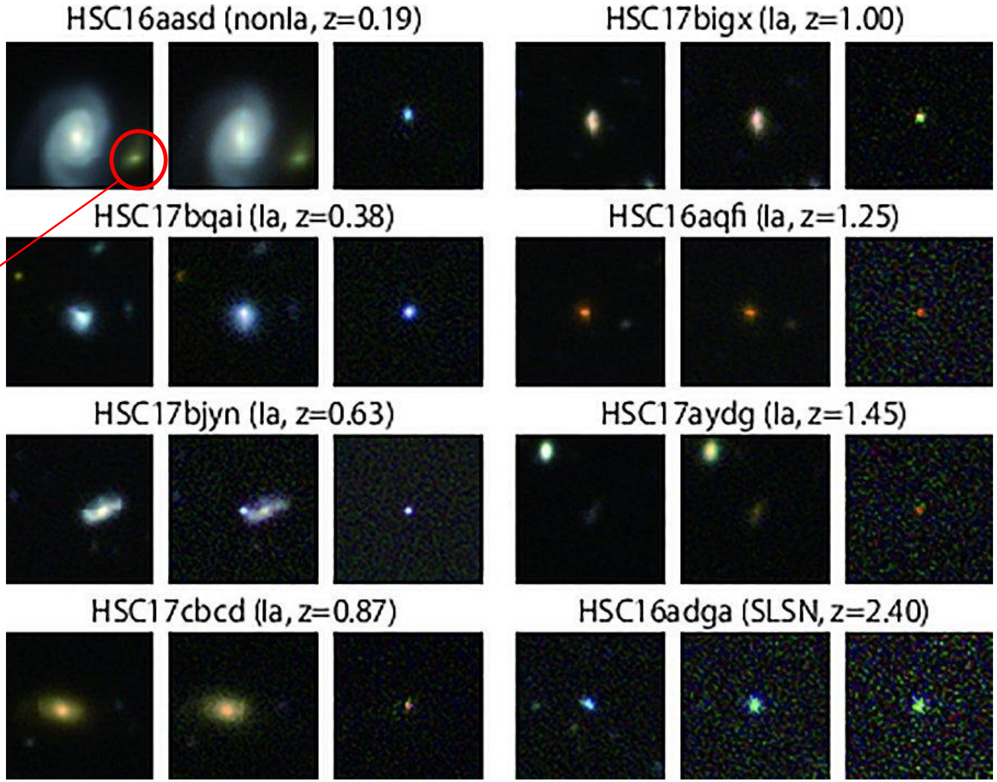
Estimating cosmological distances with incomplete supernovae surveys

Dylan KUHN, Congrès des Doctorants 2024

1- Context of my work

Type Ia supernovae

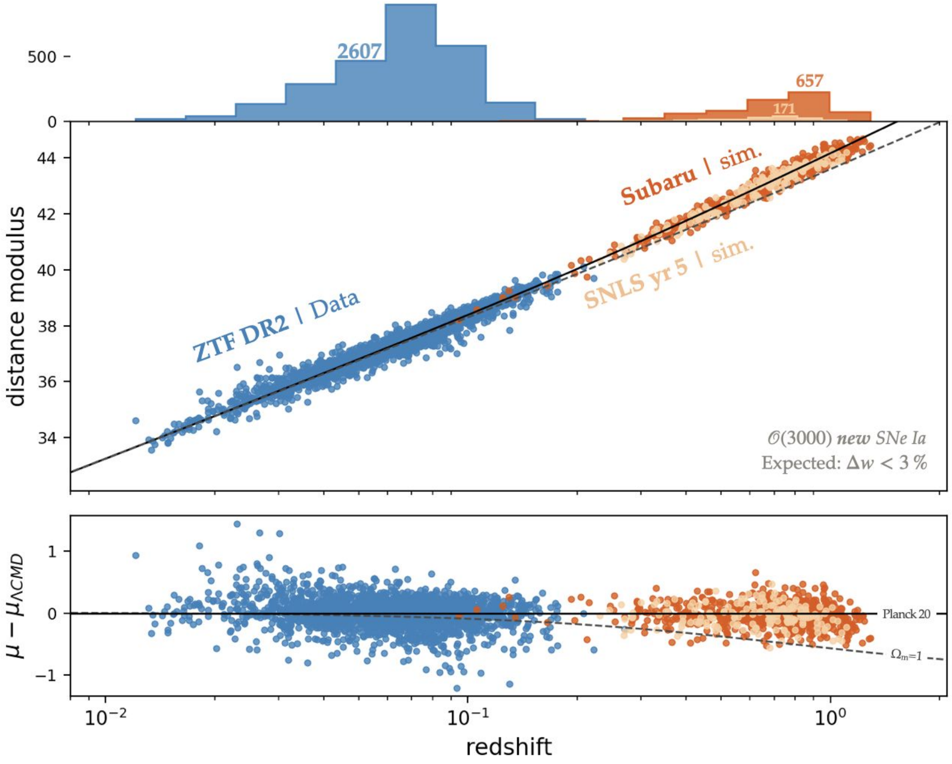
This is a supernova!



source: NAOJ, Japan

What we want to do: the "LEMAITRE (Hubble) diagram"

(theorized the expansion of the Universe)



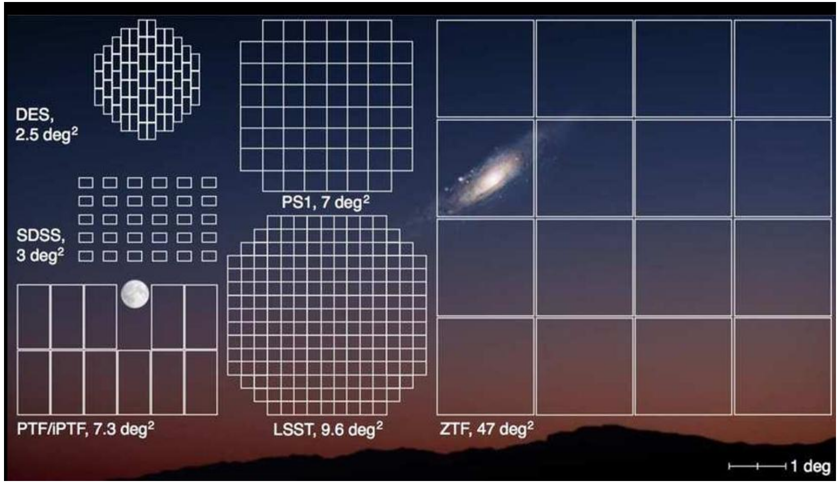
3 new sets of supernovae (ZTF, SNLS, HSC/Subaru):

Never been used in a cosmological analysis before!

source: Mickaël Rigault (IP2I, Lyon) & Nicolas Regnault (LPNHE, Paris)

ZTF (Zwicky Transient Facility)

Palomar P48 (ZTF observing system) is here



A few numbers...

- $47\ deg^2$ = 235 times the full Moon size!
- average of 1000 science exposures per observing night

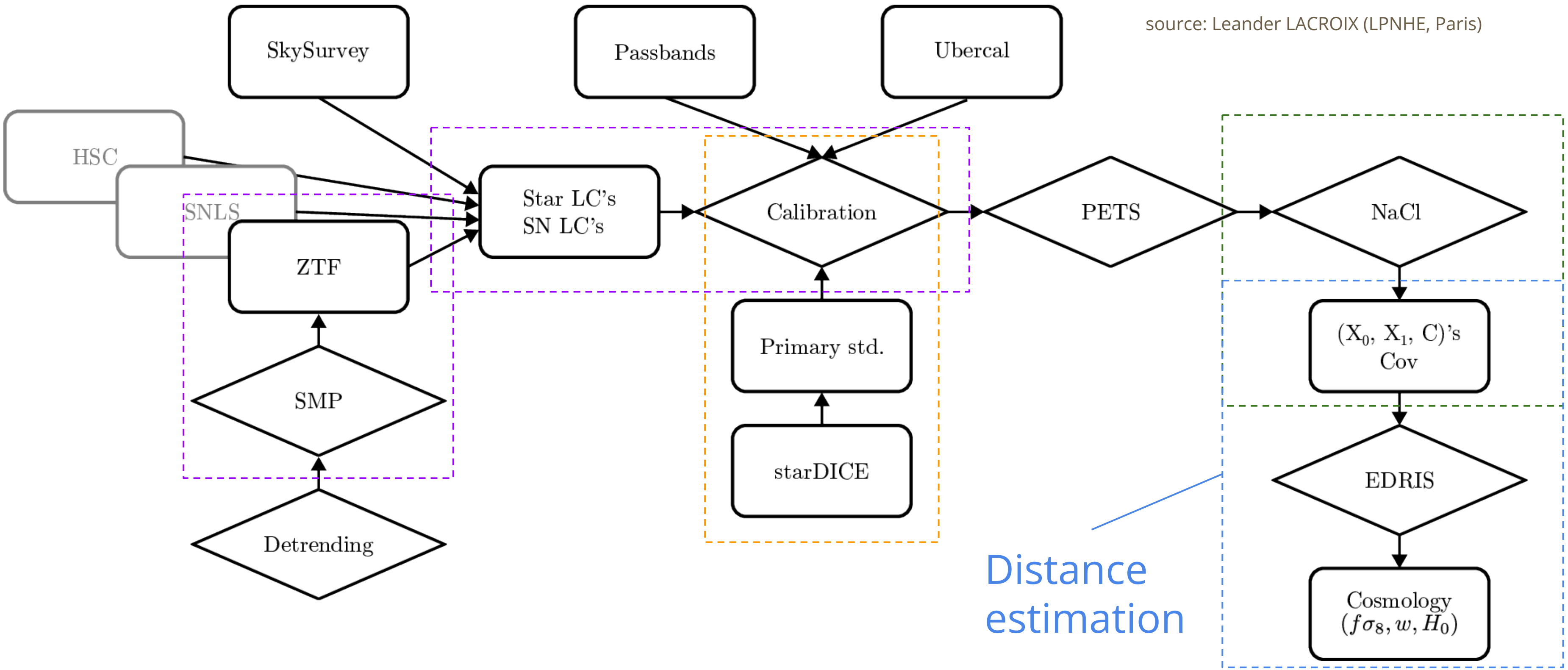
Subaru/HSC (Hyper-Suprime Cam)

Located on Mauna Kea (Hawaii)

- 8.2 m effective diameter
- high sensitivity to infrared light

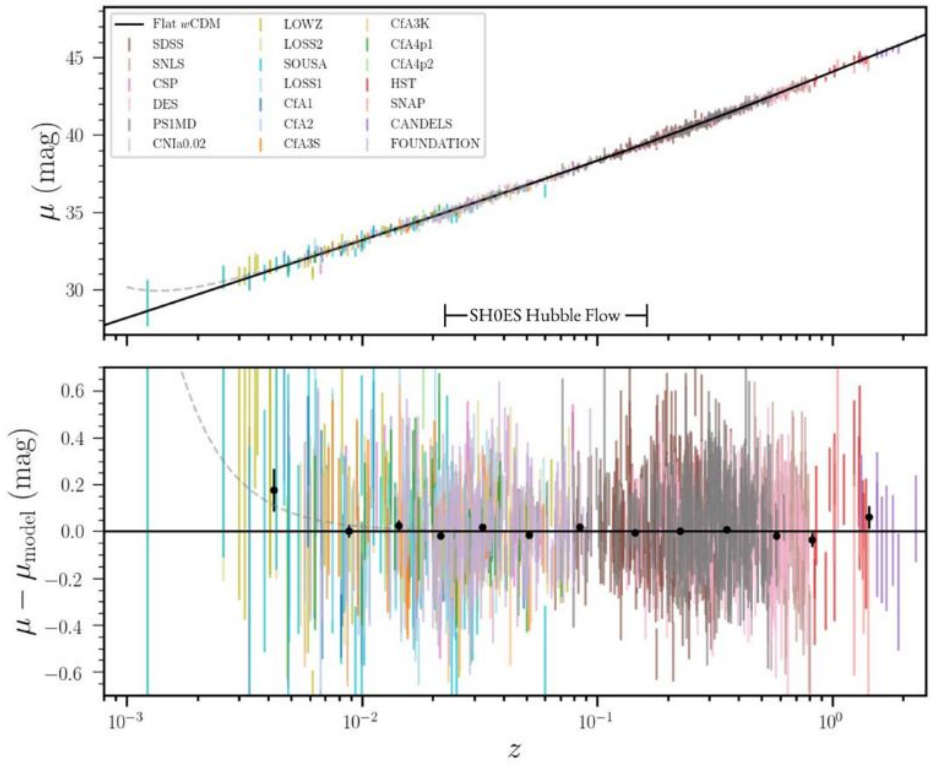
The LEMAITRE analysis pipeline

source: Leander LACROIX (LPNHE, Paris)



2- Issues when estimating distances

Huge increase of the statistics



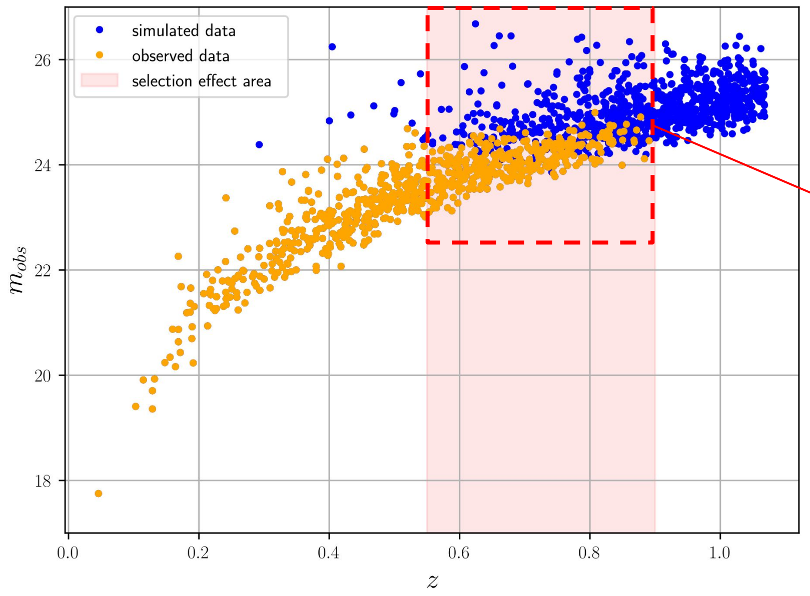
Pantheon+ (2022):
20 surveys, ~1500 SN

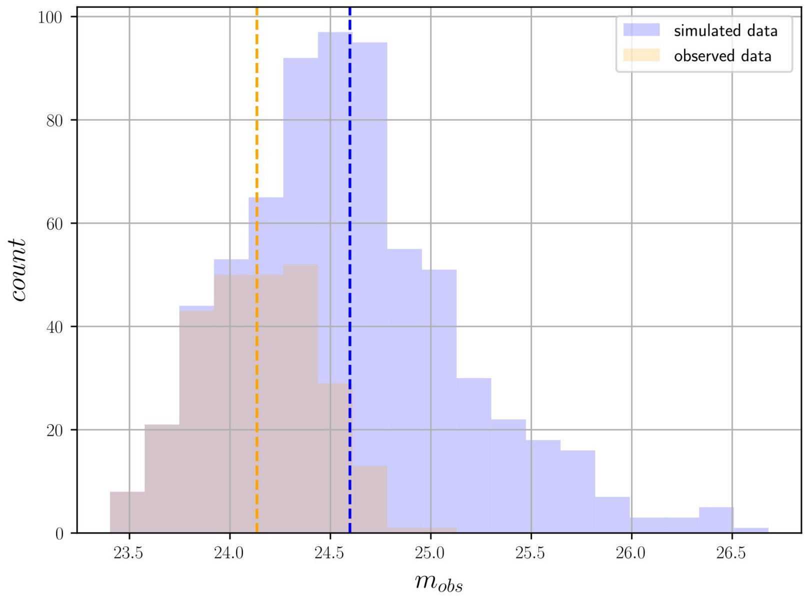
LEMAITRE (2024):
3 surveys, ~4000 SN

ZTF: ~3000 SN
SNLS: ~400 SN
HSC/Subaru: ~600 SN

source: The Pantheon+ collaboration, 2022

Instrumental selection bias: the “Malmquist bias”



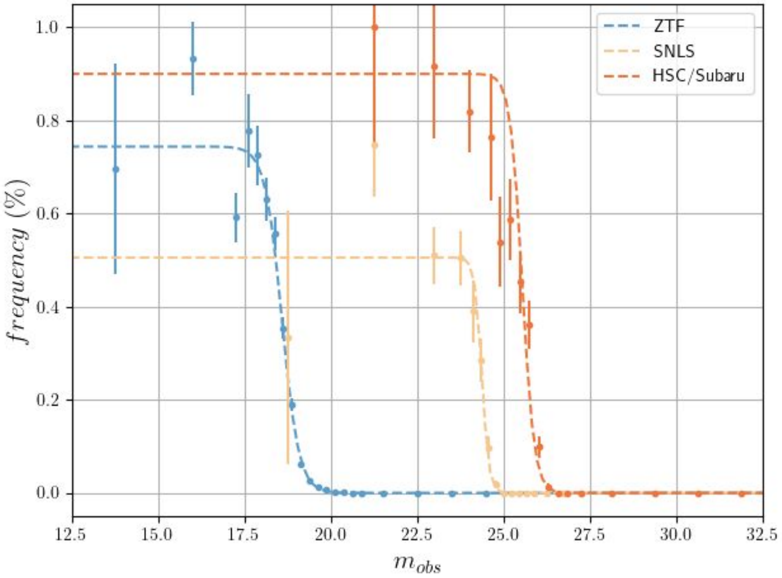


In practice, only the brightest supernovae are detected!

3- EDRIS: French for “Distance Estimator for Incomplete Supernovae Surveys”

Modeling the Malmquist bias (1): the model

Tripp model: $m_{obs,i}^{*} = M^{*} + \mu_i(z,\theta) + \alpha x_{1,i}^{*} + \beta c_i^{*} + \varepsilon_i \text{ with } \varepsilon_i \sim \mathcal{N}(0,\sigma^2)$



$m_{obs,i} = m_{obs,i}^{*} + \eta_i \text{ if } m_{obs,i}^{*} \leq m_{lim} + \kappa_i$

$\text{with } \eta_i \sim \mathcal{N}(0, C_i) \text{ and } \kappa_i \sim \mathcal{N}(0, \sigma_d^2)$

$m_{obs,i}$ *is unobserved otherwise*

One survey = one sigmoid

$$S(m_{obs}) = \frac{c}{1 + e^{\frac{m_{obs} - m_{lim}}{\sigma_d}}}$$

Modeling the Malmquist bias (2): the likelihood

classic likelihood for multivariate normal distributions

term that takes into account the truncation of data

$$\Gamma = -\ln(|W|) + r^t W r + \sum_i 2\ln\left(\Phi\left(\frac{m_{lim} - M^* - \mu_i - \alpha x_{1,i}{}^* - \beta c_i{}^*}{\sqrt{\sigma^2 + \sigma_d{}^2}}\right)\right) - 2\ln\left(\Phi\left(\frac{m_{lim} - m_{obs,i}}{\sqrt{\sigma_d{}^2 + f(C_i)}}\right)\right)$$

$$with\ \Phi(z) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{z}{\sqrt{2}}\right)\right)\ and\ r = m_{obs} - M^* - \mu - \alpha x_1 - \beta c$$

CDF of the normal distribution

Acceleration of the computation (1): some maths tricks

1- size (3N, 3N)
2- need to invert it at each step of the minimization

$$W=\begin{pmatrix} C_{mm}+\sigma^2 I_N & C_1 \\ C_1^t & C_2 \end{pmatrix}^{-1} \longrightarrow S^{-1}=Q\left(\Lambda+\sigma^2 I_N\right)^{-1}Q^t$$

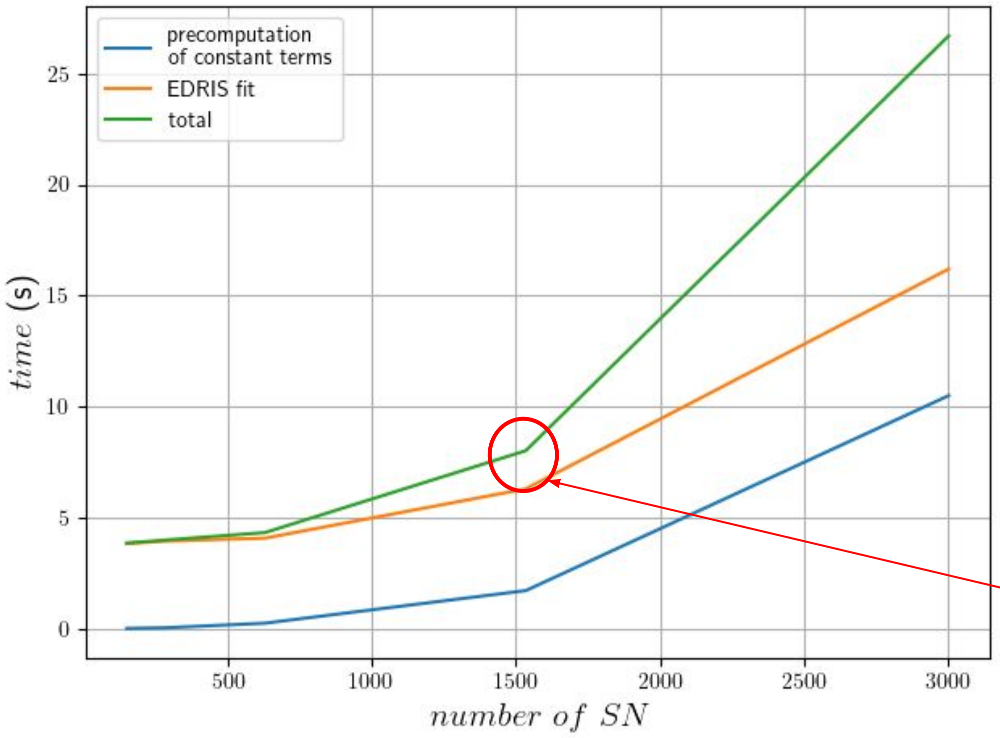
Schur complement of C_2 *in* $C=W^{-1}$

At the end, only matrix-to-vector products: computation in $O(N^2)$!

$$-\ln(|W|)=\ln(|C_2|)+\sum_i \ln\left(\Lambda_i+\sigma^2\right)$$

$$r=\begin{pmatrix} r_1 & r_2 \end{pmatrix} \longrightarrow r^tWr=r_1^tS^{-1}r_1-2r_1^tS^{-1}C_1C_2^{-1}r_2+r_2^tC_2^{-1}r_2+r_2^tC_2^{-1}C_1^tS^{-1}C_1C_2^{-1}r_2$$

Acceleration of the computation (2): time scaling



Computation of the likelihood in $O(N^2)$
+
auto-differentiation & hessian-free optimization with *JAX*

O(10s) for ~1500 SN very fast indeed!

3- Conclusion

Conclusion

1. Type Ia supernovae surveys are affected by a selection bias (Malmquist bias)

   Solution: take into account the selection in the likelihood

2. We multiply the current worldwide statistics by 3 (and more in a few years)

   Solution: use maths tricks and clever optimization methods

3. Robustness of the estimator not completely certified: stay tuned!

Thanks for your attention!