Axion emission from strange matter in core-collapse SNe

Maël Cavan - LAPTh (France)

based on work with Diego Guadagnoli (LAPTh), Micaela Oertel (LUTH), Hyeonseok Seong (DESY), Ludovico Vittorio (LAPTh)

Motivation

The QCD axion is one of the best motivated BSM particles

- solves the strong CP problem by a symmetry
- constitutes a viable Dark Matter candidate
- may address additional problems such as flavor
  [Ema *et al*, Calibbi *et al*]

Axion-quark interaction

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Dense and hot enough astro-objects may radiate axions $\Rightarrow$ cool down faster than expected from established mechanisms

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For Q_ν, difficult to go beyond a crude estimate

$$Q_\nu \sim 1.5 \times 10^{19} \frac{\mathrm{erg}}{\mathrm{s.g}} \times \rho \quad \text{with} \quad \rho \sim \rho_{core} \sim 3 - 8 \times 10^{14}\,\mathrm{g/cm^3}$$

Emissivity

$$Q_a = \int \begin{bmatrix} elem.\ of \\ phase\ space \end{bmatrix} \times E_a \times \begin{vmatrix} matrix\ elem.\ of \\ axion - producing \\ process \end{vmatrix}^2 \times \begin{bmatrix} ext.\ states' \\ distr.\ function \end{bmatrix}$$

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Calculable within assumptions :

- distribution functions not obvious away from ideal-gas assumption
- effective quantities (μ^*, m^*, E^*) $\rightarrow$ in-medium effects

Processes

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[Carenza et al, 2020]

ρ		$\bar{g}_{aN}$ ($\times 10^{-9}$)	m_a (meV)	f_a ($\times 10^{8}$ GeV)
ρ_0	only NN	0.81	21.02	2.71
	$\pi N + NN$	0.46	11.99	4.75
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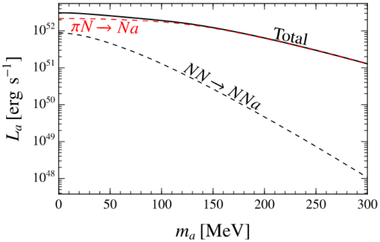
[Lella et al, 2022]



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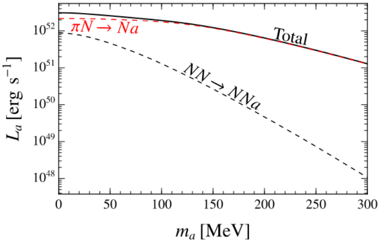

Also discussed $\Lambda \rightarrow n + a$ [Camalich et al]

Processes beyond first generation

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Our conclusions must be proved robust w.r.t.

- axion-matter modeling
  we address this question within a consistent EFT approach
- SN-core modeling
  we consider different EoS and also vary the thermodynamic parameters

Processes beyond first generation

We calculate axion emission from the full meson (M) and baryon (B) octets via :

- $B_i + M \to B_f + a$ ("Compton")
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All contribute positively to Q_a^{tot} $\to$ even if B_i, B_f, M fractions are "small", the large number of processes ($\sim$ 100) yields a relevant constraint axion-quark interactions $\xrightarrow[\text{energy}]{\text{low}}$ axion-hadron interactions

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Interactions are fixed by the global symmetries (Noether currents) and the couplings are defined unambiguously from axion-quark couplings

$$\mathcal{L}_{axion-hadron} = \frac{\partial_\mu a}{f_a} \left(x_R^b(k_R) J_R^{\mu,b}(U;B) + x_L^b(k_L) J_L^{\mu,b}(U;B) \right)$$

↑ axion-hadron couplings parametrized in terms of the fundamental k-couplings

↑ ↑ meson-octet field

↑ ↑ baryon-octet field

Axion-matter couplings

High energy : $\mu \gg \Lambda_{QCD}$	Low energy : $\mu \ll \Lambda_{QCD}$
$\mathcal{L}_{aqq} = \frac{\partial_\mu a}{f_a} \sum_{i=L,R} \bar{q}_i \gamma^\mu k_i q_i$	$\mathcal{L}_{aUB} = \frac{\partial_\mu a}{f_a} \sum_{i=L,R} x_i^b(k_i) J_i^{\mu,b}(U;B)$

By charge conservation and hermiticity we have 10 parameters $k_{V,A}^{11,\,22,\,33,\,23\,\&\,32}$

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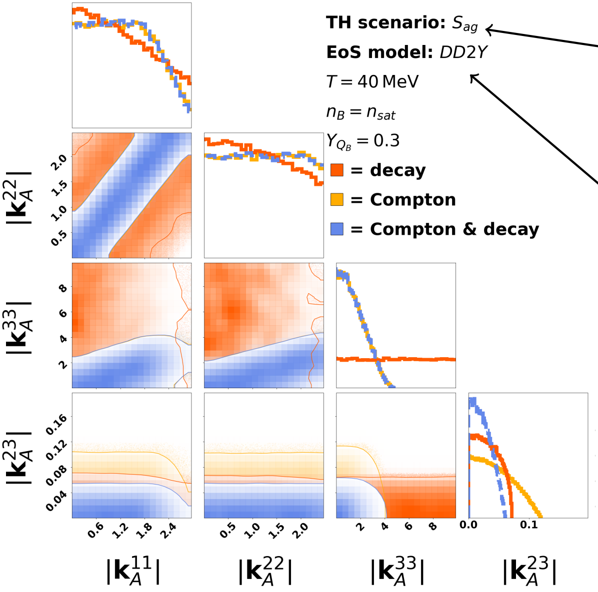
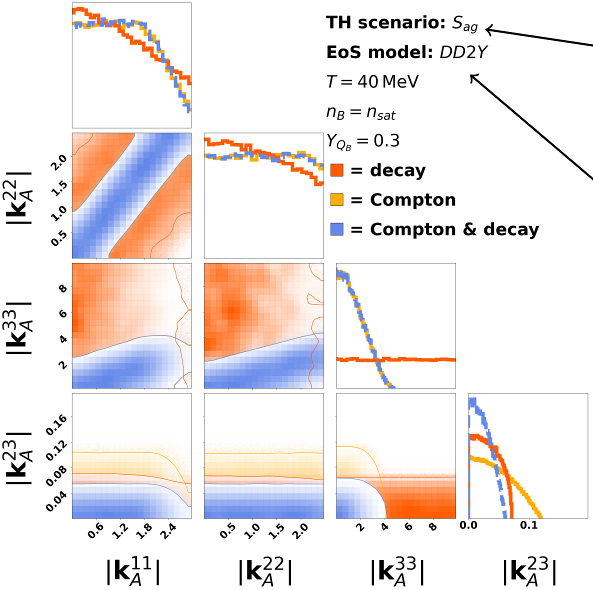
k_A^{ii} and $|k_A^{23}|$: constrained by Q_a

Results



S_{ag} ("agnostic") : couplings only constrained by data

Two alternative EoS models, within different strangeness content, DD2Y vs. SFHoY



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Main findings :

- bounds on $|k_A^{23}|$
- $k_A^{ii} \leftrightarrow k_A^{jj}$

Results

TABLE II: Q_a bounds on $|(\boldsymbol{k}_A)_{23,33}|$ according to the theory scenario (in parentheses) and the SN model. Bounds assume $f_a = 10^9$ GeV. The larger boldfaced (smaller) value quoted in each table entry refers to DD2Y (SFHoY).

$\boldsymbol{k}$ coupling (scenario)	$n_B = n_{\rm sat}$		$n_B = 1.5\, n_{\rm sat}$	
	30 MeV	40 MeV	30 MeV	40 MeV
$\lvert(\boldsymbol{k}_A)_{23}\rvert$ ($\mathrm{S_{ag}}$)	0.35 **0.15**	0.12 **0.061**	0.38 **0.097**	0.14 **0.052**
$\lvert(\boldsymbol{k}_A)_{33}\rvert$ ($\mathrm{S_{ag}}$)	8.8 **8.9**	4.4 **4.8**	5.9 **3.9**	3.1 **2.9**

k_A^{11} & k_A^{22} constrained by isolated neutron star data [Buschmann *et al*]

These bounds are "transferred" to k_A^{33} by Q_a

Conclusion

- SNe are excellent probes of fundamental physics, in particular of well-motivated SM extensions
- SNe probe axion interactions not only with ordinary, but also beyond-1st-generation matter
- Progress in the understanding of the sources is necessary to go beyond crude bounds

Outlook

- Better understand axion-strahlung
- Further explore the thermodynamic dependence
- ...

Thank you for your attention

Emissivity formula

$$dQ_a = \left(\prod_{i=1}^{n_i} \frac{f_i d^3\vec{p}_i}{(2\pi)^3 2E_i}\right)\left(\prod_{f=1}^{n_f} \frac{\bar{f}_f d^3\vec{p}_f}{(2\pi)^3 2E_f}\right)|\mathcal{M}_{fi}|^2(2\pi)^4\delta^4(P_i - P_f)$$

Type of particle	f_k	$\bar{f}_k$
Axion	0	1
Bosons	$f_k = \left(e^{(E_k-\mu_k)/T} - 1\right)^{-1}$	$\bar{f}_k = 1 + f_k$
Fermions	$f_k = \left(e^{(E_k-\mu_k)/T} - 1\right)^{-1}$	$\bar{f}_k = 1 - f_k$

Axion-quark interaction

$$\mathcal{L}_{aqq} = \frac{\partial_\mu a}{f_a}\left(\bar{q}_R k_R \gamma^\mu q_R + \bar{q}_L k_L \gamma^\mu q_L\right) \tag{1}$$

$$= \frac{\partial_\mu a}{f_a}\bar{q}\gamma^\mu\left(k_V + k_A\gamma_5\right)q \tag{2}$$

$$= \frac{a}{f_a}\bar{q}\left([k_V; M_q] + \{k_A; M_q\}\gamma_5\right)q + \partial_\mu O^\mu \tag{3}$$

$$k_V = k_R + k_L \quad \& \quad k_A = k_R - k_L \tag{4}$$

Conservation of charge, $k_{A,V}$ hermitian and (3) implie we can choose

$$k_A = \begin{pmatrix} k_A^{11} & 0 & 0 \\ 0 & k_A^{22} & k_A^{23} \\ 0 & \bar{k}_A^{23} & k_A^{33} \end{pmatrix}, \quad k_V = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & k_V^{23} \\ 0 & \bar{k}_V^{23} & 0 \end{pmatrix} \tag{5}$$

7 free parameters !

Theoretical models

To interpret our results we used two theoretical models (for QCD axion $m_a << T$ and $f_a = 10^9\,\mathrm{GeV}$) :

- **Flavor model** where we assume the flaxion/axiflavon model
- **Agnostic model** where we assume experimental bounds from previous analysis ($|k_V^{23}| < 3 \times 10^{-3}$, $|k_A^{23}| < 100$, $|k_A^{11}| < 3$, $|k_A^{22}| < 2.5$, $|k_A^{33}|$?)

In the agnostic model, the bounds over k_A^{11} and k_A^{22} propagate on k_A^{33} via the correlations.

SNe model

The SNe core is understood as a thermodynamic system therefore dependent on local variables T, n_B and Y_e.

Local values T, n_B, Y_e make it possible to determine point per point the effective quantities according to the equation of state (EoS) here DD2Y or SFHoY (more strange matter than DD2Y) [Oertel et al, 2016]

We consider the global values T, n_B, Y_{Q_B} (Y_{Q_B} correlates with Y_e).

Different standard scenarios :

- DD2Y or SFHoY
- $T \in \{30; 40\}\,\mathrm{MeV}$
- $n_B \in \{1; 1.5\}\, n_{sat}$
- $Y_{Q_B} = 0.3$

which corresponds to $2 \times 2 \times 1 = 8$ SNe scenarios

Results

$$Q_a = \alpha_V \left(\frac{|k_V^{23}|}{f_a}\right)^2 + \alpha_A \left(\frac{|k_A^{23}|}{f_a}\right)^2 + \sum_{i=1}^{2} \alpha_i \left(\sum_{j=1}^{3} \beta_{ij} \frac{k_A^{jj}}{f_a}\right)^2$$

$Q_a < Q_\nu$ (Raffelt bound) implies

- bounds over $|k_{A,V}^{23}|$
- new correlations between k_A^{ii}

$B_i + M \to B_f + a$ diagrams

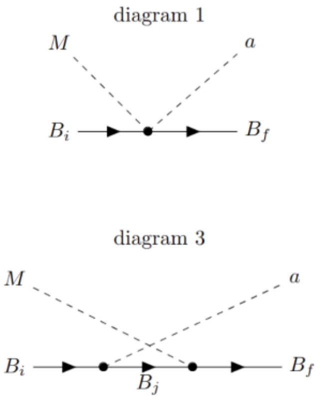
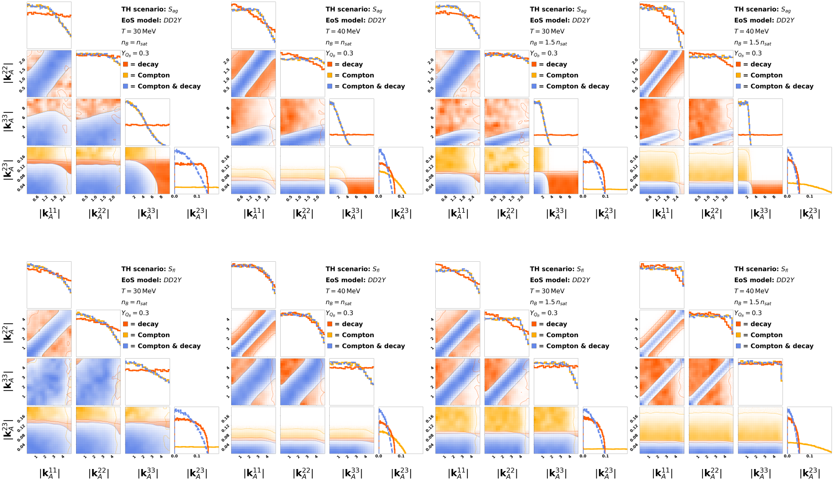


FIG. 1: The diagrams contributing to $B_i\,M \to B_f\,a$, with $B_{i,f}$ initial- or final-state octet baryons, M octet mesons, and a the axion.

DD2Y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EoS model: DD2Y
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$n_B = n_{sat}$
$Y_{Q_B} = 0.3$
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= Compton
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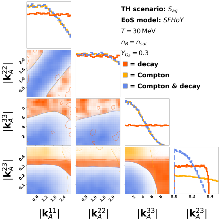
SFHoY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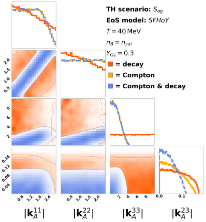
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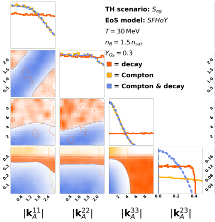
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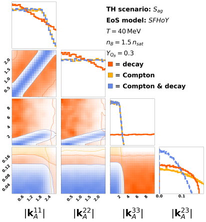
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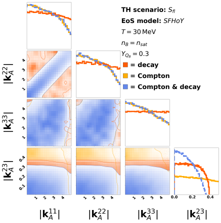
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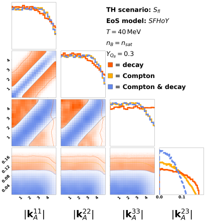
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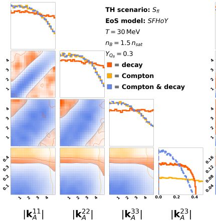
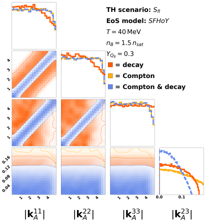

TH scenario: S_{fl}
EoS model: SFHoY
$T = 30$ MeV
$n_B = 1.5\,n_{sat}$
$Y_{Q_B} = 0.3$
= decay
= Compton
= Compton & decay
$|\mathbf{k}_A^{11}|$ $|\mathbf{k}_A^{22}|$ $|\mathbf{k}_A^{33}|$ $|\mathbf{k}_A^{23}|$


TH scenario: S_{fl}
EoS model: SFHoY
$T = 40$ MeV
$n_B = 1.5\,n_{sat}$
$Y_{Q_B} = 0.3$
= decay
= Compton
= Compton & decay
$|\mathbf{k}_A^{11}|$ $|\mathbf{k}_A^{22}|$ $|\mathbf{k}_A^{33}|$ $|\mathbf{k}_A^{23}|$