



Observatoire  
de la CÔTE d'AZUR



MANITOU  
summer school

# Ground based GW detectors

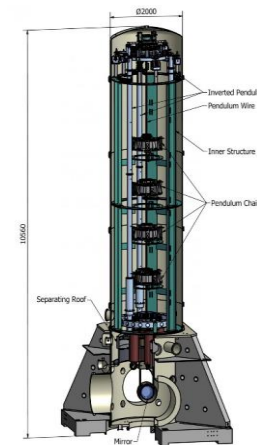
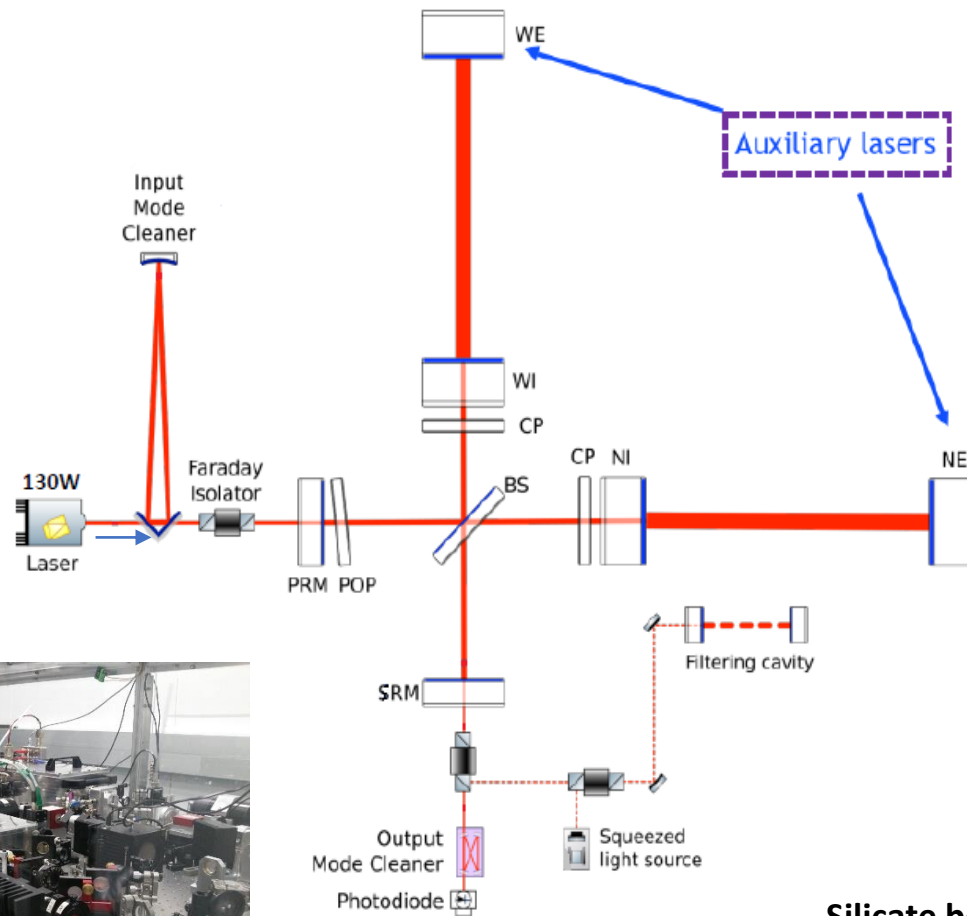
Walid Chaibi

# French-Italian-(Dutch) ground based Gravitational wave detector



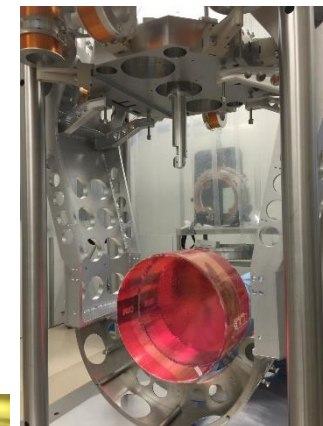
Virgo

Cascina-Pisa-Italy

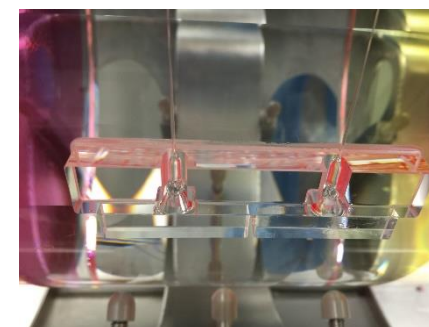


Super-attenuator

Monolithic suspension



Silicate bonding



Squeezed light source



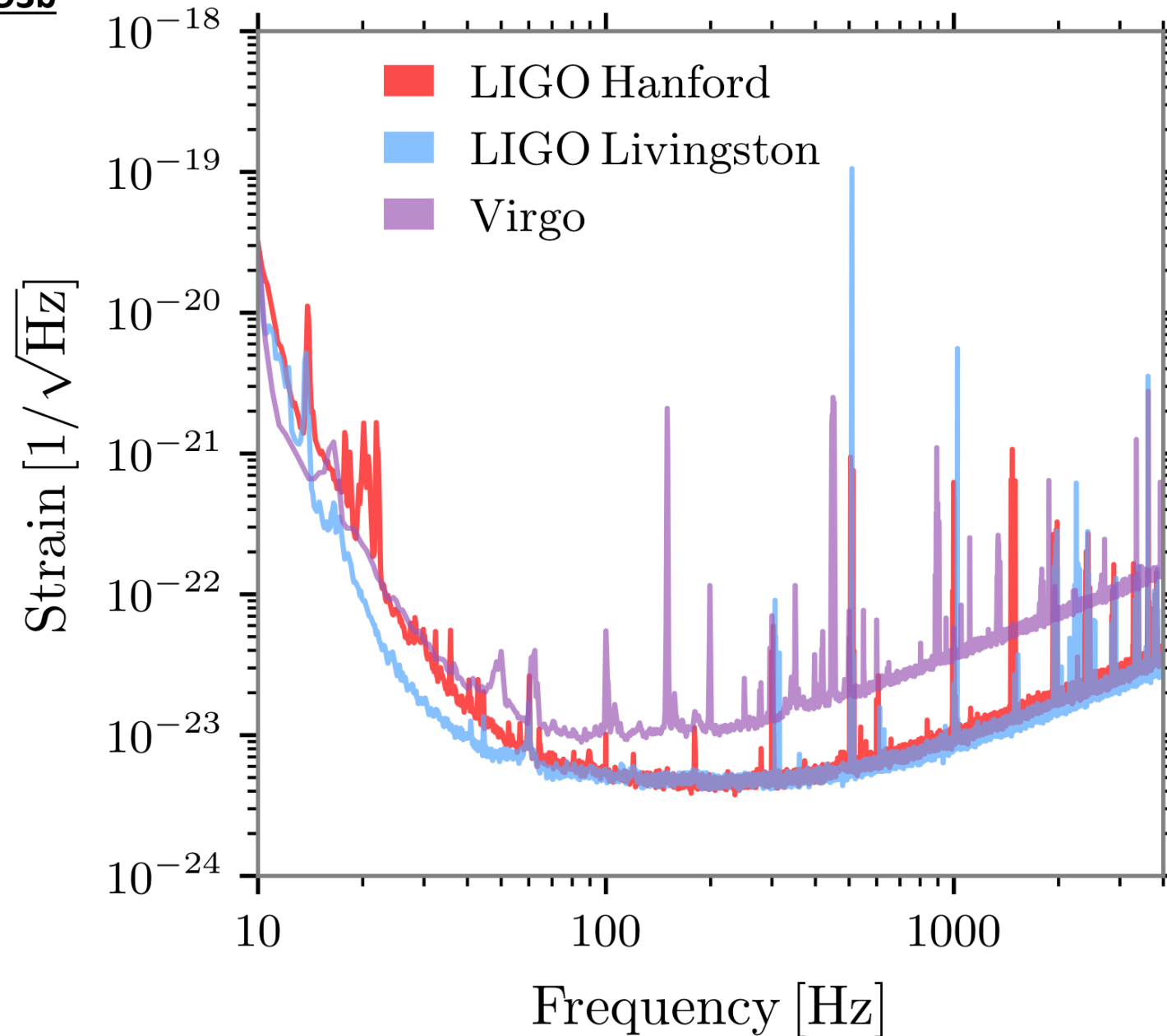
## How does it work?

### High power laser

- O4 : 45W
- O5 : 80-150W
- Post-O5 : 450W
- E-T : 700W

# Sensitivity curves

Observing run: O3b



**What does this correspond to?**

**The noise (for an experimental physicist)**

**Detection principle**

**Interferometer output and tuning :**

- **The Michelson interferometer**
- **Sensitivity enhancement : The Fabry Perot cavity**
- **Shot noise limited detector**

**Noise contribution:**

- **Harmonic oscillator model**
- **Seismic noise**
- **Thermal noise**
- **Quantum noise**

**Conclusion**

**1<sup>st</sup> lecture**

**2<sup>nd</sup> lecture**

## The noise (for an experimental physicist)

### Detection principle

#### Interferometer output and tuning :

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#### Noise contribution:

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### Conclusion

# The noise: Definition

Noise : Random signal which is combined to a physical quantity to be measured

Specific case : additive noise

$$s = x + \varepsilon$$

↑                      ↑                      ↑  
signal                      Quantity                      noise  
to be  
measured

$$\varepsilon \rightarrow p(\varepsilon)$$

Probability  
density

Specific case : time-dependent noise

$$s(t) = x(t) + \varepsilon(t)$$

$$\varepsilon(t) \rightarrow p_t(\varepsilon)$$

Time dependent  
probability density

# Noise characterization

Mean value :

$$\mu_{\varepsilon}(t) = \langle \varepsilon(t) \rangle = \int_{-\infty}^{+\infty} \varepsilon \times p_t(\varepsilon) d\varepsilon$$

$\langle \dots \rangle$  computed by repeating the experiments

Variance :

$$\mu_{\varepsilon,2}(t) = \sigma_{\varepsilon}^2(t) = \langle (\varepsilon(t) - \mu_{\varepsilon}(t))^2 \rangle = \int_{-\infty}^{+\infty} (\varepsilon(t) - \mu_{\varepsilon}(t))^2 \times p_t(\varepsilon) d\varepsilon$$

Moment of order  $n$ :

$$\mu_{\varepsilon,n}(t) = \langle (\varepsilon(t) - \mu_{\varepsilon}(t))^n \rangle = \int_{-\infty}^{+\infty} (\varepsilon(t) - \mu_{\varepsilon}(t))^n \times p_t(\varepsilon) d\varepsilon$$

↓ .....

Mean value of  $f(\varepsilon)$ :

$$\mu_{f(\varepsilon)}(t) = \langle f(\varepsilon(t)) \rangle = \int_{-\infty}^{+\infty} f(\varepsilon(t)) \times p_t(\varepsilon) d\varepsilon$$

 But this is not enough to characterize the whole process...

# Autocorrelation

What is the relationship between  $\varepsilon(t_1)$  and  $\varepsilon(t_2)$  ?

$$\Gamma_{\varepsilon}(t_1, t_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varepsilon_1 \times \varepsilon_2 \times \underbrace{p_{t_1, t_2}(\varepsilon_1, \varepsilon_2)}_{\substack{\text{joint probability} \\ \text{density function}}} d\varepsilon_1 d\varepsilon_2 = \langle \varepsilon(t_1) \varepsilon(t_2) \rangle$$

In principle, we would need to know all autocorrelation functions of order  $n$ :

$$\Gamma_{\varepsilon}(t_1, t_2, \dots, t_n) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \varepsilon_1 \times \varepsilon_2 \times \dots \times \varepsilon_n \times p_{t_1, t_2, \dots, t_n}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) d\varepsilon_1 d\varepsilon_2 \dots d\varepsilon_n$$

In practice, the second order autocorrelation function already gives enough information

# Stationary noise

Stationary noise : The noise characteristics do not change during time

$$\text{For all } T \quad p_{t_1, t_2, \dots, t_n}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) = p_{t_1+T, t_2+T, \dots, t_n+T}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$$

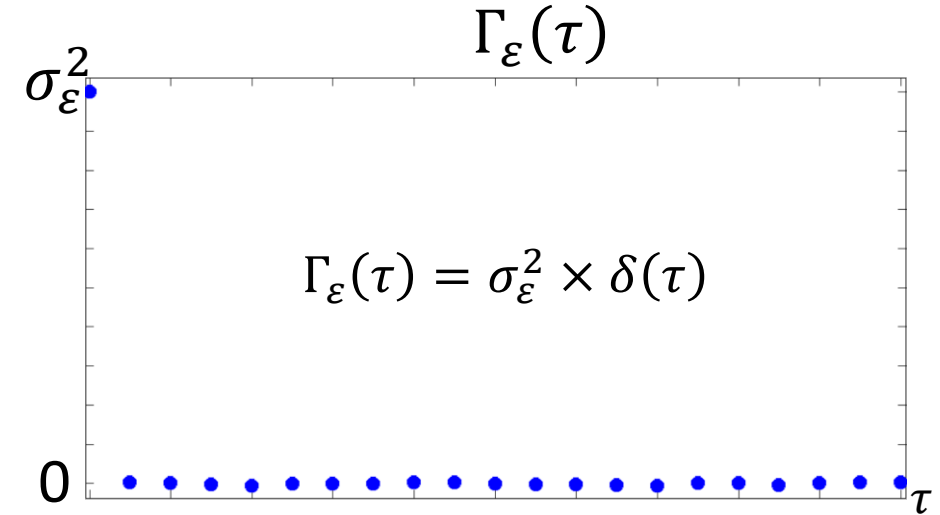
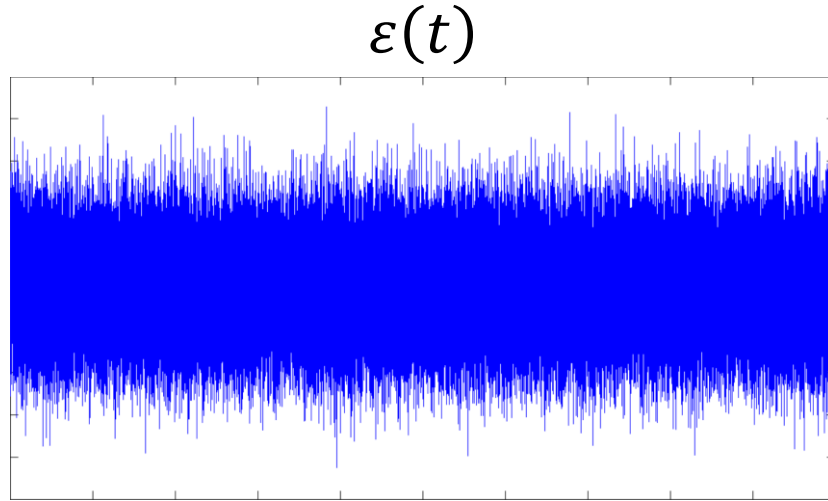
One interesting case: centered and 2<sup>nd</sup> order stationary noise:  $p_t(\varepsilon) = p_{t+T}(\varepsilon) = p(\varepsilon) \quad p_{t_1, t_2}(\varepsilon_1, \varepsilon_2) = p_{t_1+T, t_2+T}(\varepsilon_1, \varepsilon_2)$

Zero mean value  $\mu_\varepsilon(t) = \langle \varepsilon(t) \rangle = 0$

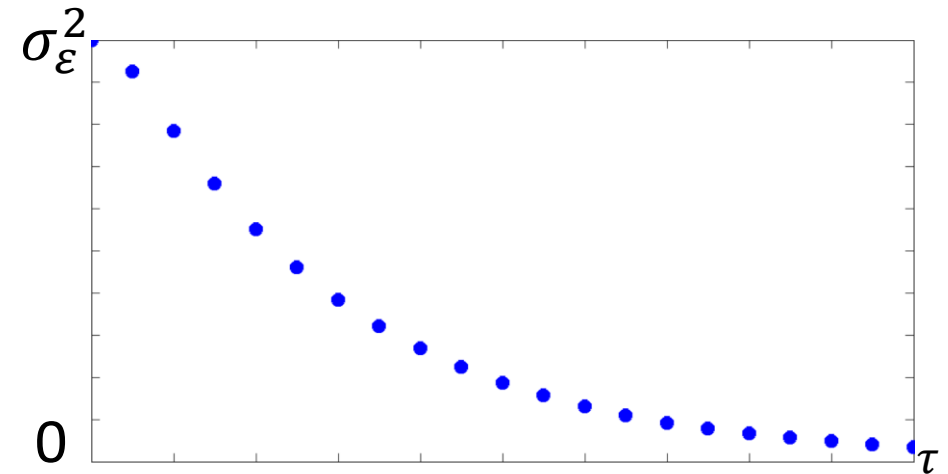
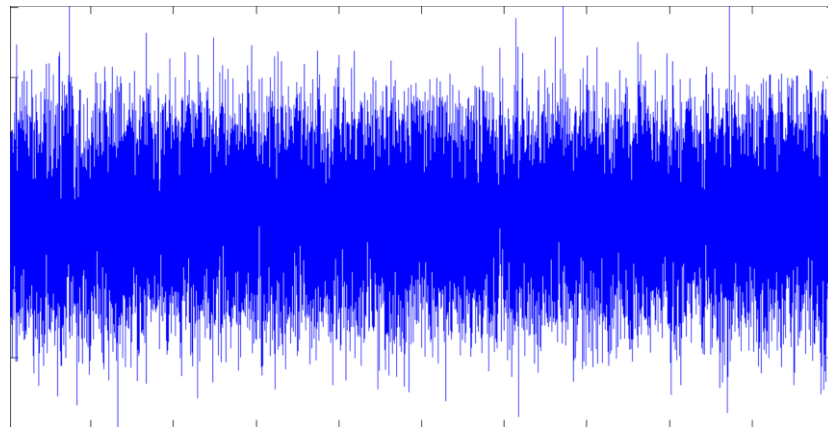
Standard deviation  $\sigma_\varepsilon^2(t) = \sigma_\varepsilon^2$

Autocorrelation 
$$\begin{cases} \Gamma_\varepsilon(t, t + \tau) = \Gamma_\varepsilon(0, \tau) = \Gamma_\varepsilon(\tau) \\ \sigma_\varepsilon^2 = \Gamma_\varepsilon(0) \geq \Gamma_\varepsilon(\tau) \end{cases}$$

White noise



Colored noise



# Temporal average

for one experiment

$$m_f = \overline{f(\varepsilon)} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(\varepsilon(t)) dt$$

$$m_\varepsilon = \bar{\varepsilon} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \varepsilon(t) dt$$

$$s_\varepsilon^2 = \overline{\varepsilon^2} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \varepsilon^2(t) dt$$

$$C_\varepsilon(\tau) = \overline{\varepsilon(t)\varepsilon(t+\tau)} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \varepsilon(t)\varepsilon(t+\tau) dt$$

2<sup>nd</sup> order ergodicity : all experiments are equivalent

$$m_{\varepsilon_i} = m_{\varepsilon_j} = m_\varepsilon = \bar{\varepsilon}$$

$$s_{\varepsilon_i}^2 = s_{\varepsilon_j}^2 = s_\varepsilon^2 = \overline{\varepsilon^2}$$

$$C_{\varepsilon_i}(\tau) = C_{\varepsilon_j}(\tau) = C_\varepsilon(\tau) = \overline{\varepsilon(t)\varepsilon(t+\tau)}$$

# Stationary and ergodic process

➔ All considered process are centered, stationary and ergodic

$$\mu_{f(\varepsilon)}(t) = \langle f(\varepsilon(t)) \rangle = \int_{-\infty}^{+\infty} f(\varepsilon) \times p_t(\varepsilon) d\varepsilon \quad m_{f(\varepsilon)} = \overline{f(\varepsilon)} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(\varepsilon(t)) dt$$

$$\mu_{f(\varepsilon)} = \overbrace{\mu_{f(\varepsilon)}(t)}^{\text{stationarity}} = \overbrace{\langle f(\varepsilon_i(t)) \rangle}^{\text{linearity}} = \overbrace{\langle \overline{f(\varepsilon_i(t))} \rangle}^{\text{ergodicity}} = \langle m_{f(\varepsilon_i)} \rangle = \langle m_{f(\varepsilon)} \rangle = m_{f(\varepsilon)}$$

➔ Averages on different experiments is equivalent to averages over time

$$m_\varepsilon = \mu_\varepsilon ; s_\varepsilon^2 = \sigma_\varepsilon^2 ; C_\varepsilon(\tau) = \Gamma_\varepsilon(\tau)$$

Finite energy process:  $\mathcal{E} = \int_{-\infty}^{+\infty} |f(t)|^2 dt$

its Fourier transform exists:  $\tilde{f}(\nu) = \int_{-\infty}^{+\infty} e^{-i2\pi\nu t} f(t) dt$

Parseval equality:  $\mathcal{E} = \int_{-\infty}^{+\infty} |\tilde{f}(\nu)|^2 d\nu$



$|\tilde{f}(\nu)|^2$  represents the energy density per spectral interval  $d\nu$ :  
the **Energy Spectral Density (ESD)**

# Harmonic analysis : The Fourier transform

Continuous process has an infinite energy, example : a laser beam  $\int_{-\infty}^{+\infty} |f(t)|^2 dt = \infty$

But...it has a finite mean power:  $\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |f(t)|^2 dt$

For noise, the mean power is given by the variance:  $\sigma_\varepsilon^2 = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \varepsilon(t)^2 dt$

If it exists, we're interested in the quantity:  $S_\varepsilon(\nu) = \lim_{T \rightarrow \infty} \frac{1}{2T} |\tilde{\varepsilon}_T(\nu)|^2 = \lim_{T \rightarrow \infty} \frac{1}{2T} \left| \int_{-T}^T e^{-i2\pi\nu t} \varepsilon(t) dt \right|^2$

Summed over the frequency  $\nu$ :

$$\int_{-\infty}^{+\infty} S_\varepsilon(\nu) d\nu = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-\infty}^{+\infty} |\tilde{\varepsilon}_T(\nu)|^2 d\nu = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-\infty}^{+\infty} \varepsilon_T(t)^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \varepsilon(t)^2 dt = \sigma_\varepsilon^2$$



$S_\varepsilon(\nu)$  represents the power density per spectral interval  $d\nu$ :  
the **Power Spectral Density (PSD)**

# Harmonic analysis : The Wiener-Khintchine Theorem

For a stationary and ergodic process (noise), the PSD exists and is given by the Fourier transform of the corresponding autocorrelation function:

$$S_{\varepsilon}(\nu) = \lim_{T \rightarrow \infty} \frac{1}{2T} \left| \int_{-T}^T e^{-i2\pi\nu t} \varepsilon(t) dt \right|^2 = \tilde{\Gamma}_{\varepsilon}(\nu) = \int_{-\infty}^{+\infty} e^{-i2\pi\nu\tau} \Gamma_{\varepsilon}(\tau) d\tau$$

➡ Very useful for analytic calculations

➡ For measurements : 
$$S_{\varepsilon}(\nu) \simeq \frac{1}{2T_{acq}} \left\langle \left| \int_{-T_{acq}}^{T_{acq}} e^{-i2\pi\nu t} \varepsilon(t) dt \right|^2 \right\rangle_N ; T_{acq} \gg f_{min}$$

Unity...? example:

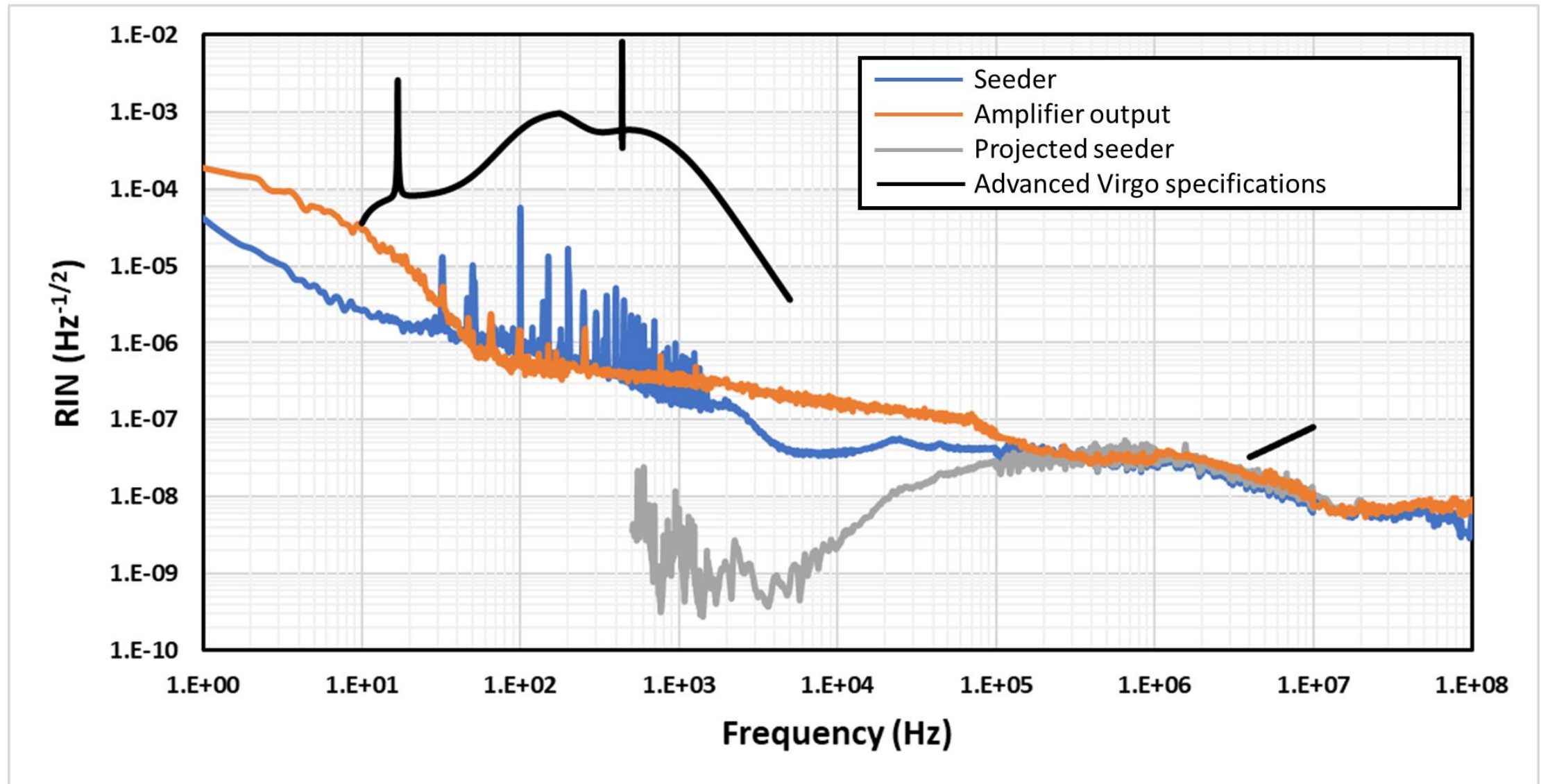
Voltage noise:  $\delta V(t) \rightarrow V$

$\Gamma_{\delta V}(\tau) \rightarrow V^2$

PSD :  $S_{\delta V}(\nu) \rightarrow V^2/\text{Hz}$

$\delta V(\nu) = \sqrt{S_{\delta V}(\nu)} \rightarrow V/\sqrt{\text{Hz}}$

# Example: the RIN spectrum of a laser



**Some math : The noise**

**Detection principle**

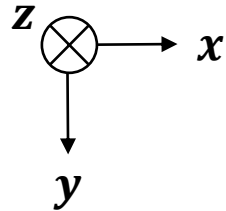
**Interferometer output and tuning :**

- **The Michelson interferometer**
- **Sensitivity enhancement : The Fabry Perot cavity**
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**Noise contribution:**

- **Harmonic oscillator model**
- **Seismic noise**
- **Thermal noise**
- **Quantum noise**

**Conclusion**



(+) polarization



(x) polarization



Space time metric in the TT gauge

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Minkowski

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & +h & 0 & 0 \\ 0 & 0 & -h & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

GW, (+) polarization

Light follows the geodesic

$$g_{\mu\nu} dx^\mu dx^\nu = 0$$

$$dx^\mu = (cdt, dx, dy, dz)$$

$$c^2 dt^2 + dx^2(-1 + h) + dy^2(-1 - h) - dz^2 = 0$$

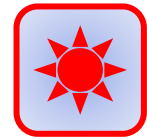
# Effect on a photon

Photon propagating along  $x \rightarrow dy = 0 ; dz = 0$

$$dx = \pm c \sqrt{\frac{1}{1-h}} dt \simeq \pm c \left( 1 + \frac{1}{2} h \right) dt$$

Photon propagating along  $y \rightarrow dx = 0 ; dz = 0$

$$dy = \pm c \left( 1 - \frac{1}{2} h \right) dt$$



**LASER**

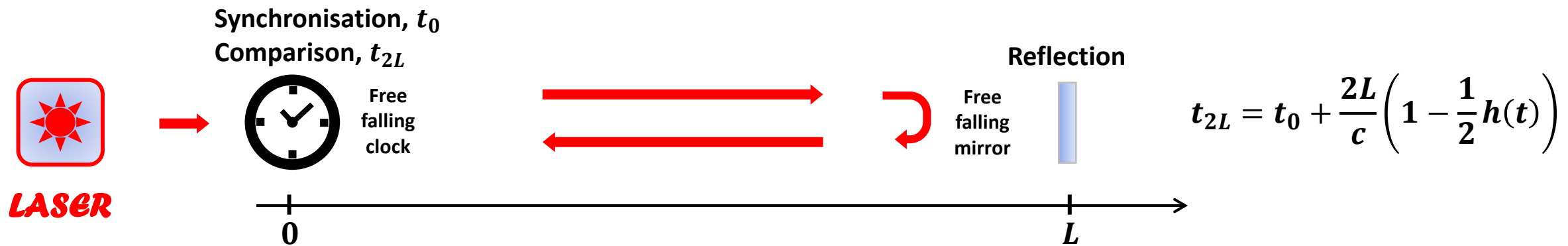
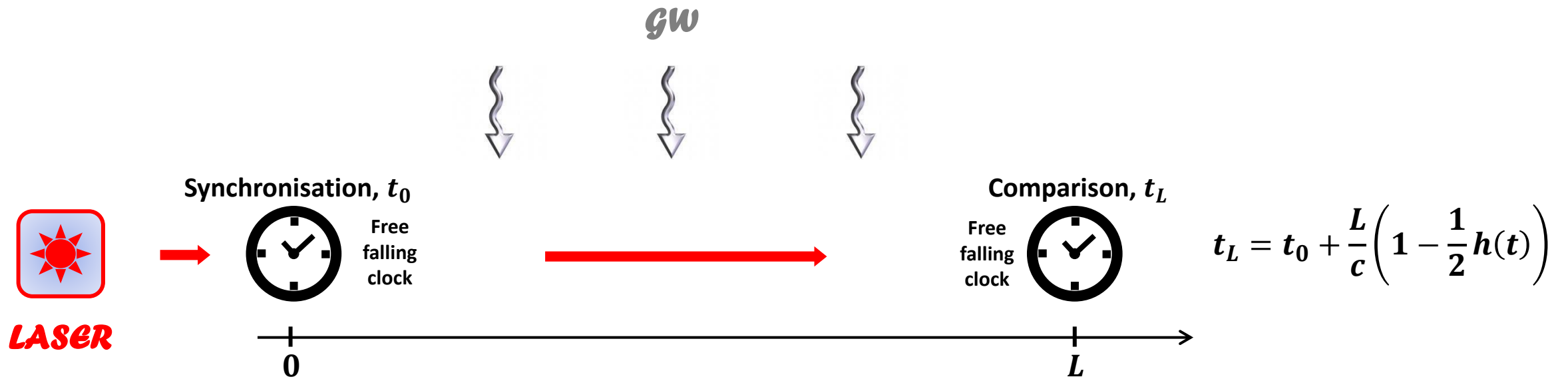


Towards  $x > 0$ , from 0 to  $L$

$$L = \int_0^L dx = c \int_{t_0}^{t_L} \left( 1 + \frac{1}{2} h(t) \right) dt = c \left( \int_{t_0}^{t_L} dt + \frac{1}{2} \int_{t_0}^{t_L} h(t) dt \right)$$

$$t_L = t_0 + \frac{L}{c} \left( 1 - \frac{1}{2} h(t) \right) \quad \text{if} \quad t[h] \gg \frac{L}{c}$$

# Detection by time delay measurement



# What is a clock?

Oscillator synchronized on a reference (atomic transition, optical cavity)

$$\text{Oscillator } E(t) = E_0 \left( 1 + a(t) \right) \times e^{-i(2\pi\nu_0 t + \varphi(t))} \rightarrow E(t) = E_0 \left( 1 + a_{ref}(t) \right) \times e^{-i(2\pi\nu_0 t + \varphi_{ref}(t))}$$

↓ Amplitude     ↓ Amplitude noise     ↓ Phase     ↓ Phase noise

➔

Single frequency laser  $\nu_0$

Time propagation measurement



Phase measurement

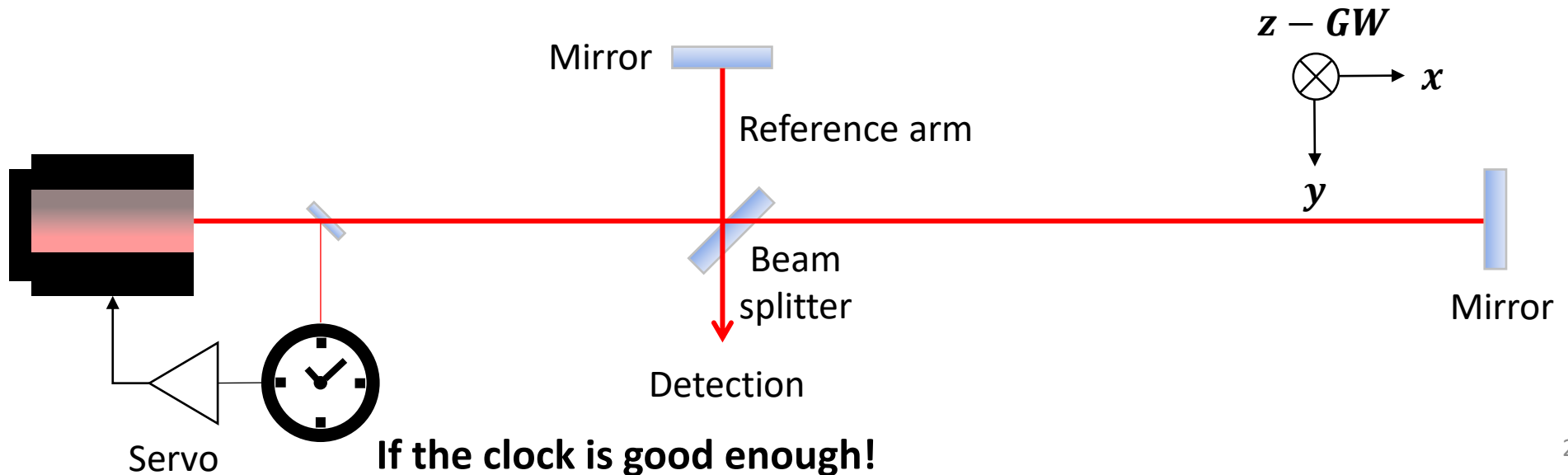


Interference

Synchronization



Locking techniques (servo loops)



**Some math : The noise**

**Detection principle**

**Interferometer output and tuning :**

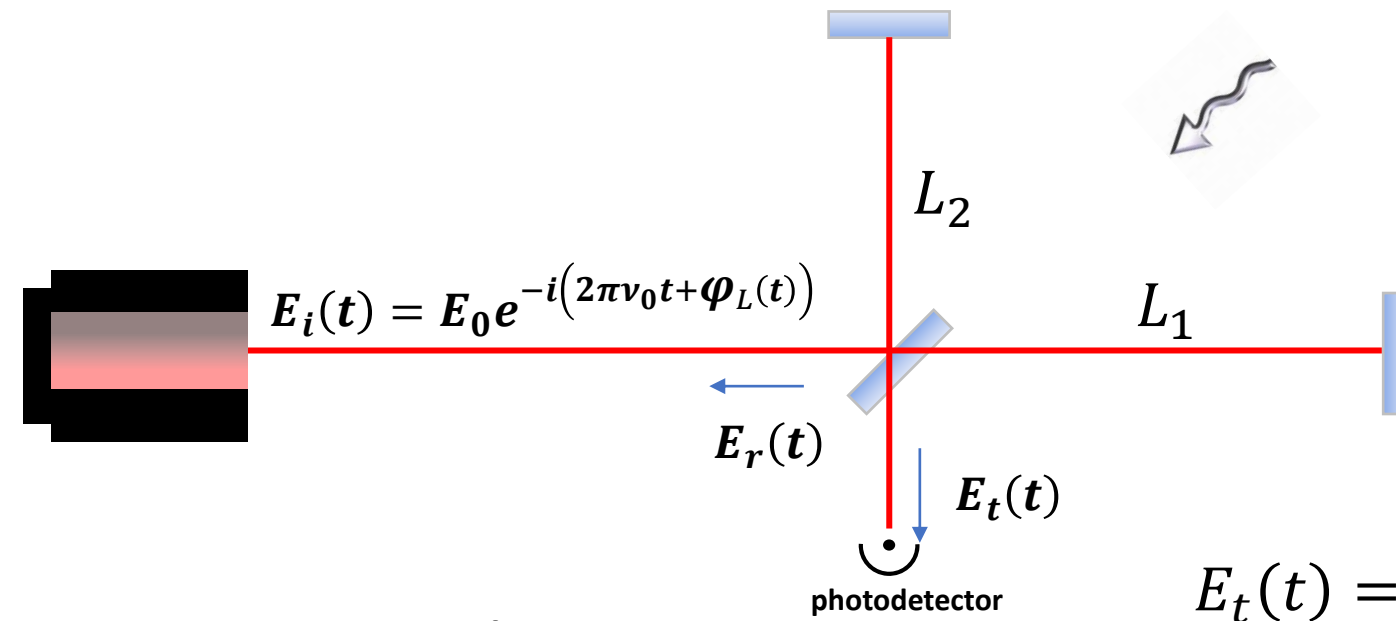
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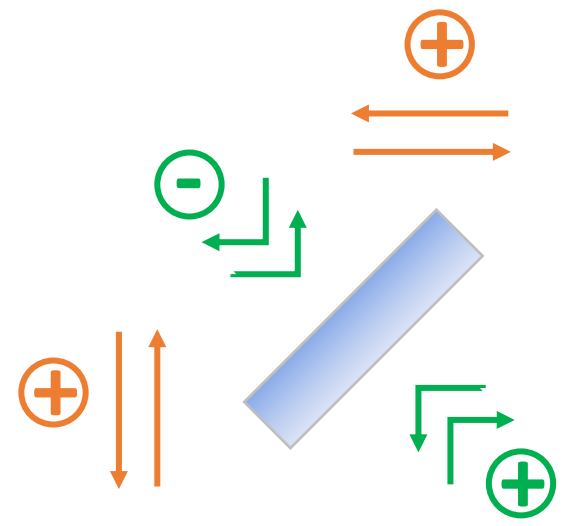
- **Harmonic oscillator model**
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- **Quantum noise**

**Conclusion**

# Michelson interferometer



**Beam splitter 50/50**



$$E_t(t) = \frac{1}{2} E_0 e^{-i2\pi\nu_0 t} (e^{i\phi_1} - e^{i\phi_2})$$

$$E_r(t) = \frac{1}{2} E_0 e^{-i2\pi\nu_0 t} (e^{i\phi_1} + e^{i\phi_2})$$

$$\phi_1 = \frac{4\pi\nu_0}{c} L_1 \left( 1 - \frac{h(t)}{2} \right) - \varphi_L \left( t - \frac{2L_1}{c} \right)$$

$$\phi_2 = \frac{4\pi\nu_0}{c} L_2 \left( 1 + \frac{h(t)}{2} \right) - \varphi_L \left( t - \frac{2L_2}{c} \right)$$

# Michelson interferometer

$$\phi = \frac{\phi_2 + \phi_1}{2} \quad ; \quad \Delta\phi = \frac{\phi_2 - \phi_1}{2} \quad ; \quad L = \frac{L_2 + L_1}{2} \quad ; \quad \Delta L = \frac{L_2 - L_1}{2}$$

$$E_t(t) = -iE_0 e^{-i2\pi\nu_0 t} \times e^{i\phi} \sin(\Delta\phi)$$

$$E_r(t) = E_0 e^{-i2\pi\nu_0 t} \times e^{i\phi} \cos(\Delta\phi)$$

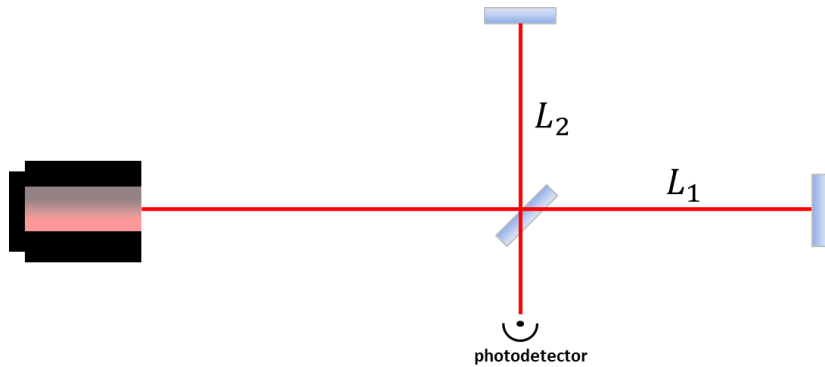
Frequency noise

$$\phi = \frac{1}{2} \left( \frac{8\pi\nu_0 L}{c} + \frac{4\pi\nu_0 \Delta L}{c} h(t) - 2\varphi_L(t) + \frac{8\pi\delta\nu(t)L}{c} \right)$$

$$\delta\nu(t) = \frac{1}{2\pi} \frac{d\varphi_L(t)}{dt}$$

$$\Delta\phi = \frac{1}{2} \left( \frac{8\pi\nu_0 \Delta L}{c} + \frac{4\pi\nu_0 L}{c} h(t) - \frac{2\pi\delta\nu(t)\Delta L}{c} \right)$$

# Michelson interferometer : Dark fringe



$$P_t(t) = |E_t(t)|^2 = P_0 \times \sin^2 \left( 4\pi \frac{\Delta L}{\lambda} + 2\pi \frac{Lh(t)}{\lambda} - \pi \frac{\delta\nu(t)}{\nu_0} \frac{\Delta L}{\lambda} \right)$$

Signal  $\propto \frac{Lh(t)}{\lambda}$  : increase the arm length of the interferometer

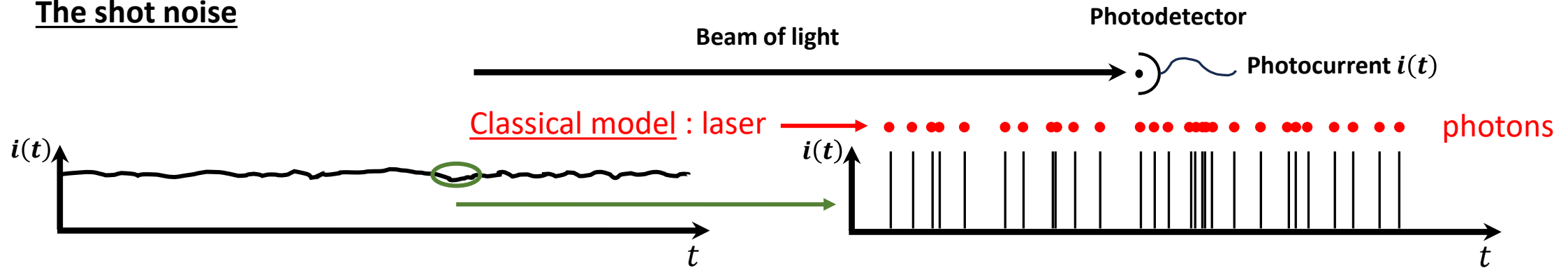
Contamination by the frequency noise : reduced for  $\Delta L \rightarrow 0$ , dark fringe

Dark fringe  $\Delta L \simeq 0$ ,  $P_t \propto \left( \frac{L \times h}{\lambda} \right)^2$  : need an offset  $\Delta L_{\text{offset}}$

How to choose the offset?

# Detection noise

## The shot noise



Ideal laser : Number of photons  $N(t)$  during  $\Delta t$  follows a Poissonian law

Mean value  $\bar{N}$

Variance  $\sigma_N^2 = \bar{N}$

Autocorrelation  $\Gamma(\tau) = \sigma_N^2 \delta(\tau)$

Power:  $P_{\Delta t}(t) = \frac{h_P \nu N(t)}{\Delta t}$ ; Mean Power:  $P_0 = \frac{h_P \nu \bar{N}}{\Delta t}$ ; Variance:  $\sigma_{P, \Delta t}^2 = \frac{h_P \nu_0 P_0}{\Delta t} = \int_{1/\Delta t} h_P \nu_0 P_0 df$ ; PSD:  $S_{\text{shot}}(f) = h_P \nu_0 P$

$P = 10 \text{ mW}$ ;  $\sqrt{S_{\text{shot}}(f)} \simeq 4 \times 10^{-11} \frac{\text{W}}{\sqrt{\text{Hz}}}$  @  $\lambda = 1064 \text{ nm}$ ,  $\text{RIN}(f) \simeq \frac{\sqrt{S_{\text{shot}}(f)}}{P} = 4 \times 10^{-9} \text{ Hz}^{-1/2}$

## The photodetector dark noise

$\sqrt{S_{\text{shot}}(f)} \simeq 300 \frac{\text{nV}}{\sqrt{\text{Hz}}}$ ;  $\sqrt{S_D(f)} \simeq 15 \frac{\text{nV}}{\sqrt{\text{Hz}}}$ ;  $\rho_D = \frac{\sqrt{S_D(f)}}{\sqrt{S_{\text{shot}}(f)}} \simeq 0.1$

# Signal to noise ratio: SNR

$$P_t(t) = |E_t(t)|^2 = P_0 \times \sin^2 \left( 4\pi \frac{\Delta L}{\lambda} + 2\pi \frac{Lh(t)}{\lambda} \right) = P_0 \times \sin^2 \left( 4\pi \underbrace{\frac{\Delta L_0}{\lambda}}_{\Delta} + 4\pi \underbrace{\frac{\delta L(t)}{\lambda}}_{\text{Length noise}} + 2\pi \underbrace{\frac{Lh(t)}{\lambda}}_{\text{Signal}} \right)$$

On the photodetector  $\frac{\delta L(t)}{\lambda}, \frac{Lh(t)}{\lambda} \ll \Delta \ll 1$

$$P_{PD}(t) = 16\pi^2 \Delta^2 P_0(t) + 16\pi^2 P_0 \Delta \frac{Lh(t)}{\lambda} + 16\pi^2 P_0 \Delta \frac{\delta L}{\lambda} + \delta P_{\text{shot}}(16\pi^2 \Delta^2 P_0, t) + \delta P_D(t)$$

$h$  as a noise

$$S_{P_{PD}}(f) = 256\pi^4 \Delta^4 S_{\delta P}(f) + 256\pi^4 \Delta^2 P_0^2 \frac{L^2}{\lambda^2} S_h(f) + 256\pi^4 \Delta^2 P_0^2 \frac{1}{\lambda^2} S_{\delta L}(f) + h_P \nu_0 16\pi^2 \Delta^2 P_0 + S_{\delta P_D}(f)$$

SNR

$$\rho^2(f) = \frac{256\pi^4 \Delta^2 P_0^2 \frac{L^2}{\lambda^2} S_h(f)}{h_P \nu_0 16\pi^2 \Delta^2 P_0 + 256\pi^4 \Delta^2 P_0^2 \frac{1}{\lambda^2} S_{\delta L}(f) + 256\pi^4 \Delta^4 S_{\delta P}(f) + S_{\delta P_D}(f)} \xrightarrow{\text{Shot noise limited}} \rho_{\text{shot}}^2(f) = \frac{16\pi^2 P_0 L^2}{h_P \nu_0 \lambda^2} S_h(f)$$

$\Delta$  choice :

- Not to be limited by the dark noise
- What technically possible  $\overline{P_{PD}} = 16\pi^2 \Delta^2 P_0 < 100 \text{ mW}$
- Not too high to reduce power noise contribution after Pstab

**Does not depend on the offset!**

**Some math : The noise**

**Detection principle**

**Interferometer output and tuning :**

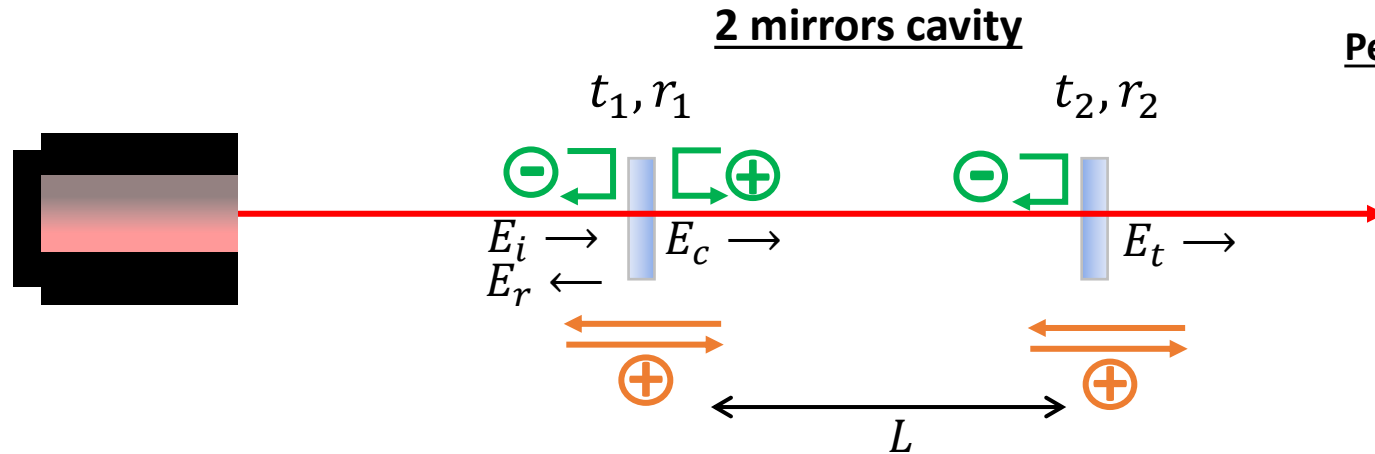
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**Conclusion**

# Enhancement of the interferometer sensitivity: Optical Cavities



**Perfect mirrors : energy conservation**

$$t_1^2 + r_1^2 = 1$$

$$t_2^2 + r_2^2 = 1$$

## Cavity equation

$$\left\{ \begin{array}{l} E_c(t) = t_1 E_i(t) - r_2 r_1 E_c \left( t - \frac{2L}{c} \right) \\ E_t(t) = t_2 E_c \left( t - \frac{L}{c} \right) \\ E_r(t) = -r_1 E_i(t) - r_2 t_1 E_c \left( t - \frac{L}{c} \right) \\ E_{(i,r,c,t)}(t) = E_{(i,c,r,t)_0}(t) e^{-i2\pi\nu_0 t} \end{array} \right.$$

Steady state

$$E_{(i,c,r,t)_0}(t)$$



$$\left\{ \begin{array}{l} E_{c0} = t_1 E_{i0} - r_2 r_1 e^{i\frac{4\pi\nu_0 L}{c}} E_{c0} \\ E_{t0} = t_2 e^{i\frac{2\pi\nu_0 L}{c}} E_{c0} \\ E_{r0} = -r_1 E_{i0} - r_2 t_1 E_{c0} e^{i\frac{4\pi\nu_0 L}{c}} \end{array} \right.$$

# Intracavity wave

$\phi = \frac{4\pi\nu_0 L}{c}$ : Round trip propagation phase

$$E_{c0} = \frac{t_1}{1+r_2r_1e^{i\phi}} E_{i0} = \Sigma(\phi) E_{i0}$$

$\Sigma(\phi)$ : Enhancement factor

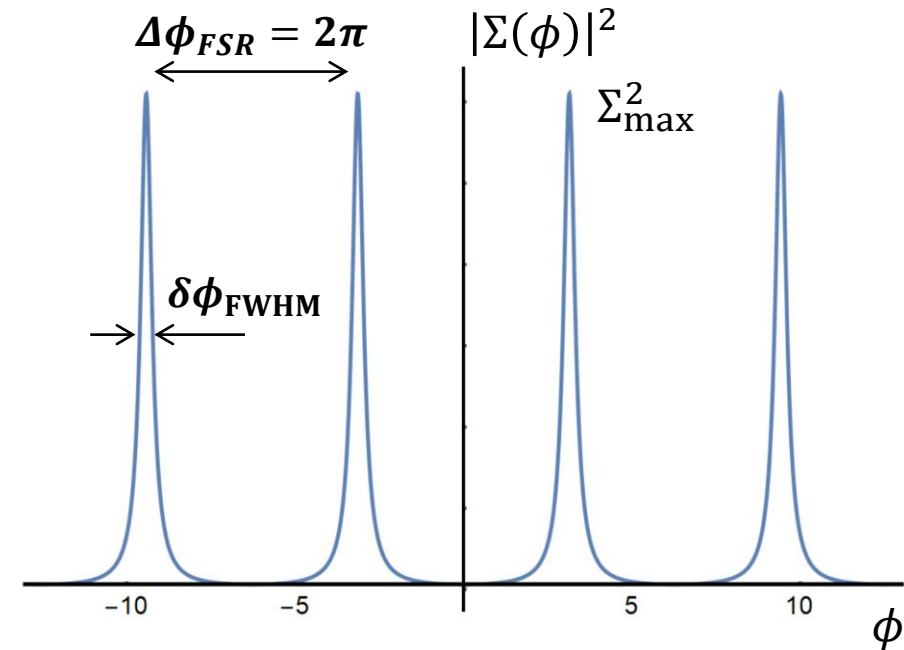
$$|\Sigma(\phi)|^2 = \frac{\Sigma_{\max}^2}{1 + \frac{4F^2}{\pi^2} \cos^2\left(\frac{\phi}{2}\right)}$$

$$\Sigma_{\max}^2 = \frac{t_1^2}{(1-r_2r_1)^2} \simeq_{r_1=r_2} \frac{1}{t_1^2} \gg 1$$

$$F = \frac{\pi\sqrt{r_1r_2}}{1-r_2r_1} \simeq_{r_1=r_2} \frac{\pi}{t_1^2} \gg 1, F = 10 \rightarrow 10^6$$

$$\Delta\phi_{FSR} = 2\pi; \Delta\nu_{FSR} = \frac{c}{2L}; \Delta L_{FSR} = \frac{\lambda}{2}$$

$$\delta\phi_{FWHM} = \frac{2\pi}{F}; \delta\nu_{FWHM} = \frac{\Delta\nu_{FSR}}{F}; \delta L_{FWHM} = \frac{\lambda}{2F}$$



Resonance condition

$$\phi = \pi + 2l\pi$$

# Gaussian beams : solution of the paraxial equation

Fundamental mode

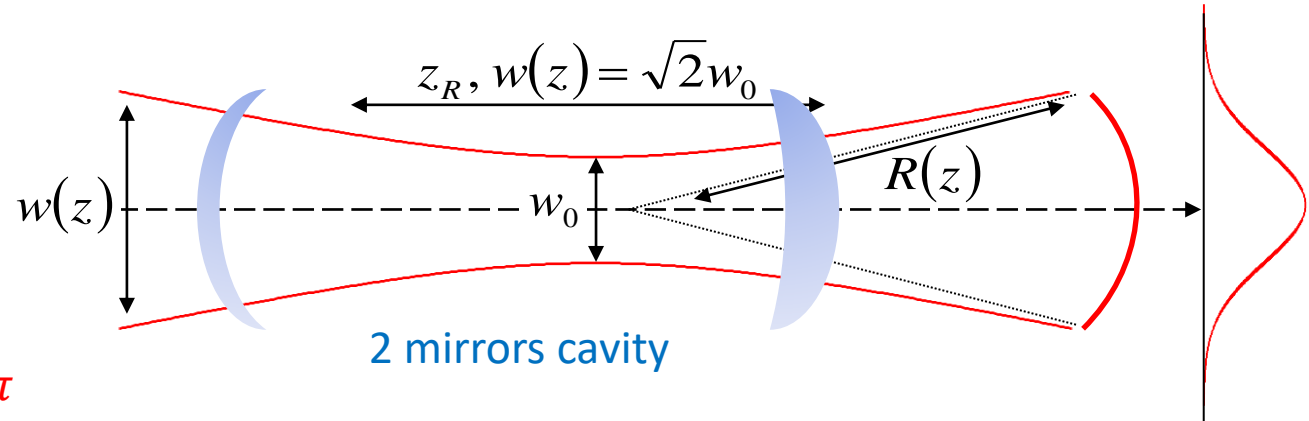
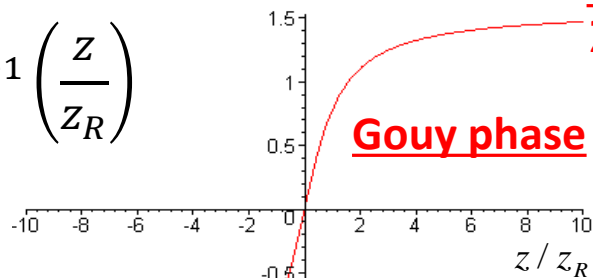
$$E(x, y, z, t) = E_0 \underbrace{\frac{1}{w(z)}}_{\text{Beam radius}} \times e^{i\left(\underbrace{\frac{2\pi}{\lambda}z}_{\text{Propagation phase}} + \underbrace{\psi_G(z)}_{\text{Gouy phase}}\right)} \times e^{-i \underbrace{\frac{2\pi}{\lambda} \frac{x^2+y^2}{2R(z)}}_{\text{Spherical wave front}}} \times e^{-\underbrace{\frac{x^2+y^2}{w^2(z)}}_{\text{Gaussian profile}}}$$

$$z_R = \frac{\pi w_0^2}{\lambda} : \text{Rayleigh range}$$

$$w(z) = w_0 \sqrt{1 + \frac{z^2}{z_R^2}}$$

$$R(z) = z + \frac{z_R^2}{z}$$

$$\psi_G(z) = \tan^{-1}\left(\frac{z}{z_R}\right)$$



**Resonance condition**

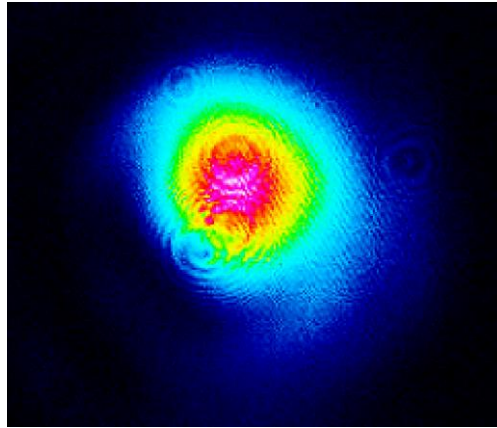
$$2\phi_{res} + 2\Delta\psi_G = \pi + 2l\pi$$

# High order modes

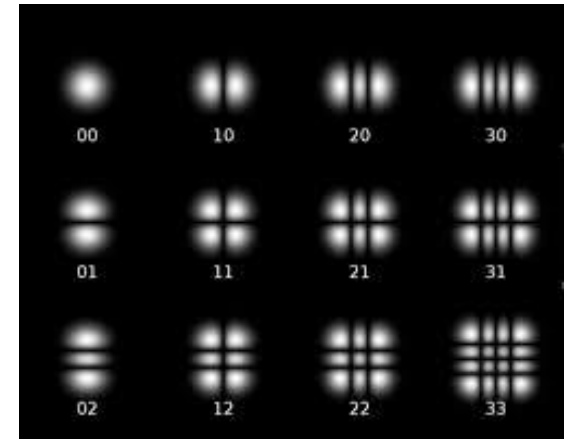
Gaussian mode : fundamental mode of a set of high order modes ; example : Hermite-Gauss modes

$$E_{n,m}(x, y, z, t) = E_{n,m,0} \frac{1}{w(z)} \times e^{i\left(\frac{2\pi}{\lambda}z + (n+m+1)\psi_G(z)\right)} \times H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_n\left(\frac{\sqrt{2}y}{w(z)}\right) e^{-i\frac{2\pi}{\lambda}\frac{x^2+y^2}{2R(z)}} \times e^{-\frac{x^2+y^2}{w^2(z)}}$$

$H_j$ : Hermite polynomial of order  $j$



decomposition

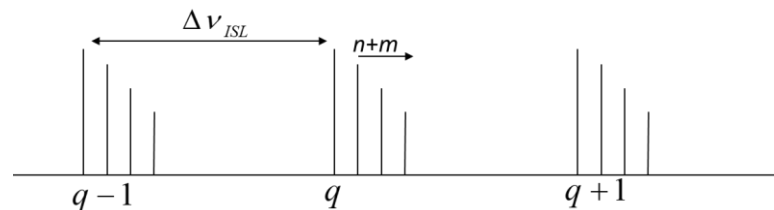


**Resonance condition**

$$\phi_{n,m,res} + 2(m + n + 1) \Delta\psi_G = \pi + 2l\pi$$



**depends on the mode order**



# Cavity transmission

Single mode injected  $\rightarrow$  Transmission:  $|T(\phi)|^2 = \left| \frac{E_t}{E_i} \right|^2 = \frac{T_{\max}^2}{1 + \frac{4F^2}{\pi^2} \cos^2\left(\frac{\phi}{2}\right)}$  Resonance  $\rightarrow$  Maximum transmission

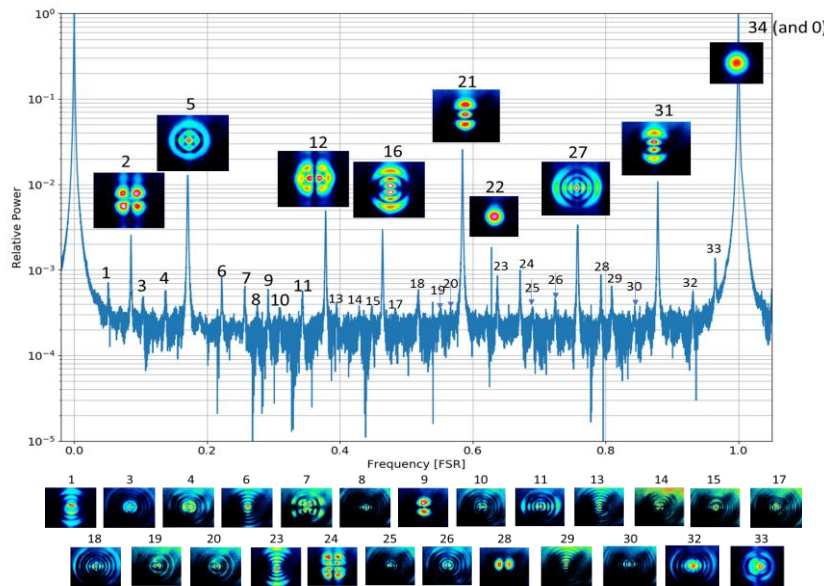
$r_2 = r_1 ; t_2 = t_1 \rightarrow T_{\max}^2 = 1$  **Totally transmissive cavity**

Real beam:

$$E_i = \sum_{m,n} c_{m,n,i} \times E_{m,n,i}$$

$\rightarrow$  Resonance on the fundamental mode

$$\begin{cases} |E_t|_{0,0}^2 = |c_{0,0,i}|^2 \\ |E_t|_{m \neq 0, n \neq 0}^2 \propto \frac{|c_{m,n,i}|^2}{F^2} \simeq 0 \end{cases}$$



**Mode cleaner cavity**

# Cavity reflectivity

$$R(\phi) = \frac{E_r}{E_i}$$

Specific configuration :  $r_2 = 1 ; t_2 = 0$



$$T = 0 ; |R(\phi)|^2 = 1$$

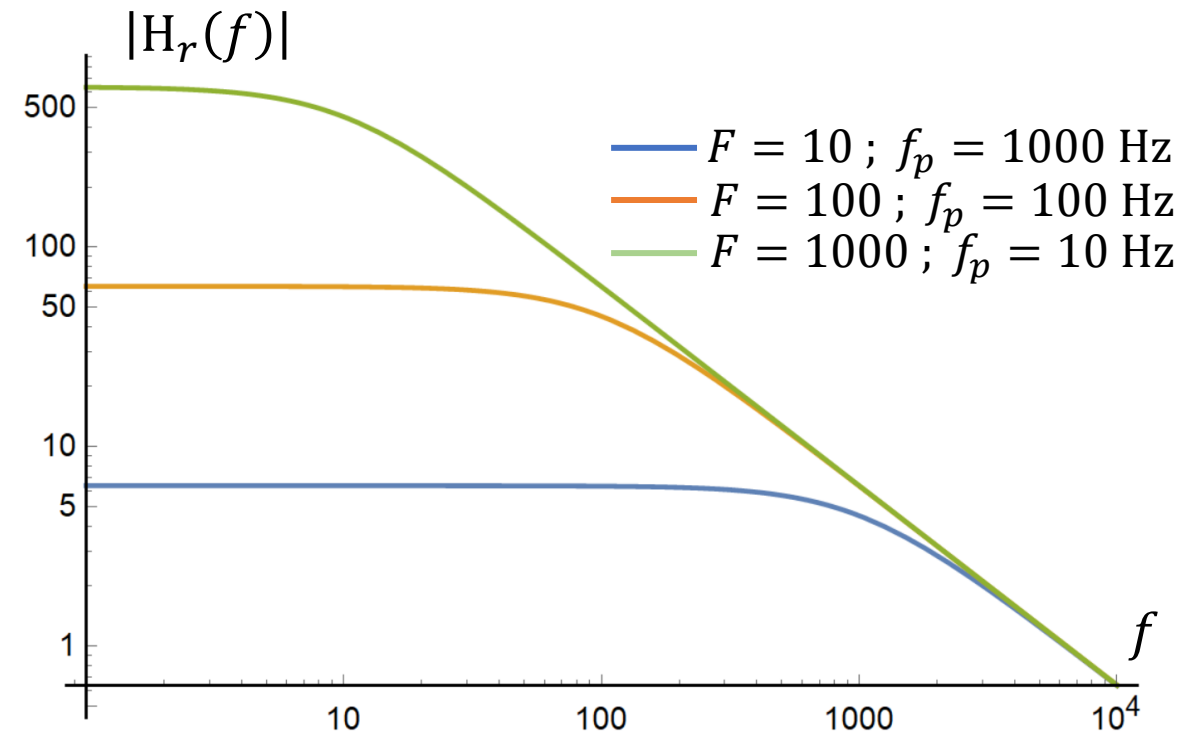
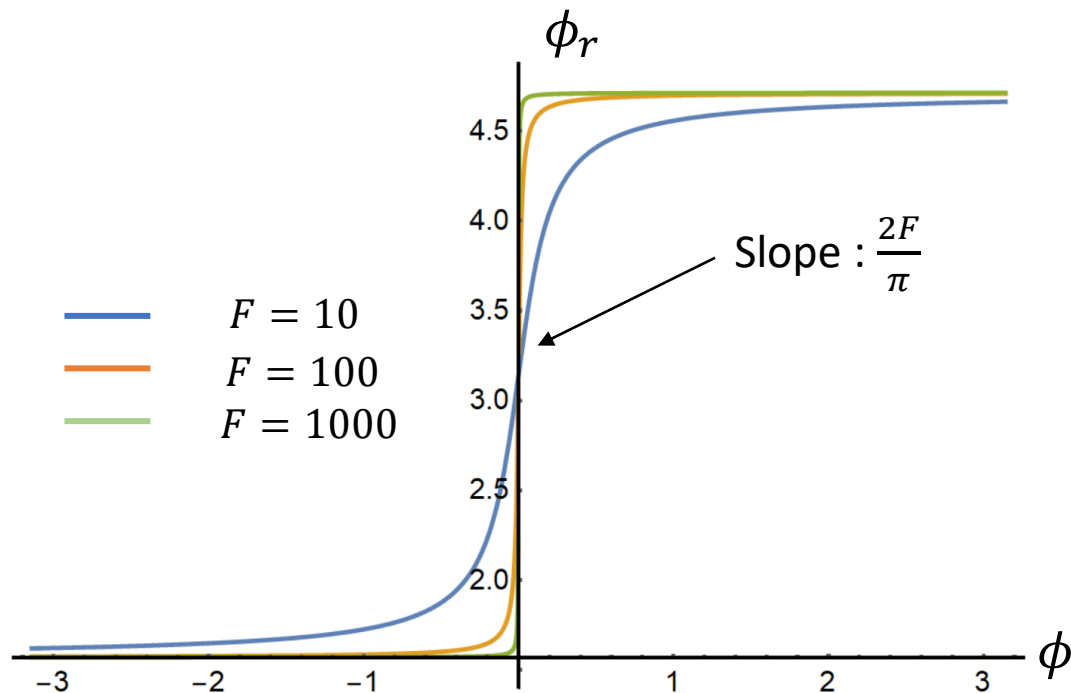
Phase without cavity

What about the phase response?

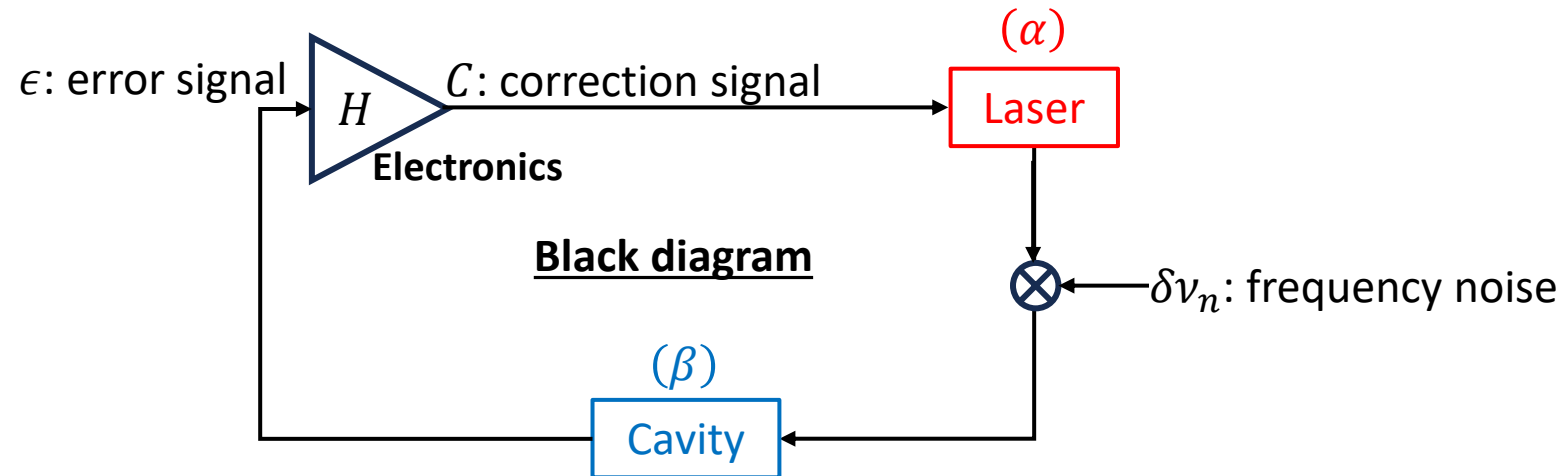
$$\phi_r = \text{Arg}(R(\phi)) = \pi + \tan^{-1}\left(\frac{2F}{\pi} \phi\right)$$

$$H_r(f) = \frac{\phi_r}{\phi}(f) \simeq \frac{2F/\pi}{1+i\frac{f}{f_p}} ; f_p = \frac{\delta\nu_{FWHM}}{2} : \text{cavity pole}$$

1<sup>st</sup> order filter



# Cavity as a frequency reference : servo loop



## Equations:

$$C = H \times \epsilon$$

$$\delta\nu = \alpha \times C + \delta\nu_n$$

$$\epsilon = \beta \times \delta\nu = \beta \times \delta\nu_n + \alpha\beta H \times \epsilon$$

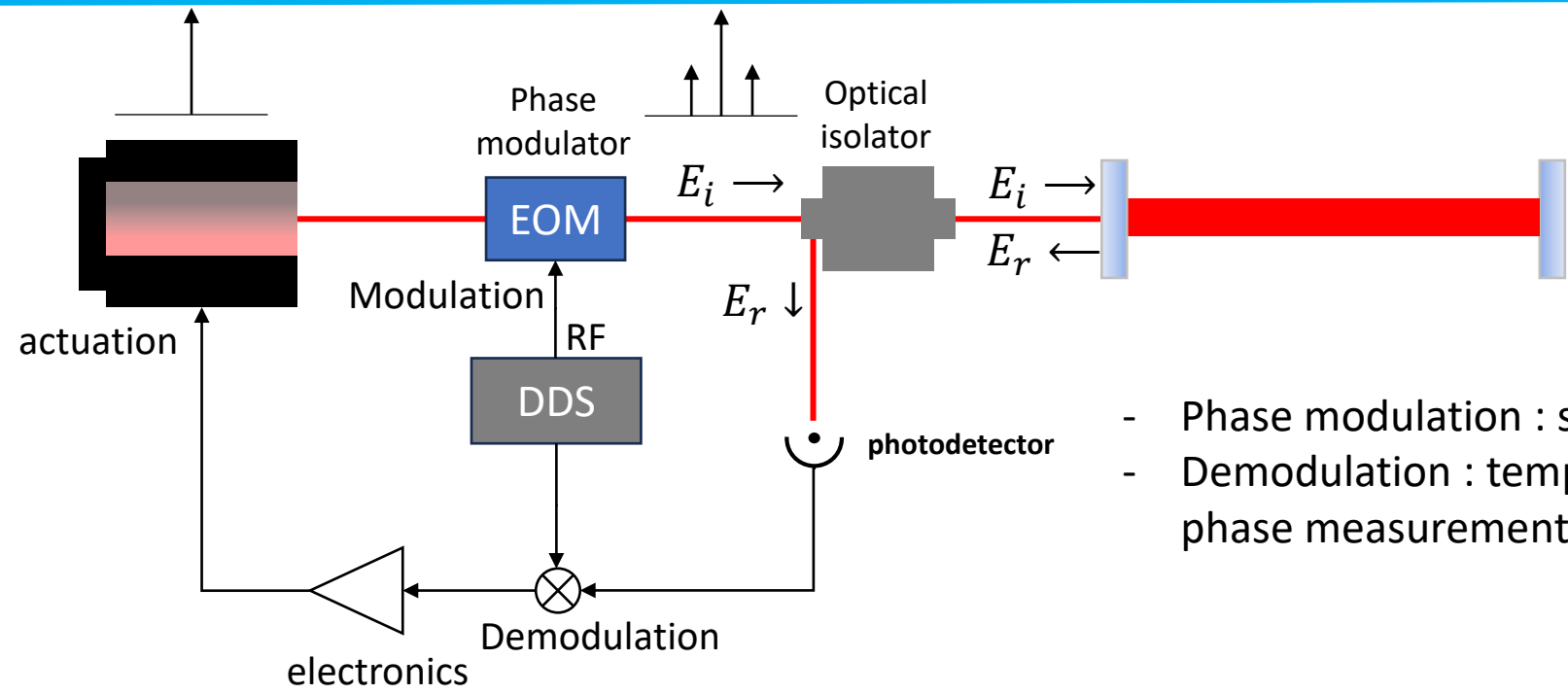


$$\epsilon = \frac{\beta \times \delta\nu_n}{1 - \alpha\beta H}$$

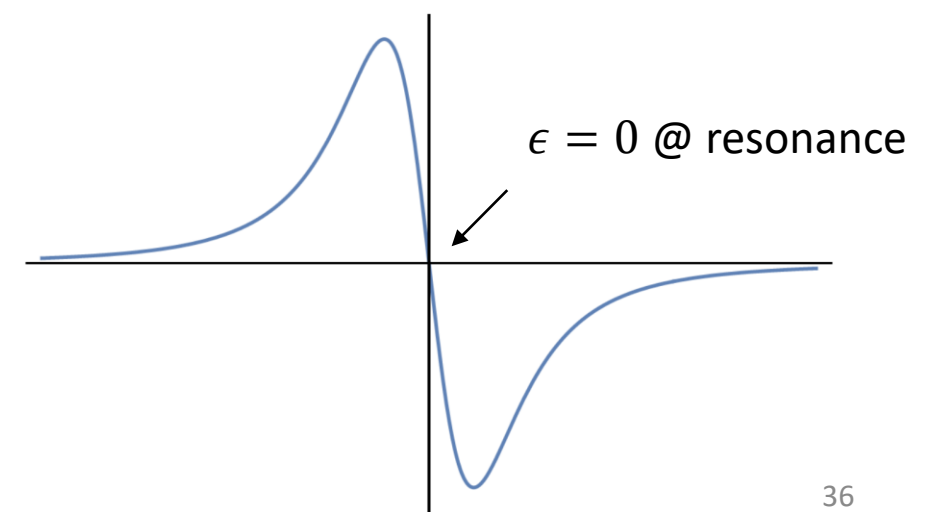
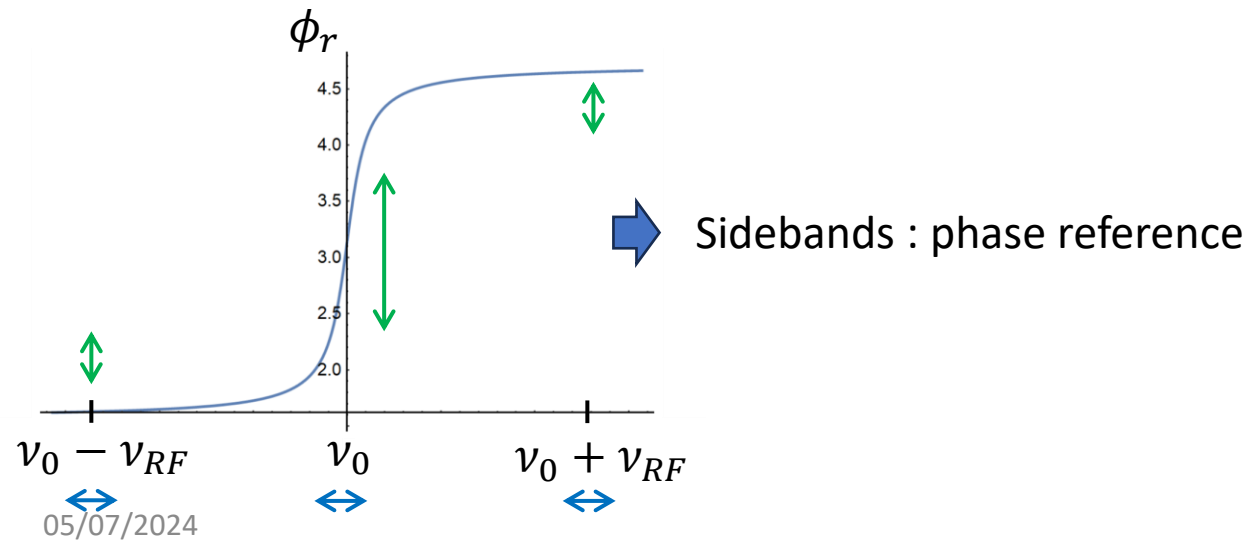
$$\alpha\beta H \gg 1 \quad \Rightarrow \quad \epsilon \simeq \frac{\delta\nu_n}{\alpha H} \rightarrow 0$$

**Problem : How to generate the error signal?**

# Pound Drever Hall (PDH) technique



- Phase modulation : sidebands generation
- Demodulation : temporal interference for phase measurement



**Some math : The noise**

**Detection principle**

**Interferometer output and tuning :**

- **The Michelson interferometer**
- **Sensitivity enhancement : The Fabry Perot cavity**
- **Shot noise limited detector**

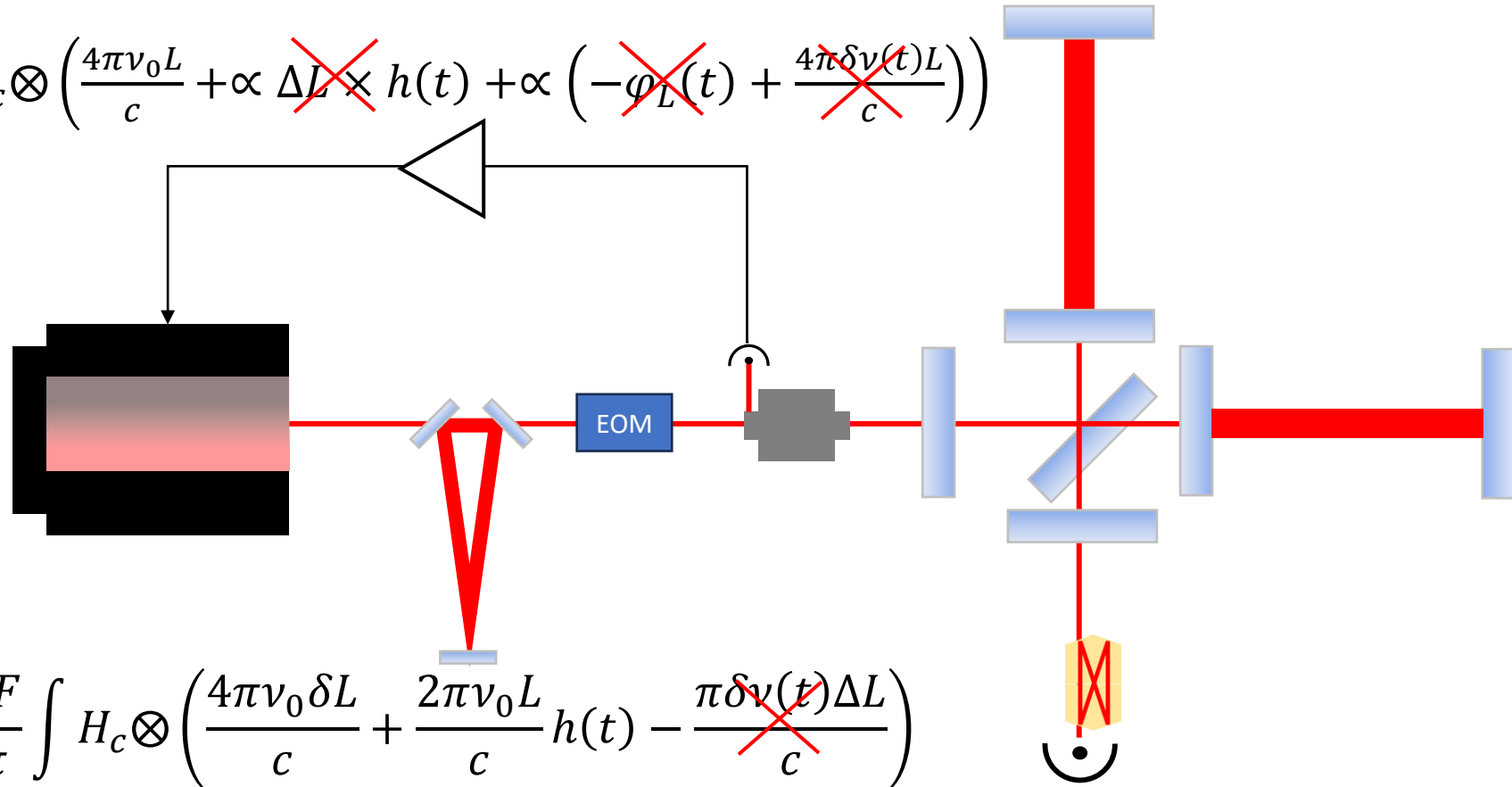
**Noises contribution:**

- **Harmonic oscillator model**
- **Seismic noise**
- **Thermal noise**
- **Quantum noise**

**Conclusion**

# Shot noise limited detector

$$\phi = \frac{2F}{\pi} \int H_c \otimes \left( \frac{4\pi\nu_0 L}{c} + \alpha \Delta L \times h(t) + \alpha \left( -\cancel{\varphi_L(t)} + \frac{4\pi\cancel{\delta\nu(t)L}}{c} \right) \right)$$



$$\Delta\phi = \frac{2F}{\pi} \int H_c \otimes \left( \frac{4\pi\nu_0 \delta L}{c} + \frac{2\pi\nu_0 L}{c} h(t) - \frac{\pi\cancel{\delta\nu(t)\Delta L}}{c} \right)$$

$P_0 = 25 \text{ W} ; G = 50 ; P_{PD} = 1 \text{ mW} ; F = 400 ; h = 10^{-23} \text{ Hz}^{-1/2} @ 100 \text{ Hz} ; \delta\nu < \mu\text{Hz} \text{ Hz}^{-1/2}$

$$\frac{\delta\nu}{\nu_0} \simeq 3 \times 10^{-21} \text{ Hz}^{-1/2}$$



The interferometer common mode is good enough !

# Shot noise limited detector : sensitivity curve

$$\rho_{shot}^2(f) = \frac{16\pi^2 P_0 L^2}{h_p \nu_0 \lambda^2} S_h(f)$$

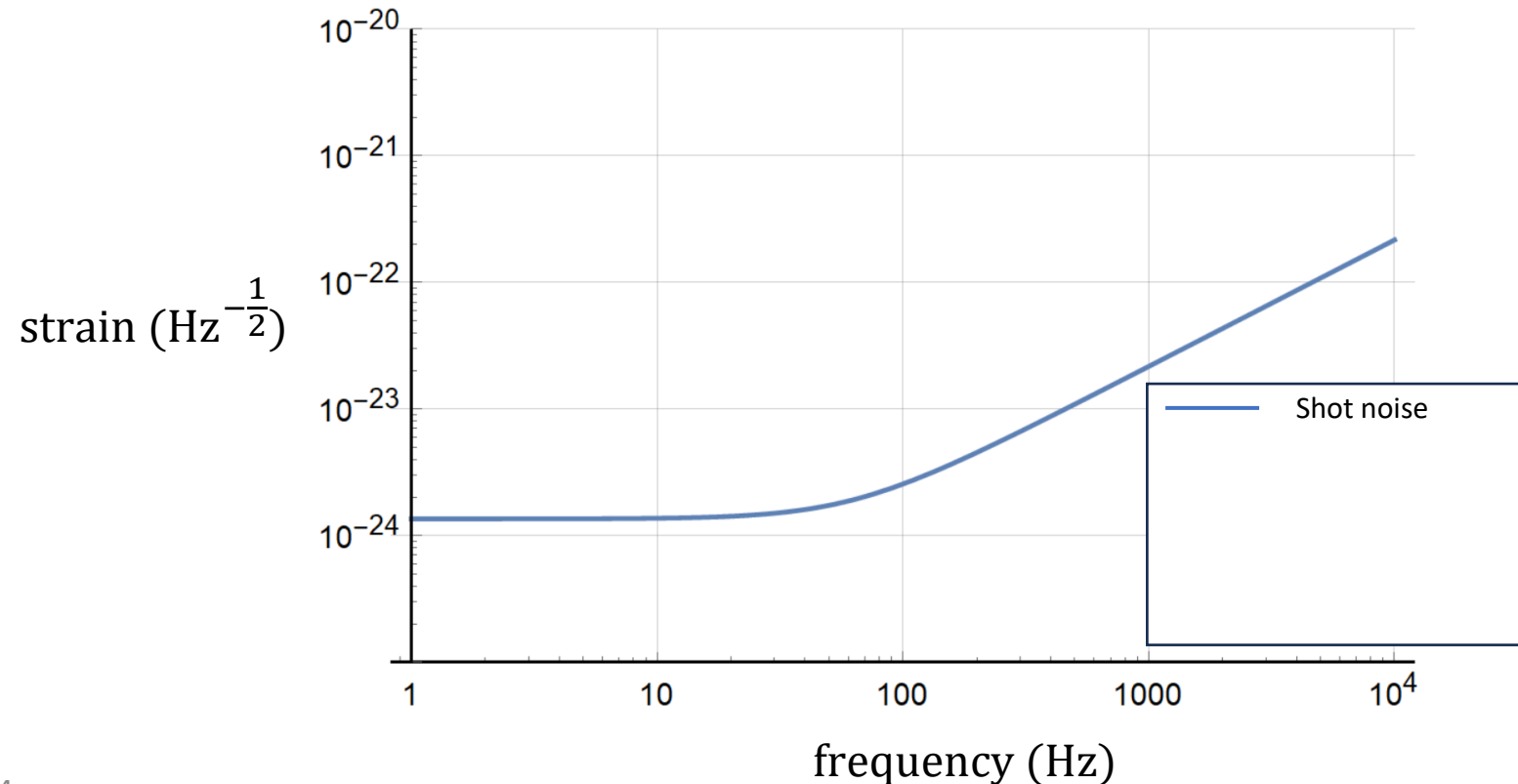
cavities  
➔

$$\rho_{shot}^2(f) = \frac{64F^2 G P_0 L^2}{h_p \nu_0 \lambda^2} \left| \frac{1}{1 + i \frac{f}{f_p}} \right|^2 S_h(f)$$

Sensitivity defined by  $\rho_{shot}^2(f) = 1$

➔

$$S_{h,shot}(f) = \frac{h_p \nu_0}{64F^2 G P_0} \frac{L^2}{\lambda^2} \left( 1 + \frac{f^2}{f_p^2} \right)$$



**Some math : The noise**

**Detection principle**

**Interferometer output and tuning :**

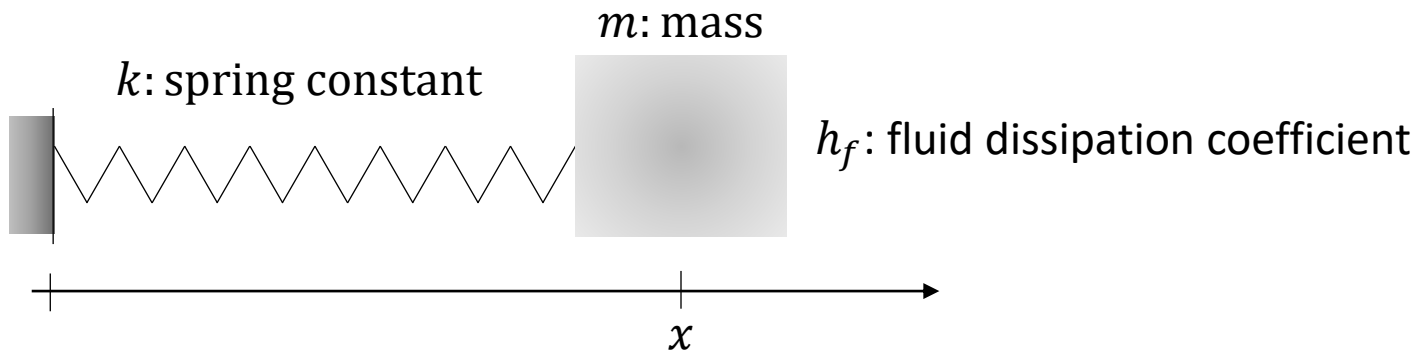
- **The Michelson interferometer**
- **Sensitivity enhancement : The Fabry Perot cavity**
- **Shot noise limited detector**

**Noise contribution:**

- **Harmonic oscillator model**
- **Seismic noise**
- **Thermal noise**
- **Quantum noise**

**Conclusion**

# Harmonic oscillator



Newton 2<sup>nd</sup> law:  $m \frac{d^2x}{dt^2} = F_0 - k(x - x_0) - h_f \frac{dx}{dt}$

$\chi(f) = \frac{x(f)}{F_0(f)/m + \omega_0^2 x_0(f)} = \frac{1/m}{\omega_0^2 - \omega^2 + i \frac{\omega \omega_0}{Q}}$ : mechanical susceptibility

$\omega = 2\pi f$ : angular frequency

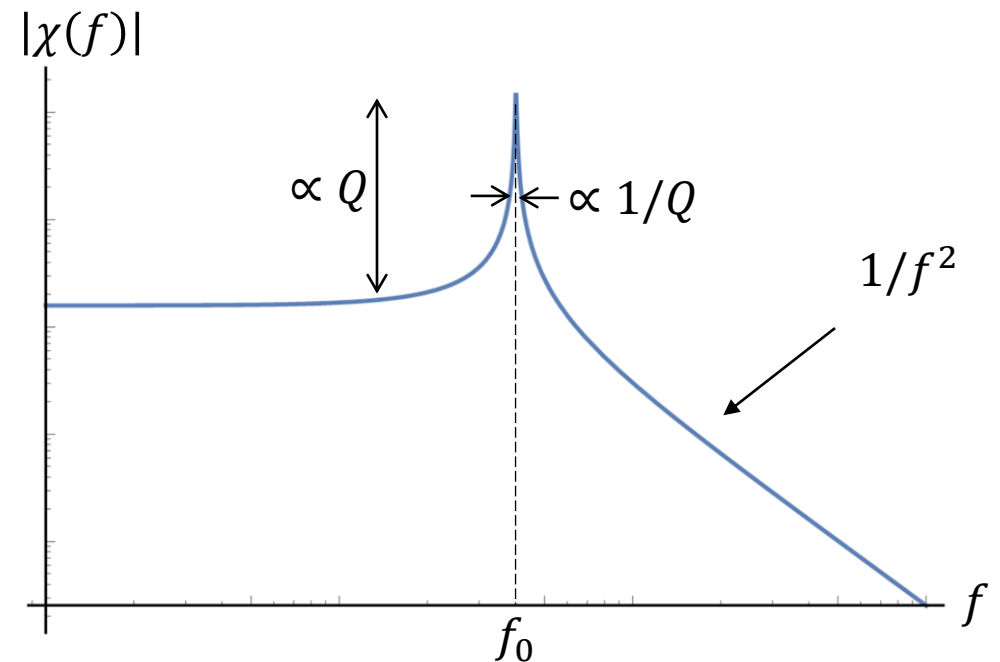
$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$ ,  $f_0$ : resonance frequency

$Q = \omega_0 \frac{m}{h_f}$ : mechanical quality factor

$p = m \frac{dx}{dt}$ : momentum

$H = \frac{p^2}{2m} + m \frac{\omega_0^2 x^2}{2}$ : Energy

05/07/2024



**Some math : The noise**

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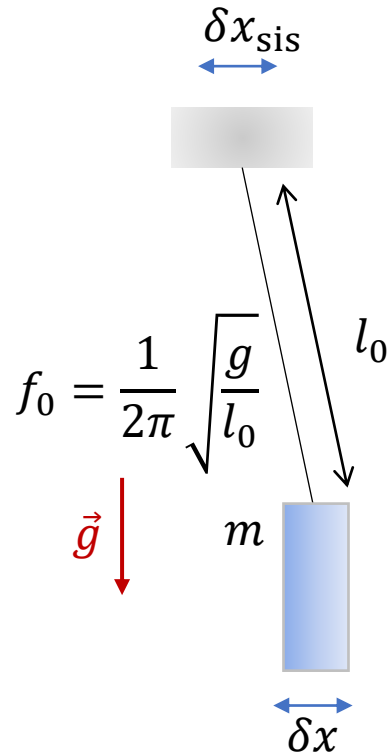
**Conclusion**

# Seismic noise

Free falling mirrors → Isolated from the external environment : seismic noise

Seismic noise : white acceleration noise for  $f \geq 1$  Hz  $\delta a(f) \simeq 4 \times 10^{-6} \text{ ms}^{-2} / \sqrt{\text{Hz}}$

$$\rightarrow \delta x_{\text{sis}}(f) \simeq 10^{-7} \left( \frac{1 \text{ Hz}}{f} \right)^2 \text{ m} / \sqrt{\text{Hz}}$$

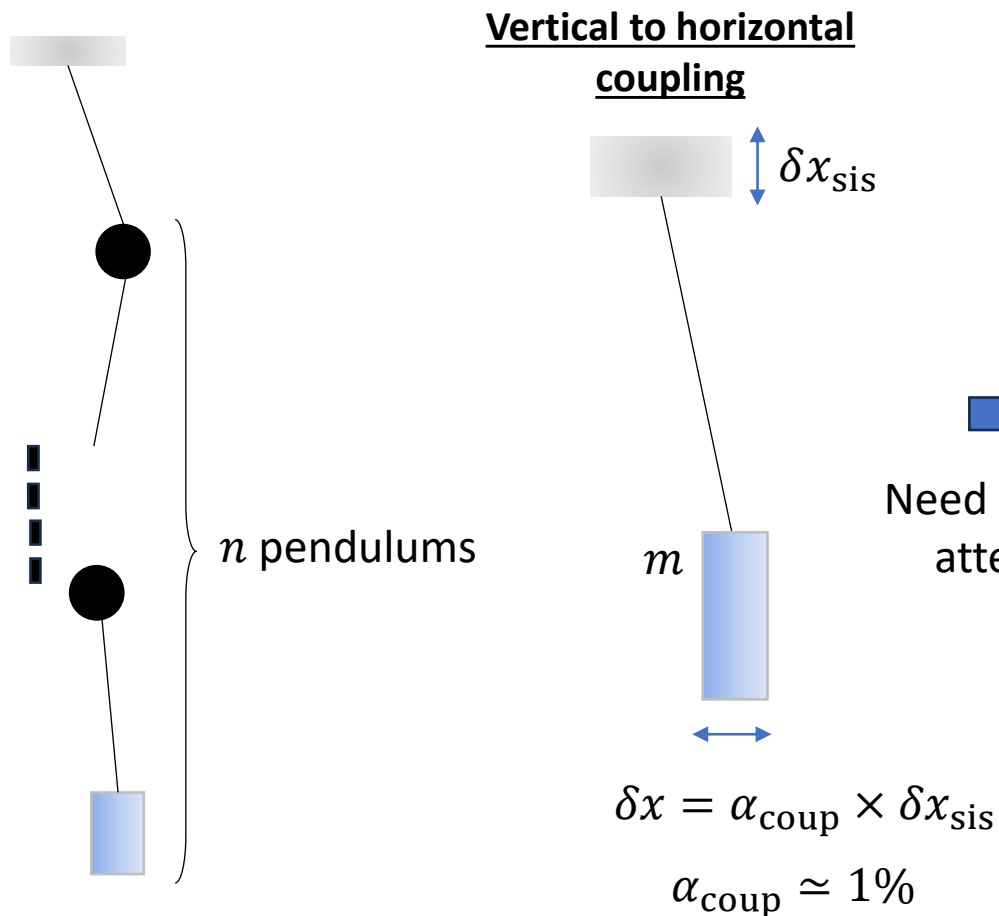


$$l_0 = 70 \text{ cm} ; f_0 = 0.6 \text{ Hz} \quad \delta x(f) = \chi(f) \times m \omega_0^2 \delta x_{\text{sis}}(f)$$

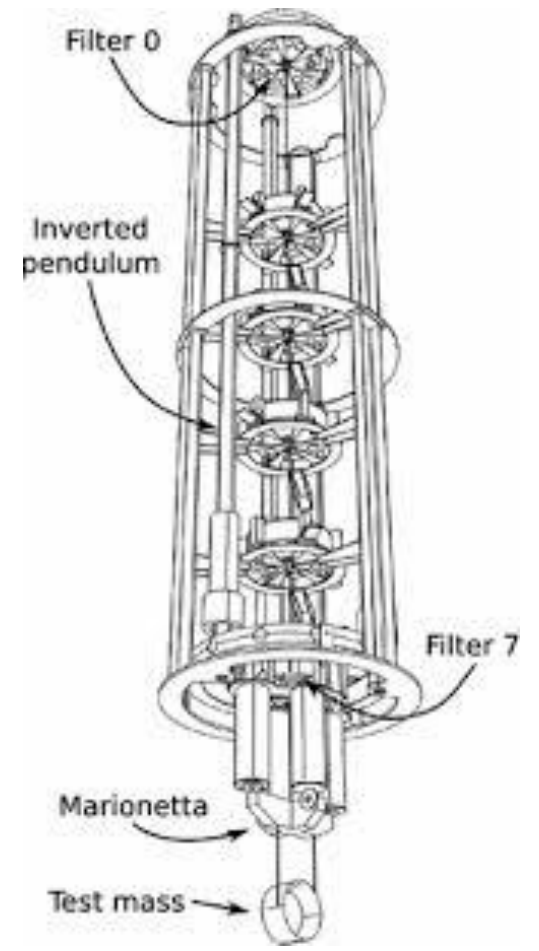
$$f \gg f_0 \quad \delta x(f) \simeq \frac{f_0^2}{f^2} \delta x_{\text{sis}}(f) \simeq 10^{-7} (1 \text{ Hz})^2 \times f_0^2 \times \left( \frac{1}{f} \right)^{2+2} \text{ m} / \sqrt{\text{Hz}}$$

$$\text{for 2 pendulums} \quad \delta x(f) \simeq 10^{-7} (1 \text{ Hz})^2 \times f_0^4 \times \left( \frac{1}{f} \right)^{2+4} \text{ m} / \sqrt{\text{Hz}}$$

# Seismic noise : Superattenuators



Need of vertical attenuator

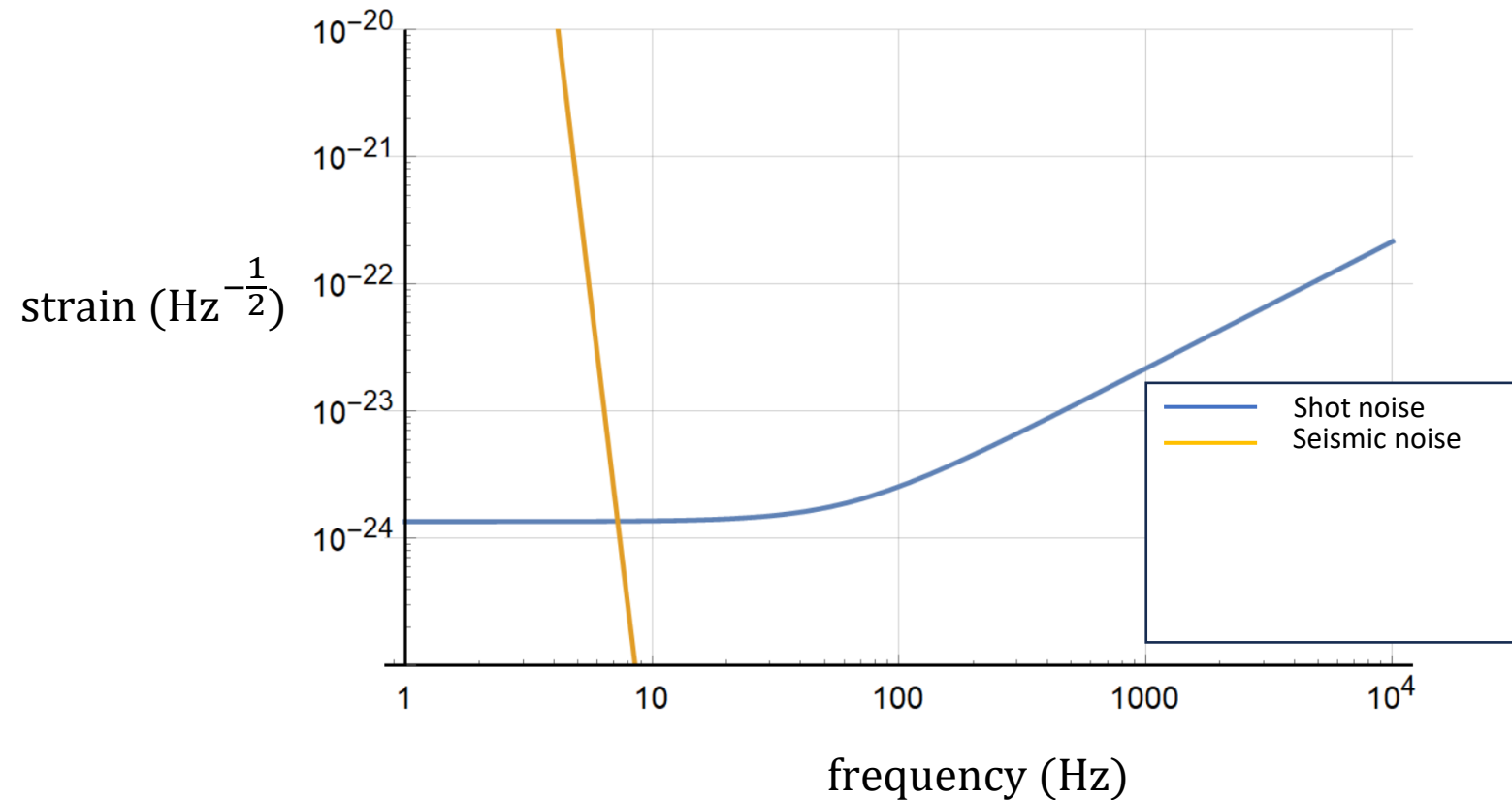


$$\delta x(f) \simeq 10^{-7} (1 \text{ Hz})^2 \times f_0^{2n} \times \left(\frac{1}{f}\right)^{2n+2} \text{ m}/\sqrt{\text{Hz}}$$

$$\delta x(f) \simeq \sqrt{\left[ \delta x_{\text{sis,h}}(f) \times \left(\frac{f_0}{f}\right)^{14} \right]^2 + \left[ \delta x_{\text{sis,v}}(f) \times \left(\frac{f_0}{f}\right)^{10} \times \alpha_{\text{coup}}^5 \right]^2}$$

# Seismic noise : sensitivity curve

$$S_{sism}(f) \simeq \frac{1}{L^2} \left[ \delta x_{sis,h}(f) \times \left( \frac{f_0}{f} \right)^{14} \right]^2 + \left[ \delta x_{sis,v}(f) \times \left( \frac{f_0}{f} \right)^{10} \times \alpha_{dis}^5 \right]^2$$



**Some math : The noise**

**Detection principle**

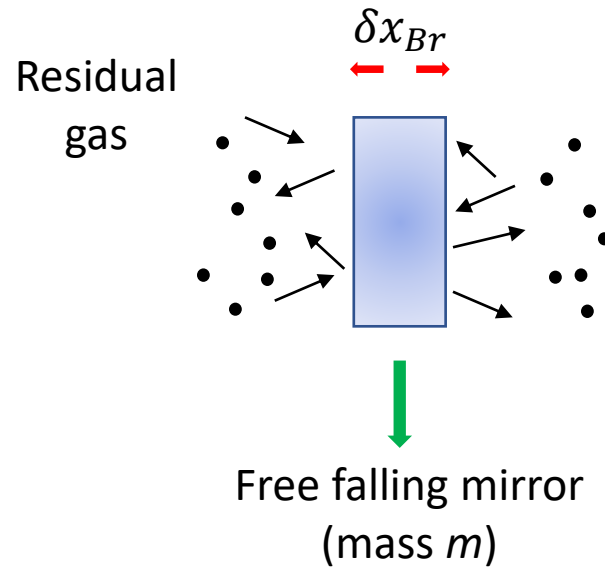
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**Noise contribution:**

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- **Seismic noise**
- **Thermal noise**
- **Quantum noise**

**Conclusion**



## Brownian Motion

Langevin theory 1908

$$m \frac{d^2 x}{dt^2} = m \frac{dv}{dt} = -m\gamma v + F_L(t)$$

➔ Stochastic differential equation

$F_L(t)$ : Stochastic Langevin force, results from the momentum transfer of the surrounding particles

Centered process :  $\langle F_L(t) \rangle = 0$

Markovian process :  $\Gamma_{F_L}(\tau) = \sigma_{F_L}^2 \delta(\tau)$

$\gamma$  and  $F_L$  have the same physical origin, but what is the relationship between them?

# Langevin equation resolution

$$\frac{dv}{dt} + \gamma v = F_L(t)$$

Homogeneous equation

$$\frac{dv}{dt} + \gamma v = 0 \quad \Rightarrow \quad v(t) = A \times e^{-t/\tau}$$

General solution

$$v(t) = A(t) \times e^{-t/\tau} \quad \Rightarrow \quad \frac{dA}{dt} = \frac{e^{\gamma t} \times F_L(t)}{m}$$

$$\left\{ \begin{array}{l} A(t) = \frac{1}{m} \int_0^t e^{\gamma t'} F_L(t') dt' + A_0 \\ v(t) = v_0 e^{-\gamma t} + \frac{1}{m} \int_0^t e^{-\gamma(t-t')} F_L(t') dt' \\ F_L(t=0) = 0 ; v(t) = v_0 \end{array} \right.$$

# Langevin force

$$\text{Mean value: } \langle v \rangle(t) = v_0 e^{-\gamma t} + \frac{1}{m} \int_0^t e^{-\gamma(t-t')} \langle F_L(t') \rangle dt' = v_0 e^{-\gamma t}$$

$$\text{Variance: } \sigma_v^2(t) = \left\langle \left( \frac{1}{m} \int_0^t e^{-\gamma(t-t')} F_L(t') dt' \right)^2 \right\rangle = \frac{1}{m^2} \int_0^t dt' \int_0^{t'} dt'' \langle F_L(t') F_L(t'') \rangle e^{-\gamma(t-t')} e^{-\gamma(t-t'')}$$



$$\sigma_v^2(t) = \frac{\sigma_{F_L}^2}{4\gamma m^2} (1 - e^{-2\gamma t})$$

Mirror in thermodynamic equilibrium with a bath at temperature  $T$

$$\langle U_k \rangle(t) = \frac{1}{2} m \langle v^2 \rangle(t) \xrightarrow[t \rightarrow \infty]{} \langle U_k \rangle(t) = \frac{1}{2} m \sigma_v^2(t \rightarrow \infty) = \frac{k_B T}{2}$$

$$\sigma_{F_L}^2 = 4m\gamma k_B T \quad S_{F_L}(f) = 4m\gamma k_B T \quad k_B: \text{ Boltzman constant ; } T: \text{ temperature}$$

$$\text{mechanical susceptibility: } \chi(f) = \frac{1/m}{-\omega^2 + i\omega\gamma} = \frac{1}{m} \frac{-\omega - i\gamma}{\omega(\omega^2 + \gamma^2)} \quad \Rightarrow \quad S_x(f) = |\chi(f)|^2 S_{F_L}(f) = \frac{4k_B T}{\omega} |\text{Im}(\chi(f))|$$

Generalized by Callen & Welton(1952) : Fluctuation-Dissipation Theorem

Dissipation  $\Rightarrow$  Fluctuation :

- Any mechanical system
- Any dissipation mechanism

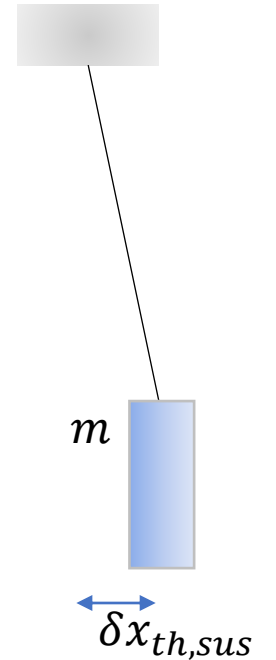
$$S_{F_L}(f) = \frac{4k_B T}{\omega} \left| \text{Im} \left( \frac{1}{\chi(f)} \right) \right|$$

# Suspension thermal noise

Internal dissipation :  $k \rightarrow k(1 + i\phi_l)$       $\phi_l$ : loss angle, frequency independent

$$\rightarrow \chi_{sus}(f) = \frac{1/m}{\omega_0^2 - \omega^2 + i\phi_l\omega_0^2} = \frac{1}{m} \frac{\omega_0^2 - \omega^2 - i\phi_l\omega_0^2}{(\omega_0^2 - \omega^2)^2 + \phi_l^2\omega_0^4}$$

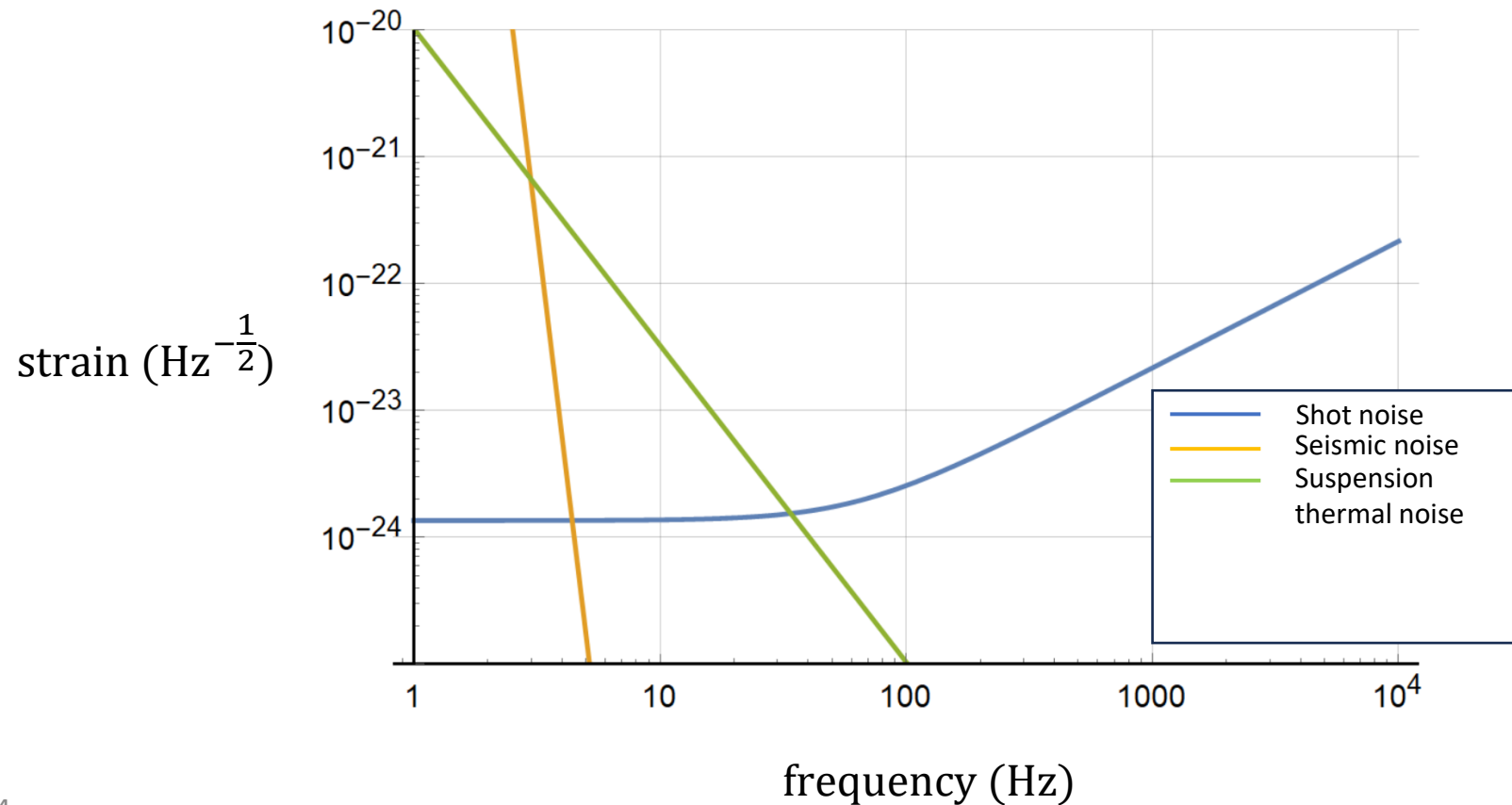
$$|\mathcal{I}m(\chi_{sus}(f))| = \frac{1}{m} \frac{\phi_l\omega_0^2}{(\omega_0^2 - \omega^2)^2 + \phi_l^2\omega_0^4} \xrightarrow{\omega \gg \omega_0} |\mathcal{I}m(\chi_{sus}(f))| = \frac{1}{m} \frac{\omega_0^2}{\omega^4} \phi_{l,sus}$$



# Suspension thermal noise : sensitivity curve

$$S_{th,sus}(f) \simeq \frac{1}{L^2} \underbrace{(4)}_{\text{mirrors}} \times \underbrace{(4)}_{\text{wires}} \frac{4k_B T f_0^2}{m(2\pi)^3 f^5} \phi_{sus}$$

$$m = 40 \text{ kg}; \phi_{sus} = 10^{-10}$$



# Mirror thermal noise

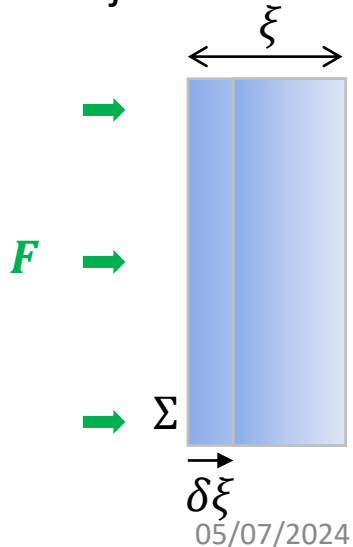
Multiple resonances system  $\chi_{mir}(f) = \sum_j \frac{1/m_j}{\omega_j^2 - \omega^2 + i\phi_{l,mir}\omega_j^2} = \sum_j \frac{1}{m_j} \frac{\omega_j^2 - \omega^2 - i\phi_{l,mir}\omega_j^2}{(\omega_j^2 - \omega^2)^2 + \phi_{l,mir}^2\omega_j^4}$

First resonance 1 kHz  $\Rightarrow |\mathcal{I}m(\chi_{mir}(f))| \approx \phi_{l,mir} \sum_j \frac{1}{m_j \omega_j^2}$  For  $\omega \ll \omega_1 < \omega_2 \dots$

And...  $|\chi_{mir}(f)| \approx \sum_j \frac{1}{m_j \omega_j^2} \Rightarrow |\mathcal{I}m(\chi_{mir})| = \phi_{l,mir} |\chi_{mir}|$

Interaction energy for a low frequency applied force:  $\mathcal{E} = F \times x_{mir} = |\chi_{mir}(f)| F^2 \Rightarrow |\mathcal{I}m(\chi_{mir})| = \phi_{l,mir} |\chi_{mir}| = \phi_{l,mir} \frac{\mathcal{E}}{F^2}$

We just need to compute the elastic energy when we apply a constant force



$Y$ : Young modulus

$$\frac{\delta\xi}{\xi} = \frac{F/\Sigma}{Y}$$

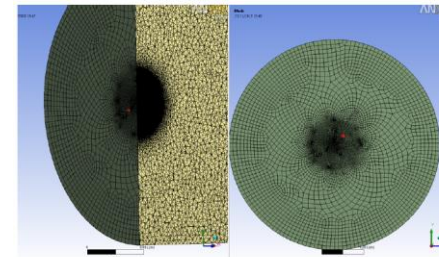
Elastic energy

$$\mathcal{E} = \frac{1}{2} F \times \delta\xi$$



$$\frac{\mathcal{E}}{F^2} = \frac{1}{2Y \times \Sigma} \xi$$

- We need to consider the beam shape
- In practice, FEM simulations

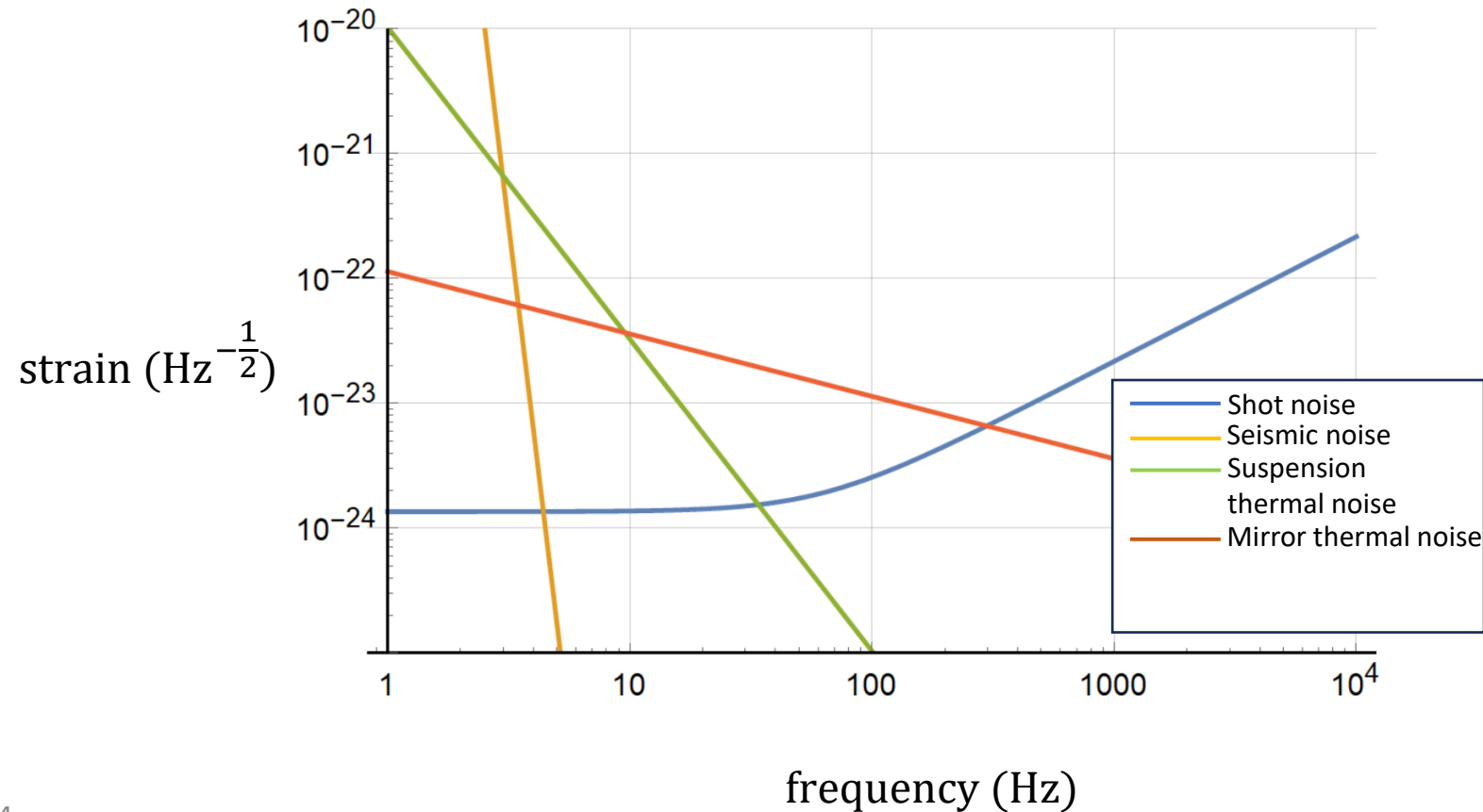


# Mirror thermal noise : Sensitivity curve

$$S_{th,sus}(f) \simeq \frac{1}{L^2} \times 4 \times \frac{k_B T}{\pi f} \frac{\xi}{Y \pi r_{mir}^2} \phi_{mir}$$

↑  
mirrors

$$\xi = 20 \text{ cm} ; r_{mir} = 20 \text{ cm} ; Y = 72 \text{ GPa} ; \phi_{mir} = 10^{-6}$$



**Some math : The noise**

**Detection principle**

**Interferometer output and tuning :**

- **The Michelson interferometer**
- **Sensitivity enhancement : The Fabry Perot cavity**
- **Shot noise limited detector**

**Noise contribution:**

- **Harmonic oscillator model**
- **Seismic noise**
- **Thermal noise**
- **Quantum noise**

**Conclusion**

# Quantum harmonic oscillator

$$\text{Hamiltonian : } H = \frac{p^2}{2m} + m \frac{\omega_0^2 x^2}{2}$$

$x$  and  $p$  : conjugate canonical variables

quantification



$$\left\{ \begin{array}{l} [\hat{x}, \hat{p}] = i\hbar = i \frac{h_P}{2\pi} \\ \hat{H} = \frac{\hat{p}^2}{2m} + m \frac{\omega_0^2 \hat{x}^2}{2} \end{array} \right.$$

## Annihilation and creation operators

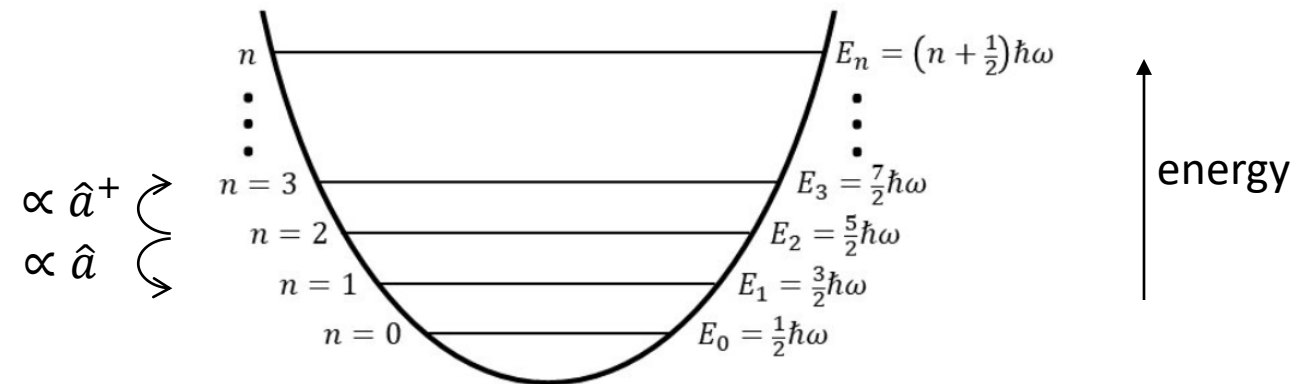
$$\hat{a} = \frac{1}{\sqrt{2}} \left( \sqrt{\frac{m\omega_0}{\hbar}} \hat{x} + \frac{i}{\sqrt{\hbar m \omega_0}} \hat{p} \right)$$

$$\hat{a}^+ = \frac{1}{\sqrt{2}} \left( \sqrt{\frac{m\omega_0}{\hbar}} \hat{x} - \frac{i}{\sqrt{\hbar m \omega_0}} \hat{p} \right)$$

## Hamiltonian

$$\hat{H} = \hbar\omega_0 \left( \hat{a}^+ \hat{a} + \frac{1}{2} \right)$$

Non-zero energy  
fundamental state



# Classical electromagnetic radiation

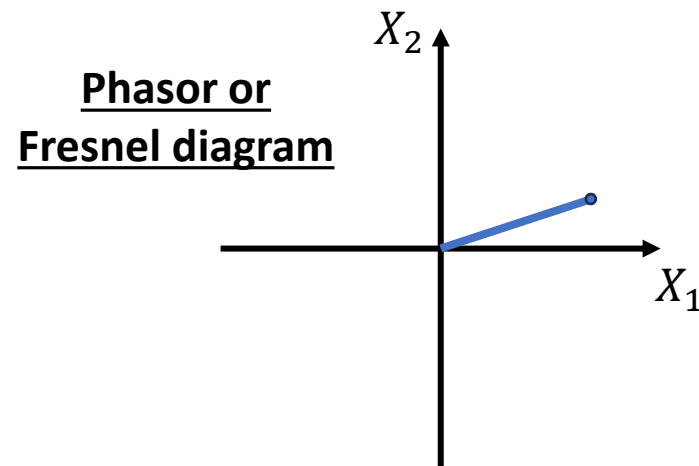
Single mode electric field ; plane wave ; in vacuum

$$\mathfrak{E}(t) = \mathfrak{E}_0(\alpha(t)e^{-i2\pi\nu t} + \alpha^*(t)e^{+i2\pi\nu t}) \quad \alpha(t) = \alpha_0(t)e^{-i\varphi(t)}$$

$$\left\{ \begin{array}{l} X_1(t) = \alpha(t) + \alpha^*(t) \\ X_2(t) = -i(\alpha(t) - \alpha^*(t)) \end{array} \right. \Rightarrow \mathfrak{E}(t) = \mathfrak{E}_0(X_1(t) \cos(2\pi\nu t) + X_2(t) \sin(2\pi\nu t))$$

Single mode energy

$$\mathcal{E} = \frac{h_P\nu}{4} (X_1^2 + X_2^2)$$



$X_1, X_2$ : field quadratures

# Electromagnetic radiation quantification

Electromagnetic radiation quantification

$$\mathcal{E} = \frac{h_p \nu}{4} (X_1^2 + X_2^2)$$

$X_1$  and  $X_2$  : conjugate canonical variables

quantification



$$[\hat{X}_1, \hat{X}_2] = i\hbar$$

$$\hat{H} = \frac{h_p \nu}{4} (\hat{X}_1^2 + \hat{X}_2^2)$$

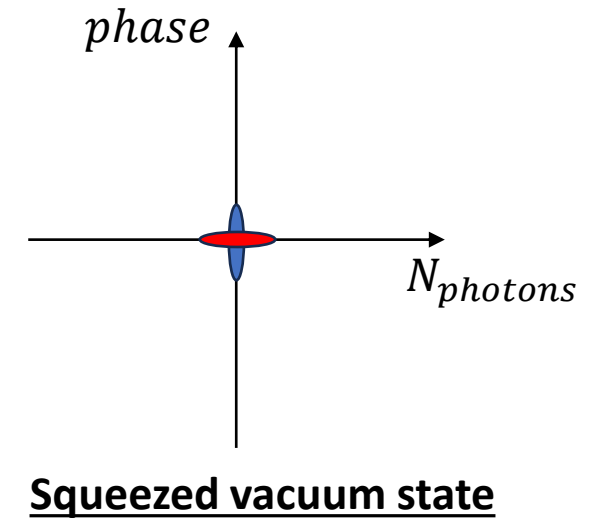
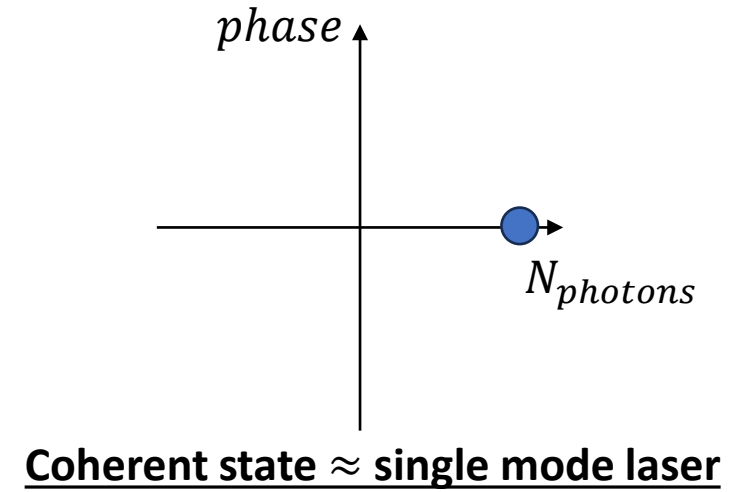
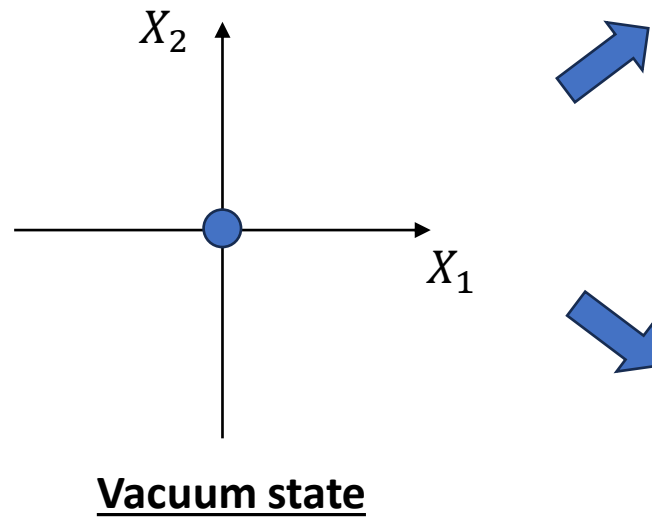
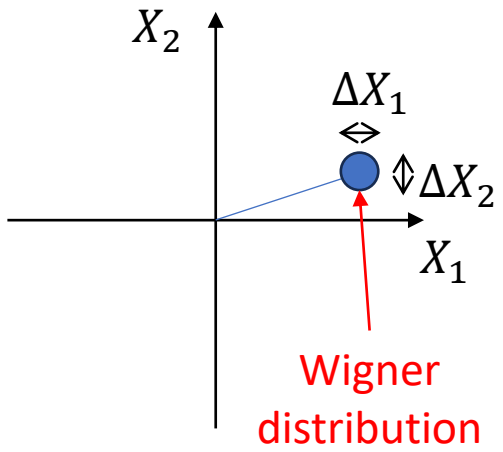
Using annihilation and creation operators

$$\hat{H} = \left( \hat{a}^+ \hat{a} + \frac{1}{2} \right) = h_p \nu \left( \hat{N}_j + \left( \frac{1}{2} \right) \right)$$

Photon energy      Number of photons

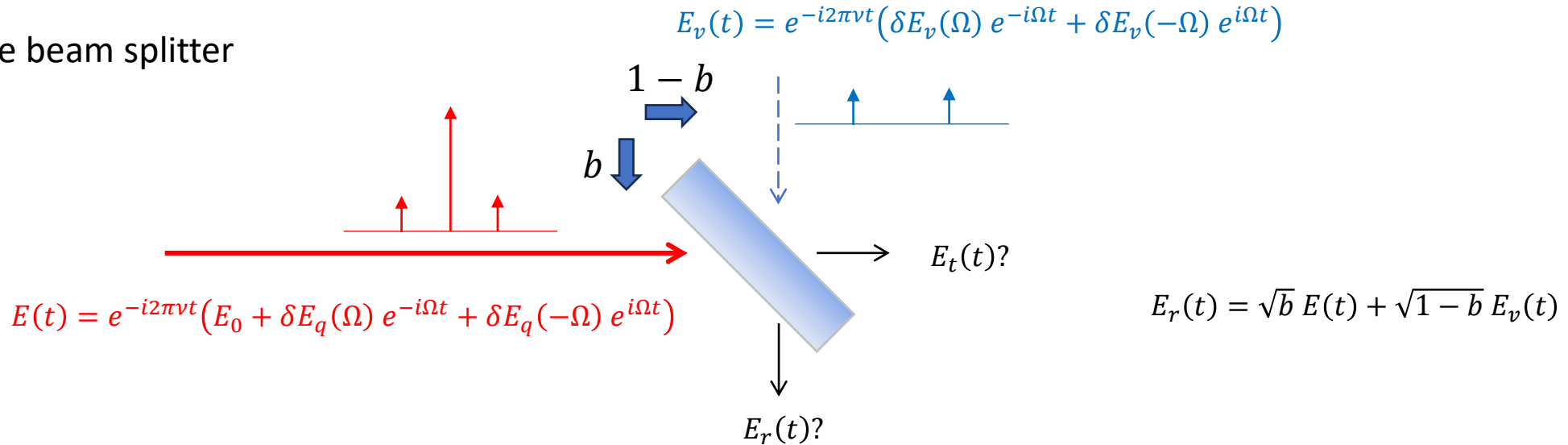
Zero photons energy :  
vacuum is not empty

# Wigner distribution



# Quantum noise as sidebands : semiclassical approach

The beam splitter



$$P_r(t) = b P_0 + \underbrace{\sqrt{b} E_0 \left( \sqrt{b} \delta E_q^*(-\Omega) + \sqrt{1-b} \delta E_v^*(-\Omega) + \sqrt{b} \delta E_q(\Omega) + \sqrt{1-b} \delta E_v(\Omega) \right)}_{\delta P(\Omega)} e^{-i\Omega t} + c.c$$

$$S_{\delta P}(\Omega) = \langle \delta P(\Omega) \delta P(\Omega)^* \rangle = b P_0 \left( \underbrace{b \langle \delta E_q^*(-\Omega) \delta E_q(-\Omega) \rangle}_{\frac{h_p \nu}{2}} + \underbrace{b \langle \delta E_q(\Omega) \delta E_q^*(\Omega) \rangle}_{\frac{h_p \nu}{2}} + \underbrace{(1-b) \langle \delta E_v^*(-\Omega) \delta E_v(-\Omega) \rangle}_{\frac{h_p \nu}{2}} + \underbrace{(1-b) \langle \delta E_v(\Omega) \delta E_v^*(\Omega) \rangle}_{\frac{h_p \nu}{2}} \right)$$

$$S_{\delta P}(\Omega) = h_p \nu b P_0 = h_p \nu P_r \quad \Rightarrow \quad \text{Shot noise}$$

# Radiation pressure noise

Radiation pressure on a mirror: transfer of photon momentum

$$F_{rad,p} = \frac{dp_{tot}}{dt} = \frac{dN_{photons}}{dt} \times \underset{\substack{\uparrow \\ \text{Total} \\ \text{reflection}}}{2} \times p_{photon} = \frac{2P_{inc}}{c} \quad \Rightarrow \quad S_{\delta F_{rad,p}} = \frac{4}{c^2} \times S_{\delta P}$$

$$S_{\delta x_{rad,p}} = \underset{\substack{\uparrow \\ \text{Mirrors motion are} \\ \text{anti-correlated}}}{4} \times S_{\delta F_{rad,p}} \times |\chi(f)|^2 = \frac{16}{c^2} \times S_{\delta P} \times |\chi(f)|^2$$

$$|\chi(f)| \simeq \frac{1}{m \omega^2}$$

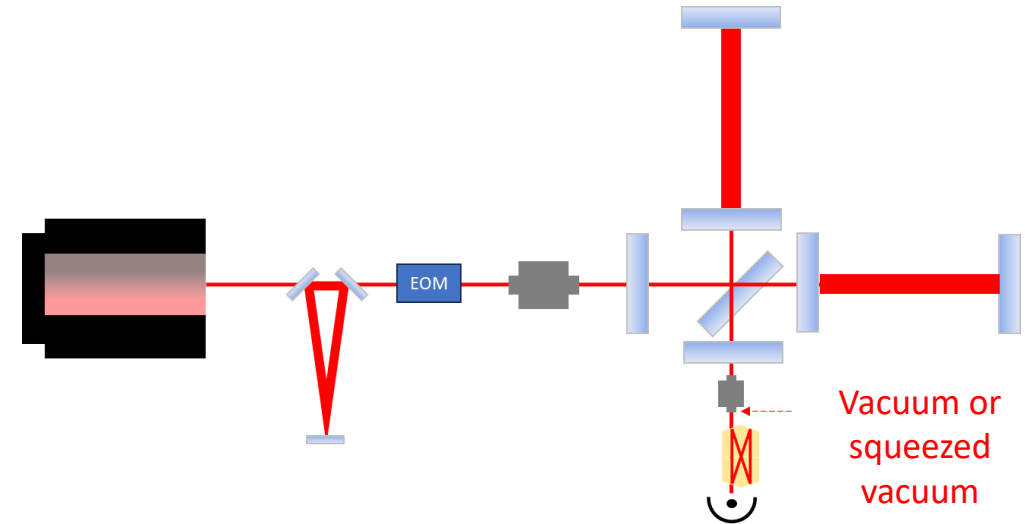
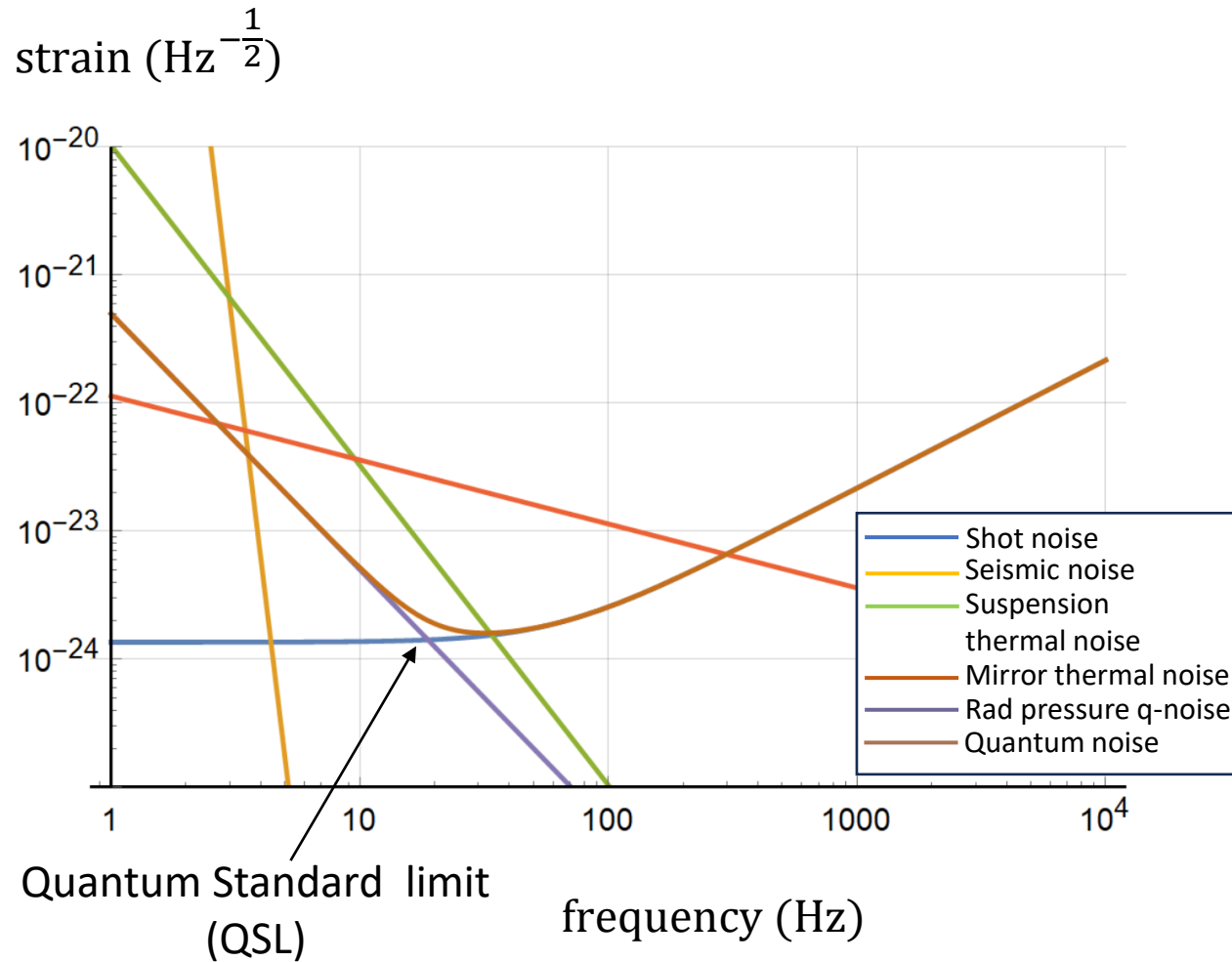
@ frequencies higher than the pendulum resonance

$$S_{\delta P}(f) \simeq h\nu \frac{2F}{\pi} \frac{P_0}{2}$$

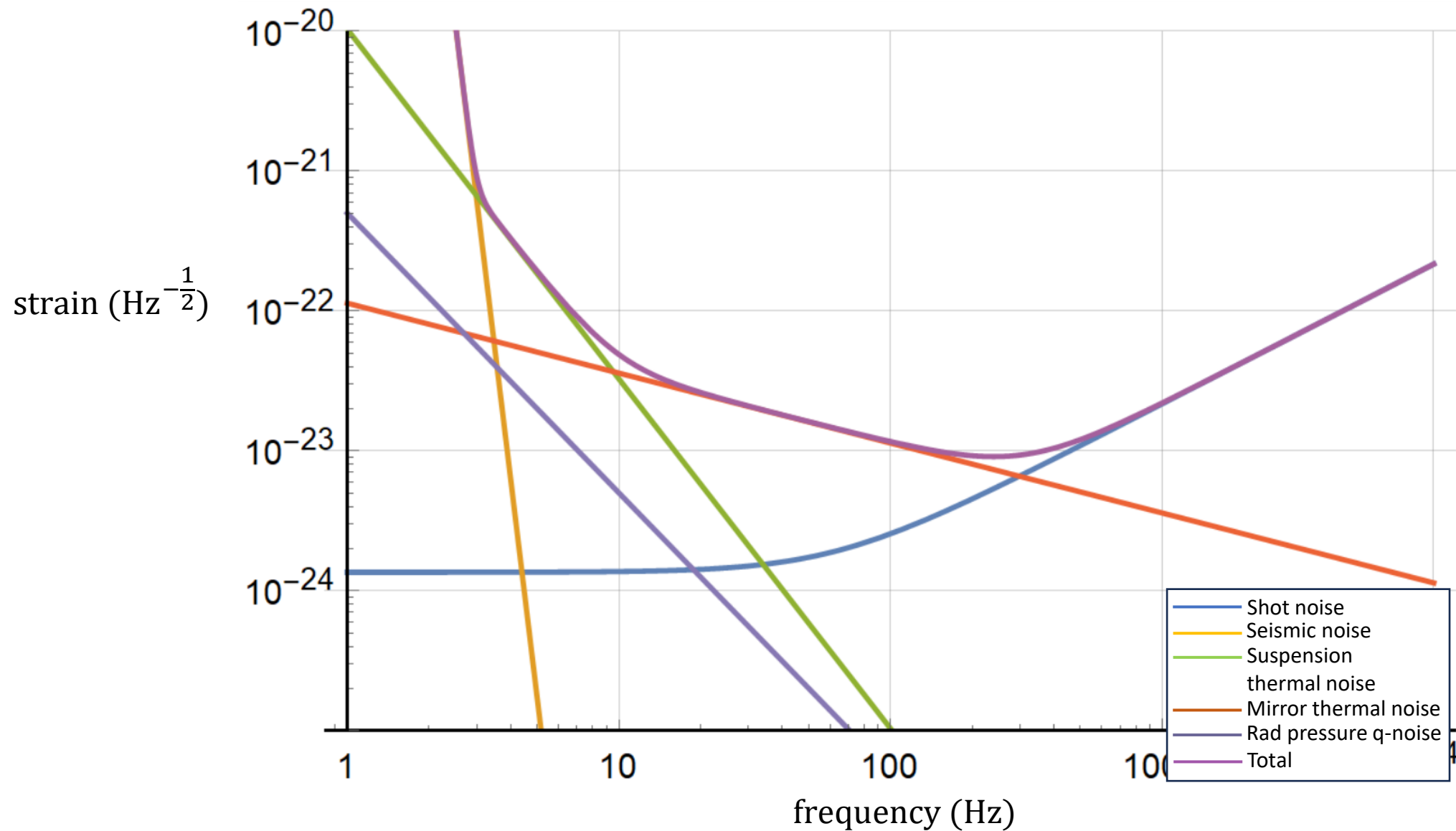
@ frequencies lower than the cavity pole

# Quantum noise : sensitivity curve

$$S_{rad,p}(f) \simeq \frac{1}{L^2} \times 2 \times \frac{h_p \nu_0}{m^2 c^2 \pi^5 f^4} GF \frac{P_0}{2}$$



# Sensitivity curve



**Some math : The noise**

**Detection principle**

**Interferometer output and tuning :**

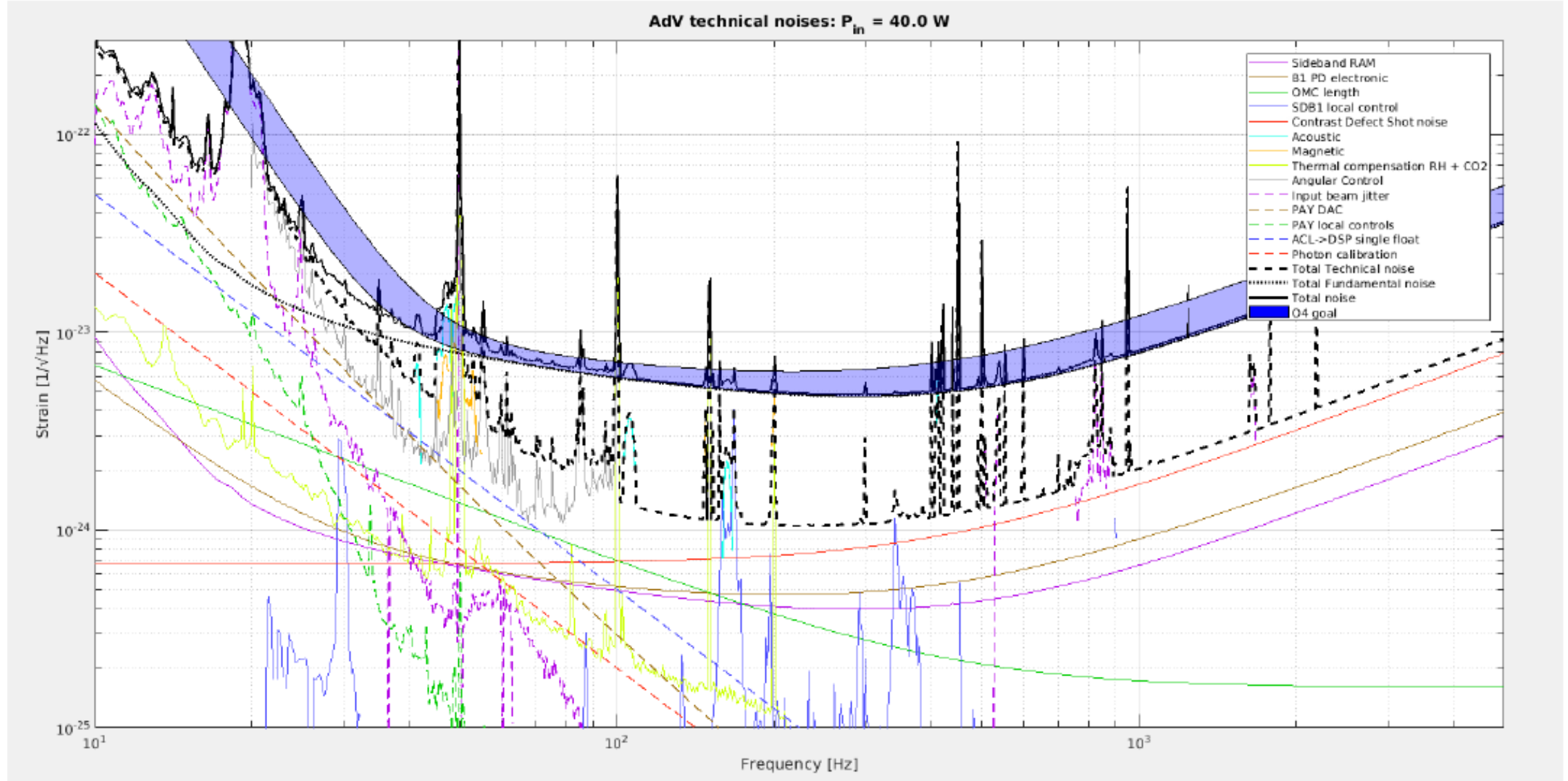
- **The Michelson interferometer**
- **Sensitivity enhancement : The Fabry Perot cavity**
- **Shot noise limited detector**

**Noise contribution:**

- **Harmonic oscillator model**
- **Seismic noise**
- **Thermal noise**
- **Quantum noise**

**Conclusion**

# A lot of other contributions



Thermal compensation system

Controls

High power laser

Electronics

Vacuum

Parametric instabilities

Newtonian noise

Squeezing, dependent and independent

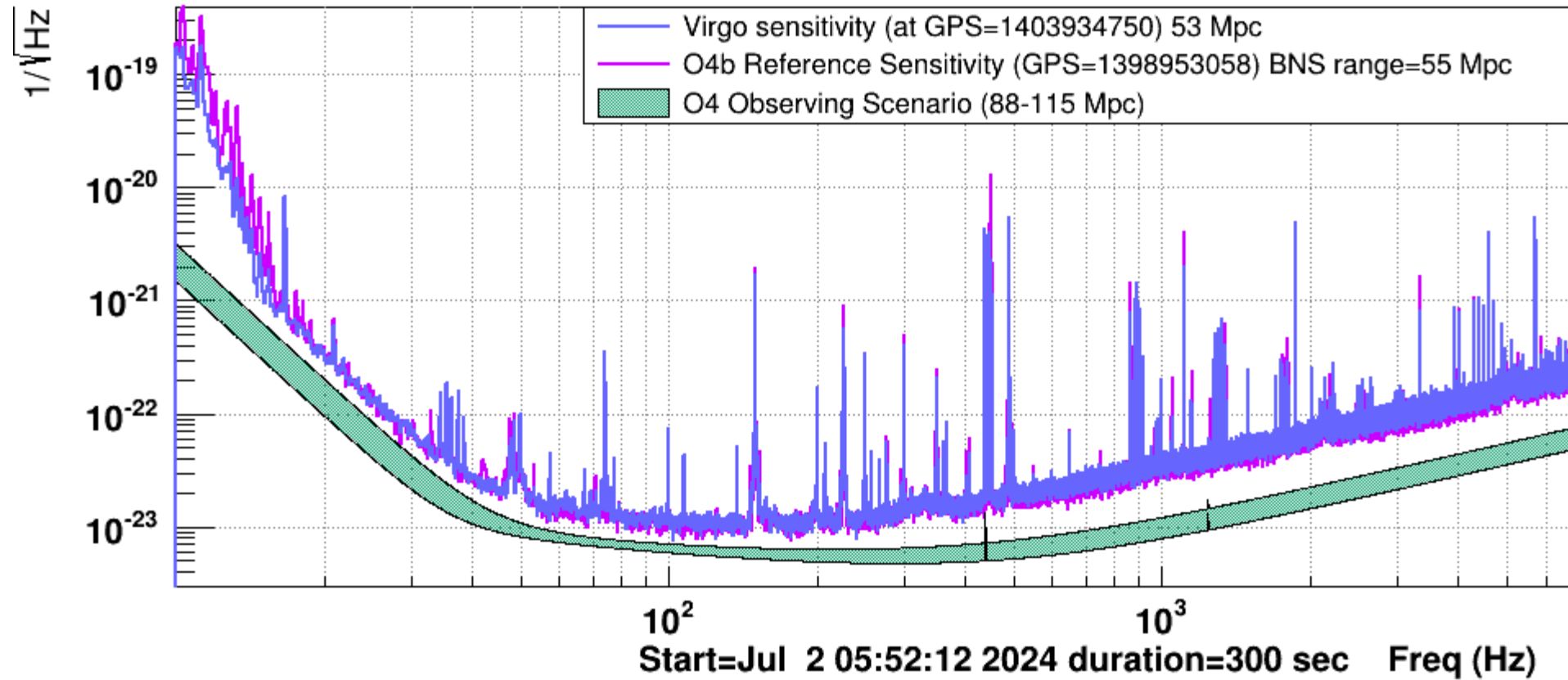
Calibration

Coatings

Simulations

....and R&D

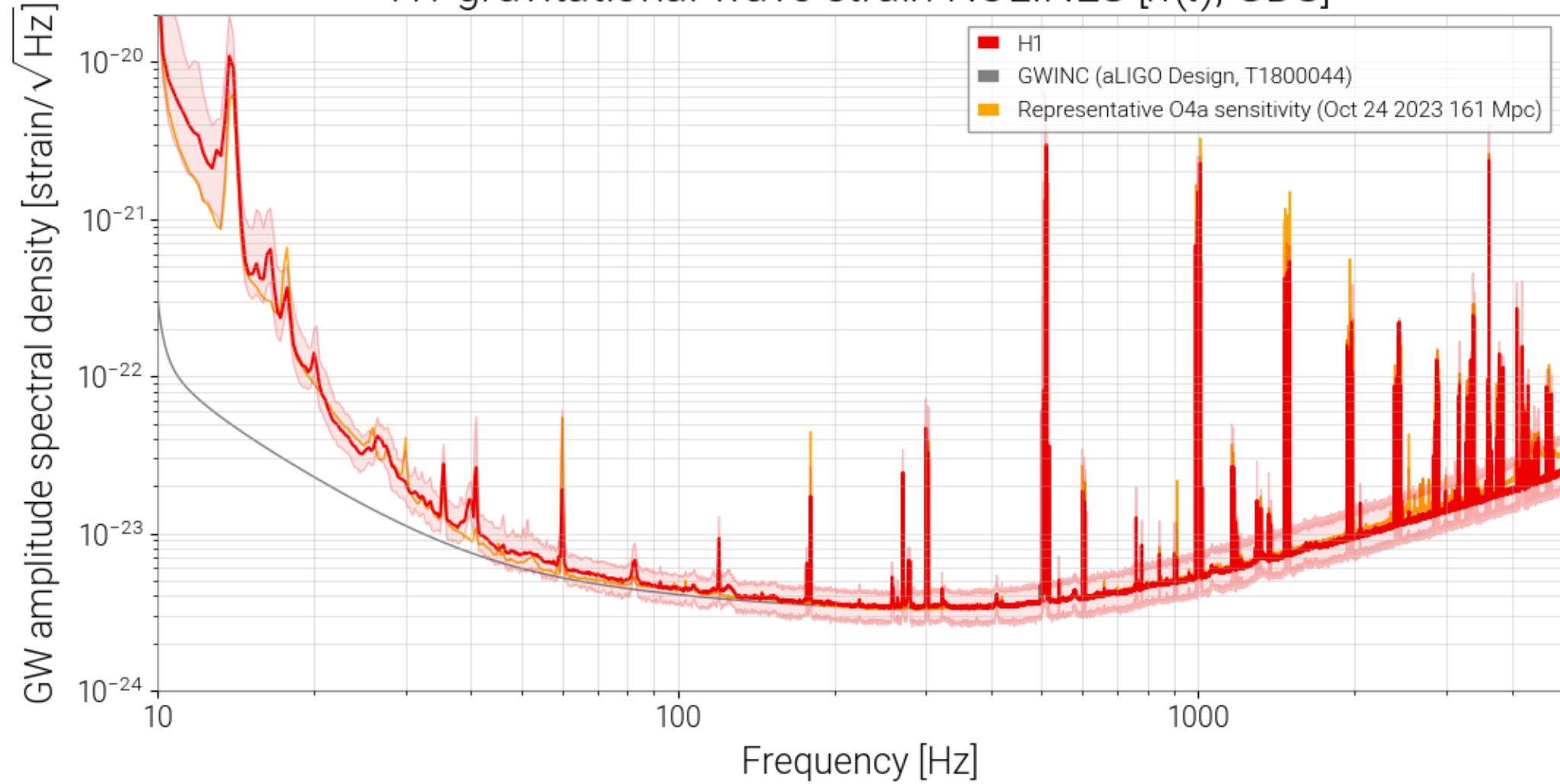
Last Sensitivity (Tue Jul 2 05:52:12 2024 UTC)



# Current sensitivity of LIGO Hanford

[1403913618-1404000018, state: Locked]

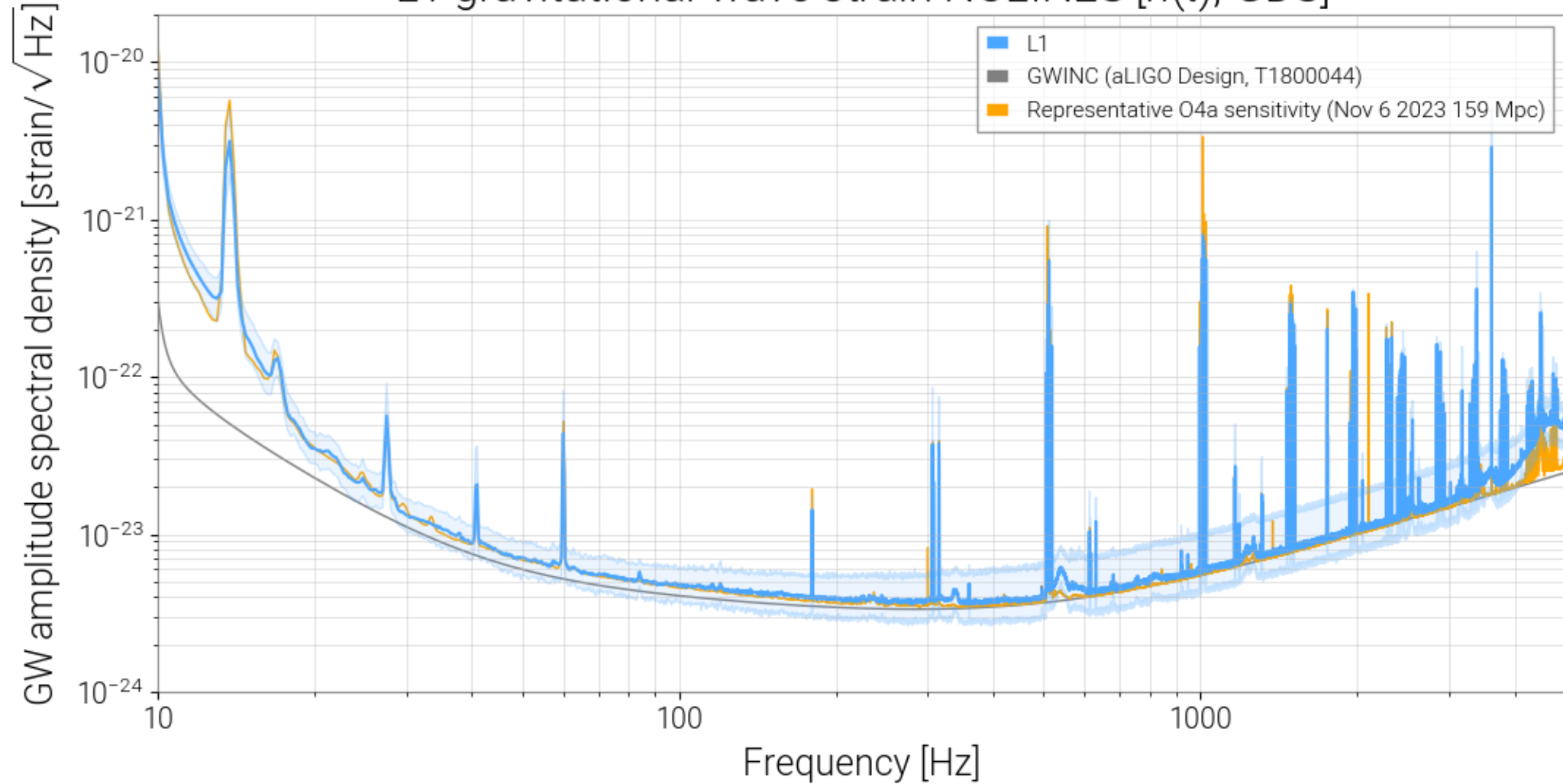
## H1 gravitational-wave strain NOLINES [ $h(t)$ , GDS]



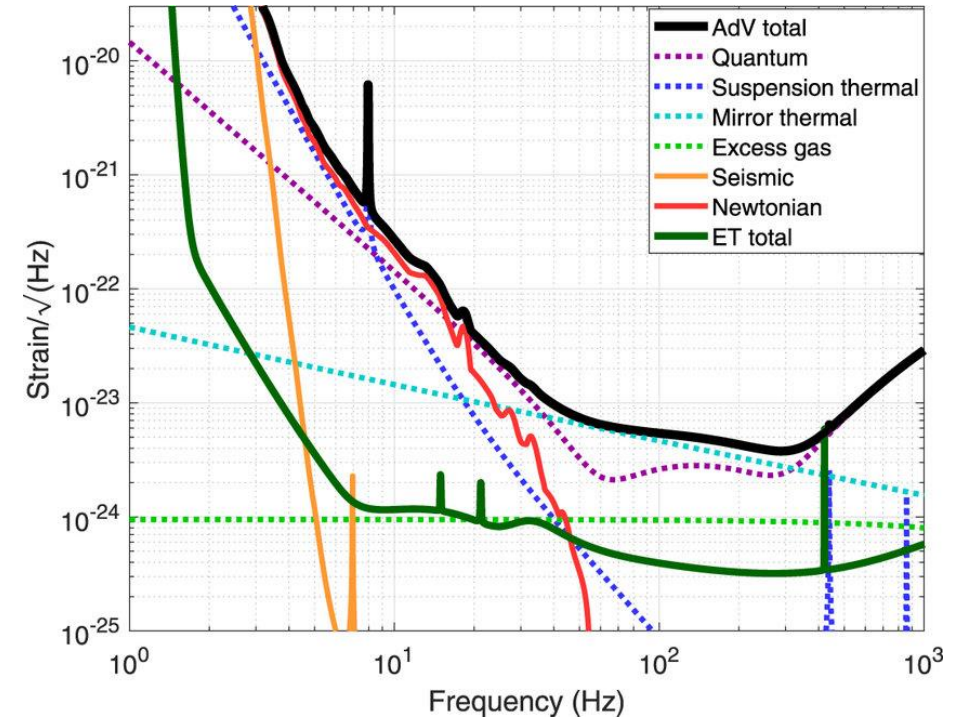
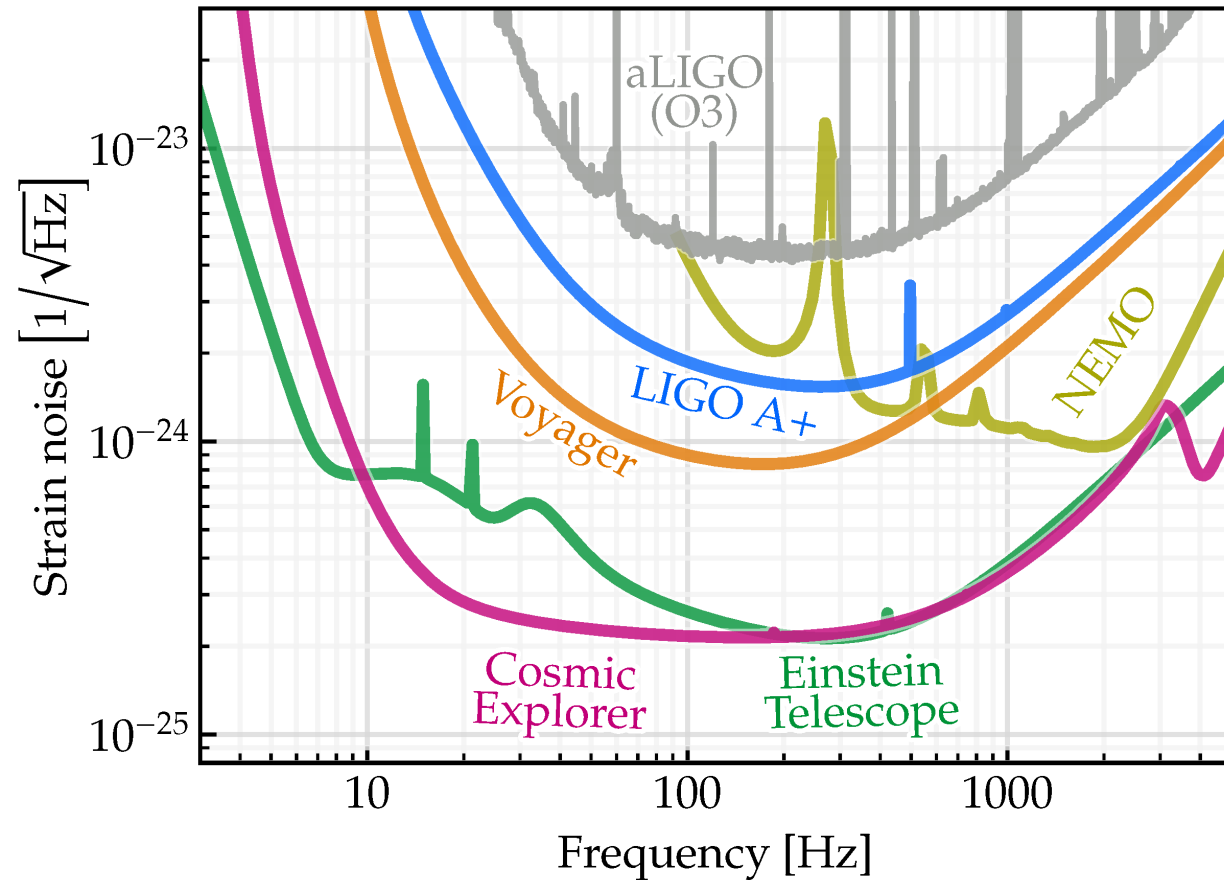
# Current sensitivity of LIGO Livingston

[1403913618-1404000018, state: Locked]

## L1 gravitational-wave strain NOLINES [ $h(t)$ , GDS]

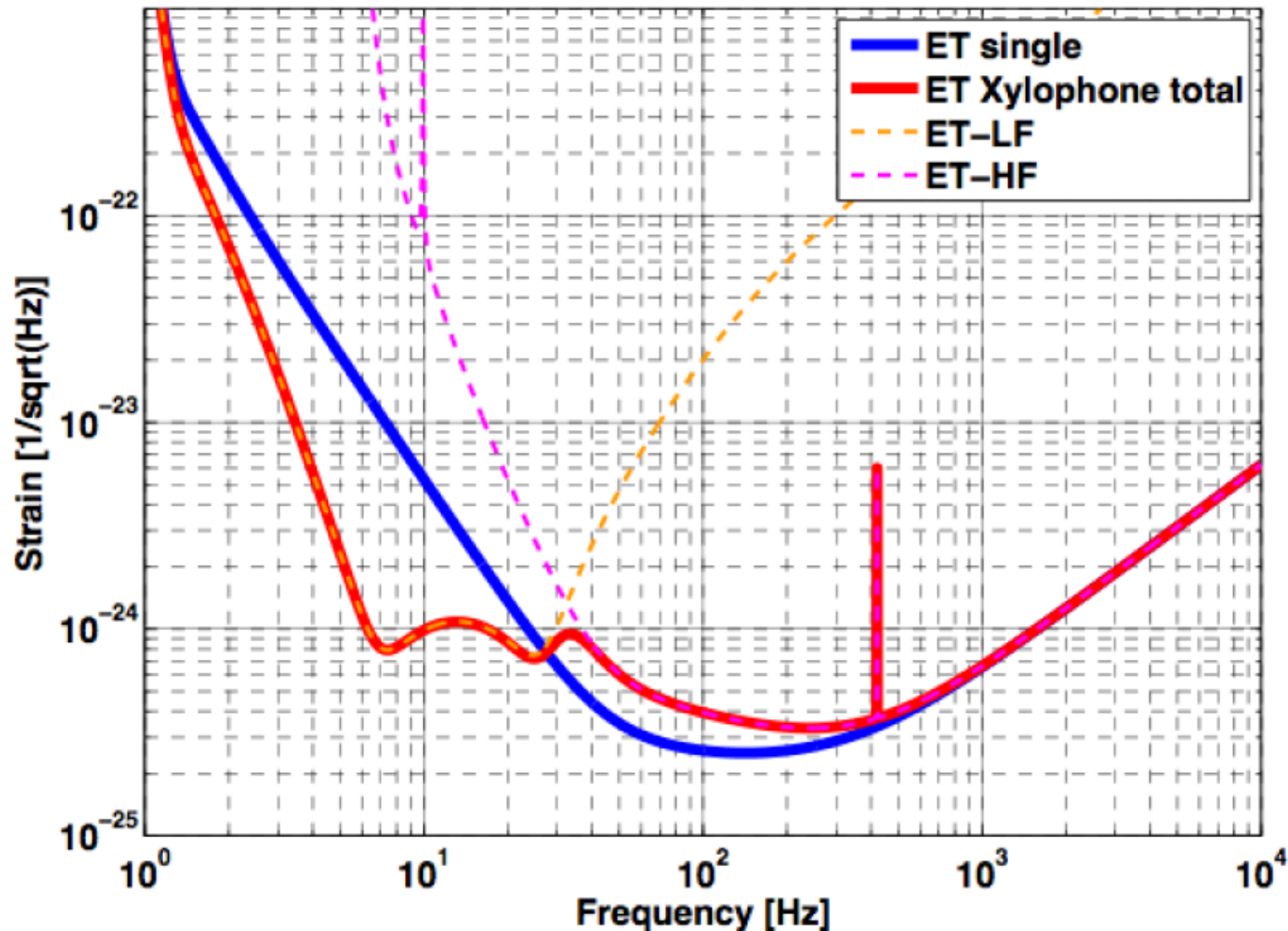


# Third generation detectors



➡ Towards low frequency : technological issues

# Third generation detectors : The Einstein Telescope



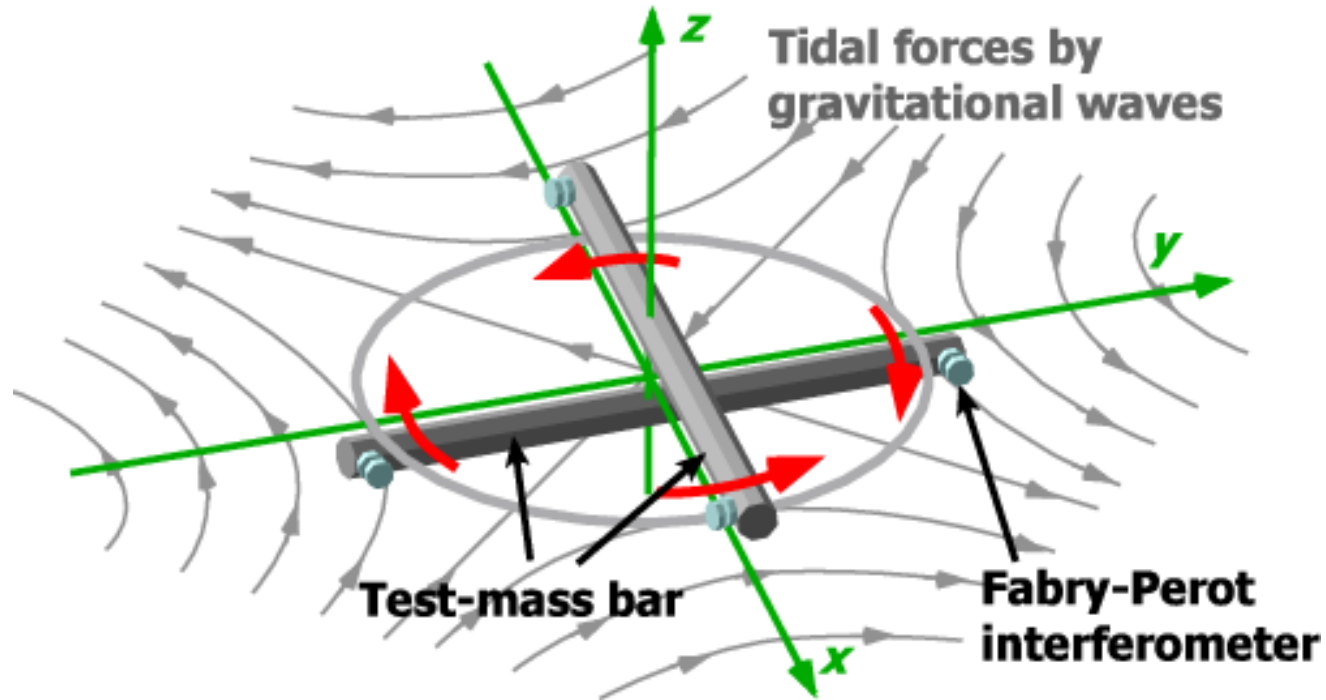
ET-HF :

- Same wavelength : 1064nm
- High power : 700W
- Squeezing

ET-LF :

- Low frequency super-attenuators
- Newtonian noise subtraction
- Silicon mirrors @ 10K, low power  
1550nm laser
- Squeezing

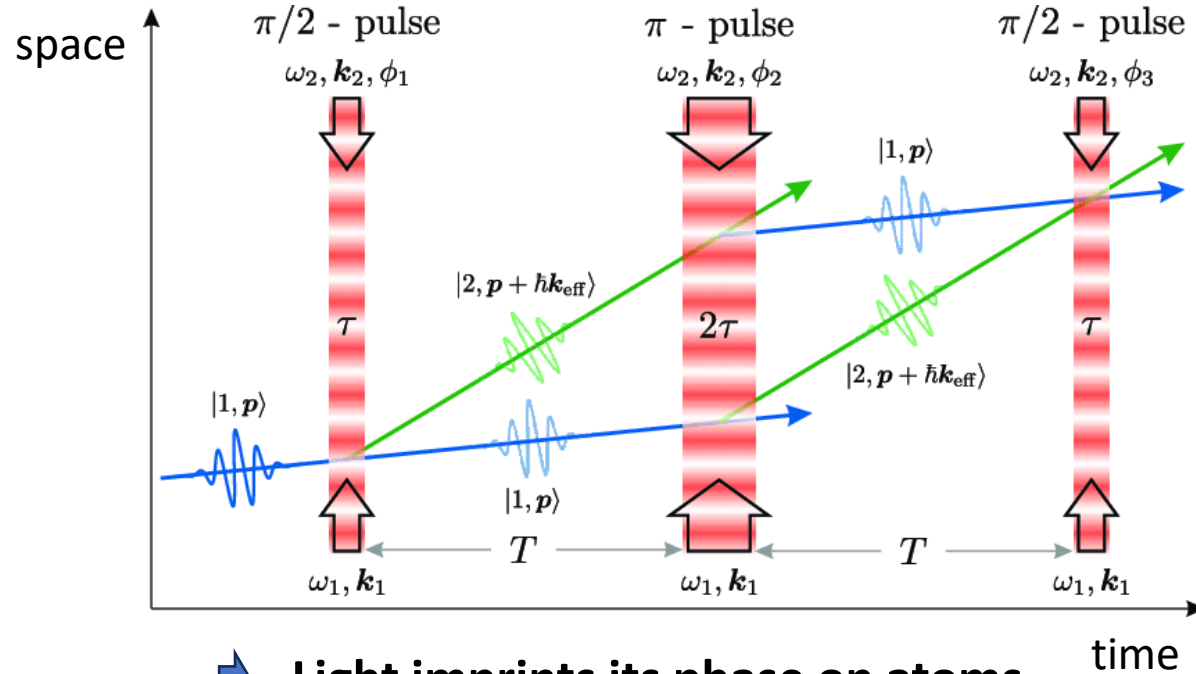
# Low frequency detectors : Torsion bars



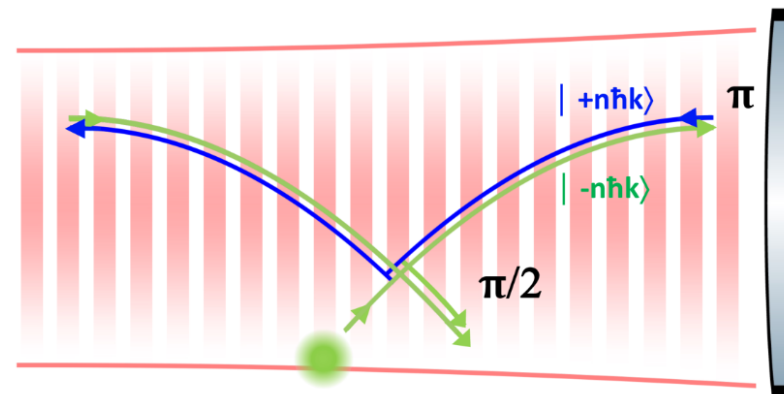
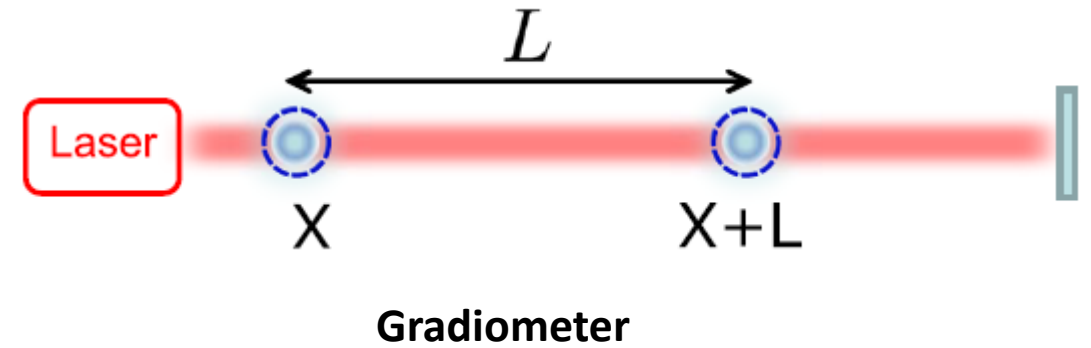
- Torsion pendulum : very low frequency
- Technically limited by the size of the bars

# Low frequency detectors : Atom interferometers

## Free falling atoms



## Light imprints its phase on atoms



Bragg configuration



Thank you!