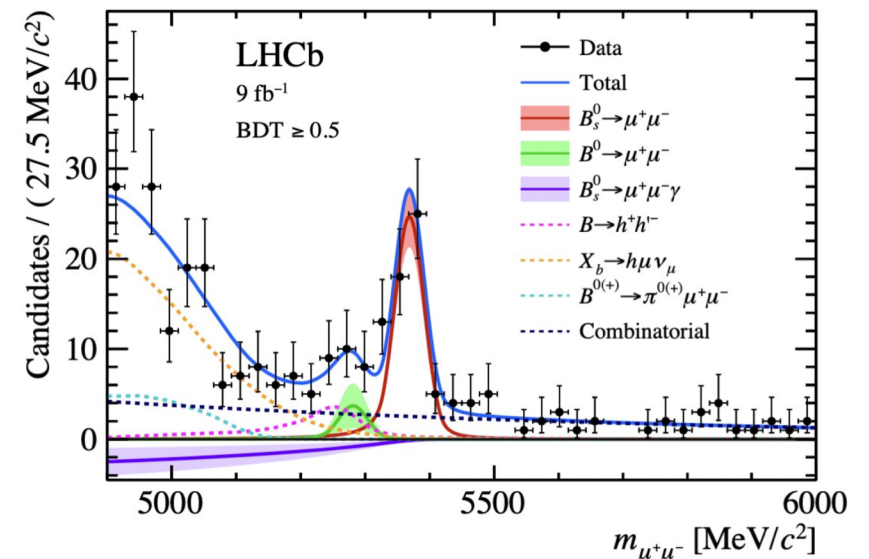


The relevance of $B_s \rightarrow \gamma FFs$ within and beyond the Standard Model

Work in collaboration with Diego Guadagnoli, Camille Normand and Silvano Simula
[based on JHEP '23 (2303.02174) and JHEP '23 (2308.00034)]

Ludovico Vittorio (LAPTh & CNRS, Annecy, France)

Workshop on radiative leptonic B decays, Marseille – 29th February 2024



(from LHCb-PAPER-2021-007)

Radiative-and-leptonic Bs decays

A novel possibility to analyze $b \rightarrow s$ quark transitions is the study of rare radiative-and-leptonic Bs decays. This is experimentally challenging, and yet LHCb has recently set a limit (very close to the SM signal):

$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma)_{m_{\mu\mu} > 4.9 \text{ GeV}} = (-2.5 \pm 1.4 \pm 0.8) \times 10^{-9} < 2.0 \times 10^{-9}$$

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- 2. Sensitivity to a larger set of WCs:** not only $O_{10}(')$, also $O_7(')$ and $O_9(')$
(reminder: $O_9(')$ and $O_{10}(')$ are particularly relevant @ high- q^2)
- 3. Two ways to detect it experimentally:**
 - *directly* (i.e. w/ photon reconstruction)
 - *indirectly* (i.e. w/out photon reconstruction) [[Dettori et al, PLB '17 \(1610.00629\)](#)]

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See talks by Irene Bachiller and Camille Normand

Radiative-and-leptonic decays in Effective Field Theory

Standard WET approach:

$$\mathcal{H} = -\frac{4G_F}{\sqrt{2}}\lambda_{\text{CKM}} \left[\frac{\alpha_{em}}{4\pi} \left(\sum_i C_i \mathcal{O}_i + \sum_i C'_i \mathcal{O}'_i \right) \right]$$

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Melikhov and Nikitin, Phys. Rev. D 70 (2004) 114028

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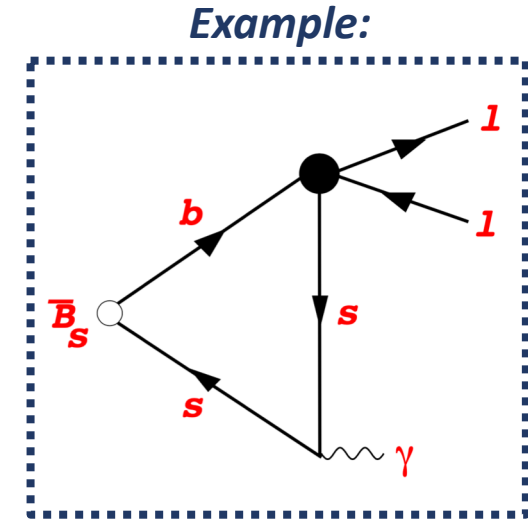
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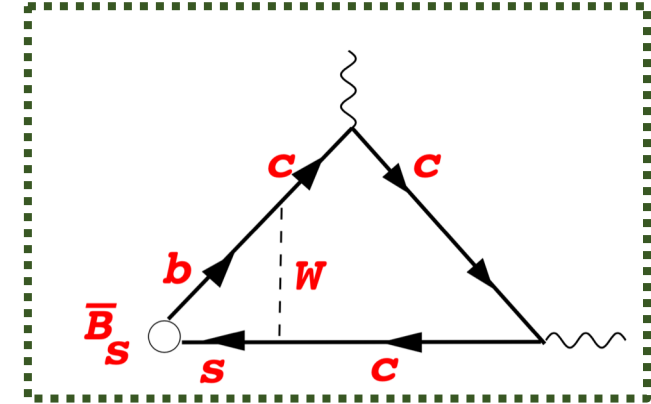
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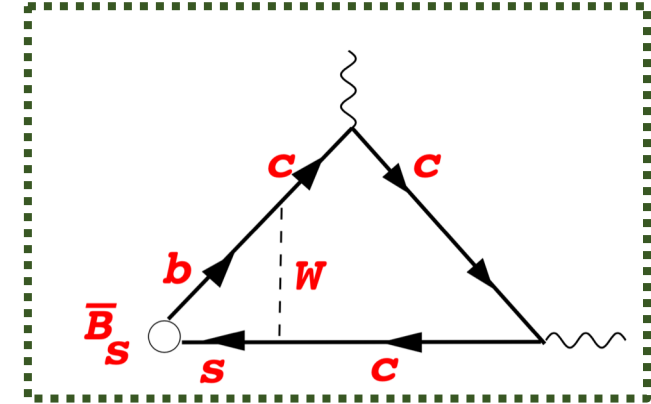
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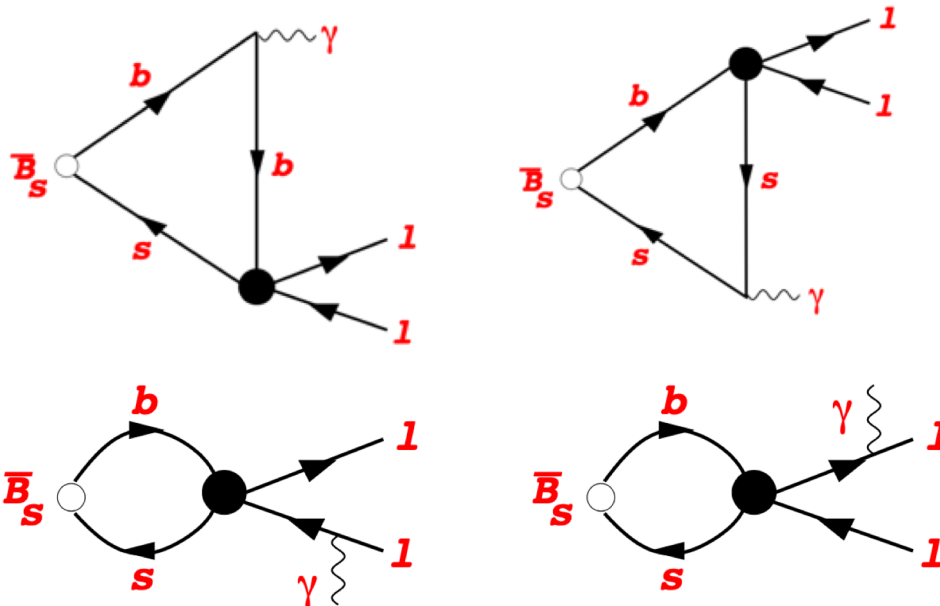
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Important issue: only 4 diagrams give the most important contribution at high- q^2 , i.e.



$\bullet = \mathcal{O}(q, q_0)$

Available results for the $B_s \rightarrow \gamma$ hadronic FFs

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arXiv:2402.03262 [hep-lat]



See talk by Giuseppe Gagliardi

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Main idea: HQET scaling of FFs parameters from the D_s -sector to the B_s -sector, starting from Lattice QCD (LQCD) data available for the D_s -sector @ high- q^2

Methodological overview of the GNSV approach to hadronic FFs

Each FF obeys a dispersion relation of this form:

$$V_{\perp[\llbracket]}^{D_s}(q^2) = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im}[V_{\perp[\llbracket]}^{D_s}(t)]}{t - q^2} = \frac{r_{\perp[\llbracket]}^{D_s^*[D_{s1}]}}{1 - q^2/m_{D_s^*[D_{s1}]}^2} + \dots$$

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Opportunity to test different pole structures ($\chi = \perp, \parallel$): for instance

P fit

$$V_{\chi}(q^2) = \frac{r_{\chi 1}}{1 - q^2/m_{\text{ph1}}^2},$$

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General methodology: the free parameters of each ansaetze can be determined through fits to lattice data !

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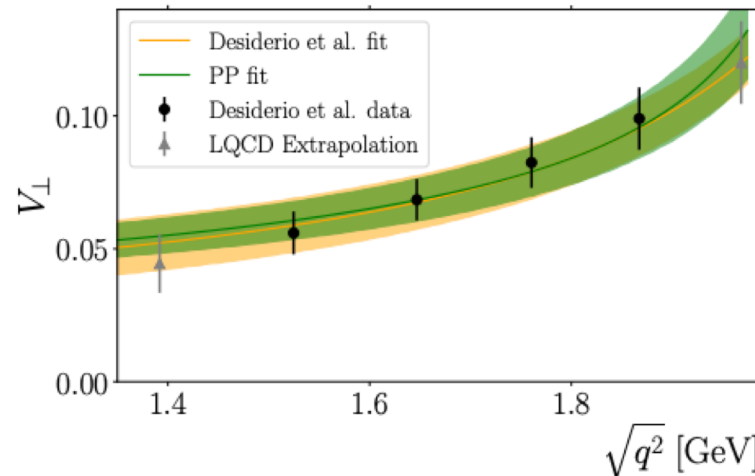
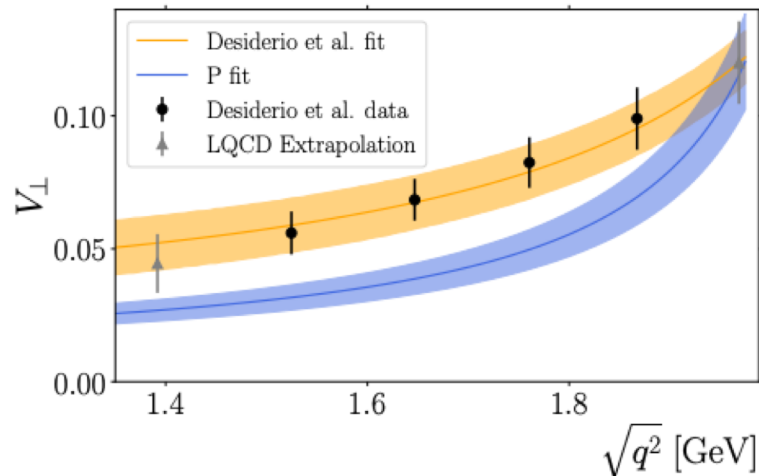
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LQCD data from Desiderio et al., PRD '21 (2006.05358)

P fit

$$r_{\perp}^{D_s^*} = 0.015(2),$$

$$\chi^2 = 32$$

PP fit

$$r_{\perp 1} = 0.009 \pm 0.003,$$

$$r_{\perp 2} = 0.029 \pm 0.005$$

$$\chi^2 = 1.5$$

Methodological overview of the GNSV approach to hadronic FFs

Once inferred the residues in the Ds sector, the extrapolation to the Bs-sector is based on the 3-couplings:

$$\begin{aligned} g_{D_s^* D_s \gamma} &= Q_s \mu_s^{\perp 1} + Q_c \mu_c^{\perp 1}, & g_{D_{s1} D_s \gamma} &= -Q_s \mu_s^{\parallel} + Q_c \mu_c^{\parallel}, \\ g_{B_s^* B_s \gamma} &= Q_s \mu_s^{\perp 1} + Q_b \mu_b^{\perp 1}, & g_{B_{s1} B_s \gamma} &= -Q_s \mu_s^{\parallel} + Q_b \mu_b^{\parallel}, \end{aligned} \quad \left[\mu_c^{\parallel} = \frac{m_s}{m_c} \mu_s^{\parallel}, \quad \mu_b^{\parallel} = \frac{m_s}{m_b} \mu_s^{\parallel} \right]$$

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Quark magnetic moments		$B_s \rightarrow \gamma$ FFs parameters	
$\mu_s^{\perp 1}$	-0.22(8)	$r_{\perp 1}^{B_s}$	0.017 ± 0.006
$\mu_b^{\perp 1}$	-0.019(6)	$r_{\perp 2}^{B_s}$	0.088 ± 0.030
$\mu_s^{\perp 2}$	-2.6(8)	$r_{\parallel}^{B_s}$	-0.043 ± 0.004
$\mu_b^{\perp 2}$	-0.22(6)	$\rho(r_{\perp 1}, r_{\perp 2})$	-0.21
μ_s^{\parallel}	-0.46(4)		
μ_b^{\parallel}	-0.038(3)		

**Starting from LQCD data in
Desiderio et al., PRD '21
(2006.05358)**

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Final ingredient for the computation of $\text{BR}(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2 : charmonia !

All **broad-charmonium states** are included as properly normalized Breit-Wigner (BW) poles, that **shift the Wilson coefficient C_9** in this way:

$$C_9 \rightarrow C_9 - \frac{9\pi}{\alpha^2} \bar{C} \sum_V |\eta_V| e^{i\delta_V} \frac{\hat{m}_V \mathcal{B}(V \rightarrow \mu^+ \mu^-) \hat{\Gamma}_{\text{tot}}^V}{\hat{q}^2 - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_{\text{tot}}^V}$$

Kruger and Sehgal, **PLB '96 (hep-ph/9603237)**

$$\bar{C} = C_1 + C_2/3 + C_3 + C_4/3 + C_5 + C_6/3$$

Values taken from

Beneke, Bobeth and Wang, JHEP '20 [2008.12494]

$m_{\psi(2S)}$	3.686 GeV	PDG '22	$\Gamma_{\psi(2S)}$	0.294×10^{-3} GeV	PDG '22
$m_{\psi(3770)}$	3.774 GeV		$\Gamma_{\psi(3770)}$	27.2×10^{-3} GeV	
$m_{\psi(4040)}$	4.039 GeV		$\Gamma_{\psi(4040)}$	80×10^{-3} GeV	
$m_{\psi(4160)}$	4.191 GeV		$\Gamma_{\psi(4160)}$	70×10^{-3} GeV	
$m_{\psi(4415)}$	4.421 GeV		$\Gamma_{\psi(4415)}$	62×10^{-3} GeV	
$\mathcal{B}(\psi(2S) \rightarrow \ell\ell)$	8.0×10^{-3}	PDG '22	$\delta_{\psi(2S)}$	0	BES Coll. ↓
$\mathcal{B}(\psi(3770) \rightarrow \ell\ell)$	9.6×10^{-6}		$\delta_{\psi(3770)}$	0	
$\mathcal{B}(\psi(4040) \rightarrow \ell\ell)$	10.7×10^{-6}		$\delta_{\psi(4040)}$	$133 \times \pi/180$	
$\mathcal{B}(\psi(4160) \rightarrow \ell\ell)$	6.9×10^{-6}		$\delta_{\psi(4160)}$	$301 \times \pi/180$	
$\mathcal{B}(\psi(4415) \rightarrow \ell\ell)$	2.0×10^{-5}		$\delta_{\psi(4415)}$	$246 \times \pi/180$	

arXiv:0705.4500

The full formula for BR(Bs → μμγ)

$$\frac{d^2\Gamma^{(1)}}{d\hat{s}d\hat{t}} = \frac{G_F^2 \alpha_{em}^3 M_1^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \left[x^2 B_0(\hat{s}, \hat{t}) + x \xi(\hat{s}, \hat{t}) \tilde{B}_1(\hat{s}, \hat{t}) + \xi^2(\hat{s}, \hat{t}) \tilde{B}_2(\hat{s}, \hat{t}) \right], \quad (6.1)$$

Emission of the photon from valence quarks or FCNC vertex (DE component)

$$B_0(\hat{s}, \hat{t}) = (\hat{s} + 4\hat{m}_l^2) (F_1(\hat{s}) + F_2(\hat{s})) - 8\hat{m}_l^2 |C_{10A}(\mu)|^2 (F_V^2(q^2) + F_A^2(q^2)),$$

$$\tilde{B}_1(\hat{s}, \hat{t}) = 8 \left[\hat{s} F_V(q^2) F_A(q^2) \text{Re} \left(C_{9V}^{eff*}(\mu, q^2) C_{10A}(\mu) \right) + \hat{m}_b F_V(q^2) \text{Re} \left(C_{7\gamma}^*(\mu) \bar{F}_{TA}^*(q^2) C_{10A}(\mu) \right) + \hat{m}_b F_A(q^2) \text{Re} \left(C_{7\gamma}^*(\mu) \bar{F}_{TV}^*(q^2) C_{10A}(\mu) \right) \right],$$

$$\tilde{B}_2(\hat{s}, \hat{t}) = \hat{s} (F_1(\hat{s}) + F_2(\hat{s})),$$

$$F_1(\hat{s}) = \left(|C_{9V}^{eff}(\mu, q^2)|^2 + |C_{10A}(\mu)|^2 \right) F_V^2(q^2) + \left(\frac{2\hat{m}_b}{\hat{s}} \right)^2 |C_{7\gamma}(\mu) \bar{F}_{TV}(q^2)|^2 + \frac{4\hat{m}_b}{\hat{s}} F_V(q^2) \text{Re} \left(C_{7\gamma}(\mu) \bar{F}_{TV}(q^2) C_{9V}^{eff*}(\mu, q^2) \right),$$

$$F_2(\hat{s}) = \left(|C_{9V}^{eff}(q^2, \mu)|^2 + |C_{10A}(\mu)|^2 \right) F_A^2(q^2) + \left(\frac{2\hat{m}_b}{\hat{s}} \right)^2 |C_{7\gamma}(\mu) \bar{F}_{TA}(q^2)|^2 + \frac{4\hat{m}_b}{\hat{s}} F_A(q^2) \text{Re} \left(C_{7\gamma}(\mu) \bar{F}_{TA}(q^2) C_{9V}^{eff*}(\mu, q^2) \right).$$

$$\frac{d^2\Gamma^{(2)}}{d\hat{s}d\hat{t}} = \frac{G_F^2 \alpha_{em}^3 M_1^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \left(\frac{8 f_{B_q}}{M_B} \right)^2 \hat{m}_l^2 |C_{10A}(\mu)|^2 \left[\frac{\hat{s} + x^2/2}{(\hat{u} - \hat{m}_l^2)(\hat{t} - \hat{m}_l^2)} - \left(\frac{x \hat{m}_l}{(\hat{u} - \hat{m}_l^2)(\hat{t} - \hat{m}_l^2)} \right)^2 \right] \quad (6.2)$$

Bremsstrahlung

$$\frac{d^2\Gamma^{(12)}}{d\hat{s}d\hat{t}} = -\frac{G_F^2 \alpha_{em}^3 M_1^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \frac{16 f_{B_q}}{M_B} \hat{m}_l^2 \frac{x^2}{(\hat{u} - \hat{m}_l^2)(\hat{t} - \hat{m}_{l34}^2)} \quad (6.3)$$

Interference

$$\times \left[\frac{2x\hat{m}_b}{\hat{s}} \text{Re} \left(C_{10A}^*(\mu) C_{7\gamma}(\mu) \bar{F}_{TV}(q^2, 0) \right) + x F_V(q^2) \text{Re} \left(C_{10A}^*(\mu) C_{9V}^{eff}(\mu, q^2) \right) + \xi(\hat{s}, \hat{t}) F_A(q^2) |C_{10A}(\mu)|^2 \right].$$

Melikhov and Nikitin, Phys. Rev. D 70 (2004) 114028

Kozachuk, Melikhov and Nikitin, Phys. Rev. D 97 (2018) 053007

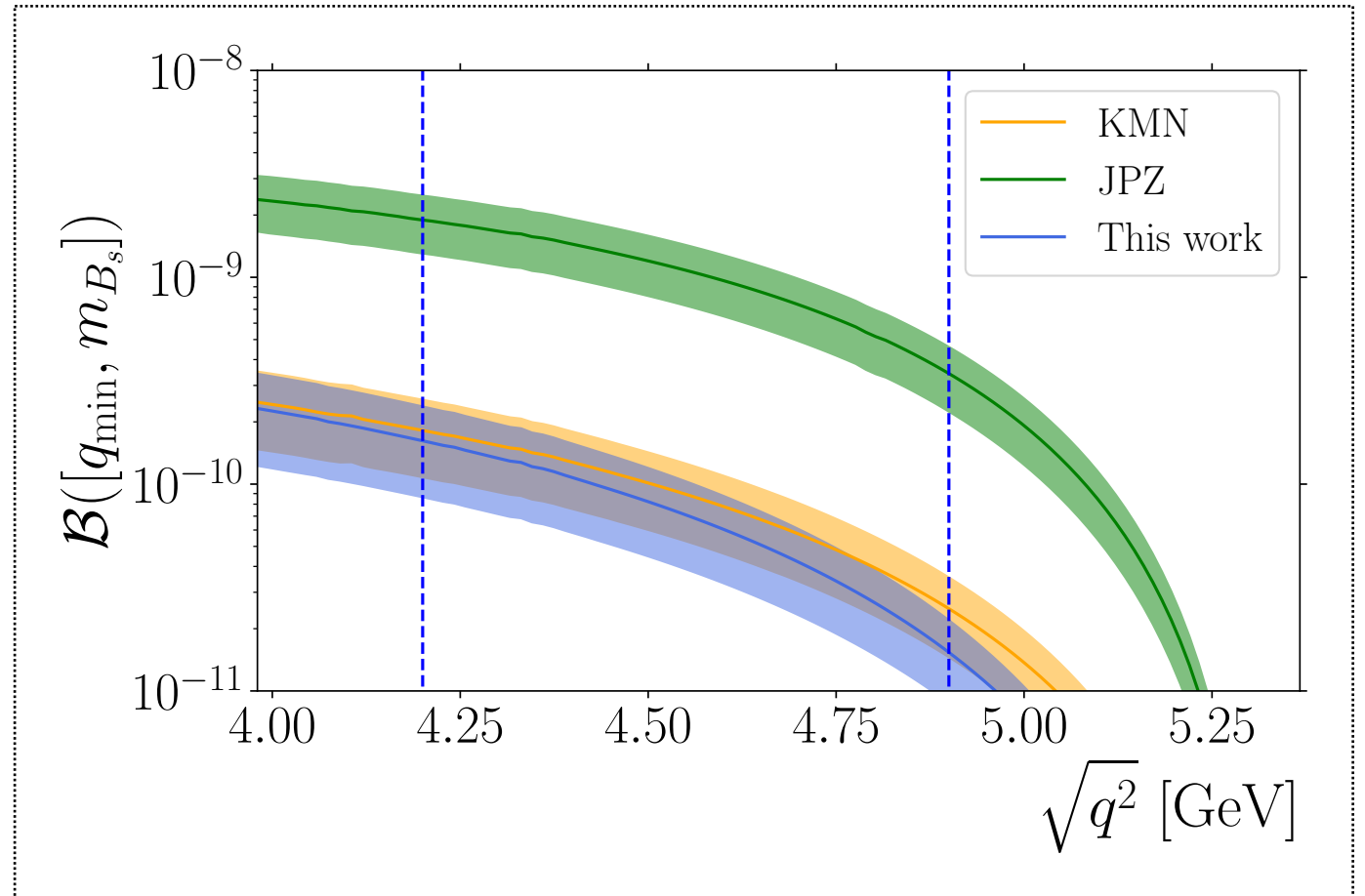
Guadagnoli, Melikhov and Reboud, Phys.Lett.B 760 (2016) 442-447 [fixes a typo in the interference term Γ^{12}]

Impact of charmonium resonances on BR prediction

By focusing on the DE-only component of $\text{BR}(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2 ,
we find the following values for the BR:

(REMARK: DDILST computation gives results very similar to JPZ ones!)

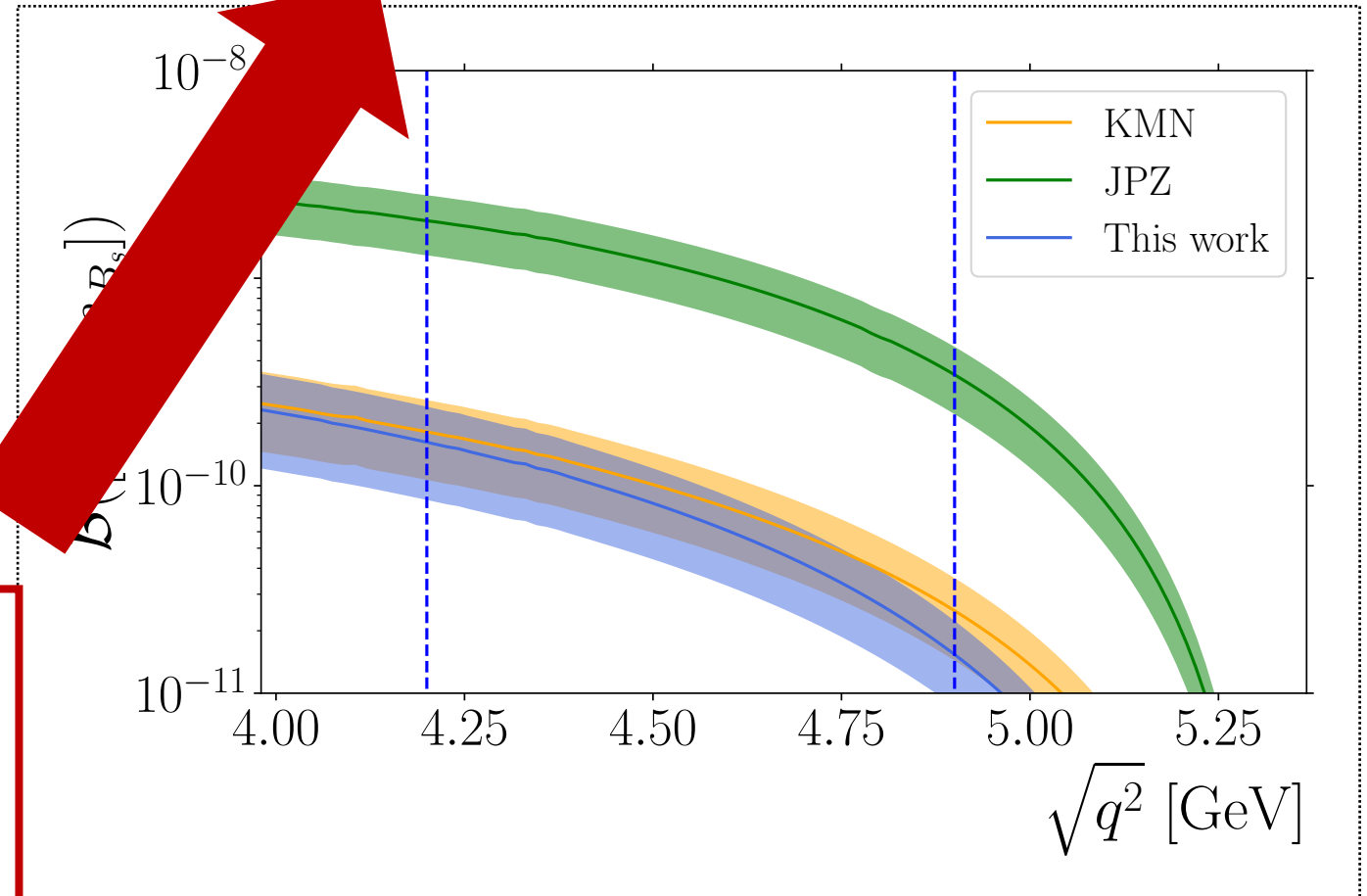
$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma)[4.2 \text{ GeV}, m_{B_s^0}]$	
GNSV	$(1.63 \pm 0.80) \times 10^{-10}$
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Influence of the choice of $T_{\perp,\parallel}$ (with $V_{\perp,\parallel}$ from this work)	
$T_{\perp,\parallel}$ from KMN	$(1.22 \pm 0.70) \times 10^{-10}$
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$T_{\perp,\parallel} = 0$	$(1.63 \pm 0.80) \times 10^{-10}$



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Negligible impact of tensor FFs ...

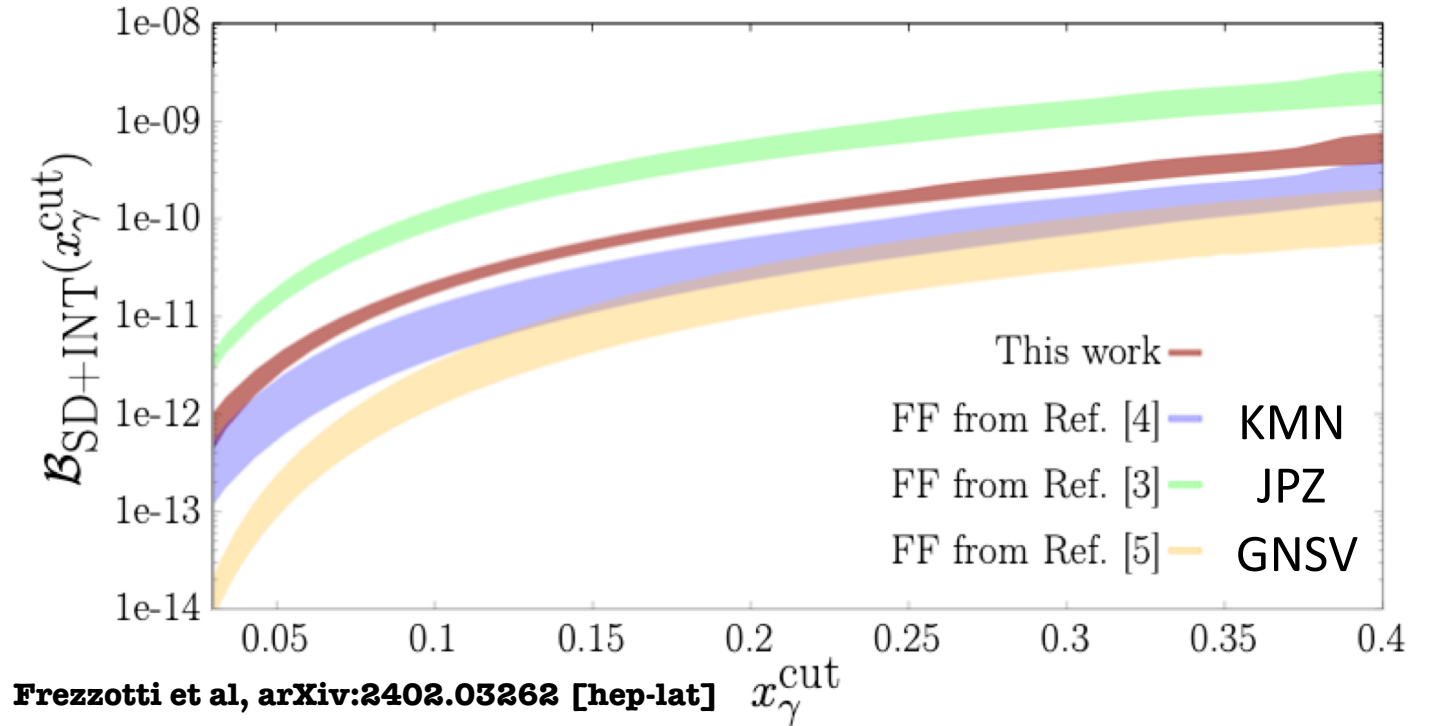
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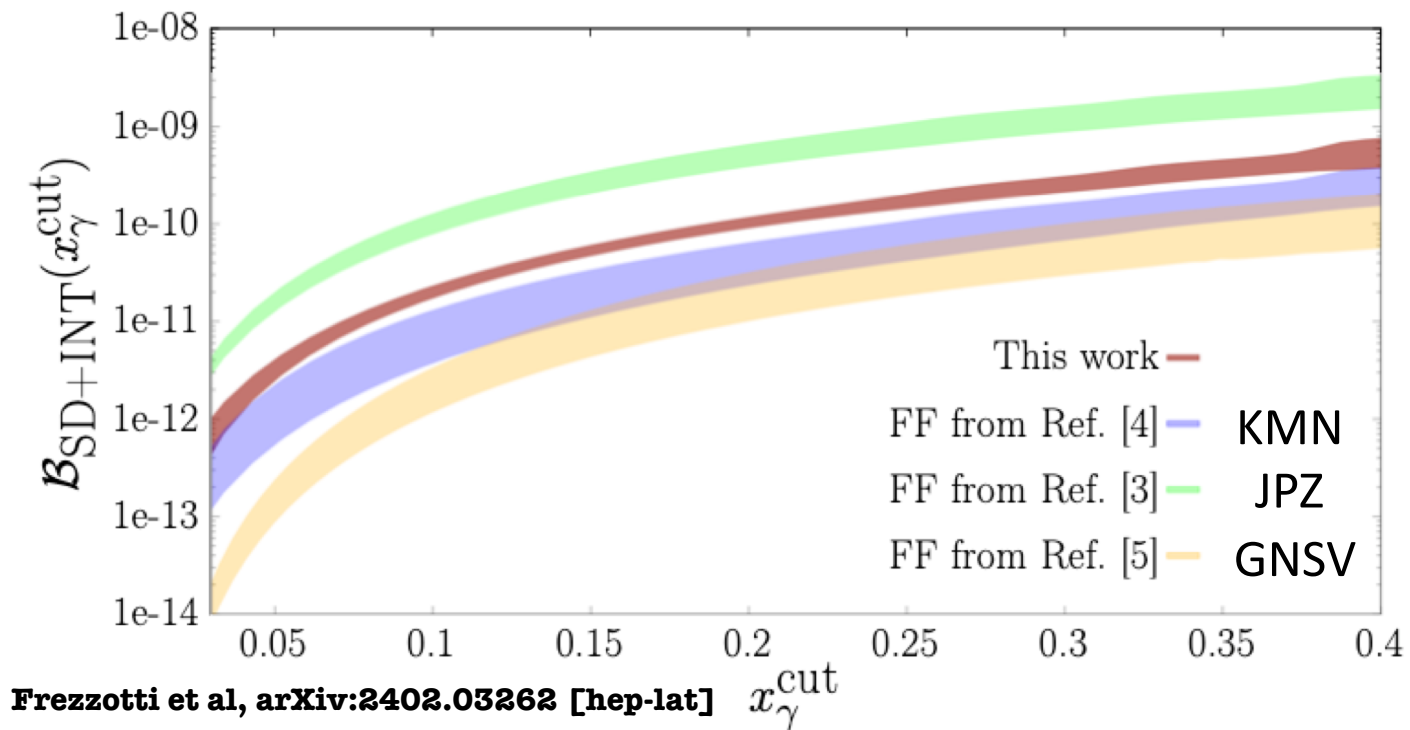


ETMC number (see Table VI): $\text{BR} \times 10^{10} = 5.3(1.7)$

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Differences @ the $1.9\sigma - 2.2\sigma$ level despite the differences in the FFs values
Fundamental topic of discussion: treatment of broad-charmonium parameters!

Brief recap of the GNSV procedure within the SM

The procedure presented in **Guadagnoli, Normand, Simula, LV, JHEP '23 (2303.02174)** is based on a HQET scaling of FFs parameters from the Ds-sector to the Bs-sector, starting from LQCD data available for the Ds-sector @ high- q^2 .

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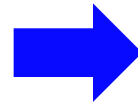
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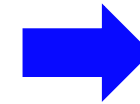
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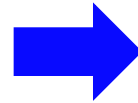
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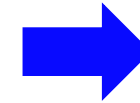
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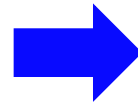
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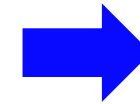
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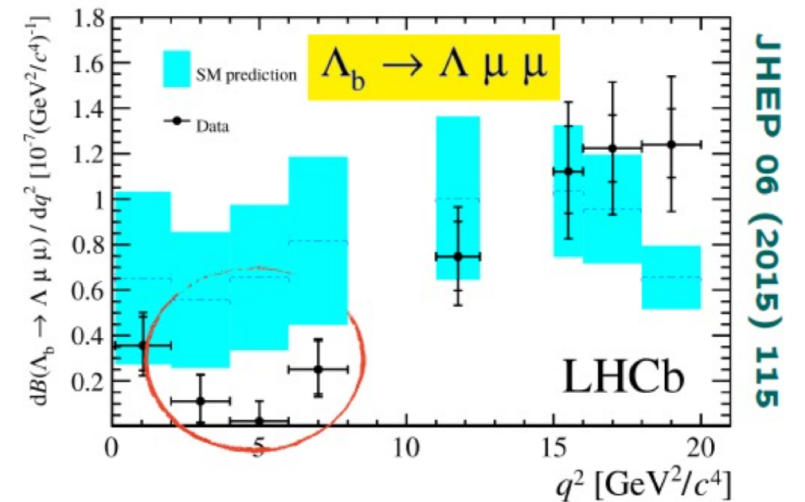
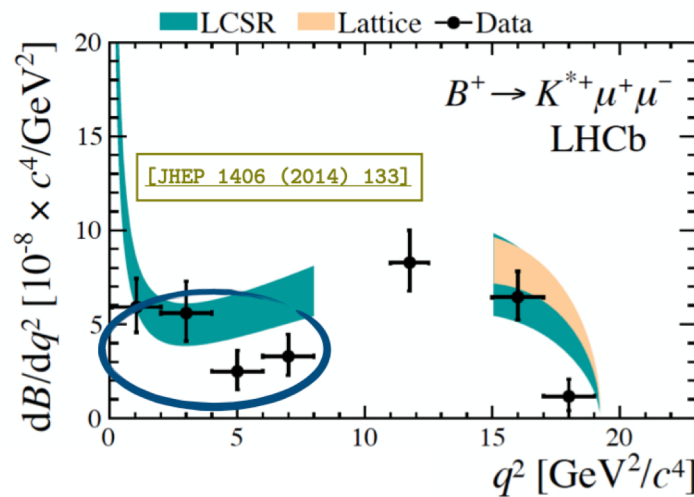
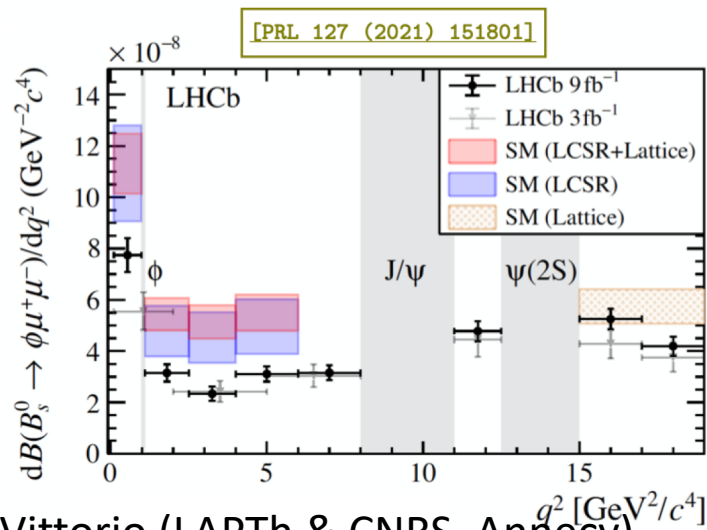
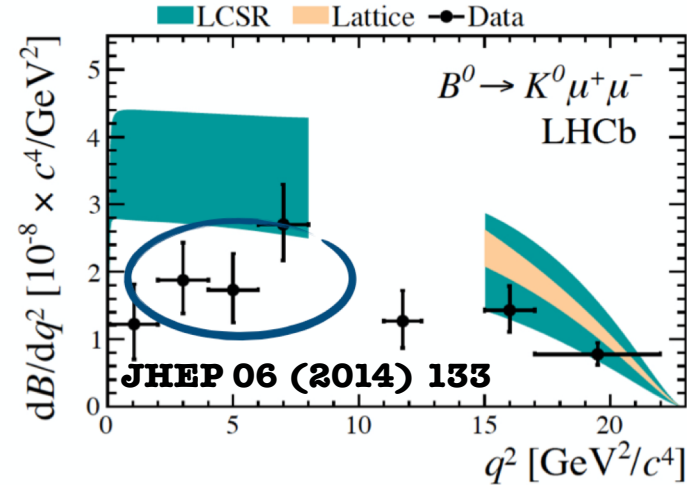
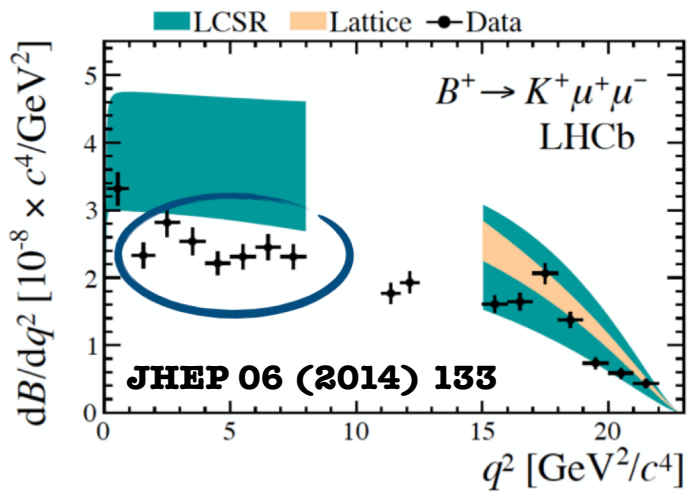
And what about Bs → γ beyond the Standard model ??

The relevance of $B_s \rightarrow \gamma$ beyond the SM

The $B_s \rightarrow \mu\mu\gamma$ channel can be used to study hypothetical New Physics (NP) effects affecting $b \rightarrow s$ quark *transition*. In fact, despite the disappearance of the $R(K^*)$ anomalies, we have several discrepancies among theory and experiments in semileptonic neutral-current B decays:

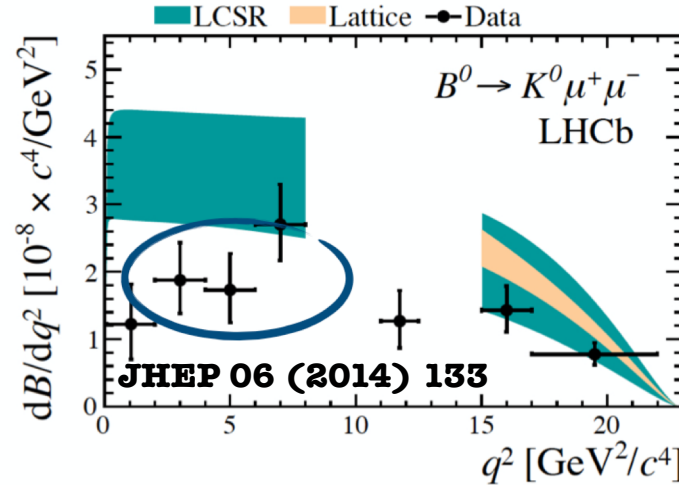
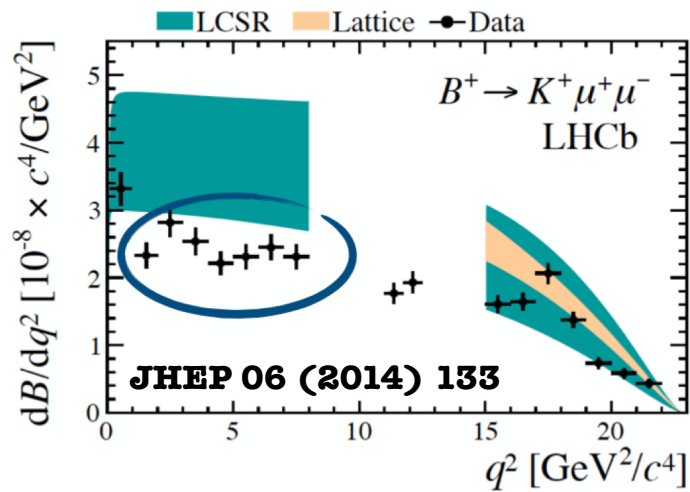
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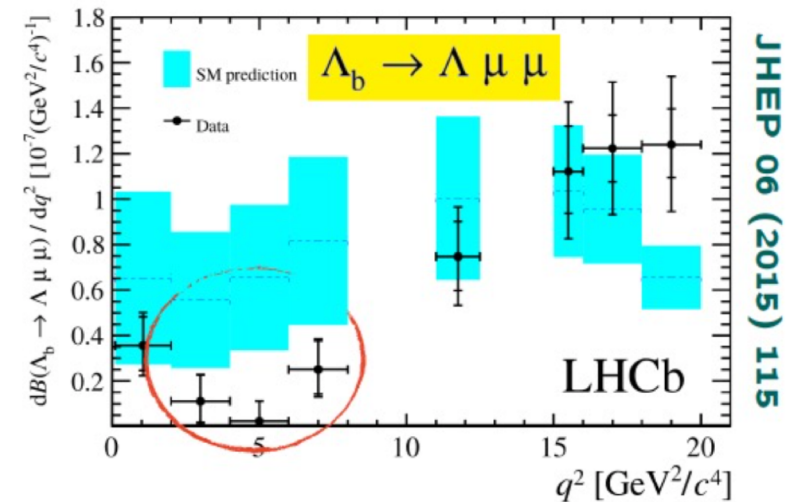
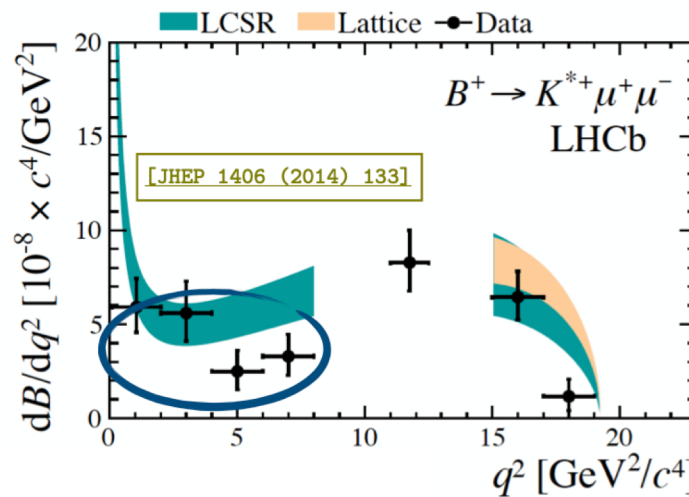
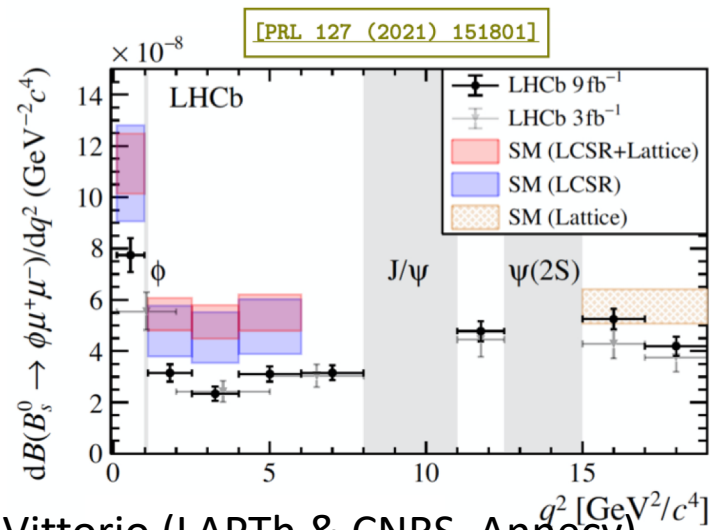


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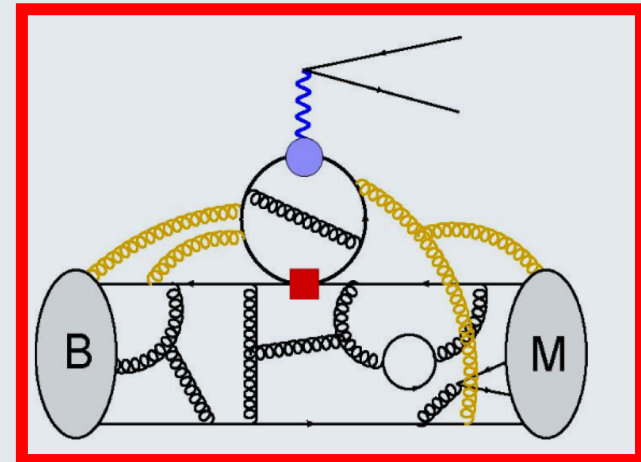
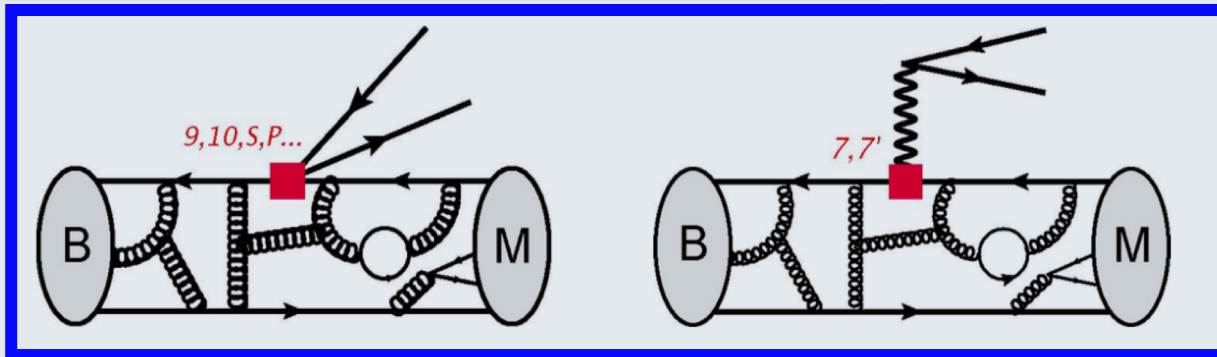


SM expectations seem to be always above experimental data ...



Semileptonic B decays theory in a nutshell

$$\mathcal{H}(b \rightarrow sll) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu)$$



$$A_\lambda^{L,R}(B \rightarrow M_\lambda ll) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

$$\mathcal{H}_\mu(k, q) = i \int d^4x e^{iq \cdot x} \langle \bar{M}(k) | T \{ \mathcal{J}_\mu^{\text{em}}(x), C_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

Non-local form-factors

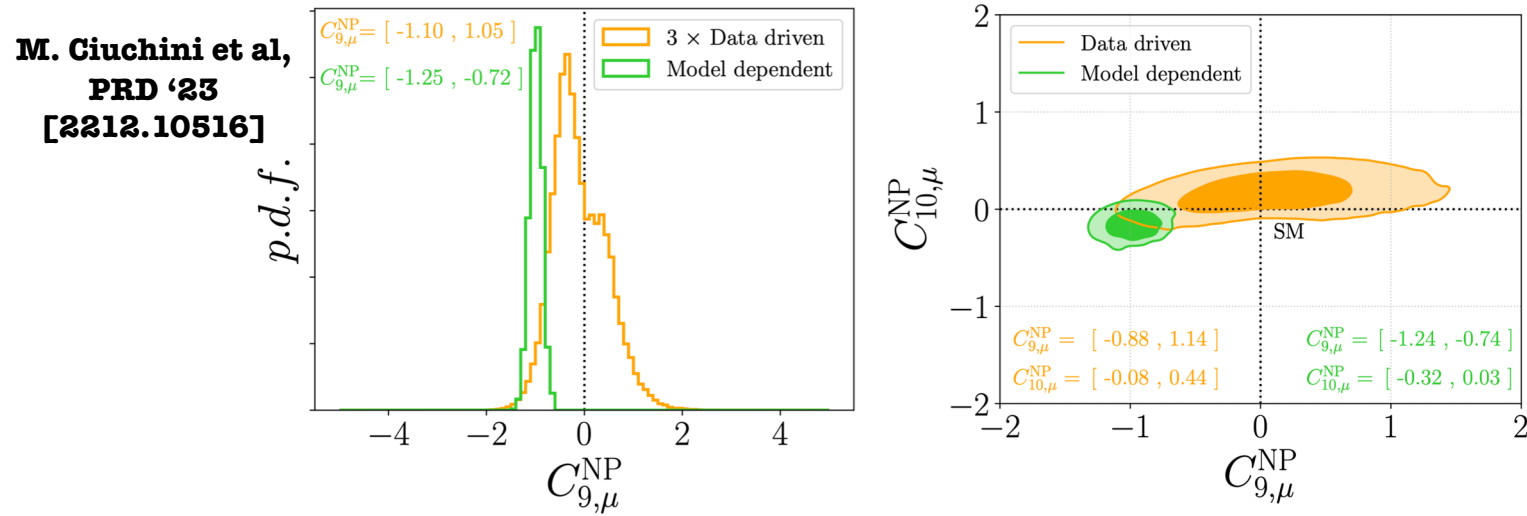
M. Reboud's talk @ LHC Implication Workshop 2022 @ CERN

$$\mathcal{F}_\mu(k, q) = \langle \bar{M}(k) | \bar{s} \gamma_\mu b_L | \bar{B}(q+k) \rangle$$

$$Q_2^c = (\bar{s}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L)$$

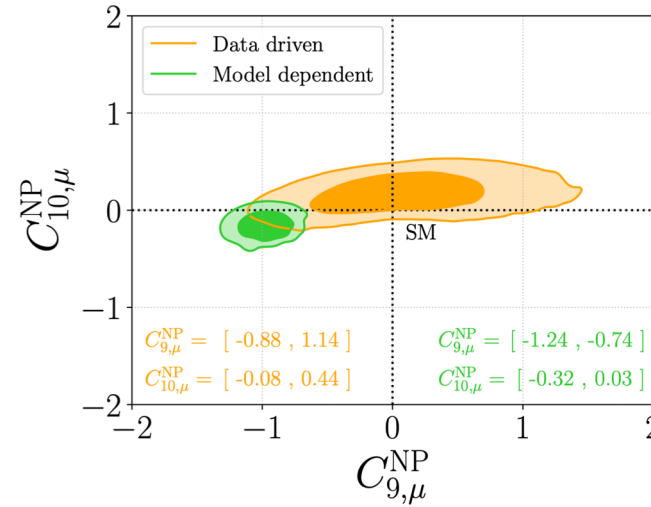
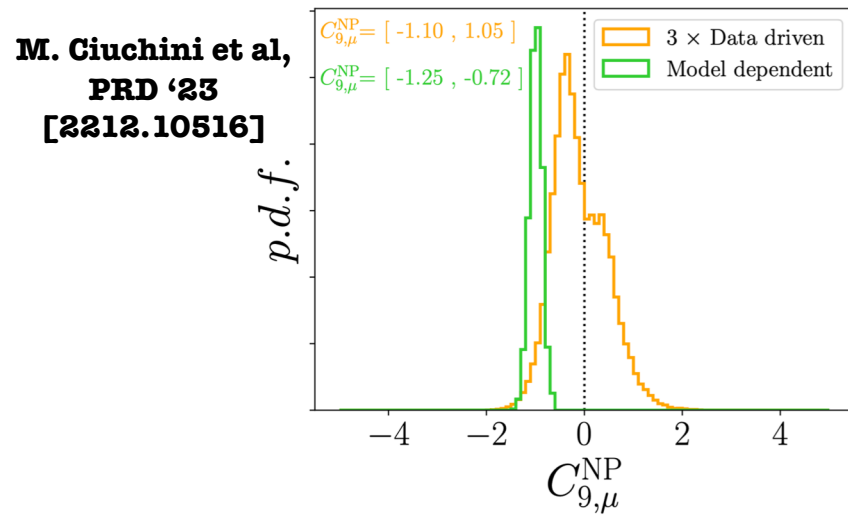
Important issue concerning non-local FFs

There is **not** general consensus on the magnitude of the contributions coming from these non-local FFs:



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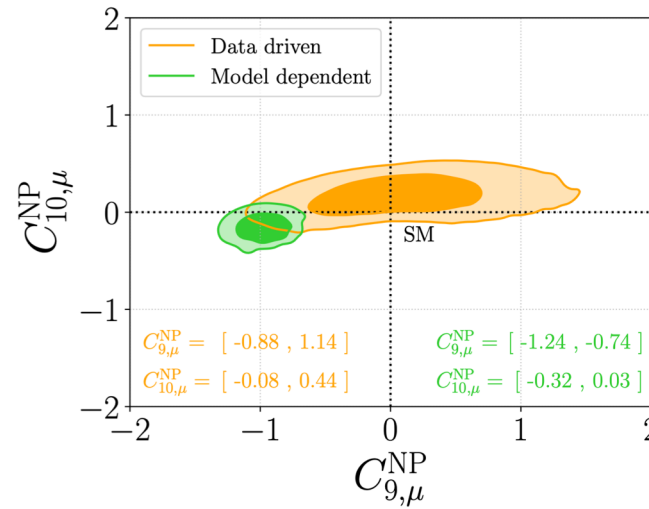
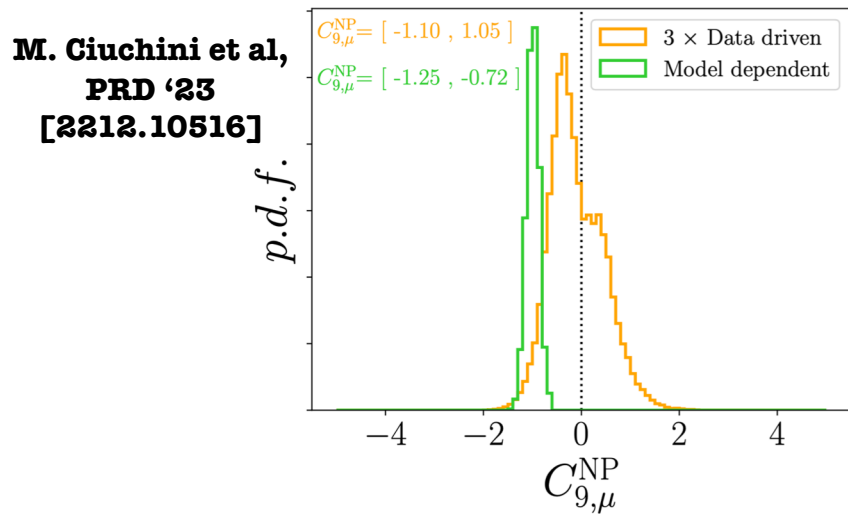
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**The «hypothetical»
presence of NP seems
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The «hypothetical» presence of NP seems to depend on the way in which the non-local FFs are described !

Data-driven: naïve q^2 -expansion of the form

$$H_V^- \propto \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(C_7^{\text{SM}} + h_-^{(0)} \right) \tilde{T}_{L-} - 16\pi^2 h_-^{(2)} q^4 \right] + \left(C_9^{\text{SM}} + h_-^{(1)} \right) \tilde{V}_{L-},$$

Clear advantage: it is transparent the interplay between hadronic and possible NP contributions!

M. Ciuchini et al, JHEP '16 [1512.07157], EPJC '17 [1704.05447], EPJC '19 [1903.09632], PRD '21 [2011.01212], EPJC '23 [2110.10126], PRD '23 [2212.10516]

Model-dependent: it assumes the h -terms on the left $[h_-^{(0)}, h_-^{(1)}, h_-^{(2)}]$ to be negligible.

Underlying idea: this assumption is supported at present by the **application of dispersion relations, analyticity and unitarity (together with LCSR data)** to the description of non-local FFs !

**C. Bobeth et al, EPJC '18 [1707.07305]
M. Chrzaszcz et al, JHEP '19 [1805.06378]
N. Gubernari et al, JHEP '21 [2011.09813], JHEP '22 [2206.03797], 2305.06301**

$B_s \rightarrow \mu\mu\gamma$ as a golden channel to study NP

KEY IDEA: high- q^2 observables are sensitive to the very same short-distance physics present in $B \rightarrow K(^*)$ decays, without being affected by the same long-distance effects !

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$B_s \rightarrow \mu\mu\gamma$ as a perfect candidate: if there is really NP in $B \rightarrow K(^*)$ decays, *i.e.* if there is really a NP contribution to C_9 , this effect must influence as well the $BR(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2 !

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However, we do not have a direct measurement of $BR(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2 , but only an experimental bound:

$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma)_{m_{\mu\mu} > 4.9 \text{ GeV}} = (-2.5 \pm 1.4 \pm 0.8) \times 10^{-9} < 2.0 \times 10^{-9}$$

LHCb Collaboration, LHCb-PAPER-2021-007 & LHCb-PAPER-2021-008

Thus, the best that we can do at this stage is a sensitivity study !

$B_s \rightarrow \mu\mu\gamma$ as a golden channel to study NP

Main ingredients of our sensitivity study:

- 1. Identification of NP benchmarks**
from semileptonic neutral-current B decays:

	NP shift		ℓ -specific + ℓ -univ. parts
$(k = 9, 10)$	δC_k^{bsee}	\equiv	$\delta C_k^{(e)} + \delta C_k^{u(e,\mu)}$
	$\delta C_k^{bs\mu\mu}$	\equiv	$\delta C_k^{(\mu)} + \delta C_k^{u(e,\mu)}$

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$\boxed{\delta C_9^{(\ell)} = -\delta C_{10}^{(\ell)} \equiv \delta C_{LL}^{(\ell)}/2}$	

by fitting the data
(see back-up slides
for their list)



Scenario	Best-fit point	1σ Interval	$\sqrt{\chi^{2,SM} - \chi^2}$
$(\delta C_9^{u(e,\mu)}, \delta C_{10}^{u(e,\mu)}) \in \mathbb{R}$	$(-0.88, +0.30)$	$([-1.08, -0.56], [0.15, 0.46])$	5.5
$\delta C_{LL}^{u(e,\mu)}/2 \in \mathbb{C}$	$-0.70 - 1.36i$	$[-1.00, -0.54] + i[-1.77, -0.54]$	5.8
$\delta C_9^{u(e,\mu)} \in \mathbb{C}$	$-1.08 + 0.10i$	$[-1.31, -0.85] + i[-0.70, +0.85]$	6.4
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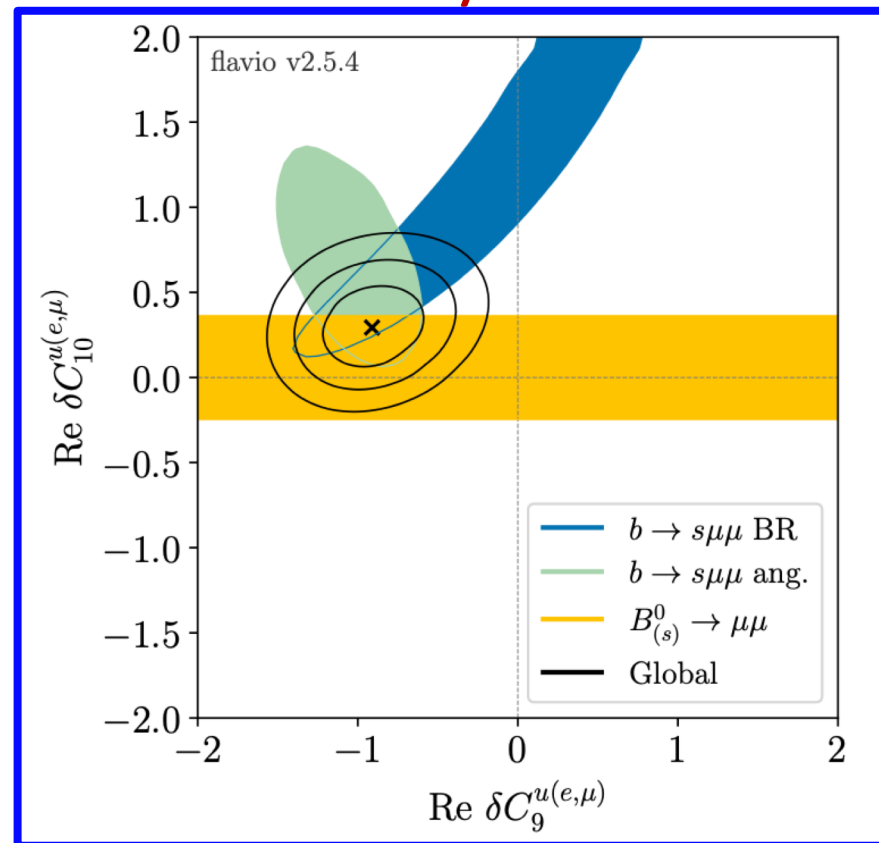
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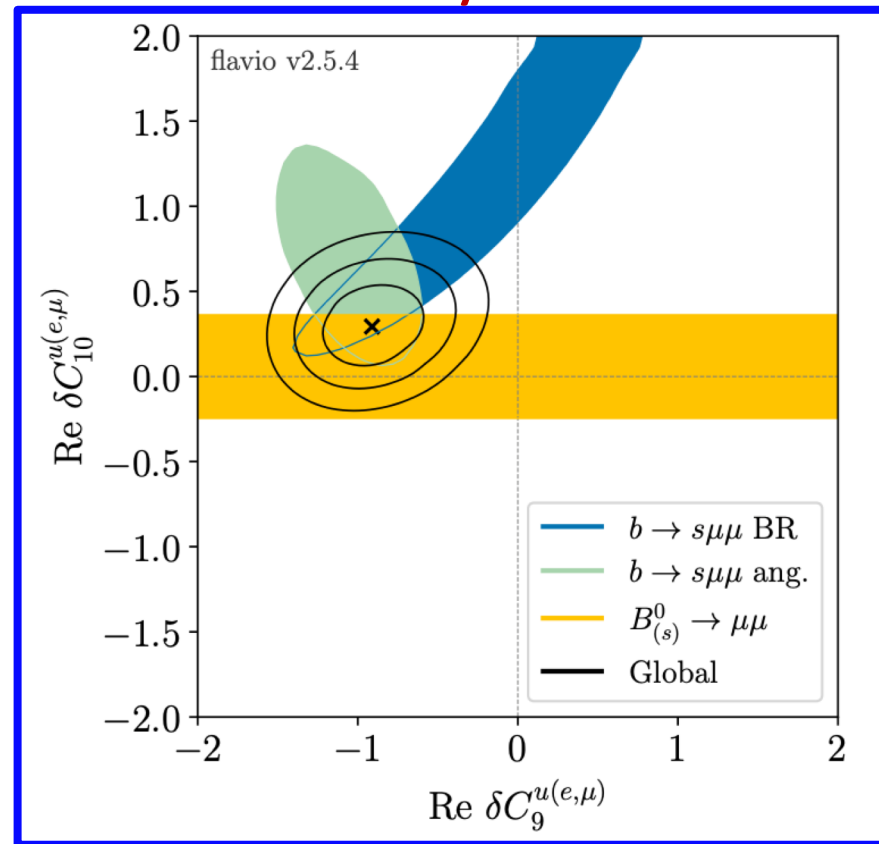
EXAMPLE

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EXAMPLE

REMARK: a complete calculation of non-local FFs may, or may not, show that the NP shifts previously shown, in particular those involving C_9 , are actually due to SM long-distance dynamics !!

(See also the global analyses in JHEP '23 [2212.10497], PRD '23 [2212.10516] ...)

$B_s \rightarrow \mu\mu\gamma$ as a golden channel to study NP

Main ingredients of our sensitivity study:

2. **Experimental uncertainties:** we will assume that all the backgrounds are under control, i.e. that their uncertainties will eventually fall safely below the signal yield ("**no-background**" hypothesis).

Thus, the $B_s \rightarrow \mu\mu\gamma$ -signal uncertainty will be dominated by the sheer amount of data collected

(Many effects to be taken into account: efficiencies, optimal choice of $(q_{min})^2$, ...)

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Motivated by precision on $D_s \rightarrow \gamma$ FFs as from the lattice study **PRD '23 (2306.05904)**, confirmed by the $B_s \rightarrow \gamma$ results in **2402.03262**

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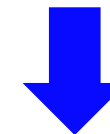
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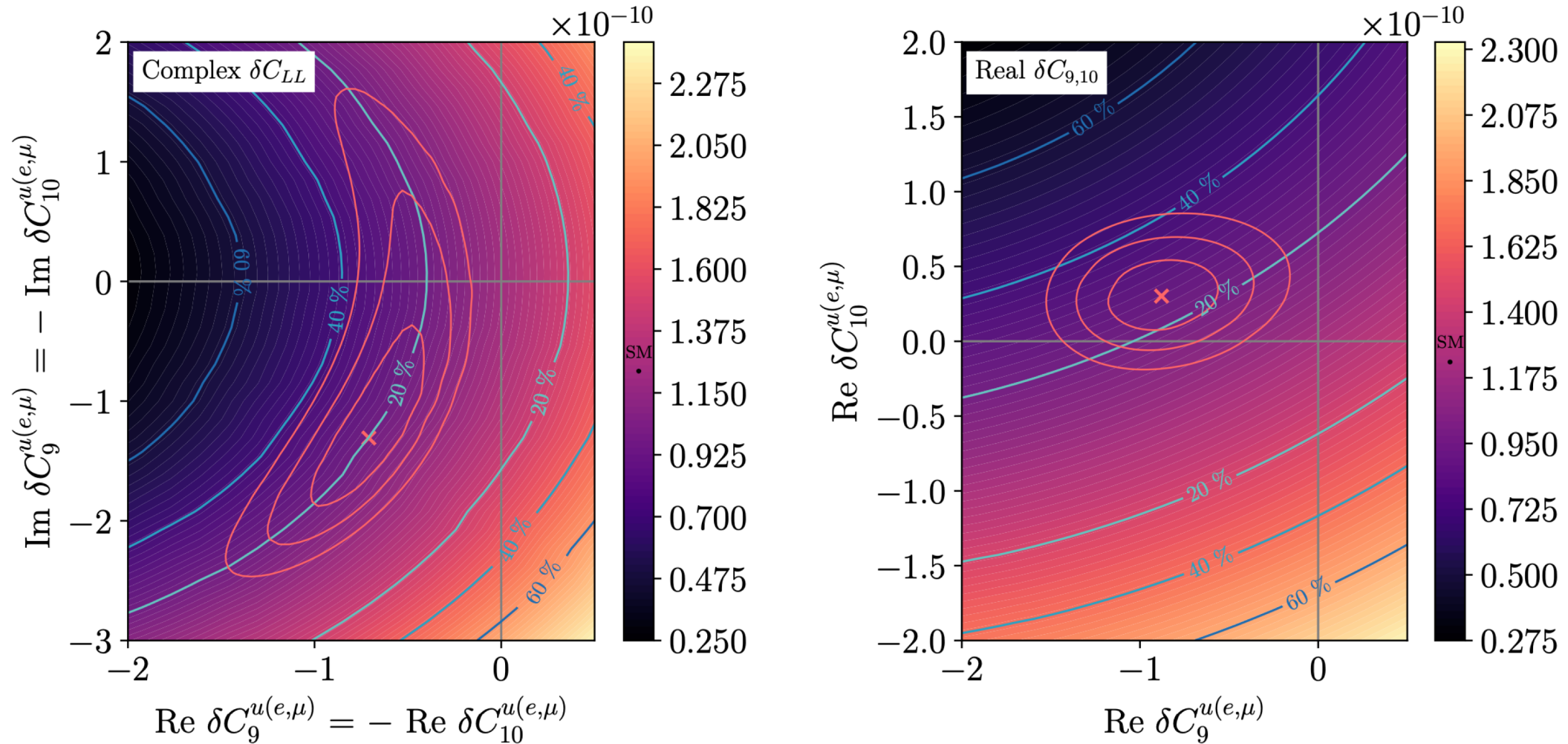
Recall that we need to resolve

$$\delta C_9^{(\mu)} / C_9^{(\mu),SM} \simeq 15\% !!$$

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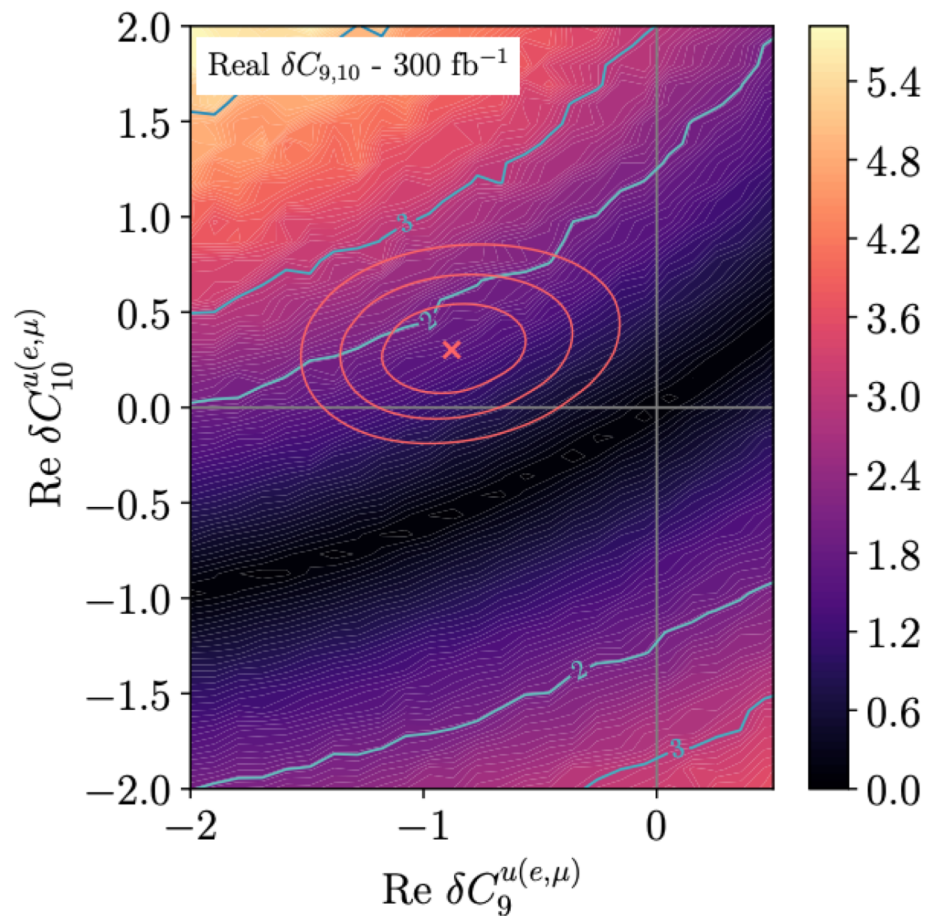
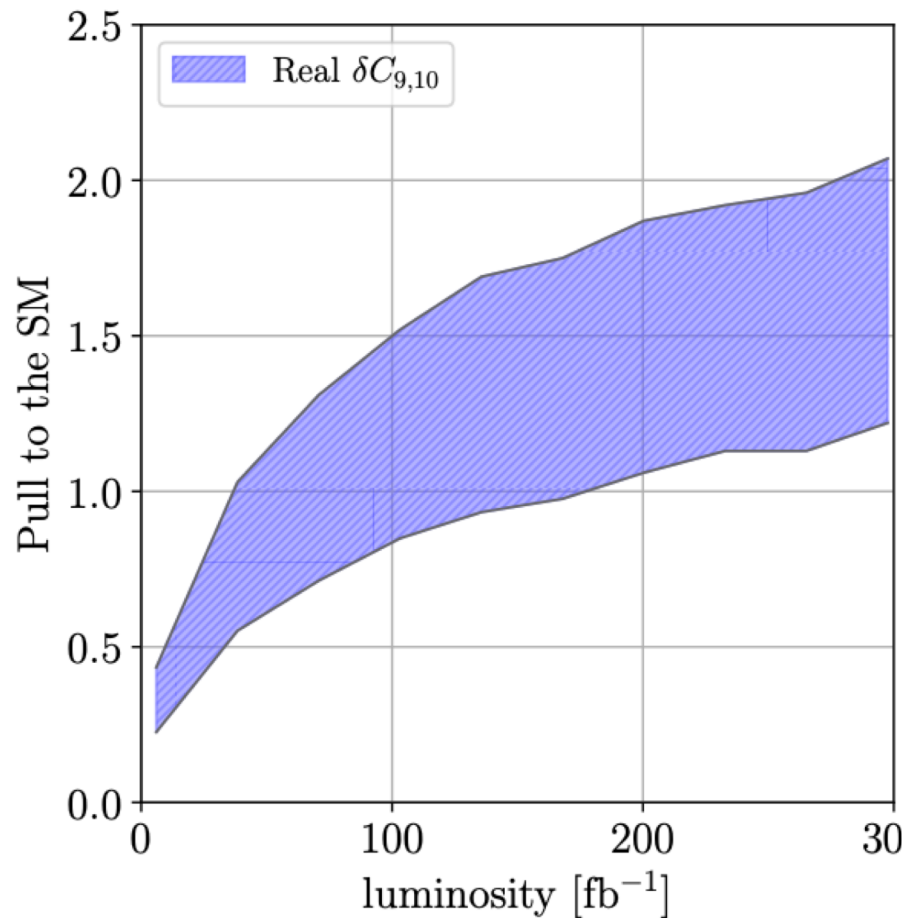


$B_s \rightarrow \mu\mu\gamma$ as a golden channel to study NP



Guadagnoli, Normand, Simula, LV, JHEP '23 [2308.00034]

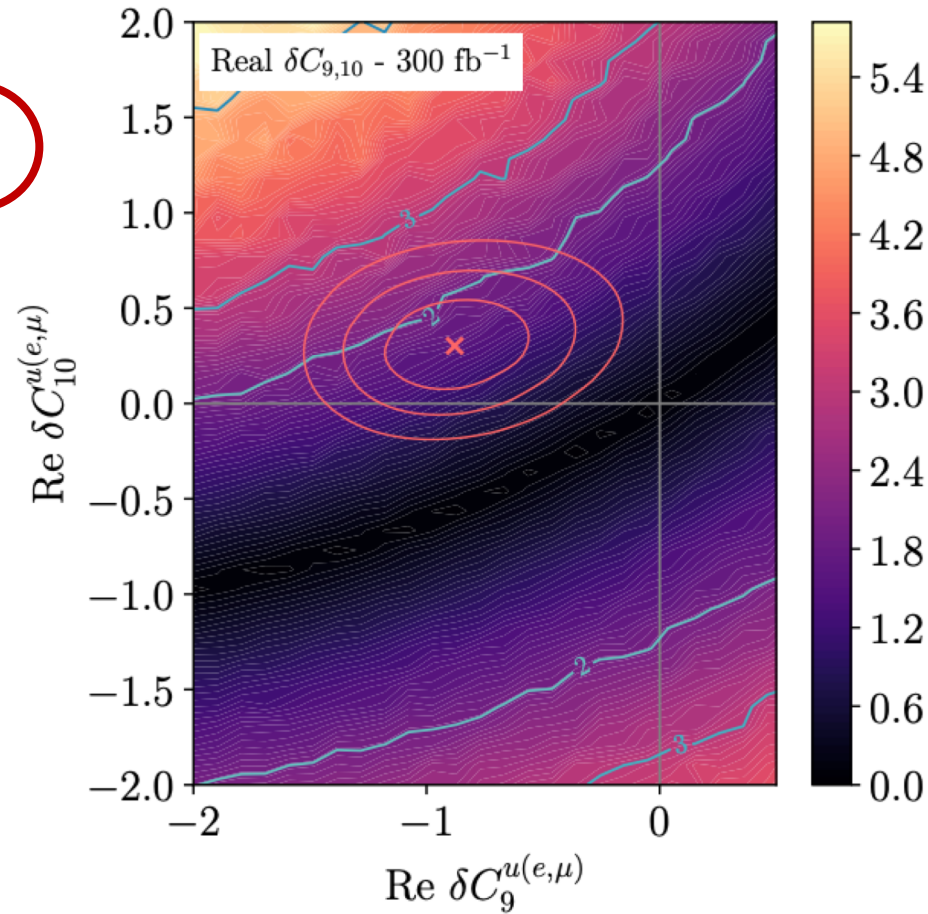
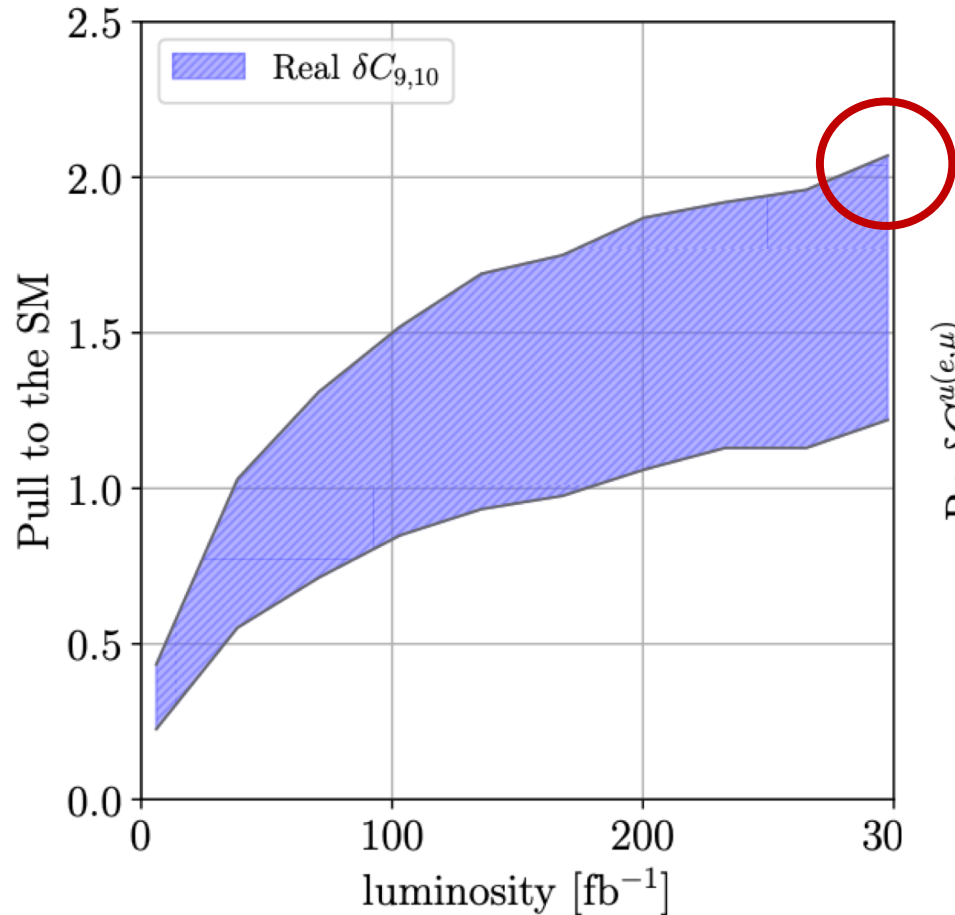
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Pull to the SM of the BR **assuming NP in both C_9 , C_{10}**

Guadagnoli, Normand, Simula, LV, JHEP '23 [2308.00034]

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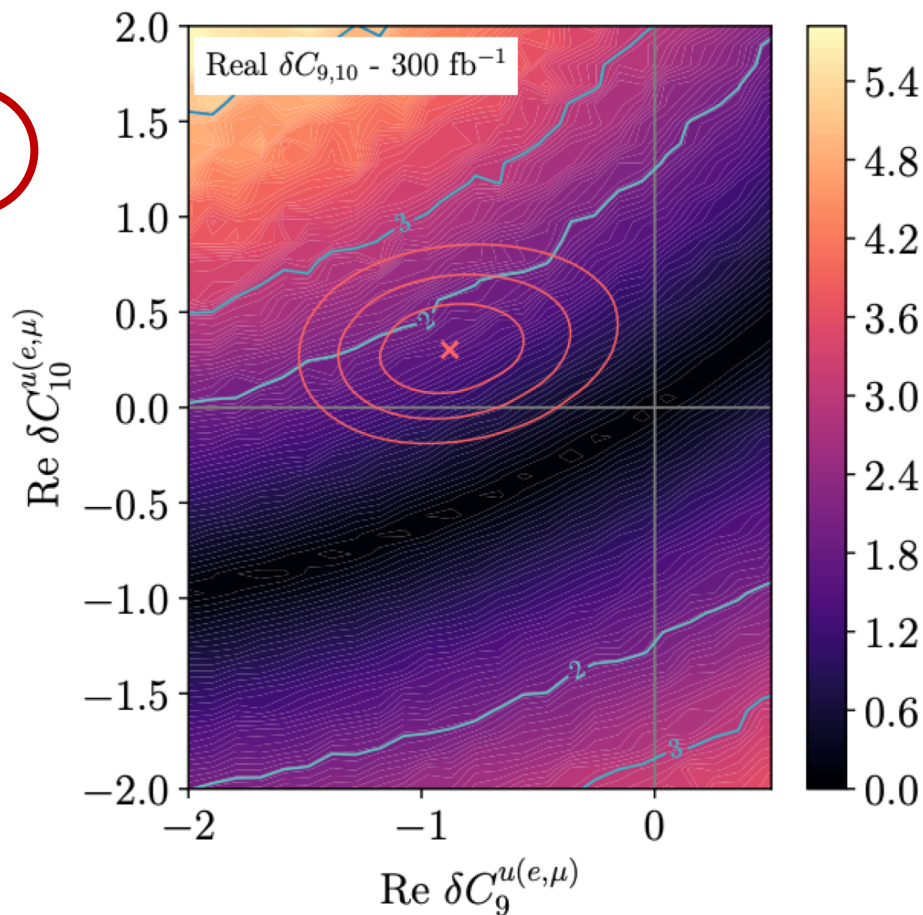
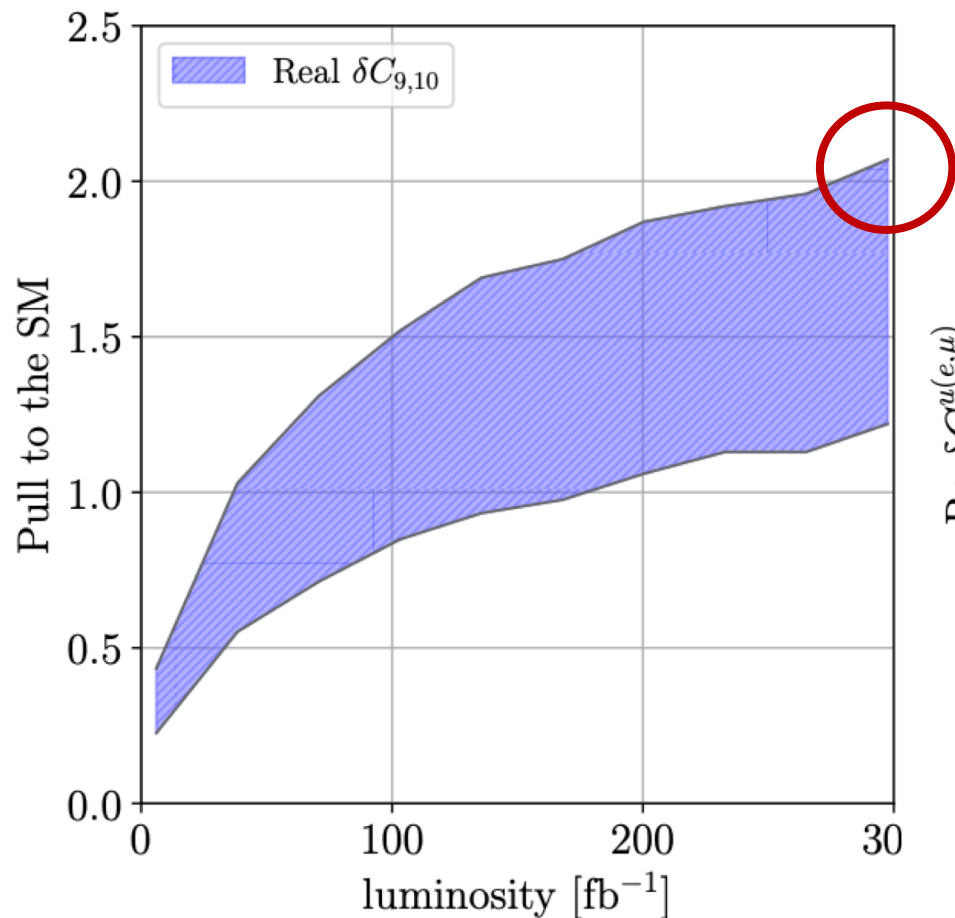


It reaches the 2σ level at the border of the 1σ region for the WC shift preferred by our global fit !

Pull to the SM of the BR **assuming NP in both C_9, C_{10}**

Guadagnoli, Normand, Simula, LV, JHEP '23 [2308.00034]

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Methodological issue: this exercise can be repeated and updated in the future with other FFs determinations (as, for instance, the ETMC results in 2402.03262)

Pull to the SM of the BR **assuming NP in both C_9, C_{10}**

Guadagnoli, Normand, Simula, LV, JHEP '23 [2308.00034]

Conclusions

Radiative-and-leptonic $B_s \rightarrow \mu\mu\gamma$ decay is an important channel to be investigated at present. Huge efforts have been and are being developed by **both the theoretical and the experimental communities to have new data!** In this talk:

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 - **inter-connection with non-leptonic radiative decays** (see *e.g.* the determination of the 3-couplings)
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 - **possible method for global analyses of all the lattice data available in the future**
- 2. Study of $B_s \rightarrow \mu\mu\gamma$ beyond the SM:** complementary way to investigate **hypothetical New Physics (NP)** effects affecting $b \rightarrow s$ quark transitions
 - **key issue: same short-distance effects present in semileptonic neutral-current B decays, while being independent (and free) of the long-distance ones !** Complementary insight on the debate on non-local FFs
 - **the pull to the SM can reach the 2σ level** at the border of the 1σ region for the WC C_9 shift preferred by our global fit of BRs and angular observables in $B \rightarrow K(^*)$ decays
 - **assumption on the exp./th. uncertainties improvable in the future**

THANKS FOR
YOUR ATTENTION!

BACK-UP SLIDES

The “indirect” method to detect radiative-and-leptonic decays

The basic idea is to reconstruct the radiative signal from the non-radiative counterpart, namely

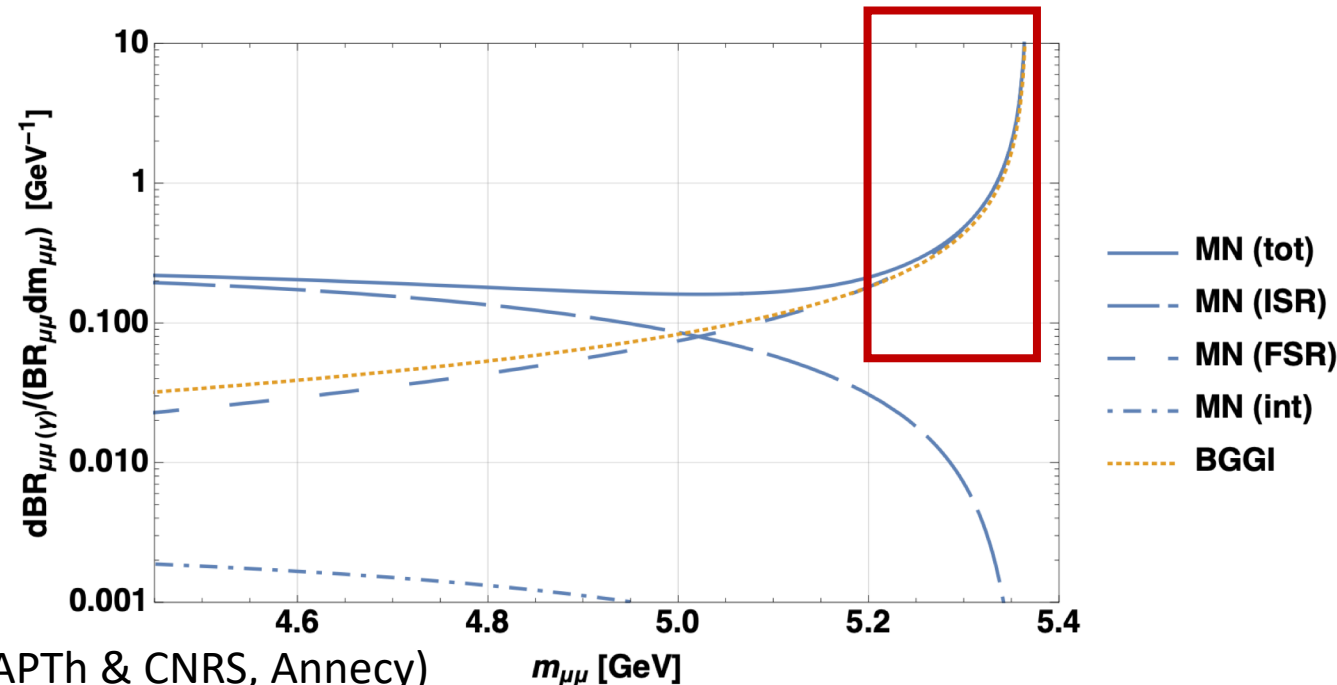
$$B_s^0 \rightarrow \mu^+ \mu^- \gamma \text{ from } B_s^0 \rightarrow \mu^+ \mu^-$$

Dettori, Guadagnoli, Reboud, Phys.Lett.B 768 (2017) 163-167

How? Enlarging the **dilepton invariant mass** below the Bs-peak (it works IF the bkg are well under control!)

The problem is in other words

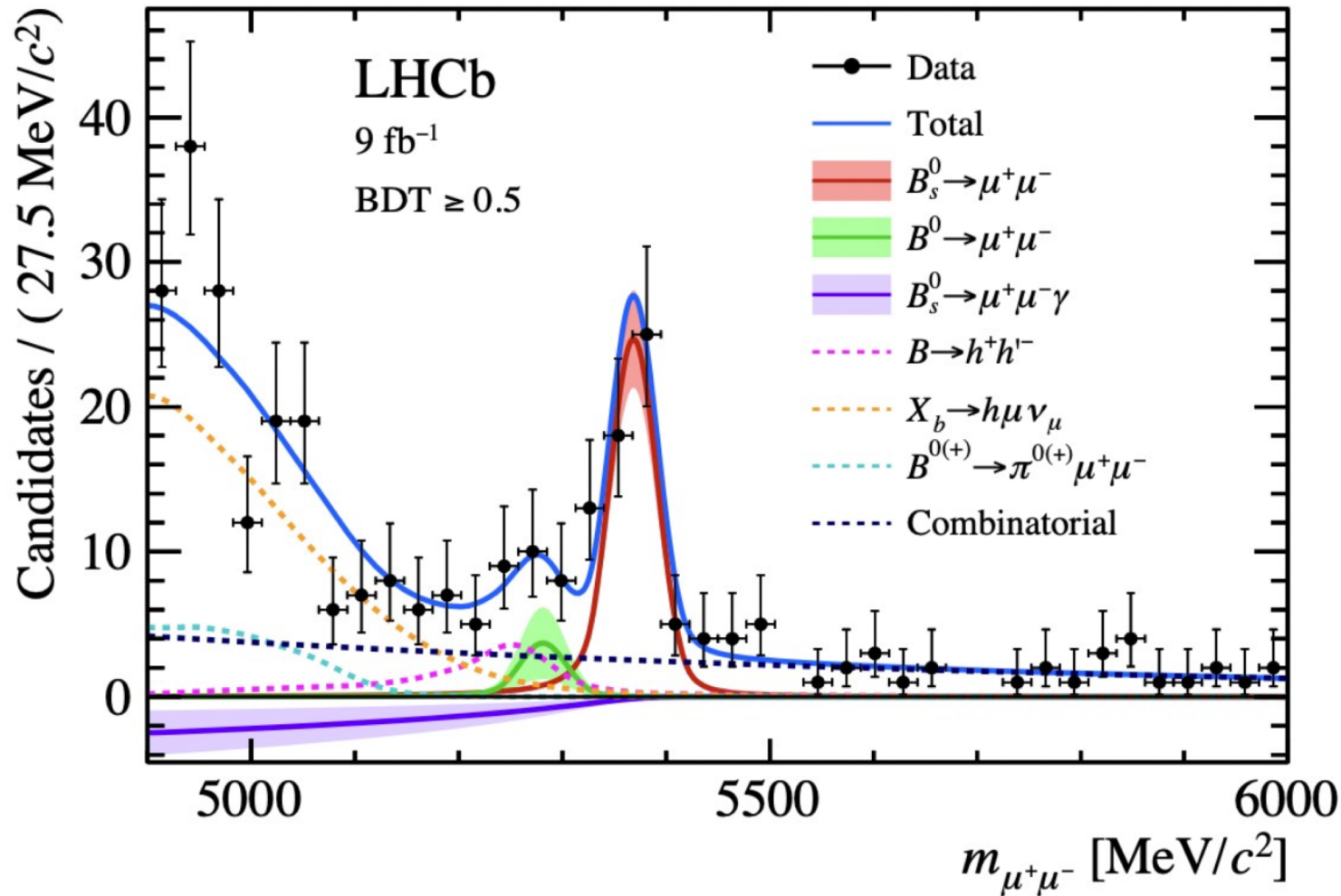
$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- + n\gamma) \text{ VS } \mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma)$$



For very soft photons, the single-photon component of the former should be equal to the latter!

The “indirect” method to detect radiative-and-leptonic decays

$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma)_{m_{\mu\mu} > 4.9 \text{ GeV}} = (-2.5 \pm 1.4 \pm 0.8) \times 10^{-9} < 2.0 \times 10^{-9}$$



(from LHCb-PAPER-2021-007, LHCb-PAPER-2021-008)

Some pros:

- i) No reconstruction of the photon, whose efficiency is inherently small
- ii) Measur. at high- q^2 , which is the best region for Lattice QCD and is also the region least affected by resonances
- iii) Sensitivity to C9, C10

Some cons:

- i) Signal as a «shoulder», i.e. requires reliable estimation of all other «shoulders»
- ii) Difficult below $(4.2 \text{ GeV})^2$
- iii) Mass resolution crucial !!

Relevant data for $b \rightarrow s$ global fits

$b \rightarrow s\mu^+\mu^-$ BR obs.
$\left\langle \frac{d\mathcal{B}}{dq^2} \right\rangle (B^+ \rightarrow K^{(*)}\mu\mu)$
$\left\langle \frac{d\mathcal{B}}{dq^2} \right\rangle (B_0 \rightarrow K\mu\mu)$
$\left\langle \frac{d\mathcal{B}}{dq^2} \right\rangle (B_s \rightarrow \phi\mu\mu)$
$\left\langle \frac{d\mathcal{B}}{dq^2} \right\rangle (B_0 \rightarrow K^*\mu\mu)$
$\langle \mathcal{B} \rangle (B \rightarrow X_s\mu\mu)$
$b \rightarrow s\mu^+\mu^-$ angular and CPV obs.
$\langle F_L, P_1, P'_{4,5}, A_{\text{FB}} \rangle (B_0 \rightarrow K^*\mu^+\mu^-)$
$\langle F_L, P_{1,2}, P'_{4,5} \rangle (B^+ \rightarrow K^{*+}\mu^+\mu^-)$
$\langle F_L, S_{3,4,7} \rangle (B_s \rightarrow \phi\mu\mu)$
$A_{3-9}(B_0 \rightarrow K^*\mu^+\mu^-)$

R_{K/K^*}
$\mathcal{B}(B_{d,s} \rightarrow \mu\mu)$
$b \rightarrow s\gamma$ obs.
$\langle \mathcal{B}, A_{CP} \rangle (B \rightarrow X_s\gamma)$
$\mathcal{B}(B^0 \rightarrow K^{*0}\gamma)/\mathcal{B}(B_s^0 \rightarrow \phi\gamma)$
$\mathcal{B}(B \rightarrow K^*\gamma)$
$\mathcal{B}(B_s^0 \rightarrow \phi\gamma)$
$A_{\Delta\Gamma}, S(B_s^0 \rightarrow \phi\gamma)$
$S_{K^{*0}\gamma}$

Disappearance of $R(K)$ and $R(K^*)$ anomalies

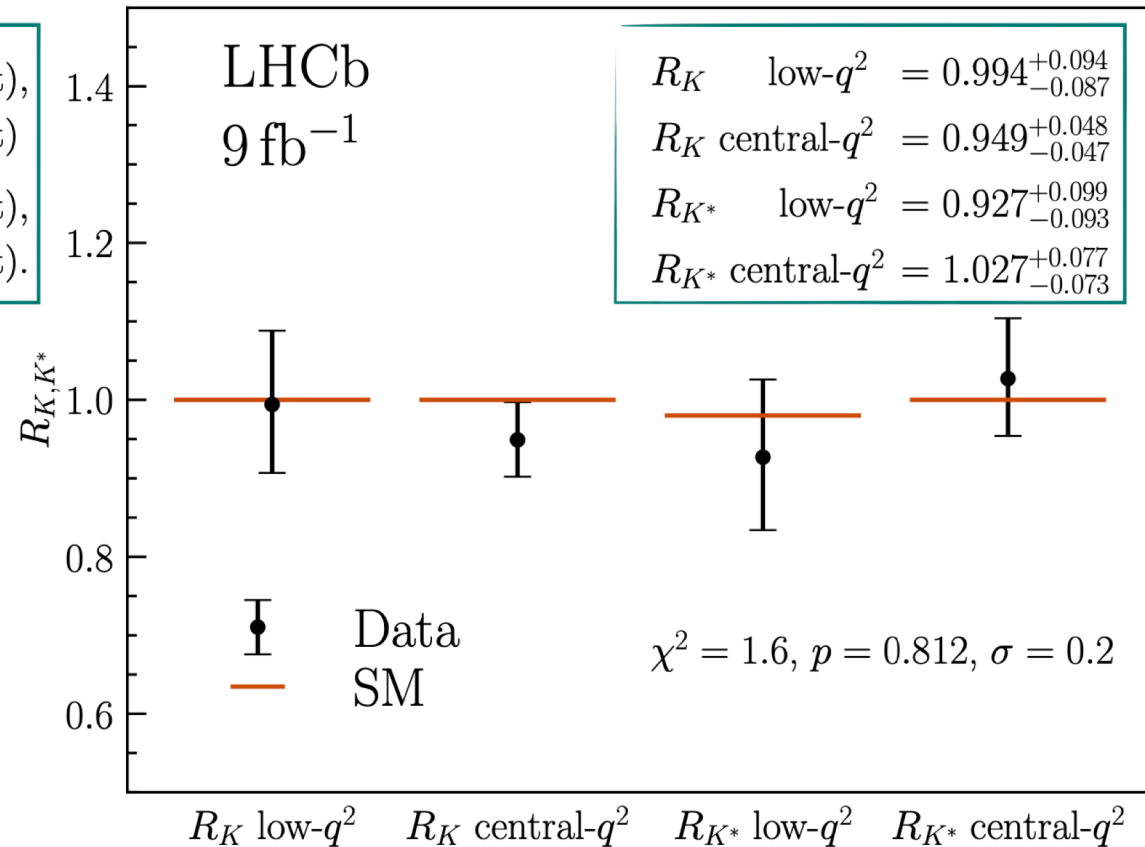
Analysis: results

Results

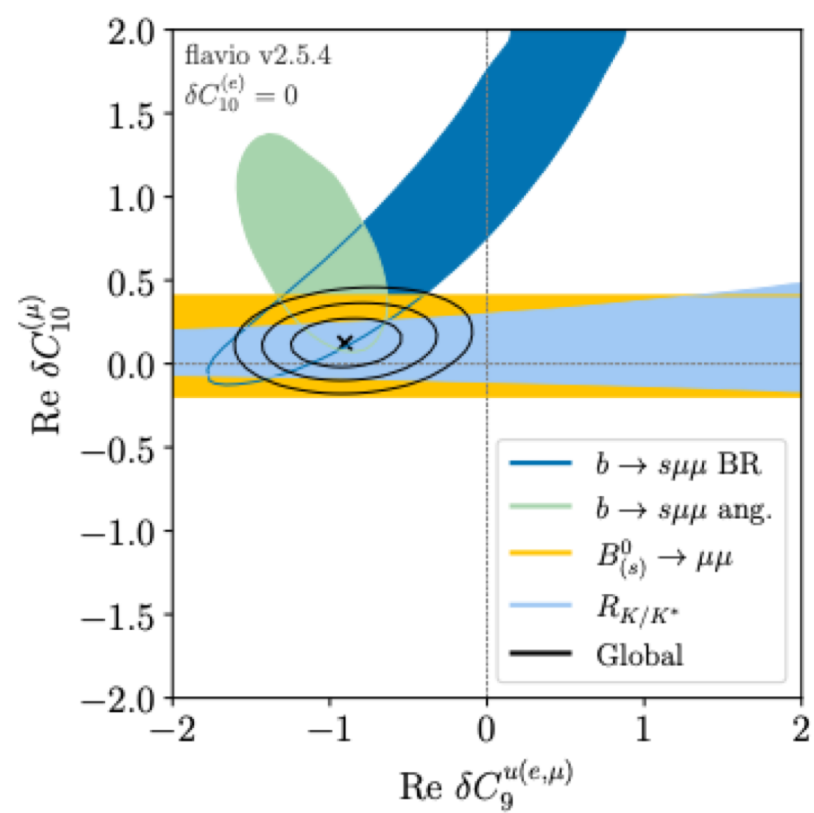
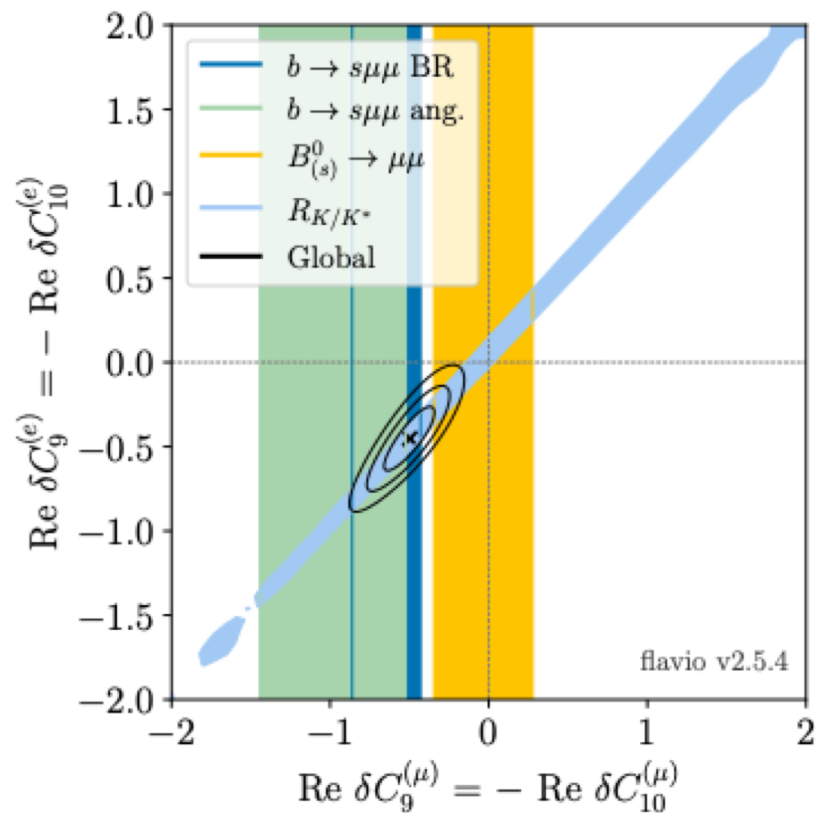
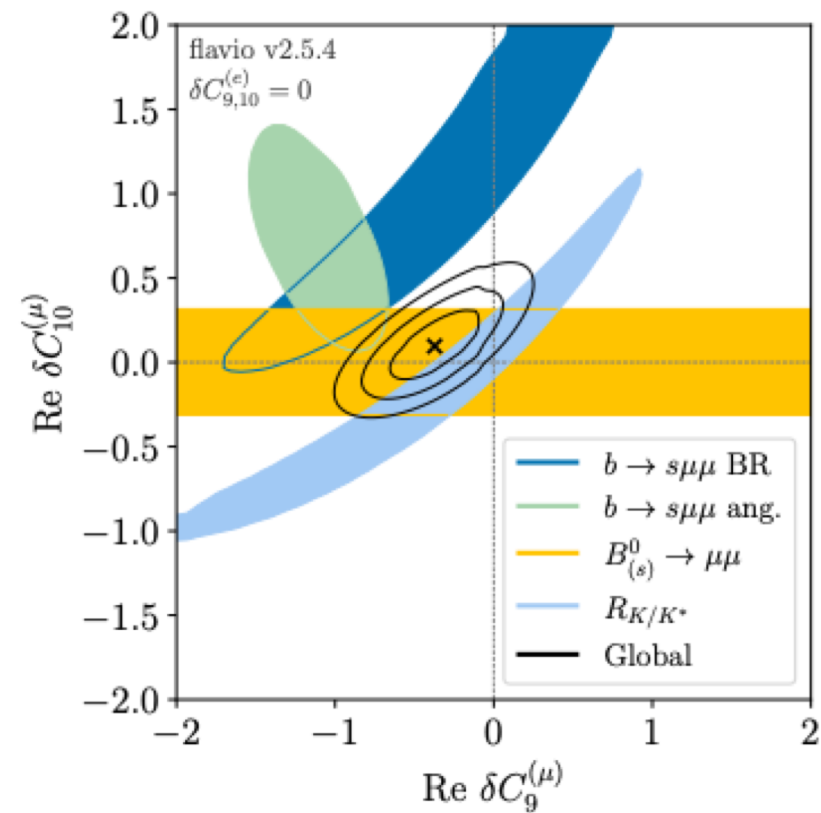
$$\text{low-}q^2 \begin{cases} R_K & = 0.994^{+0.090}_{-0.082} \text{ (stat)} \ ^{+0.027}_{-0.029} \text{ (syst)}, \\ R_{K^*} & = 0.927^{+0.093}_{-0.087} \text{ (stat)} \ ^{+0.034}_{-0.033} \text{ (syst)} \end{cases}$$

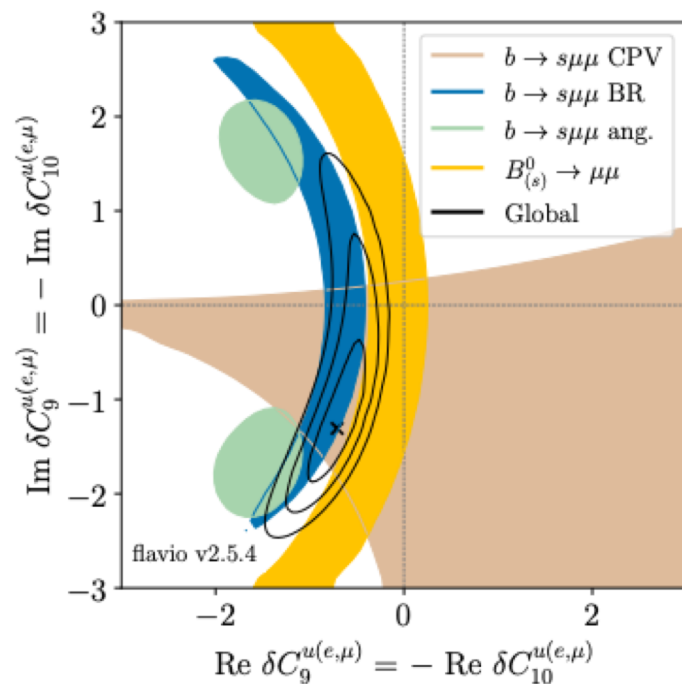
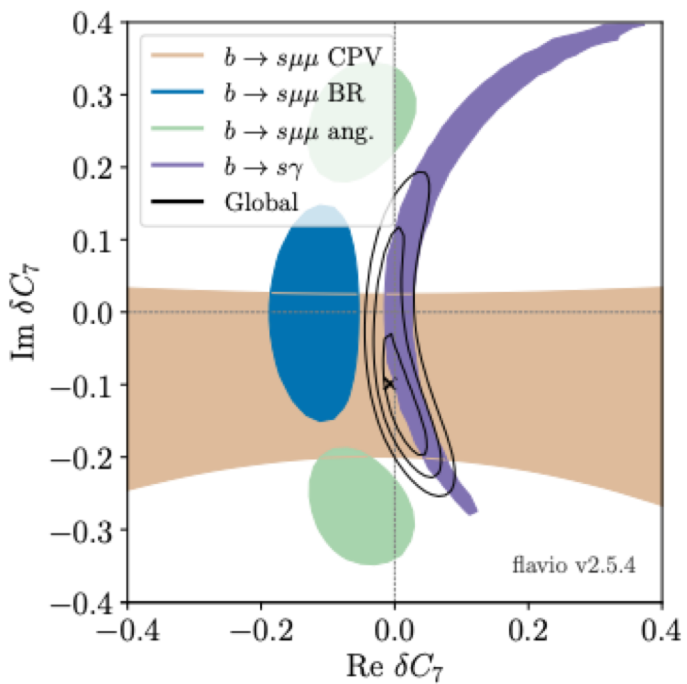
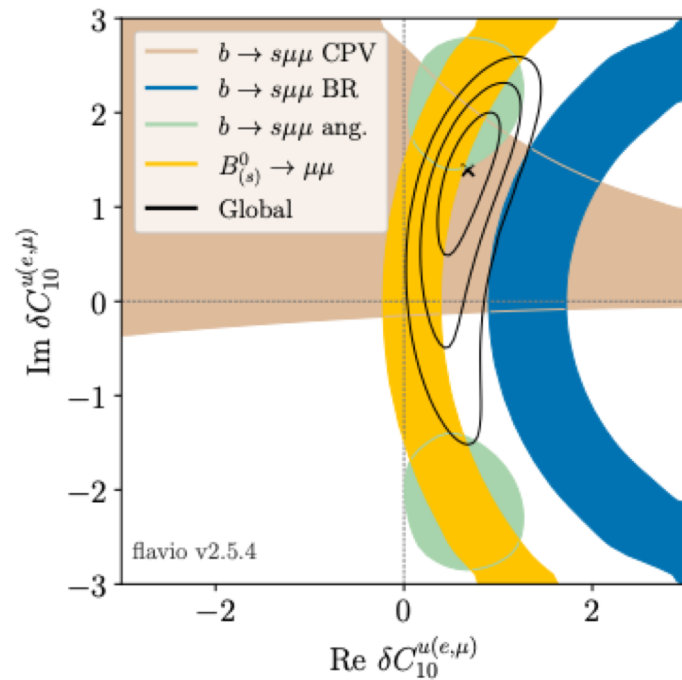
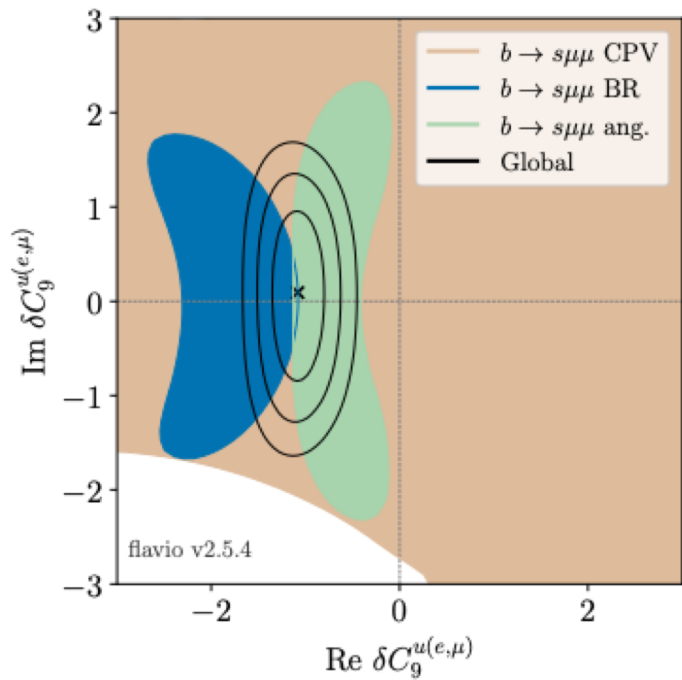
$$\text{central-}q^2 \begin{cases} R_K & = 0.949^{+0.042}_{-0.041} \text{ (stat)} \ ^{+0.023}_{-0.023} \text{ (syst)}, \\ R_{K^*} & = 1.027^{+0.072}_{-0.068} \text{ (stat)} \ ^{+0.027}_{-0.027} \text{ (syst)}. \end{cases}$$

- ◆ Most precise and accurate LFU test in $b \rightarrow s\ell\ell$ transition
- ◆ Compatible with SM with a simple χ^2 test on 4 measurement at 0.2σ



Other plots concerning fits to the WCs





Connection with $b \rightarrow c$?

- $C_9^{\text{univ.}}$ of the correct size can be generated through RGE effects
 [Bobeth-Haisch, 2011] [Crivellin et al., 2018] [Aebischer et al., 2019]

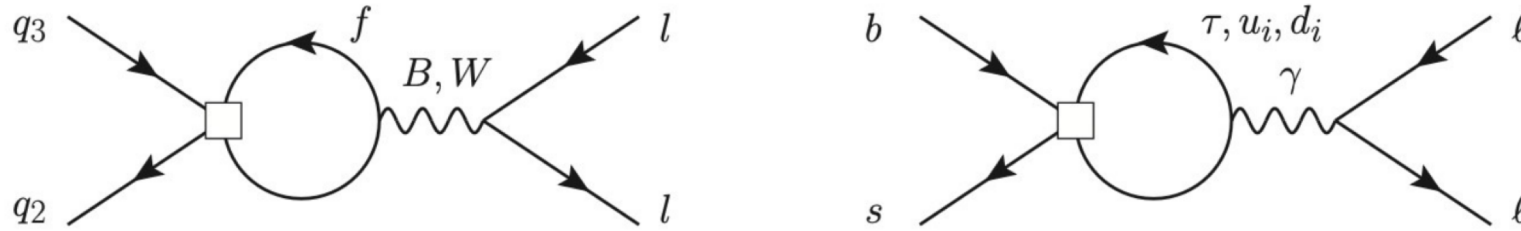


Figure 3: Diagrams inducing a contribution to C_9 through RG running above (left panel) and below (right panel) the EW scale. A sizeable contribution to C_9 is obtained when $f = u_{1,2}, d_{1,2,3}$ or l_3 , see text for details.

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 Workshop 2023, Pisa

$$2\mathcal{N} C_9^{bsl_i l_i} = [C_{qe}]_{23ii} + [C_{lq}^{(1)}]_{ii23} + [C_{lq}^{(3)}]_{ii23} - \zeta c_Z,$$

$$2\mathcal{N} C_{10}^{bsl_i l_i} = [C_{qe}]_{23ii} - [C_{lq}^{(1)}]_{ii23} - [C_{lq}^{(3)}]_{ii23} + c_Z,$$



$$[O_{qe}]_{23ii} = (\bar{q}_2 \gamma_\mu q_3) (\bar{e}_i \gamma^\mu e_i),$$

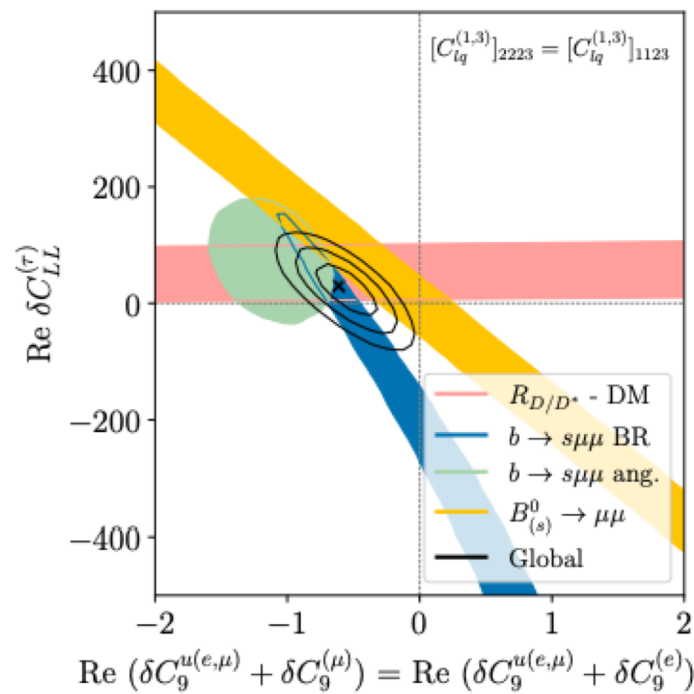
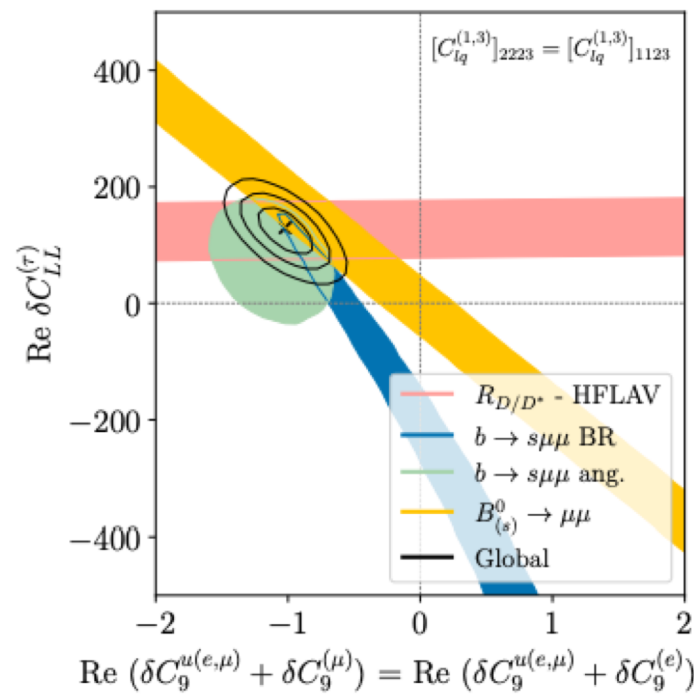
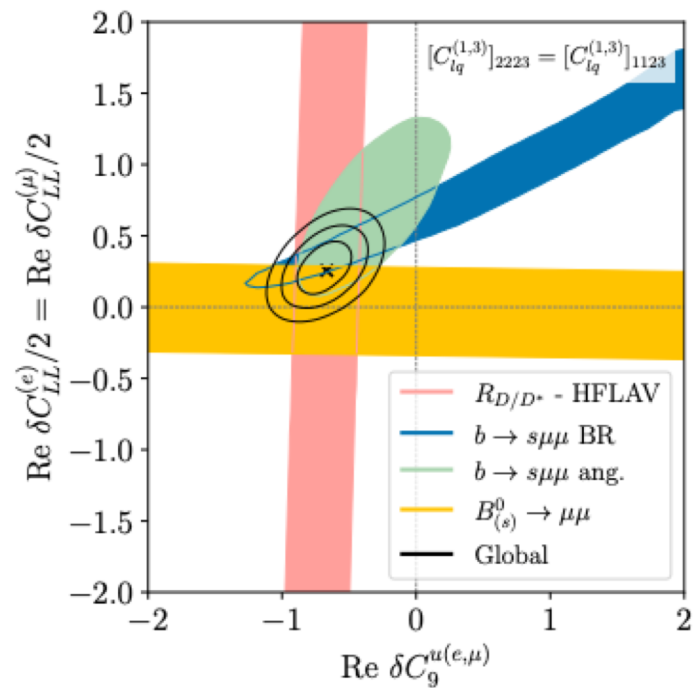
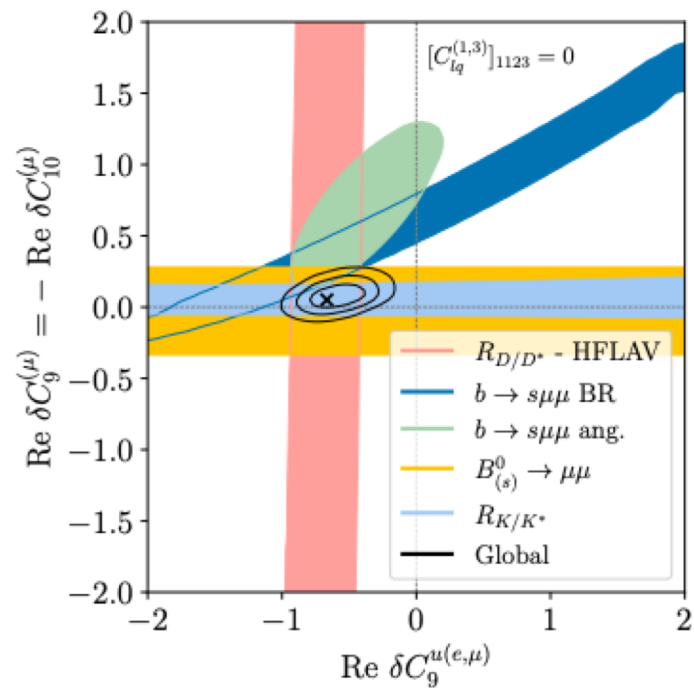
$$[O_{lq}^{(1)}]_{ii23} = (\bar{l}_i \gamma_\mu l_i) (\bar{q}_2 \gamma^\mu q_3),$$

$$[O_{\varphi q}^{(1)}]_{23} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{q}_2 \gamma^\mu q_3),$$

$$[O_{lq}^{(3)}]_{ii23} = (\bar{l}_i \gamma_\mu \tau^I l_i) (\bar{q}_2 \gamma^\mu \tau^I q_3),$$

$$[O_{\varphi q}^{(3)}]_{23} = (\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi) (\bar{q}_2 \gamma^\mu \tau^I q_3),$$

Intriguingly, a large value for $[C_{lq}^{(3)}]_{3323}$, that can explain the hints for LFU violation in charged-current $b \rightarrow c$ transitions (R_D and R_{D^*}), also induces a LFU effect in C_9 that goes in the right direction to solve the $b \rightarrow s \mu \mu$ anomalies in branching ratios and angular observables.



$B \rightarrow K^{(*)} \nu \bar{\nu}$ in the SMEFT

Four fermion operators

If the NP contribution is heavy enough, $\Lambda > v$, we can work in the SMEFT

$$\mathcal{L}_{\text{SMEFT}}^{(6)} \supset \frac{1}{\Lambda^2} \left\{ \left(\mathcal{C}_{lq}^{(1)} + \mathcal{C}_{lq}^{(3)} \right)_{ij} (\bar{s}_L \gamma^\mu b_L) (\bar{e}_{Li} \gamma_\mu e_{Lj}) + \left(\mathcal{C}_{lq}^{(1)} - \mathcal{C}_{lq}^{(3)} \right)_{ij} (\bar{s}_L \gamma^\mu b_L) (\bar{\nu}_{Li} \gamma_\mu \nu_{Lj}) \right. \\ \left. + 2 V_{cs} \left[\mathcal{C}_{lq}^{(3)} \right]_{ij} (\bar{c}_L \gamma^\mu b_L) (\bar{e}_{Li} \gamma_\mu \nu_{Lj}) + [\mathcal{C}_{ld}]_{ij} (\bar{s}_R \gamma^\mu b_R) [(\bar{\nu}_{Li} \gamma_\mu \nu_{Lj}) + (\bar{e}_{Li} \gamma_\mu e_{Lj})] + \text{h.c.} \right\}$$

Matching to the low-energy NP couplings

$$\delta C_L^{\nu_i \nu_j} = \frac{\pi}{\alpha_{\text{em}} \lambda_t} \frac{v^2}{\Lambda^2} \left\{ [\mathcal{C}_{lq}^{(1)}]_{ij} - [\mathcal{C}_{lq}^{(3)}]_{ij} \right\}$$

$$\delta C_R^{\nu_i \nu_j} = \frac{\pi}{\alpha_{\text{em}} \lambda_t} \frac{v^2}{\Lambda^2} [\mathcal{C}_{ld}]_{ij}$$

Contributions to $B \rightarrow K \nu \bar{\nu}$ will have an impact on observables with charged leptons!