Modern Machine Learning **Tools for Unfolding**

[arXiv: soon!] Nathan Huetsch, Javier Mariño Villadamigo, Anja Butter, Theo Heimel, Tilman Plehn



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Traditionally:

Matrix-based unfolding

$$g(s) = \int R(s \mid t) f(t) \, \mathrm{d}t$$



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With neural networks:

- **ML-based** unfolding
 - Unbinned: advantageous if one wants to derive quantities from the unfolding observables Allows to unfold (and account for correlations in) many dimensions



$$r_i = \sum_j R_{ij} \cdot t_j$$





► (*)

Omnifold [1911.09107]

Distribution mapping

- Direct Diffusion [2311.17175]
- Schrödinger Bridge [2308.12351]

(*) These are not comprehensive lists. For a more extensive catalogue see for example the <u>HEP ML Living Review</u>

Several approaches

Conditional phase space sampling

- GANs [1912.00477]
- Latent Diffusion [2305.10399]
- Conditional Flow Matching [2305.10475]
- cINN [<u>2212.08674</u>, <u>2006.06685</u>]
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$x_0 \sim p_{\text{model}}(x_{\text{hard}})$

 $= v_{\theta}(x(t), t)$



 $x_1 \sim p_{\text{reco}}(x_{\text{reco}})$







• Connect x_0 and x_1 with a linear trajectory:



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 $x(t) = (1 - t)x_0 + tx_1$







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• Loss:
$$\mathscr{L}_{\text{DiDi}} = \left\langle [v_{\theta}((1-t)x_0 + tx_1, t) \right\rangle$$



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 $-(x_1 - x_0)]^2 \rangle_{t \sim \mathcal{U}([0,1]), (x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})}$







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Conditional Flow Matching (CFM)





 $= v_{\theta}(x(t), t \,|\, x_{\text{reco}})$





Conditional Flow Matching (CFM)



- Connect x_0 and ϵ with a linear trajectory:
- The NN is regressed to predict the velocity field:
- For sampling, solve ODE starting from ϵ :

• Loss:
$$\mathscr{L}_{\text{CFM}} = \Big\langle [v_{\theta}((1-t)x_0 + t\epsilon, t, x_0)] + t\epsilon \Big\rangle \Big\rangle$$



$$v_{\theta}(x(t), t \mid x_{\text{reco}})$$

 $\epsilon = z \sim p_{\text{latent}}(z)$

$$x(t) = (1-t)x_0 + t\epsilon$$

ty field: $v_{\theta}(x(t), t | x_{reco}) \approx \frac{dx(t)}{dt} = \epsilon - x_0$

$$x_0 = \epsilon + \int_1^0 v_{\theta}(x(t), t | x_{\text{reco}}) dt$$

 $x_{\text{reco}}) - (\epsilon - x_0)]^2 \rangle_{t \sim \mathcal{U}([0,1]), (x_0, x_{\text{reco}}) \sim p(x_{\text{hard}}, x_{\text{reco}}), \epsilon \sim \mathcal{N}(0,1)}$





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Conditional INN (cINN)



$x_0 \sim p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}})$



 $z \sim p_{\text{latent}}(z)$



• Bijective function between $p_{\text{latent}}(z)$ and p

 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z)$



hard
$$x_{reco}$$



$$z \sim p_{\text{latent}}(z)$$

$$p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}):$$

 $\left| \det \frac{\partial g_{\theta}(x_{\text{h}}, x_{\text{r}})}{\partial x_{\text{hard}}} \right| = p_{\text{lat.}}(z) \left| \det J_{g_{\theta}} \right|$



Bijective function between $p_{\text{latent}}(z)$ and p

 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z)$

> Pairs (x_{hard}, x_{reco}) are passed through the NN to the latent space:



$$S_{\theta}(x_{hard} | x_{reco})$$

$$= g_{\theta}^{-1}(z | x_{reco})$$



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$$\mathcal{P}_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}):$$

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 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$



Bijective function

between
$$p_{\text{latent}}(z)$$
 and $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}})$:
 $p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) = p_{\text{latent}}(z) \left| \det \frac{\partial g_{\theta}(x_{\text{h}}, x_{\text{r}})}{\partial x_{\text{hard}}} \right| = p_{\text{lat.}}(z) \left| \det J_{g_{\theta}} \right|$

• Pairs (x_{hard}, x_{reco}) are passed through the NN to the latent space: • Once trained, one can sample -conditioned on reco- from the latent: $p_{hard}(x) \approx p_{model}(x_{hard} | x_{reco})$



hard
$$|x_{reco}\rangle$$

 $|(z|x_{reco})|$



 $z \sim p_{\text{latent}}(z)$

 $z = g_{\theta}(x_{\text{hard}} | x_{\text{reco}})$





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> Pairs (x_{hard}, x_{reco}) are passed through the NN to the latent space: $\mathscr{L}_{\text{cINN}} = -\langle \log p_{\text{model}}(x_{\text{hard}} | x_{\text{reco}}) \rangle_{(x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})}$ Loss:



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Z + jets events

$Z(p_T > 200 \text{ GeV})$ + jets events generated at $\sqrt{s} = 14 \text{ TeV}$ with Pythia 8.244 and Delphes simulation 3.5.0 available on Zenodo. Slight modification from [1911.09107] dataset

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Six widely-used jet substructure observables:

- Jet mass *m*
- Jet width *w*
- Jet constituents multiplicity N

 $Z(p_T > 200 \text{ GeV})$ + jets events generated at $\sqrt{s} = 14 \text{ TeV}$ with Pythia 8.244 and Delphes

• Groomed mass $\log \rho = 2 \log (m_{SD} / p_T)$

• Groomed momentum fraction $z_g = \tau_1^{\beta=1}$

N-subjettiness ratio $\tau_{21} = \tau_2^{\beta=1} / \tau_1^{\beta=1}$



Unfolded observables (DiDi)





Unfolded observables (CFM & cINN)



Matrix elements are evaluated at $\sqrt{s} = 13$ TeV using MadGraph_aMC@NLO. Showering and hadronization are simulated with Pythia8, and detector response is simulated with Delphes with the standard CMS card. For a detailed description see [2305.10399].

$$q\bar{q}/gg \rightarrow t$$

 $t\bar{t} \rightarrow (bl^+\nu_l)(bqq)$



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Unfolding from 6 final-state particles $(bl\nu)(bqq)$:

- 4 DoFs for the lepton
- > 3 DoFs for the missing \vec{p}^{ν}
- 5 DoFs per jet (4-momentum + b-tag)

hadronization are simulated with Pythia8, and detector response is simulated with Delphes with

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27 DoFs at reco-level



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 $t\bar{t} \rightarrow (bl^+\nu_l)(bqq)$

27 DoFs at reco-level

19 DoFs at parton-level





Much harder problem:

Unfolding to parton-level means inverting the entire forward simulation chain



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- ► Faithful modeling of complex correlations at parton-level, i.e., W boson and top mass



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Adding transformers:

correlations at reco and parton-level.

Tra-CFM as an extension to CFM [2310.07752]. A transformer is employed to encode



Results: naive parametrization



Results: mass parametrization



 $(m_t, p_{T,t}^L, \eta_t^L, \phi_t^L, m_W, \eta_W^T, \phi_W^T, (m_{d_1}^W), \eta_{d_1}^W, \phi_{d_1}^W)$

Originally introduced in [2308.00027]





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What is next?

Compare to other methods, single-event unfolding, model dependence, and more...



Thanks for your attention!





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- The NN is regressed to predict the velocity
- For

For sampling, solve ODE starting from
$$x_1$$
: $x_0 = x_1 + \int_1^0 v_\theta(x(t), t) dt$
Loss:
$$\mathscr{L}_{\text{DiDi-P}} = \left\langle \left[v_\theta((1-t)x_0 + tx_1, t) - (x_1 - x_0) \right]^2 \right\rangle_{t \sim \mathscr{U}([0,1]), (x_0, x_1) \sim p(x_{\text{hard}}, x_{\text{reco}})} \\ \mathscr{L}_{\text{DiDi-U}} = \left\langle \left[v_\theta((1-t)x_0 + tx_1, t) - (x_1 - x_0) \right]^2 \right\rangle_{t \sim \mathscr{U}([0,1]), x_0 \sim p(x_{\text{hard}}, x_1 \sim p(x_{\text{reco}}))} \right\rangle_{t \sim \mathscr{U}([0,1]), x_0 \sim p(x_{\text{hard}}, x_1 \sim p(x_{\text{reco}}))}$$



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Networks of ~3M parameters 19M training events and 1M validation events ~4M events for testing





The dataset



Optimal transport (DiDi)



N-subjettiness ratio τ_{21}







Optimal transport (CFM)





Optimal transport (cINN)











Single event unfolding





Single event unfolding





Single event unfolding





Calibration





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Non-bayesian networks

cINN ~ 8M parameters, CFM ~ 6M, Tra-CFM, Transfermer ~ 3M

10M training events and 1M testing events

hadronization are simulated with Pythia8, and detector response is simulated with Delphes with

Total: 27 DoFs at reco-level and 19 DoFs at parton-level



Adding transformers:

- dimensions and reco-level event: $p_{\text{model}}(x_{\text{part}} | x_{\text{reco}}) = \prod_{i=1}^{i} p_{\text{model}}$
- each different dimension:

 $v(x_{\text{part}}(t), t \mid x_{\text{reco}}) = ($

For transfermer, likelihoods are factorized autoregressively on all previous parton-level

$$x_{\text{podel}}^{(i)}(x_{\text{part}}^{(i)} \mid c(x_{\text{part}}^{(0)}, \dots, x_{\text{part}}^{(i-1)}, x_{\text{reco}}))$$

For Tra-CFM, the transformer is made time-dependent and a small CFM predicts velocities at

$$\left(v^{(1)}(c^{(1)},t), \dots, v^{(n)}(c^{(n)},t)\right)$$





Transfermer





Tra-CFM





