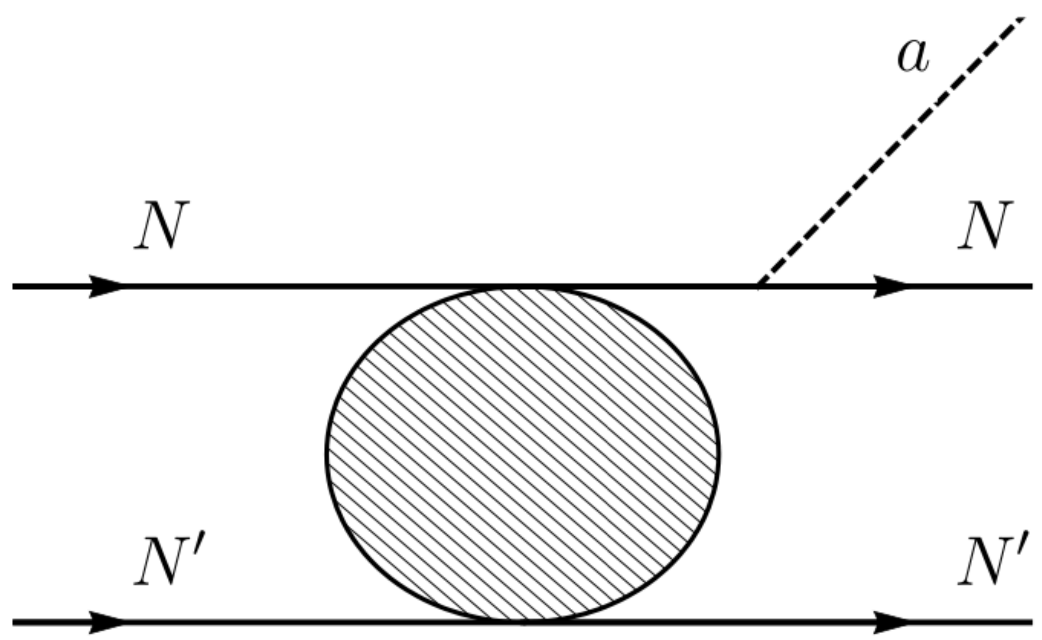
Forecast for future surveys (Simons Observatory and CMB-S4) in the m_a vs $\sum m_\nu$ plane competitive with current constraints from astrophysics.



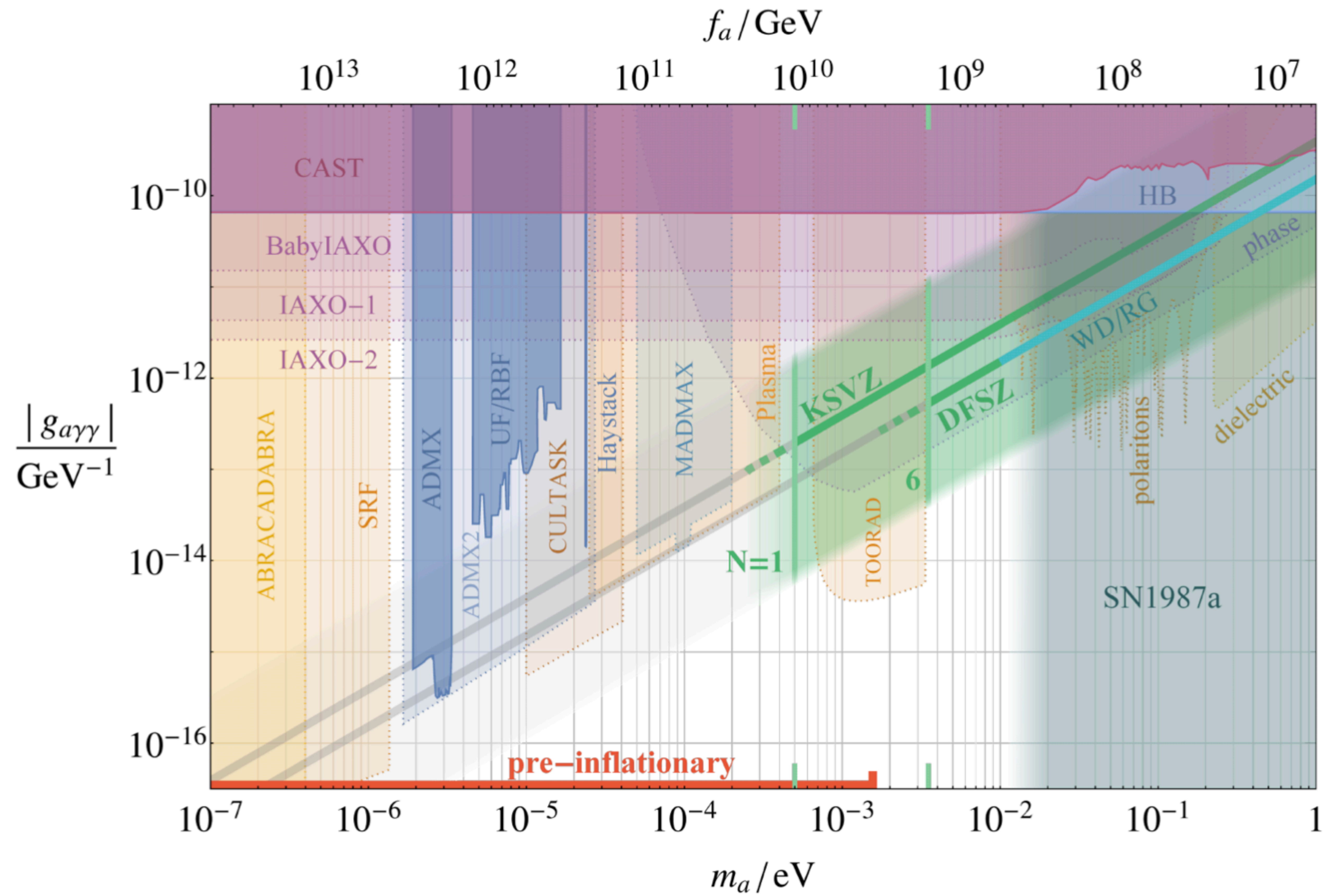
BACKUP

Bounds





[Gorghetto, Hardy, Villadoro 2020]



Cooling of Horizontal Branch stars, Supernovae, neutron stars give

$f_a \gtrsim (10^7 - 10^8)$ GeV

[see e.g. Di Luzio+ 2020]

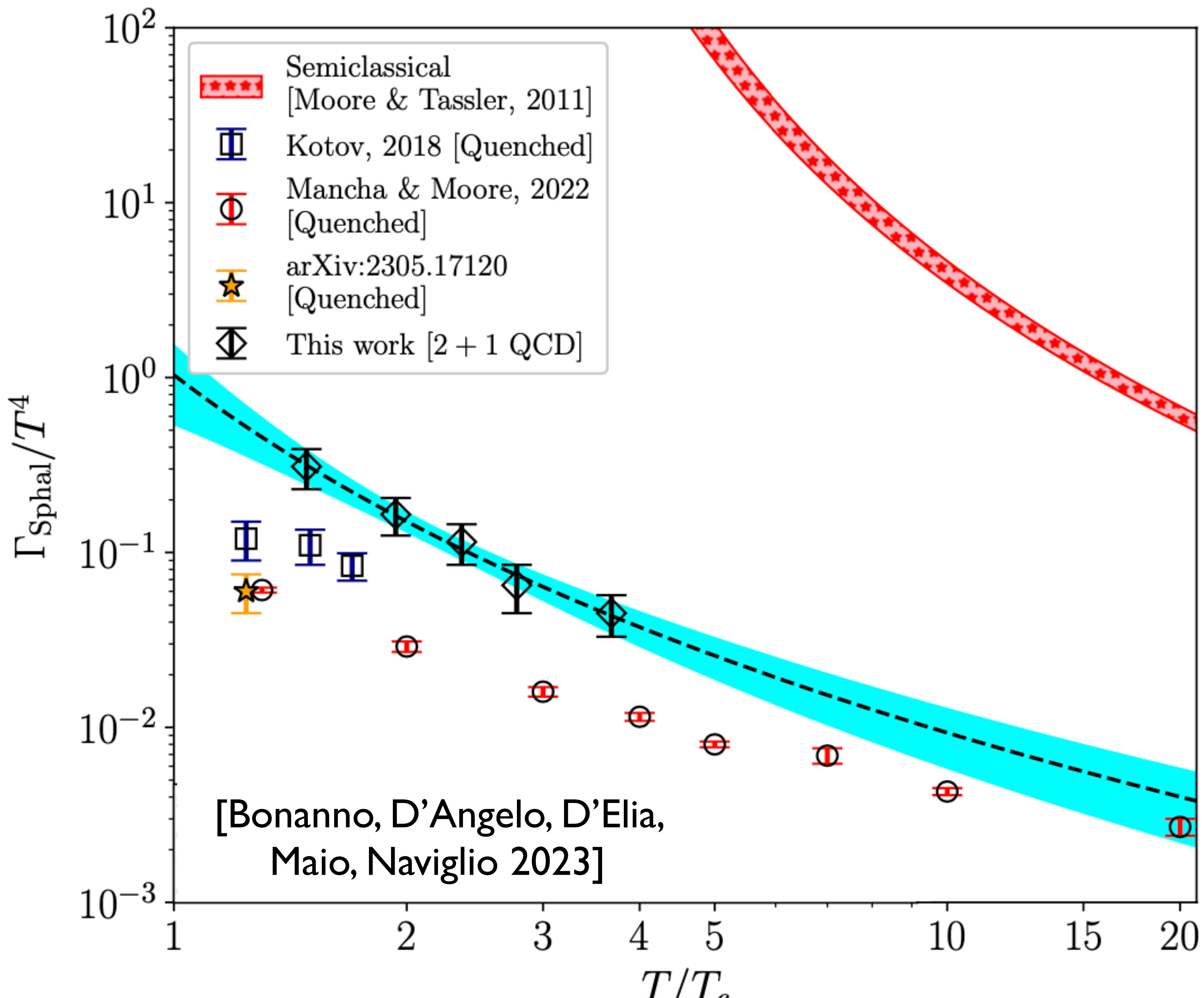
Caveat: strong debate on validity/ uncertainties.

[Chang+ 2018, Bar+ 2019, Carenza+ 2020, …]

Future

$$\Gamma_a = \int d^4x e^{ikx} \langle \mathcal{Q}(x)\mathcal{Q}(0) \rangle$$

$\mathcal{Q} = \frac{\alpha_s}{8\pi} G\tilde{G}$ is the topological charge



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1. Expected sensitivity of Simons Observatory and CMB-S4
2. ΛCDM cosmology with three degenerate neutrinos with $m_{\nu_i} = 0.02\ \mathrm{eV}$
3. QCD axion in thermal equilibrium: $Y_a(600\,\mathrm{MeV}) \simeq Y_{\mathrm{eq}}(T_{EW})$
4. Conservatively estimate rate at zero momentum via strong sphalerons
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