Model-Independent Radiative Symmetry Breaking: Gravitational Waves, Primordial Black Holes and New Physics

Alberto Salvio

Università di Roma Tor Vergata

INFN

16 April 2024

IRN Terascale meeting
Laboratory Nazionali di Frascati

Main topic of the talk and motivations

If symmetries are broken and masses are generated radiatively one always has first-order phase transitions (PTs) with corresponding

- observable gravitational waves (GWs)
- primordial black holes (PBHs)

also, if observed, they would signal new physics.

This radiative symmetry breaking (RSB) can be a feature of a Standard Model (SM) extension or of a dark sector (weakly coupled to the SM)

Main topic of the talk and motivations

If symmetries are broken and masses are generated radiatively one always has first-order phase transitions (PTs) with corresponding

- observable gravitational waves (GWs)
- primordial black holes (PBHs)

also, if observed, they would signal new physics.

This radiative symmetry breaking (RSB) can be a feature of a Standard Model (SM) extension or of a dark sector (weakly coupled to the SM)

We discuss a model-independent approach that is

- valid for large-enough supercooling
- can be implemented perturbatively

to quantitatively describe phase transitions phenomena in terms of few parameters, which are computable once the model is specified

A further advantage: models with RSB are more predictive!

Radiative symmetry breaking (RSB) mechanism

[Coleman-Weinberg (1973)], *[Gildener, S. Weinberg (1976)]*

To illustrate this general result we consider the general matter Lagrangian density

$$\mathscr{L}^{\rm ns}_{\rm matter} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \frac{D_\mu\phi_a\, D^\mu\phi_a}{2} + \bar{\psi}_j i\not{D}\psi_j - \frac{1}{2}(Y^a_{ij}\psi_i\psi_j\phi_a + \text{h.c.}) - V_{\rm ns}(\phi),$$

with

$$V_{\rm ns}(\phi) = \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

In the RSB mechanism masses emerge radiatively: there is an energy $\tilde{\mu}$ at which $V_{\rm ns}$ develops a flat direction, $\phi_a = \nu_a\chi$, with $\nu_a\nu_a = 1$, and χ a single scalar field
$\Longrightarrow$ RG-improved potential V along ν_a reads

$$V(\chi) = \frac{\lambda_\chi(\mu)}{4}\chi^4, \qquad \left(\lambda_\chi(\mu) \equiv \frac{1}{3!}\lambda_{abcd}(\mu)\nu_a\nu_b\nu_c\nu_d, \quad \lambda_\chi(\tilde{\mu}) = 0\right)$$

Radiative symmetry breaking (RSB) mechanism

[Coleman-Weinberg (1973)], *[Gildener, S. Weinberg (1976)]*

To illustrate this general result we consider the general matter Lagrangian density

$$\mathscr{L}^{\rm ns}_{\rm matter} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \frac{D_\mu\phi_a\, D^\mu\phi_a}{2} + \bar\psi_j i\not{D}\psi_j - \frac{1}{2}(Y^a_{ij}\psi_i\psi_j\phi_a + \text{h.c.}) - V_{\rm ns}(\phi),$$

with

$$V_{\rm ns}(\phi) = \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

In the RSB mechanism masses emerge radiatively: there is an energy $\tilde\mu$ at which $V_{\rm ns}$ develops a flat direction, $\phi_a = \nu_a\chi$, with $\nu_a\nu_a = 1$, and χ a single scalar field $\Longrightarrow$ RG-improved potential V along ν_a reads

$$V(\chi) = \frac{\lambda_\chi(\mu)}{4}\chi^4, \qquad \left(\lambda_\chi(\mu) \equiv \frac{1}{3!}\lambda_{abcd}(\mu)\nu_a\nu_b\nu_c\nu_d, \quad \lambda_\chi(\tilde\mu) = 0\right)$$

Including the one-loop correction the quantum effective potential can always be written

$$V_q(\chi) = \frac{\bar\beta}{4}\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)\chi^4, \qquad \begin{cases} \lambda_\chi(\tilde\mu) = 0 & \text{(flat direction)}, \\ \bar\beta \equiv \left[\mu\frac{d\lambda_\chi}{d\mu}\right]_{\mu=\tilde\mu} > 0 & \text{(minimum condition)}, \end{cases}$$

Radiative symmetry breaking (RSB) mechanism

[Coleman-Weinberg (1973)], *[Gildener, S. Weinberg (1976)]*

To illustrate this general result we consider the general matter Lagrangian density

$$\mathscr{L}^{\rm ns}_{\rm matter} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \frac{D_\mu\phi_a\, D^\mu\phi_a}{2} + \bar\psi_j i\not{D}\psi_j - \frac{1}{2}(Y^a_{ij}\psi_i\psi_j\phi_a + \text{h.c.}) - V_{\rm ns}(\phi),$$

with

$$V_{\rm ns}(\phi) = \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

In the RSB mechanism masses emerge radiatively: there is an energy $\tilde\mu$ at which $V_{\rm ns}$ develops a flat direction, $\phi_a = \nu_a\chi$, with $\nu_a\nu_a = 1$, and χ a single scalar field $\Longrightarrow$ RG-improved potential V along ν_a reads

$$V(\chi) = \frac{\lambda_\chi(\mu)}{4}\chi^4, \qquad (\lambda_\chi(\mu) \equiv \frac{1}{3!}\lambda_{abcd}(\mu)\nu_a\nu_b\nu_c\nu_d, \quad \lambda_\chi(\tilde\mu) = 0)$$

Including the one-loop correction the quantum effective potential can always be written

$$V_q(\chi) = \frac{\bar\beta}{4}\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)\chi^4, \qquad \begin{cases} \lambda_\chi(\tilde\mu) = 0 & \text{(flat direction)}, \\ \bar\beta \equiv \left[\mu\frac{d\lambda_\chi}{d\mu}\right]_{\mu=\tilde\mu} > 0 & \text{(minimum condition)}, \end{cases}$$

The fluctuations of χ around χ_0 have mass $m_\chi = \sqrt{\bar\beta}\,\chi_0$

$\chi_0 \neq 0$ can break global and/or local symmetries and generate the particle masses.

E.g. a term in $\mathscr{L}$ of the form $\mathscr{L}_{\chi h} = \lambda_{\chi h}(\tilde\mu)\chi^2|H|^2/2$ can contribute to electroweak (EW) symmetry breaking

Thermal effective potential and PT

$$V_{\rm eff}(\chi,T) = V_q(\chi) + \frac{T^4}{2\pi^2}\left(\sum_b n_b J_B(m_b^2(\chi)/T^2) - 2\sum_f J_F(m_f^2(\chi)/T^2)\right) + \Lambda_0$$

The thermal functions J_B and J_F are

$$J_B(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 - e^{-\sqrt{p^2+x}}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\log\left(\frac{x}{a_B}\right) + O(x^3),$$

$$J_F(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 + e^{-\sqrt{p^2+x}}\right) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\log\left(\frac{x}{a_F}\right) + O(x^3),$$

Thermal effective potential and PT

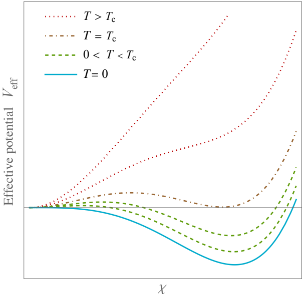
$$V_{\text{eff}}(\chi, T) = V_q(\chi) + \frac{T^4}{2\pi^2}\left(\sum_b n_b J_B(m_b^2(\chi)/T^2) - 2\sum_f J_F(m_f^2(\chi)/T^2)\right) + \Lambda_0$$

The thermal functions J_B and J_F are

$$J_B(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 - e^{-\sqrt{p^2+x}}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\log\left(\frac{x}{a_B}\right) + O(x^3),$$

$$J_F(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 + e^{-\sqrt{p^2+x}}\right) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\log\left(\frac{x}{a_F}\right) + O(x^3),$$

The PT associated with a RSB always turns out to be of first order! *[Salvio (2023)]*



Thermal effective potential and PT

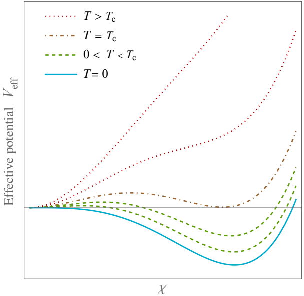
$$V_{\rm eff}(\chi,T) = V_q(\chi) + \frac{T^4}{2\pi^2}\left(\sum_b n_b J_B(m_b^2(\chi)/T^2) - 2\sum_f J_F(m_f^2(\chi)/T^2)\right) + \Lambda_0$$

The thermal functions J_B and J_F are

$$J_B(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 - e^{-\sqrt{p^2+x}}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\log\left(\frac{x}{a_B}\right) + O(x^3),$$

$$J_F(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 + e^{-\sqrt{p^2+x}}\right) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\log\left(\frac{x}{a_F}\right) + O(x^3),$$

The PT associated with a RSB always turns out to be of first order! *[Salvio (2023)]*



The decay rate per unit of spacetime volume, Γ, of the false vacuum into the true vacuum can be computed with the formalism of *[Coleman (1977); Callan, Coleman (1980); Linde (1981); Linde (1983)]*

Supercooling and model-independent approach

As long as perturbation theory holds, for all RSB theories, when $T < T_c$ the scalar field χ is trapped in the false vacuum $\langle\chi\rangle = 0$ until T is much below T_c, in other words the universe features a phase of supercooling *[Witten (1981); Salvio (2023)]*

Explanation: If the theory is scale invariant Γ must scale as T^4 and, therefore, the smaller T, the smaller Γ. At quantum level scale invariance is broken by perturbative loop corrections, which introduce another dependence of T in the bounce action. This dependence, however, is logarithmic and can become large only when T is very small compared to the other scale of the problem, χ_0.

Supercooling and model-independent approach

As long as perturbation theory holds, for all RSB theories, when $T < T_c$ the scalar field χ is trapped in the false vacuum $\langle\chi\rangle = 0$ until T is much below T_c, in other words the universe features a phase of supercooling *[Witten (1981); Salvio (2023)]*

Explanation: If the theory is scale invariant Γ must scale as T^4 and, therefore, the smaller T, the smaller Γ. At quantum level scale invariance is broken by perturbative loop corrections, which introduce another dependence of T in the bounce action. This dependence, however, is logarithmic and can become large only when T is very small compared to the other scale of the problem, χ_0.

Perturbativity: The one-loop effective potential is real and when $T \ll \chi_0$ the vacuum mass is much larger than the thermal mass so the problem discussed in *[Linde (1980)]* is absent in this case.

Supercooling and model-independent approach

As long as perturbation theory holds, for all RSB theories, when $T < T_c$ the scalar field χ is trapped in the false vacuum $\langle\chi\rangle = 0$ until T is much below T_c, in other words the universe features a phase of supercooling *[Witten (1981); Salvio (2023)]*

Explanation: If the theory is scale invariant Γ must scale as T^4 and, therefore, the smaller T, the smaller Γ. At quantum level scale invariance is broken by perturbative loop corrections, which introduce another dependence of T in the bounce action. This dependence, however, is logarithmic and can become large only when T is very small compared to the other scale of the problem, χ_0.

Perturbativity: The one-loop effective potential is real and when $T \ll \chi_0$ the vacuum mass is much larger than the thermal mass so the problem discussed in *[Linde (1980)]* is absent in this case.

If enough supercooling occured a **model-independent approach** is possible! *[Salvio (2023) I; Salvio (2023) II]*

The amount of supercooling needed is quantified by

$$\epsilon \equiv \frac{g^4}{6\bar{\beta}\log\frac{\chi_0}{T}},$$

with

$$g^2 \equiv \sum_b n_b m_b^2(\chi)/\chi^2 + \sum_f m_f^2(\chi)/\chi^2$$

Small ϵ case *[Salvio (2023) I]*

$$\bar{V}_{\rm eff}(\chi,T) \equiv V_{\rm eff}(\chi,T) - V_{\rm eff}(0,T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{\lambda(T)}{4}\chi^4$$

$$m^2(T) \equiv \frac{g^2T^2}{12}, \qquad \lambda(T) \equiv \bar{\beta}\log\frac{\chi_0}{T}$$

$$\Gamma \approx T^4\left(\frac{S_3}{2\pi T}\right)^{3/2}\exp(-S_3/T), \quad \text{with} \quad S_3 = -8\pi\int_0^\infty dr\, r^2 \bar{V}_{\rm eff}(\chi,T)$$

Corrections are easily computable in a small-ϵ expansion

Small ϵ case *[Salvio (2023) I]*

$$\bar{V}_{\text{eff}}(\chi, T) \equiv V_{\text{eff}}(\chi, T) - V_{\text{eff}}(0, T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{\lambda(T)}{4}\chi^4$$

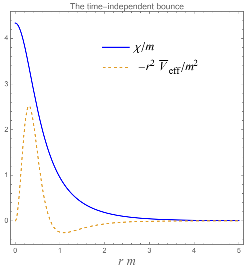
$$m^2(T) \equiv \frac{g^2 T^2}{12}, \qquad \lambda(T) \equiv \bar{\beta} \log \frac{\chi_0}{T}$$

$$\Gamma \approx T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} \exp(-S_3/T), \quad \text{with} \quad S_3 = -8\pi \int_0^\infty dr\, r^2 \bar{V}_{\text{eff}}(\chi, T)$$

where χ is the time-independent bounce configuration:

$$\chi'' + \frac{2}{r}\chi' = \frac{d\bar{V}_{\text{eff}}}{d\chi}, \qquad \chi'(0) = 0, \quad \lim_{r\to\infty} \chi(r) = 0$$

one finds $S_3 \approx c_3 \frac{m}{\lambda}$ with $c_3 = 18.9...$ and for $\lambda = 1$ $\longrightarrow$



Corrections are easily computable in a small-ϵ expansion

Small ϵ case *[Salvio (2023) I]*

$$\bar{V}_{\rm eff}(\chi,T) \equiv V_{\rm eff}(\chi,T) - V_{\rm eff}(0,T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{\lambda(T)}{4}\chi^4$$

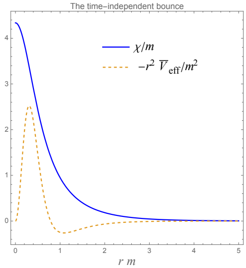
$$m^2(T) \equiv \frac{g^2T^2}{12}, \qquad \lambda(T) \equiv \bar{\beta}\log\frac{\chi_0}{T}$$

$$\Gamma \approx T^4\left(\frac{S_3}{2\pi T}\right)^{3/2}\exp(-S_3/T), \quad \text{with} \quad S_3 = -8\pi\int_0^\infty dr\, r^2\bar{V}_{\rm eff}(\chi,T)$$

where χ is the time-independent bounce configuration:

$$\chi'' + \frac{2}{r}\chi' = \frac{d\bar{V}_{\rm eff}}{d\chi}, \qquad \chi'(0) = 0, \quad \lim_{r\to\infty}\chi(r) = 0$$

one finds $S_3 \approx c_3\frac{m}{\lambda}$ with $c_3 = 18.9...$ and for $\lambda = 1$ $\longrightarrow$



The nucleation temperature defined as the solution of $\Gamma = H_I$ is

$$T_n \approx \chi_0\exp\left(\frac{\sqrt{c^2-16a}-c}{8}\right), \quad \text{with} \quad a \equiv \frac{c_3 g}{\sqrt{12\bar{\beta}}}, \quad c \equiv 4\log\frac{4\sqrt{3}\bar{M}_{\rm Pl}}{\sqrt{\bar{\beta}}\,\chi_0}$$

One always has a very strong PT and a small inverse duration β: $\quad \frac{\beta}{H_n} \approx \frac{a}{\log^2(\chi_0/T_n)} - 4$

Corrections are easily computable in a small-ϵ expansion

$\epsilon \sim 1$

$\epsilon \sim 1$. Simple case: several d.o.f. with dominant couplings to χ

The formulæ we have seen in the small ϵ case still hold

$\epsilon \sim 1$. General case *[Salvio (2023) II]*

$$\bar{V}_{\rm eff}(\chi, T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{k(T)}{3}\chi^3 - \frac{\lambda(T)}{4}\chi^4, \quad \text{with} \quad k(T) \equiv \frac{\tilde{g}^3 T}{4\pi}$$

and

$$\tilde{g}^3 \equiv \sum_b n_b m_b^3(\chi)/\chi^3$$

$\epsilon \sim 1$. **General case** *[Salvio (2023) II]*

$$\bar{V}_{\rm eff}(\chi, T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{k(T)}{3}\chi^3 - \frac{\lambda(T)}{4}\chi^4, \quad \text{with} \quad k(T) \equiv \frac{\tilde{g}^3 T}{4\pi}$$

and

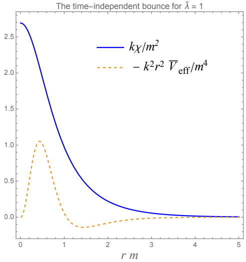
$$\tilde{g}^3 \equiv \sum_b n_b m_b^3(\chi)/\chi^3$$

The relation between Γ and S_3 we have seen still holds, but

$$S_3 = -\frac{8\pi m^3}{k^2}\int_0^\infty d\rho\, \rho^2 \left(\frac{1}{2}\varphi^2 - \frac{1}{3}\varphi^3 - \frac{\tilde{\lambda}}{4}\varphi^4\right)$$

where

$$\varphi \equiv \frac{k\chi}{m^2} \quad \text{and} \quad \tilde{\lambda} \equiv \frac{\lambda m^2}{k^2} > 0$$



$\epsilon \sim 1$. General case *[Salvio (2023) II]*

$$\bar{V}_{\rm eff}(\chi, T) \approx \frac{m^2(T)}{2}\chi^2 - \frac{k(T)}{3}\chi^3 - \frac{\lambda(T)}{4}\chi^4, \quad \text{with} \quad k(T) \equiv \frac{\tilde{g}^3 T}{4\pi}$$

and

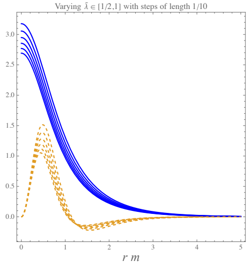
$$\tilde{g}^3 \equiv \sum_b n_b m_b^3(\chi)/\chi^3$$

The relation between Γ and S_3 we have seen still holds, but

$$S_3 = -\frac{8\pi m^3}{k^2}\int_0^\infty d\rho\, \rho^2 \left(\frac{1}{2}\varphi^2 - \frac{1}{3}\varphi^3 - \frac{\tilde{\lambda}}{4}\varphi^4\right)$$

where

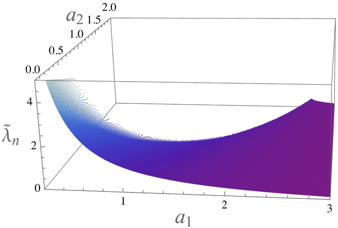
$$\varphi \equiv \frac{k\chi}{m^2} \quad \text{and} \quad \tilde{\lambda} \equiv \frac{\lambda m^2}{k^2} > 0$$

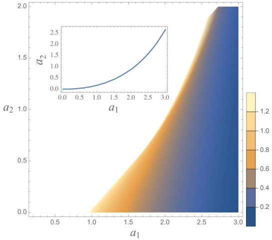


$\epsilon \sim 1$. General case: nucleation temperature T_n

T_n can be numerically computed once and for all as the solution $\tilde{\lambda}_n$ of

$$a_1 - a_2\tilde{\lambda} = F(\tilde{\lambda}) \equiv \frac{1 + \exp(-1/\sqrt{\tilde{\lambda}})}{2/9 + \tilde{\lambda}}, \quad \text{where} \quad a_1 \equiv \frac{c\, c_3 k^2}{3\pi a\, \bar{\beta}\, m^2}, \qquad a_2 \equiv \frac{4 c_3 k^4}{3\pi a\, \bar{\beta}^2 m^4}$$





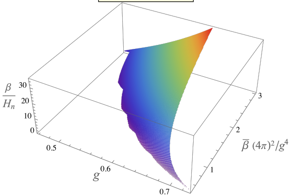
The inset in the right plot gives the maximal value of a_2 for a given a_1 such that $\tilde{\lambda}_n$ exists

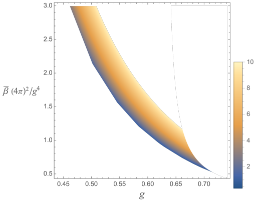
$\epsilon \sim 1$. General case: inverse duration β.

$$\frac{\beta}{H_n} \approx \frac{\pi^3 g^5}{6\sqrt{3}\tilde{g}^8} \frac{(4\pi)^2 \bar{\beta}}{\tilde{g}^4} (-F'(\tilde{\lambda}_n)) - 4$$

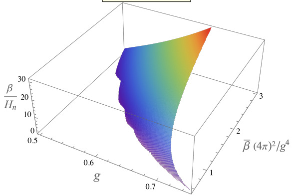
$\epsilon \sim 1$. General case: inverse duration β. Imposing $\tilde{g} = g$ and $\epsilon < 3$

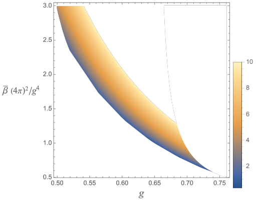
$\chi_0 = 10^{-1}$ GeV



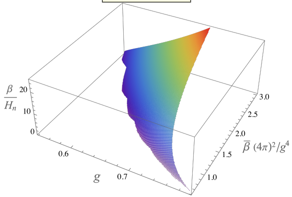


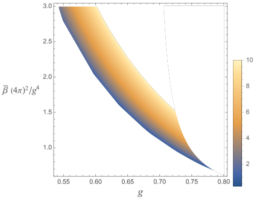
$\chi_0 = 10^{1}$ GeV



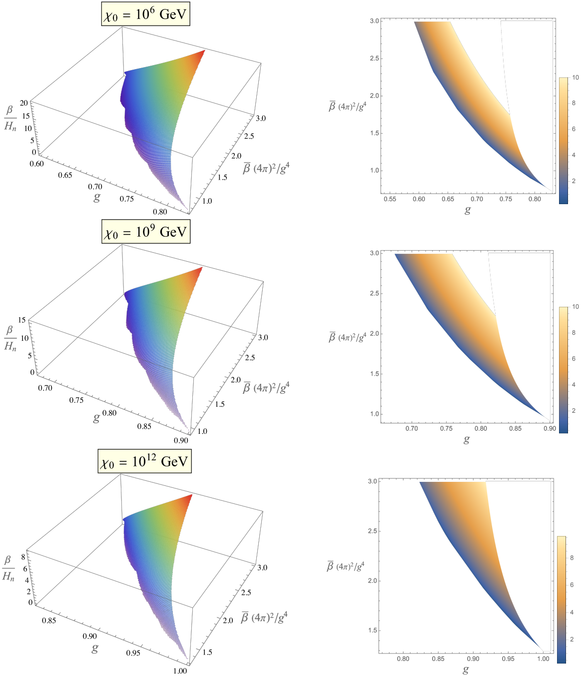


$\chi_0 = 10^{4}$ GeV





$\epsilon \sim 1$. General case: inverse duration β. Imposing $\tilde{g} = g$ and $\epsilon < 3$

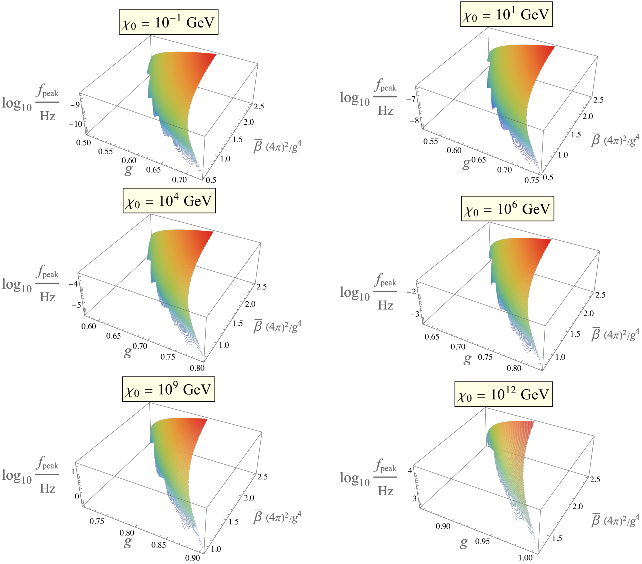


Gravitational waves

$$h^2\Omega_{\rm GW}(f) \approx 1.29\times10^{-6}\left(\frac{H_r}{\beta}\right)^2\left(\frac{100}{g_*(T_r)}\right)^{1/3}\frac{3.8(f/f_{\rm peak})^{2.8}}{1+2.8(f/f_{\rm peak})^{3.8}}$$

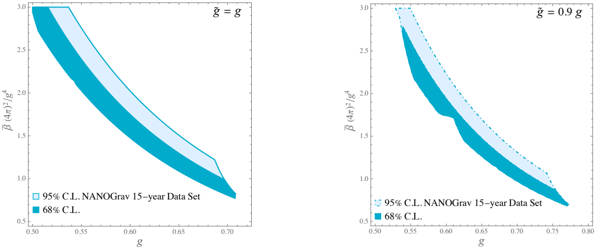
$$f_{\rm peak} \approx 3.79\,\frac{\beta}{H_r}\left(\frac{g_*(T_r)}{100}\right)^{1/6}\frac{T_r}{10^8{\rm GeV}}\,{\rm Hz}$$

Gravitational waves: peak frequency

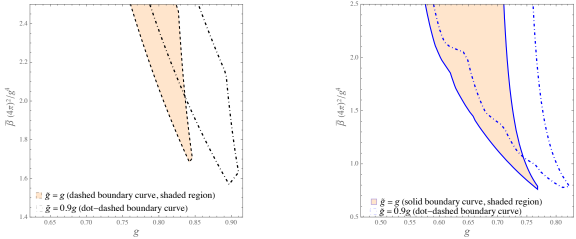


The peak frequency as a function of g and $\bar{\beta}$ in the case of fast reheating and fixing $g_*(T_r) = 110$. Also, $\tilde{g} = g$ and $\epsilon < 3$ has been imposed.

Gravitational waves: comparison with experiments



Regions corresponding to the GW background detected by pulsar timing arrays. In both plots $\chi_0 = 10$ GeV, $g_*(T_r) = 110$ and fast reheating is assumed. Here $\epsilon < 3$ has been imposed.



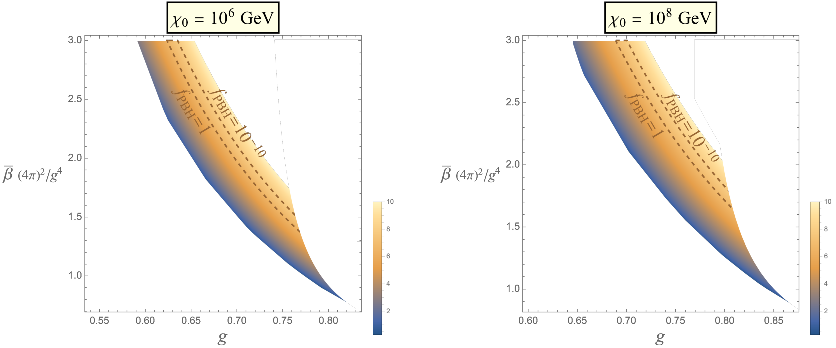
Regions where $\Omega_{\rm GW}(f_{\rm peak})$ is above the sensitivities of LIGO-VIRGO O3 (left plot, where $\chi_0 = 2 \times 10^9$ GeV) and LISA (right plot, where $\chi_0 = 10^4$ GeV). In both plots $g_*(T_r) = 110$ and fast reheating is assumed. Here $\epsilon < 3$ has been imposed.

Primordial black holes

Late-blooming mechanism: Since the bubble formation process is statistical for both quantum and thermal reasons, distinct causal patches percolate at different times. Patches that percolate the latest undergo the longest vacuum-dominated stage and, therefore, develop large over-densities triggering their collapse into PBHs (see e.g. *[Gouttenoire, Volansky (2023)]*)

Primordial black holes

Late-blooming mechanism: Since the bubble formation process is statistical for both quantum and thermal reasons, distinct causal patches percolate at different times. Patches that percolate the latest undergo the longest vacuum-dominated stage and, therefore, develop large over-densities triggering their collapse into PBHs (see e.g. *[Gouttenoire, Volansky (2023)]*)



Density plots giving the values of β/H_n varying g and $\bar{\beta}$. On the lower dashed line the whole dark matter is due to PBHs generated through the late-blooming mechanism ($f_{\rm PBH} = 1$); the upper dashed line corresponds instead to $f_{\rm PBH} = 10^{-10}$. Here $\tilde{g} = g$ and $\epsilon < 3$ has been imposed.

Conclusions

- SM extensions or dark sectors with RSB (where symmetries are broken and masses are then generated radiatively) feature strong and long first-order PTs
- high predictivity

Conclusions

- SM extensions or dark sectors with RSB (where symmetries are broken and masses are then generated radiatively) feature strong and long first-order PTs
- high predictivity
- All theories with RSB lead to
  - observable GWs
  - PBHs that can account for a fraction or the entire dark matter

Thank you very much for your attention!