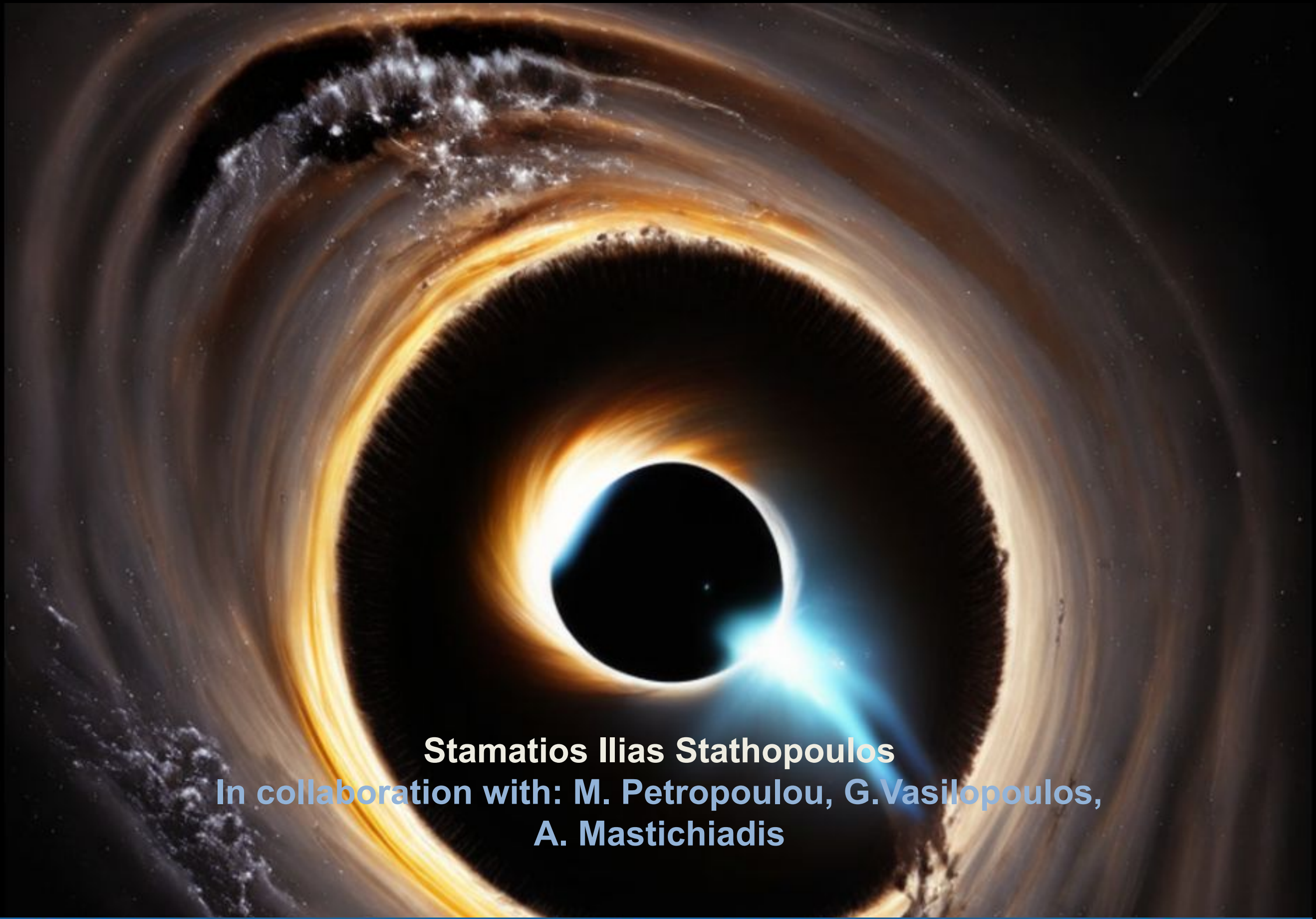
LeHaMoC : A Versatile and Efficient Time-Dependent Lepto-Hadronic Code for Astrophysical Sources



Stamatios Ilias Stathopoulos
In collaboration with: M. Petropoulou, G.Vasilopoulos, A. Mastichiadis

National and Kapodistrian University of Athens

Workshop on Numerical Multi-messenger Modeling

UNTRAPHOB

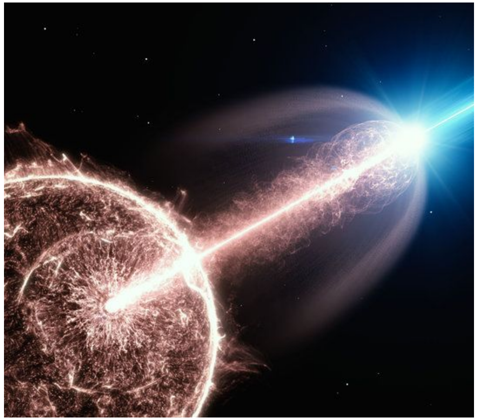
H.F.R.I.
Hellenic Foundation for Research & Innovation

High energy astrophysical sources

AGNs

- Accretion into SMBH
- Thermal+non-thermal radiation
- High-energy neutrino association

GRBs



- Shock acceleration in relativistic outflows
- Thermal+non-thermal radiation

Pulsars

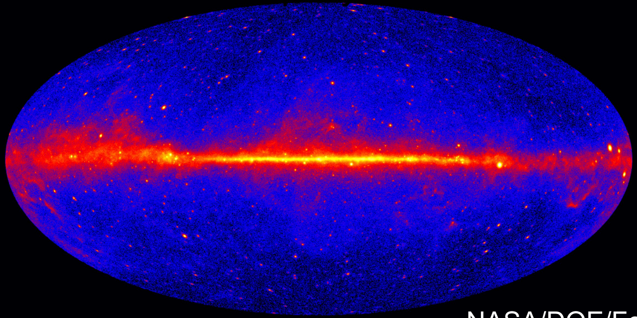
- Acceleration of pairs in the magnetosphere + termination shocks
- Thermal+non-thermal radiation

TDEs

- Tidal disruption of a star
- Acceleration of particles through shocks
- Thermal+non-thermal radiation
- High-energy neutrino association

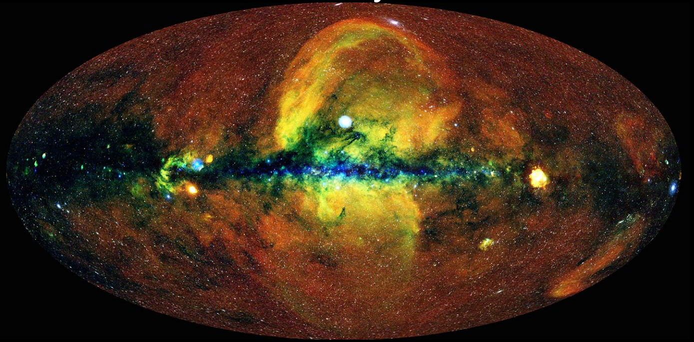
Motivation and aims

γ-rays



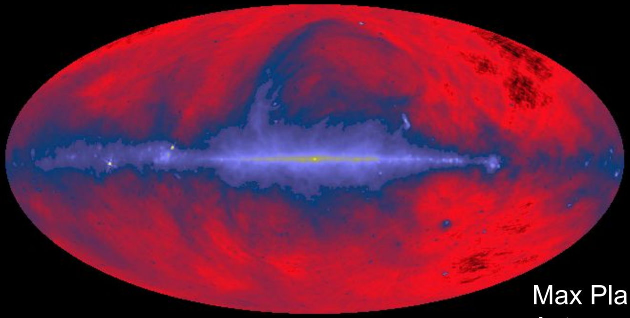
NASA/DOE/Fermi LAT Collaboration

X-rays

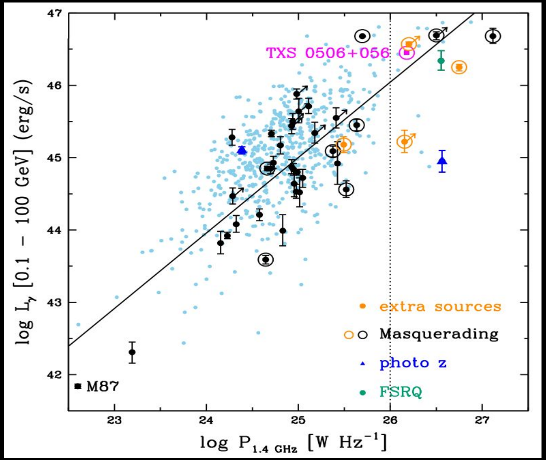


eROSITA

Radio



Max Planck Institute for Radio Astronomy



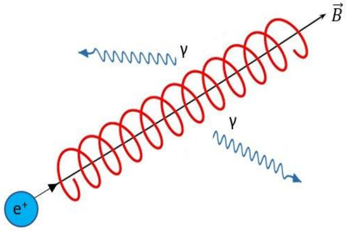
P.Padovani et.al. 2021

- Association between AGN, high energy astrophysical sources and HE neutrinos
- Rapid increase in the amount of multi-wavelength data of these sources

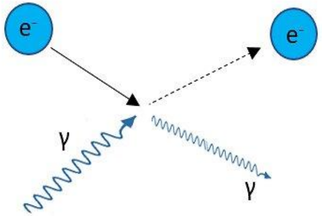
Underscores the need for efficient computational tools to analyze and interpret this information

Physical processes in multi-messenger source modelling

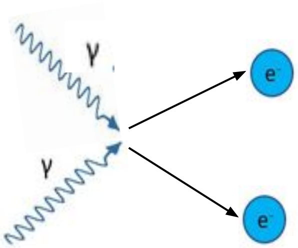
e-syn

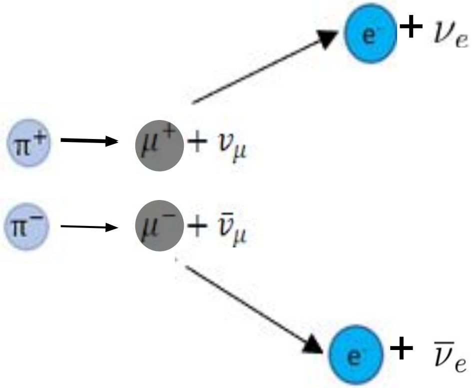


e-ICS

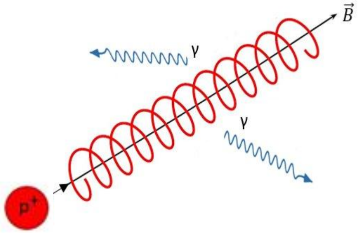


γγ⟶pair production

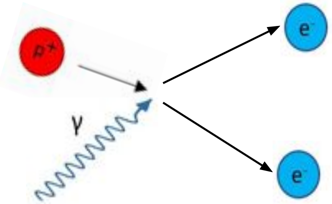




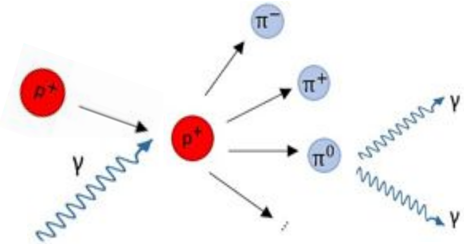
p-syn



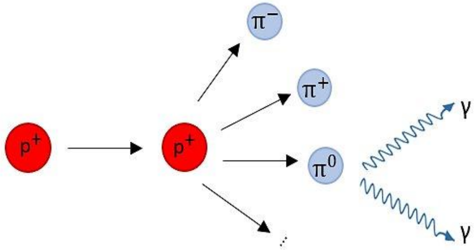
Bethe-Heitler
Pair production



Photopion
production



Proton-proton
collision



The kinetic equation

The **Boltzmann equation:**

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

- Collisionless
- No transport
- Interactions with particles escape and injection

The **kinetic equation:**

$$\frac{\partial N_i(E,t)}{\partial t} + \frac{\partial}{\partial E}\Big(b(E)N_i(E,t)\Big) + \frac{N_i(E,t)}{t_{esc}(E,t)} = Q_{inj}(E,t) + Q_{ext}(E,t)$$

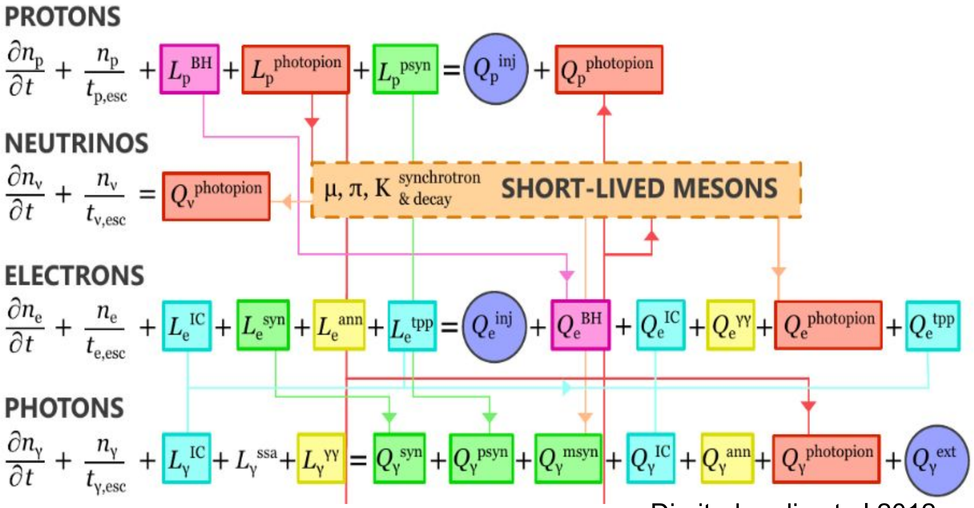
The kinetic equation and challenges

- Particles occupy a region (spherical)
- Description of the numerical density of particles in time and energy through partial differential equations

$$\frac{\partial N_i(E,t)}{\partial t} + \frac{\partial}{\partial E}\left(b(E)N_i(E,t)\right) + \frac{N_i(E,t)}{t_{esc}(E,t)} = Q_{inj}(E,t) + Q_{ext}(E,t)$$

Evolution in time | Energy gain-losses | Escape term | Injection terms

- ➢ System of coupled kinetic equations for protons, electrons , photons and neutrinos



PROTONS

$$\frac{\partial n_p}{\partial t} + \frac{n_p}{t_{p,esc}} + L_p^{BH} + L_p^{photopion} + L_p^{psyn} = Q_p^{inj} + Q_p^{photopion}$$

NEUTRINOS

$$\frac{\partial n_\nu}{\partial t} + \frac{n_\nu}{t_{\nu,esc}} = Q_\nu^{photopion}$$

μ, π, K synchrotron & decay **SHORT-LIVED MESONS**

ELECTRONS

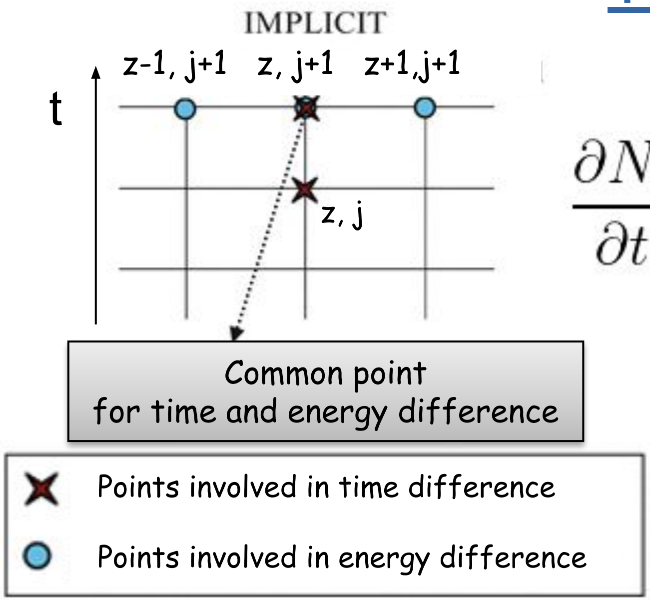
$$\frac{\partial n_e}{\partial t} + \frac{n_e}{t_{e,esc}} + L_e^{IC} + L_e^{syn} + L_e^{ann} + L_e^{tpp} = Q_e^{inj} + Q_e^{BH} + Q_e^{IC} + Q_e^{\gamma\gamma} + Q_e^{photopion} + Q_e^{tpp}$$

PHOTONS

$$\frac{\partial n_\gamma}{\partial t} + \frac{n_\gamma}{t_{\gamma,esc}} + L_\gamma^{IC} + L_\gamma^{ssa} + L_\gamma^{\gamma\gamma} = Q_\gamma^{syn} + Q_\gamma^{psyn} + Q_\gamma^{msyn} + Q_\gamma^{IC} + Q_\gamma^{ann} + Q_\gamma^{photopion} + Q_\gamma^{ext}$$

Dimitrakoudis et.al 2012

The kinetic equation and **LeHaMoC**



Common point
for time and energy difference

✕ Points involved in time difference

● Points involved in energy difference

$$\frac{\partial N_i}{\partial t} + \frac{\partial}{\partial x}(\mathcal{P}_i(x,t)N_i) + \frac{N_i}{t^i_{esc}(t)} = Q_i$$

+

Implicit Method

Faster computational time
(No constraint on dt)
+
Unconditional stable

Discretization

$$\frac{\partial N_i}{\partial t} = \frac{N^{j+1}_{i,z} - N^{j}_{i,z}}{\Delta t}$$ (Euler's Method)

$t = (t_1,t_2,..,t_n)$, $j \in (1,n)$
$x = (x_1,x_2,..,x_m)$, $z \in (1,m)$

$$\frac{\partial}{\partial x}(\mathcal{P}_i(x,t)N_i) = \frac{F^{j+1}_{i,z+\frac{1}{2}} - F^{j+1}_{i,z-\frac{1}{2}}}{\Delta x}, \ F^{j+1}_{i,z\pm\frac{1}{2}} \equiv \mathcal{P}_i(x_{z\pm\frac{1}{2}}, t^j)N^{j+1}_{i,z\pm\frac{1}{2}}$$

The kinetic equation and **LeHaMoC**

$$V1_{i,z}N^{j+1}_{i,z-1} + V2_{i,z}N^{j+1}_{i,z} + V3_{i,z}N^{j+1}_{i,z+1} = S^{j}_{i,z}$$

Coefficients

$$V1_{i,z} = 0$$

$$V2_{i,z} = 1 + \frac{\Delta t}{t_{i,esc}} + \frac{\Delta t}{\Delta \gamma_z} \sum_p \left(\frac{d\gamma_i}{dt} \right)^{j+1}_{p,z}$$

$$V3_{i,z} = -\frac{\Delta t}{\Delta \gamma_z} \sum_p \left(\frac{d\gamma_i}{dt} \right)^{j+1}_{p,z+1}$$

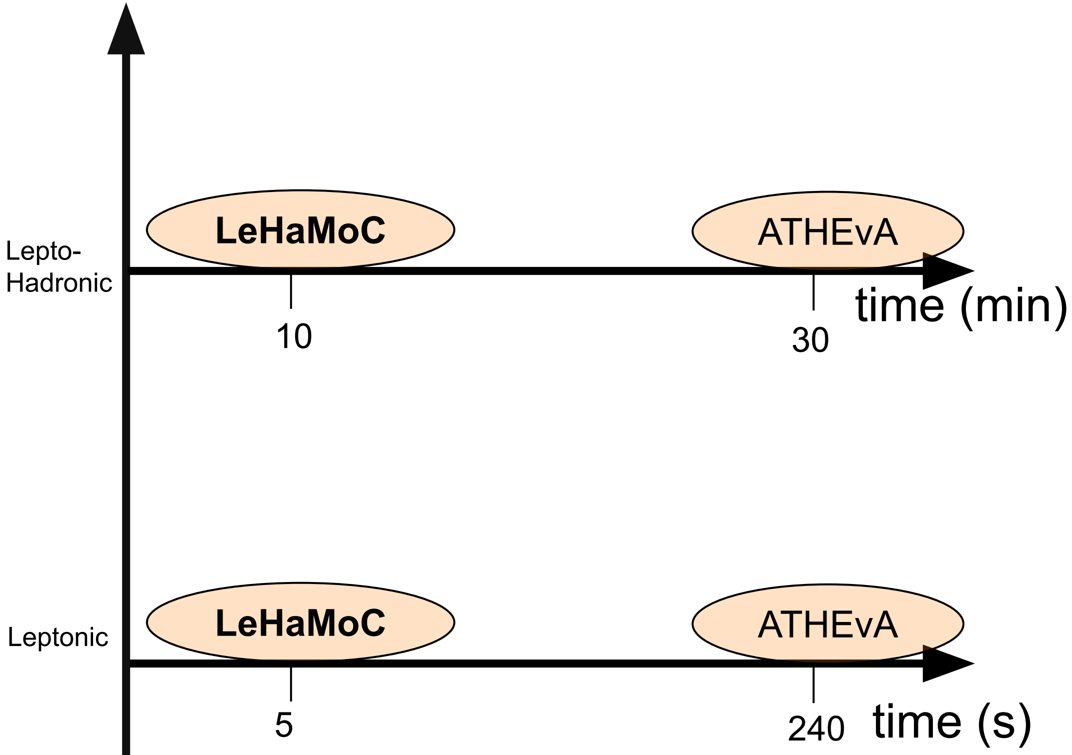
Source term

$$S^{j}_{i,z} = N^{j}_{i,z} + Q^{j}_{i,z}\Delta t$$

Tridiagonal matrix (Thomas, L. H. 1949)

Evolution of particles

LeHaMoC: speed advantage and accuracy



Lepto-
Hadronic

LeHaMoC

ATHEvA

10

30

time (min)

Leptonic

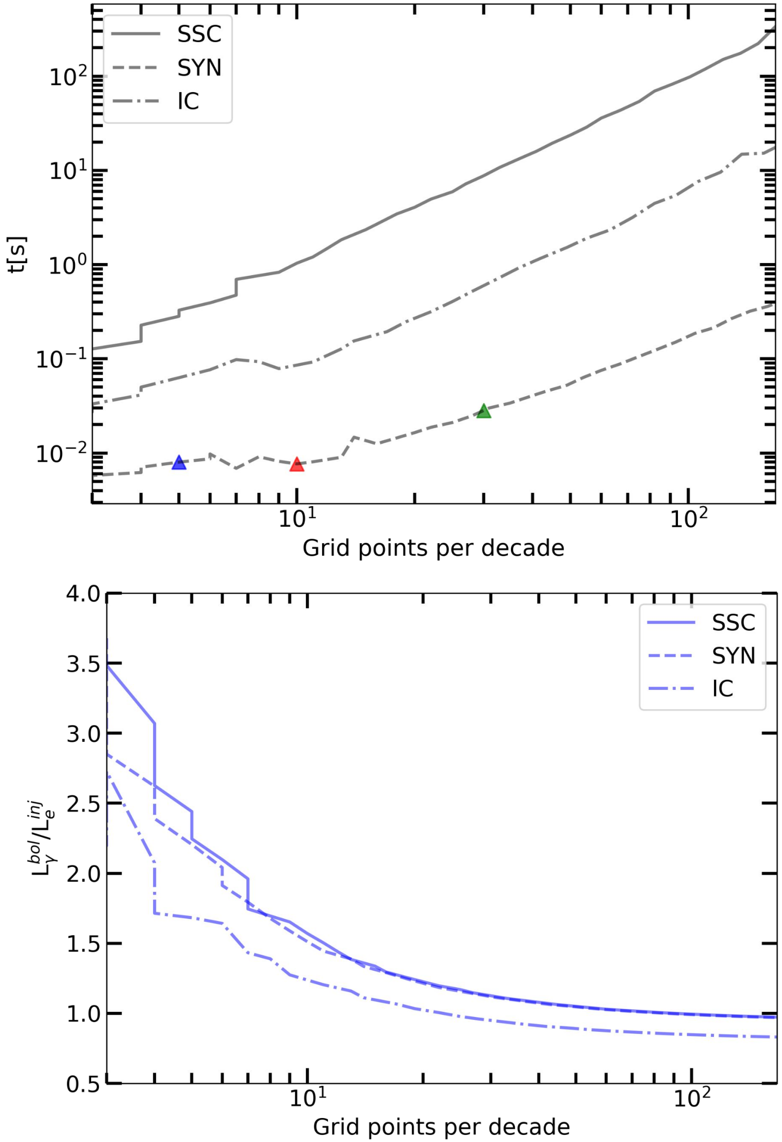
LeHaMoC

ATHEvA

5

240

time (s)



Comparison with the ATHE*v*A code (a time-dependent one-zone lepto-hadronic code)

Mastichiadis & Kirk 1995; Dimitrakoudis et. al. 2012

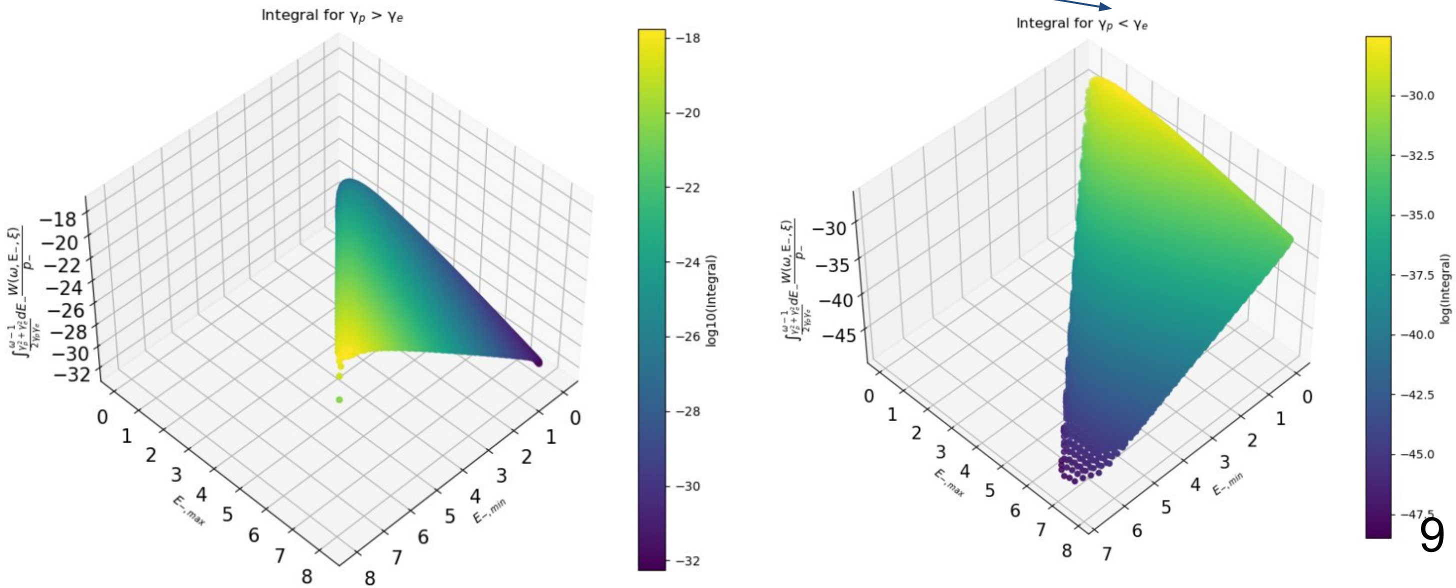
Time consuming processes: Bethe-Heitler

Energy distribution of pairs (direct computation of 3 integrals):

Target photons Emission angle Cross-section

$$\frac{dN}{d\gamma_e} = \frac{1}{2\gamma_p^3} \int_{\frac{(\gamma_p+\gamma_e)^2}{4\gamma_p^2\gamma_e}}^{\frac{m_p}{\gamma_p m_e}} d\epsilon \frac{f_{ph}(\epsilon)}{\epsilon^2} \int_{\frac{(\gamma_p+\gamma_e)^2}{2\gamma_p\gamma_e}}^{2\gamma_p\epsilon} d\omega\, \omega \int_{\frac{(\gamma_p^2+\gamma_e^2)}{2\gamma_p\gamma_e}}^{\omega-1} dE_- \frac{W(\omega, E_-, \xi)}{p_-}$$

Kelner & Aharonian 2008



Interpolation of tabulated integral ->30 times faster

Stathopoulos et al. 2023 ,A&A accepted

Time consuming processes: Bethe-Heitler

$$\frac{dN}{d\gamma_e} = \int_{\frac{(\gamma_p+\gamma_e)^2}{4\gamma_p^2\gamma_e}}^{\frac{m_p}{\gamma_p m_e}} d\epsilon \frac{f_{ph}(\epsilon)}{\epsilon^2} \underbrace{\xcancel{\int_{\frac{(\gamma_p+\gamma_e)^2}{2\gamma_p\gamma_e}}^{2\gamma_p\epsilon} d\omega\, \omega \int_{\frac{(\gamma_p^2+\gamma_e^2)}{2\gamma_p\gamma_e}}^{\omega-1} dE_- \frac{W(\omega,E_-,\xi)}{p_-}}}$$

Empirical function: $q_{\rm BH}(\gamma_e) = A(\gamma_p,\epsilon)\cdot\exp\left[-\frac{\left[\log_{10}\left(\frac{\gamma_e}{\gamma_{e,\rm pk}}\right)\right]^{p(\gamma_p\epsilon)}}{2{a_1}^2} - a_2^2\left(\frac{\gamma_{e,\rm pk}}{\gamma_e}-1\right)^2 - a_3\frac{\gamma_e}{\gamma_{e,\rm cr}}\right]$

D. Karavola & M. Petropoulou JCAP submitted, arXiv:2401.05534

https://github.com/Des0053/Bethe-Heitler-Injection-Rate-Analytical-Approximation

~2 times faster than using the interpolation to calculate the last integral

Time consuming processes:Photo meson production

Energy distribution of secondaries:

$$\frac{dN_l}{dtdVdE_l} = \int n_p(E_p)n_{ph}(\epsilon)\Phi_l(\eta, x)\frac{dE_p}{E_p}d\epsilon$$

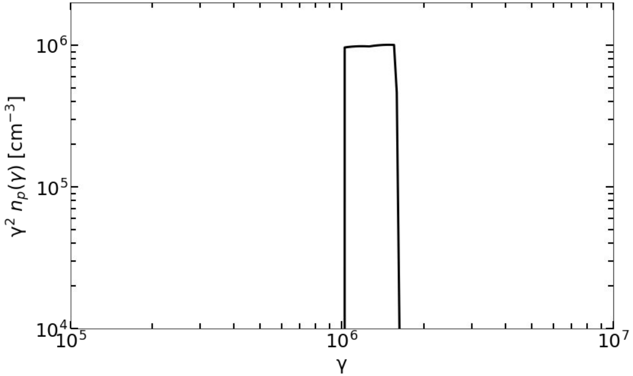
Kelner & Aharonian 2008

$\Phi_l(\eta, x)$: Energy distribution of the 8 species l

Time consumption: Each species emissivity is calculated in the same process separately (3 seconds each)

Solution: Parallel computing (implemented in the future)

LeHaMoC compared against other codes



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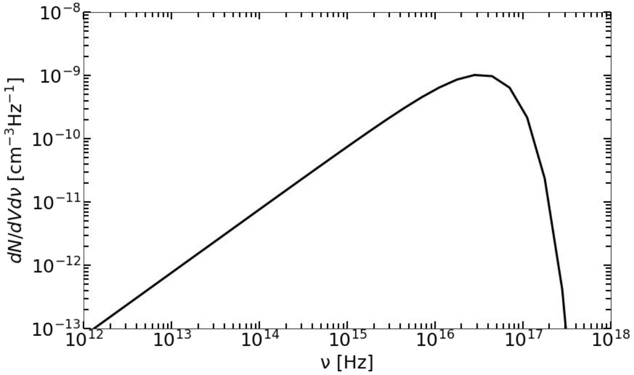


M. Cerruti
21/02 16:30

Bethe-Heitler emissivity

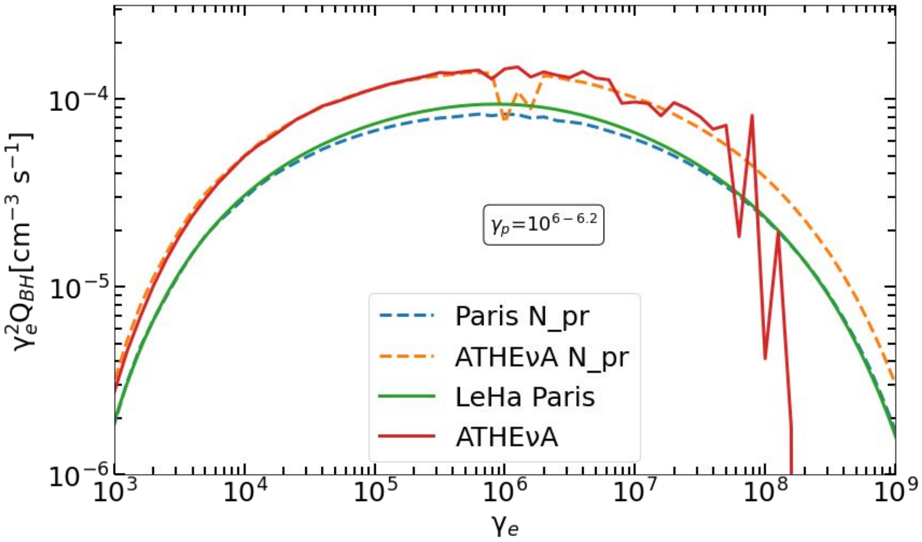
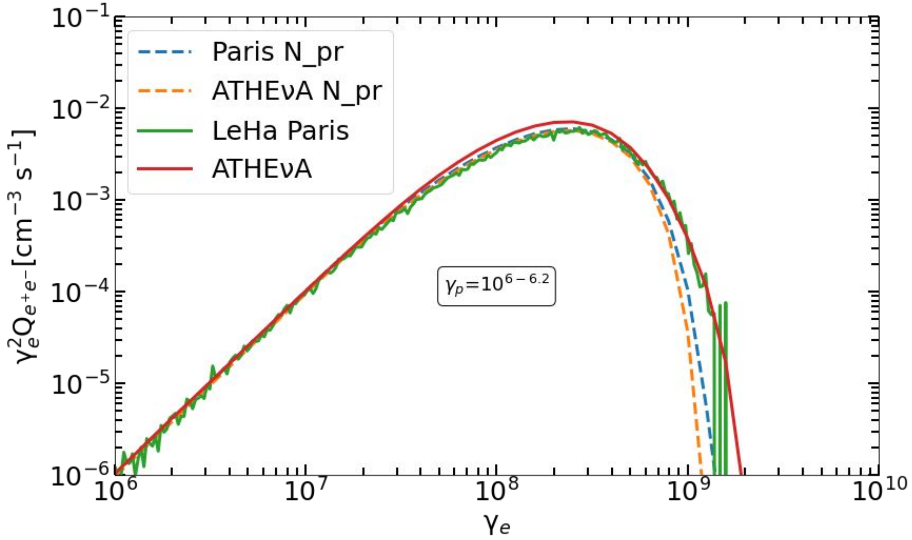
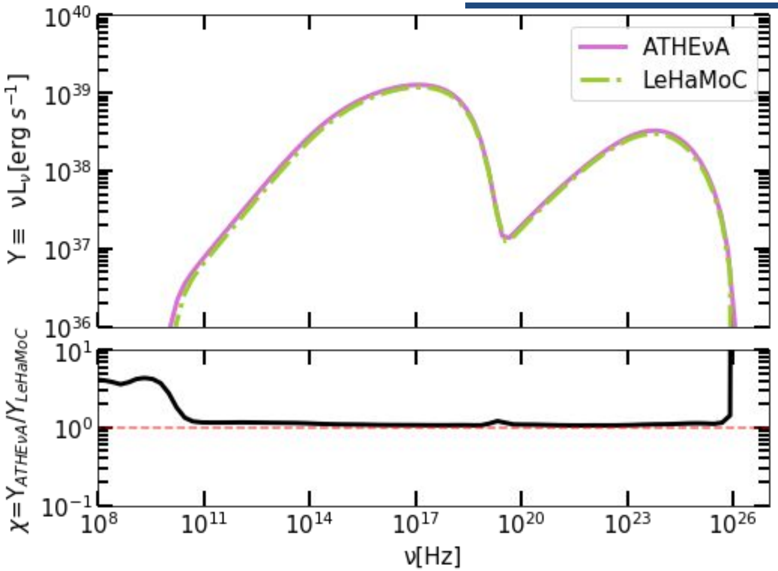


Photo-meson pair emissivity

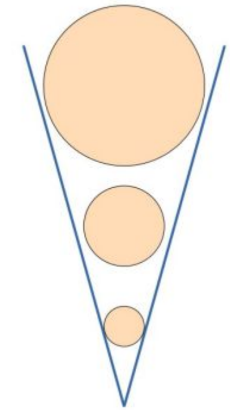


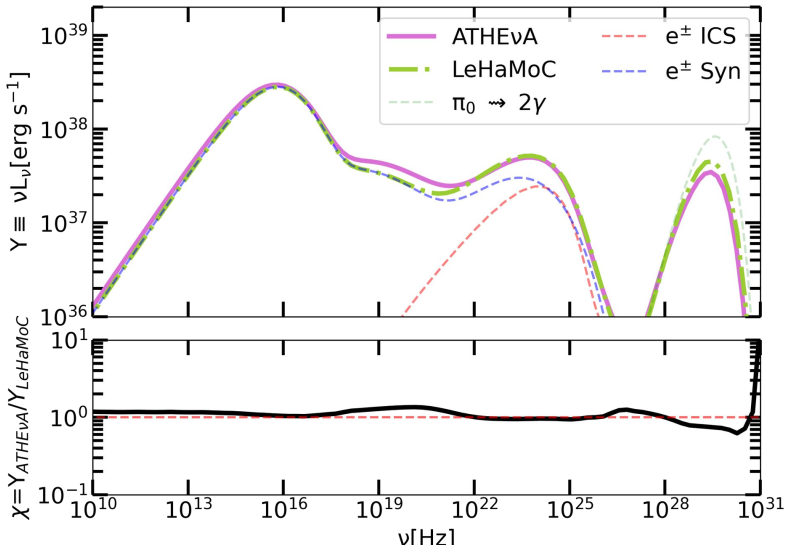
LeHaMoC results-Validation and Comparison



Leptonic model

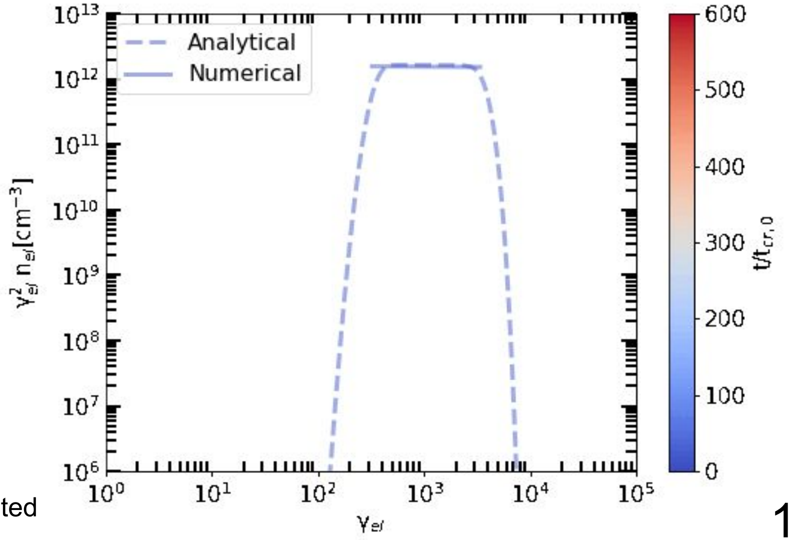
Analytical model





Lepto-hadronic model

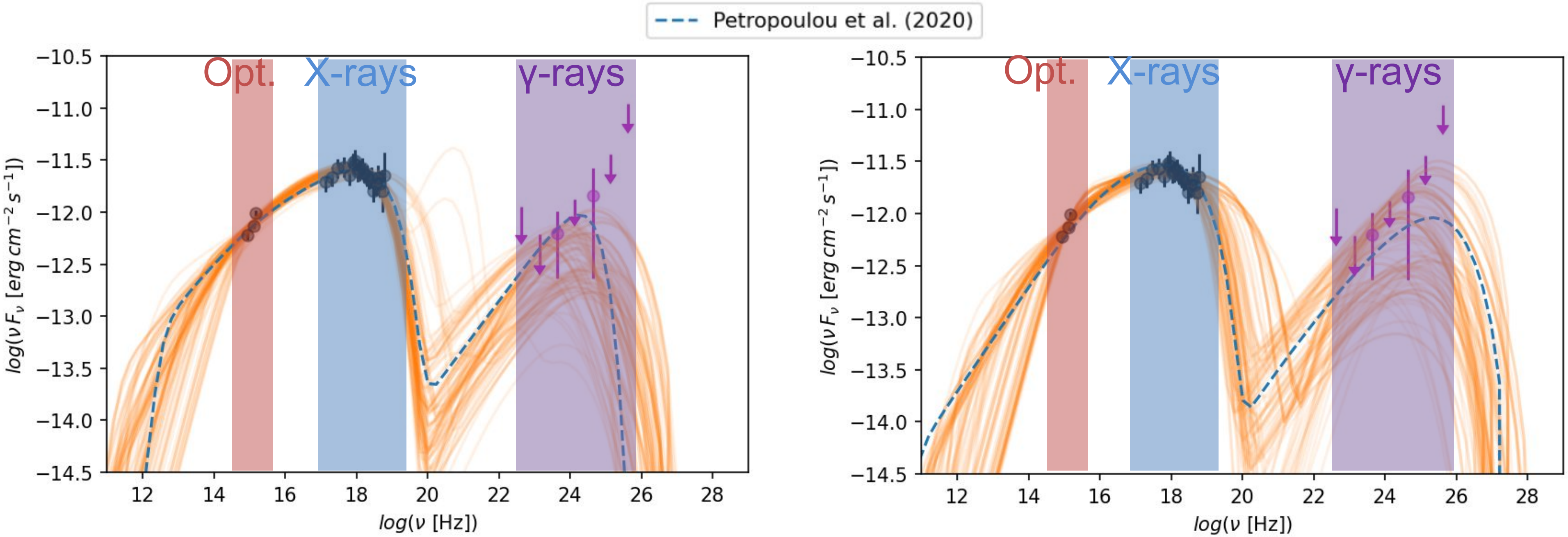
Stathopoulos et al. 2023 ,A&A accepted



LeHaMoC results-Illustrative Example 1: Blazar SED Fitting

LeHaMoC + emcee → Better understanding of the physics inside the emitting region

3HSP J095507.9+355101



Proton synchrotron model

Leptonic model

Small execution time: Able to scan the parameter space in a short timeframe

Stathopoulos et al. 2023 ,A&A accepted

LeHaMoC results-Illustrative Example 2: NGC 1068 SED Fitting

Source characteristics

M_{SMBH} ~$10^{7.3}M_{SUN}$

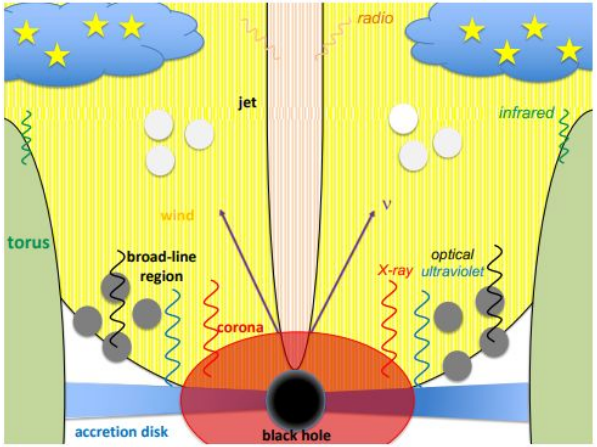
R_s ~6 10^{12} [cm]

Intrinsic X-ray luminosity ~ 10^{44} [erg s^{-1}]

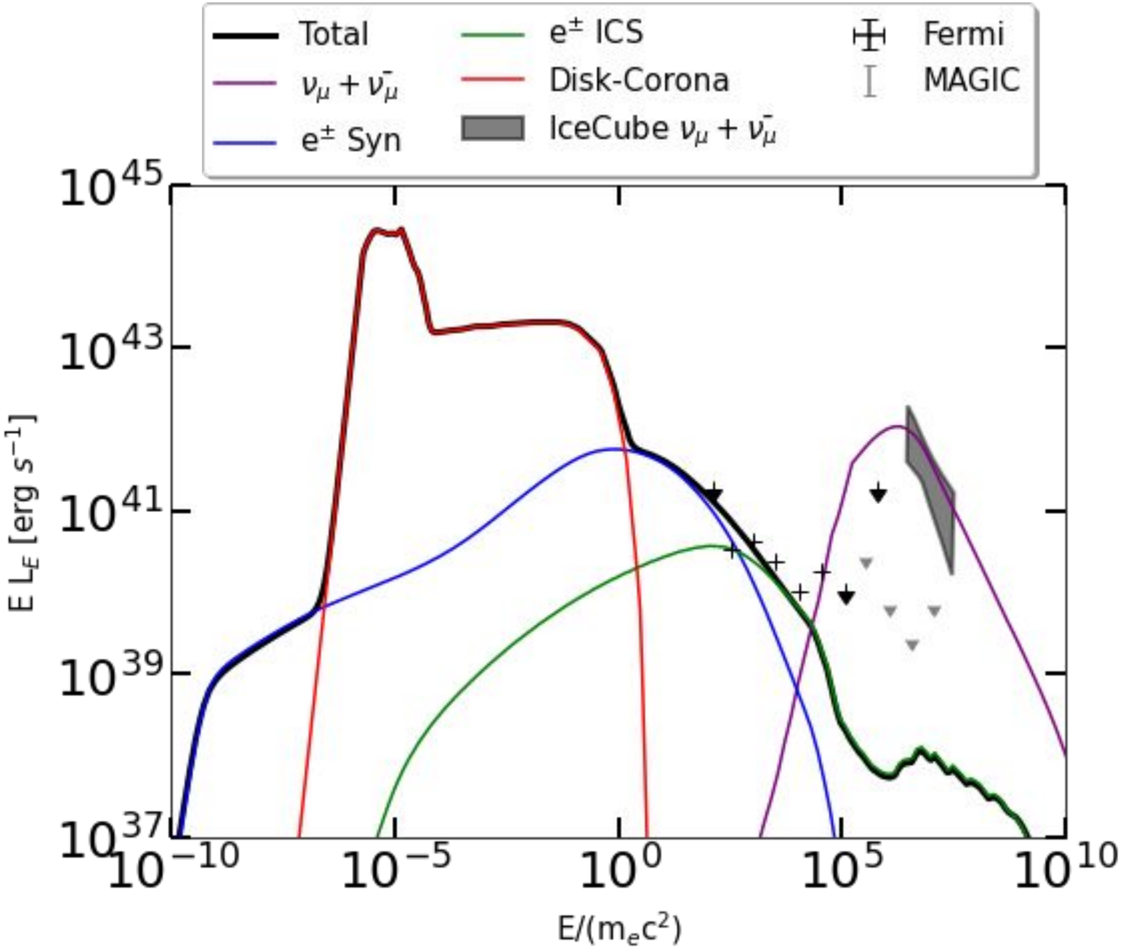
Opaque in 0.1-10GeV ($\tau_{\gamma\gamma}$>1) → R<100R_s

Conclusions

Neutrinos are produced in the vicinity of the SMBH → Corona-disk region



Murase, 2022

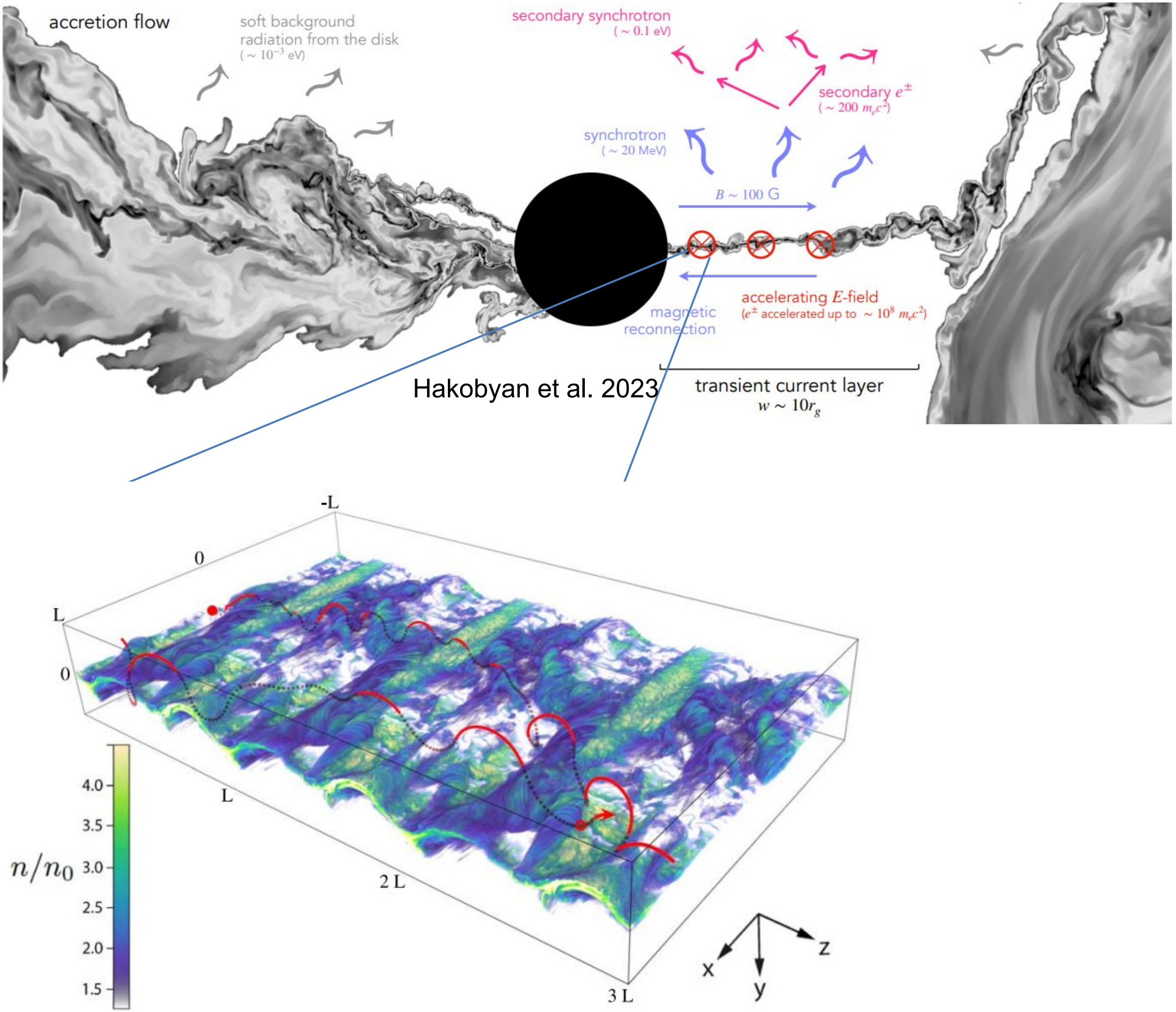


Stathopoulos et al. 2023 ,A&A accepted

On going projects with LeHaMoC

Cosmic-ray acceleration in M87 current sheets

in collaboration with M.Petropoulou, D.Giannios and L.Sironi



Hakobyan et al. 2023

Non thermal pairs:

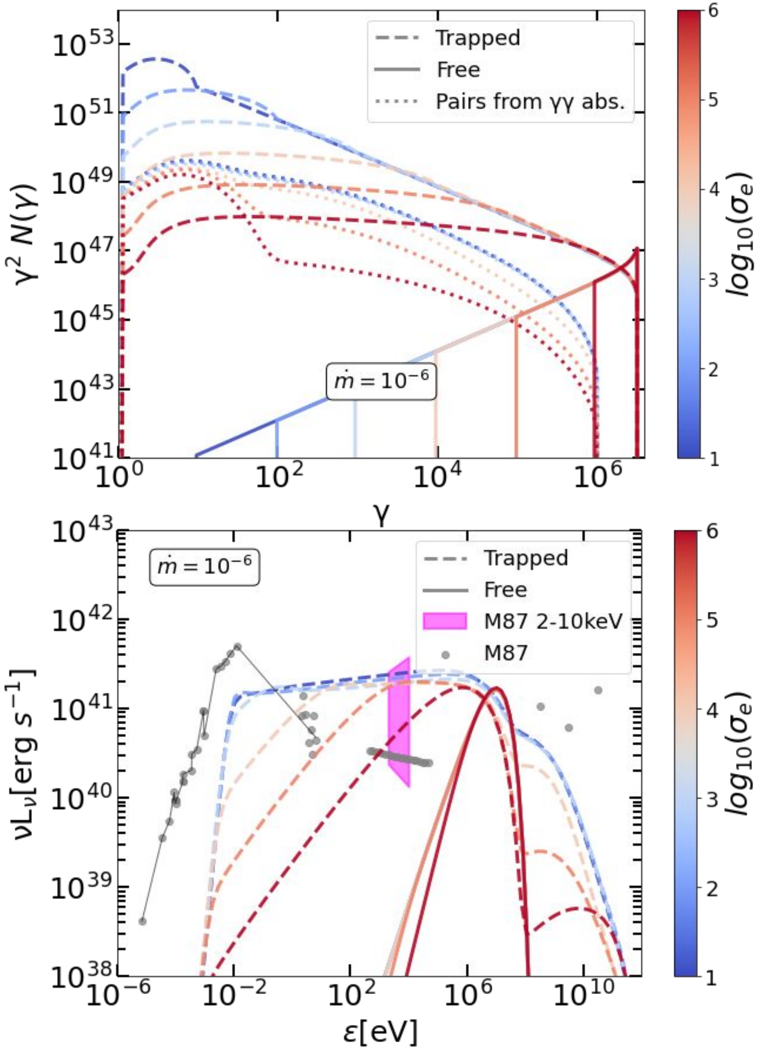
- ➢ Trapped pairs (pairs within plasmoids accelerated in the X-points $1 < \gamma < \sigma$)
- ➢ Free pairs (pairs accelerated in the upstream region $\sigma < \gamma < \gamma_{rad}$)
- ➢ Secondary pairs (injected from γγ absorption)

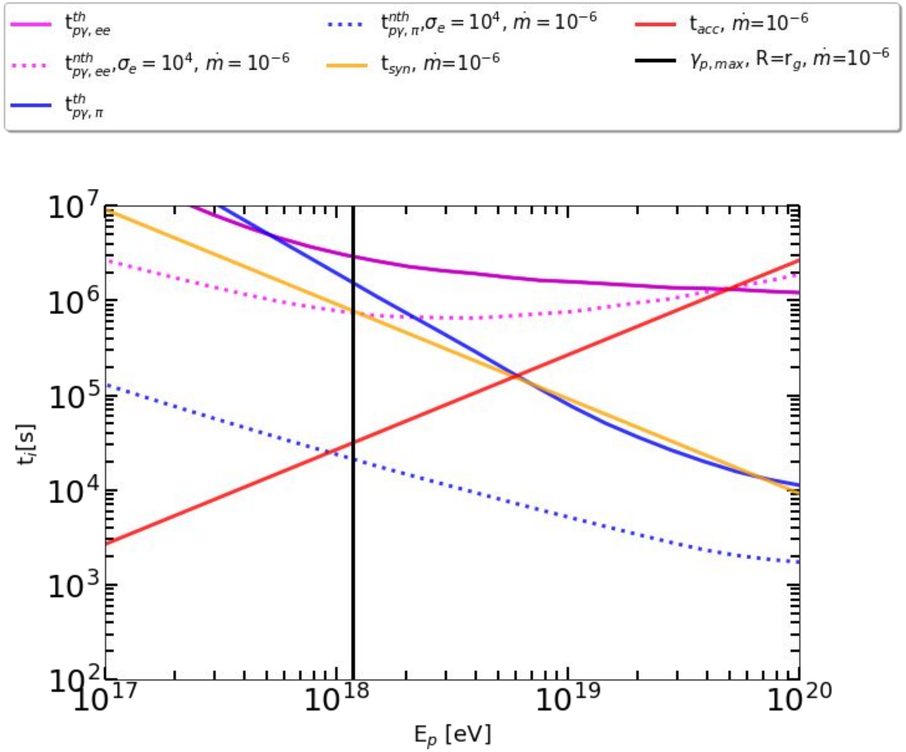
Pairs can be accelerated up to:

$$\gamma_{rad} \simeq \sqrt{3} \cdot 10^6$$

Cosmic-ray acceleration in M87 current sheets

in collaboration with M.Petropoulou, D.Giannios and L.Sironi



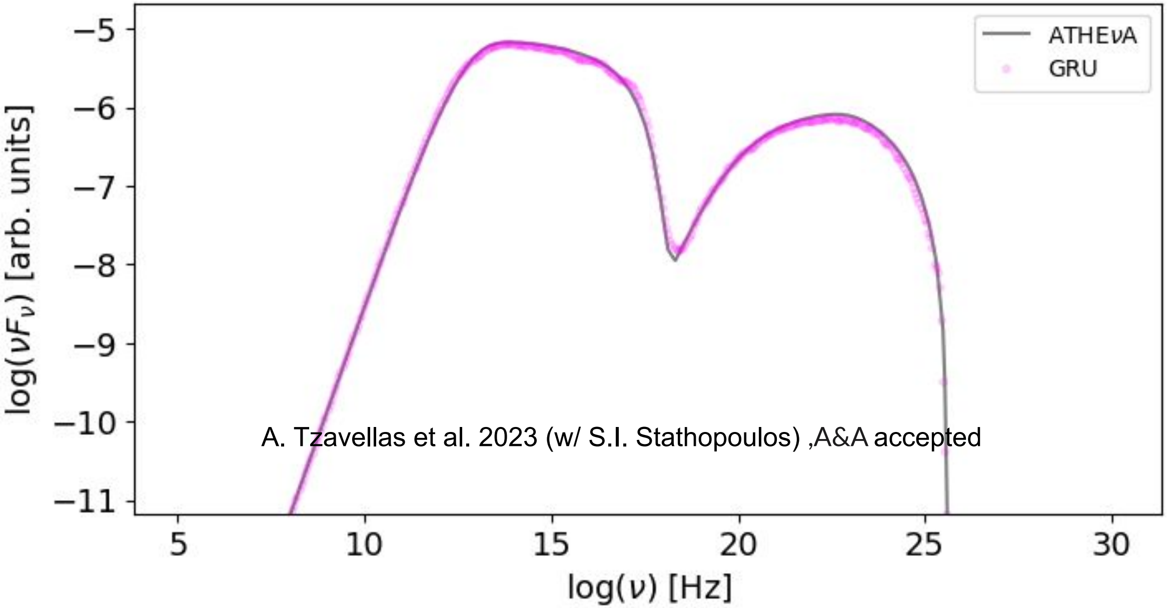


Protons can be accelerated up to ~0.1EeV

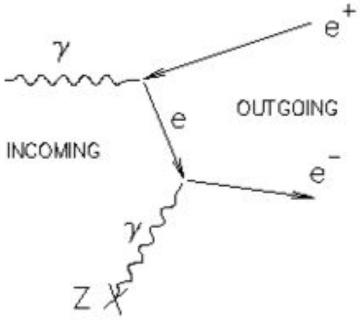
LeHaMoC: Application to Neural Networks

ML: speed up the modelling of synchro-Compton blazar emission
Application of NN -> replace computational tools

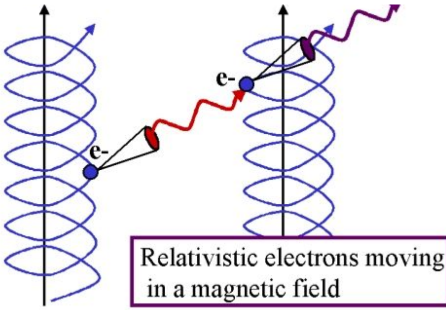
A. Tzavellas
22/02 15:30



A. Tzavellas et al. 2023 (w/ S.I. Stathopoulos) ,A&A accepted



Include γγ pair production, SSA and external photon fields using simulated SEDs from **LeHaMoC**



Summary

- ☐ LeHaMoC: Versatile and efficient numerical tool for calculating spectra for high energy astrophysical sources
- ☐ The code's versatility and speed make it well suited for modelling GRBs, Blazars and other high-energy sources
- ☐ Download the code in the following link

M. Chatzis
22/02 10:00

A Study of Broadband Variability in the Context of Hybrid Leptohadronic Models for TeV Blazars

https://github.com/mariapetro/LeHaMoC

Thank you!

Backup slides

Kelner & Aharonian 2009 ①

Kelner & Aharonian 2008 ②

Kirk & Mastichiadis 1995 ③

Rybicki & Lightman 1985 ④

Leptons

ENERGY LOSSES

- ④ Synchrotron
- ③ Inverse Compton
- Adiabatic

INJECTION

- ① BH pair production
- ① Photopion production
- ② p-p collisions
- ③ Pair creation

Protons

ENERGY LOSSES

- ④ Synchrotron
- ④ Inverse Compton
- ① BH pair production
- ① Photopion production
- ② p-p collisions
- Adiabatic

Photons

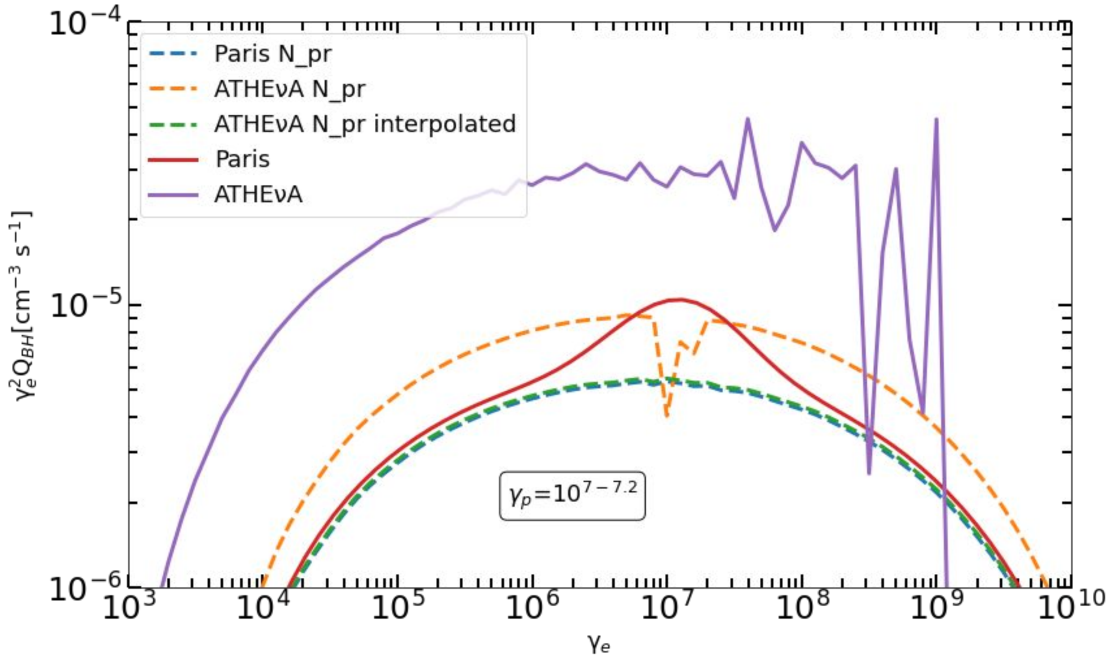
INJECTION-LOSSES

- ④ Synchrotron
- ③ Inverse Compton
- ① Photopion production
- ② p-p collisions
- ③ γ-γ absorption
- ④ SSA

The expression:

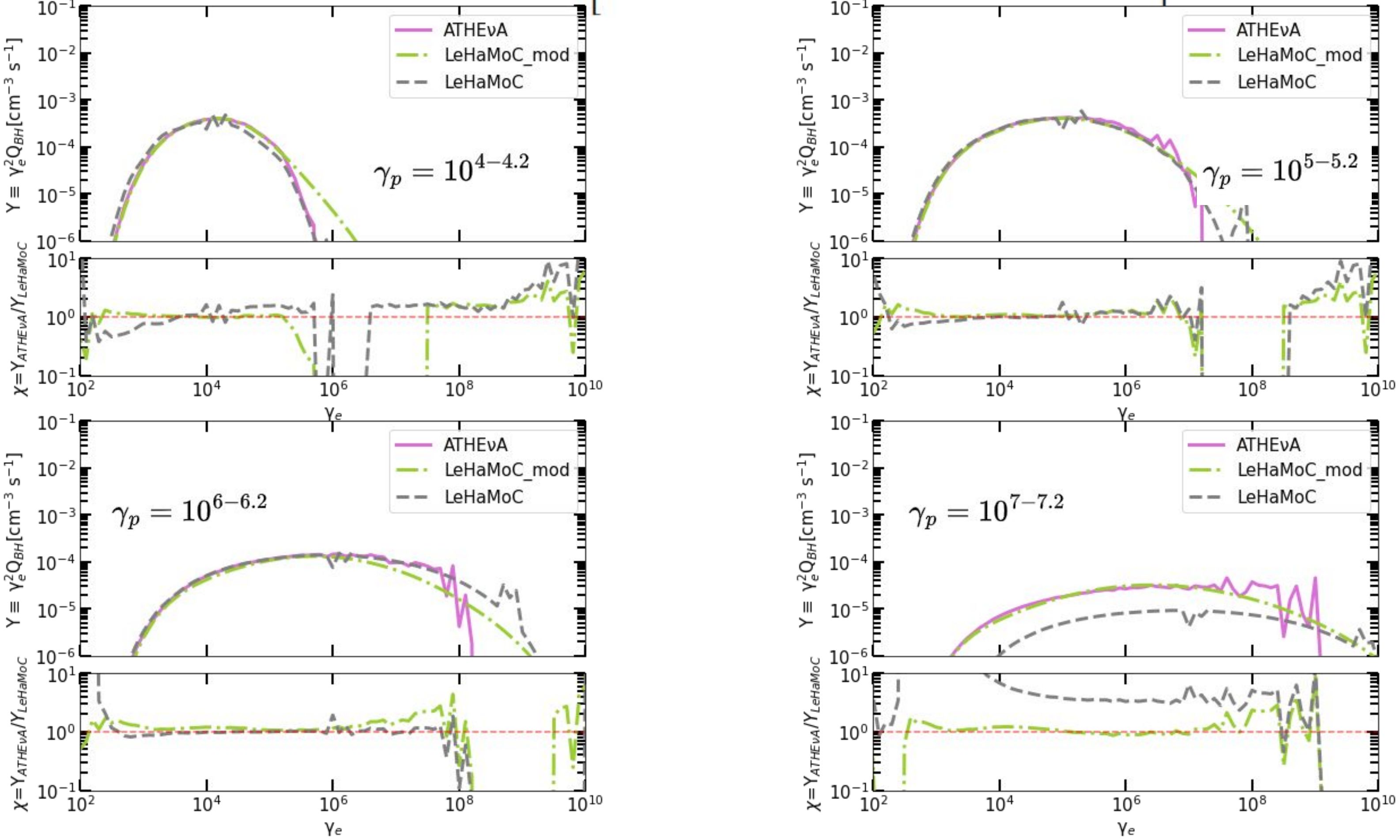
$$\frac{dN}{d\gamma_e} = \int_{\frac{(\gamma_p+\gamma_e)^2}{4\gamma_p^2\gamma_e}}^{\frac{m_p}{\gamma_p m_e}} d\epsilon \frac{n_\gamma(\epsilon)}{\epsilon^2} \int_{\frac{(\gamma_p+\gamma_e)^2}{2\gamma_p\gamma_e}}^{2\gamma_p\epsilon} d\omega\, \omega \int_{\frac{(\gamma_p^2+\gamma_e^2)}{2\gamma_p\gamma_e}}^{\omega-1} dE_- \frac{W(\omega, E_-, \xi)}{p_-}$$

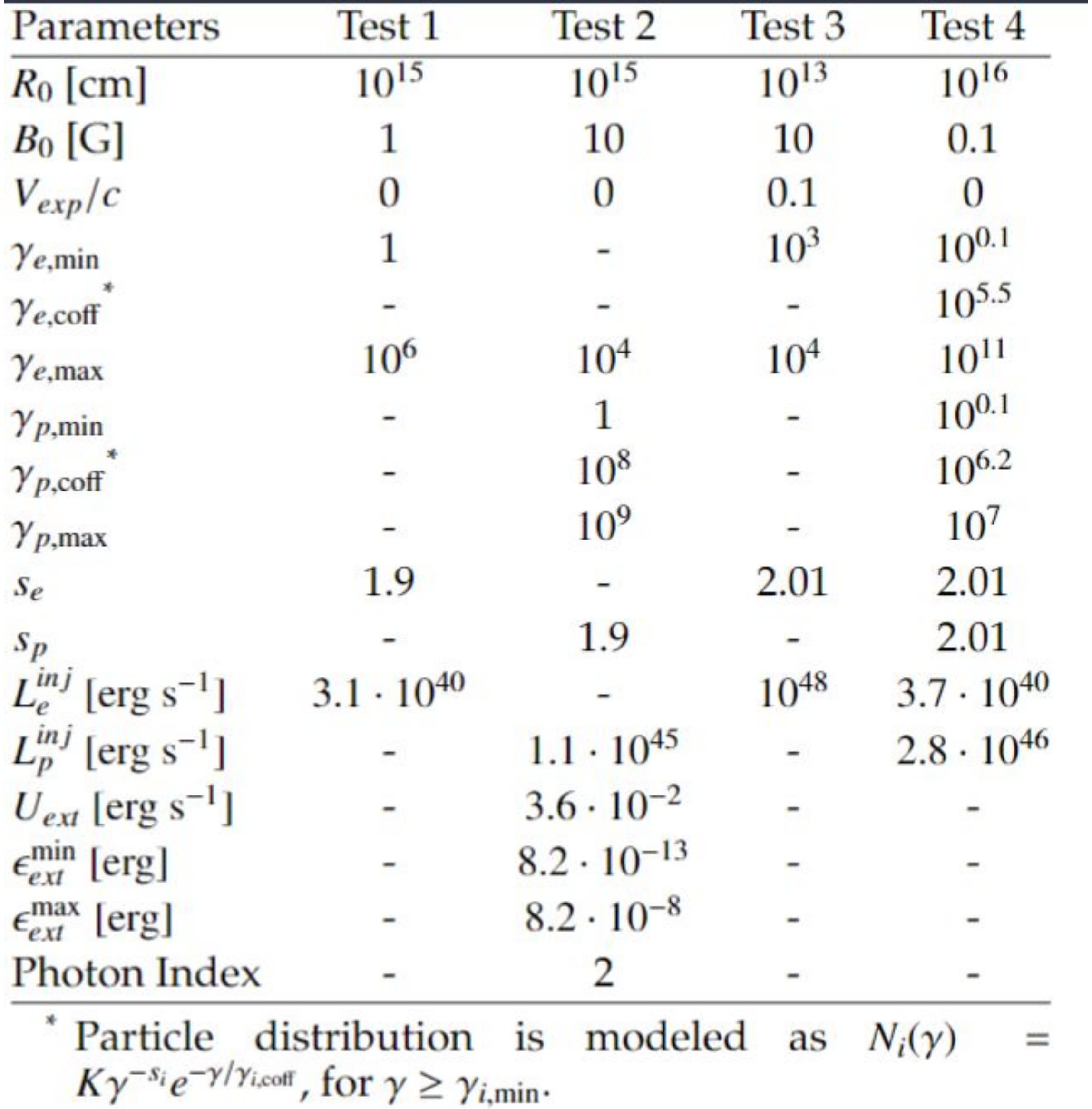
is valid when: $m_e c^2 \ll \gamma_p \epsilon \ll m_p c^2$



Comparison between the empirical function (δ function +GB)

Empirical function: $q_{\rm BH}(\gamma_e) = A(\gamma_p, \epsilon)\cdot\exp\left[-\frac{\left[\log_{10}\left(\frac{\gamma_e}{\gamma_{e,\rm pk}}\right)\right]^{p(\gamma_p\epsilon)}}{2{a_1}^2} - a_2^2\left(\frac{\gamma_{e,\rm pk}}{\gamma_e}-1\right)^2 - a_3\frac{\gamma_e}{\gamma_{e,\rm cr}}\right]$





Parameters	Test 1	Test 2	Test 3	Test 4
R_0 [cm]	10^{15}	10^{15}	10^{13}	10^{16}
B_0 [G]	1	10	10	0.1
V_{exp}/c	0	0	0.1	0
$\gamma_{e,\rm min}$	1	-	10^3	$10^{0.1}$
$\gamma_{e,\rm coff}{}^*$	-	-	-	$10^{5.5}$
$\gamma_{e,\rm max}$	10^6	10^4	10^4	10^{11}
$\gamma_{p,\rm min}$	-	1	-	$10^{0.1}$
$\gamma_{p,\rm coff}{}^*$	-	10^8	-	$10^{6.2}$
$\gamma_{p,\rm max}$	-	10^9	-	10^7
s_e	1.9	-	2.01	2.01
s_p	-	1.9	-	2.01
L_e^{inj} [erg s^{-1}]	$3.1 \cdot 10^{40}$	-	10^{48}	$3.7 \cdot 10^{40}$
L_p^{inj} [erg s^{-1}]	-	$1.1 \cdot 10^{45}$	-	$2.8 \cdot 10^{46}$
U_{ext} [erg s^{-1}]	-	$3.6 \cdot 10^{-2}$	-	-
$\epsilon_{ext}^{\rm min}$ [erg]	-	$8.2 \cdot 10^{-13}$	-	-
$\epsilon_{ext}^{\rm max}$ [erg]	-	$8.2 \cdot 10^{-8}$	-	-
Photon Index	-	2	-	-

[*] Particle distribution is modeled as $N_i(\gamma) = K\gamma^{-s_i} e^{-\gamma/\gamma_{i,\rm coff}}$, for $\gamma \geq \gamma_{i,\rm min}$.

Introduction to the “emcee” Sampler: Exploring Posterior Distributions with MCMC

- The “emcee” sampler is a Python package for MCMC sampling.
- It uses ensemble sampling with multiple walkers to explore the parameter space.
- Walkers propose new parameter sets based on current positions, accepting or rejecting based on data likelihood.
- Through iterations, walkers converge towards regions of higher posterior probability.
- Samples generated by “emcee” provide estimates of statistical properties (mean, median, etc.) for inference and uncertainty quantification.

Cosmic-ray acceleration in M87 current sheets (work in progress)

Reconnection rate from 3D PIC: $\eta_{rec} = \frac{v_{rec}}{c} \sim 0.06$

Typical size of current sheets: $l \sim (5-10) \cdot r_g$

Acceleration timescale from 3D PIC:

$$\frac{d\gamma_{acc}}{dt} \approx \frac{eE_{rec}v_z}{mc^2} = \frac{e\eta_{rec}B_0\beta_z}{mc} = \eta_{rec}\beta_z\omega_0$$

In the MAD regime the dimensionless magnetic flux, threading the black hole horizon takes the maximum value 50:

$$\varphi_{BH} = \frac{\Phi_{BH}}{\sqrt{cr_g{}^2\dot{M}}} \xrightarrow{MAD} \Phi_{BH} \sim 50\sqrt{cr_g{}^2\dot{M}}$$

$$\Phi_{BH} \simeq 4\pi r_H^2 B_0$$

$$\dot{M} = \dot{m}\,\dot{M}_{EDD} = \dot{m}\frac{L_{EDD}}{\eta_c c^2} \overset{\eta_c=0.1}{\simeq} 10^{27}\dot{m}M_9\ [g/s]$$

$$B_0 = 10^{5.24}\frac{\dot{m}^{1/2}(M_9\eta_{c,-1})^{-1/2}}{(1+\sqrt{(1-\alpha_s)})^2}[G]$$