cuHARM : general relativistic radiation magneto-hydrodynamic simulations accelerated by GPUs

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Bar Ilan University

With
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Bar-Ilan University
אוניברסיטת בר־אילן

Layout of the presentation

- Physical motivations : why GRMHD ?

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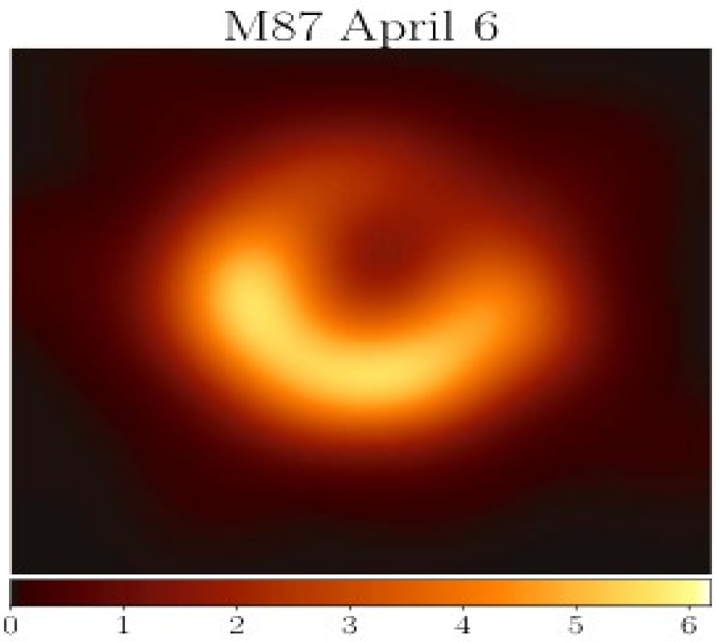
Physical motivations: accretion disks



The Event Horizon Telescope Collaboration (April 10, 2019).

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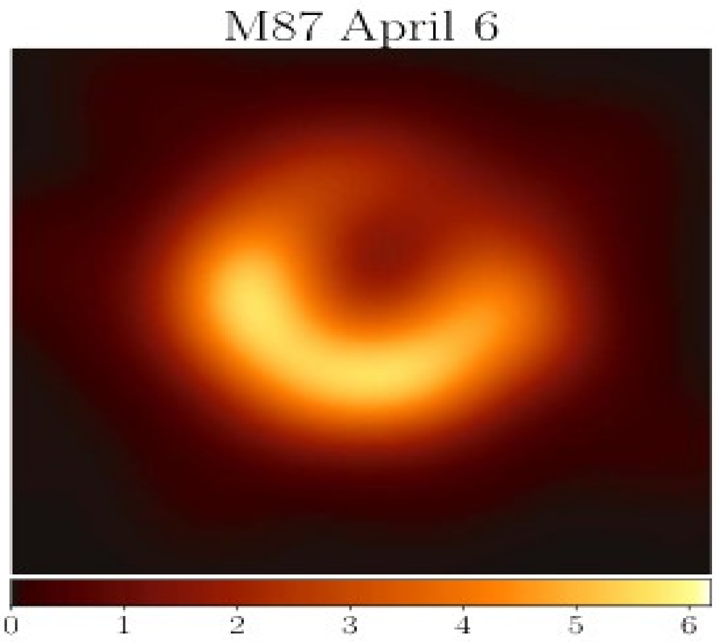

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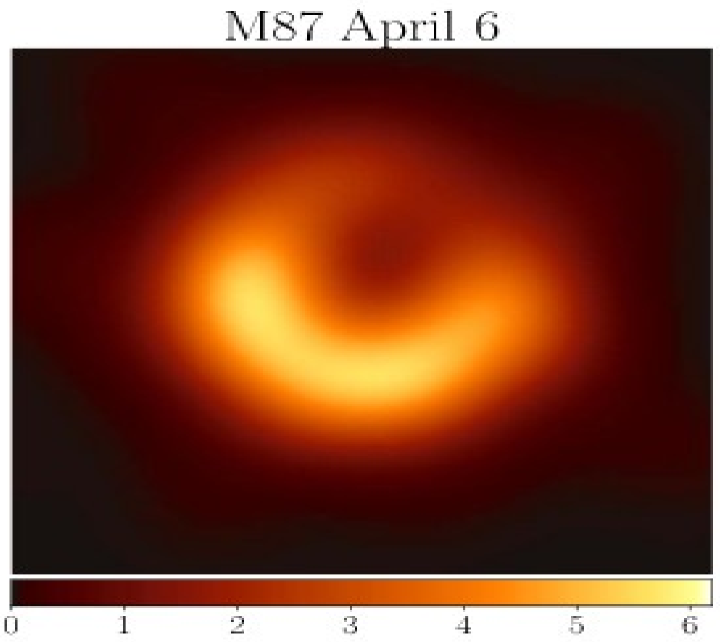

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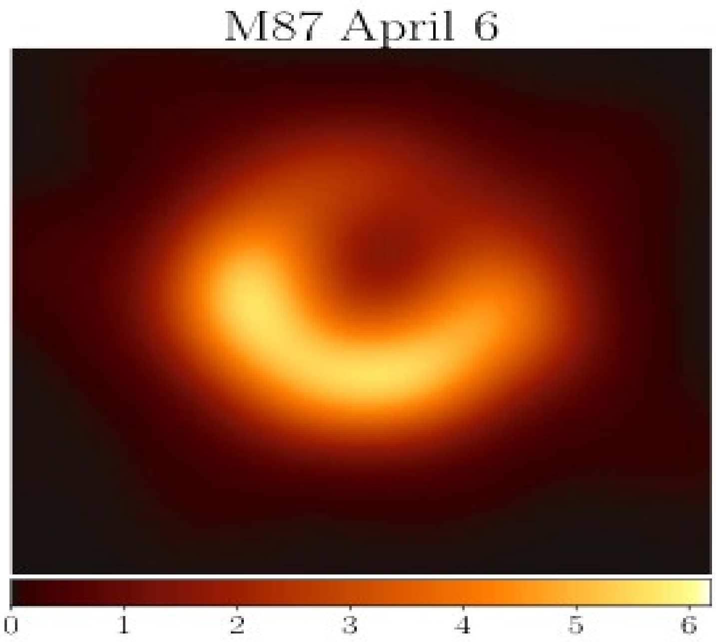

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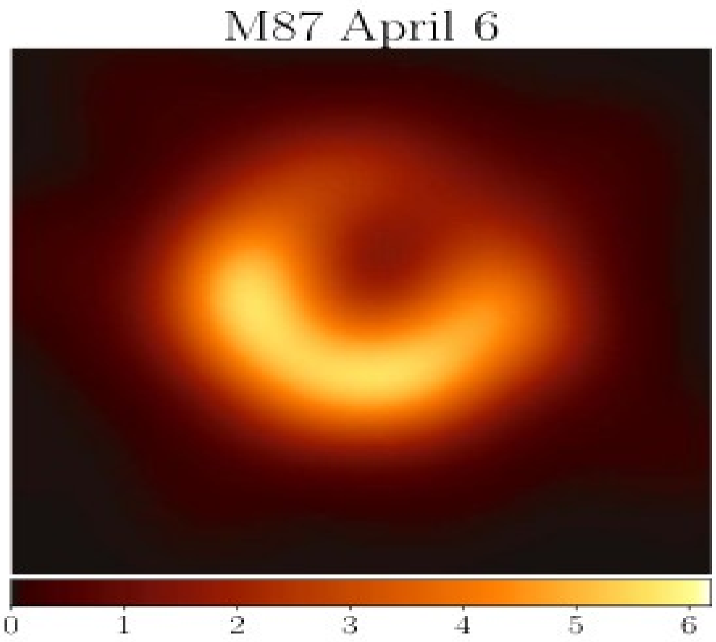

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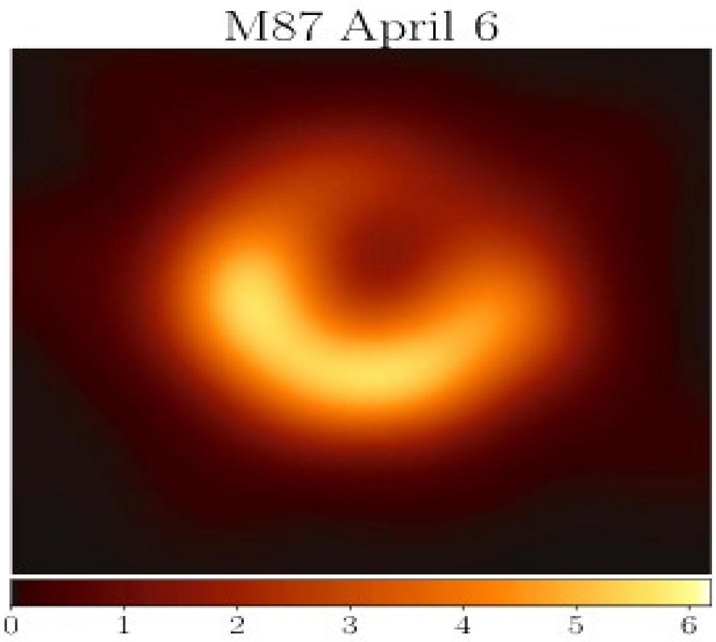

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- Emission mechanisms and properties.

Numerical GR MHD

Study the motion of magnetized plasma in the close vicinity of a black-hole

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Conservation of gas density

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\rho u^\mu\right) = 0 \quad (1)$$

Energy and momentum conservation

$$T^\mu{}_{\nu;\mu} = 0 \quad (2)$$

Maxwell Equations

$$\nabla_\mu({}^*F^{\mu\nu}) = 0 \quad (3)$$

Numerical motivations

Code specificities:

- Conservative, shock-capturing code
- Piece-wise linear / PPM reconstruction
- Constrained transport to maintain div B = 0

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We started from harm [harmpi] (Gammie et al 2003):

- Well tested
- Used by many groups
- Original experience with this code (see e.g. O’Riordan, **Pe’er**, McKinney 2016, 2017, 2018).

Numerical motivations: GPU acceleration

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→ Very good for GPUs

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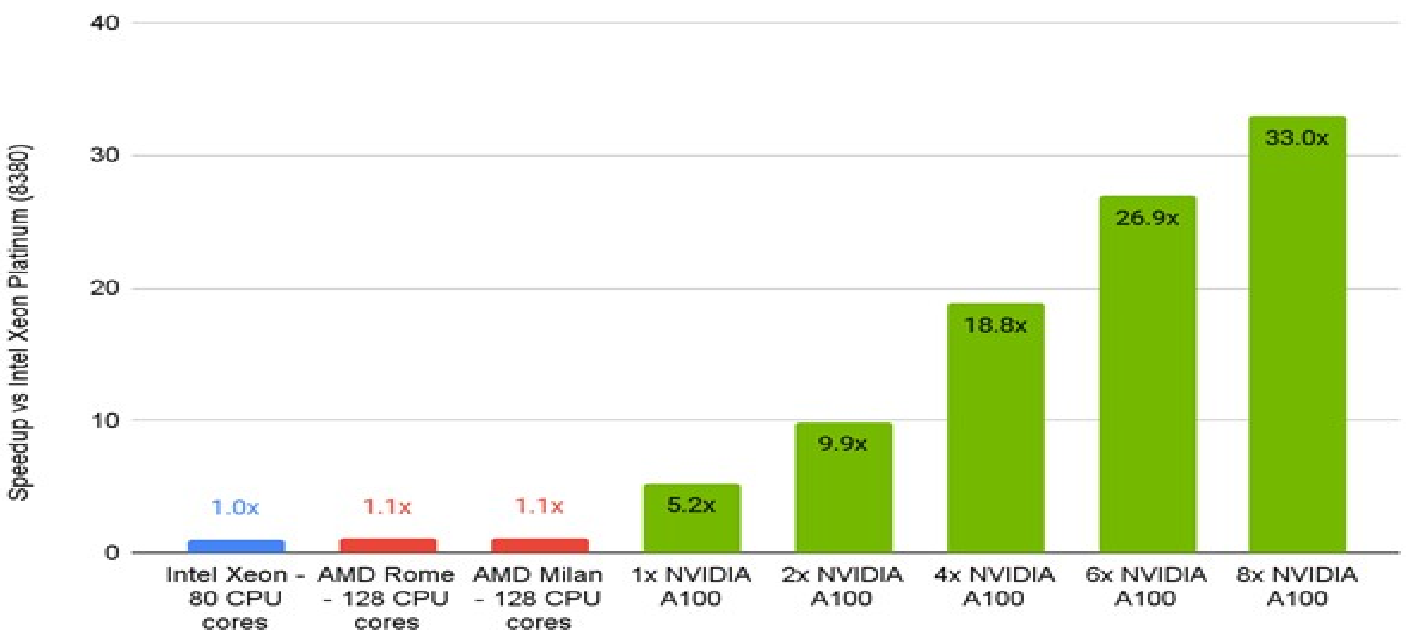

Figure 4. Performance of Ansys FLUENT 2022 beta1 for a 105M cell car model server vs. CPU-only servers.

May 26, 2022

https://developer.nvidia.com/blog/computational-fluid-dynamics-revolution-driven-by-gpu-acceleration/

cuHARM in a nutshell

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  - Finite volume
  - Flux-CT to preserve div B = 0

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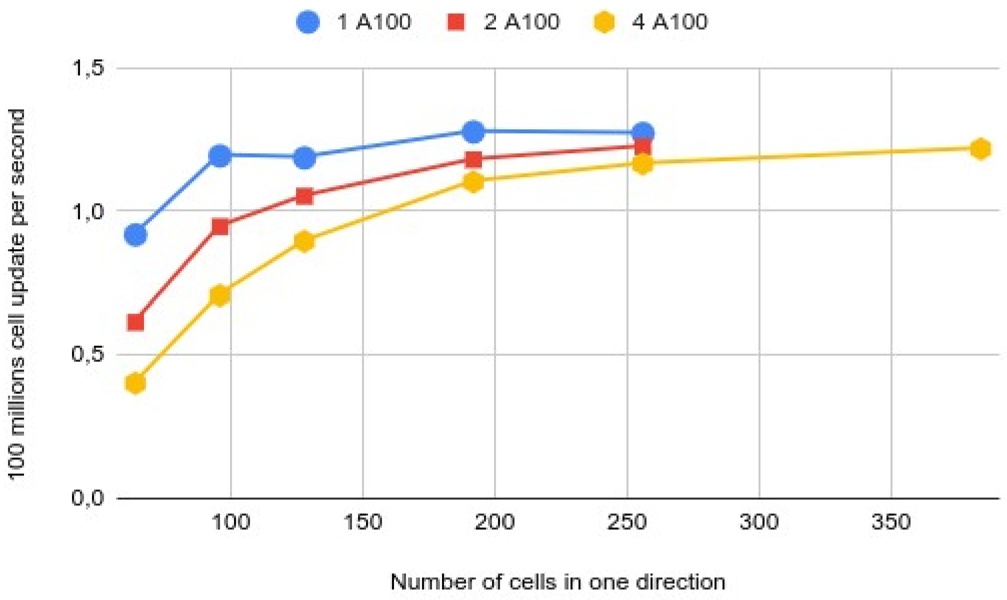
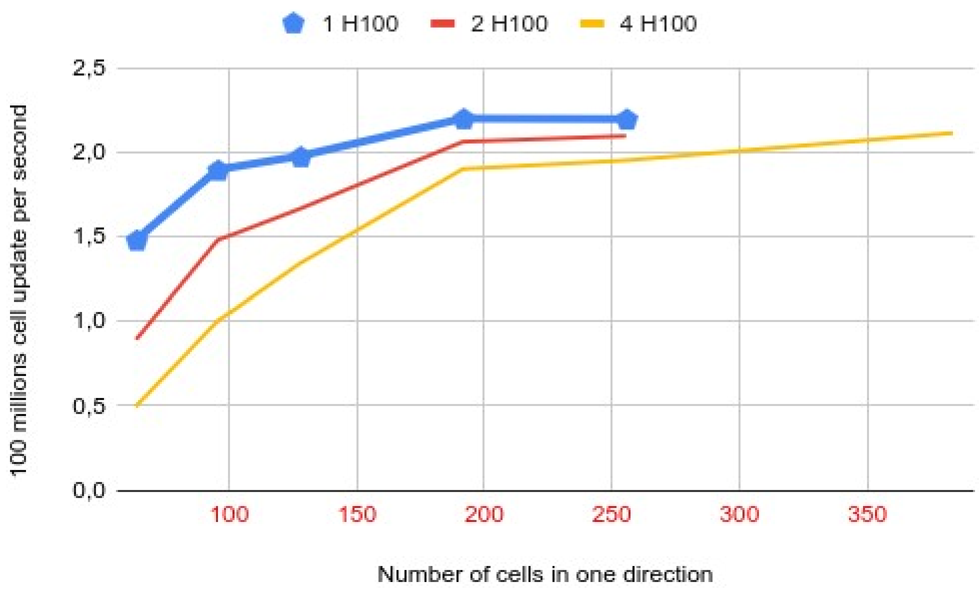
  - Two versions: highly optimized vs easily modifiable

cuHARM in a nutshell

A100



H100



Numerical study of SANE/MAD accretion disks

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Standard And Normal Evolution

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Why SANE ?

- Numerically simpler than MAD
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Why MAD ?

- More interesting phenomenology,
- Produces powerful jets,
- More relevant for EHT results,
- Role of magnetic fields not fully understood.

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Fishbone and Moncrief (1976)

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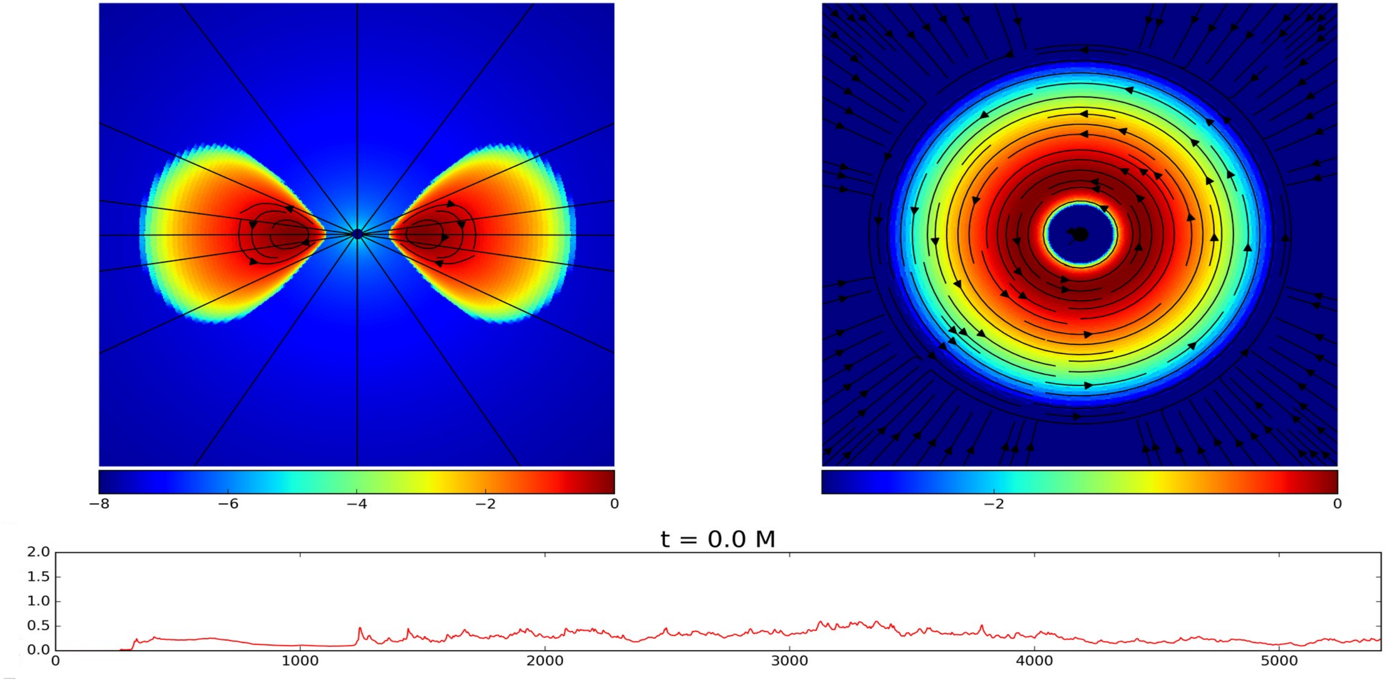

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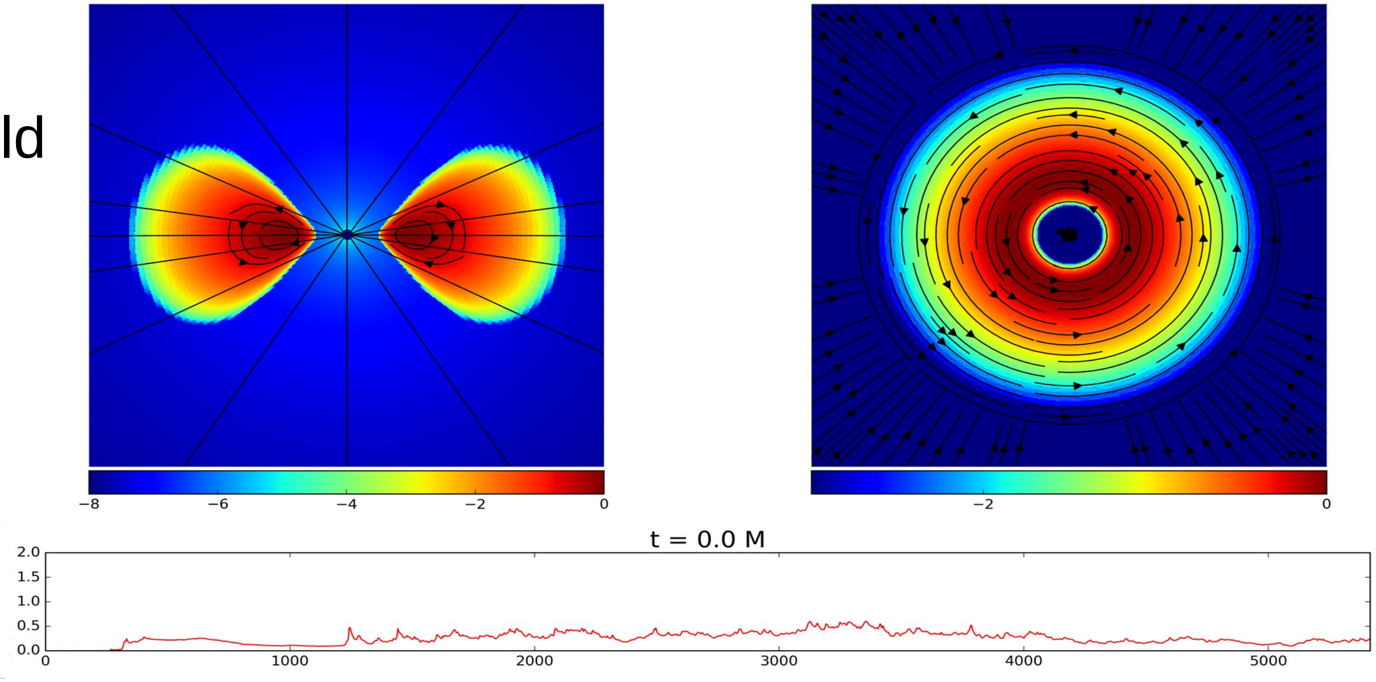
Polar cut
Toroidal field



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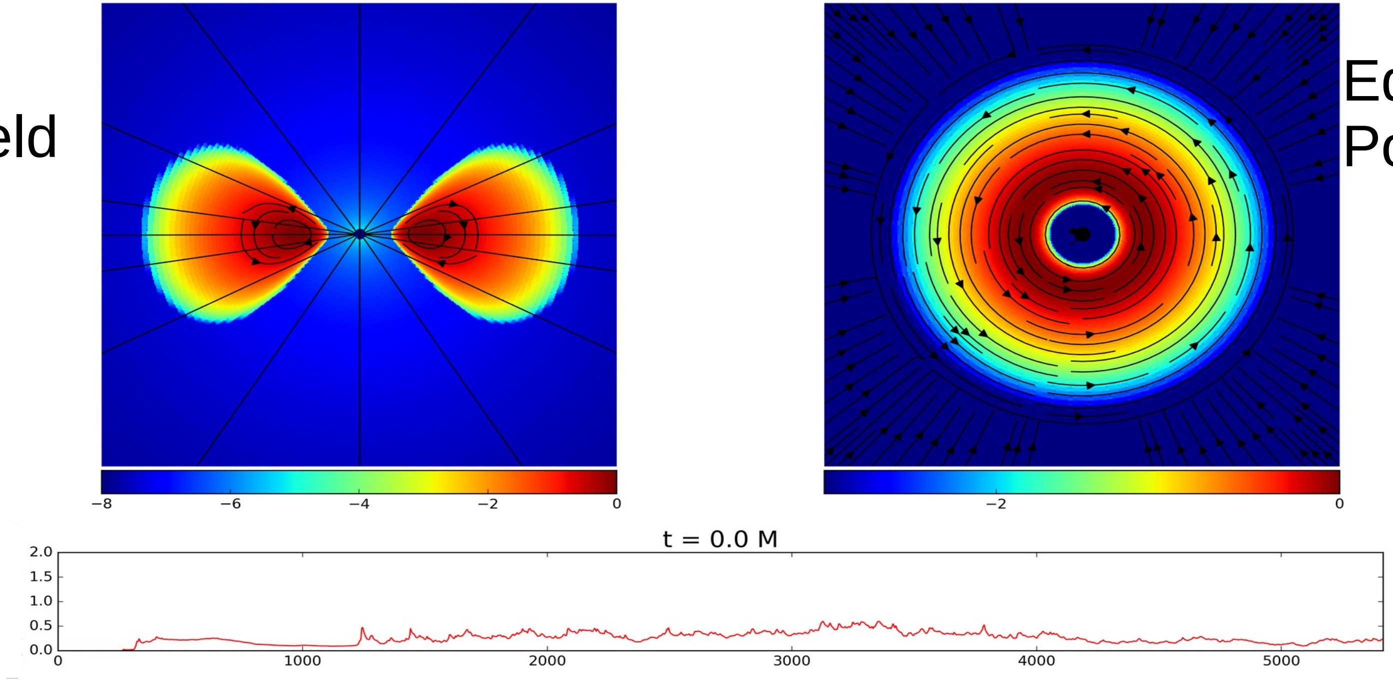

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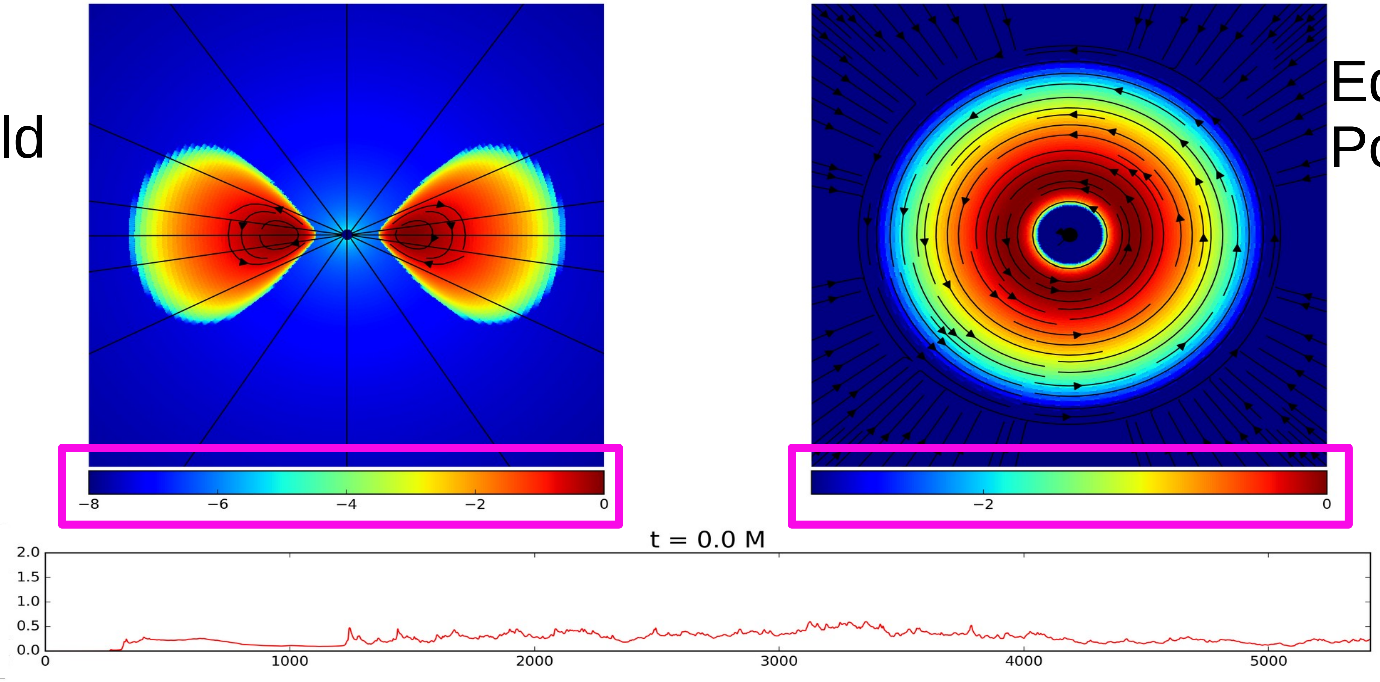

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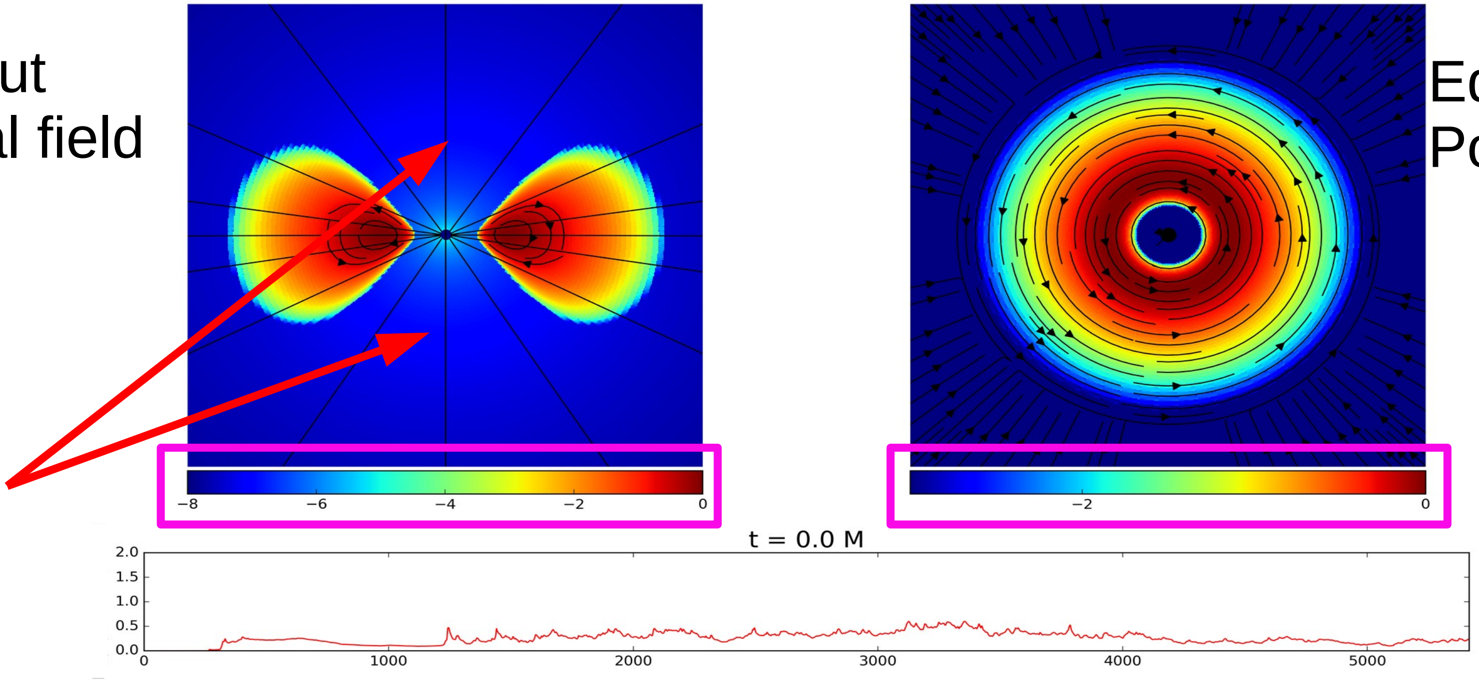

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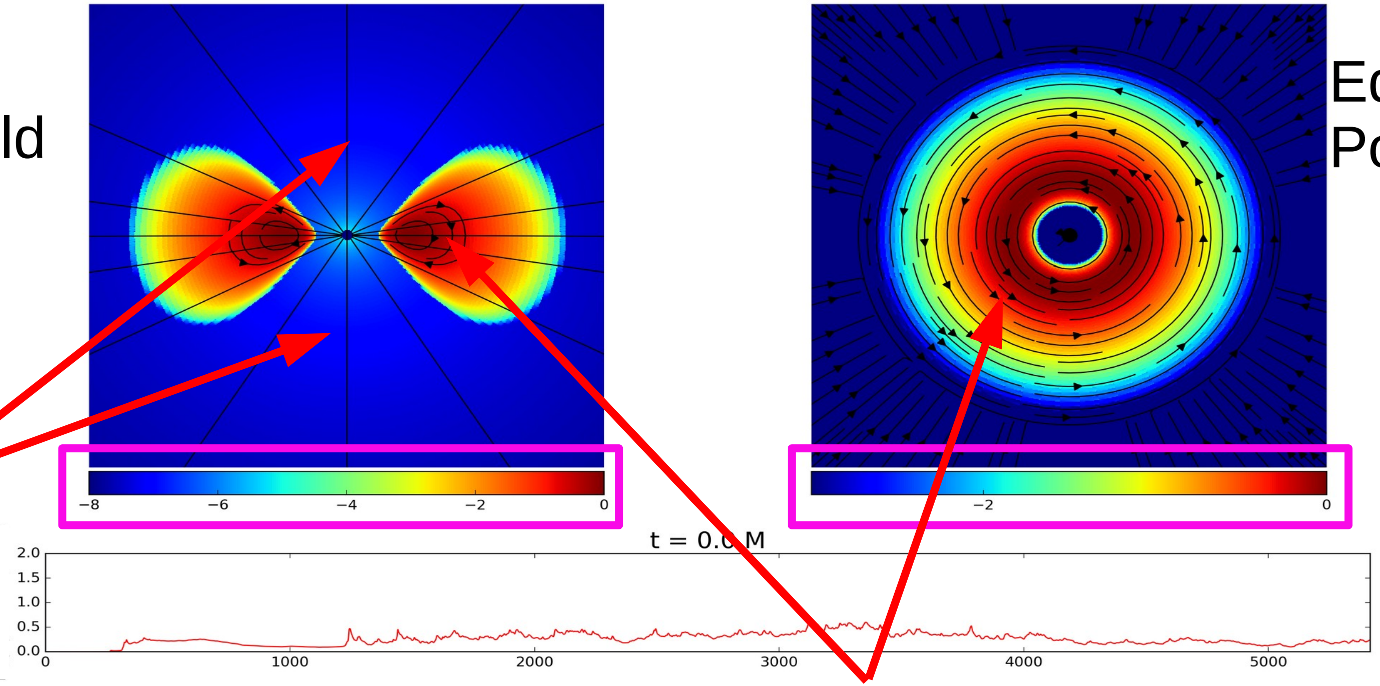

Jets

Turbulence

SANE accretion mode

The video shown on this slide can be found on youtube:

https://youtu.be/FBabQZNcyhE

MAD accretion mode

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https://youtu.be/PS2sjELxULs

MAD parameter

1. Mass accretion rate

$$\dot{M} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \sqrt{-g}\, \rho u^r d\theta d\phi$$

2. Magnetic flux threading the horizon

$$\varphi_B = \frac{1}{2} \int_{\theta=0}^{\pi} \int_{0}^{2\pi} \sqrt{-g} |{}^*F^{rt}| \, d\theta d\phi$$

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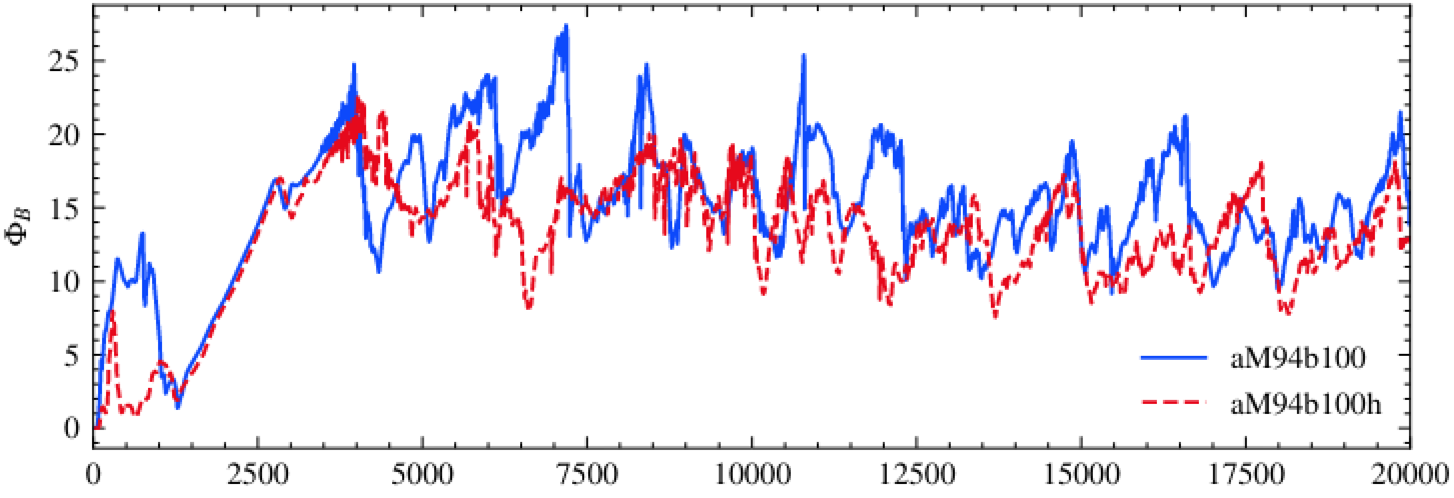

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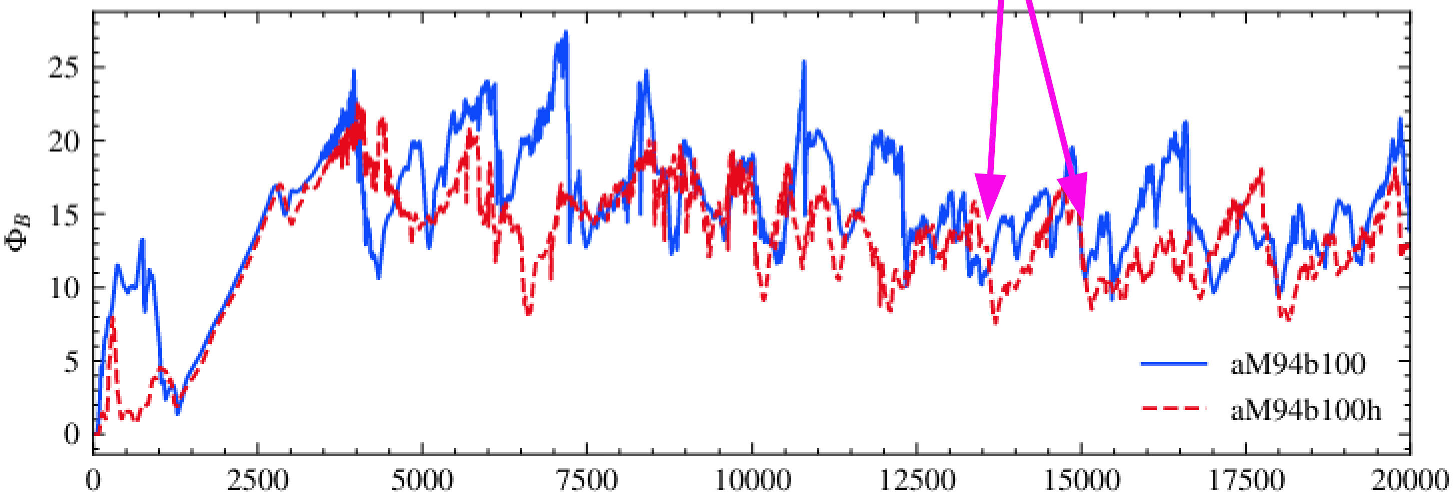

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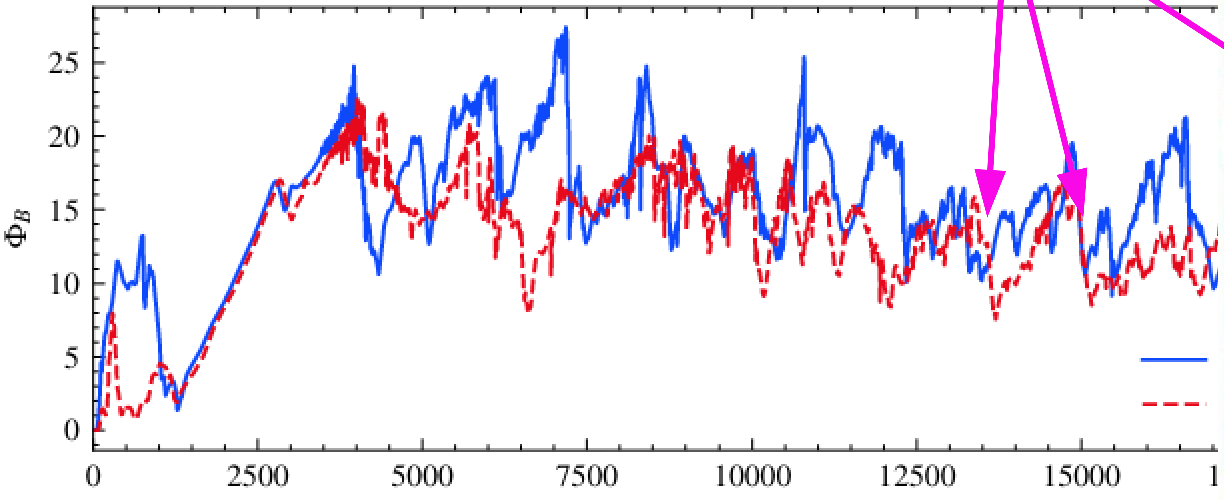
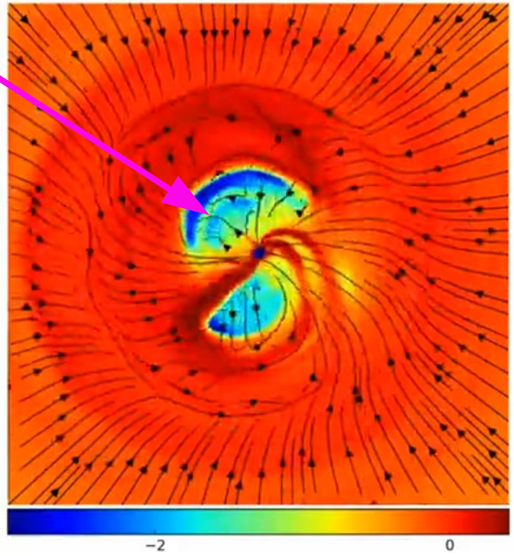
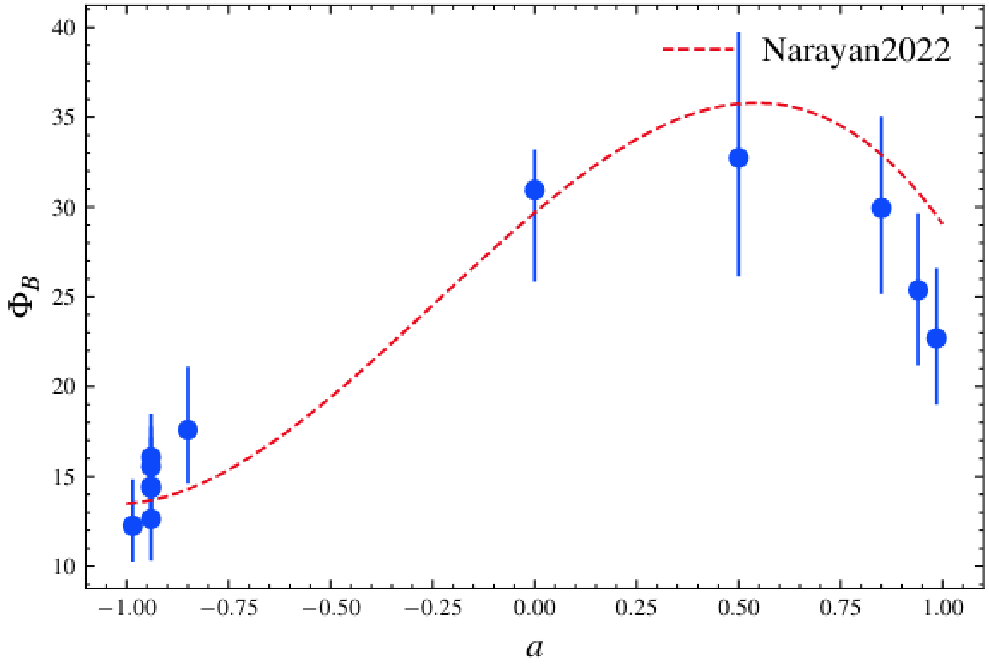




Spin dependence for MAD



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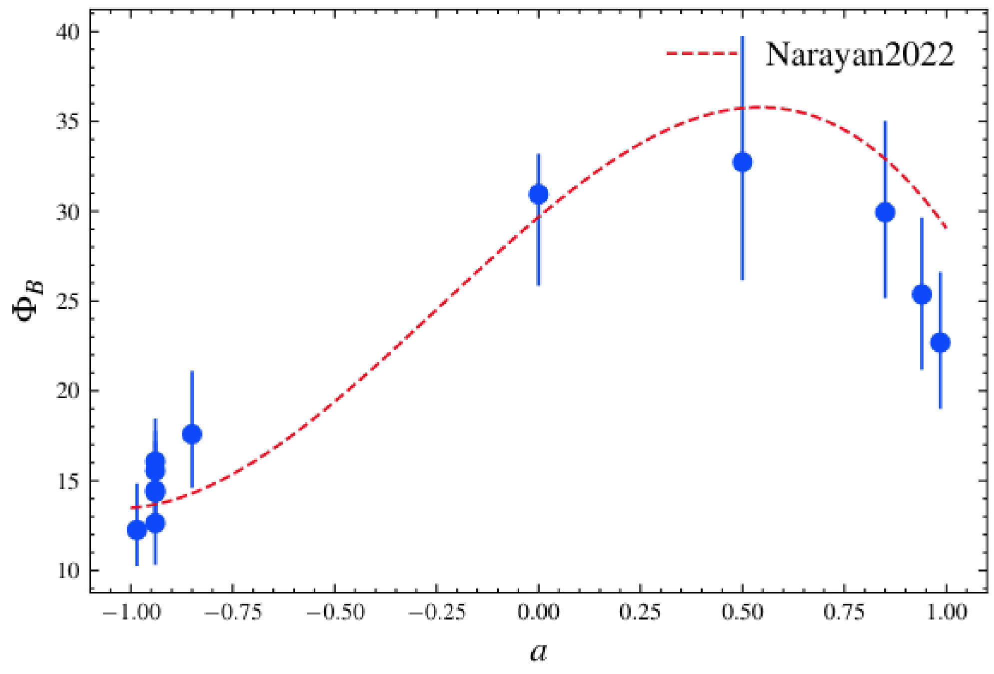

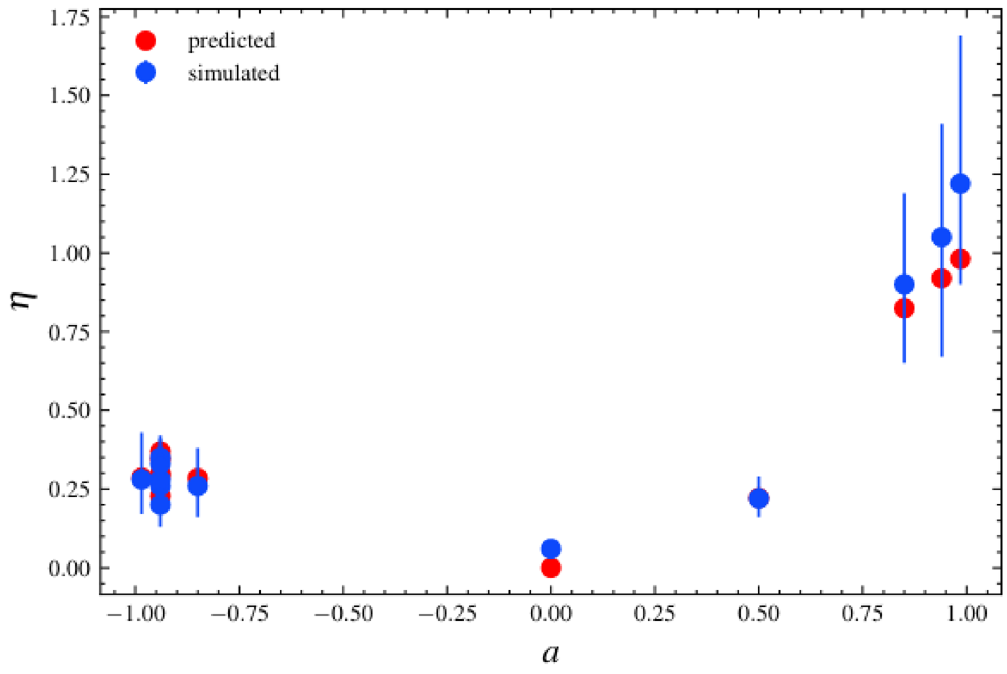

Predicted = Tchekhovskoy et al. (2010)

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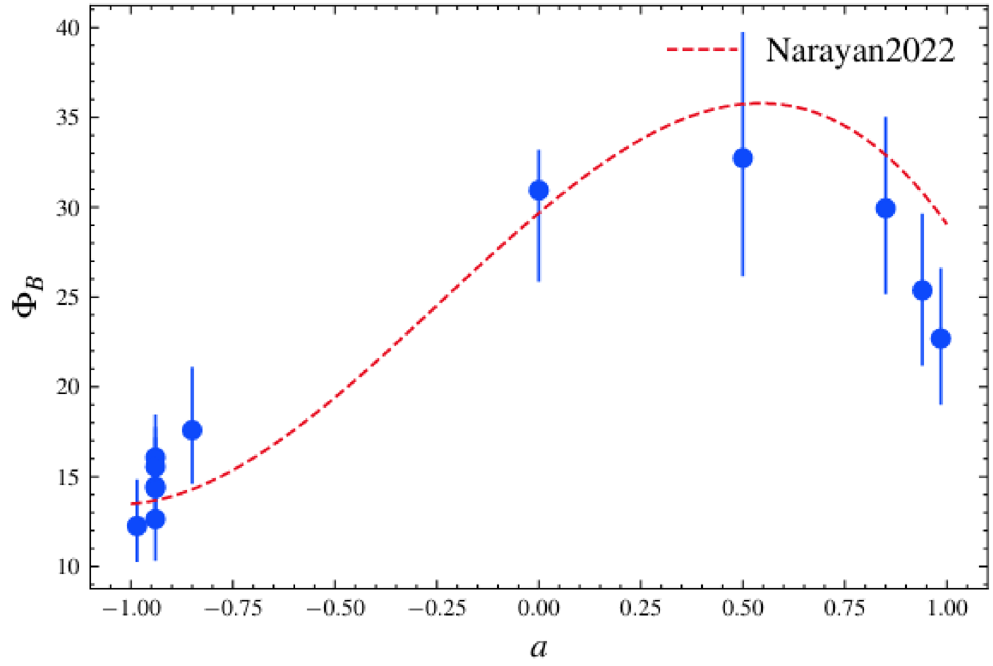

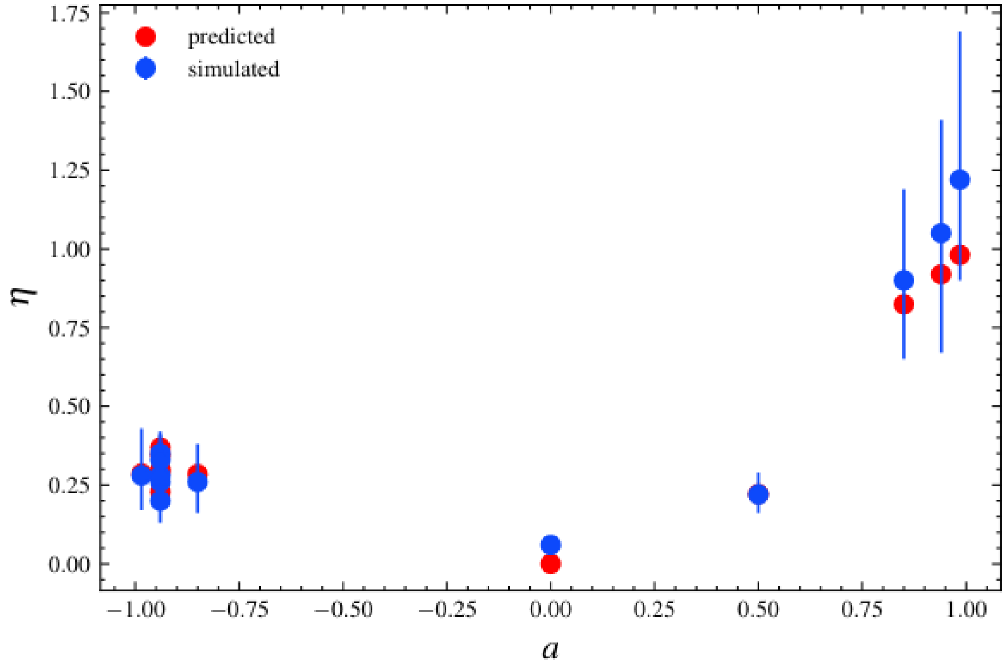

Bégué, Pe’er et al. (2023)
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General relativistic radiation MHD

John Wallace.

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- GRB jets (neutrino vs magnetic)

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Mihalas and Mihalas (84)

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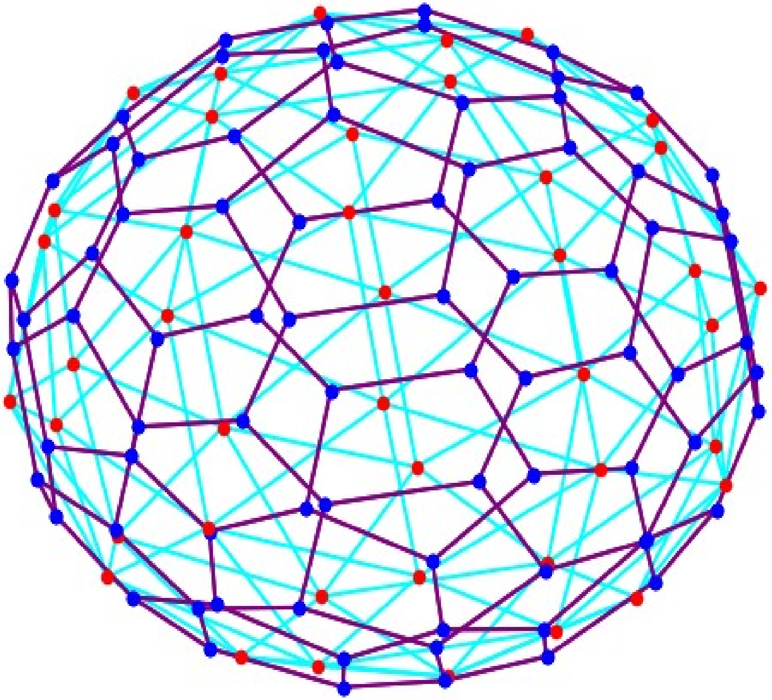

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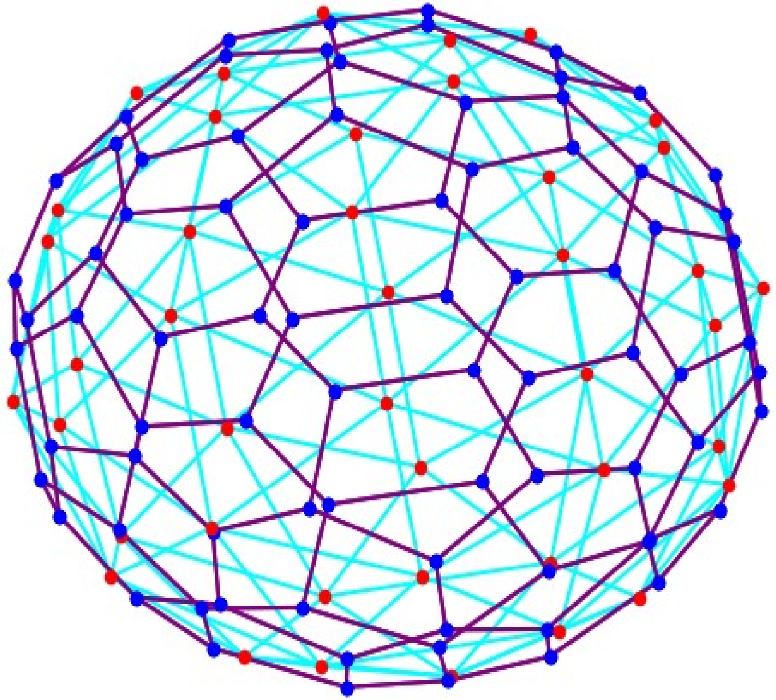

Grid level 2: 162 exagons and pentagons

What it takes to solve for I_ν ?

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Gravitational Redshift

Change of direction with the coordinate system

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Same as MHD on a different grid

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Different/hard

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Same as MHD on a different grid

Change of direction with the coordinate system

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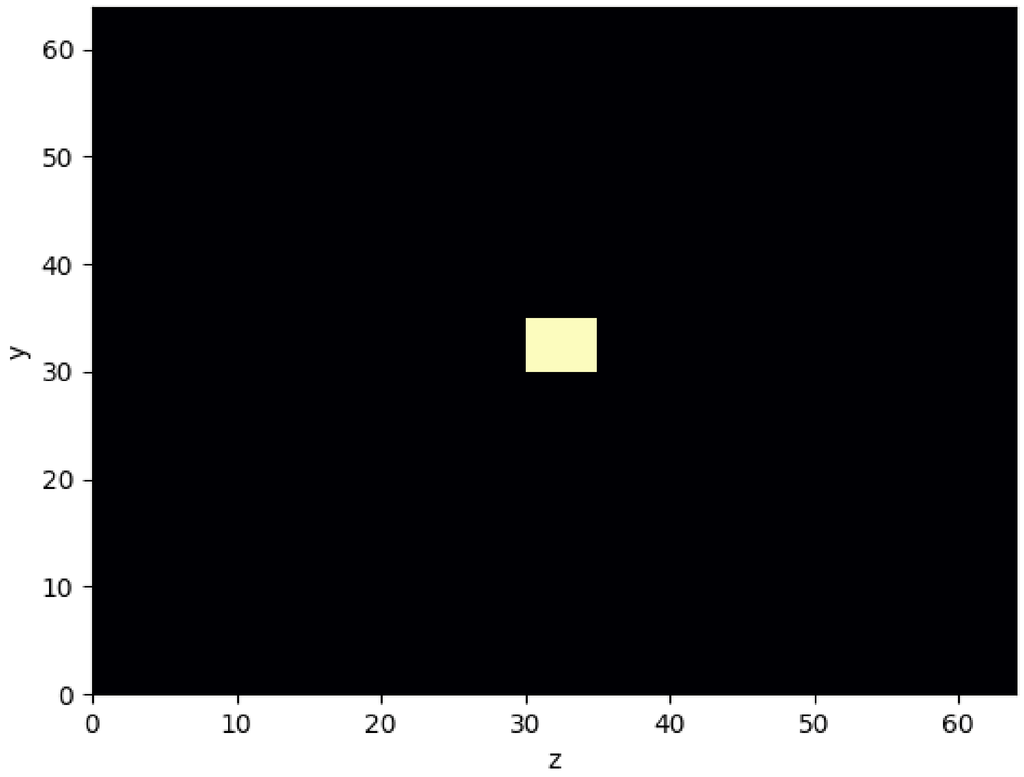
Interaction term

Similar to Soprano

Davis and Gammie (2000), White et al. (2023).

First light !



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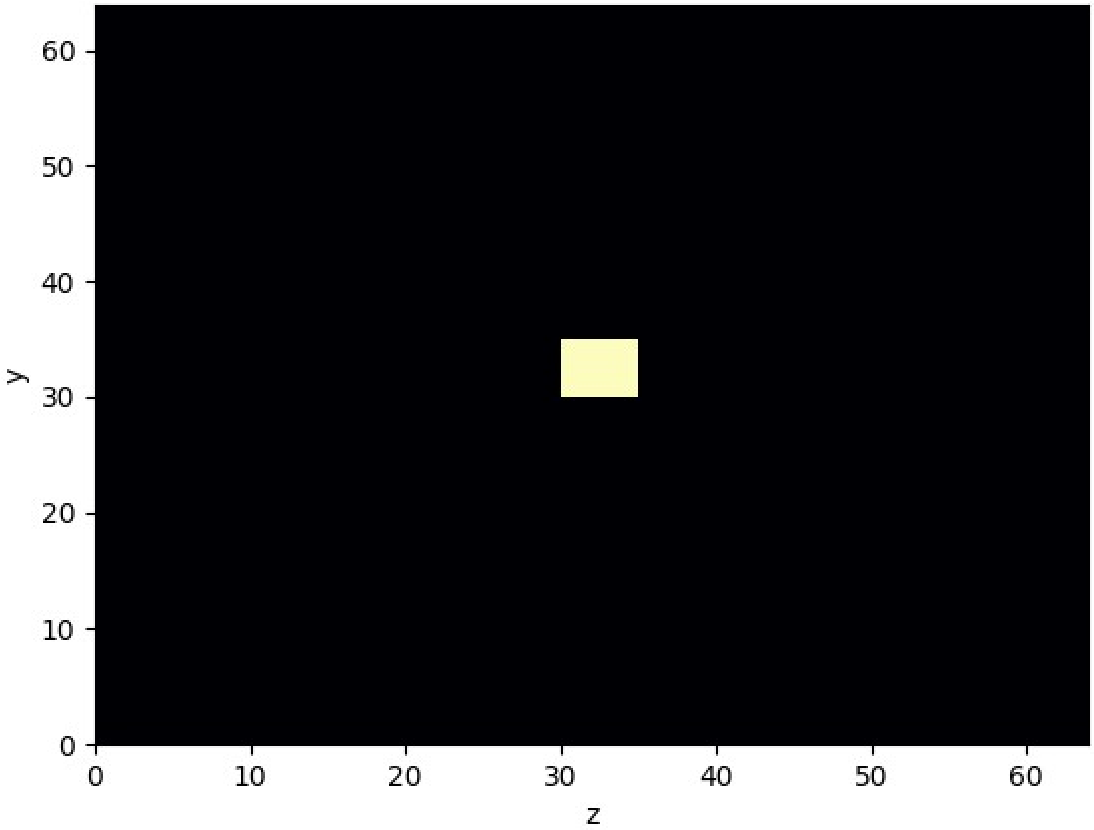

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- We have tested the code on many test problems, including accretion in both SANE and MAD regime.
- We used it to study SANE and MAD accretion regimes.
- We are in the process of adding the radiation sector:
  - Requires new algorythms for the angular discretization
  - The interaction part is the next (and last) large bottleneck.